HINTS & SOLUTIONS

EXERCISE - 1

Single Choice

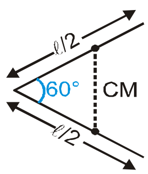
1. Equation of line joining the CM of two rods

$$\frac{x}{L/2}+\frac{y}{L/2}=1$$

coordinate $\left(\frac{L}{3},\frac{L}{6}\right)$ satisfies this equation.

2. $$\bar{x}=\frac{\int x\,dm}{\int dm}=\frac{\int_0^L x\frac{kx^2}{L}dx}{\int \frac{kx^2}{L}dx}=\frac{3L}{4}$$

3. $$\bar{x}=\frac{\ell}{4}\cos 30°=\frac{\sqrt{3}\ell}{8}$$



4. $$\Delta\bar{x}=\frac{m_1.\Delta x_1+m_2.\Delta x_2}{m_1+m_2}$$

$$\Rightarrow\quad 0=\frac{m_1 a+m_2\Delta x_2}{m_1+m_2}$$

$$\Rightarrow\quad \Delta x_2=-\frac{m_1 a}{m_2}$$

5. Let x_p = x shift of plank to the right

$$\Delta\bar{x}=\frac{m_A\Delta x_A+m_B\Delta x_B+m_c\Delta x_c+m_p\Delta x_p}{m_A+m_B+m_C+m_P}$$

$$0=\frac{40(x+4)+50x+60(x-4)+90x}{40+50+60+90}\Rightarrow x=\frac{1}{3}m$$

6. $$\vec{v}_{cm}=\frac{m_1\vec{v}_1+m_2\vec{v}_2}{m_1+m_2}=\frac{1\times 2\hat{i}+2\times(2\cos 30\hat{i}-2\sin 30\hat{j})}{3}$$

$$=\left(\frac{2+2\sqrt{3}}{3}\right)\hat{i}-\frac{2}{3}\hat{j}$$

7. CM remains at rest if initially it is at rest.

8. $$\Delta\bar{y}=\frac{m_1\Delta y_1+m_2\Delta y_2}{m_1+m_2}$$

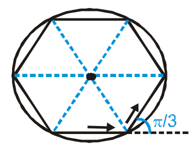
$$\Rightarrow\quad 0=\frac{m}{4}(15)+\frac{3m}{4}(y_2)$$

$$\Rightarrow\quad y_2=-5\text{ cm}$$

9. $$v_{CM}=\frac{(1)(5)+(1)(-3)}{1+1}=1\text{ m/s}$$

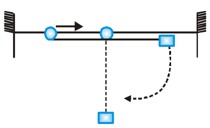
Position of centre of mass at t = 1s

$$X_{CM}=\frac{(1)(2)+(1)(8)}{1+1}+(1)(1)=5+1=6m$$

10. 

$$\Delta p=2p\cos\frac{\pi}{3}=2mv_0\sin\left(\frac{\pi}{6}\right)$$

11. $$\Delta\bar{x}=\frac{m_1.\Delta x_1+m_2.\Delta x_2}{m_1+m_2}$$



Let x = distance moved by ring

$$0=\frac{mx+2m(1.2-x)}{m+2m}\Rightarrow x=0.8\text{ m}$$

12. Impulse $=p_f-p_i=1\times 10-1\times(-25)=35\text{kg m/s}\,(\uparrow)$

13. $$\Delta KE=\frac{1}{2}mv_2^2-\frac{1}{2}mv_1^2$$

$$=\frac{1}{2}m(\vec{v}_2-\vec{v}_1).(\vec{v}_2+\vec{v}_1)=\frac{1}{2}\vec{I}.(\vec{v}_1+\vec{v}_2)$$

14. For the I[st] ball : $\frac{h}{4}=e_1^2h$

For the II[nd] ball : $\frac{h}{16}=e_2^2h$

Impulse on first ball $=I_1=mv_0(1+e_1)=\frac{3}{2}mv_0$

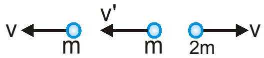
Impulse on second ball $=I_2=mv_0(1+e_2)=\frac{5}{4}mv_0$

$$\Rightarrow\quad \frac{I_1}{I_2}=\frac{3/2\ mv_0}{5/4\ mv_0}=\frac{6}{5}\Rightarrow 5I_1=6I_2$$

15. COLM : $3\times 2=(3+2)v\Rightarrow v=\frac{6}{5}$ m/s

COME : $\frac{1}{2}\times 3\times 2^2=\frac{1}{2}\times 5\times\left(\frac{6}{5}\right)^2+\frac{1}{2}\times 480\times x^2$

$$\Rightarrow\quad x=\frac{1}{10}m$$

16. 

$0 = 2mv - mv + mv'$

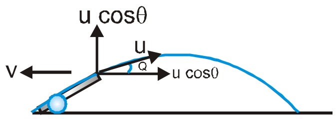
$\Rightarrow$ $v' = -v$

Total mechanical energy released

$$= \frac{1}{2}(m+m+2m)v^2 = 2mv^2$$

17. COLM :

Along horizontal



$0 = m(u\cos\theta - v) - 4mv$

$\Rightarrow$ $v = \frac{u\cos\theta}{5}$

$\therefore$ velocity of shell along horizontal w.r.t ground

$$= u\cos\theta - \frac{u\cos\theta}{5} = \frac{4}{5}(u\cos\theta)$$

Time of flight $T = \frac{2u\sin\theta}{g}$

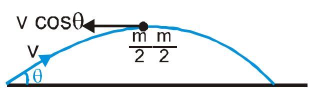
$\therefore$ x = horizontal displacement

$$= \left(\frac{4}{5}u\cos\theta\right)\left(\frac{2u\sin\theta}{g}\right) = \frac{4u^2\sin 2\theta}{5g}$$

18. The ball & the earth forms a system and no external force acts on it. Hence total momentum remains constant.

19. COME : $mv\cos\theta = \frac{m}{2}(-v\cos\theta) + \frac{m}{2}v'$

$\Rightarrow$ $\frac{3mv\cos\theta}{2} = \frac{m}{2}v'$ $\Rightarrow$ $v' = 3v\cos\theta$



20. COME : $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

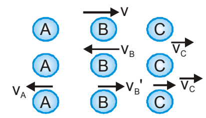
$1 \times u + 0 = 1 \times \frac{u}{4} + mv_2$

$\Rightarrow$ $\frac{3}{4}u = mv_2$ ....(i)

$$e = -\left(\frac{v_2 - v_1}{u_2 - u_1}\right) = -\left(\frac{v_2 - u/4}{0 - u}\right) = \frac{v_2 - u/4}{u}$$

$\Rightarrow$ $v_2 = \frac{u}{4} + u = \frac{5u}{4}$ ....(ii)

(i) & (ii) $\frac{3}{4}u = m\left(\frac{5}{4}u\right)$ $\Rightarrow$ $m = \frac{3}{5} = 0.6\text{kg}$

21. 

For collision between B and C :

$$v_B = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 + \frac{m_2(1+e)}{(m_1+m_2)}u_2$$

$$= \left(\frac{m - 4m}{5m}\right)v + 0 = -3/5\ v$$

$$v_C = \frac{m_1(1+e)}{(m_1+m_2)}u_1 + \left(\frac{m_2 - em_1}{m_1+m_2}\right)v_2 = \frac{m(1+1)}{4m}v + 0 = \frac{v}{2}$$

For collision between A and B :

$$v_A = 0 + \frac{m(1+1)}{5m} \times \left(\frac{3v}{5}\right) = \frac{6}{25}v$$

$$v_B' = 0 + \left(\frac{m - 4m}{5m}\right)\left(-\frac{3v}{5}\right) = -\frac{9v}{25}$$

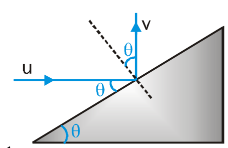
$\because$ $v_B' < v_C$

$\therefore$ B will not collide with C.

Therefore there will be only two collisions.

22. Along tangent

$u\cos\theta = v\sin\theta$ ...(i)



Along normal

$$e = \frac{v\cos\theta}{u\sin\theta} = \cot^2\theta = \cot^2 60 = \frac{1}{3}$$

23. At the lowest position

COME : $M\sqrt{2gL} = (M+m)v$ ....(i)

COME : $\frac{1}{2}(M+m)v^2 = (M+m)gh$ ....(ii)

$\Rightarrow$ $v = \sqrt{2gh} = \frac{M\sqrt{2gL}}{(M+m)}$ $\Rightarrow$ $h = \left(\frac{M}{m+M}\right)^2 L$

24. After 1s ,

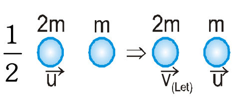
$v_A = 20 - 10 \times 1 = 10$ m/s and $v_B = 0 + 10 \times 1 = 10$ m/s

At the time of collision , $V_A = V_B = 5$m/s

after collision, velocity gets interchanged.

25. COLM $\Rightarrow$ $2mu + 0 = 2mv + mu$ $\Rightarrow$ $v = \frac{u}{2}$

$$e = -\left(\frac{v_2 - v_1}{u_2 - u_1}\right) = -\left(\frac{u - u/2}{0 - u}\right) = \frac{1}{2}$$



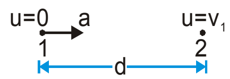
26. $e = -\left(\dfrac{v_2 - v_1}{u_2 - u_1}\right)$

$\Rightarrow \quad 1 = -\dfrac{5 - v_1}{5 - (-10)} = \dfrac{v_1 - 5}{15} \quad \Rightarrow \quad v_1 = 20 \text{ m/s}$

$\therefore$ Impulse on ball= $m(\vec{v} - \vec{u})$ =1× [20–(10)]=30N-s

27. For second object

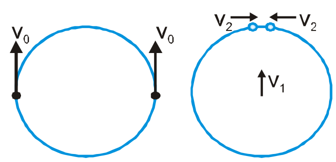
$2v_1 = 0 + at;\ t = \dfrac{2v_1}{a}$ ...(i)



$s_1 = s_2 + d;\ \dfrac{1}{2}at^2 = ut + d$

$\Rightarrow \quad \dfrac{1}{2}a \times \left(\dfrac{2v_1}{a}\right)^2 = v_1\left(\dfrac{2v_1}{a}\right) + d \quad \Rightarrow \quad d = \dfrac{2v_1^2}{a} - \dfrac{2v_1^2}{a} = 0$

28.



COLM $\Rightarrow \quad 2mv_0 = 3mv_1 \quad \Rightarrow \quad v_1 = \dfrac{2}{3}v_0$

COME $\Rightarrow \quad 2 \times \dfrac{1}{2}mv_0^2 = 2 \times \dfrac{1}{2}m(v_1^2 + v_2^2) + \dfrac{1}{2}mv_1^2$

$\Rightarrow \quad 2mv_0^2 = 2m\left(\dfrac{2}{3}v_0\right)^2 + 2mv_2^2 + m\left(\dfrac{2}{3}v_0\right)^2$

$\Rightarrow \quad 2v_2^2 = 2v_0^2 - 3\left(\dfrac{2}{3}v_0\right)^2 \quad \Rightarrow \quad v_2 = \dfrac{v_0}{\sqrt{3}}$

Velocity of particle $= \sqrt{v_1^2 + v_2^2} = \sqrt{\dfrac{4v_0^2}{9} + \dfrac{v_0^2}{3}} = \dfrac{\sqrt{7}}{3}v_0$

29. Average power

$\dfrac{\Delta W}{\Delta t} = \left(\dfrac{\Delta K + \Delta U}{\Delta t}\right) = \dfrac{1}{2}\dfrac{(\lambda \Delta x)v^2}{\Delta t} + \dfrac{\lambda \Delta x g(\Delta x / 2)}{\Delta t}$

$< P > = \dfrac{1}{2}\lambda v^3 + \dfrac{\lambda \ell}{2}vg$

EXERCISE - 2

Part # I : Multiple Choice

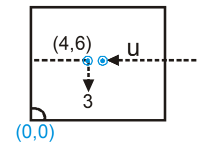
1. Momentum of the coin perpendicular to the common normal remains constant.

$\therefore \quad v_y = -3$ (constant)

$v_y t = -6$

$\Rightarrow \quad t = 2$ sec & $v_x t = -4$

$v_x = -2$ m/s



Which is given by striker.

So initial velocity of striker = $2ms^{-1}$

Final velocity of the striker = 0.

2. In the absence of external forces, the linear momentum of the system remains constant.

3. $F_{ext} = m\dfrac{dv}{dt} + v_{rel}\dfrac{dm}{dt}$

$\Rightarrow \quad 0 = m\dfrac{dv}{dt} + 2\dfrac{dm}{dt} \quad \Rightarrow -\displaystyle\int_{+m}^{m/2}\dfrac{2dm}{m} = \int_0^v dv$

$\Rightarrow \quad v = 2\ell n2$

4. Force exerted by one leg on the ground

$N \quad = \dfrac{1}{4} \times$ [Total force]

$= \dfrac{1}{4}$ [wt + rate of change of momentum]

$= \dfrac{1}{4}$ [Mg + n (mv cos 60°)× 2]= 1N

5. In the ground frame

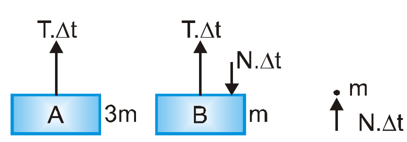
$: m_A\Delta x_A + m_B\Delta x_B + m_p\Delta x_p = 0$

$\Rightarrow \quad 40 \times 60 + 0 + 40 \times \Delta x_p = 0$

$\Rightarrow \quad \Delta x_p = -60$ (to the left)

Hence A & B meet at the right end.

6.



$-N\Delta t = m(v - u);\ N\Delta t - T\Delta t = m(v - 0)$

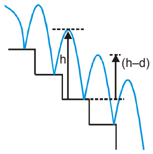
$T\Delta t = 3m(v - 0)$

$\Rightarrow \quad v = u/5$

$\therefore$ Impulsive tension $T\Delta t = \dfrac{3mu}{5}$

7. COLM : $m_R(0.8) + m_S(0) = m_R(0.2) + m_S(1.0)$

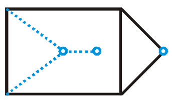
$\Rightarrow 0.6\, m_R = m_S \quad \Rightarrow \quad m_R > m_S$

8. 

$\sqrt{2g(h-d)} = e\sqrt{2gh} \Rightarrow \frac{h}{d} = \frac{1}{1-e^2}$

9. $\bar{x} = \frac{M_1x_1 + M_2x_2}{M_1 + M_2} = \frac{\frac{M}{2}\left(-\frac{a}{2}\right) + \frac{M}{2}\left(\frac{a}{3}\right)}{M} = -\frac{a}{12}$

10. $Nmv = (M + Nm)v_f \quad \Rightarrow \quad v_f = \frac{mvN}{M + Nm}$

11. 

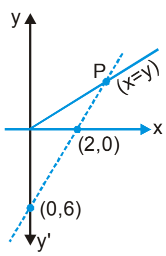
$\bar{x} = \frac{(M \times 0) + \left(-\frac{M}{4} \times \frac{-L}{3}\right) + \left(\frac{M}{4} \times \frac{4L}{6}\right)}{M} = \frac{L}{4}$

12. $\Delta\bar{x} = \frac{3M.x + M(x+2)}{4M} = 0 \Rightarrow x = -\frac{1}{2}$

13. Let x = displacement of ring to the left.

$\Rightarrow \Delta x_{cm}$

$= \frac{2mx + m(x + L - L\cos\theta)}{3m} = 0 \Rightarrow x = -\frac{L}{3}(1-\cos\theta)$

14. 

$\frac{x}{2} - \frac{y}{6} = 1$

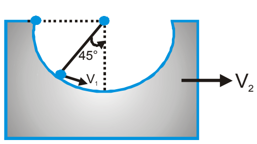
Co-ordinate of P = (3,3)

$\therefore$ Speed of 3rd particle $= 3\sqrt{2}$ m/s

15. COLM $\Rightarrow m(v_1\cos 45° + v_2) + mv_2 = 0$

$\Rightarrow v_1 = -2\sqrt{2}.v_2$

COME $\Rightarrow K_i + U_i = K_f + U_f$



$\Rightarrow 0 + \frac{mgR}{\sqrt{2}} = \frac{1}{2}mv_2^2 + \frac{1}{2}m\left[\frac{v_1^2}{2} + \left(\frac{v_1}{\sqrt{2}} - \frac{v_1}{2\sqrt{2}}\right)^2\right]$

$\Rightarrow v_2 = \sqrt{\frac{gR}{3\sqrt{2}}}$

16. $Mv = 0 + \frac{M}{10}v_2 \Rightarrow v_2 = 10v$

17. Initially when the shell is empty the C.M. lies at its geometric centre. Also when the shell is filled with sand CM lies at its geometric centre.

18. Δp = change in momentum = 2mv

Δt = time between two collision $= \frac{2(L-d)}{v}$

$\therefore$ Force exerted on wall $= \frac{\Delta p}{\Delta t} = \frac{mv^2}{(L-d)}$

19. $a_{cm} = \frac{F_{net}}{\text{Total mass}} = \frac{(0.2)(3)(10)}{1+2} = 2\,ms^{-2}$

OR

Acceleration of 1kg w.r.t. ground

$= (0.1)(10) = 1ms^{-2}$

Acceleration of 2 kg w.r.t. ground

$= \frac{(0.2)(3)(10) - (0.1)(10)}{2} = \frac{5}{2}ms^{-2}$

$a_{cm} = \frac{m_1a_1 + m_2a_2}{m_1 + m_2} = \frac{(1)(1) + (2)(5/2)}{1+2} = 2ms^{-2}$

20. In elastic head on collision if the masses of the colliding bodies are equal, the velocities after collision are interchanged.

For Ist bead, $Fd = \frac{1}{2}mu^2 \Rightarrow u = \sqrt{\frac{2Fd}{m}}$

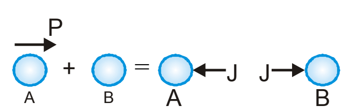
21. $t_1 = \frac{L}{v}$ (time for Ist collision)

$t_2 = \frac{2L}{v}$ (time for IInd collision)

$t_3 = \frac{3L}{v}$ (time for 3rd collision)

$t_{(n-1)} = \frac{L}{v}(n-1)$ (time for (nth) collision)

$\sum_{i-1}^{h} t_i = \frac{n(n-1)}{2}\frac{L}{v}$

22. 

For A : $P - J = mv_1$ ....(i) for B : $J = mv_2$ ....(ii)

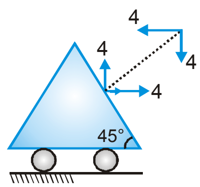
$\therefore e = -\left(\frac{v_2 - v_1}{u_2 - u_1}\right) = -\left[\frac{\frac{J}{m} - \left(\frac{P-J}{m}\right)}{0 - \frac{P}{m}}\right] = \frac{2J}{p} - 1$

23. $\Delta\bar{x} = \dfrac{m_1\Delta x_1 + m_2\Delta x_2 + m_3\Delta x_3}{m_1 + m_2 + m_3}$

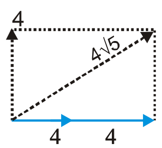
$\Delta\bar{x} = \dfrac{(80\times 2)-(50\times 2)+(70\times 0)}{80+50+70}$

= 30cm towards right

24. Cart frame :



Ground frame :



The velocity of rebound = $4\sqrt{5}$ m/s

25. $mu - I_1 = -mu$

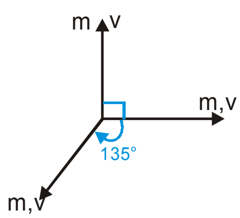
$\therefore$ $I_1 = 2mu$ & $mu - I_2 = 0$

$\Rightarrow$ $I_2 = mu$ (for IInd ball) $\therefore I_2 = I_1/2$

26. COLM $\Rightarrow$

$$mv\hat{i} + mv\hat{j} + m\left(\frac{-v}{\sqrt{2}}\hat{i} - \frac{v}{\sqrt{2}}\hat{j}\right) + m\vec{v}_4 = 0$$

$$\vec{v}_4 = -v\left(1-\frac{1}{\sqrt{2}}\right)\hat{i} - v\left(1-\frac{1}{\sqrt{2}}\right)\hat{j}$$



Total energy released

$$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}m\left[v^2\left(1-\frac{1}{\sqrt{2}}\right)^2 \times 2\right]$$

$= mv^2(3-\sqrt{2})$

27. COME : $-MV + m(v\cos 60 - V) = 0$ $\Rightarrow$ $v = 10$ m/s

28. $F_{ext} = \Delta mv$ (for first body)

$\Rightarrow$ $F_{ext} = \dfrac{10\times(15-0)}{3} = 50N$

COME : $m_1u_1 + m_2u_2 = (m_1+m_2)v$

$\Rightarrow$ $10\times 15 + 25\times u_2 = (10+25)5$

$\Rightarrow$ $u_2 = 1$ m/s

29. $x_{cm} = \dfrac{m\times 0 + m\times R}{m+m} = \dfrac{R}{2}$

$x_{CG} = \dfrac{W_1(0)+W_2(R)}{W_1+W_2} = \dfrac{mgR}{mg} = R$

$\therefore$ $x_{CG} - x_{CM} = R - \dfrac{R}{2} = \dfrac{R}{2}$

30. $\Delta\bar{x} = \dfrac{m\Delta x_1 + M\Delta x_2}{m+M} = 0$

$\Rightarrow$ $\dfrac{1(\ell\sin 30° + \ell\sin 30° - x) + 4x}{1+4} = 0$

$\Rightarrow$ Displacement of bar x = 0.2

31. $v_1 = \dfrac{(m_1-m_2)}{(m_1+m_2)}u_1 + \left(\dfrac{2m_2}{m_1+m_2}\right)u_2$

$v_2 = \dfrac{2m_1}{(m_1+m_2)}u_1 + \left(\dfrac{m_2-m_1}{m_1+m_2}\right)u_2$

For C : $v_C = \dfrac{2mu}{3m} = \dfrac{2}{3}u$

32. Velocity before strike $u = \sqrt{2gh}$

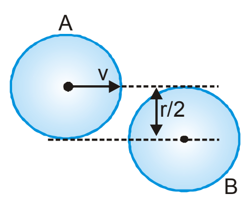
Impulse = $F\Delta t = m(v-u)$

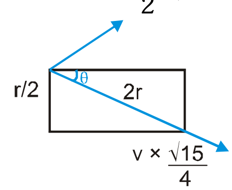
$\Rightarrow$ $F = \dfrac{m(v-u)}{t} = \dfrac{w(0-\sqrt{2gh})}{g\times 0.15} = 5.21$ W

33. COLM $\Rightarrow$ $m_1u_1 + m_2u_2 = (m_1+m_2)v$

$5\times 10^3\times 1.2 + 0 = (5+1)\times 10^3\times v$ $\Rightarrow$ $v = 1$ m/s

34. $2r\sin\theta = \dfrac{r}{2}$; $\sin\theta = \dfrac{1}{4}$



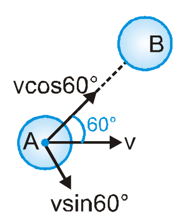


$\cos\theta = \dfrac{\sqrt{15}}{4}$

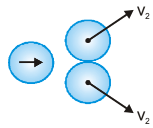
35. From COLM $Mv_{x_A} + 2Mv_{x_B} = 0 \Rightarrow v_{x_A} = -2v_{x_B}$

So $\vec{v}_A = -2v_x\hat{i} + v\hat{k}$

36. Component of velocity of A along common normal is v cos 60° and this velocity of A after collision with B is interchanged. Hence A moves along v sin 60° which is normal to common normal.



37. $m \times 1 = mv_1 + (2mv_2 \times \cos 60°)$



$\Rightarrow \; v_2 = \frac{1}{2}$ & $v_1 = \frac{1}{2}$

$(KE)_{initial} = \frac{1}{2} \times 1 \times 1^2 = 0.5$ J

$(KE)_{final} = \frac{1}{2}(1) \times \left(\frac{1}{2}\right)^2 + \left\{\frac{1}{2} \times 1 + \times \left(\frac{1}{2}\right)^2\right\} \times 2$

$= 0.25 + 0.125 = 0.3755$

$\Delta KE = 0.5 - 0.375 = 0.125$ J

38. At the time of maximum compression,

COLM : $mu = 2mv$ (for A & B) $\Rightarrow v = u/2$

COME : $\frac{1}{2}mu^2 = \frac{1}{2}2mv^2 + \frac{1}{2}kx^2 \Rightarrow x = v\sqrt{\frac{m}{2k}}$

39. At the time of collision both particles have common velocity and hence the system has minimum kinetic energy.

COME : $mu + 0 = 3mv \;\Rightarrow\; v = u/3$

$KE_{initial} = \frac{1}{2}mu^2 = 3J$

$KE_{collision} = \frac{1}{2}(3m)v^2 = \frac{1}{2}(3m)\frac{u^2}{9} = 1J$

$PE_{collision} = (3-1) = 2J$

Total energy remains constant and hence KE of system First decreases & then increases.

40. COLM : $mu + 0 = (m + M)v \Rightarrow v = \left(\frac{m}{M+m}\right)u$

KE after collision $= \frac{1}{2}(m+M) \times \left(\frac{m}{m+M}\right)^2 u^2$

$= \frac{m^2u^2}{2(m+M)}$

41. PE of solid sphere $= mgR = mg\frac{D}{2} = \rho g D^4\left(\frac{\pi}{12}\right)$

PE of solid cube $= mg\frac{D}{2} = \rho g D^4\left(\frac{1}{2}\right)$

PE of solid cone $= mg\frac{D}{4} = \rho g D^4\left(\frac{\pi}{48}\right)$

PE of solid cylinder $= mg\frac{D}{2} = \rho g D^4\left(\frac{\pi}{8}\right)$

42. $p_i = -mv$, $p_f = m(v + 2u)$ $\quad\therefore\; \Delta p = 2m(v+u)$

$\therefore\;$ Force $= \frac{\Delta p}{\Delta t} = \frac{2m(v+u)}{\Delta t}$

$KE_{initial} = \frac{1}{2}mu^2$, $KE_{final} = \frac{1}{2}m(2v+u)^2$

$\therefore\; \Delta KE = \frac{1}{2}m[4v^2 + u^2 + 4uv - u^2]$

$= \frac{1}{2}m[4v(v+u)] = 2mv(u+v)$

Part # II : Assertion & Reason

1. A **2.** A **3.** A **4.** A **5.** A **6.** C
7. C **8.** B **9.** B

EXERCISE - 3

Part # I : Matrix Match Type

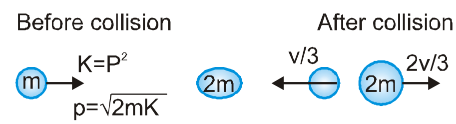
1. Impulse = change in momentum

(A) For body M : $p = |\vec{p}_f - \vec{p}_\ell| \;\Rightarrow\; p_f = p$

(B) For body 2M : $p = |\vec{p}_f - \vec{p}_\ell| \Rightarrow p_f = 2p$

(C) $e = \frac{v_2 - v_1}{u_1 - u_2} \Rightarrow \frac{\frac{p}{M} - \frac{p}{M}}{\frac{2p}{M} - \frac{p}{2M}} = 0 \Rightarrow e = 0$

2. By applying conservation of momentum



$m_1u_1 + 2m(0) = mv_1 + 2mv_2$ ......(i)

Also $u = v_2 - v_1$ .....(ii)

$v_2 = \frac{2v}{3}$ and $v_1 = -\frac{v}{3}$; $p_2 = \frac{4mv}{3} = \frac{4p}{3}$,

$p_1 = -\frac{mv}{3} = -\frac{p}{3}$; $K_2 = \frac{8K}{9}$; $K_1 = \frac{K}{9}$

3. For 1kg $v_1 = (2t)\hat{i} = 4\hat{i}$; $a_i = 2\hat{i} = 2\hat{i}$

For 2kg $v_2 = t^2\hat{j} = 4\hat{j}$; $\quad a_2 = 2t\hat{j} = 4\hat{j}$

(A) Acceleration of centre of mass $= \frac{m_1a_1 + m_2a_2}{m_1 + m_2}$

$\vec{a}_{cm} = \frac{2}{3}\hat{i} + \frac{8}{3}\hat{j} \Rightarrow a_{cm} = \sqrt{\frac{4}{9} + \frac{64}{9}} \Rightarrow \frac{\sqrt{68}}{3}$ m/s²

$f = ma_{cm} = \sqrt{68}$ N

(B) Velocity of centre of mass

$\vec{v}_{cm} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \left(\frac{4}{3}\hat{i} + \frac{8}{3}\hat{j}\right)$ m/s

$\Rightarrow\; |\vec{v}_{cm}| = \sqrt{\frac{16}{9} + \frac{64}{9}} \Rightarrow \frac{\sqrt{80}}{3}$

(C) $\vec{v}_{cm} = \dfrac{1(2t)\tilde{i} + 2(t^2\tilde{j})}{1+2} = \dfrac{2}{3}t\tilde{i} + \dfrac{2}{3}t^2\tilde{j}$

$$\text{Displacement} = \int_0^2 \vec{v}_{cm}dt = \left[\frac{3}{2}\left(\frac{t^2}{2}\right)^2 \hat{i} + \frac{2}{3}\left(\frac{t^3}{3}\right)\hat{j}\right]_0^2$$

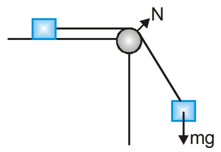
$$= \frac{4}{3}\hat{i} + \frac{16}{9}\hat{j}$$

$$\Rightarrow \quad |Dispalcement| = \sqrt{\left(\frac{4}{3}\right)^2 + \left(\frac{16}{9}\right)^2} = \frac{20}{9} units$$

4. **(A)** Net force on block m acts in downward direction

$\therefore$ Acceleration of centre of mass is in downward direction.

(B) Net force acts in downward as well as in horizontal direction.



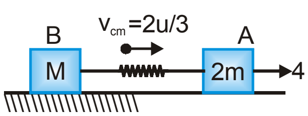
$\therefore$ a_{cm} moves both in horizontal & vertical direction.

(C) As the mass of monkey & block is same both moves upward.

$\therefore$ Centre of mass moves upward

(D) Centre of mass of the system does not moves

5. As no external force acts on the system velocity of centre of mass remain same



$$v_{cm} = \frac{2m(u)}{3m} = \frac{2u}{3}$$

In frame of centre of mass velocity of B is 2u/3 and It oscillates from $\left(-\dfrac{2u}{3}, \dfrac{2u}{3}\right)$

In frame of centre of mass velocity of A is u/3 and It oscillates from $\left(-\dfrac{u}{3}, \dfrac{u}{3}\right)$. In ground frame velocity of B $\left[0, \dfrac{4u}{3}\right]$. In ground frame velocity of A $\left[\dfrac{u}{3}, u\right]$

and by conservation of energy $\Delta K.E. = \Delta U_S$

Part # II : Comprehension

Comprehension # 1

By applying conservation of momentum

$2(6) + 1(4) = 1v_2 + 2v_1$; $16 = v_2 + 2v_1$ .....(i)

By applying newton law of collision

$1 = \dfrac{v_2 - v_1}{2} \Rightarrow v_2 - v_1 = 2$ .....(ii) $\Rightarrow v_2 = \dfrac{20}{3}, \; v_1 = \dfrac{14}{3}$

1. Impulse = change in momentum $= \dfrac{20}{3} - 4 = \dfrac{8}{3}$ N–s

2. To change the direction of a block impulse should be greater than 12 N–s

Comprehension # 2

1. By applying conservation of momentum

$mv_1 + Mv_2 = 0 \Rightarrow \quad v_1 = -4v_2$ .....(i)

By applying conservation of energy

$\dfrac{1}{2}mv_1^2 + \dfrac{1}{2}Mv_2^2 = mgh \Rightarrow \dfrac{v_1^2}{2} + 2v_2^2 = 20$ .....(ii)

$v_2 = \sqrt{2}m/s$; $v_1 = 4\sqrt{2}$ m/s

2. When 'm' leaves the wedge 'M' then wedge moves distance 'x' in left side

$\therefore$ $m(4-x) = Mx \Rightarrow x = 0.8$ m

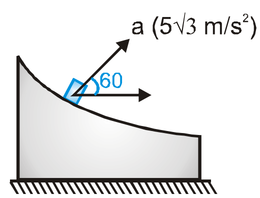
$\therefore$ Co-ordinate where block will leave wedge

$x = 4 - 0.8 = 3.2$

Time for m will strike the ground is $= \sqrt{\dfrac{2\times 2}{10}}$

$\therefore$ $x_f = 3.2 + 4\sqrt{2} \times \dfrac{2}{\sqrt{10}} = 6.8$ m

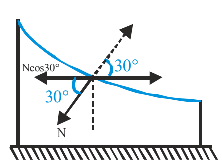
3. $a_{cm} = \dfrac{m_1a_1 + m_2a_2}{m_1 + m_2}$



(1) a cos 60 = 4a

$a_M = \dfrac{5\sqrt{3}}{8}$ m/s²

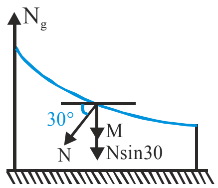
$N\cos 30 = 4ma_M$



$N\dfrac{\sqrt{3}}{2} = 4(1)\dfrac{5\sqrt{3}}{8}$

$\Rightarrow N = 5$ Newton

4. $N_g = N\sin 30 + 40 \Rightarrow N_g = 42.5$



Comprehension # 3

As horizontal velocity (i.e. velocity along the surface) remains constant so required time $= \frac{30}{5} = 6s$

Comprehension # 4

1. $\vec{r}_{cm} = \frac{m_1\vec{r}_{10} + m_2\vec{r}_{20}}{m_1 + m_2} = \frac{1(3\vec{i}) + 2(9\vec{j})}{1+2} = (\vec{i} + 6\vec{j})m$

$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} = \frac{1(3\vec{i}) + 2(6\vec{j})}{1+2}(\vec{i} + 4\vec{j})m/s$

Now $\Delta\vec{r}_{cm} = \vec{v}_{cm}t \Rightarrow \vec{r}_{cm} - \vec{r}_{cm_0} = \vec{v}_{cm}t$

$\Rightarrow (x-1)\hat{i} + (y-6)\hat{j} = (\hat{i} + 4\hat{j})t$

$\Rightarrow x = 1 + t$ and $y = 6 + 4t \Rightarrow y = 4x + 2$

2. $\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2} = \frac{1(-2\vec{i}) + 2(-2\vec{j})}{1+2}$

$= -\frac{2}{3}(\vec{i} + 2\vec{j})m/s^2$

By using $\vec{v}_{cm} = \vec{u}_{cm} + \vec{a}_{cm}t$; we get t = 3s

3. By using $s = ut + \frac{1}{2}at^2$ for individual particles

For I[st] particle : $s_x = (3)(1.5) + \frac{1}{2}(2)(1.5)^2 = 2.25$ m

For II[nd] particle $= s_y = (6)(3) + \frac{1}{2}(-2)(3)^2 = 9$m

Therefore $x_{cm} = \frac{2.25}{3} + 1 = 1.75$ m and

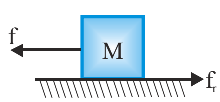
$y_{cm} = \frac{2(9)}{3} + 6 = 12$m

Comprehension # 5

1. $f = v\frac{dm}{dt} = 2(20) = 40 \Rightarrow f_r = \mu mg$

$m = 40 = M_0 - 2(t) \Rightarrow t = 5$ sec

so $(0.1)(50 - 2t) = 40 \Rightarrow t = 5$ sec



2. $v = v\,\ell n\left(\frac{m}{m_0}\right) - gt = 20\,\ell n\left[\frac{4}{3}\right] - gt = 20(0.28) - 5$

$= 5.6 - 5 = 0.6$m/s

Comprehension # 6

1. By COLM

$0 = m_A v_A + m_B v_B;\ \vec{v}_B = -\frac{m_A}{m_B}\vec{v}_A$

Both velocity are opposite in direction

$\therefore$ II, IV, V

2. If $m_A = m_B \Rightarrow \vec{v}_B = -\vec{v}_A \Rightarrow$ Graph II

; $m_A > m_B \Rightarrow v_B > v_A \Rightarrow \tan\theta_B > \tan\theta_A \Rightarrow$ Graph IV

If $m_A < m_B \Rightarrow v_A > v_B \Rightarrow$ Graph IV

3. v_{cm} is not zero in graph (I), (III) and (VI)

Comprehension # 7

1. As $ma_{cm} = f \Rightarrow a_{cm} = \frac{f}{2m}$ so $s_{cm} = \frac{1}{2}at^2 = \left(\frac{f}{4m}\right)t^2$

2. $x_{cm} = \frac{m_1x_1 + m_2x_2}{2m} \Rightarrow x_1 + x_2 = \frac{f}{2m}t^2 \Rightarrow x_1 - x_2 = x_0$

Therefore $x_1 = \frac{f}{4m}t^2 + \frac{x_0}{2}$

From above equations $x_2 = \frac{ft^2}{4m} - \frac{x_0}{2}$

Comprehension # 8

1. As no external force acts on the two blocks friction acts like an internal forces and the two blocks will move with common velocity. By applying conservation of momentum

(1kg) (15 m/s) = 1v = 2v $\Rightarrow$ v = 5 m/s

$P_1 = 5$ N–s ; $P_2 = 10$ N–s

2. $\frac{dp}{dt} = f_{ext}$

For block of 1 kg

friction $f_r = \mu mg = 0.4 \times 1 \times 10 = 4$ N

3. $v = u + at \Rightarrow 5 = 15 - (4)t \Rightarrow t = 2.5$ sec

EXERCISE - 4

Subjective Type

1. $x_{cm} = \dfrac{(m\times 0)+(2m\times a)+(3m\times a)+(4m\times 0)}{m+2m+3m+4m} = \dfrac{a}{2}$

2. **(i)** The centre of mass remains at O as the excluded masses are symmetrically placed.

(ii) CM shifts from 0 to 3 diagonally

(iii) CM shifts along OY

(iv) CM does not shift.

(v) CM shifts diagonally from 0 to 4.

(vi) CM does not shift.

$y_{cm} = \dfrac{(m\times 0)+(2m\times 0)+(3m\times a)+(4m\times a)}{m+2m+3m+4m} = \dfrac{7a}{10}$

3. Length of rod = $\sqrt{(4-2)^2+(2-5)^2} = \sqrt{13}m$

$x_{cm} = \dfrac{(3\times 2)+(2\times 4)}{3+2} = \dfrac{14}{5}; y_{cm} = \dfrac{(3\times 5)+(2\times 2)}{3+2} = \dfrac{19}{5}$

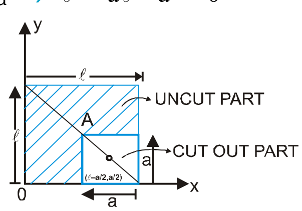
4. $y_{cm} = \dfrac{\left\{\rho\dfrac{\pi(2R)^2}{3}\times 4R\right\}\times R + \left\{12\rho\times\dfrac{4}{3}\pi R^3\right\}\times 5R}{\left\{\rho\dfrac{\pi(2R)^2}{3}\times 4R\right\}+\left\{12\rho\times\dfrac{4}{3}\pi 4R^3\right\}} = 4R$

5. $x_{cm} = \dfrac{\left(M\times\dfrac{a}{2}\right)+(M\times 0)+\left(M\times\dfrac{a}{2}\right)}{M+M+M} = \dfrac{a}{3}$

$y_{cm} = \dfrac{(M\times 0)+\left(M\times\dfrac{a}{2}\right)+\left(M\times\dfrac{a}{2}\right)}{M+M+M} = \dfrac{a}{3}$

6. $\bar{y} = \dfrac{(\rho\ell^2)\times\dfrac{\ell}{2}-(\rho a^2)\dfrac{a}{2}}{\rho\ell^2-\rho a^2} = a \Rightarrow \ell^2 - a\ell - a^2 = 0$

$\Rightarrow a = \dfrac{(\sqrt{5}-1)\ell}{2}$ x

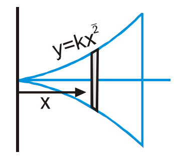


7. $x_{cm} = \dfrac{\left(\rho\dfrac{\pi\times 56^2}{4}\right)\times 28+\left(\rho\dfrac{\pi\times 42^2}{4}\right)\times 35}{\left(\rho\dfrac{\pi\times 56^2}{4}\right)-\left(\rho\dfrac{\pi\times 42^2}{4}\right)}$

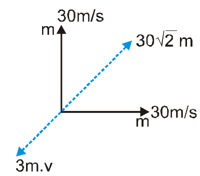
= 9cm from left edge

8. $x_{cm} = \dfrac{\left(\rho 2r^2\right)\dfrac{r}{2}-\left(\rho\dfrac{\pi r^2}{2}\right)\left(\dfrac{4r}{3\pi}\right)}{\left(\rho 2r^2\right)-\left(\rho\dfrac{\pi r^2}{2}\right)} = \dfrac{2r}{3(4-\pi)}$

9. $x_{cm} = \dfrac{\int x\,dm}{\int dm} = \dfrac{\int_0^a x\rho y\,dx}{\int_0^a \rho y\,dx} = \dfrac{3}{4}a$



10. COLM $\Rightarrow 3mv = 30\sqrt{2}m \Rightarrow v = 10\sqrt{2}$ m / s



11. **(i)** CM does not shift

(ii) Plank moves towards right.

(iii) $\Delta x_{CM} = \dfrac{m_1\Delta x_1 + m_2\Delta x_2 + M\Delta x}{m_1+m_2+M} = 0$

$\Rightarrow 0 = \dfrac{50(x+2)+70(x-2)+80x}{50+70+80}$

$\Rightarrow x = 0.2m$ (right)

(iv) $\Delta x_{m_1} = x+2 = 2.2$ m (right)

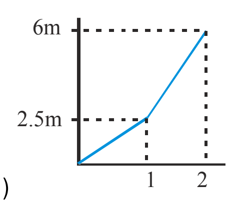
(v) $\Delta x_{m_2} = x-2 = -1.8$m (left)

12. For $0 \le t < 1$ from $v_{cm} = \dfrac{m_1v_1+m_2v_2}{m_1+m_2}$

$\Rightarrow 2 = \dfrac{(1)(1)+(2)v_2}{3}$

$\Rightarrow v_2 = 2.5\ ms^{-1}$

$\Rightarrow x_2 = 2.5t$



for $1 \le t < 2$, $2 = \dfrac{(1)(-1)+2(v_2)}{3}$

$\Rightarrow v_2 = 3.5ms^{-1} \Rightarrow x_2 = 2.5 + 3.5\,(t-1)$

13. From work energy theorem $W_F + W_g = \Delta KE$

$\Rightarrow F_{avg}(h_2 - h_1) - mg(h_3 - h_1) = 0$

$\Rightarrow F_{avg} = \dfrac{mg(h_3-h_1)}{(h_2-h_1)}$

14. Velocity of mass-1 when string is in normal length.

$$v_A = \sqrt{6g\ell - 2g\ell} = 2\sqrt{g\ell}$$

Now impulsive tension acts on both bodies to come to common velocity $v_{common} = \frac{v_A}{2} = \sqrt{g\ell}$

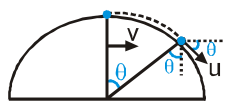
Displacement of C.M. when string becomes taut.

$$\Delta y_{1cm} = \frac{m \times \ell + m \times 0}{m+m} = \frac{\ell}{2}$$

Displacement of CM when masses reach the max. height

$$\Delta y_{2m} = \frac{v^2}{2g} = \frac{\ell}{2} \quad \therefore \Delta y_{cm} = \frac{\ell}{2} + \frac{\ell}{2} = \ell$$

15. Let v = velocity of wedge and u = velocity of particle relative to wedge



COLM

$\Rightarrow$ $m(v + u\cos\theta) + 4mv = 0$

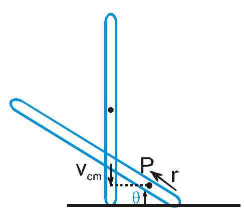
$\Rightarrow$ $u\cos\theta = -5v$; $\omega = \frac{u}{R} = \frac{5v}{R\cos\theta}$

16. In the presence of gravity, the CM shifts along vertically downward direction.

For point P : $y = r\sin\theta$

$x = (\ell/2 - r)\cos\theta$

$$1 = \left(\frac{x}{\frac{\ell}{2} - r}\right)^2 + \left(\frac{y}{r}\right)^2$$

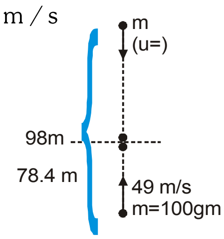


17. $\left(\frac{1}{2}gt^2\right) + \left(49t - \frac{1}{2}gt^2\right) = 98$

$\Rightarrow$ $t = 2s$ $\quad \vec{u}_1 = gt(\hat{j}) = -19.6\hat{j}\,m/s$

$\vec{u}_2 = (u - gt)\hat{j} = +29.4\hat{j}\,m/s$

$$v_f = \frac{\vec{u}_1 + \vec{u}_2}{2} = 4.9\,m/s$$

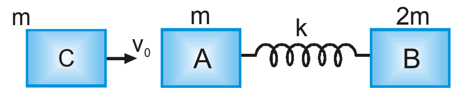


$h = 4.9 \times 2 - \frac{1}{2} \times 9.8 \times 2 \times 2 = 78.4m$

For the combined mass $x = ut + \frac{1}{2} \times 9.8\,t^2, t = 4.53$

$\therefore$ Total time of height $= 2 + 4.53 = 6.53$ sec.

18.



After collision $u_A = v_0$

(i) When $(v_{inst})_A = (v_{inst})_B$ $\Rightarrow$ $mv_0 = 3mv$ $\Rightarrow$ $v = v_0/3$

(ii) COME

$$\Rightarrow \frac{1}{2}mv_0^2 = \frac{1}{2}(3m)v^2 + \frac{1}{2}kx_0^2 \Rightarrow k = \frac{2m}{3}\left(\frac{v_0}{x_0}\right)^2$$

19. $\Delta x_{cm} = \frac{M\Delta x_1 + m\Delta x_2}{M+m} \Rightarrow 0 = \frac{Mx + m(x + R - r)}{M+m}$

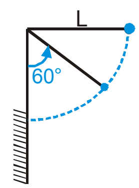
Distance moved by the cylinder $x = -\frac{m(R-r)}{M+m}$

For motion along x-axis $0 = m(v_1 + v_2) + mv_2$ ....(i)

$mg(R - r) = \frac{1}{2}m(v_1 + v_2)^2 + \frac{1}{2}Mv_2^2$ ....(ii)

$$\therefore \quad v_2 = m\sqrt{\frac{2g(R-r)}{M(M+m)}}$$

20. After I[st] collision



$$\frac{2}{\sqrt{5}}\sqrt{2g\ell} = \sqrt{2g\ell(1-\cos\theta)}$$

$\Rightarrow$ $\cos\theta = \frac{1}{5}$

For II[nd] collision

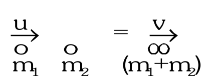
$$\frac{2}{\sqrt{5}} \cdot \frac{2}{\sqrt{5}}\sqrt{2g\ell} = \sqrt{2g\ell(1-\cos\theta)}$$

For n[th] collision

$$\left(\frac{2}{\sqrt{5}}\right)^n = \sqrt{1-\cos\theta} \Rightarrow \left(\frac{4}{5}\right)^n = 1 - \cos\theta$$

$\cos\theta = 1 - \left(\frac{4}{5}\right)^n$ Put n=0,1,2,3 and get answer

21.



COLM $\Rightarrow$ $m_1u + 0 = (m_1 + m_2)v$ ....(i)

Energy equation

$\left(\frac{1}{2}mu^2\right) \times \frac{2}{3} = \frac{1}{2}(m_1 + m_2)v^2$ ....(ii)

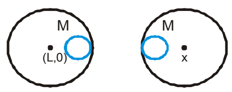
$\Rightarrow$ $\frac{m_1}{m_2} = 2:1$

22. In this elastic collision velocity of masses are exchanged.

So $v_A = 0$ $\Rightarrow$ A does not rise

23. $x_{cm} = \frac{4ML + M(L+5R)}{5M} = \frac{4MX + M(X-5R)}{5M}$

$\Rightarrow x = L + 2R$



24. No, KE is not conserved during the short time of collision.

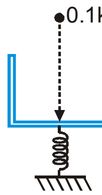
25. COME $\Rightarrow mg(h + x_0) = \frac{1}{2}kx_0^2$

$\Rightarrow mg(0.24 + 0.01)$

$= \frac{1}{2}k(0.01 \times 0.01)$ .....(i)

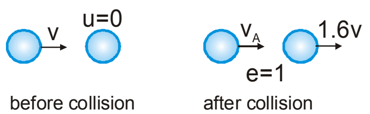
$\& \; mg(h + 0.04) = \frac{1}{2}k(0.04)^2$

.....(ii)



Equation (i) divided by (ii) h= 3.96 m

26.



COLM $= -\frac{(1.6v) - v_A}{0 - v} \; v_A = 0.6V$ ...(i)

COLM $\Rightarrow m_A v = m_A \times (0.6V) + m_B(1.6v)$

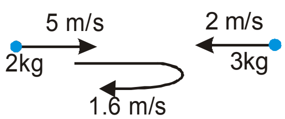
$\frac{m_A}{m_B} = 4$ ...(ii)

$\Delta K_B = \frac{1}{2}m_B(1.6v)^2 = 0 = 2m_B(0.8v)^2$

$K_A = \frac{1}{2}(4m_B)v^2 = 2m_B v^2 \Rightarrow \frac{\Delta K_B}{K_A} = 0.64 = 64\%$

27. COLM: implies that $\vec{v}_C \; \& \; \vec{v}_B$ are opposite to each other.

28.



(A) $v_{cm} = \left[\frac{(2\times5) - (3\times2)}{2+3}\right]\hat{i} = \frac{4}{5}\hat{i} \;\; m/s$

(B) COLM $\Rightarrow (2\times 5) + 3(-2) = +2(-1.6) + 3v_2$

$\Rightarrow v_2 = 2.4$ m/s

(C) $e = -\left(\frac{v_2 - v_1}{u_2 - u_1}\right) = \frac{4}{7}$

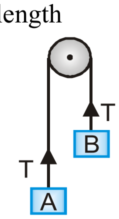
29. Velocity of B when string is in natural length

$= u_B = \sqrt{2gh} = \sqrt{2g} \; (h = 1)$

Impulse equation

$\Rightarrow -T\Delta t = m[v - \sqrt{2g}\,]$ ...(i)

$T.\Delta t = m[v - 0]$ ...(ii)



On solving eq. (ii) & (ii) : $v = \frac{\sqrt{2}g}{2} = \frac{g}{\sqrt{2}}$

$\therefore$ Distance travelled by A before coming to rest,

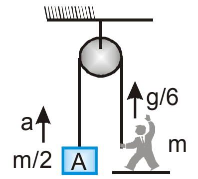
$s = 1 + \frac{v^2}{2g} = 1.25m$

30. $mg - T = m\left(a - \frac{g}{6}\right)$ ...(i)

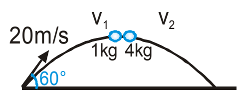
$\Rightarrow T - \frac{m}{2}g = \frac{m}{2}a$ ...(ii)

For solving eq. (ii) & (ii)

$a = \frac{4g}{9} \;\; \& \;\; T = \frac{13}{18}mg$



31. COLM : $1 \times v_1 + 4 \times v_2 = 5 \times 20 \cos 60° = 50$ ...(i)



COME $\Rightarrow \frac{1}{2}\times 1 \times v_1^2 + \frac{1}{2}\times 4 \times v_2^2$

$= \left[\frac{1}{2}5 \times (10)^2\right] \times 2$ ...(ii)

$\Rightarrow \; v_1 = -10$ or 30 m/s & $v_2 = 15$ or 5 m/s

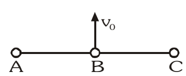
$\Rightarrow \; \Delta v = 25$ m/s

Time to fall down to ground

$= \frac{\sqrt{2h}}{g} = \frac{\sqrt{3}}{.98}$ sec.

$\therefore$ Separation between particles = $\Delta v.t = 44.2m$

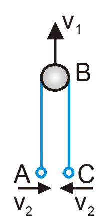
32.



COLM $\Rightarrow mv_0 = 3mv_1$

$v_1 = \frac{v_0}{3}$ ...(i)

COME



$\Rightarrow \frac{1}{2}mv_0^2 = \frac{1}{2}(3m)v_1^2 + \frac{1}{2}(2m)v_0^2 \Rightarrow v_2 = \frac{v_0}{\sqrt{3}}$

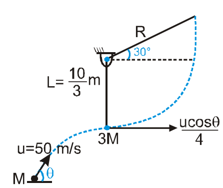
Therefore velocity of A = $\sqrt{v_1^2 + v_2^2} = 6$ m/s

33. (i) $\left(\frac{u\cos\theta}{4}\right)^2 = 2g\left(\frac{3R}{2}\right)$

$\Rightarrow \cos\theta = 4/5 \Rightarrow \theta = 37°$

(ii) $x = \frac{R}{2} = \frac{u^2 \sin^2\theta}{2g} = 120m$

$y = H = \frac{u^2 \sin^2\theta}{2g} = 45m$



34. $u_s = \sqrt{2g(1-\cos 60°)} = \sqrt{2g \times \frac{1}{2}} = 3.13\,m/s$

COLM $\Rightarrow 5u_B = 4 \times u_s \quad \Rightarrow u_B = \frac{4}{5}\sqrt{g} = 2.53$ m/s

Energy equation

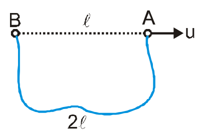
$$\frac{1}{2}m\left(\frac{4}{5}\sqrt{g}\right)^2 = \mu mg \times 0.8 \quad \Rightarrow \mu = 0.4$$

$$e = -\left(\frac{v_2 - v_1}{u_2 - u_1}\right) = 0.8$$

35. $\Delta x_{cm} = 0 = \dfrac{mx_0 + m_1(x_0 - h\cot\alpha) + m_2(x_0 - h)}{m + m_1 + m_2}$

$$\Rightarrow x_0 = \frac{h(m_2 + m_1\cot\alpha)}{(m + m_1 + m_2)}$$

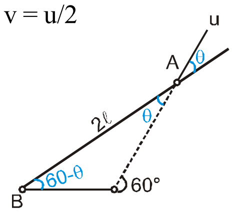
36. **(a)** $mu - T\Delta t = mv$ ...(i)



$T\Delta t = mv$ ...(ii)

On solving eq. (i) & (ii) $\Rightarrow v = u/2$

(b) $\dfrac{2\ell}{\sin 120°} = \dfrac{\ell}{\sin\theta}$



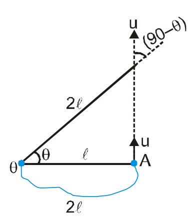
$\sin\theta = \dfrac{\sqrt{3}}{4}$; $\cos\theta = \dfrac{\sqrt{13}}{4}$

$mu\cos\theta - T\Delta t = mv$ ...(i)

$\Rightarrow T\Delta t = mv$ ...(ii)

On solving eq. **(i)** & **(ii)** $\dfrac{u\cos\theta}{2} = v$

(c) $2\ell\cos\theta = \ell \Rightarrow \theta = 60°$



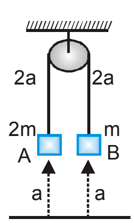
$mu\cos 30° - T\Delta t = mv$ ..(i)

$T\Delta t = \dfrac{mu\sqrt{3}}{4}$ ...(ii)

37. $2mg - T = 2m.a$ ...(i)

$T - mg = m.a$ ..(ii)

On solving eq. (i) & (ii) $a = \dfrac{g}{3}$



(a) $v_B = \sqrt{u^2 + 2as} = \sqrt{0 + 2\left(\frac{g}{3}\right)a} = \sqrt{\frac{2ag}{3}}$

(b) $s = ut + \frac{1}{2}at^2$, $a = 0 + \frac{1}{2}\left(\frac{g}{3}\right)t^2$, $t = \sqrt{\frac{6a}{g}} = \frac{3v}{g}$

(c) $t = \dfrac{2v}{g}$

38. COLM $\Rightarrow mu = (m + \rho Ax)v \Rightarrow v = \dfrac{mu}{m + \rho Ax}$

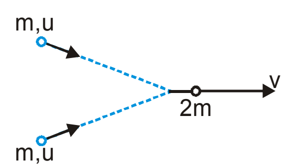
$$\Rightarrow \frac{dx}{dt} = \frac{mu}{m + \rho Ax} \Rightarrow \int_0^{150}(m + \rho Ax)dx = \int_0^{150} mudt$$

$$\Rightarrow \left(mx + \rho\frac{Ax^2}{2}\right) = mut$$

$$\Rightarrow 10^{-2}x + 10^{-3} \times \frac{10^{-4}}{2}x^2 = 10^{-2} \times 10^3 \times 150$$

$x = 10^5$ m

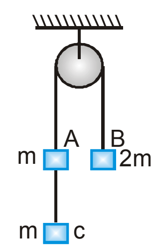
39. $2mu\cos\dfrac{\theta}{2} = 2mv$



$$\Rightarrow v = u\cos\frac{\theta}{2} = \frac{u}{2}$$

$$\Rightarrow \cos\frac{\theta}{2} = \frac{1}{2};\ \frac{\theta}{2} = 60°,\ \theta = 120°$$

40. $2mg - T = 2ma;\ T - mg = ma;\ a = \dfrac{g}{3}$



Velocity of m & 2m after falling through a distance

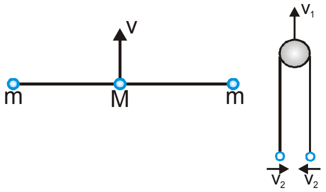
$x = \sqrt{2ax} = \sqrt{\dfrac{2gx}{3}}$

Impulse equation

$$T\Delta t = 2m\left(v - \sqrt{\frac{2gx}{3}}\right)$$

$$T\Delta t - T'\Delta t = m\left(v - \sqrt{\frac{2gx}{3}}\right)$$

$T'\Delta t = m(v-0)$, $v = \sqrt{\dfrac{3gx}{8}}$

41. 

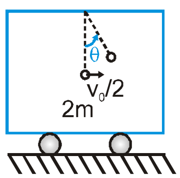
(i) COLM $\Rightarrow$ $(m+m)\times 0 + Mv = (M+2m)v_1$

$$v_1 = \left(\frac{M}{M+2m}\right)v$$

(ii) COME $\Rightarrow$ $\frac{1}{2}MV^2 = \left[\frac{1}{2}m(v_1^2+v_2^2)\right]\times 2 + \frac{1}{2}mv_1^2$

Net velocity $v_0 = \sqrt{v_1^2+v_2^2} = v\frac{\sqrt{2M(M+m)}}{(M+2m)}$

42. COLM : $mv_0 = (M+2m)\,v_1$



$$v_1 = \frac{mv_0}{6m} = \frac{v_0}{6} = 1\text{m/s}$$

COME :

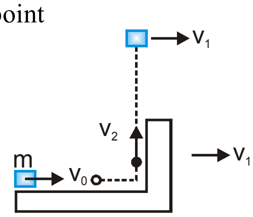
$$\Rightarrow \frac{1}{2}(2m)\left(\frac{v_0}{2}\right)^2 = \frac{1}{2}Mv_1{}^2 + 2m\times g\times h + \frac{1}{2}(2m)v_1{}^2$$

$\Rightarrow$ $h = 0.3\text{ m} = L\,(1-\cos\theta)$

$\cos\theta = 1 - \frac{3}{15} = \frac{4}{5}$ $\Rightarrow$ $\theta = 37°$

43. COLM $\Rightarrow$ $mv_0 + 0 = (m+2m)\,v_1 \Rightarrow v_1 = \frac{v_0}{3}$

After collision at highest point



$$v_x = 1\text{m/s}\left(=\frac{mv_1}{2m}\right)$$

$$v_y = 1\text{m/s}\left(=\frac{m\times 2}{2m}\right)$$

COME $\Rightarrow$ $\frac{1}{2}mv_0^2 = \frac{1}{2}m(v_1^2+v_2^2) + \frac{1}{2}(2m)v_1^2$

$\Rightarrow$ $v_2 = \sqrt{24}$ m/s

Max height attained $= \frac{v_2^2}{2g} = 1.2\text{m}$

For the block $v_x = 1\text{m/s}$
while for the wedge it has
$v_x = 2\text{m/s}$

$(v_{x_{wedge}} - v_{x_{block}})t = \ell$ & $\left(ut + \frac{1}{2}at^2\right)$ block $= 1.2$

$\Rightarrow$ $t = 0.4$ sec and $\ell = (2-1)t = 0.4\text{ m} = 40\text{ cm}$

44. Let v = velocity of the ball after collision along the normal

J = impulse on ball

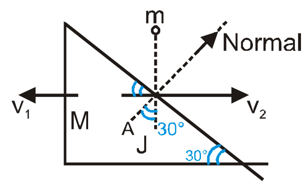
$= v - (-2\cos 30°) = v + \sqrt{3}$

Impulse on wedge

$J\sin 30° = mv_1 = 2v_1$

$\Rightarrow$ $v = 4v_1 - \sqrt{3}$ .....(i)

Coefficient of restitution



$$e = -\left(\frac{v_2 - v_1}{u_2 - u_1}\right) \Rightarrow \frac{1}{2} = \frac{\left(v + \frac{v_1}{2}\right)}{2\cos 30°}$$

$\Rightarrow$ $v = \frac{\sqrt{3}}{2} - \frac{v_1}{2}$ .....(ii)

Solving we get $v_1 = \frac{1}{\sqrt{3}}$ m/s

For the ball velocity along incline remains constant.

$\therefore$ $v' = 2\sin 30° = 1$ m/s

$\therefore$ Final velocity of ball $= \sqrt{1^2 + \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{2}{\sqrt{3}}$ m/s

45. For first collision with plate A, final velocity of ball

$v_1 = ev_0 = e\sqrt{2gh_0}$ .....(i)

For second collision $mv = 4mv'$

$\Rightarrow$ $v' = \frac{v}{4}$ $\Rightarrow$ $e\sqrt{\frac{2gh_0}{4}} = \sqrt{2gh_2}$ $\Rightarrow$ $e = \frac{2}{3}$

Height attained after first collision

$\therefore$ $h_1 = e^2h_0 = \frac{2}{3}\times\frac{2}{3}\times 9 = 4\text{m}$

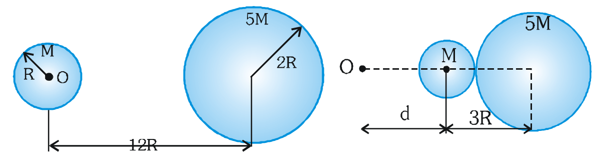
EXERCISE - 5

Part # I : AIEEE/JEE-MAIN

1. $v_{cm} = \dfrac{m(2v) + m(-v)}{m + m} = \dfrac{v}{2}$

2. Linear momentum is a vector quantity whereas kinetic energy is a scalar quantity.

3.



Initial position — Just before collision

For this system, position of centre of mass remains same

$\left[\because \vec{F}_{system} = 0\right]$

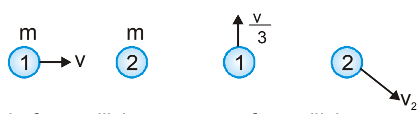
$$\frac{M(0) + 5M(12R)}{M + 5M} = \frac{M(d) + 5M(d + 3R)}{M + 5M} \Rightarrow d = 7.5R$$

4. In order to shift centre of mass, the system must experience an external force, as there is no external force responsible for explosion, hence centre of mass does not shift.

5. Let maximum momentum be p then

$$\frac{p^2}{2M} = \frac{1}{2}kL^2 \Rightarrow p = L\sqrt{Mk}$$

6.

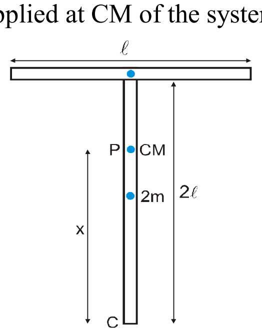


before collision — after collision

From COLM

$$mv_2 = \sqrt{(mv)^2 + \left(m\frac{v}{\sqrt{3}}\right)^2} \Rightarrow v_2 = \frac{2v}{\sqrt{3}}$$

7. The object will have translation motion without rotation, when $\vec{F}$ is applied at CM of the system.



If P is the CM then

$$m(2\ell - x) = 2m(x - \ell) \Rightarrow x = \frac{4\ell}{3}$$

8. On applying law of conservation of linear momentum

$$\vec{P}_i = \vec{P}_f \Rightarrow 16 \times \vec{0} = 4\vec{v}_4 + 12 \times 4\hat{i} \Rightarrow \vec{v}_4 = 12(-\hat{i})$$

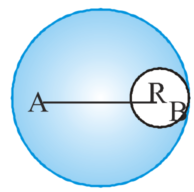
The 4 kg block will move in a direction opposite to 12 kg block with a speed of 12 m/s. The corresponding kinetic energy of 4 kg block

$$= \frac{1}{2} \times 4 \times (12)^2 = 288 \text{ J}$$

9.

Here $m_1 d = m_2 x \Rightarrow x = \dfrac{m_1}{m_2}d$

10. Since mass ∝ area



Let mass of the bigger disc = 4M

∴ mass of the smaller disc = M

∴ mass of the remaining portion = 4M – M = 3M

Now put the cut disc at its place again, centre of mass of the whole disc will be at centre O.

Centre of mass of the smaller disc is at its centre that is at B.

Suppose CM of the remaining portion is at A and AO is X.

Let O as origin

$\therefore$ 3M(x)=RM $\Rightarrow x = \dfrac{R}{3}$

This suggests that centre of mass of remaining disc will shift from the centre of original disc by a distance of (1/3)R towards left.

$$\alpha = \frac{1}{3}$$

11. Energy loss $= \dfrac{1}{2}\dfrac{m_1 m_2}{(m_1 + m_2)}(u_1 - u_2)^2$

$$= \frac{(0.5)(1)}{2[0.5 + 1]}(2 - 0)^2 = \frac{2}{3}\text{J}$$

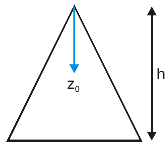
12. $X_{cm} = \dfrac{\int x\,dm}{\int dm} = \dfrac{\int_0^L k\left(\frac{x}{L}\right)^n x\,dx}{\int_0^L k\left(\frac{x}{L}\right)^n dx} = \left(\dfrac{n+1}{n+2}\right)L$

For n=0, $x_{cm} = \dfrac{L}{2}$ and for $n \to \theta$, $x_{cm} = L$

15. 1

16. $z_0 = \dfrac{3h}{4}$

(From class theory)



17. $I = \dfrac{Mx^2}{6}$

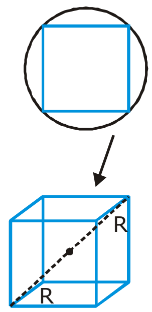
edge length : (x)

$2R = \sqrt{3}\,x \Rightarrow x = \dfrac{2R}{\sqrt{3}}$

Now, mass of cube :

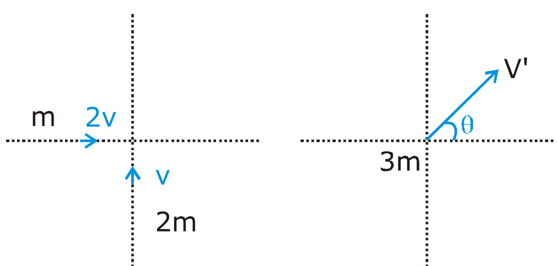
$m = \dfrac{M}{\left(\dfrac{4}{3}\pi R^3\right)}\left(\dfrac{2R}{\sqrt{3}}\right)^3$

$\left(\dfrac{3M}{4\pi R^3}\right)\left(\dfrac{8R^3}{3\sqrt{3}}\right) \quad m = \dfrac{2M}{\sqrt{3}\pi}$



$I = \dfrac{1}{3}\left(\dfrac{2M}{\sqrt{3}\pi}\right)\left[\dfrac{4R^2}{3}\right] = \dfrac{4MR^2}{9\sqrt{3}\pi}$

18.



before collision $\quad$ After collision

$\vec{P}_x = 2mv\,\hat{i} \quad \vec{P}_x = 3mv'\cos\theta$

$\vec{P}_y = 2mv\,\hat{j} \quad \vec{P}_y = 3\,v'\sin\theta$

By momentum conservation ;

in horizontal $\rightarrow 2mv = 3mv'\cos\theta$ ...(i)

in vertical $\rightarrow 2mv = 3mv'\sin\theta$ ...(ii)

from (i) and (ii) $\tan\theta = 1;\ \theta = 45°$

final speed $v' = \dfrac{2\sqrt{2}v}{3}$

initial K.E. ;$\rightarrow 1/2(m)(2v)^2 + 1/2(2m)(v)^2 = 3mv^2$

final K.E. ; $\rightarrow 1/2\,(3m)\left(\dfrac{2\sqrt{2}v}{3}\right)^2 = 4/3\ mv^2$

% loss $\rightarrow \dfrac{(KE)_i - (KE)_f}{(KE)_i} \times 100\%$

$= 55.55 \simeq 56\%$

Part # II : IIT-JEE ADVANCED

Sol. **1 or 3**

By right hand thumb rule

1. By applying impulse-momentum theorem

$= \left|\left(m_1\vec{v}_1' + m_2\vec{v}_2'\right) - \left(m_1\vec{v}_2'\right)\right|$

$= \left|\left(m_1 + m_2\right)\vec{g}\left(2t_0\right)\right| = 2\left(m_1 + m_2\right)gt_0$

2. Just after collision

$v_c = \dfrac{10 \times 14 + 4 \times 0}{10 + 4} = 10\ m/s$

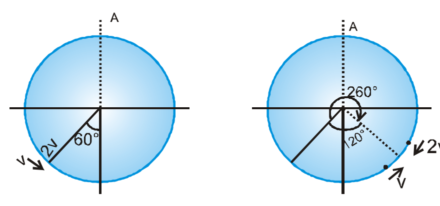
since spring force is internal force, it cannot change the linear momentum of the (two mass + spring) system. Therefore v_c remains the same.

3. $\vec{p}(t) = A[\hat{i}\cos(kt) - \hat{j}\sin(kt)]$

$\vec{F} = \dfrac{d\vec{p}}{dt} = Ak[-\hat{i}\sin(kt) - \hat{j}\cos(kt)]$

$\vec{F}.\vec{p} = Fp\cos\theta$ But $\vec{F}.\vec{p} = 0 \Rightarrow \cos\theta = 0 \Rightarrow \theta = 90°.$

4.



1st collision $\quad$ IInd collision

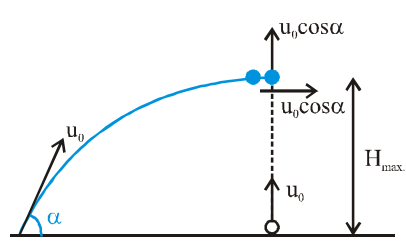
Particle with velocity 'v' covers and angle of 120° and after collision its velocity become '2v'.

It will cover angle of 240°

5. $Y_{cm} = \dfrac{m_1y_1 + m_2y_2 + m_3y_3}{M_1 + M_2 + M_3}$

$Y_{cm} = \dfrac{6m(0) + m(+a) + m(a) + m(-a) + m(0)}{10m}$; $Y_{cm} = \dfrac{a}{10}$

6.



Maximum height of first particle $H_{max} = \dfrac{u_0^2\sin^2\alpha}{2g}$

Speed of 2nd particle at height H_{max} given as

$v_y^2 = u_0^2 - 2gH_{max} = u_0^2 - u_0^2\sin^2\alpha \Rightarrow v_y = u_0\cos\alpha$

By Momentum Conseravtion

$\vec{p}_f = \vec{p}_i \Rightarrow 2m\vec{v}_f = mv_0\cos\alpha\hat{i} + mv_0\cos\alpha\hat{j}$

$\Rightarrow \ \vec{v}_f = \dfrac{v_0 \cos\alpha}{2}(\hat{i}+\hat{j})$

$\Rightarrow$ Angle with horizontal immediately after the collision

$= \dfrac{\pi}{4}$

Multiple Choice Questions

1. As $\vec{p}_1 + \vec{p}_2 = \vec{0}$ so $\vec{p}_1' + \vec{p}_2' = \vec{0}$

For (A) $\vec{p}_1' + \vec{p}_2' = (a_1 + a_2)\,\tilde{i} + (b_1 + b_2)\,\tilde{j} + c_1\,\tilde{k}$

For (B) $\vec{p}_1' + \vec{p}_2' = (a_1 + a_2)\,\tilde{i} + (b_1 + b_2)\,\tilde{j}$

For (C) $\vec{p}_1' + \vec{p}_2' = (c_1 + c_2)\,\tilde{k}$

For (D) $\vec{p}_1' + \vec{p}_2' = (a_1 + a_2)\,\tilde{i} + 2b_1\,\tilde{j}$

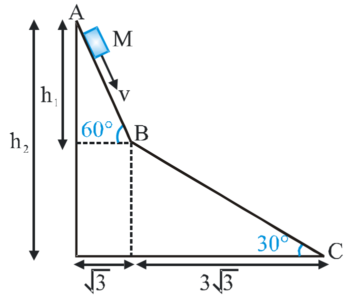
But $a_1, b_1, c_1, a_2, b_2, c_2 \neq 0$

Therefore (A) & (D) is not possible to get

$\vec{p}_1' + \vec{p}_2' = \vec{0}$

Comprehension type questions

1.

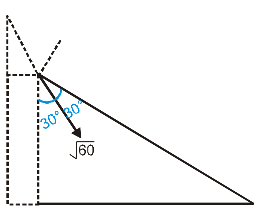


$\dfrac{h_1}{\sqrt{3}} = \tan 60° \Rightarrow h_1 = 3\text{m}$

$\dfrac{h_2 - h_1}{3\sqrt{3}} = \tan 30° \Rightarrow h_2 - h_1 = 3 \Rightarrow h_2 = 6\text{m}$

Velocity of block just before collision at B

$= \sqrt{2gh} = \sqrt{2 \times 10 \times 3} = \sqrt{60}\ \text{ms}^{-1}$



For totally inelastic collision velocity of block along normal to BC becomes zero and since there is no impulse along BC so momentum (velocity) along BC remains unchanged

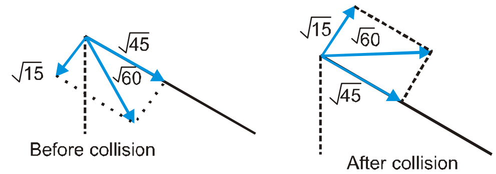
Speed of block just after collision

$v_B = \sqrt{60}\cos 30° = \sqrt{60} \times \dfrac{\sqrt{3}}{2} = \sqrt{45}\text{ms}^{-1}$

2. $v_c^{\,2} = v_B^{\,2} + 2g(h_2 - h_1)$

$\Rightarrow \ v_c = \sqrt{45^2 + 2 \times 10 \times 3} = \sqrt{105}\ \text{ms}^{-1}$

3.



$\Rightarrow$ vertical component of velocity is zero

Subjective Questions

1. For body of mass m from A to B

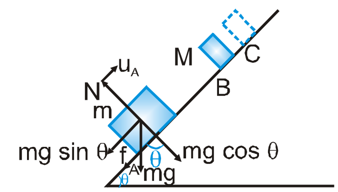
u = 10 m/s (given)

$a = -\left[\dfrac{mg\sin\theta + f}{m}\right] = -\left[\dfrac{mg\sin\theta + \mu mg\cos\theta}{m}\right]$

$= -[g\sin\theta + \mu g\cos\theta] = -g[\sin\theta + \mu\cos\theta]$

$= -10[0.05 + 0.25 \times 0.99] = -2.99\ \text{m/s}^2$

$v^2 - u^2 = 2as \Rightarrow v = \sqrt{100 + 2 - (-2.99) \times 6} = 8\ \text{m/s}$



After collision

Let v_1 be the velocity of mass m after collision and v_2 be the velocity of mass M after collision.Body of mass M moving from B to C and coming to rest.

$u = v_2;\quad v = 0,\quad a = -2.99\ \text{m/s}^2$

and $s = 0.5\quad v^2 - u^2 = 2as$

$\Rightarrow (0)^2 - v_2^2 = 2(-2.99) \times 0.5 \Rightarrow v_2^{\,2} = 1.73\ \text{m/s}$

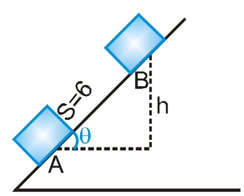
Body of mass m moving from B to A after collision

$u = v_1;\ v = +1\ \text{m/s}$

$(\text{K.E.} + \text{P.E.})_{\text{initial}} = (\text{K.E.} + \text{P.E.})_{\text{final}} + W_{\text{frication}}$

$\dfrac{1}{2}mv_1^2 + mgh = \dfrac{1}{2}mv^2 + 0 + \mu mgs$

$\dfrac{1}{2}v_1^2 + 10 \times (6 \times 0.05) = \dfrac{1}{2}(1)^2 + 0.25 \times 10 \times 6$



$v_1 = -5\text{m/s}$

$\sin\theta = \dfrac{h}{6}$

$h = 6\sin\theta = 6 \times 0.05$

$\therefore$ **Coefficient of restitution**

$e = \left|\dfrac{\text{Relative velocity of seperation}}{\text{Relative velocity of approach}}\right| = \left|\dfrac{-5 - 1.73}{8 - 0}\right| = 0.84$

2. Consider the vertical motion of the cannon ball

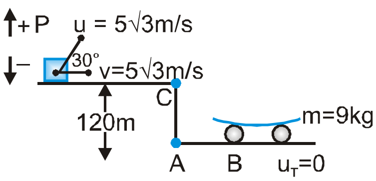
$\because \quad S = ut + \frac{1}{2}at^2 \qquad \therefore \quad -120 = 50t_0 - 5t_0^2$

$\Rightarrow \quad 5t_0^2 - 50t_0 - 120 = 0 \qquad \Rightarrow \quad t_0^2 - 10\,t_0 - 24 = 0$

$\therefore \quad t_0 = -\frac{(-10) \pm \left[\sqrt{100 - 4(1)(-24)}\right]}{2} = 12 \text{ or } -2$

The horizontal velocity of the cannon ball remains the same

$\therefore \quad v_x = 100\cos 30° + 5\sqrt{3} = 55\sqrt{3}$ m/s



$\therefore$ Apply conservation of linear momentum to the cannon ball-trolley system in horizontal direction. If m is the mass of cannon ball and M is the mass of the trolley then

$mv_x + M \times 0 = (m + M)\,V_x \quad \therefore \; V_x = \frac{mv_x}{m + M}$

where v_x is the velocity of the (cannon ball- trolley) system $V_x = \frac{1 \times 55\sqrt{3}}{1 + 9} = 5.5\sqrt{3}$ m/s

The second ball was projected after 12 second.

Horizontal distance covered by the car

$P = 12 \times 5\sqrt{3} = 60\sqrt{3}$m

Since the second ball also struck the trolley

$\therefore$ In time 12 seconds the trolley covers a distance of $60\sqrt{3}$

For trolley in 12 sec

From

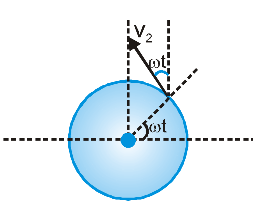
$$s = \left(\frac{u+v}{2}\right)60\sqrt{3} = \left(\frac{5.5\sqrt{3} + v}{2}\right)(12) \Rightarrow v = 7.8 \text{ m/s}$$

To find the final velocity of the carriage after the second impact we again apply conservation of linear momentum in the horizontal direction

$m\,v_x + (M + m)7.8 = (M + 2m)\,v_f$

$\therefore 1 \times 55\sqrt{3} + (9 + 1)7.8 = (9 + 2)v_f \Rightarrow v_f = 15.75$ m/s

3.



$\vec{v}_2 = (-v_2 \sin\omega t\hat{i} + v_2\cos\omega t\hat{j})$ and $\vec{v}_1 = v_1\hat{j}$

$\vec{v}_{21} = \vec{v}_2 - \vec{v}_1 = -v_2\sin\omega t\hat{i} + (v_2\cos\omega t - v_1)\hat{j}$

$\vec{p}_{21} = m\vec{v}_{21} = -mv_2\sin\omega t\hat{i} + m(v_2\cos\omega t - v_1)\hat{j}$

where $\omega = \frac{v_2}{R}$

4. The string snaps and the spring force comes into play. The spring force being an internal force for the two mass-spring system will not be able to change the velocity of centre of mass. This means the location of centre of mass at time t will be v_0 t

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = v_0 t$$

$\Rightarrow \quad m_1[v_0t - A(1 - \cos\omega t)] + m_2x_2 = v_0tm_1 + v_0tm_2$

$\Rightarrow \quad m_2x_2 = v_0tm_1 + v_0tm_2 - v_0tm_1 + m_1A(1 - \cos\omega t)$

$\Rightarrow \quad m_2x_2 = v_0tm_2 + m_1A(1 - \cos\omega t)$

$\Rightarrow \quad x_2 = v_0t + \frac{m_1}{m_2}A(1 - \cos\omega t)$

(b) Given that $x_1 = v_0t - A(1 - \cos\omega t)$

$\therefore \; \frac{dx_1}{dt} = v_0 - A\omega\sin\omega t \Rightarrow \therefore \frac{d^2x_1}{dt^2} = -A\omega^2\cos\omega t \qquad$ ... (i)

This is the acceleration of mass m_1. When the spring comes to its natural length instantaneously then

$\frac{d^2x_1}{dt^2} = 0$ and $x_2 - x_1 = \ell_0$

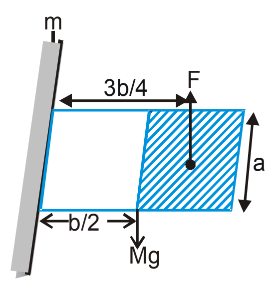
$\therefore \; \left[v_0t + \frac{m_1}{m_2}A(1 - \cos\omega t)\right] - [v_0t - A(1 - \cos\omega t)] = \ell_0$

$\left(\frac{m_1}{m_2} + 1\right)A(1 - \cos\omega t) = \ell_0$

Also when $\frac{d^2x_1}{dt^2} = 0$; $\cos\omega t = 0$ from (1)

$\therefore \quad \ell_0 = \left(\frac{m_1}{m_2} + 1\right)A$

5. Since the plate is held horizontal therefore net torque acting on the plate is zero.



$\Rightarrow \quad Mg \times \frac{b}{2} \times F \times \frac{3b}{4} \qquad$ ..... (i)

$$F = n\frac{dp}{dt} \times (\text{Area}) = n \times (2mv) \times a \times \frac{b}{2} \qquad ..... (ii)$$

From (i) and (ii) $Mg \times \frac{b}{2} = n \times (2mv) \times a \times \frac{b}{2} \times \frac{3b}{4}$

$\Rightarrow \quad 3 \times 10 = 100 \times 2 \times 0.01 \times v \times 1 \times \frac{3 \times 2}{4}$

$\Rightarrow \quad v = 10$ m/s

6. For collision between A & B

$$v_A = \frac{(m-2m)}{(m+2m)}(9) = -3ms^{-1} \Rightarrow v_B = \frac{2(m)}{(m+2m)}(9) = +6ms^{-1}$$

For collision between B and C

$$v_C = \left(\frac{2m}{2m+m}\right)(6) = 4ms^{-1}$$

MOCK TEST

1. Since $\Sigma \vec{F}_{ext} = \vec{0}$

$\therefore$ Momentum of system will remain conserved, equal to zero.

2. The line of impact for duration of collision is parallel to x-axis.

The situation of striker and coin just before the collision is given as

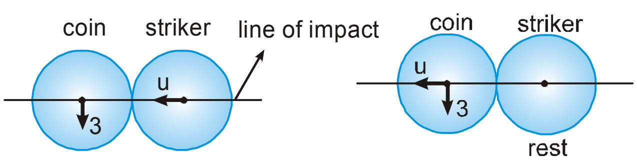
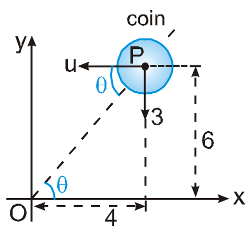


Figure (A) before collision

Figure (B) after collision

Because masses of coin and striker are same, their components of velocities along line of impact shall exchange. Hence the striker comes to rest and the x-y component of velocities of coin are u and 3 m/s as shown in figure.



For coin to enter hole,
its velocity must be along PO

$\therefore \quad \tan\theta = \frac{6}{4} = \frac{3}{u} \quad$ or $\quad u = 2$ m/s

3. If we treat the train as a ring of mass 'M' then its COM will be at a distance $\frac{2R}{\pi}$ from the centre of the circle.

Velocity of centre of mass is :

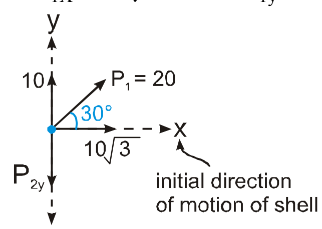
$$V_{CM} = R_{CM}.\omega = \frac{2R}{\pi}.\omega = \frac{2R}{\pi}\cdot\left(\frac{V}{R}\right) \quad (\because \omega = \frac{V}{R})$$

$$\Rightarrow V_{CM} = \frac{2V}{\pi} \Rightarrow MV_{CM} = \frac{2MV}{\pi}$$

As the linear momentum of any system $= MV_{CM}$

$\therefore$ The linear momentum of the train $= \frac{2MV}{\pi}$

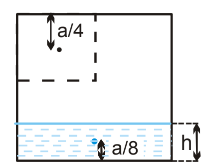
4. As shown in figure the component of momentum of one shell along initial direction and perpendicular to initial direction are $P_{1X} = 10\sqrt{3}$ Ns and $P_{1y} = 10$Ns.



For momentum of the system to be zero in y-direction P_{2y} must be 10 Ns. 2[nd] part of shell may or may not have momentum in x-direction

5. Let h be the height of water surface, finally

$a^2h = a\,.\,\frac{a}{2}\,.\,\frac{a}{2}\;;\; h = \frac{a}{4}$



$\therefore$ C.M. gets lowered by

$$a - \left(\frac{a}{4} + \frac{a}{8}\right) = a - \frac{3a}{8} = \frac{5a}{8}$$

$\therefore$ Work done by gravity $= mg\frac{5a}{8}$

6. Neglecting gravity,

$$v = u\ell n\left(\frac{m_0}{m_t}\right);$$

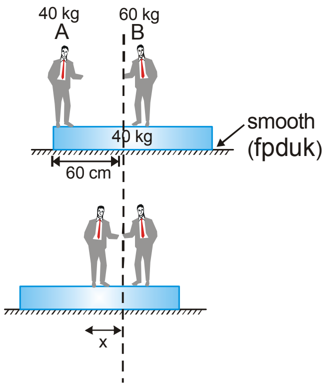
u = ejection velocity w.r.t. balloon .
m_0 = initial mass
m_t = mass at any time t.

$$= 2\ell n\left(\frac{m_0}{m_0/2}\right) = 2\ell n2.$$

7. Let the tube displaced by x towards left, then

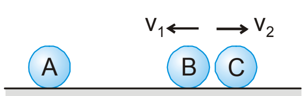
$$mx = m(R-x) \Rightarrow x = \frac{R}{2}$$

8. Taking the origin at the centre of the plank.

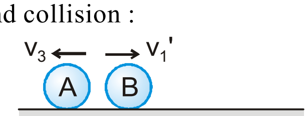


$m_1\Delta x_1 + m_2 \Delta x_2 + m_3\Delta x_3 = 0 \quad (\because \Delta x_{CM} = 0)$
(Assuming the centres of the two men are exactly at the axis shown.)
$60(0) + 40(60) + 40(-x) = 0$, x is the displacement of the block.
$\Rightarrow \quad x = 60$ cm
i.e. A & B meet at the right end of the plank.
$m_1\Delta x_1 + m_2 \Delta x_2 + m_3\Delta x_3 = 0 \quad (\because \Delta x_{CM} = 0)$
$60(0) + 40(60) + 40(-x) = 0, \quad \Rightarrow x = 60$ cm

9. For 1st collision
Since e = 1;



$v = v_1 + v_2 \Rightarrow v_2 = v - v_1$ .... **(1)**
By momentum conservation :
$m_B v = - m_B v_1 + m_C v_2$
or $m_B v = - m_B v_1 + 4 m_B v_2 \quad (\because m_C = 4 m_B)$
$\Rightarrow v_2 = \dfrac{v_1 + v}{4}$ .... **(2)**
From (1) to (2) : $v_1 = \dfrac{3}{5}v$ and $v_2 = \dfrac{2}{5}v$
For second collision :

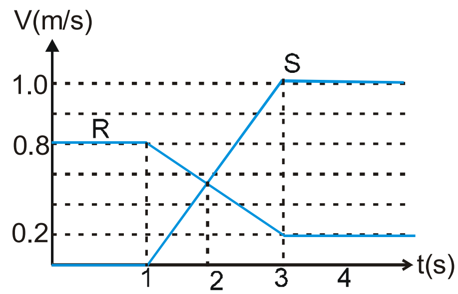


$e = 1 \Rightarrow v_1 = v_1' + v_3 \Rightarrow v_3 = v_1 - v_1'$ .... **(3)**
By momentum conservation :
$- m_B v_1 = m_B v_1' - m_A v_3$
or $- m_B v_1 = m_B v_1' - 4 m_B v_3 \quad (\because m_A = 4 m_B)$
$\Rightarrow v_3 = \dfrac{v_1' + v_1}{4}$ .... **(4)**
From **(3)** and **(4)** :
$$v_1' = \frac{3}{5}v_1 = \frac{3}{5}\left(\frac{3}{5}v\right) = \frac{9}{25}v$$

Clearly % $\dfrac{9}{25}v < \dfrac{2}{5}v$
Therefore 'B' can not collide with 'C' for the second time.
Hence ; total number of collisions is 2.

10. Force on table due to collision of balls :
$$F_{dynamic} = \frac{dp}{dt} = 2 \times 20 \times 20 \times 10^{-3} \times 5 \times 0.5 = 2\text{ N}$$
Net force on one leg $= \dfrac{1}{4}(2 + 0.2 \times 10) = 1$ N

11.



(i) Since, both have positive final velocities, hence, both moved in the same direction after collision.
(ii) at t = 2 sec, both had equal velocities.
(iii) by conservation of linear momentum, we can say that mass of R was greater than mass of S.

12. By conservation of linear momentum along the string,
$mu = (m + m + 3m)\, v$ or $v = \dfrac{u}{5}$
and impulse on the block A $= 3m\,(v - 0) = \dfrac{3mu}{5}$

13. Let the three mutually perpendicular directions be along x, y and z-axis respectively,
$\vec{p}_1 = mv_0\hat{i}$
$\vec{p}_2 = mv_0\hat{j}$ where, $\dfrac{1}{2}mv_0^2 = E_0$
$\vec{p}_3 = mv_0\hat{k}$ and $\vec{p}_4 = m\vec{v}$
By linear momentum conservation,
$0 = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \vec{p}_4$ or $\vec{v} = -v_0(\hat{i} + \hat{j} + \hat{k})$
or $v = v_0\sqrt{1^2 + 1^2 + 1^2} = v_0\sqrt{3}$

total energy $= 3\left(\dfrac{1}{2}mv_0^2\right) + \dfrac{1}{2}mv^2 = 3E_0 + 3E_0 = 6E_0$

14. Let v_1, v_2 and v_3 be velocities of blocks 1, 2 and 3 after suffering collision each.

$mv = mv_1 + Mv_2$ and $v_1 - v_2 = -v$

solving we get $v_1 = \frac{m-M}{m+M} < 0 \quad \therefore m < M$

$\therefore \quad |v_1| = \frac{M-m}{m+M} v$ .... **(1)**

and $v_2 = \frac{2mv}{m+M}$

Similarly, $v_3 = \frac{2m}{m+M} \times v_2 = \frac{4Mmv}{(m+M)^2}$ ....**(2)**

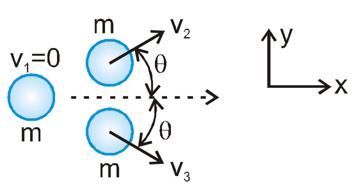
$\therefore \quad \frac{M-m}{M+m} v = \frac{4Mmv}{(M+m)^2}$ or $M^2 - m^2 = 4Mm$.

$\frac{M}{m} = 2 + \sqrt{5}$ **Ans.**

15. After collision by momentum conservation :

Along y-axis

$0 = 0 + mv_2 \sin\theta - mv_3 \sin\theta$



$\Rightarrow \quad v_2 = v_3$

Along x-axis

$mv = 0 + mv_2 \cos\theta + mv_3 \cos\theta$

$mv = 2\, m\, v_2 \cos\theta$

$v_2 = \frac{v}{2} \frac{1}{\cos\theta}$

So $v_2 = v_3 > \frac{v}{2} \quad \because \quad \cos\theta < 1$

16. $\int dp = p_f - p_i = \int F\,dt$ = Area under the curve.

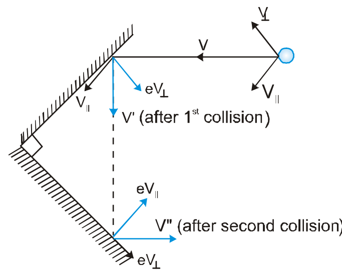
$p_i = 0$

Net Area $= 16 - 2 - 1 = 13$ N-s

$\frac{p_f}{M} = V_f = \frac{13}{2} = 6.5\ \hat{i}$ m/s

[As momentum is positive, car is moving along positive x axis.]

17. During 1st collision perpendicular component of v, $v_\perp$ becomes e times, while IInd component $v_{||}$ remains unchanged and similarly for second collision. The end result is that both $v_{||}$ and $v_\perp$ becomes e times their initial value and hence $v'' = -ev$ (the (–) sign indicates the reversal of direction).



18. It can be shown that

$K_0 = K_{cm} + \frac{1}{2} MV_{cm}^2$ where M is the total mass of the system and V_{cm} is velocity of centre of mass with respect to ground.

Due to internal changes K_{cm} can change but V_{cm} will remain same. Hence only K_{CM} portion of kinetic energy can be transformed to some other form of energy. Thus D is the wrong statement.

19. For first collision $v = 10$ m/s. $t_1 = \frac{\pi(5)}{10} = \pi/2$ sec.

velocity of sep = e. velocity of opp.

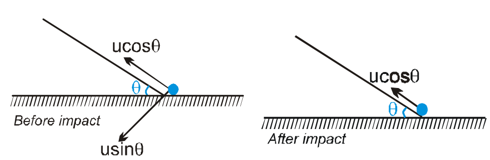
$v_2 - v_1 = \frac{1}{2}(10) \Rightarrow v_2 - v_1 = 5$ m/s

for second collision

$\therefore \quad t_2 = \frac{2\pi(5)}{5} = 2\pi$

$\therefore \quad$ total time $t = t + t_2 = \pi/2 + 2\pi \Rightarrow t = \frac{5}{2}\pi$

20. Just before the particle transfers to inclined surface, we resolve its velocity along and normal to the plane.



For the trajectory of the particle to sharply change from the horizontal line to the inclined line, the impact of the particle with inclined plane should reduce the usinθ component of velocity to zero. Hence the particle starts to move up the incline with speed u cosθ.

Hence as θ increases, the height to which the particle rises shall decrease.

21. Friction force between wedge and block is internal i.e. will not change motion of COM. Friction force on the wedge by ground is external and causes COM to move towards right. Gravitational force (mg) on block brings it downward hence COM comes down.

22. $U_Q > U_S > U_P > U_R$

$\because$ $M_Q > M_S > M_P > M_R$ and CM of cone is on smallest height

23. Since, $F_{ext} = 0$

Hence, momentum will remain conserved equal to mv.

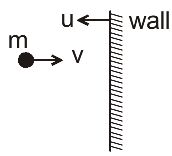
$mv = (m + M)\, v'$ or $v' = \dfrac{mv}{m+M}$

and final kinetic energy is $\frac{1}{2}(m + M)\, v'^2$

$= \dfrac{1}{2}(m+M)\left(\dfrac{mv}{m+M}\right)^2 = \dfrac{m^2v^2}{2(m+M)}$

24. (B), (C)

in an elastic collision



$v_{sep} = v_{app}$

or $v' - u = v + u$

or $v' = v + 2u$

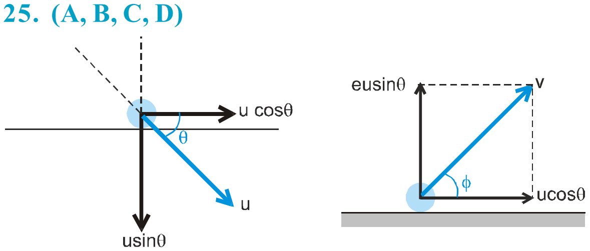
change in momentum of ball is $|p_f - p_i|$

$= |m(-v') - mv| = m(v' + v) = 2m(u + v)$

average force $= \dfrac{\Delta p}{\Delta t} = \dfrac{2m(u+v)}{\Delta t}$

change in $KE = K_f - K_i = \dfrac{1}{2} mv'^2 - \dfrac{1}{2} mv^2 = 2mu(u + v)$

25. (A, B, C, D)



Impulse $(J) = \Delta P = mv\sin\phi - m(-u\sin\theta)$

$= m(v\sin\phi + u\sin\theta) = m(V_{sep} + V_{app})$

$= m(eV_{app} + V_{app})$ $\quad [e = \dfrac{V_{sep}}{V_{app}}]$

$= m V_{app}(e + 1)$

$J = m(u\sin\theta)(1 + e)$

In horizontal direction, momentum is conserved :

$u\cos\theta = v\cos\phi$ or $v = \dfrac{u\cos\theta}{\cos\phi}$

or $e = \dfrac{V_{sep}}{V_{app}} = \dfrac{v\sin\phi}{u\sin\theta} = \dfrac{\tan\phi}{\tan\theta}$

or $\tan\phi = e\tan\theta$

in vertical direction, $e = \dfrac{v\sin\phi}{u\sin\theta}$

or $v\sin\phi = eu\sin\theta$,

$v = \sqrt{(eu\sin\theta)^2 + (u\cos\theta)^2} = u\sqrt{e^2\sin^2\theta + \cos^2\theta}$

$v = u\sqrt{1 - (1 - e^2)\sin^2\theta}$

final kinetic energy $= \dfrac{1}{2} mv^2$

initial kinetic energy $= \dfrac{1}{2} mu^2$

ratio $= \dfrac{v^2}{u^2} = e^2\sin^2\theta + \cos^2\theta$

26,. Sphere A moving with velocity v has a component v/2 along the line joining the centres of the spheres at the time of collision and another component $v\sqrt{3}/2$ perpendicular to the previous direction. After collision the component along the line will interchange i.e. B will move with v/2 velocity i.e. 4 m/s along the line joining the centres and A moves with $v\sqrt{3}/2$ velocity at perpendicular direction to B.

27. For a system of two isolated sphere having non zero initial kinetic energy, the complete kinetic energy can be converted to other forms of energy if the momentum of system is zero. This is due to the fact that for an isolated system, the net momentum remains conserved. If an isolated system has nonzero momentum, for the momentum to remain constant complete kinetic energy of the system cannot become zero. Hence statement 1 is true while statement 2 is false.

28. Statement-2 contradicts Newton's third law and hence is false.

29. For sum of three non null vectors to be zero, they must be coplanar. Hence Statement-2 is a correct explanation for Statement-1.

30. During collision KE of system is not constant, hence statement-1 is false.

31. (a) The acceleration of the centre of mass is

$a_{COM} = \dfrac{F}{2m}$

The displacement of the centre of mass at time t will be

$x = \dfrac{1}{2} a_{COM}\, t^2 = \dfrac{Ft^2}{4m}$ **Ans.**

32. **(A)**

33. Suppose the displacement of the right block is x_1 and that of the left is x_2. Then,

$$x = \frac{mx_1 + mx_2}{2m} \text{ or, } \frac{Ft^2}{4m} = \frac{x_1 + x_2}{2} \quad \text{or}$$

$$x_1 + x_2 = \frac{Ft^2}{2m} \quad \text{...(i)}$$

Further, the extension of the spring is $x_1 - x_2$. Therefore, $x_1 - x_2 = x_0$ **...(ii)**

From Eqs. (i) and (ii), $x_1 = \frac{1}{2}\left(\frac{Ft^2}{2m} + x_0\right)$

and $x_2 = \frac{1}{2}\left(\frac{Ft^2}{2m} - x_0\right)$ **Ans.**

34. From conservation of momentum

$mv = mv' \cos 30° + mv' \cos 30°$

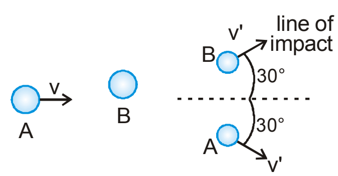
$\therefore \quad v' = \frac{v}{2\cos 30°} = \frac{v}{\sqrt{3}}$

35. Loss in kinetic energy

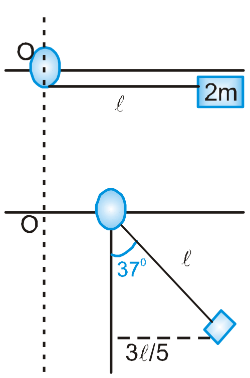
$$= \frac{1}{2}mv^2 - 2 \times \frac{1}{2}m\left(\frac{v}{\sqrt{3}}\right)^2 = \frac{1}{6}mv^2$$

36. Initially B was at rest, therefore line of impact is along final velocity of B.

$$\therefore \quad e = \frac{v' - v'\cos 60°}{v \cos 30} = \frac{\frac{1}{2}\frac{v}{\sqrt{3}}}{v \times \frac{\sqrt{3}}{2}} = \frac{1}{3}$$



37. Taking 'O' as the origin ;



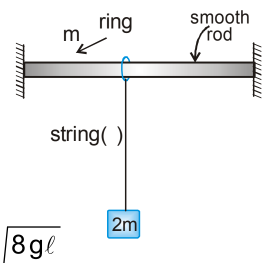
$$X_{CM(i)} = \frac{m(0) + 2m(\ell)}{3m} = \frac{2\ell}{3}$$

and $$X_{CM\,(f)} = \frac{m(x) + 2m(x + \frac{3\ell}{5})}{m + 2m}$$

As $\Sigma F_x = 0 \Rightarrow X_{CM}(i) = X_{CM}(f)$

$$\Rightarrow X = \frac{4\ell}{15} \quad \textbf{Ans.}$$

38. By momentum conservation :
(Inhorizontal direction) $mv_1 = 2\,mv_2$
& by energy conservation:



$$2mg\ell = \frac{1}{2}mv_1^2 + \frac{1}{2}2mv_2^2$$

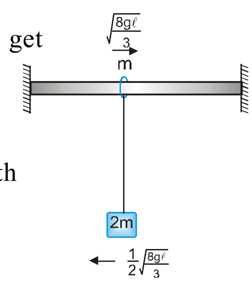
$$\Rightarrow 2\,g\ell = \frac{1}{2}v_1^2 + \left(\frac{v_1}{2}\right)^2$$

$$\Rightarrow \ 2gl = \frac{3}{4}\,v_1^{\,2} \ \Rightarrow v_1 = \sqrt{\frac{8g\ell}{3}}$$

39. From momentum conservation
$0 = m\,V_r + 2m\,V_B$
Energy conservation

$$2mg\ell = \frac{1}{2}mV_r^{\,2} + \frac{1}{2}mV_B^{\,2}$$

Solving above equations, we get



$$V_r = \sqrt{\frac{8g\ell}{3}},\ V_B = \frac{1}{2}\sqrt{\frac{8g\ell}{3}}$$

Relative velocity of block with

respect to ring is $\frac{3}{2}\sqrt{\frac{8g\ell}{3}}$.

Applying newton's law equations on the block

$$T - 2mg = \frac{(2m)\left(\frac{3}{2}\sqrt{\frac{8g\ell}{3}}\right)^2}{\ell} \qquad T = 14\ mg.$$

40. **(A)** If velocity of block A is zero, from conservation of momentum, speed of block B is 2u. Then K.E. of block B $= \frac{1}{2}m(2u)^2 = 2mu^2$ is greater than net mechanical energy of system. Since this is not possible, velocity of A can never be zero.

(B) Since initial velocity of B is zero, it shall be zero for many other instants of time.

(C) Since momentum of system is non-zero, K.E. of system cannot be zero. Also KE of system is minimum at maximum extension of spring.

(D) The potential energy of spring shall be zero whenever it comes to natural length. Also P.E. of spring is maximum at maximum extension of spring.

41. $A \rightarrow P,R,T \quad \Sigma F = 0$

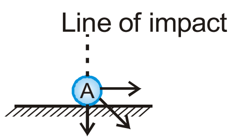
So, linear momentum conservation and centre of mass will not move.

$B \rightarrow Q,S$

So, linear momentum will not be conserved and centre of mass will accelerate $W_{ext} = \Delta E$.

$C \rightarrow P,S,T$; $D \rightarrow P,S,T$

42. (P)



(A) Normal force from ground lies along line of impact. Hence (A) is not answer.

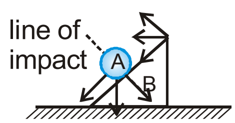
(B) Since no external force act perpendicular to the line of impact. (B) is an answer.

(C) Horizontal direction is same as direction perpendicular to the line of impact. (C) is an answer.

(D) Normal impulse from ground lies in vertical direction.

(S) is not an answer.

(Q)



(A) The component of normal force from ground lies along the line of impact. Hence not an answer.

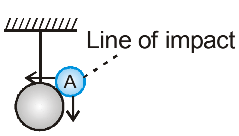
(B) No external force perpendicular to the line of impact for A.

(C) For the system A + B there is no external force along horizontal direction. Hence an answer.

(D) For B the normal force from ground is balanced by the impulsive force by A. Initial and final momentum is zero.

Hence an answer.

(R)



(A) The component of tension force of thread lies along the line of impact. Hence not an answer.

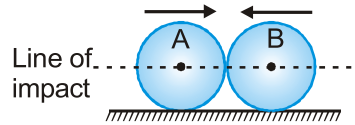
(B) No external force perpendicular to the line of impact for A.

(C) For system A + B there is no external force along horizontal direction. Hence an answer.

(D) For B the tension force from thread is balanced by the impulsive force by A. Initial and final momentum is zero.

Hence an answer.

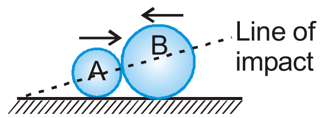
(S)



(A) & **(C)** are the same direction and there is no external force for the system A + B. Hence answer.

(B) & **(D)**are the same direction and there is no net force for the system A + B. Hence answer.

(T)



(A) The component of normal force from ground lies along the line of impact. Hence not an answer.

(B) Is answer because the normal force from the ground is balanced for A. Hence an answer.

(C) For the system A + B there is no external force along horizontal direction. Hence an answer.

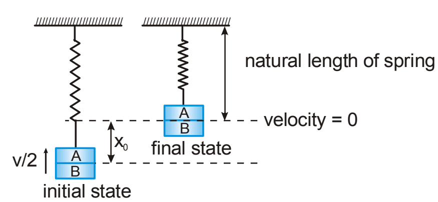
(D) For A the normal force from ground is balanced by the impulsive force by B. Initial and final momentum is zero.

Hence an answer.

43. The initial extension in spring is $x_0 = \dfrac{mg}{k}$

Just after collision of B with A the speed of combined mass is $\dfrac{v}{2}$.

For the spring to just attain natural length the combined mass must rise up by $x_0 = \dfrac{mg}{k}$(sec fig.) and comes to rest.



Applying conservation of energy between initial and final states

$$\frac{1}{2}\,2m\left(\frac{v}{2}\right)^2 + \frac{1}{2}\,k\left(\frac{mg}{k}\right)^2 = 2mg\left(\frac{mg}{k}\right)$$

Solving we get $\quad v = \sqrt{\dfrac{6mg^2}{k}}$

Alternative solution by SHM

$$\frac{v}{2} = \sqrt{\frac{k}{2m}}\sqrt{\left(\frac{2mg}{k}\right)^2 - \left(\frac{mg}{k}\right)^2} \;;$$

$$v = \sqrt{\frac{2k}{m}}\sqrt{\frac{3m^2g^2}{k^2}} = \sqrt{\frac{6mg^2}{k}} = 6 \text{ m/sec}$$

44. For the duration of collision the pendulum does not exert any force on the sphere in the horizontal direction. Hence the horizontal momentum of bullet + sphere is conserved for the duration of collision. Let v' be the velocity of bullet and sphere just after the collision.

$\therefore$ from conservation of momentum

$(m+m)\,v' = mv$ or $v' = \dfrac{v}{2} = 1\text{m/sec}$

45. Force F on plate = force exerted by dust particles

= force on dust particles by the plate

= rate of change of momentum of dust particles

= mass of dust particles striking the plate per unit time × change in velocity of dust particles. $= A(v+u)\,\rho \times (v+u)$

$= A\rho\,(v+u)^2$

46. Maximum compression will take place when the blocks move with equal velocity. As no net external horizontal force acts on the system of the two blocks, the total linear momentum will remain constant. If V is the common speed at maximum compression, we have,

(1 kg) (2 m/s) = (1 kg)V + (1 kg)V or, V = 1 m/s.

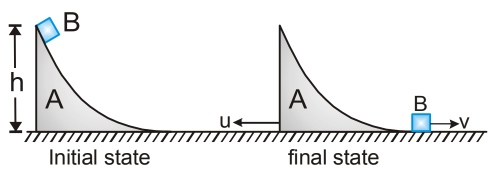
Initial kinetic energy $= \dfrac{1}{2}$ (1 kg) (2 m/s)^2 = 2 J.

Final kinetic energy

$= \dfrac{1}{2}$ (1 kg) (1m/s)^2 + $\dfrac{1}{2}$ (1 kg) (1 m/s)^2 = 1 J

The kinetic energy lost is stored as the elastic energy in the spring.

Hence, $\dfrac{1}{2}$ (50 N/m) x^2 = 2J – 1J = 1 J or, x = 0.2 m.

47. 

(figure - 1)

Let u and v be the speed of wedge A and block B at just after the block B gets off the wedge A. Applying conservation of momentum in horizontal direction, we get.

mu = mv ......(1)

Applying conservation of energy between initial and final state as shown in figure (1), we get

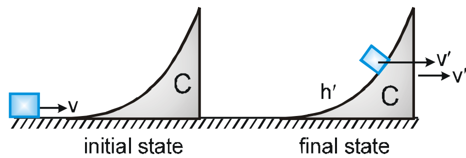
$mgh = \dfrac{1}{2}mu^2 + \dfrac{1}{2}mv^2$ ......(2)

solving (1) and (2) we get

$v = \sqrt{gh}$ ......(3)

At the instant block B reaches maximum height h′ on the wedge C (figure 2), the speed of block B and wedge C are v′. Applying conservation of momentum in horizontal direction, we get

$mv = (m+m)\,v'$ ......(4)



Applying conservation of energy between initial and final state

$\dfrac{1}{2}mv^2 = \dfrac{1}{2}(m+m)\,v'^2 + mgh'$ ......(5)

Solving equations (3), (4) and (5) we get

$h' = \dfrac{h}{4} = 25$ cm **Ans.**

48. Since $e = \dfrac{1}{5}$

$\therefore$ Final normal component of velocity $= \dfrac{v\cos 37^0}{5}$.

As the angle of rebound is equal to the angle before impact.

Therefore, both normal & tangential components of velocities must change by the same factor.

$\therefore$ Tangential velocity after impact becomes $\dfrac{v\sin 37^0}{5}$.

Let the time of impact be Δt.

$$N = \frac{m\left(v\cos 37^0 + \dfrac{v\cos 37^0}{5}\right)}{\Delta t} = \frac{6mv\cos 37^0}{5\Delta t}$$

where N is the normal force imparted on the ball by the wall.

Frictional force $= \mu N = \dfrac{6}{5}\dfrac{\mu mv\cos 37^0}{\Delta t}$

Also frictional force $= \dfrac{m\left[v\sin 37^0 - \dfrac{v\sin 37^0}{5}\right]}{\Delta t}$

$$\Rightarrow \frac{m\left[v\sin 37^0 - \dfrac{v\sin 37^0}{5}\right]}{\Delta t} = \frac{6}{5}\frac{\mu mv\cos 37^0}{\Delta t}$$

$\Rightarrow \mu = \dfrac{2}{3}\tan 37^0 \Rightarrow \mu = \dfrac{2}{3}\cdot\dfrac{3}{4} = \dfrac{1}{2}$ **Ans.**