SOLVED EXAMPLES

Ex. 1 Find the equation to the ellipse whose centre is origin, axes are the axes of co-ordinate and passes through the points (2, 2) and (3, 1).

Sol. Let the equation to the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Since it passes through the points (2, 2) and (3, 1)

$\therefore \quad \frac{4}{a^2} + \frac{4}{b^2} = 1$ ......**(i)**

and $\quad \frac{9}{a^2} + \frac{1}{b^2} = 1$ ......**(ii)**

from **(i)** – 4 **(ii)**, we get

$$\frac{4-36}{a^2} = 1-4 \quad \Rightarrow \quad a^2 = \frac{32}{3}$$

from **(i)**, we get

$$\frac{1}{b^2} = \frac{1}{4} - \frac{3}{32} = \frac{8-3}{32}$$

$$b^2 = \frac{32}{5}$$

$\therefore$ Ellipse is $3x^2 + 5y^2 = 32$

Ex. 2 If α, β are eccentric angles of end points of a focal chord of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then tan α/2. tan β/2 is equal to -

Sol. Equation of line joining points 'α' and 'β' is $\frac{x}{a}\cos\frac{\alpha+\beta}{2} + \frac{y}{b}\sin\frac{\alpha+\beta}{2} = \cos\frac{\alpha-\beta}{2}$

If it is a focal chord, then it passes through focus (ae, 0), so $e\cos\frac{\alpha+\beta}{2} = \cos\frac{\alpha-\beta}{2}$

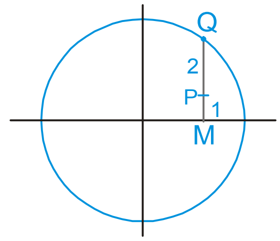
$$\Rightarrow \quad \frac{\cos\frac{\alpha-\beta}{2}}{\cos\frac{\alpha+\beta}{2}} = \frac{e}{1}$$

$$\Rightarrow \quad \frac{\cos\frac{\alpha-\beta}{2} - \cos\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2} + \cos\frac{\alpha+\beta}{2}} = \frac{e-1}{e+1}$$

$$\Rightarrow \quad \frac{2\sin\alpha/2 \;\; \sin\beta/2}{2\cos\alpha/2 \;\; \cos\beta/2} = \frac{e-1}{e+1} \quad \Rightarrow \quad \tan\frac{\alpha}{2}\tan\frac{\beta}{2} = \frac{e-1}{e+1}$$

using (–ae, 0), we get $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} = \frac{e+1}{e-1}$

Ex. 3 From a point Q on the circle $x^2 + y^2 = a^2$, perpendicular QM are drawn to x-axis, find the locus of point 'P' dividing QM in ratio 2 : 1.



Sol. Let $Q \equiv (a\cos\theta, a\sin\theta)$

$M \equiv (a\cos\theta, 0)$

Let $P \equiv (h, k)$

$\therefore \quad h = a\cos\theta,\ k = \dfrac{a\sin\theta}{3}$

$\therefore \quad \left(\dfrac{3k}{a}\right)^2 + \left(\dfrac{h}{a}\right)^2 = 1$

$\Rightarrow$ Locus of P is $\dfrac{x^2}{a^2} + \dfrac{y^2}{(a/3)^2} = 1$

Ex. 4 Find the equation of axes, directrix, co-ordinate of focii, centre, vertices, length of latus - rectum and eccentricity of an ellipse $\dfrac{(x-3)^2}{25} + \dfrac{(y-2)^2}{16} = 1$.

Sol. Let $x - 3 = X$, $y - 2 = Y$, so equation of ellipse becomes as $\dfrac{X^2}{5^2} + \dfrac{Y^2}{4^2} = 1$.

equation of major axis is $Y = 0$ $\Rightarrow$ $y = 2$.

equation of minor axis is $X = 0$ $\Rightarrow$ $x = 3$.

centre $(X = 0, Y = 0)$ $\Rightarrow$ $x = 3, y = 2$

$C \equiv (3, 2)$

Length of semi-major axis $a = 5$ Length of major axis $2a = 10$

Length of semi-minor axis $b = 4$ Length of minor axis $= 2b = 8$.

Let 'e' be eccentricity

$\therefore \quad b^2 = a^2(1 - e^2)$

$e = \sqrt{\dfrac{a^2 - b^2}{a^2}} = \sqrt{\dfrac{25-16}{25}} = \dfrac{3}{5}$.

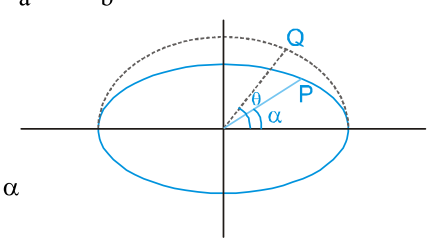
Length of latus rectum $= LL' = \dfrac{2b^2}{a} = \dfrac{2\times16}{5} = \dfrac{32}{5}$

Co-ordinates focii are $X = \pm ae$, $Y = 0$

$\Rightarrow \quad S \equiv (X = 3, Y = 0)$ & $S' \equiv (X = -3, Y = 0)$

$\Rightarrow \quad S \equiv (6, 2)$ & $S' \equiv (0, 2)$

Ea. 5 Find the distance from centre of the point P on the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ whose radius makes angle α with x-axis.



Sol. Let $P \equiv (a\cos\theta, b\sin\theta)$

$\therefore \quad m_{(op)} = \dfrac{b}{a}\tan\theta = \tan\alpha \quad \Rightarrow \quad \tan\theta = \dfrac{a}{b}\tan\alpha$

$$OP = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} = \sqrt{\frac{a^2 + b^2\tan^2\theta}{\sec^2\theta}}$$

$$= \sqrt{\frac{a^2 + b^2\tan^2\theta}{1+\tan^2\theta}} = \sqrt{\frac{a^2 + b^2 \times \frac{a^2}{b^2}\tan^2\alpha}{1+\frac{a^2}{b^2}\tan^2\alpha}}$$

$$OP = \frac{ab}{\sqrt{a^2\sin^2\alpha + b^2\cos^2\alpha}}$$

Ex. 6 The tangent at a point P on an ellipse intersects the major axis in T and N is the foot of the perpendicular from P to the same axis. Show that the circle drawn on NT as diameter intersects the auxiliary circle orthogonally.

Sol. Let the equation of the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Let P(acosθ, bsinθ) be a point on the ellipse. The equation of the tangent at P is $\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$. It meets the major axis at T ≡ (a secθ, 0).

The coordinates of N are (a cosθ, 0). The equation of the circle with NT as its diameter is $(x - a\sec\theta)(x - a\cos\theta) + y^2 = 0$.

⇒ $x^2 + y^2 - ax(\sec\theta + \cos\theta) + a^2 = 0$

It cuts the auxiliary circle $x^2 + y^2 - a^2 = 0$ orthogonally if

$2g \cdot 0 + 2f \cdot 0 = a^2 - a^2 = 0$, which is true.

Ex. 7 If P(α) and P(β) are extremities of a focal chord of ellipse then prove that its eccentricity

$$e = \left|\frac{\cos\left(\frac{\alpha-\beta}{2}\right)}{\cos\left(\frac{\alpha+\beta}{2}\right)}\right|.$$

Sol. Let the equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

∴ equation of chord is $\frac{x}{a}\cos\left(\frac{\alpha+\beta}{2}\right) + \frac{y}{b}\sin\left(\frac{\alpha+\beta}{2}\right) = \cos\left(\frac{\alpha-\beta}{2}\right)$

Since above chord is focal chord,

∴ it passes through focus (ae, 0) or (– ae, 0)

∴ $\pm e\cos\left(\frac{\alpha+\beta}{2}\right) = \cos\left(\frac{\alpha-\beta}{2}\right)$

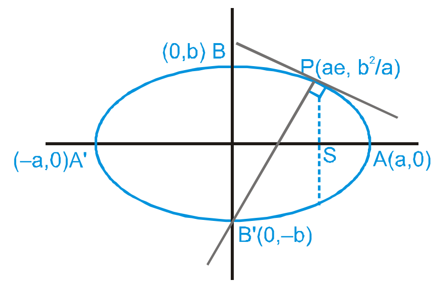
∴ $e = \left|\frac{\cos\left(\frac{\alpha-\beta}{2}\right)}{\cos\left(\frac{\alpha+\beta}{2}\right)}\right|$

Ex. 8 If the normal at an end of a latus-rectum of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ passes through one extremity of the minor axis, show that the eccentricity of the ellipse is given by $e=\sqrt{\frac{\sqrt{5}-1}{2}}$

Sol. The co-ordinates of an end of the latus-rectum are $(ae, b^2/a)$.

The equation of normal at $P(ae, b^2/a)$ is

$$\frac{a^2x}{ae}-\frac{b^2(y)}{b^2/a}=a^2-b^2 \quad \text{or} \quad \frac{ax}{e}-ay=a^2-b^2$$



It passes through one extremity of the minor axis whose co-ordinates are (0, –b)

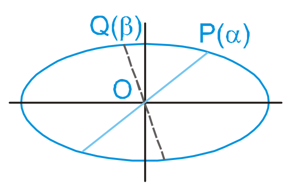
$\therefore \quad 0+ab=a^2-b^2 \quad \Rightarrow \quad a^2b^2)=(a^2-b^2)^2$

$\Rightarrow \quad a^2.a^2(1-e^2)=(a^2e^2)^2 \quad \Rightarrow \quad 1-e^2=e^4$

$\Rightarrow \quad e^4+e^2-1=0 \quad \Rightarrow \quad (e^2)^2+e^2-1=0$

$\therefore \quad e^2=\frac{-1\pm\sqrt{1+4}}{2} \quad \Rightarrow \quad e=\sqrt{\frac{\sqrt{5}-1}{2}}$ (taking positive sign)

Ex.9 Find the angle between two diameters of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Whose extremities have eccentricity angle α and $\beta=\alpha+\frac{\pi}{2}$.



Sol. Let ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Slope of OP $=m_1=\frac{b\sin\alpha}{a\cos\alpha}=\frac{b}{a}\tan\alpha$

Slope of OQ $=m_2=\frac{b\sin\beta}{a\cos\beta}=-\frac{b}{a}\cot\alpha$ given $\beta=\alpha+\frac{\pi}{2}$

$$\therefore \quad \tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|=\left|\frac{\frac{b}{a}(\tan\alpha+\cot\alpha)}{1-\frac{b^2}{a^2}}\right|=\left|\frac{2ab}{(a^2-b^2)\sin2\alpha}\right|$$

Ex. 10 Find the set of value(s) of 'α' for which the point P(α, – α) lies inside the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$.

Sol. If P(α, – α) lies inside the ellipse

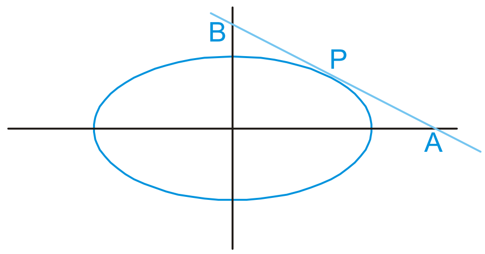
$\therefore \quad S_1<0$

$\Rightarrow \quad \frac{\alpha^2}{16}+\frac{\alpha^2}{9}-1<0 \quad \Rightarrow \quad \frac{25}{144}.\alpha^2<1 \quad \Rightarrow \quad \alpha^2<\frac{144}{25}$

$\therefore \quad \alpha\in\left(-\frac{12}{5},\frac{12}{5}\right)$.

Ex. 11 A tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ touches at the point P on it in the first quadrant and meets the co-ordinate axes in A and B respectively. If P divides AB in the ratio 3 : 1, find the equation of the tangent.

Sol. Let $P \equiv (a\cos\theta, b\sin\theta)$



$\therefore$ equation of tangent is

$$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$$

$A \equiv (a\sec\theta, 0)$

$B \equiv (0, b\,\text{cosec}\theta)$

$\because$ P divide AB internally in the ratio 3 : 1

$\therefore$ $a\cos\theta = \frac{a\sec\theta}{4}$

$\Rightarrow \cos^2\theta = \frac{1}{4} \Rightarrow \cos\theta = \frac{1}{2}$

and $b\sin\theta = \frac{3b\,\text{cosec}\theta}{4} \Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$

$\therefore$ tangent is $\frac{x}{2a} + \frac{\sqrt{3}y}{2b} = 1 \Rightarrow bx + \sqrt{3}\,ay = 2ab$

Ex. 12 A tangent to the ellipse $x^2 + 4y^2 = 4$ meets the ellipse $x^2 + 2y^2 = 6$ at P and Q. Prove that the tangents at P and Q of the ellipse $x^2 + 2y^2 = 6$ are at right angles.

Sol. Given ellipse are $\frac{x^2}{4} + \frac{y^2}{1} = 1$ ......**(i)**

and, $\frac{x^2}{6} + \frac{y^2}{3} = 1$ ......**(ii)**

any tangent to **(i)** is $\frac{x\cos\theta}{2} + \frac{y\sin\theta}{1} = 1$ ......**(iii)**

It cuts **(ii)** at P and Q, and suppose tangent at P and Q meet at (h, k) Then equation of chord of contact of (h, k) with respect to ellipse **(ii)** is $\frac{hx}{6} + \frac{ky}{3} = 1$ ......**(iv)**

comparing **(iii)** and **(iv)**, we get $\frac{\cos\theta}{h/3} = \frac{\sin\theta}{k/3} = 1$

$\Rightarrow \cos\theta = \frac{h}{3}$ and $\sin\theta = \frac{k}{3} \Rightarrow h^2 + k^2 = 9$

locus of the point (h, k) is $x^2 + y^2 = 9 \Rightarrow x^2 + y^2 = 6 + 3 = a^2 + b^2$

i.e. director circle of second ellipse. Hence the tangents are at right angles.

Ex.13 Prove that the locus of the point of intersection of tangents to an ellipse at two points whose eccentric angle differ by a constant α is an ellipse.

Sol. Let P (h, k) be the point of intersection of tangents at A(θ) and B(β) to the ellipse.

$$\therefore \quad h = \frac{a\cos\left(\frac{\theta+\beta}{2}\right)}{\cos\left(\frac{\theta-\beta}{2}\right)} \quad \& \quad k = \frac{b\sin\left(\frac{\theta+\beta}{2}\right)}{\cos\left(\frac{\theta-\beta}{2}\right)}$$

$$\Rightarrow \quad \left(\frac{h}{a}\right)^2 + \left(\frac{k}{b}\right)^2 = \sec^2\left(\frac{\theta-\beta}{2}\right)$$

but given that $\theta - \beta = \alpha$

$$\therefore \quad \text{locus is } \frac{x^2}{a^2\sec^2\left(\frac{\alpha}{2}\right)} + \frac{y^2}{b^2\sec^2\left(\frac{\alpha}{2}\right)} = 1$$

Ex. 14 A point moves so that the sum of the squares of its distances from two intersecting straight lines is constant. Prove that its locus is an ellipse.

Sol. Let two intersecting lines OA and OB, intersect at origin O and let both lines OA and OB makes equal angles with x axis.

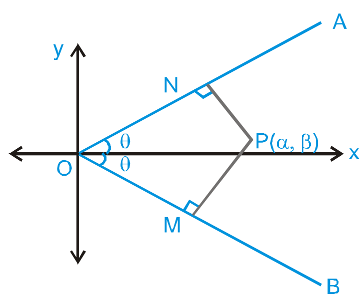
i.e., $\angle XOA = \angle XOB = \theta$.

$\therefore$ Equations of straight lines OA and OB are

$y = x\tan\theta$ and $y = -x\tan\theta$

or $x\sin\theta - y\cos\theta = 0$ .... **(i)**

and $x\sin\theta + y\cos\theta = 0$ .... **(ii)**



Let P(α, β) is the point whose locus is to be determine.

According to the example $(PM)^2 + (PN)^2 = 2\lambda^2$ (say)

$\therefore \quad (\alpha\sin\theta + \beta\cos\theta)^2 + (\alpha\sin\theta - \beta\cos\theta)^2 = 2\lambda^2$

$\Rightarrow \quad 2\alpha^2\sin^2\theta + 2\beta^2\cos^2\theta = 2\lambda^2$

or $\quad \alpha^2\sin^2\theta + \beta^2\cos^2\theta = \lambda^2 \quad \Rightarrow \quad \dfrac{\alpha^2}{\lambda^2\text{cosec}^2\theta} + \dfrac{\beta^2}{\lambda^2\sec^2\theta} = 1$

$$\Rightarrow \quad \frac{\alpha^2}{(\lambda\ \text{cosec}\ \theta)^2} + \frac{\beta^2}{(\lambda\sec\theta)^2} = 1$$

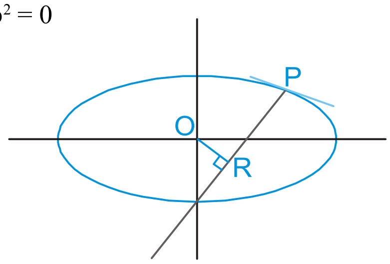
Hence required locus is $\dfrac{x^2}{(\lambda\ \text{cosec}\ \theta)^2} + \dfrac{y^2}{(\lambda\sec\theta)^2} = 1$

Ex.15 Prove that, in an ellipse, the distance between the centre and any normal does not exceed the difference between the semi-axes of the ellipse.

Sol. Let the equation of ellipse is $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

Equation of normal at P (θ) is $(a \sec\theta)x - (b\,\text{cosec}\,\theta)y - a^2 + b^2 = 0$

distance of normal from centre



$$= OR = \frac{|a^2 - b^2|}{\sqrt{a^2 + b^2 + (a\tan\theta)^2 + (b\cot\theta)^2}}$$

$$= \frac{|a^2 - b^2|}{\sqrt{(a+b)^2 + (a\tan\theta - b\cot\theta)^2}}$$

∵ $(a + b)^2 + (a\tan\theta - b\cot\theta)^2 \geq (a + b)^2$

or $\leq \frac{|a^2 - b^2|}{\sqrt{(a+b)^2}}$

$|OR| \leq (a - b)$

Ex. 16 Find the condition on 'a' and 'b' for which two distinct chords of the ellipse $\frac{x^2}{2a^2} + \frac{y^2}{2b^2} = 1$ passing through (a, –b) are bisected by the line x + y = b.

Sol. Let (t, b – t) be a point on the line x + y = b.

Then equation of chord whose mid point (t, b – t) is

$$\frac{tx}{2a^2} + \frac{y(b-t)}{2b^2} - 1 = \frac{t^2}{2a^2} + \frac{(b-t)^2}{2b^2} - 1 \quad \text{......(i)}$$

(a, –b) lies on **(i)** then $\frac{ta}{2a^2} - \frac{b(b-t)}{2b^2} = \frac{t^2}{2a^2} + \frac{(b-t)^2}{2b^2}$

⇒ $t^2(a^2 + b^2) - ab(3a + b)t + 2a^2b^2 = 0$

Since t is real $B^2 - 4AC \geq 0$

⇒ $a^2b^2(3a + b)^2 - 4(a^2 + b^2)2a^2b^2 \geq 0$ ⇒ $a^2 + 6ab - 7b^2 \geq 0$

⇒ $a^2 + 6ab \geq 7b^2$, which is the required condition.

Ex. 17 Find the locus of point of intersection of perpendicular tangents to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Sol. Let P(h, k) be the point of intersection of two perpendicular tangents

equation of pair of tangents is $SS_1 = T^2$

⇒ $\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right)\left(\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1\right) = \left(\frac{hx}{a^2} + \frac{ky}{b^2} - 1\right)^2$

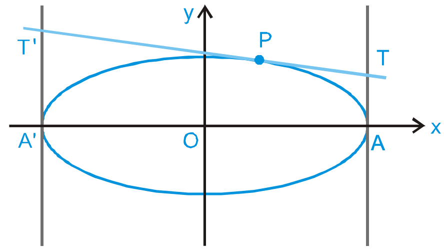
⇒ $\frac{x^2}{a^2}\left(\frac{k^2}{b^2} - 1\right) + \frac{y^2}{b^2}\left(\frac{h^2}{a^2} - 1\right) + \text{........} = 0$ ......**(i)**

Since equation **(i)** represents two perpendicular lines

∴ $\frac{1}{a^2}\left(\frac{k^2}{b^2} - 1\right) + \frac{1}{b^2}\left(\frac{h^2}{a^2} - 1\right) = 0$

⇒ $k^2 - b^2 + h^2 - a^2 = 0$ ⇒ locus is $x^2 + y^2 = a^2 + b^2$

Ex. 18 Any tangent to an ellipse is cut by the tangents at the ends of the major axis in T and T '. Prove that circle on TT' as diameter passes through foci.



Sol. Let ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

and let P(acosϕ, bsinϕ) be any point on this ellipse

∴ Equation of tangent at P(acosϕ, bsinϕ) is

$$\frac{x}{a}\cos\phi+\frac{y}{b}\sin\phi=1 \qquad ......(i)$$

The two tangents drawn at the ends of the major axis are x = a and x = –a

Solving (i) and x = a we get $T=\left\{a,\frac{b(1-\cos\phi)}{\sin\phi}\right\}\equiv\left\{a, b\tan\left(\frac{\phi}{2}\right)\right\}$

and solving (i) and x = – a we get $T'=\left\{-a,\frac{b(1+\cos\phi)}{\sin\phi}\right\}\equiv\left\{-a, b\cot\left(\frac{\phi}{2}\right)\right\}$

Equation of circle on TT' as diameter is $(x-a)(x+a)+(y-b\tan(\phi/2))(y-b\cot(\phi/2))=0$

or $x^2+y^2-by(\tan(\phi/2)+\cot(\phi/2))-a^2+b^2=0 \qquad ......(ii)$

Now put x = ± ae and y = 0 in LHS of (ii), we get

$a^2e^2+0-0-a^2+b^2=a^2-b^2-a^2+b^2=0=$ RHS

Hence foci lie on this circle

Ex. 19 Find the condition on 'a' and 'b' for which two distinct chords of the ellipse $\frac{x^2}{2a^2}+\frac{y^2}{2b^2}=1$ passing through (a, – b) are bisected by the line x + y = b.

Sol. Let the line x + y = b bisect the chord at P(α, b – α)

∴ equation of chord whose mid-point is P(α, b – α)

$$\frac{x\alpha}{2a^2}+\frac{y(b-\alpha)}{2b^2}=\frac{\alpha^2}{2a^2}+\frac{(b-\alpha)^2}{2b^2}$$

Since it passes through (a, –b)

$$\therefore \quad \frac{\alpha}{2a}-\frac{(b-\alpha)}{2b}=\frac{\alpha^2}{2a^2}+\frac{(b-\alpha)^2}{2b^2}$$

$$\Rightarrow \quad \left(\frac{1}{a}+\frac{1}{b}\right)\alpha-1=\alpha^2\left(\frac{1}{a^2}+\frac{1}{b^2}\right)-\frac{2}{b}\alpha+1$$

$$\Rightarrow \quad \alpha^2\left(\frac{1}{a^2}+\frac{1}{b^2}\right)-\left(\frac{3}{b}+\frac{1}{a}\right)\alpha+2=0$$

since line bisect two chord

∴ above quadratic equation in α must have two distinct real roots

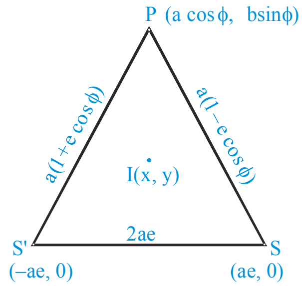
$\therefore \quad \left(\frac{3}{b}+\frac{1}{a}\right)^2 - 4\left(\frac{1}{a^2}+\frac{1}{b^2}\right) . 2 > 0$

$\Rightarrow \quad \frac{9}{b^2} + \frac{1}{a^2} + \frac{6}{ab} - \frac{8}{a^2} - \frac{8}{b^2} > 0 \quad \Rightarrow \quad \frac{1}{b^2} - \frac{7}{a^2} + \frac{6}{ab} > 0$

$\Rightarrow \quad a^2 - 7b^2 + 6ab > 0$

$\Rightarrow \quad a^2 > 7b^2 - 6ab$ which is the required condition.

Ex. 20 A variable point P on an ellipse of eccentricity e, is joined to its foci S, S'. Prove that the locus of the incentre of the triangle PSS' is an ellipse whose eccentricity is $\sqrt{\left(\frac{2e}{1+e}\right)}$.



Sol. Let the given ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Let the co-ordinates of P are $(a \cos\phi, b \sin\phi)$

By hypothesis

$b^2 = a^2(1 - e^2)$ and S(ae, 0), S'(–ae, 0)

$\therefore \quad$ SP = focal distance of the point P = $a - ae \cos \phi$

and $\quad$ S'P = $a + ae \cos \phi$

Also SS' = 2ae

If (x, y) be the incentre of the ΔPSS' then

$$\therefore \quad x = \frac{(2ae)a\cos\phi + a(1-e\cos\phi)(-ae) + a(1+e\cos\phi)ae}{2ae + a(1-e\cos\phi) + a(1+e\cos\phi)}$$

$$x = ae\cos\phi \qquad \text{....... (i)}$$

$$y = \frac{2ae(b\sin\phi) + a(1+e\cos\phi).0 + a(1-e\cos\phi).0}{2ae + a(1-e\cos\phi) + a(1+e\cos\phi)}$$

$$\Rightarrow \quad y = \frac{eb\sin\phi}{(e+1)} \qquad \text{....... (ii)}$$

Eliminating ϕ from equations **(i)** and **(ii)**, we get $\frac{x^2}{a^2e^2} + \frac{y^2}{\left[\frac{be}{e+1}\right]^2} = 1$

which represents an ellipse.

Let e_1 be its eccentricity.

$$\therefore \quad \frac{b^2e^2}{(e+1)^2} = a^2e^2(1-e_1^2)$$

$$\Rightarrow \quad e_1^2 = 1 - \frac{b^2}{a^2(e+1)^2} = 1 - \frac{1-e^2}{(e+1)^2} = 1 - \frac{1-e}{1+e} = \frac{2e}{1+e}$$

$$\Rightarrow \quad e_1 = \sqrt{\left(\frac{2e}{1+e}\right)}$$

Exercise # 1

[Single Correct Choice Type Questions]

1. The curve represented by $x = 3(\cos t + \sin t)$, $y = 4(\cos t - \sin t)$, is -

(A) ellipse **(B)** parabola **(C)** hyperbola **(D)** circle

2. Let 'E' be the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ & 'C' be the circle $x^2 + y^2 = 9$. Let P & Q be the points (1 , 2) and (2, 1) respectively. Then :

(A) Q lies inside C but outside E **(B)** Q lies outside both C & E

(C) P lies inside both C & E **(D)** P lies inside C but outside E.

3. If PQR is an equilateral triangle inscribed in the auxiliary circle of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, (a > b), and P'Q'R' is the corresponding triangle inscribed withint the ellipse, then the centroid of triangle P'Q'R' lies at

(A) center of ellipse **(B)** focus of ellipse

(C) between focus and center on major axis **(D)** none of these

4. An ellipse having foci at (3, 3) and (–4, 4) and passing through the origin has eccentricity equal to-

(A) $\frac{3}{7}$ **(B)** $\frac{2}{7}$ **(C)** $\frac{5}{7}$ **(D)** $\frac{3}{5}$

5. The length of the major axis of the ellipse $(5x - 10)^2 + (5y + 15)^2 = \frac{(3x - 4y + 7)^2}{4}$ is

(A) 10 **(B)** 20/3 **(C)** 20/7 **(D)** 4

6. The normal at P(θ) on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ cuts the axis of x at G and PG is produced to Q so that GQ = 2PG then the locus of Q is given by

(A) $\frac{a^2x^2}{(3b^2 - a^2)^2} + \frac{y^2}{b^2} = 1$ **(B)** $\frac{a^2x^2}{(a^2 + 3b^2)^2} + \frac{y^2}{4b^2} = 1$

(C) $\frac{a^2x^2}{(a^2 - 3b^2)^2} + \frac{y^2}{4b^2} = 1$ **(D)** $\frac{x^2}{(a^2 - 3b^2)^2} + \frac{y^2}{b^2} = 1$

7. A tangent having slope of $-\frac{4}{3}$ to the ellipse $\frac{x^2}{18} + \frac{y^2}{32} = 1$ intersects the major & minor axes in points A & B respectively. If C is the centre of the ellipse then the area of the triangle ABC is :

(A) 12 sq. units **(B)** 24 sq. units **(C)** 36 sq. units **(D)** 48 sq. units

8. The auxiliary circle of a family of ellipses passes through the origin and makes intercepts of 8 and 6 units on the x- and the y-axis, respectively. If the eccentricity of all such ellipse is 1/2, then the locus of the focus will be

(A) $\frac{x^2}{16} + \frac{y^2}{9} = 25$ **(B)** $4x^2 + 4y^2 - 32x - 24y + 75 = 0$

(C) $\frac{x^2}{16} - \frac{y^2}{9} = 25$ **(D)** none of these

9. $x-2y+4=0$ is a common tangent to $y^2=4x$ & $\frac{x^2}{4}+\frac{y^2}{b^2}=1$. Then the value of b and the other common tangent are given by :

(A) $b=\sqrt{3}$; $x+2y+4=0$ **(B)** $b=3$; $x+2y+4=0$

(C) $b=\sqrt{3}$; $x+2y-4=0$ **(D)** $b=\sqrt{3}$; $x-2y-4=0$

10. The equation to the locus of the middle point of the portion of the tangent to the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ included between the co-ordinate axes is the curve-

(A) $9x^2+16y^2=4x^2y^2$ **(B)** $16x^2+9y^2=4x^2y^2$

(C) $3x^2+4y^2=4x^2y^2$ **(D)** $9x^2+16y^2=x^2y^2$

11. Tangents are drawn to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $(a>b)$ and the circle $x^2+y^2=a^2$ at the points where a common ordinate cuts them (on the same side of the x-axis). Then the greatest acute angle between these tangents is given by

(A) $\tan^{-1}\left(\frac{a-b}{2\sqrt{ab}}\right)$ **(B)** $\tan^{-1}\left(\frac{a+b}{2\sqrt{ab}}\right)$ **(C)** $\tan^{-1}\left(\frac{2ab}{\sqrt{a-b}}\right)$ **(D)** $\tan^{-1}\left(\frac{2ab}{\sqrt{a+b}}\right)$

12. The area of the rectangle formed by the perpendiculars from the centre of the standard ellipse to the tangent and normal at its point whose eccentric angle is $\pi/4$ is :

(A) $\frac{(a^2-b^2)ab}{a^2+b^2}$ **(B)** $\frac{(a^2-b^2)}{(a^2+b^2)ab}$ **(C)** $\frac{(a^2-b^2)}{ab(a^2+b^2)}$ **(D)** $\frac{a^2+b^2}{(a^2-b^2)ab}$

13. Which of the following is the common tangent to the ellipses $\frac{x^2}{a^2+b^2}+\frac{y^2}{b^2}=1$ & $\frac{x^2}{a^2}+\frac{y^2}{a^2+b^2}=1$?

(A) $ay=bx+\sqrt{a^4-a^2b^2+b^4}$ **(B)** $by=ax-\sqrt{a^4+a^2b^2+b^4}$

(C) $ay=bx-\sqrt{a^4+a^2b^2+b^4}$ **(D)** $by=ax-\sqrt{a^4-a^2b^2+b^4}$

14. Let P be any point on a directrix of an ellipse of eccentricity e, S be the corresponding focus, and C the center of the ellipse. The line PC meets the ellipse at A. The angle between PS and tangent at A is α, then α is equal to

(A) $\tan^{-1} e$ **(B)** $\pi/2$ **(C)** $\tan^{-1}(1-e^2)$ **(D)** none of these

15. A conic passes through the point (2, 4) and is such that the segment of any of its tangents at any point contained between the co-ordinate axes is bisected at the point of tangency. Then the foci of the conic are

(A) $(2\sqrt{2}, 0)$ & $(-2\sqrt{2}, 0)$ **(B)** $(2\sqrt{2}, 2\sqrt{2})$ & $(-2\sqrt{2}, -2\sqrt{2})$

(C) $(4,4)$ & $(-4,-4)$ **(D)** $(4\sqrt{2}, 4\sqrt{2})$ & $(-4\sqrt{2}, -4\sqrt{2})$

16. The point of intersection of the tangents at the point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, and its corresponding point Q on the auxiliary circle meet on the line -

(A) $x=a/e$ **(B)** $x=0$ **(C)** $y=0$ **(D)** none

17. If the ellipse $\frac{x^2}{a^2-7}+\frac{y^2}{13-5a} = 1$ is inscribed in a square of side length $\sqrt{2}$ a, then a is equal to

(A) 6/5 **(B)** $(-\infty, -\sqrt{7}) \cup (\sqrt{7}, 13/5)$

(C) $(-\infty, -\sqrt{7}) \cup (13/5, \sqrt{7})$ **(D)** no such a exists

18. Imagine that you have two thumbtacks placed at two points, A and B. If the ends of a fixed length of string are fastened to the thumbtacks and the string is drawn taut with a pencil, the path traced by the pencil will be an ellipse. The best way to maximise the area surrounded by the ellipse with a fixed length of string occurs when

I the two points A and B have the maximum distance between them.

II two points A and B coincide.

III A and B are placed vertically.

IV The area is always same regardless of the location of A and B.

(A) I **(B)** II **(C)** III **(D)** IV

19. The equation of the normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the positive end of latus rectum is -

(A) $x + ey + e^2a = 0$ **(B)** $x - ey - e^3a = 0$

(C) $x - ey - e^2a = 0$ **(D)** none of these

20. Let P be any point on any directrix of an ellipse. Then the chords of contact of point P with respect to the ellipse and its auxiliary circle intersect at

(A) some point on the major axis depending upon the position of point P

(B) the midpoint of the line segment joining the center to the corresponding focus

(C) the corresponding focus

(D) none of these

21. PQ is a double ordinate of the ellipse $x^2 + 9y^2 = 9$, the normal at P meets the diameter through Q at R, then the locus of the mid point of PR is

(A) a circle **(B)** a parabola **(C)** an ellipse **(D)** a hyperbola

22. PQ is a double ordinate of the ellipse $x^2 + 9y^2 = 9$, the normal at P meets the diameter through Q at R, then the locus of the mid point of PR is -

(A) a circle **(B)** a parabola **(C)** an ellipse **(D)** a hyperbola

23. A parabola is drawn with focus at one of the foci of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$, a > b, and directrix passing through the other focus and perpendicular to the major axis of the ellipse. If the latus rectum of the ellipse and that of the parabola are same, then the eccentricity of the ellipse is

(A) $1 - \frac{1}{\sqrt{2}}$ **(B)** $2\sqrt{2} - 2$ **(C)** $\sqrt{2} - 1$ **(D)** none of these

24. If α & β are the eccentric angles of the extremities of a focal chord of an standard ellipse, then the eccentricity of the ellipse is :

(A) $\frac{\cos\alpha + \cos\beta}{\cos(\alpha+\beta)}$ **(B)** $\frac{\sin\alpha - \sin\beta}{\sin(\alpha-\beta)}$ **(C)** $\frac{\cos\alpha - \cos\beta}{\cos(\alpha-\beta)}$ **(D)** $\frac{\sin\alpha + \sin\beta}{\sin(\alpha+\beta)}$

25. If F_1 & F_2 are the feet of the perpendiculars from the foci S_1 & S_2 of an ellipse $\frac{x^2}{5}+\frac{y^2}{3}=1$ on the tangent at any point P on the ellipse , then $(S_1F_1) . (S_2F_2)$ is equal to

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5

26. The set of values of m for which it is possible to draw the chord $y = \sqrt{m}\,x + 1$ to the curve $x^2 + 2xy + (2 + \sin^2\alpha)\ y^2 = 1$, which subtends a right angle at the origin for some value of α, is

(A) [2, 3] **(B)** [0, 1] **(C)** [1, 3] **(D)** none of these

27. An ellipse is inscribed in a circle and a point within the circle is chosen at random. If the probability that this point lies outside the ellipse is 2/3 then the eccentricity of the ellipse is :

(A) $\frac{2\sqrt{2}}{3}$ **(B)** $\frac{\sqrt{5}}{3}$ **(C)** $\frac{8}{9}$ **(D)** $\frac{2}{3}$

28. The number of values of c such that the straight line $y = 4x + c$ touches the curve $(x^2/4) + y^2 = 1$ is -

(A) 0 **(B)** 1 **(C)** 2 **(D)** infinite

29. An ellipse having foci at (3, 3) and (– 4, 4) and passing through the origin has eccentricity equal to

(A) $\frac{3}{7}$ **(B)** $\frac{2}{7}$ **(C)** $\frac{5}{7}$ **(D)** $\frac{3}{5}$

30. The point of intersection of the tangents at the point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and its corresponding point Q on the auxiliary circle meet on the line :

(A) $x = a/e$ **(B)** $x = 0$ **(C)** $y = 0$ **(D)** None

Exercise # 2

Part # I [Multiple Correct Choice Type Questions]

1. $\frac{x^2}{r^2-r-6}+\frac{y^2}{r^2-6r+5}=1$ will represent an ellipse if r lies in the interval

(A) $(-\infty,-2)$ **(B)** $(3,\infty)$ **(C)** $(5,\infty)$ **(D)** $(1,\infty)$

2. On the x-y plane, the eccentricity of an ellipse is fixed (in size and position) by

(A) both foci

(B) both directrices

(C) one focus and the corresponding directrix

(D) the length of major axis

3. The tangent at any point P on a standard ellipse with foci as S & S' meets the tangents at the vertices A & A' in the points V & V', then -

(A) $l(AV).l(A'V') = b^2$ **(B)** $l(AV).l(A'V') = a^2$

(C) $\angle V'SV = 90°$ **(D)** V'S' VS is a cyclic quadrilateral

4. If the tangent at the point P(θ) to the ellipse $16x^2 + 11y^2 = 256$ is also a tangent to the circle $x^2 + y^2 - 2x = 15$, then θ =

(A) 2π/3 **(B)** 4π/3 **(C)** 5π/3 **(D)** π/3

5. If P is a point of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, whose foci are S and S'. Let $\angle PSS' = \alpha$ and $\angle PS'S = \beta$, then -

(A) PS + PS' = 2a, if a > b **(B)** PS + PS' = 2b, if a < b

(C) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}=\frac{1-e}{1+e}$ **(D)** $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}=\frac{\sqrt{a^2-b^2}}{b^2}[a-\sqrt{a^2-b^2}]$ when a > b

6. The coordinates (2, 3) and (1, 5) are the foci of an ellipse which passes through the origin. Then the equation of the

(A) tangent at the origin is $(3\sqrt{2}-5)x+(1-2\sqrt{2})y=0$

(B) tangent at the origin is $(3\sqrt{2}+5)x+(1+2\sqrt{2}\,y)=0$

(C) normal at the origin is $(3\sqrt{2}+5)x-(2\sqrt{2}+1)y=0$

(D) normal at the origin is $x(3\sqrt{2}-5)-y(1-2\sqrt{2})=0$

7. If the chord through the points whose eccentric angles are θ and ϕ on the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ passes through a focus, then the value of tan(θ/2) tan(ϕ/2) is

(A) 1/9 **(B)** –9 **(C)** –1/9 **(D)** 9

8. If a pair of variable straight line $x^2+4y^2+axy=0$ (where a is a real parameter) cuts the ellipse $x^2+4y^2=4$ at two points A and B, then the locus of the point of intersection of tangents at A and B is

(A) $x-2y=0$ **(B)** $2x-y=0$ **(C)** $x+2y=0$ **(D)** $2x+y=0$

9. Which of the following is/are true ?

(A) There are infinite positive integral values of a for which $(13x-1)^2+(13y-2)^2=\left(\frac{5x+12y-1}{a}\right)^2$ represents an ellipse.

(B) The minimum distance of a point (1, 2) from the ellipse $4x^2+9y^2+8x-36y+4=0$ is 1.

(C) If from a point P(0, a) two normals other than the axes are drawn to the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$, then $|a|<9/4$.

(D) If the length of the latus rectum of an ellipse is one-third of its major axis, then its eccentricity is equal to $1/\sqrt{3}$.

10. Extremities of the latera recta of the ellipses $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (a > b) having a given major axis 2a lies on

(A) $x^2=a(a-y)$ **(B)** $x^2=a(a+y)$ **(C)** $y^2=a(a+x)$ **(D)** $y^2=a(a-x)$

11. If the chord through the points whose eccentric angles are θ & φ on the ellipse, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ passes through the focus, then the value of tan (θ/2) tan (φ/2) is -

(A) $\frac{e+1}{e-1}$ **(B)** $\frac{e-1}{e+1}$ **(C)** $\frac{1+e}{1-e}$ **(D)** $\frac{1-e}{1+e}$

12. Which of the following is/are true about the ellipse $x^2+4y^2-2x-16y+13=0$?

(A) The latus rectum of the ellipse is 1.

(B) The distance between the foci of the ellipse is $4\sqrt{3}$

(C) The sum of the focal distances of a point P(x, y) on the ellipse is 4.

(D) Line y = 3 meets the tangents drawn at the vertices of the ellipse at points P and Q. Then PQ subtends a right angle of any of its foci.

13. The locus of the image of the focus of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$, (a > b), with respect to any of the tangents to the ellipse is

(A) $(x+4)^2+y^2=100$ **(B)** $(x+2)^2+y^2=50$

(C) $(x-4)^2+y^2=100$ **(D)** $(x-2)^2+y^2=50$

14. Let E_1 and E_2, respectively, be two ellipse $\frac{x^2}{a^2}+y^2=1$ and $x^2+\frac{y^2}{a^2}=1$ (where a is a parameter). Then the locus of the points of intersection of the ellipses E_1 and E_2 is a set of curves comprising

(A) two straight line **(B)** one straight line

(C) one circle **(D)** one parabols

15. If point P(α + 1, α) lies between the ellipse $16x^2+9y^2-16x=0$ and its auxiliary circle, then -

(A) [α] = 0 **(B)** [α] = –1

(C) no such real α exist **(D)** [α] = 1

where [.] denotes greatest integer function.

16. Consider the ellipse $\frac{x^2}{f(k^2+2k+5)}+\frac{y^2}{f(k+11)}=1$. If f(x) is a positive decreasing function, then

(A) The set of values of k for which the major axis is the x-axis is $(-3, 2)$

(B) The set of values of k for which the major axis is the y-axis is $(-\infty, 2)$

(C) The set of values of k for which the major axis is the y-axis is $(-\infty, -3) \cup (2, \infty)$

(D) The set of values of k for which the major axis is the y-axis is $(-3, \infty)$

17. Two concentric ellipses are such that the foci of one are on the other and their major axes are equal. Let e and e' be their eccentricities. Then,

(A) The quadrilateral formed by joining the foci of the two ellipse is a parallelogram

(B) The angle θ between their axes is given by $\theta = \cos^{-1}\sqrt{\frac{1}{e^2}+\frac{1}{e'^2}-\frac{1}{e^2e'^2}}$

(C) If $e^2 + e'^2 = 1$, then the angle between the axes of the two ellipse is 90°.

(D) none of these

18. If latus rectum of an ellipse $\frac{x^2}{16}+\frac{y}{b^2}=1$ $\{0< b < 4\}$, subtend angle 2θ at farthest vertex such that $\operatorname{cosec}\theta = \sqrt{5}$, then -

(A) $e = \frac{1}{2}$

(B) no such ellipse exist

(C) $b = 2\sqrt{3}$

(D) area of Δ formed by LR and nearest vertex is 6 sq. units

19. If a number of ellipse be described having the same major axis 2a but a variable minor axis then the tangents at the ends of their latera recta pass through fixed points which can be

(A) (0, a) **(B)** (0, 0) **(C)** (0, – a) **(D)** (a, a)

20. If the tangent drawn at point $(t^2, 2t)$ on the parabola $y^2 = 4x$ is the same as the normal drawn at point $(\sqrt{5}\cos\theta, 2\sin\theta)$ on the ellipse $4x^2 + 5y^2 = 20$, then

(A) $\theta = \cos^{-1}\left(-\frac{1}{\sqrt{5}}\right)$

(B) $\theta = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$

(C) $t = -\frac{2}{\sqrt{5}}$

(D) $t = -\frac{1}{\sqrt{5}}$

Part # II [Assertion & Reason Type Questions]

These questions contains, Statement I (assertion) and Statement II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.

(C) Statement-I is true, Statement-II is false.

(D) Statement-I is false, Statement-II is true.

1. **Statement-I :** Tangent drawn at a point $P\left(\frac{4\sqrt{5}}{3}, 2\right)$ on the ellipse $9x^2 + 16y^2 = 144$ intersects a straight line $x = \frac{16}{\sqrt{7}}$ at M, then PM subtends a right angle at $(-\sqrt{7}, 0)$

Statement-II : The portion of the tangent to an ellipse between the point of contact and the directrix subtends a right angle at the corresponding focus.

2. **Statement-I :** Tangents are drawn to the ellipse $\frac{x^2}{4}+\frac{y^2}{2} = 1$ at the points where it is intersected by the line $2x + 3y = 1$. The point of intersection of these tangents is (8, 6)

Statement-II : The equation of the chord of contact to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ from an external point is given by $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}-1=0$

3. **Statement-I :** Any chord of the ellipse $x^2 + y^2 + xy = 1$ through (0, 0) is bisected at (0, 0)

Statement-II : The centre of an ellipse is a point through which every chord is bisected.

4. **Statement-I :** If a and b are real numbers and c > 0, then the locus represented by the equation $|ay - bx| = c\sqrt{(x-a)^2+(y-b)^2}$ is an ellipse.

Statement-II : An ellipse is the locus of a point which moves in a plane such that the ratio of its distances from a fixed point (i.e., focus) to that from the fixed line (i.e. directrix) is constant and less than 1.

5. **Statement-I :** If $P\left(\frac{3\sqrt{3}}{2}, 1\right)$ is a point on the ellipse $4x^2 + 9y^2 = 36$. Circle drawn AP as diameter touches another circle $x^2 + y^2 = 9$, where $A \equiv (-\sqrt{5}, 0)$

Statement-II : Circle drawn with focal radius as diameter touches the auxilliary circle.

Exercise # 3 Part # I [Matrix Match Type Questions]

Following question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled as A, B, C and D while the statements in Column-II are labelled as p, q, r and s. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II.

1.

	Column - I		Column - II
(A)	If the vertices of a rectangle of maximum area inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are extremities of latus rectum, then the eccentricity of the ellipse is	**(p)**	$\frac{2}{\sqrt{5}}$
(B)	If the extremities of the diameter of the circle $x^2 + y^2 = 16$ are the foci of the ellipse, then the eccentricity of the ellipse, if its size is just sufficient to contain the circle, is	**(q)**	$\frac{1}{\sqrt{2}}$
(C)	If the normal at point (6, 2) to the ellipse passes through its nearest focus (5, 2), having center at (4, 2), then its eccentricity is	**(r)**	$\frac{1}{3}$
(D)	It the extremities of the latus rectum of the parabola $y^2 = 24x$ are the foci of ellipse, and if the ellipse passes through the vertex of the parabola, then its eccentricity is	**(s)**	$\frac{1}{2}$

2.

	Column - I		Column - II
(A)	The minimum and maximum distance of a point (2, 6) from the ellipse are $9x^2 + 8y^2 - 36x - 16y - 28 = 0$	**(p)**	0
(B)	The minimum and maximum distance of a point $\left(\frac{9}{5},\frac{12}{5}\right)$ from the ellipse $4(3x + 4y)^2 + 9(4x - 3y)^2 = 900$ are	**(q)**	2
(C)	If E : $2x^2 + y^2 = 2$ and director circle of E is C_1, director circle of C_1 is C_2 director circle of C_2 is C_3 and so on. If r_1, r_2, r_3 .... are the radii of C_1, C_2, C_3 ... respectively then G.M. of r_1^2, r_2^2, r_3^2 is	**(r)**	6
(D)	Minimum area of the triangle formed by any tangent to the ellipse $x^2 + 4y^2 = 16$ with coordinate axes is	**(s)**	8

Part # II [Comprehension Type Questions]

Comprehension # 1

A coplanar beam of light emerging from a point source has the equation $\lambda x - y + 2(1 + \lambda) = 0, \lambda \in R$. The rays of the beam strike an elliptical surface and get reflected. The reflected rays from another convergent beam having equation $\mu x - y + 2(1 - \mu) = 0, \mu \in R$. Further, it is found that the foot of the perpendicular from the point (2, 2) upon any tangent to the ellipse lies on the circle $x^2 + y^2 - 4y - 5 = 0$.

1. The eccentricity of the ellipse is equal to

(A) 1/3 **(B)** $1/\sqrt{3}$ **(C)** 2/3 **(D)** 1/2

2. The area of the largest triangle that an incident ray and the corresponding reflected ray can enclose with the axis of the ellipse is equal to

(A) $4\sqrt{5}$ **(B)** $2\sqrt{5}$ **(C)** $\sqrt{5}$ **(D)** None of these

3. The total distance travelled by an incident ray and the corresponding reflected ray is the least if the point of incidence coincides with

(A) an end of the minor axis **(B)** an end of the major axis

(C) an end of the latus rectum **(D)** none of these

Comprehension # 2

An ellipse whose distance between foci S and S' is 4 units is inscribed in the triangle ABC touching the sides AB, AC and BC at P, Q and R. If centre of ellipse is at origin and major axis along x-axis, SP + S'P = 6.

On the basis of above information, answer the following questions :

1. If $\angle BAC = 90°$, then locus of point A is -

(A) $x^2 + y^2 = 12$ **(B)** $x^2 + y^2 = 4$ **(C)** $x^2 + y^2 = 14$ **(D)** none of these

2. If chord PQ subtends 90° angle at centre of ellipse, then locus of A is -

(A) $25x^2 + 81y^2 = 620$ **(B)** $25x^2 + 81y^2 = 630$ **(C)** $9x^2 + 16y^2 = 25$ **(D)** none of these

3. If difference of eccentric angles of points P and Q is 60°, then locus of A is -

(A) $16x^2 + 9y^2 = 144$ **(B)** $16x^2 + 45y^2 = 576$ **(C)** $5x^2 + 9y^2 = 60$ **(D)** $5x^2 + 9y^2 = 15$

Exercise # 4 [Subjective Type Questions]

1. A circle is concentric with the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and passes through the foci F_1 and F_2 of the ellipse. Two curves intersect at four points. Let P be any point of intersection. If the major axis of the ellipse is 15 and the area of triangle PF_1F_2 is 26, then find the value of $4a^2 - 4b^2$.

2. Find the set of value(s) of α for which the point $\left(7-\frac{5}{4}\alpha,\ \alpha\right)$ lies inside the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$.

3. The tangent at any point P of a circle $x^2 + y^2 = a^2$ meets the tangent at a fixed point A (a, 0) in T and T is joined to B, the other end of the diameter through A. Prove that the locus of the intersection of AP and BT is an ellipse whose eccentricity is $\frac{1}{\sqrt{2}}$.

4. The tangent at the point α on a standard ellipse meets the auxiliary circle in two points which subtends a right angle at the centre. Show that the eccentricity of the ellipse is $(1 + \sin^2 \alpha)^{-1/2}$.

5. Let P be a point on an ellipse with eccentricity 1/2, such that $\angle PS_1S_2 = \alpha$, $\angle PS_2S_1 = \beta$, and $\angle S_1PS_2 = \gamma$, where S and S_2 are the foci of the ellipse. Then prove that $\cot(\alpha/2)$, $\cot(\gamma/2)$, and $\cot(\beta/2)$ are in AP.

6. The tangent at $P\left(4\cos\theta,\ \frac{16}{\sqrt{11}}\sin\theta\right)$ to the ellipse $16x^2 + 11y^2 = 256$ is also a tangent to the circle $x^2 + y^2 - 2x - 15 = 0$. Find θ. Find also the equation to the common tangent.

7. ABC is an isosceles triangle with its base BC twice its altitude. A point P moves within the triangle such that the square of its distance from BC is half the area of rectangle contained by its distances from the two sides. Show that the locus of P is an ellipse with eccentricity $\sqrt{\frac{2}{3}}$ passing through B & C.

8. Prove that the length of the focal chord of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ which is inclined to the major axis at angle θ is $\frac{2ab^2}{a^2\sin^2\theta + b^2\cos^2\theta}$.

9. 'O' is the origin & also the centre of two concentric circles having radii of the inner & the outer circle as 'a' & 'b' respectively. A line OPQ is drawn to cut the inner circle in P & the outer circle in Q. PR is drawn parallel to the y-axis & QR is drawn parallel to the x– axis. Prove that the locus of R is an ellipse touching the two circles. If the foci of this ellipse lie on the inner circle, find the ratio of inner : outer radii & find also the eccentricity of the ellipse.

10. A tangent is drawn to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ to cut the ellipse $\frac{x^2}{c^2}+\frac{y^2}{d^2}=1$ at the points P and Q. If the tangents at P and Q to the ellipse $\frac{x^2}{c^2}+\frac{y^2}{d^2}=1$ intersect at right angle, then prove that $\frac{a^2}{c^2}+\frac{b^2}{d^2}=1$.

11. If the normals at the points P, Q, R with eccentric angles α, β, γ on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are concurrent, then show that $\begin{vmatrix} \sin\alpha & \cos\alpha & \sin 2\alpha \\ \sin\beta & \cos\beta & \sin 2\beta \\ \sin\gamma & \cos\gamma & \sin 2\gamma \end{vmatrix} = 0$.

12. If tangent drawn at a point $(t^2, 2t)$ on the parabola $y^2 = 4x$ is same as the normal drawn at a point $\left(\sqrt{5}\cos\phi,\ 2\sin\phi\right)$ on the ellipse $4x^2 + 5y^2 = 20$, then find the values of t & ϕ.

13. Let d be the perpendicular distance from the centre of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ to the tangent drawn at a point P on the ellipse. If F_1 & F_2 are the two foci of the ellipse, then show that $(PF_1 - PF_2)^2 = 4a^2\left[1 - \frac{b^2}{d^2}\right]$

14. The tangent and normal to the ellipse $x^2 + 4y^2 = 4$ at a point $P(\theta)$ on it meet the major axis in Q and R respectively. If QR = 2, show that the eccentric angle θ of P is given by $\cos\theta = \pm(2/3)$.

15. If a triangle is inscribed in an ellipse and two of its sides are parallel to the given straight lines, then prove that the third side touches the fixed ellipse.

16. Consider the family of circles, $x^2 + y^2 = r^2$, $2 < r < 5$. If in the first quadrant, the common tangent to a circle of the family and the ellipse $4x^2 + 25y^2 = 100$ meets the co-ordinate axes at A & B , then find the equation of the locus of the mid–point of AB.

17. If the normal at a point P on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ of semi axes a , b & centre C cuts the major & minor axes at G & g , show that $a^2.(CG)^2 + b^2.(Cg)^2 = (a^2 - b^2)^2$. Also prove that $CG = e^2CN$, where PN is the ordinate of P. (N is foot of perpendicular from P on its major axis.)

18. Find the equation of the largest circle with centre (1, 0) that can be inscribed in the ellipse $x^2 + 4y^2 = 16$.

19. A ray emanating from the point (–4, 0) is incident on the ellipse $9x^2 + 25y^2 = 225$ at the point P with abscissa 3. Find the equation of the reflected ray after first reflection.

20. Tangents are drawn to the ellipse from the point $\left(a^2/\sqrt{a^2 - b^2}, \sqrt{a^2 + b^2}\right)$. Prove that the tangents intercept on the ordinate througbh the nearer focus a distance equal to the major axis.

Exercise # 5 Part # I [Previous Year Questions] [AIEEE/JEE-MAIN]

1. If distance between the foci of an ellipse is equal to its minor axis, then eccentricity of the ellipse is- **[AIEEE-2002]**

(1) $e = \frac{1}{\sqrt{2}}$ **(2)** $e = \frac{1}{\sqrt{3}}$ **(3)** $e = \frac{1}{\sqrt{4}}$ **(4)** $e = \frac{1}{\sqrt{6}}$

2. The equation of an ellipse, whose major axis = 8 and eccentricity = 1/2 is- (a > b) **[AIEEE-2002]**

(1) $3x^2 + 4y^2 = 12$ **(2)** $3x^2 + 4y^2 = 48$ **(3)** $4x^2 + 3y^2 = 48$ **(4)** $3x^2 + 9y^2 = 12$

3. The eccentricity of an ellipse, with its centre at the origin, is 1/2. If one of the directirices is x = 4, then the equation of the ellipse is- **[AIEEE-2004]**

(1) $3x^2 + 4y^2 = 1$ **(2)** $3x^2 + 4y^2 = 12$ **(3)** $4x^2 + 3y^2 = 12$ **(4)** $4x^2 + 3y^2 = 1$

4. An ellipse has OB as semi minor axis, F and F' its focii and the angle FBF' is a right angle. Then the eccentricity of the ellipse is- **[AIEEE-2005, IIT-1997]**

(1) $\frac{1}{\sqrt{2}}$ **(2)** $\frac{1}{2}$ **(3)** $\frac{1}{4}$ **(4)** $\frac{1}{\sqrt{3}}$

5. In an ellipse, the distance between its foci is 6 and minor axis is 8. Then its eccentricity is- **[AIEEE-2006]**

(1) $\frac{1}{2}$ **(2)** $\frac{4}{5}$ **(3)** $\frac{1}{\sqrt{5}}$ **(4)** $\frac{3}{5}$

6. A focus of an ellipse is at the origin. The directrix is the line x = 4 and the eccentricity is 1/2. Then the length of the semi-major axis is- **[AIEEE-2008]**

(1) 8/3 **(2)** 2/3 **(3)** 4/3 **(4)** 5/3

7. The ellipse $x^2 + 4y^2 = 4$ is inscribed in a rectangle aligned with the coordinate axes, which in turn is inscribed in another ellipse that passes through the point (4, 0). Then the equation of the ellipse is :- **[AIEEE-2009]**

(1) $4x^2 + 48y^2 = 48$ **(2)** $4x^2 + 64y^2 = 48$ **(3)** $x^2 + 16y^2 = 16$ **(4)** $x^2 + 12y^2 = 16$

8. Equation of the ellipse whose axes are the axes of coordinates and which passes through the point (–3, 1) and has eccentricity $\sqrt{2/5}$ is :- **[AIEEE-2011]**

(1) $3x^2 + 5y^2 - 15 = 0$ **(2)** $5x^2 + 3y^2 - 32 = 0$ **(3)** $3x^2 + 5y^2 - 32 = 0$ **(4)** $5x^2 + 3y^2 - 48 = 0$

9. An ellipse is drawn by taking a diameter of the circle $(x - 1)^2 + y^2 = 1$ as its semi-minor axis and a diameter of the circle $x^2 + (y - 2)^2 = 4$ as its semi-major axis. If the centre of the ellipse is at the origin and its axes are the coordinate axes, then the equation of the ellipse is : **[AIEEE-2012]**

(1) $x^2 + 4y^2 = 16$ **(2)** $4x^2 + y^2 = 4$ **(3)** $x^2 + 4y^2 = 8$ **(4)** $4x^2 + y^2 = 8$

10. **Statement–I :** An equation of a common tangent to the parabola $y^2 = 16\sqrt{3}\,x$ and the ellipse $2x^2 + y^2 = 4$ is $y = 2x + 2\sqrt{3}$.

Statement–II : If the line $y = mx + \frac{4\sqrt{3}}{m}$, $(m \neq 0)$ is a common tangent to the parabola $y^2 = 16\sqrt{3}\,x$ and the ellipse $2x^2 + y^2 = 4$, then m satisfies $m^4 + 2m^2 = 24$. **[AIEEE-2012]**

(1) Statement–I is true, Statement–II is false.

(2) Statement–I is false, Statement–II is true.

(3) Statement–I is true, Statement–II is true ; Statement–II is a correct explanation for Statement–I.

(4) Statement–I is true, Statement–II is true ; Statement–II is not a correct explanation for Statement–I.

11. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ and having centre at (0, 3) is : **[JEE (Main)-2013]**

(1) $x^2 + y^2 - 6y - 7 = 0$ **(2)** $x^2 + y^2 - 6y + 7 = 0$ **(3)** $x^2 + y^2 - 6y - 5 = 0$ **(4)** $x^2 + y^2 - 6y + 5 = 0$

12. The locus of the foot of perpendicular drawn from the centre of the ellipse $x^2 + 3y^2 = 6$ on any tangent to it is : **[JEE Main 2014]**

(1) $(x^2 - y^2) = 6x^2 + 2y^2$ **(2)** $(x^2 - y^2)^2 = 6x^2 - 2y^2$

(3) $(x^2 + y^2)^2 = 6x^2 + 2y^2$ **(4)** $(x^2 + y^2)^2 = 6x^2 - 2y^2$

13. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latera recta to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$, is **[JEE Main 2015]**

(1) $\frac{27}{2}$ **(2)** 27 **(3)** $\frac{27}{4}$ **(4)** 18

Part # II [Previous Year Questions][IIT-JEE ADVANCED]

1. Let ABC be an equilateral triangle inscribed in the circle $x^2 + y^2 = a^2$.Suppose perpendiculars from A, B, C to the major axis of the ellipse, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $(a > b)$ meet the ellipse respectively at P,Q,R so that P, Q,R lie on the same side of the major axis as A, B,C respectively . Prove that the normals to the ellipse drawn at the points P, Q and R are concurrent. **[JEE 2000 (Mains)]**

2. Let C_1 and C_2 be two circles with C_2 lying inside C_1. A circle C lying inside C_1 touches C_1 internally and C_2 externally. Identify the locus of the centre of C. **[JEE 2001 (Mains)]**

3. Prove that, in an ellipse, the perpendicular from a focus upon any tangent and the line joining the centre of the ellipse to the point of contact meet on the corresponding directrix. **[JEE 2002 (Mains)]**

4. Tangent is drawn to ellipse $\frac{x^2}{27}+y^2=1$ at $(3\sqrt{3}\cos\theta, \sin\theta)$ (where $\theta \in (0, \pi/2)$). Then the value of θ, such that sum of intercepts on axes made by this tangent is least is - **[JEE 2003 (Screening)]**

(A) $\frac{\pi}{3}$ **(B)** $\frac{\pi}{6}$ **(C)** $\frac{\pi}{8}$ **(D)** $\frac{\pi}{4}$

5. The area of the quadrilateral formed by the tangents at the end points of the latus rectum of the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$, is - **[JEE 2003 (Screening)]**

(A) 27/4 sq. units **(B)** 9 sq. units **(C)** 27/2 sq. units **(D)** 27 sq. units

6. Find a point on the curve $x^2 + 2y^2 = 6$ whose distance from the line $x + y = 7$, is as small as possible.

[JEE 2003 (Main)]

7. Locus of the mid points of the segments which are tangents to the ellipse $\frac{1}{2}x^2 + y^2 = 1$ and which are intercepted between the coordinate axes is - **[JEE 2004 (Screening)]**

(A) $\frac{1}{2}x^2 + \frac{1}{4}y^2 = 1$ **(B)** $\frac{1}{4}x^2 + \frac{1}{2}y^2 = 1$ **(C)** $\frac{1}{3x^2} + \frac{1}{4y^2} = 1$ **(D)** $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$

8. The minimum area of triangle formed by tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and coordinate axes -

(A) ab **(B)** $\frac{a^2 + b^2}{2}$

(C) $\frac{(a+b)^2}{2}$ **(D)** $\frac{a^2 + ab + b^2}{3}$ **[JEE 2005 (Screening)]**

9. Find the equation of the common tangent in 1^{st} quadrant to the circle $x^2 + y^2 = 16$ and the ellipse $\frac{x^2}{25} + \frac{y^2}{4} = 1$. Also find the length of the intercept of the tangent between the coordinate axes. **[JEE 2005 (Mains)]**

10. Let $P(x_1, y_1)$ and $Q(x_2, y_2)$, $y_1 < 0$, $y_2 < 0$, be the end points of the latus rectum of the ellipse $x^2+4y^2=4$. The equations of parabolas with latus rectum PQ are - **[JEE 2008]**

(A) $x^2 + 2\sqrt{3}\,y = 3 + \sqrt{3}$ **(B)** $x^2 - 2\sqrt{3}\,y = 3 + \sqrt{3}$

(C) $x^2 + 2\sqrt{3}\,y = 3 - \sqrt{3}$ **(D)** $x^2 - 2\sqrt{3}\,y = 3 - \sqrt{3}$

11. The line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse $x^2 + 9y^2 = 9$ meets its auxiliary circle at the point M. Then the area of the triangle with vertices at A, M and the origin O is :- **[JEE 2009]**

(A) $\frac{31}{10}$ **(B)** $\frac{29}{10}$ **(C)** $\frac{21}{10}$ **(D)** $\frac{27}{10}$

12. The normal at a point P on the ellipse $x^2 + 4y^2 = 16$ meets the x-axis at Q. If M is the mid point of the line segment PQ, then the locus of M intersects the latus rectums of the given ellipse at the points -

[JEE 2009]

(A) $\left(\pm\frac{3\sqrt{5}}{2}, \pm\frac{2}{7}\right)$ **(B)** $\left(\pm\frac{3\sqrt{5}}{2}, \pm\frac{\sqrt{19}}{4}\right)$

(C) $\left(\pm 2\sqrt{3}, \pm\frac{1}{7}\right)$ **(D)** $\left(\pm 2\sqrt{3}, \pm\frac{4\sqrt{3}}{7}\right)$

Paragraph for Question 13 to 15 [JEE 2010]

Tangents are drawn from the point P(3, 4) to the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ touching the ellipse at points A and B.

13. The coordinates of A and B are

(A) (3, 0) and (0, 2)
(B) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ and $\left(-\frac{9}{5}, \frac{8}{5}\right)$
(C) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ and (0, 2)
(D) (3, 0) and $\left(-\frac{9}{5}, \frac{8}{5}\right)$

14. The orthocenter of the triangle PAB is

(A) $\left(5, \frac{8}{7}\right)$ **(B)** $\left(\frac{7}{5}, \frac{25}{8}\right)$ **(C)** $\left(\frac{11}{5}, \frac{8}{5}\right)$ **(D)** $\left(\frac{8}{25}, \frac{7}{5}\right)$

15. The equation of the locus of the point whose distances from the point P and the line AB are equal, is

(A) $9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$
(B) $x^2 + 9y^2 + 6xy - 54x + 62y - 241 = 0$
(C) $9x^2 + 9y^2 - 6xy - 54x - 62y - 241 = 0$
(D) $x^2 + y^2 - 2xy + 27x + 31y - 120 = 0$

16. The ellipse $E_1 : \frac{x^2}{9}+\frac{x^2}{4}=1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point (0,4) circumscribes the rectangle R. The eccentricity of the ellipse E_2 is - **[JEE 2012]**

(A) $\frac{\sqrt{2}}{2}$ **(B)** $\frac{\sqrt{3}}{2}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{3}{4}$

17. A vertical line passing through the point (h,0) intersects the ellipse $\frac{x^2}{4}+\frac{y^2}{3}=1$ at the points P and Q. Let the tangents to the ellipse at P and Q meet at the point R. If Δ(h) = area of the triangle PQR, $\Delta_1 = \max\limits_{1/2\le h\le 1} \Delta(h)$ and $\Delta_2 = \min\limits_{1/2\le h\le 1} \Delta(h)$, then $\frac{8}{\sqrt{5}}\Delta_1 - 8\Delta_2 =$ **[JEE-Ad. 2013]**

18. Suppose that the foci of the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$ are $(f_1, 0)$ and $(f_2, 0)$ where $f_1 > 0$ and $f_2 < 0$. Let P_1 and P_2 be two parabolas with a common vertex at (0, 0) and with foci at $(f_1, 0)$ and $(2f_2, 0)$, respectively. Let T_1 be a tangent to P_1 which passes through $(2f_2, 0)$ and T_2 be a tangent to P_2 which passes through $(f_1, 0)$. If m_1 is the slope of T_1 and m_2 is the slope of T_2, then the value of $\left(\frac{1}{m_1^2}+m_2^2\right)$ is **[JEE-Ad. 2015]**

19. Let E_1 and E_2 be two ellipses whose centres are at origin. The major axes of E_1 and E_2 lie along the x-axis and the y-axis, respectively. Let S be the circle $x^2 + (y-1)^2 = 2$. The straight line $x + y = 3$ touches the curves S, E_1 and E_2 at P, Q and R, respectively. Suppose that $PQ = \frac{2\sqrt{2}}{3}$. If e_1 and e_2 are the eccentricites of E_1 and E_2, respectively, then the correct expression(s) is (are) **[JEE-Ad. 2015]**

(A) $e_1^2 + e_2^2 = \frac{43}{40}$

(B) $e_1 + e_2 = \frac{\sqrt{7}}{2\sqrt{10}}$

(C) $\left|e_1^2 + e_2^2\right| = \frac{5}{8}$

(D) $e_1 + e_2 = \frac{\sqrt{3}}{4}$

Comprehension

Let $F_1(x_1, 0)$ and $F_2(x_2, 0)$ for $x_1 < 0$ and $x_2 > 0$, be the foci of the ellipse $\frac{x^2}{9} + \frac{y^2}{8} = 1$. Suppose a parabola having vertex at the origin and focus at F_2 intersects the ellipse at point M in the first quadrant and at point N in the fourth quadrant.

20. The orthocentre of the triangle F_1MN is

(A) $\left(-\frac{9}{10}, 0\right)$ **(B)** $\left(\frac{2}{3}, 0\right)$ **(C)** $\left(\frac{9}{10}, 0\right)$ **(D)** $\left(\frac{2}{3}, \sqrt{6}\right)$

21. If the tangents to the ellipse at M and N meet at R and the normal to the parabola at M meets the x-axis at Q, then the ratio of area of the triangle MQR to area of the quadrilateral MF_1NF_2 is

(A) 3 : 4 **(B)** 4 : 5 **(C)** 5 : 8 **(D)** 2 : 3

MOCK TEST

SECTION - I : STRAIGHT OBJECTIVE TYPE

1. Find the locus of point of intersection of pair of tangents to the ellipse if the sum of the ordinates of the point of contact is b.

(A) $\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)\frac{b}{4y}=1$ **(B)** $\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)\frac{b}{2y}=1$

(C) $\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)\frac{2b}{y}=1$ **(D)** $\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)\frac{b}{2y}=4$

2. If α, β are eccentric angles of the extremities of a focal chord of an ellipse, then eccentricity of the ellipse is

(A) $\frac{\cos\alpha+\cos\beta}{\cos(\alpha+\beta)}$ **(B)** $\frac{\sin\alpha-\sin\beta}{\sin(\alpha-\beta)}$ **(C)** $\sec\alpha+\sec\beta$ **(D)** $\frac{\sin\alpha+\sin\beta}{\sin(\alpha+\beta)}$

3. If f(x) is a decreasing function then the set of values of 'k', for which the major axis of the ellipse

$\frac{x^2}{f(k^2+2k+5)}+\frac{y^2}{f(k+11)}=1$ is the x-axis, is:

(A) $k\in(-2,3)$ **(B)** $k\in(-3,2)$

(C) $k\in(-\infty,-3)\cup(2,\infty)$ **(D)** $k\in(-\infty,-2)\cup(3,\infty)$

4. If circumcentre of an equilateral triangle inscribed in $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, with vertices having eccentric angles α, β, γ respectively is (x_1, y_1), then $\Sigma\cos\alpha\cos\beta+\Sigma\sin\alpha\sin\beta=$

(A) $\frac{9x_1^2}{a^2}+\frac{9y_1^2}{b^2}+\frac{3}{2}$ **(B)** $9x_1^2-9y_1^2+a^2b^2$ **(C)** $\frac{9x_1^2}{a}+\frac{9y_1^2}{b}+3$ **(D)** $\frac{9x_1^2}{2a^2}+\frac{9y_1^2}{2b^2}-\frac{3}{2}$

5. Q is a point on the auxiliary circle corresponding to the point P of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. If T is the foot of the perpendicular dropped from the focus S onto the tangent to the auxiliary circle at Q then the ΔSPT is:

(A) isosceles **(B)** equilateral **(C)** right angled **(D)** right isosceles

6. The distance of the point $(\sqrt{6}\cos\theta,\sqrt{2}\sin\theta)$ on the ellipse $x^2/6+y^2/2=1$ from the centre of the ellipse is 2, if

(A) $\theta=\pi/3$ **(B)** $\theta=\pi/6$ **(C)** $\theta=5\pi/4$ **(D)** $\theta=5\pi/3$

7. If circumcentre of an equilateral triangle inscribed in $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, with vertices having eccentric angles α, β, γ respectively is (x_1, y_1), then $\Sigma\cos\alpha\cos\beta+\Sigma\sin\alpha\sin\beta=$

(A) $\frac{9x_1^2}{a^2}+\frac{9y_1^2}{b^2}+\frac{3}{2}$ **(B)** $9x_1^2-9y_1^2+a^2b^2$ **(C)** $\frac{9x_1^2}{a}+\frac{9y_1^2}{b}+3$ **(D)** $\frac{9x_1^2}{2a^2}+\frac{9y_1^2}{2b^2}-\frac{3}{2}$

8. The locus of extremities of latus rectum of the family of ellipse $b^2x^2 + y^2 = a^2b^2$ where b is a parameter $(b^2 < 1)$, is-

(A) $x^2 \pm a^2y^2 = a^2$ **(B)** $x^2 \pm ay = a^2$ **(C)** $x \pm ay^2 = a^2$ **(D)** none of these

9. **S_1 :** If from a point $P(0, \alpha)$ two normals other than axes are drawn to ellipse $\frac{x^2}{25}+\frac{y^2}{16} = 1$, where $|\alpha| \le k$, then least value of k is $\frac{9}{4}$

S_2 : The minimum and maximum distances of a point (1, 2) from the ellipse $4x^2 + 9y^2 + 8x - 36y + 4 = 0$ are L and G, then G – L is equal to 4

S_3 : If the length of latus rectum of an ellipse is one-third of its major axis. Its eccentricity is equal to $\frac{2}{3}$

S_4 : The set of all positive values of a for which $(13x - 1)^2 + (13y - 2)^2 = \left(\frac{5x+12y-1}{a}\right)^2$ represents an ellipse is (1, 2)

(A) TTFF **(B)** TTFT **(C)** TTTF **(D)** TFTF

10. A series of concentric ellipses $E_1, E_2, ..., E_n$ are drawn such that E_n touches the extremities of the major axis of E_{n-1} and the foci of E_n coincide with the extremities of minor axis of E_{n-1}. If the eccentricity of the ellipses is independent of n, then the value of the eccentricity, is

(A) $\frac{\sqrt{5}}{3}$ **(B)** $\frac{\sqrt{5}-1}{2}$ **(C)** $\frac{\sqrt{5}+1}{2}$ **(D)** $\frac{1}{\sqrt{5}}$

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

11. If P is a point of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, whose focii are S and S′. Let $\angle PSS' = \alpha$ and $\angle PS'S = \beta$, then

(A) $PS + PS' = 2a$, if $a > b$

(B) $PS + PS' = 2b$, if $a < b$

(C) $\tan\frac{\alpha}{2} \tan\frac{\beta}{2} = \frac{1-e}{1+e}$

(D) $\tan\frac{\alpha}{2} \tan\frac{\beta}{2} = \frac{\sqrt{a^2-b^2}}{b^2} [a - \sqrt{a^2-b^2}\,]$ when $a > b$

12. The parametric angle θ, where $-\pi - \theta \le \pi$, of the point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at which the tangent drawn cuts the intercept of minimum length on the coordinate axes, is/are

(A) $\tan^{-1}\sqrt{\frac{b}{a}}$ **(B)** $-\tan^{-1}\sqrt{\frac{b}{a}}$ **(C)** $\pi - \tan^{-1}\sqrt{\frac{b}{a}}$ **(D)** $\pi + \tan^{-1}\sqrt{\frac{b}{a}}$

13. Let F_1, F_2 be two focii of the ellipse and PT and PN be the tangent and the normal respectively to the ellipse at point P then

(A) PN bisects $\angle F_1PF_2$ **(B)** PT bisects $\angle F_1PF_2$

(C) PT bisects angle $(180° - \angle F_1PF_2)$ **(D)** None of these

14. Let F_1, F_2 be two focii of the ellipse and PT and PN be the tangent and the normal respectively to the ellipse at point P then

(A) PN bisects $\angle F_1PF_2$ **(B)** PT bisects $\angle F_1PF_2$

(C) PT bisects angle $(180° - \angle F_1PF_2)$ **(D)** None of these

15. Let $A(\alpha)$ and $B(\beta)$ be the extremeties of a chord of an ellipse . If the slope of AB is equal to the slope of the tangent at a point $C(\theta)$ on the ellipse, then the value of θ, is

(A) $\frac{\alpha+\beta}{2}$ **(B)** $\frac{\alpha-\beta}{2}$ **(C)** $\frac{\alpha+\beta}{2}+\pi$ **(D)** $\frac{\alpha-\beta}{2}-\pi$

SECTION - III : ASSERTION AND REASON TYPE

16. **Statement-I:** If $P\left(\frac{3\sqrt{3}}{2}, 1\right)$ is a point on the ellipse $4x^2 + 9y^2 = 36$. Circle drawn taking AP as diameter touches another circle $x^2 + y^2 = 9$, where $A \equiv (-\sqrt{5}, 0)$.

Statement-II : Circle drawn with focal radius as diameter touches the auxiliary circle.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

17. Tangents are drawn from the point $P\left(-\sqrt{3}, \sqrt{2}\right)$ to an ellipse $4x^2 + y^2 = 4$.

Statement-I : The tangents are mutually perpendicular.

Statement-II : The locus of the points from which mutually perpendicular tangents can be drawn to given ellipse is $x^2 + y^2 = 5$.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

18. **Statement-I :** Let tangent at a point P on the ellipse, which is not an extremity of major axis, meets a directrix at T. If circle drawn on PT as diameter cuts the directrix at Q, then PQ = ePS, where S is the focus corresponding to the directrix.

Statement-II : Let tangent at a point P on an ellipse, which not an extremity of major axis, meets the directrices at T′ and T. Then PT substends a right angle at the focus corresponding the directrix at which T lies.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

19. **Statement-I:** The ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ and $\frac{x^2}{9}+\frac{y^2}{16}=1$ are congruent.

Statement-II: The ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ and $\frac{x^2}{9}+\frac{y^2}{16}=1$ have the same eccentricity.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

20. **Statement-I :** A triangle ABC right angled at A moves so that its perpendicular sides touch the curve

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$ all the time. Then loci of the points A, B and C are circle.

Statement-II : Locus of point of intersection of two perpendicular tangents to the conic is director circle
(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

SECTION - IV : MATRIX - MATCH TYPE

21.

Column-I		Column-II	
(A)	The eccentricity of the ellipse which meets the straight line $2x-3y=6$ on the X-axis and the straight line $4x+5y=20$ on the Y-axis and whose principal axes lie along the coordinate axes, is	**(p)**	$\frac{1}{2}$
(B)	A bar of length 20 units moves with its ends on two fixed straight lines at right angles. A point P marked on the bar at a distance of 8 units from one end describes a conic whose eccentricity is	**(q)**	$\frac{1}{\sqrt{2}}$
(C)	If one extremity of the minor axis of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the foci form an equilateral triangle, then its eccentricity, is	**(r)**	$\frac{\sqrt{5}}{3}$
(D)	There are exactly two points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose distance from the centre of the ellipse are greatest and equal to $\sqrt{\frac{a^2+2b^2}{2}}$. Eccentricity of this ellipse is equal to	**(s)**	$\frac{\sqrt{7}}{4}$

22. **Column – I** | **Column – II**

	Column – I		Column – II
(A)	If the angle between the straight angle lines joining foci and one of the ends of the minor axis of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is 90º. Find its eccentricity.	**(p)**	4
(B)	For an ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ with vertices A and A′, tangent drawn at the point P in the first quadrant meets the y-axis in Q and the chord A′P meets the y-axis in M. If 'O' is the origin, then $OQ^2 - MQ^2$ equals to	**(q)**	2
(C)	The x-coordinate of points on the axis of the parabola $4y^2 - 32x + 4y + 65 = 0$ from which all the three normals to the parabola are real is	**(r)**	$\frac{1}{\sqrt{2}}$
(D)	The area of the parallelogram inscribed in the ellipse $\frac{x^2}{2^2}+\frac{y^2}{(1/2)^2}=1$ whose diagonals are the conjugate diameters of the ellipse is given by	**(s)**	7
		(t)	8

SECTION - V : COMPREHENSION TYPE

23. **Read the following comprehensions carefully and answer the questions.**

Consider the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the parabola $y^2 = 2x$. They intersect at P and Q in the first and fourth quadrants respectively. Tangents to the ellipse at P and Q intersect the x-axis at R and tangents to the parabola at P and Q intersect the x-axis at S.

1. The ratio of the areas of the triangles PQS and PQR, is

(A) 1 : 3 **(B)** 1 : 2 **(C)** 2 : 3 **(D)** 3 : 4

2. The area of quadrilateral PRQS, is

(A) $\frac{3\sqrt{15}}{2}$ **(B)** $\frac{15\sqrt{3}}{2}$ **(C)** $\frac{5\sqrt{3}}{2}$ **(D)** $\frac{5\sqrt{15}}{2}$

3. The equation of circle touching the parabola at upper end of its latus rectum and passing through its vertex, is

(A) $2x^2 + 2y^2 - x - 2y = 0$ **(B)** $2x^2 + 2y^2 + 4x - \frac{9}{2}y = 0$

(C) $2x^2 + 2y^2 + x - 3y = 0$ **(D)** $2x^2 + 2y^2 - 7x + y = 0$

24. Read the following comprehensions carefully and answer the questions.

Second degree equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents an ellipse if

$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} \neq 0$ & $h^2 < ab$. Intersection of major axis and minor axis gives centre of ellipse

1. There are exactly 'n' integral values of λ for which equation $x^2 + \lambda xy + y^2 = 1$ represents an ellipse then 'n' must be

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3

2. Length of the longest chord of the ellipse $x^2 + y^2 + xy = 1$ is __

(A) $\sqrt{2}$ **(B)** $\frac{1}{\sqrt{2}}$ **(C)** $2\sqrt{2}$ **(D)** 1

3. Length of the chord perpendiuclar to longest chord as in above question and pasing through centre of ellipse is __

(A) $\frac{1}{\sqrt{2}}$ **(B)** $\frac{\sqrt{3}}{2}$ **(C)** $2\sqrt{\frac{2}{3}}$ **(D)** $\frac{1}{\sqrt{3}}$

25. Read the following comprehensions carefully and answer the questions.

A bird flies on ellipse $ax^2 + by^2 = 1$ & $z = 5\sqrt{3}$ $(b > a > 0)$ whose eccentricity is $\frac{1}{\sqrt{2}}$. An observer stands at a point $P(\alpha, \beta, 0)$ where maximum and minimum angle of elevation of the bird are 60º and 30º when bird is at Q and R respectively on its path and Q′ and R′ are projection of Q and R on x- y plane, P, Q′ R′ are collinear & the distance between Q′ and R′ is maximum Let θ be the angle elevation of the bird when it is at a point on the arc of the ellipse exactly mid-way between Q & R. It is given that $a\alpha^2 + b\beta^2 - 1 > 0$

1. If $\alpha > 0$, then equation of the line along which minimum angle of elevation is observed, is

(A) $\frac{x-\sqrt{3}}{\sqrt{3}} = \frac{z+\sqrt{3}}{-1}$ **(B)** $\frac{x-13}{\sqrt{3}} = \frac{z+\sqrt{3}}{-1}, \ y=0$

(C) $\frac{x-10}{\sqrt{3}} = \frac{z+\sqrt{3}}{-1}, y=0$ **(D)** $\frac{x-10}{\sqrt{3}} = \frac{y}{0} = \frac{z+\sqrt{3}}{-1}$

2. Equation of plane which touches the ellipse at Q and passes through P $(\alpha > 0)$ is

(A) $-\sqrt{3}x + y + z - 10\sqrt{3} = 0$ **(B)** $\sqrt{3}x + y + z - 10\sqrt{3} = 0$

(C) $\sqrt{3}x + z - 10\sqrt{3} = 0$ **(D)** $\sqrt{3}x + y - 10\sqrt{3} = 0$

3. Value of $\tan\theta$, is

(A) $\frac{2}{3}$ **(B)** $\frac{3}{2}$ **(C)** $\sqrt{\frac{2}{3}}$ **(D)** $\sqrt{\frac{6}{5}}$

SECTION - VI : INTEGER TYPE

26. Number of distinct normal lines that can be drawn to ellipse $\frac{x^2}{169}+\frac{y^2}{25}=1$ from the point P (0, 6) is

27. Find the number of integral values of parameter 'a' for which three chords of the ellipse $\frac{x^2}{2a^2}+\frac{y^2}{a^2}=1$ (other than its diameter) passing through the point $P\left(11a,-\frac{a^2}{4}\right)$ are bisected by the parabola $y^2 = 4ax$.

28. Origin O is the centre of two concentric circles whose radii are a & b respectively, a < b. A line OPQ is drawn to cut the inner circle in P & the outer circle in Q. PR is drawn parallel to the y– axis & QR is drawn parallel to the x– axis. The locus of R is an ellipse touching the two circles. If the focii of this ellipse lie on the inner circle, if eccentricity is $\sqrt{2}\,\lambda$, then find λ

29. Consider two concentric circles $S_1 : |z| = 1$ and $S_2 : |z| = 2$ on the Argand plane. A variable parabola is drawn through the points where 'S_1' meets the real axis and having arbitrary tangent drawn to 'S_2' as its directrix. If the locus of the focus of the parabola is a conic C then find the area of the quadrilateral formed by the tangents at the ends of the latus-rectum of conic C.

30. Number of points on the ellipse $\frac{x^2}{50}+\frac{y^2}{20}=1$ from which pair of perpendicular tangents may be drawn to the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ is

ANSWER KEY

EXERCISE - 1

1. A **2.** D **3.** A **4.** C **5.** B **6.** C **7.** B **8.** B **9.** A **10.** A **11.** A **12.** A **13.** B
14. B **15.** C **16.** C **17.** D **18.** B **19.** B **20.** C **21.** C **22.** C **23.** C **24.** D **25.** B **26.** A
27. A **28.** C **29.** C **30.** C

EXERCISE - 2 : PART # I

1. AC **2.** AC **3.** ACD **4.** CD **5.** ABC **6.** AC **7.** CD **8.** AC **9.** ABC
10. AB **11.** AB **12.** ACD **13.** AC **14.** AC **15.** AB **16.** AC **17.** ABC **18.** ACD
19. AC **20.** AD

PART - II

1. D **2.** A **3.** A **4.** D **5.** A

EXERCISE - 3 : PART # I

1. $A \to q$ $B \to q$ $C \to s$ $D \to p$ **2.** $A \to q,s$ $B \to p,r$ $C \to r$ $D \to s$

PART - II

Comprehension # 1 : 1. C **2.** B **3.** D **Comprehension # 2 : 1.** C **2.** B **3.** C

EXERCISE - 5 : PART # I

1. 1 **2.** 1 **3.** 2 **4.** 1 **5.** 4 **6.** 1 **7.** 4 **8.** 1 **10.** 3 **11.** 1 **12.** 3 **13.** 2

PART - II

2. Locus is an ellipse with foci as the centres of the circles C_1 and C_2.

4. B **5.** D **6.** (2,1) **7.** D **8.** A **9.** $y = -\frac{2}{\sqrt{3}}x + 4\sqrt{\frac{7}{3}}; \frac{14}{\sqrt{3}}$ **10.** B,C **11.** D **12.** C **13.** D

14. C **15.** A **16.** C **17.** 9 **18.** 4 **19.** AB **20.** A **21.** C

MOCK TEST

1. B	**2.** D	**3.** B	**4.** D	**5.** A	**6.** C	**7.** D	**8.** B	**9.** A
10. B	**11.** A,B,C	**12.** A,B,C	**13.** A,C	**14.** A,C	**15.** A,C	**16.** A	**17.** A	**18.** D

19. B **20.** D **21.** $A \to s$ $B \to r$ $C \to p$ $D \to q$ **22.** $A \to r$ $B \to p$ $C \to s,t$ $D \to q$

23. **1.** C **2.** B **3.** D **24.** **1.** D **2.** C **3.** C **25.** **1.** B **2.** C **3.** C

26. 3 **27.** 2 **28.** 1/2 **29.** 16 **30.** 4