SOLVED EXAMPLES

Ex. 1 Find the coordinates of foci, the eccentricity and latus-rectum, equations of directrices for the hyperbola $9x^2 - 16y^2 - 72x + 96y - 144 = 0$.

Sol. Equation can be rewritten as $\frac{(x-4)^2}{4^2} - \frac{(y-3)^2}{3^2} = 1$ so a = 4, b = 3

$b^2 = a^2(e^2 - 1)$ given $e = \frac{5}{4}$

Foci : $X = \pm ae, Y = 0$ gives the foci as (9, 3), (–1, 3)

Centre : $X = 0, Y = 0$ i.e. (4, 3)

Directrices : $X = \pm \frac{a}{e}$ i.e. $x - 4 = \pm \frac{16}{5}$ $\therefore$ directrices are $5x - 36 = 0$; $5x - 4 = 0$

Latus-rectum $= \frac{2b^2}{a} = 2.\frac{9}{4} = \frac{9}{2}$

Ex. 2 Find the eccentricity of the hyperbola whose latus rectum is half of its transverse axis.

Sol. Let the equation of hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Then transverse axis = 2a and latus–rectum = $\frac{2b^2}{a}$

According to question $\frac{2b^2}{a} = \frac{1}{2}(2a)$

$\Rightarrow$ $2b^2 = a^2$ $\quad$ ($\because$ $b^2 = a^2(e^2 - 1)$)

$\Rightarrow$ $2a^2(e^2 - 1) = a^2$ $\quad \Rightarrow$ $2e^2 - 2 = 1$

$\Rightarrow$ $e^2 = \frac{3}{2}$ $\quad \therefore$ $e = \sqrt{\frac{3}{2}}$

Hence the required eccentricity is $\sqrt{\frac{3}{2}}$.

Ex. 3 If (a secθ, b tanθ) and (asecϕ, b tanϕ) are the ends of a focal chord of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then $\tan\frac{\theta}{2}\tan\frac{\phi}{2}$ equal to -

Sol. Equation of chord connecting the points (asecθ, b tanθ) and (asecϕ, b tanϕ) is

$$\frac{x}{a}\cos\left(\frac{\theta-\phi}{2}\right) - \frac{y}{b}\sin\left(\frac{\theta+\phi}{2}\right) = \cos\left(\frac{\theta+\phi}{2}\right) \qquad \text{........ (i)}$$

If it passes through (ae, 0); we have, $e\cos\left(\frac{\theta-\phi}{2}\right) = \cos\left(\frac{\theta+\phi}{2}\right)$

$$\Rightarrow \quad e = \frac{\cos\left(\frac{\theta+\phi}{2}\right)}{\cos\left(\frac{\theta-\phi}{2}\right)} = \frac{1 - \tan\frac{\theta}{2}.\tan\frac{\phi}{2}}{1 + \tan\frac{\theta}{2}.\tan\frac{\phi}{2}} \quad \Rightarrow \quad \tan\frac{\theta}{2}.\tan\frac{\phi}{2} = \frac{1-e}{1+e}$$

Similarly if **(i)** passes through (–ae, 0), $\tan\frac{\theta}{2}.\tan\frac{\phi}{2} = \frac{1+e}{1-e}$

Ex. 4 Prove that the straight line $\ell x + my + n = 0$ touches the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ if $a^2\ell^2 - b^2 m^2 = n^2$.

Sol. The given line is $\ell x + my + n = 0$ or $y = -\frac{\ell}{m}x - \frac{n}{m}$

Comparing this line with $y = Mx + c$

$\therefore \quad M = -\frac{\ell}{m}$ and $c = -\frac{n}{m}$ ....**(1)**

This line (1) will touch the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ if $c^2 = a^2M^2 - b^2$

$\Rightarrow \quad \frac{n^2}{m^2} = \frac{a^2\ell^2}{m^2} - b^2$

or $\quad a^2\ell^2 - b^2m^2 = n^2$

Ex. 5 A common tangent to $9x^2 - 16y^2 = 144$ and $x^2 + y^2 = 9$ is -

Sol. $\frac{x^2}{16} - \frac{y^2}{9} = 1$, $x^2 + y^2 = 9$

Equation of tangent $y = mx + \sqrt{16m^2 - 9}$ (for hyperbola)

Equation of tangent $y = m'x + 3\sqrt{1 + m'^2}$ (circle)

For common tangent $m = m'$ and $3\sqrt{1 + m'^2} = \sqrt{16m^2 - 9}$

or $\quad 9 + 9m^2 = 16m^2 - 9$

or $\quad 7m^2 = 18 \Rightarrow m = \pm 3\sqrt{\frac{2}{7}}$

$\therefore \quad$ required equation is $y = \pm 3\sqrt{\frac{2}{7}}\,x \pm 3\sqrt{1 + \frac{18}{7}}$

or $\quad y = \pm 3\sqrt{\frac{2}{7}}\,x \pm \frac{15}{\sqrt{7}}$

Ex. 6 Find the equation and the length of the common tangents to hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$.

Sol. Tangent at $(a \sec \phi, b \tan \phi)$ on the 1st hyperbola is

$\frac{x}{a}\sec\phi - \frac{y}{b}\tan\phi = 1$ .....**(i)**

Similarly tangent at any point $(b \tan \theta, a \sec \theta)$ on 2nd hyperbola is

$\frac{y}{a}\sec\theta - \frac{x}{b}\tan\theta = 1$ .....**(ii)**

If (1) and (2) are common tangents then they should be identical. Comparing the co–effecients of x and y

$\Rightarrow \quad \frac{\sec\theta}{a} = -\frac{\tan\varphi}{b}$ .....**(iii)** and $\quad -\dfrac{\tan\theta}{b} = \dfrac{\sec\varphi}{a}$

or $\quad \sec\theta = -\dfrac{a}{b}\tan\phi \qquad$ ......(iv)

$\because \quad \sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \quad \dfrac{a^2}{b^2}\tan^2\phi - \dfrac{b^2}{a^2}\sec^2\phi = 1 \qquad$ {from (iii) and (iv)}

or $\quad \dfrac{a^2}{b^2}\tan^2\phi - \dfrac{b^2}{a^2}(1+\tan^2\phi) = 1$ or $\left(\dfrac{a^2}{b^2}-\dfrac{b^2}{a^2}\right)\tan^2\phi = 1 + \dfrac{b^2}{a^2}$

$\tan^2\phi = \dfrac{b^2}{a^2-b^2}$

and $\quad \sec^2\phi = 1 + \tan^2\phi = \dfrac{a^2}{a^2-b^2}$

Hence the point of contact are

$$\left\{\pm\frac{a^2}{\sqrt{(a^2-b^2)}}, \pm\frac{b^2}{\sqrt{(a^2-b^2)}}\right\} \text{ and } \left\{\pm\frac{b^2}{\sqrt{(a^2-b^2)}}, \pm\frac{a^2}{\sqrt{(a^2-b^2)}}\right\} \quad \{\text{from (3) and (4)}\}$$

Length of common tangent i.e., the distance between the above points is $\sqrt{2}\dfrac{(a^2+b^2)}{\sqrt{(a^2-b^2)}}$ and equation of common tangent on putting the values of $\sec\phi$ and $\tan\phi$ in (1) is

$$\pm\frac{x}{\sqrt{(a^2-b^2)}} \mp \frac{y}{\sqrt{(a^2-b^2)}} = 1 \quad \text{or} \quad x \mp y = \pm\sqrt{(a^2-b^2)}$$

Ex. 7 Line $x\cos\alpha + y\sin\alpha = p$ is a normal to the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, if -

(A) $a^2\sec^2\alpha - b^2\operatorname{cosec}^2\alpha = \dfrac{(a^2+b^2)^2}{p^2}$

(C) $a^2\sec^2\alpha + b^2\operatorname{cosec}^2\alpha = \dfrac{(a^2+b^2)^2}{p^2}$

(C) $a^2\cos^2\alpha - b^2\sin^2\alpha = \dfrac{(a^2+b^2)^2}{p^2}$

(D) $a^2\cos^2\alpha + b^2\sin^2\alpha = \dfrac{(a^2+b^2)^2}{p^2}$

Sol. Equation of a normal to the hyperbola is $ax\cos\theta + by\cot\theta = a^2 + b^2$

comparing it with the given line equation

$$\frac{a\cos\theta}{\cos\alpha} = \frac{b\cot\theta}{\sin\alpha} = \frac{a^2+b^2}{p} \quad \Rightarrow \quad \sec\theta = \frac{ap}{\cos\alpha(a^2+b^2)}, \; \tan\theta = \frac{bp}{\sin\alpha(a^2+b^2)}$$

Eliminating θ, we get

$$\frac{a^2p^2}{\cos^2\alpha(a^2+b^2)^2} - \frac{b^2p^2}{\sin^2\alpha(a^2+b^2)^2} = 1 \quad \Rightarrow \quad a^2\sec^2\alpha - b^2\operatorname{cosec}^2\alpha = \frac{(a^2+b^2)^2}{p^2}$$

Ex. 8 Find the locus of point of intersection of perpendicular tangents to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Sol. Let P(h, k) be the point of intersection of two perpendicular tangents. Equation of pair of tangents is $SS_1 = T^2$

$$\Rightarrow \quad \left(\frac{x^2}{a^2}-\frac{y^2}{b^2}-1\right)\left(\frac{h^2}{a^2}-\frac{k^2}{b^2}-1\right)=\left(\frac{hx}{a^2}-\frac{ky}{b^2}-1\right)^2$$

$$\Rightarrow \quad \frac{x^2}{a^2}\left(-\frac{k^2}{b^2}-1\right)-\frac{y^2}{b^2}\left(\frac{h^2}{a^2}-1\right)+\ldots\ldots=0 \qquad \ldots\ldots\ldots\text{(i)}$$

Since equation **(i)** represents two perpendicular lines

$$\therefore \quad \frac{1}{a^2}\left(-\frac{k^2}{b^2}-1\right)-\frac{1}{b^2}\left(\frac{h^2}{a^2}-1\right)=0$$

$\Rightarrow \quad -k^2-b^2-h^2+a^2=0 \qquad \Rightarrow \quad$ locus is $x^2+y^2=a^2-b^2$

Ex. 9 Find the hyperbola whose asymptotes are $2x-y=3$ and $3x+y-7=0$ and which passes through the point (1, 1).

Sol. The equation of the hyperbola differs from the equation of the asymptotes by a constant

$\Rightarrow$ The equation of the hyperbola with asymptotes $3x+y-7=0$ and $2x-y=3$ is $(3x+y-7)(2x-y-3)+k=0$

It passes through (1, 1)

$\Rightarrow \quad k=-6.$

Hence the equation of the hyperbola is $(2x-y-3)(3x+y-7)=6$.

Ex. 10 Find the condition on 'a' and 'b' for which two distinct chords of the hyperbola $\frac{x^2}{2a^2}-\frac{y^2}{2b^2}=1$ passing through (a, b) are bisected by the line $x+y=b$.

Sol. Let the line $x+y=b$ bisect the chord at $P(\alpha, b-\alpha)$

$\therefore$ equation of chord whose mid-point is $P(\alpha, b-\alpha)$

$$\frac{x\alpha}{2a^2}-\frac{y(b-\alpha)}{2b^2}=\frac{\alpha^2}{2a^2}-\frac{(b-\alpha)^2}{2b^2}$$

Since it passes through (a, b)

$$\therefore \quad \frac{\alpha}{2a}-\frac{(b-\alpha)}{2b}=\frac{\alpha^2}{2a^2}-\frac{(b-\alpha)^2}{2b^2}$$

$$\alpha^2\left(\frac{1}{a^2}-\frac{1}{b^2}\right)+\alpha\left(\frac{1}{b}-\frac{1}{a}\right)=0$$

$$\alpha=0, \quad \alpha=\frac{1}{\frac{1}{a}+\frac{1}{b}} \qquad \therefore \quad a\neq\pm b$$

Ex. 11 Chords of the circle $x^2 + y^2 = a^2$ touch the hyperbola $x^2/a^2 - y^2/b^2 = 1$. Prove that locus of their middle point is the curve $(x^2 + y^2)^2 = a^2x^2 - b^2y^2$.

Sol. Let (h, k) be the mid-point of the chord of the circle $x^2 + y^2 = a^2$,

so that its equation by $T = S_1$ is $hx + ky = h^2 + k^2$

or $$y = -\frac{h}{k}x + \frac{h^2 + k^2}{k} \quad \text{i.e. of the form } y = mx + c$$

It will touch the hyperbola if $c^2 = a^2m^2 - b^2$

$$\therefore \quad \left(\frac{h^2 + k^2}{k}\right)^2 = a^2\left(-\frac{h}{k}\right)^2 - b^2 \quad \text{or} \quad (h^2 + k^2)^2 = a^2h^2 - b^2k^2$$

Generalising, the locus of mid-point (h, k) is $(x^2 + y^2)^2 = a^2x^2 - b^2y^2$

Ex. 12 Find the locus of the mid point of the chords of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ which subtend a right angle at the origin.

Sol. let (h,k) be the mid–point of the chord of the hyperbola. Then its equation is

$$\frac{hx}{a^2} - \frac{ky}{b^2} - 1 = \frac{h^2}{b^2} - \frac{k^2}{b^2} - 1 \quad \text{or} \quad \frac{hx}{a^2} - \frac{ky}{b^2} = \frac{h^2}{a^2} - \frac{k^2}{b^2} \quad \text{........(1)}$$

The equation of the lines joining the origin to the points of intersection of the hyperbola and the chord (1) is obtained by making homogeneous hyperbola with the help of (1)

$$\therefore \quad \frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{\left(\frac{hx}{a^2} - \frac{ky}{b^2}\right)^2}{\left(\frac{h^2}{a^2} - \frac{k^2}{b^2}\right)^2}$$

$$\Rightarrow \quad \frac{1}{a^2}\left(\frac{h^2}{a^2} - \frac{k^2}{b^2}\right)^2 x^2 - \frac{1}{b^2}\left(\frac{h^2}{a^2} - \frac{k^2}{b^2}\right)^2 y^2 = \frac{h^2}{a^4}x^2 + \frac{k^2}{b^4}y^2 - \frac{2hk}{a^2b^2}xy \quad \text{.......(2)}$$

The lines represented by (2) will be at right angle if coefficient of x^2 + coefficient of $y^2 = 0$

$$\Rightarrow \quad \frac{1}{a^2}\left(\frac{h^2}{a^2} - \frac{k^2}{b^2}\right)^2 - \frac{h^2}{a^4} - \frac{1}{b^2}\left(\frac{h^2}{a^2} - \frac{k^2}{b^2}\right)^2 - \frac{k^2}{b^4} = 0$$

$$\Rightarrow \quad \left(\frac{h^2}{a^2} - \frac{k^2}{b^2}\right)^2\left(\frac{1}{a^2} - \frac{1}{b^2}\right) = \frac{h^2}{a^4} + \frac{k^2}{b^4}$$

hence, the locus of (h,k) is $\left(\frac{x^2}{a^2} - \frac{y^2}{b^2}\right)^2\left(\frac{1}{a^2} - \frac{1}{b^2}\right) = \frac{x^2}{a^4} + \frac{y^2}{b^4}$

Ex. 13 C is the centre of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. The tangent at any point P on this hyperbola meets the straight lines bx – ay = 0 and bx + ay = 0 in the points Q and R respectively. Show that CQ. CR = $a^2 + b^2$.

Sol. P is $(a\sec\theta,\ b\tan\theta)$

Tangent at P is $\frac{x\sec\theta}{a}-\frac{y\tan\theta}{b}=1$

It meets bx – ay = 0 i.e. $\frac{x}{a}=\frac{y}{b}$ in Q

$\therefore$ Q is $\left(\frac{a}{\sec\theta-\tan\theta},\frac{b}{\sec\theta-\tan\theta}\right)$

It meets bx + ay = 0 i.e. $\frac{x}{a}=-\frac{y}{b}$ in R.

$\therefore$ R is $\left(\frac{a}{\sec\theta+\tan\theta},\frac{-b}{\sec\theta+\tan\theta}\right)$

$\therefore$ $CQ.CR=\frac{\sqrt{(a^2+b^2)}}{\sec\theta-\tan\theta}.\frac{\sqrt{(a^2+b^2)}}{\sec\theta+\tan\theta}=a^2+b^2$ $(\because \sec^2\theta-\tan^2\theta=1)$

Ex. 14 The asymptotes of a hyperbola having centre at the point (1, 2) are parallel to the lines 2x + 3y = 0 and 3x + 2y = 0. If the hyperbola passes through the point (5, 3), show that its equation is $(2x+3y-8)(3x+2y+7)=154$

Sol. Let the asymptotes be $2x + 3y + \lambda = 0$ and $3x + 2y + \mu = 0$. Since asymptotes passes through (1, 2), then $\lambda = -8$ and $\mu = -7$

Thus the equation of asymptotes are $2x+3y-8=0$ and $3x+2y-7=0$

Let the equation of hyperbola be

$$(2x+3y-8)(3x+2y-7)+v=0 \quad \text{......(i)}$$

It passes through (5,3), then

$$(10+9-8)(15+6-7)+v=0$$

$\Rightarrow$ $11\times 14+v=0$

$\therefore$ $v=-154$

putting the value of v in **(i)** we obtain

$$(2x+3y-8)(3x+2y-7)-154=0$$

which is the equation of required hyperbola.

Ex. 15 A circle of variable radius cuts the rectangular hyperbola $x^2-y^2=9a^2$ in points P, Q, R and S. Determine the equation of the locus of the centroid of triangle PQR.

Sol. Let the circle be $(x-h)^2+(y-k)^2=r^2$ where r is variable. Its intersection with $x^2-y^2=9a^2$ is obtained by putting $y^2=x^2-9a^2$.

$x^2 + x^2 - 9a^2 - 2hx + h^2 + k^2 - r^2 = 2k\sqrt{(x^2 - 9a^2)}$

or $\quad [2x^2 - 2hx + (h^2 + k^2 - r^2 - 9a^2)]^2 = 4k^2(x^2 - 9a^2)$

or $\quad 4x^4 - 8hx^3 + \ldots\ldots = 0$

$\therefore$ Above gives the abscissas of the four points of intersection.

$\therefore \quad \Sigma x_1 = \dfrac{8h}{4} = 2h$

$x_1 + x_2 + x_3 + x_4 = 2h$

Similarly $y_1 + y_2 + y_3 + y_4 = 2k$.

Now if (α, β) be the centroid of ΔPQR, then $3\alpha = x_1 + x_2 + x_3$, $3\beta = y_1 + y_2 + y_3$

$\therefore \quad x_4 = 2h - 3\alpha,\ y_4 = 2k - 3\beta$

But (x_4, y_4) lies on $x^2 - y^2 = 9a^2$

$\therefore \quad (2h - 3\alpha)^2 + (2k - 3\beta)^2 = 9a^2$

Hence the locus of centroid (α, β) is $(2h - 3x)^2 + (2k - 3y)^2 = 9a^2$

or $\quad \left(x - \dfrac{2h}{3}\right)^2 + \left(y - \dfrac{2k}{3}\right)^2 = a^2$

Ex. 16 A, B, C are three points on the rectangular hyperbola $xy = c^2$, find

(i) The area of the triangle ABC

(ii) The area of the triangle formed by the tangents at A, B and C.

Sol. Let co–ordinates of A,B and C on the hyperbola $xy = c^2$ are $\left(ct_1, \dfrac{c}{t_1}\right)$, $\left(ct_2, \dfrac{c}{t_2}\right)$ and $\left(ct_3, \dfrac{c}{t_3}\right)$ respectively.

(i) $\therefore$ Area of triangle ABC

$$= \frac{1}{2}\left[\begin{vmatrix} ct_1 & \dfrac{c}{t_1} \\ ct_2 & \dfrac{c}{t_2} \end{vmatrix} + \begin{vmatrix} ct_2 & \dfrac{c}{t_2} \\ ct_3 & \dfrac{c}{t_3} \end{vmatrix} + \begin{vmatrix} ct_3 & \dfrac{c}{t_3} \\ ct_1 & \dfrac{c}{t_1} \end{vmatrix}\right]$$

$$= \frac{c^2}{2}\left|\frac{t_1}{t_2} - \frac{t_2}{t_1} + \frac{t_2}{t_3} - \frac{t_3}{t_2} + \frac{t_3}{t_1} - \frac{t_1}{t_3}\right|$$

$$= \frac{c^2}{2t_1t_2t_3}\left|t_1^2t_3 - t_2^2t_3 + t_1t_2^2 - t_3^2t_1 + t_2t_3^2 - t_1^2t_2\right|$$

$$= \frac{c^2}{2t_1t_2t_3}\,|(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)|$$

(ii) Equations of tangents at A,B,C are

$$x + yt_1^2 - 2ct_1 = 0$$

$$x + yt_2^2 - 2ct_2 = 0$$

and

$$x + yt_3^2 - 2ct_3 = 0$$

$\therefore$ Required Area $= \dfrac{1}{2|C_1C_2C_3|}\begin{vmatrix} 1 & t_1^2 & -2ct_1 \\ 1 & t_2^2 & -2ct_2 \\ 1 & t_3^2 & -2ct_3 \end{vmatrix}^2$ ....**(1)**

where $C_1 = \begin{vmatrix} 1 & t_2^2 \\ 1 & t_3^2 \end{vmatrix}$, $C_2 = -\begin{vmatrix} 1 & t_1^2 \\ 1 & t_3^2 \end{vmatrix}$ and $C_3 = \begin{vmatrix} 1 & t_1^2 \\ 1 & t_2^2 \end{vmatrix}$

$\therefore$ $C_1 = t_3^2 - t_2^2$, $C_2 = t_1^2 - t_3^2$ and $C_3 = t_2^2 - t_1^2$

From (1) $= \dfrac{1}{2\left|(t_3^2 - t_2^2)(t_1^2 - t_3^2)(t_2^2 - t_1^2)\right|}\ 4c^2.(t_1 - t_2)^2 (t_2 - t_3)^2 (t_3 - t_1)^2$

$= 2c^2 \left|\dfrac{(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)}{(t_1 + t_2)(t_2 + t_3)(t_3 + t_1)}\right|$

$\therefore$ Required area is, $2c^2 \left|\dfrac{(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)}{(t_1 + t_2)(t_2 + t_3)(t_3 + t_1)}\right|$

Ex. 17 If a circle cuts a rectangular hyperbola $xy = c^2$ in A, B, C, D and the parameters of these four points be t_1, t_2, t_3 and t_4 respectively, then prove that :

(a) $t_1t_2t_3t_4 = 1$

(b) The centre of mean position of the four points bisects the distance between the centres of the two curves.

Sol. **(a)** Let the equation of the hyperbola referred to rectangular asymptotes as axes be $xy = c^2$ or its parametric equation be

$x = ct,\ y = c/t$ .... **(i)**

and that of the circle be

$x^2 + y^2 + 2gx + 2fy + k = 0$ .... **(ii)**

Solving **(i)** and **(ii)**, we get

$$c^2t^2 + \frac{c^2}{t^2} + 2gct + 2f\frac{c}{t} + k = 0$$

or $c^2t^4 + 2gct^3 + kt^2 + 2fct + c^2 = 0$ .... **(iii)**

Above equation being of fourth degree in t gives us the four parameters t_1, t_2, t_3, t_4 of the points of intersection.

$\therefore$ $t_1 + t_2 + t_3 + t_4 = -\dfrac{2gc}{c^2} = -\dfrac{2g}{c}$ .... **(iv)**

$t_1t_2t_3 + t_1t_2t_4 + t_3t_4t_1 + t_3t_4t_2$

$= -\dfrac{2fc}{c^2} = -\dfrac{2f}{c}$ .... **(v)**

$t_1t_2t_3t_4 = \dfrac{c^2}{c^2} = 1$. It proves (a) .... **(vi)**

Dividing (v) by (vi), we get

$$\frac{1}{t_1}+\frac{1}{t_2}+\frac{1}{t_3}+\frac{1}{t_4}=-\frac{2f}{c} \qquad \text{............ (vii)}$$

(b) The centre of mean position of the four points of intersection is

$$\left[\frac{c}{4}(t_1+t_2+t_3+t_4),\ \frac{c}{4}\left(\frac{1}{t_1}+\frac{1}{t_2}+\frac{1}{t_3}+\frac{1}{t_4}\right)\right]=\left[\frac{c}{4}\left(-\frac{2g}{c}\right),\ \frac{c}{4}\left(-\frac{2f}{c}\right)\right],$$ by (iv) and (vii)

$= (-g/2, -f/2)$

Above is clearly the mid-point of (0, 0) and $(-g, -f)$ i.e. the join of the centres of the two curves.

Ex. 18 Prove that the perpendicular focal chords of a rectangular hyperbola are equal.

Sol. Let rectangular hyperbola is $x^2 - y^2 = a^2$

Let equations of PQ and DE are

$$y = mx + c \qquad \text{......(i)}$$

and $$y = m_1x + c_1 \qquad \text{.......(ii)}$$

respectively.

Be any two focal chords of any rectangular hyperbola $x^2 - y^2 = a^2$ through its focus. We have to prove PQ = DE. Since PQ $\perp$ DE.

$\therefore$ $$mm_1 = -1 \qquad \text{......(iii)}$$

Also PQ passes through S $(a\sqrt{2}, 0)$ then from (1),

$$0 = ma\sqrt{2} + c$$

or $$c^2 = 2a^2m^2 \qquad \text{......(iv)}$$

Let (x_1,y_1) and (x_2,y_2) be the co–ordinates of P and Q then

$$(PQ)^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 \qquad \text{......(v)}$$

Since (x_1,y_1) and (x_2,y_2) lie on (i)

$\therefore$ $y_1 = mx_1 + c$ and $y_2 = mx_2 + c$

$\therefore$ $$(y_1 - y_2) = m(x_1 - x_2) \qquad \text{.......(vi)}$$

From (v) and (vi)

$$(PQ)^2 = (x_1 - x_2)^2 (1 + m^2) \qquad \text{.......(vii)}$$

Now solving $y = mx + c$ and $x^2 - y^2 = a^2$ then $x^2 - (mx + c)^2 = a^2$

or $$(m^2 - 1)x^2 + 2mcx + (a^2 + c^2) = 0$$

$\therefore$ $$x_1 + x_2 = -\frac{2mc}{m^2 - 1}$$

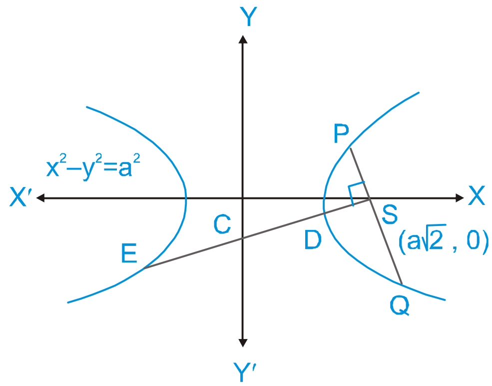
and $\quad x_1x_2 = \dfrac{a^2+c^2}{m^2-1}$

$\Rightarrow \quad (x_1-x_2)^2 = (x_1+x_2)^2 - 4x_1x_2$

$$= \frac{4m^2c^2}{(m^2-1)^2} - \frac{4(a^2+c^2)}{(m^2-1)}$$

$$= \frac{4\{a^2+c^2-a^2m^2\}}{(m^2-1)^2}$$

$$= \frac{4a^2(m^2+1)}{(m^2-1)^2} \quad \{\because c^2 = 2a^2m^2\}$$



From **(vii)**, $\quad (PQ)^2 = 4a^2\left(\dfrac{m^2+1}{m^2-1}\right)^2$

Similarly, $\quad (DE)^2 = 4a^2\left(\dfrac{m_1^2+1}{m_1^2-1}\right)^2$

$$= 4a^2\left(\frac{\left(-\frac{1}{m}\right)^2+1}{\left(-\frac{1}{m}\right)^2-1}\right)^2 = 4a^2\left(\frac{m^2+1}{m^2-1}\right)^2 = (PQ)^2 \qquad (\because mm_1 = -1)$$

Thus $(PQ)^2 = (DE)^2 \quad \Rightarrow \quad PQ = DE.$

Hence perpendicular focal chords of a rectangular hyperbola are equal.

Ex. 19 A ray originating from the point (5, 0) is incident on the hyperbola $9x^2 - 16y^2 = 144$ at the point P with abscissa 8. Find the equation of the reflected ray after first reflection and point P lying infirst quadrant.

Sol. Given hyperbola is $9x^2 - 16y^2 = 144$. This equation can be

rewritten as $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$ ......**(i)**

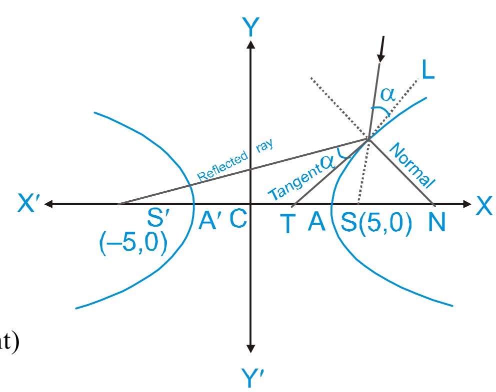
Since x co–ordinate of P is 8. Let y co–ordinate of P is α.

$\because \quad (8,\alpha)$ lies on **(i)**

$\therefore \quad \dfrac{64}{16} - \dfrac{\alpha^2}{9} = 1$

$\Rightarrow \quad \alpha^2 = 27$

$\Rightarrow \quad \alpha = 3\sqrt{3} \qquad (\because$ P lies in first quadrant)



Hence co-ordinate of point P is $(8, 3\sqrt{3})$.

$\because \quad$ Equation of reflected ray passes through P $(8, 3\sqrt{3})$ and S′(–5,0)

$\therefore \quad$ Its equation is $\quad y - 3\sqrt{3} = \dfrac{0-3\sqrt{3}}{-5-8}(x-8)$

or $\quad 13y - 39\sqrt{3} = 3\sqrt{3}\,x - 24\sqrt{3}$

or $\quad 3\sqrt{3}\,x - 13y + 15\sqrt{3} = 0.$

Exercise # 1

[Single Correct Choice Type Questions]

1. The locus of the point of intersection of the lines $\sqrt{3}x - y - 4\sqrt{3}k = 0$ and $\sqrt{3}kx + ky - 4\sqrt{3} = 0$ for different values of k is -

(A) ellipse **(B)** parabola **(C)** circle **(D)** hyperbola

2. The area of the quadrilateral with its vertices at the foci of the conics

$9x^2 - 16y^2 - 18x + 32y - 23 = 0$ and

$25x^2 + 9y^2 - 50x - 18y + 33 = 0$, is

(A) 5/6 **(B)** 8/9 **(C)** 5/3 **(D)** 16/9

3. The vertices of a hyperbola are at (0, 0) and (10,0) and one of its foci is at (18,0). The possible equation of the hyperbola is –

(A) $\frac{x^2}{25} - \frac{y^2}{144} = 1$ **(B)** $\frac{(x-5)^2}{25} - \frac{y^2}{144} = 1$ **(C)** $\frac{x^2}{25} - \frac{(y-5)^2}{144} = 1$ **(D)** $\frac{(x-5)^2}{25} - \frac{(y-5)^2}{144} = 1$

4. The equation of the transverse and conjugate axes of a hyperbola are, respectively, $x + 2y - 3 = 0$ and $2x - y + 4 = 0$, and their respective length are $\sqrt{2}$ and $2/\sqrt{3}$. The equation of the hyperbola is

(A) $\frac{2}{5}(x+2y-3)^2 - \frac{3}{5}(2x-y+4)^2 = 1$ **(B)** $\frac{2}{5}(2x-y+4)^2 - \frac{3}{5}(x+2y-3)^2 = 1$

(C) $2(2x-y+4)^2 - 3(x+2y-3)^2 = 1$ **(D)** $2(x+2y-3)^2 - 3(2x-y+4)^2 = 1$

5. AB is a double ordinate of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ such that ΔAOB (where 'O' is the origin) is an equilateral triangle, then the eccentricity e of the hyperbola satisfies -

(A) $e > \sqrt{3}$ **(B)** $1 < e < \frac{2}{\sqrt{3}}$ **(C)** $e = \frac{2}{\sqrt{3}}$ **(D)** $e > \frac{2}{\sqrt{3}}$

6. The latus rectum of a conic section is the width of the function through the focus. The positive difference between the lengths of the latus rectum of $3y = x^2 + 4x - 9$ and $x^2 + 4y^2 - 6x + 16y = 24$ is

(A) $\frac{1}{2}$ **(B)** 2 **(C)** $\frac{3}{2}$ **(D)** $\frac{5}{2}$

7. The equations to the common tangents to the two hyperbolas $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ are -

(A) $y = \pm x \pm \sqrt{b^2 - a^2}$ **(B)** $y = \pm x \pm (a^2 - b^2)$ **(C)** $y = \pm x \pm \sqrt{a^2 - b^2}$ **(D)** $y = \pm x \pm \sqrt{a^2 + b^2}$

8. The locus of a point, from where the tangents to the rectangular hyperbola $x^2 - y^2 = a^2$ contain an angle of 45°, is

(A) $(x^2 + y^2)^2 + a^2(x^2 - y^2) = 4a^2$ **(B)** $2(x^2 + y^2)^2 + 4a^2(x^2 - y^2) = 4a^2$

(C) $(x^2 + y^2)^2 + 4a^2(x^2 - y^2) = 4a^2$ **(D)** $(x^2 + y^2)^2 + a^2(x^2 - y^2) = a^4$

9. The equation of the common tangent to the parabola $y^2 = 8x$ and the hyperbola $3x^2 - y^2 = 3$ is -

(A) $2x \pm y + 1 = 0$ **(B)** $x \pm y + 1 = 0$ **(C)** $x \pm 2y + 1 = 0$ **(D)** $x \pm y + 2 = 0$

10. Locus of the middle points of the parallel chords with gradient m of the rectangular hyperbola $xy = c^2$ is

(A) $y + mx = 0$ **(B)** $y - mx = 0$ **(C)** $my - x = 0$ **(D)** $my + x = 0$

11. The asymptotes of the hyperbola $xy - 3x - 2y = 0$ are-

(A) $x - 2 = 0$ and $y - 3 = 0$ **(B)** $x - 3 = 0$ and $y - 2 = 0$

(C) $x + 2 = 0$ and $y + 3 = 0$ **(D)** $x + 3 = 0$ and $y + 2 = 0$

12. The locus of the foot of the perpendicular from the centre of the hyperbola $xy = c^2$ on a variable tangent is

(A) $(x^2 - y^2)^2 = 4c^2xy$ **(B)** $(x^2 + y^2)^2 = 2c^2xy$

(C) $(x^2 + y^2) = 4x^2xy$ **(D)** $(x^2 + y^2)^2 = 4c^2xy$

13. If the normal to the rectangular hyperbola $xy = c^2$ at the point 't' meets the curve again at 't_1' then t^3t_1 has the value equal to -

(A) 1 **(B)** –1 **(C)** 0 **(D)** none

14. For each positive integer *n*, consider the point P with abscissa *n* on the curve $y^2 - x^2 = 1$. If d_n represents the shortest distance from the point P to the line $y = x$ then $\lim\limits_{n\to\infty}(n \cdot d_n)$ has the value equal to

(A) $\frac{1}{2\sqrt{2}}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{1}{\sqrt{2}}$ **(D)** 0

15. Let P $(a\sec\theta, b\tan\theta)$ and Q $(a\sec\phi, b\tan\phi)$, where $\theta + \phi = \frac{\pi}{2}$, be two points on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If (h,k) is the point of intersection of the normals at P & Q, then k is equal to -

(A) $\frac{a^2 + b^2}{a}$ **(B)** $-\left(\frac{a^2 + b^2}{a}\right)$ **(C)** $\frac{a^2 + b^2}{b}$ **(D)** $-\left(\frac{a^2 + b^2}{b}\right)$

16. Let two points P and Q lie on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, whose center C be such that CP is perpendicular to CQ, $a < b$. Then the value of $\frac{1}{CP^2} + \frac{1}{CQ^2}$ is

(A) $\frac{b^2 - c^2}{2ab}$ **(B)** $\frac{1}{a^2} + \frac{1}{b^2}$ **(C)** $\frac{2ab}{b^2 - a^2}$ **(D)** $\frac{1}{a^2} - \frac{1}{b^2}$

17. If the normal to the rectangular hyperbola $xy = c^2$ at the point 't' meets the curve again at 't_1' then $t^3\,t_1$ has the value equal to

(A) 1 **(B)** – 1 **(C)** 0 **(D)** none

18. The exhaustive set of values of a^2 such that there exists a tangent to the ellipse $x^2 + a^2y^2 = a^2$ and the portion of the tangent intercepted by the hyperbola $a^2x^2 - y^2 = 1$ subtends a right angle at the center of the curves is

(A) $\left[\frac{\sqrt{5}+1}{2}, 2\right]$ **(B)** $(1, 2]$ **(C)** $\left[\frac{\sqrt{5}-1}{2}, 1\right)$ **(D)** $\left[\frac{\sqrt{5}-1}{2}, 1\right) \cup \left(1, \frac{\sqrt{5}+1}{2}\right]$

19. P is a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, N is the foot of the perpendicular from P on the transverse axis. The tangent to the hyperbola at P meets the transverse axis at T . If O is the centre of the hyperbola, the OT. ON is equal to :

(A) e^2 **(B)** a^2 **(C)** b^2 **(D)** b^2/a^2

20. The equation to the chord joining two points (x_1, y_1) and (x_2, y_2) on the rectangular hyperbola $xy = c^2$ is

(A) $\frac{x}{x_1 + x_2} + \frac{y}{y_1 + y_2} = 1$ **(B)** $\frac{x}{x_1 - x_2} + \frac{y}{y_1 - y_2} = 1$

(C) $\frac{x}{y_1 + y_2} + \frac{y}{x_1 + x_2} = 1$ **(D)** $\frac{x}{y_1 - y_2} + \frac{y}{x_1 - x_2} = 1$

Exercise # 2

Part # I [Multiple Correct Choice Type Questions]

1. Equations of a common tangent to the two hyperbolas $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ & $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$ is :

(A) $y=x+\sqrt{a^2-b^2}$ **(B)** $y=x-\sqrt{a^2-b^2}$

(C) $y=-x+\sqrt{a^2-b^2}$ **(D)** $-x-\sqrt{a^2-b^2}$

2. The equation $\left|\sqrt{x^2+(y-1)^2}-\sqrt{x^2+(y+1)^2}\right|=K$ will represent a hyperbola for

(A) $K \in (0, 2)$ **(B)** $K \in (-2, 1)$ **(C)** $K \in (1, \infty)$ **(D)** $K \in (0, \infty)$

3. The differential equation $\frac{dx}{dy}=\frac{3y}{2x}$ represents a family of hyperbolas (except when it represents a pair of lines) with eccentricity :

(A) $\sqrt{\frac{3}{5}}$ **(B)** $\sqrt{\frac{5}{3}}$ **(C)** $\sqrt{\frac{2}{5}}$ **(D)** $\sqrt{\frac{5}{2}}$

4. Let an incident ray $L_1 = 0$ gets reflected at point A(–2, 3) on hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ & passes through focus S(2, 0), then -

(A) equation of incident ray is $x + 2 = 0$ **(B)** equation of reflected ray is $3x + 4y = 6$

(C) eccentricity, $e = 2$ **(D)** length of latus rectum = 6

5. Circles are drawn on chords of the rectangular hyperbola $xy = c^2$ parallel to the line $y = x$ as diameters. All such circles pass through two fixed points whose co-ordinates are :

(A) (c, c) **(B)** $(c, -c)$ **(C)** $(-c, c)$ **(D)** $(-c, -c)$

6. If (5, 12) and (24, 7) are the foci of a conic passing through the origin, then the eccentricity of the conic is

(A) $\sqrt{386}/12$ **(B)** $\sqrt{386}/13$ **(C)** $\sqrt{386}/25$ **(D)** $\sqrt{386}/38$

7. For which of the hyperbolas, can we have more than one pair of perpendicular tangents ?

(A) $\frac{x^2}{4}-\frac{y^2}{9}=1$ **(B)** $\frac{x^2}{4}-\frac{y^2}{9}=-1$ **(C)** $x^2-y^2=4$ **(D)** $xy=44$

8. If θ is the angle between the asymptotes of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ with eccentricity e, then $\sec\frac{\theta}{2}$ can be -

(A) e **(B)** e/2 **(C)** e/3 **(D)** $\frac{e}{\sqrt{e^2-1}}$

9. Solutions of the differential equation $(1-x^2)\frac{dy}{dx}+xy=ax$ where $a \in R$, is

(A) a conic which is an ellipse or a hyperbola with principal axes parallel to coordinates axes.

(B) centre of the conic is (0, a)

(C) length of one of the principal axes is 1.

(D) length of one of the principal axes is equal to 2.

10. Circles are drawn on the chords of the rectangular hyperbola $xy = 4$ parallel to the line $y = x$ as diameters. All such circles pass through two fixed points whose coordinates are

(A) $(2, 2)$ **(B)** $(2, -2)$ **(C)** $(-2, 2)$ **(D)** $(-2, -2)$

11. For the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, let n be the number of points on the plane through which perpendicular tangents are drawn.

(A) If $n = 1$, then $e = \sqrt{2}$ **(B)** If $n > 1$, then $0 < e < \sqrt{2}$

(C) If $n = 0$, then $e > \sqrt{2}$ **(D)** None of these

12. Equation $(2 + \lambda)x^2 - 2\lambda xy + (\lambda - 1)y^2 - 4x - 2 = 0$ represents a hyperbola if -

(A) $\lambda = 4$ **(B)** $\lambda = 1$ **(C)** $\lambda = 4/3$ **(D)** $\lambda = -1$

13. In which of the following cases maximum number of normals can be drawn from a point P lying in the same plane

(A) circle **(B)** parabola **(C)** ellipse **(D)** hyperbola

14. If the normal at P to the rectangular hyperbola $x^2 - y^2 = 4$ meets the axes at G and g and C is the center of the hyperbola, then

(A) $PG = PC$ **(B)** $Pg = PC$ **(C)** $PG = Pg$ **(D)** $Gg = 2PC$

15. If θ is eliminated from the equations

$a \sec \theta - x \tan \theta = y$ and $b \sec \theta + y \tan \theta = x$ (a and b are constant)

then the eliminant denotes the equation of

(A) the director circle of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ **(B)** auxiliary circle of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(C) Director circle of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ **(D)** Director circle of the circle $x^2 + y^2 = \frac{a^2 + b^2}{2}$.

Part # II [Assertion & Reason Type Questions]

These questions contains, Statement I (assertion) and Statement II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.

(C) Statement-I is true, Statement-II is false.

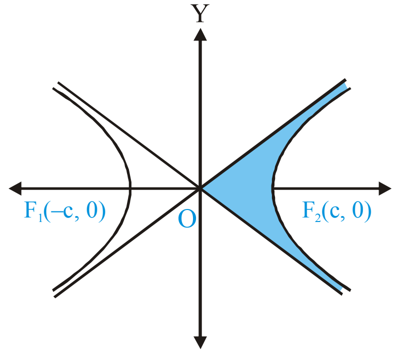
(D) Statement-I is false, Statement-II is true.

1. **Statement-I :** A bullet is fired and hit a target. An observer in the same plane heard two sounds the crack of the riffle and the thud of the ball striking the target at the same instant, then locus of the observer is hyperbola where velocity of sound is smaller than velocity of bullet.

Statement-II : If difference of distances of a point 'P' from the two fixed points is constant and less than the distance between the fixed points then locus of 'P' is a hyperbola.

2. **Statement-I :** If a point (x_1, y_1) lies in the shaded region $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, show in the figure, then $\frac{x_1^2}{a^2}-\frac{y_1^2}{b^2}<0$.

Statement-II : If $P(x_1, y_1)$ lies outside the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, then $\frac{x_1^2}{a^2}-\frac{y_1^2}{b^2}<1$.



3. **Statement-I :** If a circle S = 0 intersects a hyperbola xy = 4 at four points. Three of them are (2, 2)(4, 1) and (6, 2/3) then co-ordinates of the fourth point are (1/4, 16).

Statement-II : If a circle S = 0 intersects a hyperbola $xy = c^2$ at t_1, t_2, t_3, t_4 then $t_1 . t_2 . t_3 . t_4 = 1$.

4. **Statement-I :** Given the base BC of the triangle and the radius ratio of the excircles opposite to the angles B and C. Then the locus of the vertex A is a hyperbola.

Statement-II : |S"P – SP| = 2a, where S and S" are the two foci, 2a is the length of the transverse axis, and P is any point on the hyperbola.

Exercise # 3 Part # I [Matrix Match Type Questions]

Following question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled as A, B, C and D while the statements in Column-II are labelled as p, q, r and s. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II.

1. If e_1 and e_2 are the roots of the equation $x^2 - ax + 2 = 0$, then match the following

	Column - I		Column - II
(A)	If e_1 and e_2 are the eccentricities of ellipse and hyperbola, respectively, then the values of a are	(p)	6
(B)	If both e_1 and e_2 are the eccentricities of the hyperbolas, then the values of a are	(q)	5/2
(C)	If e_1 and e_2 are the eccentricities of hyperbola and conjugate hyperbola, then the values of a are	(r)	$2\sqrt{2}$
(D)	If e_1 is the eccentricities of the hyperbola for which there exists infinite points from which perpendicular tangents can be drawn and e_2 is the eccentricity of the hyperbola in which no such points exist, then the values of a are	(s)	5

2. Consider the hyperbola $9x^2 - 16y^2 - 36x + 96y + 36 = 0$.

	Column - I		Column - II
(A)	If directrices of the hyperbola are $y = k_1$ & $y = k_2$ then $k_1 + k_2$ is equal to	(p)	16
(B)	If foci of hyperbola are (a, b) & (a, c) then a + b + c is equal to	(q)	10
(C)	Product of the perpendiculars drawn from the foci upon its any tangent is	(r)	6
(D)	Distance between foci of the hyperbola is	(s)	8

3. Match the properties given in column-I with the corresponding curves given in the column-II.

	Column-I		Column-II
(A)	The curve such that product of the distances of any of its tangent from two given points is constant, can be	(p)	Circle
(B)	A curve for which the length of the subnormal at any of its point is equal to 2 and the curve passes through (1, 2), can be	(q)	Parabola
(C)	A curve passes through (1, 4) and is such that the segment joining any point P on the curve and the point of intersection of the normal at P with the x-axis is bisected by the y-axis. The curve can be	(r)	Ellipse
		(s)	Hyperbola
(D)	A curve passes through (1, 2) is such that the length of the normal at any of its point is equal to 2. The curve can be		

4. **Column - I** | **Column - II**

	Column - I		Column - II
(A)	A tangent drawn to hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at $P\left(\frac{\pi}{6}\right)$ forms a triangle of area $3a^2$ square units, with coordinate axes, then the square of its eccentricity is equal to	**(p)**	17
(B)	If the eccentricity of the hyperbola $x^2 - y^2 \sec^2\theta = 5$ is $\sqrt{3}$ times the eccentricity of the ellipse $x^2\sec^2\theta + y^2 = 25$ then smallest positive value of θ is $\frac{6\pi}{p}$, value of 'p' is	**(q)**	8
(C)	For the hyperbola $\frac{x^2}{3}-y^2=3$, angle between its asymptotes is $\frac{\ell\pi}{24}$ then value of 'ℓ' is	**(r)**	16
(D)	For the hyperbola xy = 8 any tangent of it at P meets co-ordinate axes at Q and R then area of triangle CQR where 'c' is centre of the hyperbola is	**(s)**	24

5.

	Column-I		Column-II
(A)	For an ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ with vertices A and A', tangent drawn at the point P in the first quadrant meets the y-axis in Q and the chord A'P meets the y-axis in M. If 'O' is the origin then $OQ^2 - MQ^2$ equals to	**(p)**	2
(B)	If the product of the perpendicular distances from any point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ of eccentricity $e=\sqrt{3}$ from its asymptotes is equal to 6, then the length of the transverse axis of the hyperbola is	**(q)**	3
(C)	The locus of the point of intersection of the lines $\sqrt{3}x - y - 4\sqrt{3}t = 0$ and $\sqrt{3}tx + ty - 4\sqrt{3} = 0$ (where t is a parameter) is a hyperbola whose eccentricity is	**(r)**	4
(D)	If F_1 & F_2 are the feet of the perpendiculars from the foci S_1 & S_2 of an ellipse $\frac{x^2}{5}+\frac{y^2}{3}=1$ on the tangent at any point P on the ellipse, then $(S_1F_1).(S_2F_2)$ is equal to	**(s)**	6

Part # II [Comprehension Type Questions]

Comprehension # 1

The graph of the conic $x^2 - (y-1)^2 = 1$ has one tangent line with positive slope that passes through the origin. the point of tangency being (a, b). Then

1. The value of $\sin^{-1}\left(\frac{a}{b}\right)$ is

(A) $\frac{5\pi}{12}$ **(B)** $\frac{\pi}{6}$ **(C)** $\frac{\pi}{3}$ **(D)** $\frac{\pi}{4}$

2. Length of the latus rectum of the conic is

(A) 1 **(B)** $\sqrt{2}$ **(C)** 2 **(D)** none

3. Eccentricity of the conic is

(A) $\frac{4}{3}$ **(B)** $\sqrt{3}$ **(C)** 2 **(D)** none

Comprehension # 2

If we rotate the axes of the rectangular hyperbola $x^2 - y^2 = a^2$ through an angle $\pi/4$ in the clockwise direction then the equation $x^2 - y^2 = a^2$ reduces to $xy = \frac{a^2}{2} = \left(\frac{a}{\sqrt{2}}\right)^2 = c^2$ (say). Since $x = ct$, $y = \frac{c}{t}$ satisfies $xy = c^2$.

$\therefore$ $(x, y) = \left(ct, \frac{c}{t}\right)$ $(t \neq c)$ is called a 't' point on the rectangular hyperbola.

On the basis of above information, answer the following questions :

1. If t_1 and t_2 are the roots of the equation $x^2 - 4x + 2 = 0$, then the point of intersection of tangents at 't_1' and 't_2' on $xy = c^2$ is -

(A) $\left(\frac{c}{2}, 2c\right)$ **(B)** $\left(2c, \frac{c}{2}\right)$ **(C)** $\left(\frac{c}{2}, c\right)$ **(D)** $\left(c, \frac{c}{2}\right)$

2. If e_1 and e_2 are the eccentricities of the hyperbolas $xy = 9$ and $x^2 - y^2 = 25$, then (e_1, e_2) lie on a circle C_1 with centre origin then the $(\text{radius})^2$ of the director circle of C_1 is -

(A) 2 **(B)** 4 **(C)** 8 **(D)** 16

3. If the normal at the point 't_1' to the rectangular hyperbola $xy = c^2$ meets it again at the point 't_2' then the value of t_1t_2 is -

(A) $-t_1^{-1}$ **(B)** $-t_1^{-2}$ **(C)** $-t_1^{-3}$ **(D)** $-t_1^{-4}$

Comprehension # 3

The vertices ΔABC lie on a rectangular hyperbola such that the orthocenter of the triangle is (3, 2) and the asymptotes of the rectangular hyperbola are parallel to the coordinate axes. The two perpendicular tangents of the hyperbola intersect at the point (1, 1).

1. The equation of the asymptotes is

(A) $xy - 1 = x - y$ **(B)** $xy + 1 = x + y$ **(C)** $2xy = x + y$ **(D)** none of these

2. The equation of the rectangular hyperbola is

(A) $xy = 2x + y - 2$ **(B)** $2xy = x + 2y + 5$ **(C)** $xy = x + y + 1$ **(D)** none of these

3. The number of real tangents that can be drawn from the point (1, 1) to the rectangular hyperbola is

(A) 4 **(B)** 0 **(C)** 3 **(D)** 2

Exercise # 4

[Subjective Type Questions]

1. The hyperbola $x^2/a^2 - y^2/b^2 = 1$ (a,b > 0) passes through the point of intersection of the lines, $7x + 13y - 87 = 0$ & $5x - 8y + 7 = 0$ and the latus rectum is $32\sqrt{2}/5$. Find 'a' & 'b'.

2. If the tangent at the point (h, k) to the hyperbola $x^2/a^2 - y^2/b^2 = 1$ cuts the auxiliary circle in points whose ordinates are y_1 and y_2 then prove that $1/y_1 + 1/y_2 = 2/k$.

3. Find the centre, the foci, the directrices, the length of the latus rectum, the length & the equations of the axes $16x^2 - 9y^2 + 32x + 36y - 164 = 0$.

4. If a rectangular hyperbola have the equation, $xy = c^2$, prove that the locus of the middle points of the chords of constant length 2 d is $(x^2 + y^2)(xy - c^2) = d^2xy$.

5. Find the eccentricity of the conic represented by $x^2 - y^2 - 4x + 4y + 16 = 0$.

6. If one axis of varying central conic (hyperbola) is fixed in magnitude and position, prove that the locus of the point of contact of a tangent drawn to it from a fixed point on the other axis is a parabola.

7. If the normal at a point P to the hyperbola $x^2/a^2 - y^2/b^2 = 1$ meets the x – axis at G, show that SG = e. SP, S being the focus of the hyperbola.

8. Prove that the locus of the middle point of the chord of contact of tangents from any point of the circle $x^2 + y^2 = r^2$ to the hyperbola $x^2/a^2 - y^2/b^2 = 1$ is given by the equation $(x^2/a^2 - y^2/b^2)^2 = (x^2 + y^2)/r^2$.

9. Show that the locus of the middle points of normal chords of the rectangular hyperbola $x^2 - y^2 = a^2$ is $(y^2 - x^2)^3 = 4a^2x^2y^2$.

10. A tangent to the parabola $x^2 = 4ay$ meets the hyperbola $xy = k^2$ in two points P & Q . Prove that the middle point of PQ lies on a parabola .

11. Find the asymptotes of the hyperbola $2x^2 - 3xy - 2y^2 + 3x - y + 8 = 0$. Also find the equation to the conjugate hyperbola & the equation of the principal axes of the curve.

12. Find the angle between the rectangular hyperbolas $(y - mx)(mx + x) = a^2$ and $(m^2 - 1)(y^2 - x^2) + 4mxy = b^2$.

13. Given the base of a triangle and the ratio of the tangent of half the base angles. Show that the vertex moves on a hyperbola whose foci are the extremities of the base.

14. If the normal to the hyperbola $x^2/a^2 - y^2/b^2 = 1$ at the point P meets the transverse axis in G & the conjugate axis in g & CF be perpendicular to the normal from the centre C , then prove that $|PF . PG| = b^2$ & $PF . Pg = a^2$ where a & b are the semi transverse & semi-conjugate axes of the hyperbola.

15. Tangents are drawn from the point (α, β) to the hyperbola $3x^2 - 2y^2 = 6$ and are inclined at angles θ & ϕ to the x –axis. If $\tan\theta . \tan\phi = 2$, prove that $\beta^2 = 2\alpha^2 - 7$.

Exercise # 5

Part # I [Previous Year Questions] [AIEEE/JEE-MAIN]

1. The latus rectum of the hyperbola $16x^2 - 9y^2 = 144$ is- **[AIEEE-2002]**

(1) 16/3 **(2)** 32/3 **(3)** 8/3 **(4)** 4/3

2. The foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide. Then the value of b^2 is- **[AIEEE-2003]**

(1) 9 **(2)** 1 **(3)** 5 **(4)** 7

3. The locus of a point $P(\alpha, \beta)$ moving under the condition that the line $y = \alpha x + \beta$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is- **[AIEEE-2005]**

(1) a hyperbola **(2)** a parabola **(3)** a circle **(4)** an ellipse

4. For the hyperbola $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$, which of the following remains constant when α varies ? **[AIEEE-2007, IIT-2003]**

(1) Abscissae of vertices **(2)** Abscissae of foci **(3)** Eccentricity **(4)** Directrix

5. The equation of the hyperbola whose foci are (–2,0) and (2, 0) and eccentricity is 2 is given by : **[AIEEE-2011]**

(1) $-3x^2 + y^2 = 3$ **(2)** $x^2 - 3y^2 = 3$ **(3)** $3x^2 - y^2 = 3$ **(4)** $-x^2 + 3y^2 = 3$

6. The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half of the distance between its foci, is : **[JEE Main 2016]**

(1) $\frac{4}{\sqrt{3}}$ **(2)** $\frac{2}{\sqrt{3}}$ **(3)** $\sqrt{3}$ **(4)** $\frac{4}{3}$

Part # II [Previous Year Questions][IIT-JEE ADVANCED]

1. The equation of the common tangent to the curve $y^2 = 8x$ and $xy = -1$ is - **[JEE 2002 Screening]**

(A) $3y = 9x + 2$ **(B)** $y = 2x + 1$ **(C)** $2y = x + 8$ **(D)** $y = x + 2$

2. For hyperbola $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$ which of the following remains constant with change in α - **[JEE 2003 Screening]**

(A) abscissae of vertices **(B)** abscissae of foci **(C)** eccentricity **(D)** directrix

3. The point of contact of the line $2x + \sqrt{6}\ y = 2$ and the hyperbola $x^2 - 2y^2 = 4$ is - **[JEE 2004 Screening]**

(A) $(4, -\sqrt{6})$ **(B)** $(\sqrt{6}, 1)$ **(C)** $\left(\frac{1}{2}, \frac{1}{\sqrt{6}}\right)$ **(D)** $\left(\frac{1}{6}, \frac{3}{2}\right)$

4. Tangents are drawn from any point on the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ to the circle $x^2 + y^2 = 9$. Find the locus of mid-point of the chord of contact. **[JEE 2005 Mains]**

5. If a hyperbola passes through the focus of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ and its transverse and conjugate axis coincides with the major and minor axis of the ellipse and product of their eccentricities is 1, then - **[JEE 2006]**

(A) equation of hyperbola $\frac{x^2}{9}-\frac{y^2}{16}=1$ **(B)** equation of hyperbola $\frac{x^2}{9}-\frac{y^2}{25}=1$

(C) focus of hyperbola (5, 0) **(D)** focus of hyperbola $(5\sqrt{3}, 3)$

6. A hyperbola, having the transverse axis of length 2sinθ, is confocal with the ellipse $3x^2 + 4y^2 = 12$. Then its equation is - **[JEE 2007]**

(A) $x^2\operatorname{cosec}^2\theta - y^2\sec^2\theta = 1$ **(B)** $x^2\sec^2\theta - y^2\operatorname{cosec}^2\theta = 1$

(C) $x^2\sin^2\theta - y^2\cos^2\theta = 1$ **(D)** $x^2\cos^2\theta - y^2\sin^2\theta = 1$

7. Match the column - **[2007]**

	Column I		Column II
(A)	Two intersecting circles	(p)	have a common tangent
(B)	Two mutually external circles	(q)	have a common normal
(C)	Two circles, one strictly inside the other	(r)	do not have a common tangent
(D)	Two branches of a hyperbola	(s)	do not have a common normal

8. Let a and b be non-zero real numbers. Then, the equation $(ax^2 + by^2 + c)(x^2 - 5xy + 6y^2) = 0$ represents -

(A) four straight lines, when c = 0 and a, b are of the same sign

(B) two straight lines and a circle, when a = b, and c is of sign opposite to that of a

(C) two straight lines and a hyperbola, when a and b are of the same sign and c is of sign opposite to that of a

(D) a circle and an ellipse, when a and b are of the same sign and c is of sign opposite to that of a **[JEE 2008]**

9. Consider a branch of the hyperbola $x^2 - 2y^2 - 2\sqrt{2}x - 4\sqrt{2}y - 6 = 0$ with vertex at the point A. Let B be one of the end points of its latus rectum. If C is the focus of the hyperbola nearest to the point A, then the area of the triangle ABC is - **[JEE 2008]**

(A) $1-\sqrt{\frac{2}{3}}$ **(B)** $\sqrt{\frac{3}{2}}-1$ **(C)** $1+\sqrt{\frac{2}{3}}$ **(D)** $\sqrt{\frac{3}{2}}+1$

10. An ellipse intersects the hyperbola $2x^2 - 2y^2 = 1$ orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the ellipse are along the coordinate axes, then :- **[JEE 2009]**

(A) equation of ellipse is $x^2 + 2y^2 = 2$ **(B)** the foci of ellipse are $(\pm1, 0)$

(C) equation of ellipse is $x^2 + 2y^2 = 4$ **(D)** the foci of ellipse are $\left(\pm\sqrt{2}, 0\right)$

Paragraph for Question 11 and 12 **[JEE 2010]**

The circle $x^2 + y^2 - 8x = 0$ and hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ intersect at the points A and B.

11. Equation of a common tangent with positive slope to the circle as well as to the hyperbola is -

(A) $2x - \sqrt{5}y - 20 = 0$ **(B)** $2x - \sqrt{5}y + 4 = 0$ **(C)** $3x - 4y + 8 = 0$ **(D)** $4x - 3y + 4 = 0$

12. Equation of the circle with AB as its diameter is -

(A) $x^2 + y^2 - 12x + 24 = 0$ **(B)** $x^2 + y^2 + 12x + 24 = 0$

(C) $x^2 + y^2 + 24x - 12 = 0$ **(D)** $x^2 + y^2 - 24x - 12 = 0$

13. The line $2x + y = 1$ is tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If this line passes through the point of intersection of the nearest directrix and the x-axis, then the eccentricity of the hyperbola is **[JEE 2010]**

14. Let the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be reciprocal to that of the ellipse $x^2 + 4y^2 = 4$. If the hyperbola passes through a focus of the ellipse, then - **[JEE 2011]**

(A) the equation of the hyperbola is $\frac{x^2}{3} - \frac{y^2}{2} = 1$ **(B)** a focus of the hyperbola is (2,0)

(C) the eccentricity of the hyperbola is $\sqrt{\frac{5}{3}}$ **(D)** the equation of the hyperbola is $x^2 - 3y^2 = 3$

15. Let P(6, 3) be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal at the point P intersects the x-axis at (9, 0), then the eccentricity of the hyperbola is - **[JEE 2011]**

(A) $\sqrt{\frac{5}{2}}$ **(B)** $\sqrt{\frac{3}{2}}$ **(C)** $\sqrt{2}$ **(D)** $\sqrt{3}$

16. Tangents are drawn to the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$, parallel to the straight line $2x - y = 1$. The points of contact of the tangents on the hyperbola are **[JEE 2012]**

(A) $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ **(B)** $\left(-\frac{9}{2\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ **(C)** $\left(3\sqrt{3}, -2\sqrt{2}\right)$ **(D)** $\left(-3\sqrt{3}, 2\sqrt{2}\right)$

17. Consider the hyperbola $H : x^2 - y^2 = 1$ and a circle S with centre $N(x_2, 0)$. Suppose that H and S touch each other at a point $P(x_1, y_1)$ with $x_1 > 1$ and $y_1 > 0$. The common tangent to H and S at P intersects the x- axis at point M. If (l, m) is the centroid of the triangle ΔPMN, then the correct expression(s) is (are) **[JEE Ad. 2015]**

(A) $\frac{d\ell}{dx_1} = 1 - \frac{1}{3x_1^2}$ for $x_1 > 1$ **(B)** $\frac{dm}{dx_1} = \frac{x}{3\left(\sqrt{x_1^2 - 1}\right)}$ for $x_1 > 1$

(C) $\frac{d\ell}{dx_1} = 1 + \frac{1}{3x_1^2}$ for $x_1 > 1$ **(D)** $\frac{dm}{dy_1} = \frac{1}{3}$ for $y_1 > 0$

MOCK TEST

SECTION - I : STRAIGHT OBJECTIVE TYPE

1. From a point P(1, 2) pair of tangent's are drawn to a hyperbola 'H' where the two tangents touch different arms of hyperbola. Equation of asymptotes of hyperbola H are $\sqrt{3}\,x - y + 5 = 0$ & $\sqrt{3}\,x + y - 1 = 0$ then eccentricity of 'H' is

(A) 2 **(B)** $\frac{2}{\sqrt{3}}$ **(C)** $\sqrt{2}$ **(D)** $\sqrt{3}$

2. The locus of the mid points of the chords passing through a fixed point (α, β) of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is :

(A) a circle with centre $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$ **(B)** an ellipse with centre $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$

(C) a hyperbola with centre $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$ **(D)** straight line passing through $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$

3. 'C' be a curve which is locus of point of intersection of lines x = 2 + m and my = 4 – m. A circle $S \equiv (x-2)^2 + (y+1)^2 = 25$ intesects the curve C at four points P, Q, R and S. If O is centre of the curve 'C', then $OP^2 + OQ^2 + OR^2 + OS^2$ is

(A) 50 **(B)** 100 **(C)** 25 **(D)** 25/2

4. If two conics $a_1x^2 + 2h_1xy + b_1y^2 = c_1$ and $a_2x^2 + 2h_2xy + b_2y^2 = c_2$ intersect in four concyclic points, then

(A) $(a_1 - b_1)h_2 = (a_2 - b_2)h_1$ **(B)** $(a_1 - b_1)h_1 = (a_2 - b_2)h_2$

(C) $(a_1 + b_1)h_2 = (a_2 + b_2)h_1$ **(D)** $(a_1 + b_1)h_1 = (a_2 + b_2)h_2$

5. The equation to the chord joining two points (x_1, y_1) and (x_2, y_2) on the rectangular hyperbola $xy = c^2$ is :

(A) $\frac{x}{x_1 + x_2} + \frac{y}{y_1 + y_2} = 1$ **(B)** $\frac{x}{x_1 - x_2} + \frac{y}{y_1 - y_2} = 1$

(C) $\frac{x}{y_1 + y_2} + \frac{y}{x_1 + x_2} = 1$ **(D)** $\frac{x}{y_1 - y_2} + \frac{y}{x_1 - x_2} = 1$

6. The chord PQ of the rectangular hyperbola $xy = a^2$ meets the x-axis at A; C is the mid point of PQ & 'O' is the origin. Then the Δ ACO is :

(A) equilateral **(B)** isosceles **(C)** right angled **(D)** right isosceles.

7. If radii of director circles of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x^2}{a^2} - \frac{y^2}{(b)^2} = 1$ are 2r and r respectively and e_e and e_h be the eccentricities of the ellipse and the hyperbola respectively then

(A) $2e_h^2 - e_e^2 = 6$ **(B)** $e_e^2 - 4e_h^2 = 6$ **(C)** $4e_h^2 - e_e^2 = 6$ **(D)** none of these

8. If AB is a double ordinate of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ such that ΔOAB (O is the origin) is an equilateral triangle, then the eccentricity 'e' of the hyperbola satisfies

(A) $e > \sqrt{3}$ **(B)** $1 < e < 2\frac{2}{\sqrt{3}}$ **(C)** $e = \frac{2}{\sqrt{3}}$ **(D)** $e > \frac{2}{\sqrt{3}}$

9. **S_1 :** If $x = 3$ & $y = 2$ are the equations of asymptotes of a hyperbola and hyperbola passes through the point (4, 6) then length of its latus rectum is $4\sqrt{2}$.

S_2 : Two concentric rectangular hyperbolas whose axes meet at an angle $\pi/4$, cut each other at an angle $\pi/2$.

S_3 : Distance between directrices of hyperbola $xy = 16$ is 4

S_4 : If line joining the points $A(x_1, 0)$ & $B(0, y_1)$ is tangent to the hyperbola $xy = c^2$ then point of contact is $\left(\frac{x_1}{2}, \frac{y_1}{2}\right)$.

(A) TTFT **(B)** TFTT **(C)** FFTT **(D)** FFTF

10. If $x \cos \alpha + y \sin \alpha = p$, a variable chord of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{2a^2} = 1$ subtends a right angle at the centre of the hyperbola, then the chords touch a fixed circle whose radius is equal to

(A) $\sqrt{2}$ a **(B)** $\sqrt{3}$ a **(C)** 2 a **(D)** $\sqrt{5}$ a

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

11. If foci of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ concide with the focii of $\frac{x^2}{25} + \frac{y^2}{9} = 1$ and eccentricity of the hyperbola is 2, then

(A) $a^2 + b^2 = 16$ **(B)** there is no director circle to the hyperbola

(C) centre of the director circle is (0, 0) **(D)** length of latus ractum of the hyperbola = 12

12. A point moves such that the sum of the squares of its distances from the two sides of length 'a' of a rectangle is twice the sum of the squares of its distances from the other two sides of length 'b'. The locus of the point can be :

(A) a circle **(B)** an ellipse **(C)** a hyperbola **(D)** a pair of lines

13. The lines $y = mx \pm \sqrt{a^2m^2 - b^2}$, $m > 0$ touches the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point's whose eccentric angle is

(A) $\sin^{-1}\left(\frac{b}{ma}\right)$ **(B)** $\pi + \sin^{-1}\left(\frac{b}{ma}\right)$ **(C)** $2\pi + \sin^{-1}\left(\frac{b}{ma}\right)$ **(D)** $-\sin^{-1}\left(\frac{b}{ma}\right)$

14. Which of the following equations in parametric form can represent a hyperbolic profile, where 't' is a parameter.

(A) $x = \frac{a}{2}\left(t + \frac{1}{t}\right)$ & $y = \frac{b}{2}\left(t - \frac{1}{t}\right)$ **(B)** $\frac{tx}{a} - \frac{y}{b} + t = 0$ & $\frac{x}{a} + \frac{ty}{b} - 1 = 0$

(C) $x = e^t + e^{-t}$ & $y = e^t - e^{-t}$ **(D)** $x^2 - 6 = 2 \cos t$ & $y^2 + 2 = 4 \cos^2 \frac{t}{2}$

15. If (a sec θ, b tan θ) and (a secϕ, b tan ϕ) are the ends of a focal chord of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then $\tan\frac{\theta}{2}\ \tan\frac{\varphi}{2}$ equals to

(A) $\frac{e-1}{e+1}$ (B) $\frac{1-e}{1+e}$ (C) $\frac{1+e}{1-e}$ (D) $\frac{e+1}{e-1}$

SECTION - III : ASSERTION AND REASON TYPE

16. **Statement -I :** A bullet is fired and hit a target. An observer in the same plane heard two sounds the crack of the rifle and the thud of the bullet striking the target at the same instant, then locus of the observer is hyperbola where velocity of sound is smallar than velocity of bullet.

Statement -II : If difference of distances of a point 'P' from the two fixed points is constant and less than the distance between the fixed points then locus of 'P' is a hyperbola.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

17. **Statement -I :** With respect to a hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$ pependicular are drawn from a point (5, 0) on the lines $3y \pm 4x = 0$, then their feet lie on circle $x^2 + y^2 = 16$.

Statement -II : If from any foci of a hyperbola perpendicular are drawn on the asymptotes of the hyperbola then their feet lie on auxiliary circle.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

18. **Statement-I :** Consider two hyperbolas $S \equiv 2x^2 - 4y^2 - 8 = 0$ and $S' \equiv 2x^2 - 4y^2 + 8 = 0$. S and S' are conjugate of each other.

Statement-II : Length of tranverse axis and conjugate axis of one of the given hyperbolas are respectively equals to length of conjugate axis and transverse axis of other hyperbola.

(A) Statement-I is true, statement-II is true; statement-II is correct explanation for statement-I.
(B) Statement-I is true, statement-II is true; statement-II is NOT a correct explanation for statement-I.
(C) Statement-I is true, statement-II is false.
(D) Statement-I is false, statement-II is true.

19. **Statement -I :** If a circle S = 0 intersects a hyperbola xy = 4 at four points. Three of them are (2, 2) (4, 1) and (6, 2/3) then co-ordinates of the fourth point are (1/4 , 16).

Statement -II : If a circle S = 0 intersects a hyperbola $xy = c^2$ at t_1, t_2, t_3, t_4, then $t_1.t_2.t_3.t_4 = 1$.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

20. **Statement -I :** If a tangent is drawn to a hyperbola $16x^2 - 9y^2 = 144$ at a point (15/4, 3) then another tangent at the point (–15/4, –3) will be parallel to the previous tangent.

Statement -II : Two parallel tangents to a hyperbola touches the hyperbola at the extremities of a diameter and converse is also true.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

SECTION - IV : MATRIX - MATCH TYPE

21.

Column-I		Column-II	
(A)	The curve such that product of the distances of any of its tangent from two given points is constant, can be	**(p)**	Circle
(B)	A curve for which the length of the subnormal at any of its point is equal to 2 and the curve passes through (1, 2), can be	**(q)**	Parabola
(C)	A curve passes through (1, 4) and is such that the segment joining any point P on the curve and the point of intersection of the normal at P with the x-axis is bisected by the y-axis. The curve can be	**(r)**	Ellipse
		(s)	Hyperbola
(D)	A curve passes through (1, 2) is such that the length of the normal at any of its point is equal to 2. The curve can be		

22.

Column – I		Column – II	
(A)	Value of c for which $3x^2 - 5xy - 2y^2 + 5x + 11y + c = 0$ are the asymptotes of the hyperbola $3x^2 - 5xy - 2y^2 + 5x + 11y - 8 = 0$	**(p)**	3
(B)	If locus of a point, whose chord of contact with respect to the circle $x^2 + y^2 = 4$ is a tangent to the hyperbola $xy = 1$ is $xy = c^2$, then value of c^2 is	**(q)**	–4
(C)	If equation of a hyperbola whose conjugate axis is 5 and distance between its foci is 13, is $ax^2 - by^2 = c$ where a and b are coprime natural numbers, then value of $\frac{ab}{c}$ is	**(r)**	–12
(D)	If the vertex of a hyperbola bisects the distance between its centre and the corresponding focus, then ratio of square of its conjugate axis to the square of its transverse axis is	**(s)**	4
		(t)	– 6

SECTION - V : COMPREHENSION TYPE

23. Read the following comprehension carefully and answer the questions.

If P is a variable point and F_1 and F_2 are two fixed points such that $|PF_1 - PF_2| = 2a$. Then the locus of the point P is a hyperbola, with points F_1 and F_2 as the two focii ($F_1F_2 > 2a$). If $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is a hyperbola, then its conjugate hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$.

Let P(x, y) is a variable point such that $\left|\sqrt{(x-1)^2+(y-2)^2} - \sqrt{(x-5)^2+(y-5)^2}\right| = 3$.

1. If the locus of the point P represents a hyperbola of eccentricity e, then the eccentricity e′ of the corresponding conjugate hyperbola is :

(A) $\frac{5}{3}$ **(B)** $\frac{4}{3}$ **(C)** $\frac{5}{4}$ **(D)** $\frac{3}{\sqrt{7}}$

2. Locus of intersection of two perpendicular tangents to the given hyperbola is

(A) $(x-3)^2 + \left(y - \frac{7}{2}\right)^2 = \frac{55}{4}$ **(B)** $(x-3)^2 + \left(y - \frac{7}{2}\right)^2 = \frac{25}{4}$

(C) $(x-3)^2 + \left(y - \frac{7}{2}\right)^2 = \frac{7}{4}$ **(D)** none of these

3. If origin is shifted to point $\left(3, \frac{7}{2}\right)$ and the axes are rotated through an angle θ in clockwise sense so that equation of given hyperbola changes to the standard form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then θ is :

(A) $\tan^{-1}\left(\frac{4}{3}\right)$ **(B)** $\tan^{-1}\left(\frac{3}{4}\right)$ **(C)** $\tan^{-1}\left(\frac{5}{3}\right)$ **(D)** $\tan^{-1}\left(\frac{3}{5}\right)$

24. Read the following comprehension carefully and answer the questions.

A conic C passes through the point (2, 4) and is such that the segment of any of its tangents at any point contained between the co-ordinate axes is bisected at the point of tangency. Let S denotes circle described on the foci F_1 and F_2 of the conic C as diameter.

1. Vertex of the conic C is

(A) (2, 2), (–2, –2) **(B)** $\left(2\sqrt{2}, 2\sqrt{2}\right), \left(-2\sqrt{2}, -2\sqrt{2}\right)$

(C) (4, 4), (–4, –4) **(D)** $\left(\sqrt{2}, \sqrt{2}\right), \left(-\sqrt{2}, -\sqrt{2}\right)$

2. Director circle of the conic is

(A) $x^2 + y^2 = 4$ **(B)** $x^2 + y^2 = 8$ **(C)** $x^2 + y^2 = 2$ **(D)** None

3. Equation of the circle S is

(A) $x^2 + y^2 = 16$ **(B)** $x^2 + y^2 = 8$ **(C)** $x^2 + y^2 = 32$ **(D)** $x^2 + y^2 = 4$

25. Read the following comprehension carefully and answer the questions.

If a circle with centre $C(\alpha, \beta)$ intersects a rectangular hyperbola with centre $L(h, k)$ at four points $P(x_1, y_1)$, $Q(x_2, y_2)$, $R(x_3, y_3)$ and $S(x_4, y_4)$, then the mean of the four points P, Q, R, S is the mean of the points C and L. In other words, the mid-point of CL coincides with the mean point of P, Q, R, S. Analytically, $\frac{x_1 + x_2 + x_3 + x_4}{4} = \frac{\alpha + h}{2}$ and $\frac{y_1 + y_2 + y_3 + y_4}{4} = \frac{\beta + k}{2}$

1. If four points are taken on the circle $x^2 + y^2 = a^2$. A rectangular hyperbola (H) passes through these four points. If the centroid of the quadrilateral formed from these four points lie on the straight line $3x - 4y + 1 = 0$ then find the locus of the centre of rectangular hyperbola (H).

(A) $3x - 4y + 2 = 0$ **(B)** $3x - 4y + 3 = 0$ **(C)** $3x - 4y + 4 = 0$ **(D)** None of these

2. A, B, C, D are the points of intersection of a circle and a rectangular hyperbola which have different centres. If AB passes through the centre of the hyperbola, then CD passes through :

(A) centre of the hyperbola

(B) centre of the circle

(C) mid-point of the centres of circle and hyperbola

(D) none of the points mentioned in the three options.

3. If the normals drawn at four concylic points on a rectangular hyperbola $xy = c^2$ meet at point (α,β) then the centre of the circle has the coordinates

(A) (α,β) **(B)** $(2\alpha, 2\beta)$ **(C)** $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$ **(D)** $\left(\frac{\alpha}{4}, \frac{\beta}{4}\right)$

SECTION - VI : INTEGER TYPE

26. If m_1 and m_2 are slopes of the tangents to the hyperbola $\frac{x^2}{25} - \frac{y^2}{16} = 1$ which passes through the point of contact of $3x - 4y = 5$ and $x^2 - 4y^2 = 5$ then $32(m_1 + m_2 - m_1m_2) =$ ....

27. Chords of the circle $x^2 + y^2 = 4$, touch the hyperbola $\frac{x^2}{4} - \frac{y^2}{16} = 1$. The locus of their middle points is the curve $(x^2 + y^2)^2 = \lambda x^2 - 16y^2$, then find λ

28. If the tangent on the point $(3 \sec \phi, 4 \tan \phi)$ (which is in first quadrant) of the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$ is perpendicular to $3x + 8y - 12 = 0$, then the value of ϕ is (in degree)

29. If a variable line has its intercepts on the co-ordinates axes e, e', where $\frac{e}{2}, \frac{e'}{2}$ are the eccentricities of a hyperbola and its conjugate hyperbola, then this line always touches the circle $x^2 + y^2 = r^2$, where r =

30. C the centre of the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$. The tangents at any point P on this hyperbola meets the striaght lines $4x - 3y = 0$ and $4x + 3y = 0$ in the points Q and R respectively. Then CQ . CR =

ANSWER KEY

EXERCISE - 1

1. D **2.** B **3.** B **4.** B **5.** D **6.** A **7.** C **8.** C **9.** A **10.** A **11.** A **12.** D **13.** B
14. A **15.** D **16.** D **17.** B **18.** A **19.** B **20.** A

EXERCISE - 2 : PART # I

1. ABCD **2.** A **3.** BD **4.** ABCD **5.** AD **6.** AD **7.** ABD **8.** AD **9.** ABD
10. AD **11.** ABC **12.** BD **13.** A **14.** ABCD **15.** CD

PART - II

1. A **2.** D **3.** D **4.** D

EXERCISE - 3 : PART # I

1. $A \rightarrow p,s$ $B \rightarrow q,r$ $C \rightarrow r$ $D \rightarrow p,s$ **2.** $A \rightarrow r$ $B \rightarrow s$ $C \rightarrow p$ $D \rightarrow q$ **3.** $A \rightarrow r,s$ $B \rightarrow q$ $C \rightarrow r$ $D \rightarrow p$
4. $A \rightarrow p$ $B \rightarrow s$ $C \rightarrow q,r$ $D \rightarrow r$ **5.** $A \rightarrow r$ $B \rightarrow s$ $C \rightarrow p$ $D \rightarrow q$

PART - II

Comprehension # 1 : 1. D **2.** C **3.** D **Comprehension # 2 : 1.** D **2.** C **3.** B
Comprehension # 3 : 1. B **2.** C **3.** D

EXERCISE - 5 : PART # I

1. 2 **2.** 4 **3.** 1 **4.** 2 **5.** 3 **6.** 2

PART - II

1. D **2.** B **3.** A **4.** $\frac{x^2}{9} - \frac{y^2}{4} = \left(\frac{x^2 + y^2}{9}\right)^2$ **5.** A, C **6.** A
7. $A \rightarrow p,q$ $B \rightarrow p,q$ $C \rightarrow q,r$ $D \rightarrow q,r$ **8.** B **9.** B **10.** A,B **11.** B **12.** A **13.** 2
14. B,D **15.** B **16.** A,B **17.** A,B,D

MOCK TEST

1. B **2.** C **3.** B **4.** A **5.** A **6.** B **7.** C **8.** D **9.** A **10.** A **11.** A, B, D **12.** C, D
13. A, B **14.** A, C, D **15.** B, C **16.** A **17.** D **18.** B **19.** D **20.** A
21. $A \rightarrow r,s$ $B \rightarrow q$ $C \rightarrow r$ $D \rightarrow p$ **22.** $A \rightarrow r$ $B \rightarrow s$ $C \rightarrow s$ $D \rightarrow p$
23. 1. C **2.** D **3.** B **24. 1.** B **2.** D **3.** C **25. 1.** A **2.** B **3.** C **26.** 22 **27.** 4
28. 30 **29.** 2 **30.** 25