HINTS & SOLUTIONS

EXERCISE - 1

Single Choice

1. $\omega = \omega_0 + \alpha t \quad \Rightarrow \quad 36 = 0 + \alpha \times 6 \quad \Rightarrow \quad \alpha = 6$

$\theta = \frac{1}{2} \times 6 \times 36 = 108$ radian.

2. Linear velocity $v = R\omega$

3. $\omega^2 = \omega_0^2 + 2\alpha\theta \,;\, \theta = \frac{1}{2} \times \alpha \times 4 = 2\alpha$

and $\left(\frac{600 \times 2\pi}{60}\right)^2 = 2 \times \alpha \times 2\alpha \quad \Rightarrow \quad \alpha = 10\pi$ rad/s^2

4. $r_1 = 0.5 \times 10^{-2} \,;\, r_2 = 0.15 \times 10^{-2}$

$\Rightarrow \quad 2\pi r_1 \times 3 = 2\pi \times r_2 \times n$

$\therefore \quad 0.5 \times 10^{-2} \times 3 = 0.15 \times 10^{-2} \times n \quad \Rightarrow \quad n = 10$

5. $2.5 \times 1000 = 2\pi \times 30 \times n \times 10^{-2}$

$n = \frac{250}{6\pi} \times 10^2 \simeq 1330$

6. $M.I. = m\ell^2 + m\left(\frac{\ell}{2}\right)^2 = \frac{5m\ell^2}{4}$

7. Moment of inertia depends on the distribution of mass about axis, more distance more inertia.

8. $v = r\omega,\ a = r\alpha \quad \therefore \quad a = 0.4 \times 1.5 \Rightarrow a = 0.6$

$v = at = 0.6 \times 20$ ($t = 20$sec.) $\quad \Rightarrow \quad v = 12$

9. $\therefore \quad dI = dmr^2 = (\text{volume} \times \text{density})\, r^2$

= M.I is greater when denser material has more distance from axis.

10. Raw egg acts as a ring (because material lies on the perimeter) and half boiled egg behaves as a disc. M.I of ring > M.I of disc.

11. M.I will be minimum about axis which passes through centre of mass.

12. $I_1 = \frac{MR_1^2}{2}$

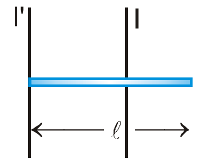
$I_2 = \frac{MR_2^2}{2} \quad \therefore \quad \frac{I_1}{I_2} = \frac{R_1^2}{R_2^2} = \frac{8.9}{7.2}$

Mass = Density × Area

$7.2 \times \pi R_1^2 = 8.9 \times \pi \times R_2^2 \quad \Rightarrow \quad \frac{R_1^2}{R_2^2} = \frac{8.9}{7.2}$

13. Mass distribution about axis is not changed so MI remains unchanged.

14. $I = \frac{M\ell^2}{12}$

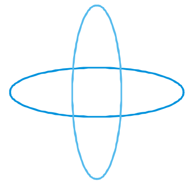


$I' = I + M\left(\frac{\ell}{2}\right)^2$

$I' = \frac{M\ell^2}{3} = \frac{M\ell^2}{12} \times 4$

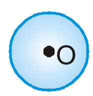
$\Rightarrow \quad I' = 4I$

15. MI of one ring about axis which is perpendicular to the plane = mr^2 and this axis is the diameter of another ring



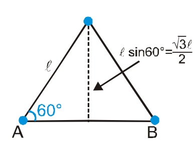
$\therefore \quad MI = \frac{mr^2}{2} \quad \therefore$ Total MI $= mr^2 + \frac{mr^2}{2} = \frac{3mr^2}{2}$

16. MI of disc about the axis at O & Perpendicular to plane is $= \frac{MR^2}{2}$ MI of four particle is $= mR^2 \times 4$



$\therefore \quad$ Total MI $= \frac{MR^2}{2} + 4mR^2$

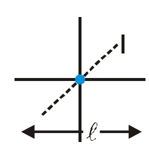
17. MI about AB is $= 0 + 0 + m \times \left(\frac{\sqrt{3}\ell}{2}\right)^2 = \frac{3m\ell^2}{4}$



18. $I.7 \times 10^{-24} \times 10^{-3} \times \left(\frac{4 \times 10^{-8} \times 10^{-2}}{2}\right)^2 \times 2$

$= 1.7 \times 10^{-24} \times \frac{16 \times 10^{-20}}{4} \times 2 \times 10^{-3} = 13.6 \times 10^{-47}$

19. $MI = \frac{2 \times m\ell^2}{12} = \frac{m\ell^2}{6}$



20. MI is more when mass is for away

$$I_{rods} = \int dm \times (2a)^2 = M \times 4a^2$$

$$I_{ring} = \int dm \ a^2 = Ma^2$$

21. $\omega = \frac{2\pi}{T} = \frac{2\pi}{\pi} = 2$ rad/sec.

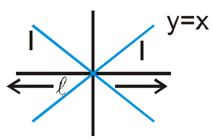
$$K.E. = \frac{1}{2} I\omega^2 \quad \Rightarrow \quad I = \frac{2K.E.}{4} \quad \Rightarrow \quad I = \frac{K.E.}{2}$$

22. $I = x^2 - 2x + 99$

MI about axis pass through centre of mass will be minimum when

$$\frac{dI}{dx} = 0 \quad \Rightarrow \quad 2x - 2 = 0 \quad \Rightarrow \quad x = 1$$

23. From perpendicular axis theorem



$\therefore \quad I + I = \frac{M\ell^2}{12} \quad \Rightarrow \quad I = \frac{M\ell^2}{24}$

$\therefore \quad$ MI of two rods $= 2 \times \frac{M\ell^2}{24} = \frac{M\ell^2}{12}$

24. $\omega = \frac{720 \times 2\pi}{60} = 24\pi$

$$0 = 24\pi - \alpha \times 8 \quad \Rightarrow \quad \alpha = 3\pi \quad \Rightarrow \quad \tau = I\alpha = \frac{24}{\pi} \times 3\pi = 72$$

25. Plane of disc is $(X - Z)$

$\therefore \quad I_y = I_x + I_z \quad \Rightarrow \quad 40 = 30 + I_z \quad \therefore \ I_z = 10$

26. Rod rotates about its one end in a horizontal plane

$\therefore \quad \tau = I\alpha \quad \Rightarrow \quad \frac{Mg}{2} \times \frac{5L}{6} = \frac{ML^2}{3} \times \alpha \Rightarrow \alpha = \frac{5g}{4L}$

27. For angular acceleration $\tau = I\alpha$

$\Rightarrow \quad 20 \times 0.2 = 0.2 \times \alpha \quad \Rightarrow \quad \alpha = 20$ rad/sec^2

$\therefore \quad \omega = \omega_0 + \alpha t = 0 + 20 \times 5 = 100$ rad/sec

28. $\therefore$ Book does not rotate so for rotational equilibrium the net torque becomes zero.

$$\vec{\tau}_{weight} + \vec{\tau}_{man} = \vec{0}$$

$$\therefore \quad \vec{\tau}_{man} = -\vec{\tau}_{weight} = -\left[W \times \frac{b}{2} \text{ anticlockwise}\right]$$

29. Angular acceleration $\alpha = \frac{\tau}{I}$

$$\tau = (10 + 9) \times 0.3 - 12 \times 0.05 = 5100\,\alpha$$

$\therefore \quad \alpha = 10^{-3}$ rad/sec^2

30. Density of steel > Density of wood

$\therefore \quad$ MI about O > MI about O

[in fig(a)] [in fig(b)]

$$\therefore \quad \alpha = \frac{\tau}{I} \quad \Rightarrow \quad \frac{\alpha_0}{\alpha_0'} = \frac{I_0'}{I_0}$$

31. Angular momentum $= \vec{r} \times \vec{p}$

As velocity is constant = p remains constant.

Along line BC angular momentum remains same.

$\therefore$ The distance of the particle momentum both having constant value about 'O'

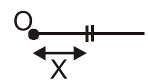
32. Centre of mass of the rod

$$= \text{C.M. from 'O'} = \frac{1}{\left(\frac{\lambda_0 L^2}{2}\right)} \int_0^L (\lambda_0 x \, dx)\, x$$

$\therefore \quad$ C.M. $= \frac{2}{3} L$ from O

Moment of inertia about O is

$$I = \int dI = \int dm\, r^2 = \int_0^L (\lambda_0 x\, dx)\, x^2 = \frac{\lambda_0 L^4}{4}$$



$\therefore \quad$ Angular acceleration

$$\alpha = \frac{\tau_{weight}}{I} = \frac{\left(\frac{\lambda_0 L^2}{2}\right) g \times \frac{2}{3} L}{\frac{\lambda_0 L^4}{4}} = \frac{4g}{3L}$$

33. $\Rightarrow I = m_{earth} r^2 = 6 \times 10^{24} \times (1.5 \times 10^8)^2$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{3.14 \times 10^7}$$

Angular momentum $= I\omega \quad \Rightarrow \quad \therefore \ 2.7 \times 10^{40}$ kg m^2/s

34. There is no exertial torque so the angular momentum remains constant as due to contraction the MI of earth decreases.

$$I_2 = \frac{2}{5} mr'^2 = \frac{2}{5} m \left(\frac{r}{n}\right)^2 \quad \frac{I_1}{n^2}$$

Now $I_1\omega_1 = I_2\omega_2 = \frac{I_1}{n^2} \quad \Rightarrow \quad \omega_2 = n^2\omega_1$

$$\therefore \quad T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{n^2\omega_1} \quad \Rightarrow \quad T_2 = \frac{T_1}{n^2} \qquad \therefore \ T_2 = \frac{24}{n^2}$$

35. As clay rod together as a system angular momentum remain constant. Angular momentum about mid point of rod before collision is

$= mv \times L/2$

36. As ice melts into water, it spreads & moment of inertia increases so angular velocity decreases.

37. From angular momentum conservation

$$\frac{I_1}{I_2} = \frac{\omega_2}{\omega_1} \Rightarrow \frac{MK_1^2}{MK_2^2} = \frac{\omega_2}{\omega_1} \quad \therefore \frac{K_1}{K_2} = \sqrt{\frac{\omega_2}{\omega_1}}$$

38. Moment of inertia decreases, then ω increases.

From the angular momentum conservation

39. From angular moment conservation $\left(\because \vec{\tau}_{ext} = \vec{0}\right)$

$Mr^2 \times \omega = (M + 4m)\, r^2 \times \omega'$

40. As he stretches hand, the MI increases so ω decreases and KE

$$= \frac{J^2}{2I} = KE\downarrow \quad (\therefore I\uparrow)$$

41. As ant approaches centre, MI decreases so ω increases and then after passing centre the MI again goes increasing then angular velocity ω will become decreasing.

42. $v = \omega r_\perp = \omega r \sin\theta \Rightarrow \omega = \dfrac{v}{r\sin\theta} = \dfrac{3}{8}$ rad/s

43. As viscous fluid spreads outwards continuously then MI increases so ω decreases.

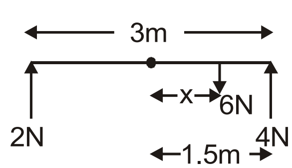
44. $K.E. = \dfrac{1}{2} I\omega^2 = \dfrac{1}{2} \times \dfrac{ML^2}{3} \times (2\pi n)^2 = \dfrac{2}{3} mL^2\pi^2 n^2$

45. $K.E. = \dfrac{J^2}{2I}$

Angular momentum is same

$$\therefore \frac{(KE)_1}{(KE)_2} \propto \frac{I_2}{I_1} \quad \therefore I_1 > I_2 \quad \therefore (KE)_I < (KE)_{II}$$

46.



or equilibrium the $\Sigma \vec{\tau}_{ext} = \vec{0}$

$\therefore \quad (4 - 2) \times 1.5 = 6x) \qquad \therefore \quad x = \dfrac{1}{2}$ m

47. $I' = MK^2 + Md^2 \; (\because I = I_{cm} + md^2)$

48. Power $= \dfrac{d(KE)}{dt} R = \dfrac{d}{dt}\left(\dfrac{1}{2}\dfrac{M\ell^2}{12} \times \omega^2\right)$

$$= \frac{1}{2}\frac{M\ell^2}{12} \times 2\omega \frac{d\omega}{dt}$$

$P \propto \omega \dfrac{d\omega}{dt}$; $P \propto \omega\alpha \qquad (\therefore \alpha$ is constant)

$\therefore \quad P \propto \omega \quad \Rightarrow \quad P \propto t$

49. $16\ell_1 = m\ell_2 \quad \Rightarrow \quad m = \dfrac{16\ell_1}{\ell_2} \Rightarrow m\ell_1 = 4\ell_2 \Rightarrow m = \dfrac{4\ell_2}{\ell_1}$

$\therefore \dfrac{16\ell_1}{\ell_2} = \dfrac{4\ell_2}{\ell_1} \quad \Rightarrow \quad \ell_1 = \dfrac{\ell_2}{2} \quad \therefore \quad m = \dfrac{4}{\ell_1} \times 2\ell_1 = 8$ kg

50. As given $(KE)_R = (KE)_{translational} = \dfrac{1}{2} I\omega^2 = \dfrac{1}{2} Mv^2$

$= \dfrac{1}{2} MK^2\omega^2 = \dfrac{1}{2} MR^2\omega^2; \; \dfrac{K^2}{R^2} = 1 \quad \Rightarrow \quad$ Hence ring

51. $a = \dfrac{g\sin\theta}{1 + \dfrac{K^2}{R^2}} \Rightarrow \dfrac{K^2}{R^2}\uparrow \Rightarrow a\downarrow = \text{time}\uparrow; \; \dfrac{K^2}{R^2} = \dfrac{2}{5}$

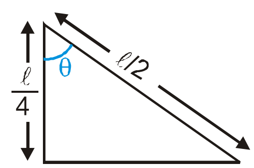
⇒ For solid sphere which is minimum in all of them. So a is maximum and time to reach bottom is minimum.

52. $\dfrac{1}{2}Mv^2 + \dfrac{1}{2}\dfrac{MR^2}{2} \times \dfrac{v^2}{R^2} = mgh \Rightarrow h = \dfrac{3v^2}{4g}$

53. $mgh = \dfrac{1}{2}Mv^2 + \dfrac{1}{2}\dfrac{MR^2}{2} \times \dfrac{v^2}{R^2}$

$\Rightarrow \; gh = \dfrac{2v^2 + v^2}{4} \Rightarrow \dfrac{3v^2}{4} = gh \Rightarrow v = \sqrt{\dfrac{4}{3}gh}$

54. Angular velocity of rod in vertical position



$\dfrac{1}{2}\dfrac{ML^2}{3}\omega^2 = \dfrac{MgL}{2}; \; \omega = \sqrt{\dfrac{3g}{L}}$

upper part of rod rotates through an angle θ its centre of mass will rise $\dfrac{L}{4}(1 - \cos\theta)$

From energy conservation

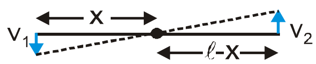
$$= \frac{1}{2}\left(\frac{M}{2}\right)\frac{(L/2)^2}{3}\left(\sqrt{\frac{3g}{2}}\right)^2 + \frac{1}{4}MgL = \frac{1}{4}MgL\cos\theta$$

$\Rightarrow \; \theta = 60^o$

55. $\frac{1}{2}Mv^2 + \frac{1}{2} \times \frac{2}{5}MR^2 \times \frac{v^2}{R^2} = Mgh$

$\Rightarrow \frac{v^2}{2}\left[1 + \frac{2}{5}\right] = gh \quad \Rightarrow v = \sqrt{\frac{10gh}{7}}$

56. $\frac{v_1}{x} = \frac{v_2}{\ell - x}$



$\therefore \; v_1\ell = v_2x + v_1x \; ; x = \frac{v_1\ell}{v_1 + v_2}$

EXERCISE - 2

Part # I : Multiple Choice

1. When ball moves towards ends of the tube MI increases

($\therefore$ For system $\Sigma \vec{f}_{ext}$ & $\Sigma \vec{\tau}_{ext}$ are zero)

$\therefore$ Angular momentum & linear momentum remains constant

$\therefore \; I_1\omega_1 = I_2\omega_2 \quad \Rightarrow I_2 > I_1 \quad \Rightarrow \omega_2 < \omega_1$

2. $I = 2 \times \left\{\frac{M\ell^2}{3} + \left[\frac{M\ell^2}{3} + M\ell^2\right]\right\} \Rightarrow I = \frac{10M\ell^2}{3}$

3. $P \times \frac{\ell}{2} = I\omega \text{ or } \omega = \frac{P\ell}{2I} = \frac{P\ell}{2 \times \frac{m\ell^2}{12}} = \frac{6P}{m\ell}$

So $t = \frac{\theta}{\omega} = \frac{\pi}{2} \times \frac{1}{6P/m\ell} = \frac{\pi m\ell}{12P}$

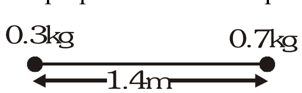
4. As rod does not slip on the disc, so the kinetic energy of the rod is $= \frac{1}{2}I\omega^2 = \frac{1}{2} \times \left[\frac{ML^2}{12} + MR^2\right]\omega^2$

I – MI of rod about axis passing through centre of disc & perpendicular to the plane.

5. The prism will be topple when the torque due to F becomes greater than the torque of weight about the front corner of the prism.

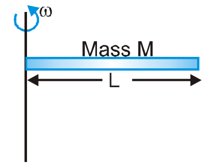
$\therefore \; F \times \frac{\sqrt{3}a}{2} = mg \times \frac{a}{2} \quad \therefore \; F = \frac{mg}{\sqrt{3}}$

6. Work required is minimum for rotation when the MI is minimum. It is minimum about that axis which pass through CM & perpendicular to the plane.



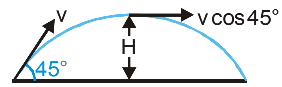
C.M from 0.3kg mass $= \frac{0.7 \times 1.4}{0.3 + 0.7} = 0.98$

7. Force exerted by liquid at the other end is the sum of centrifugal forces.



$$\int_0^L \left(\frac{M}{L}dx\right) x\,\omega^2 = \frac{M}{L}\omega^2 \int_0^L x\,dx = \frac{M}{L}\omega^2 \times \frac{L^2}{2} = \frac{M\omega^2 L}{2}$$

8. Max height $H = \frac{(v\sin 45)^2}{2g} = \frac{v^2}{4g}$

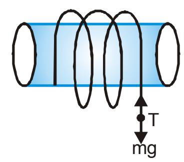


Angular momentum

$= mv\cos 45° \times H = \frac{mv}{\sqrt{2}} \times \frac{v^2}{4g} = \frac{mv^3}{4\sqrt{2}g}$

9. **D**

10. Translational equation



$mg - T = ma$

For rotational equation motion

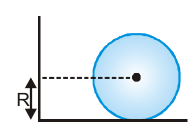
$T \times r = I \times \frac{a}{r} \quad (\alpha = \frac{a}{r}); T = \frac{aI}{r^2}$

$\therefore \; mg = a\left(m + \frac{I}{r^2}\right) \Rightarrow v = \sqrt{2 \times \frac{mg}{\left(m + \frac{I}{r^2}\right)} \times h}$

$\therefore \; v = \left[\frac{2mghr^2}{I + mr^2}\right]^{\frac{1}{2}}$

11. Angular momentum of this disc about origin

$= \frac{MR^2}{2}\omega + M(\omega R) \times R = \frac{3}{2}MR^2\omega$



12. MI of system:

$I = I_{sphere} + I_{sphere} + M \times (2R)^2$

$= 2 \times \frac{2}{5}M\left(\frac{R}{2}\right)^2 + M \times (2R)^2 = \frac{21MR^2}{5}$

13. Let the linear velocity of sphere be v $\quad \therefore \quad I = mv$

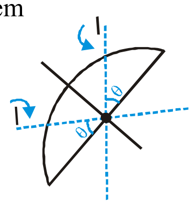
For angular momentum $J = I\omega$

$mv \times h = \frac{2}{5}mR^2\left(\frac{v}{R}\right) \quad \Rightarrow \quad \frac{h}{R} = \frac{2}{5}$

14. Acceleration of centre of mass depends on the $\Sigma \vec{f}_{ext}$

15. From the perpendicular axis theorem

$$I + I = \frac{MR^2}{2} \Rightarrow I = \frac{MR^2}{4}$$

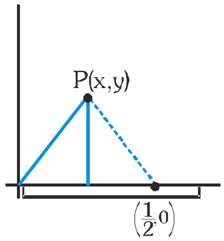


16. From the energy conservation

$$mg \times 2a = \frac{1}{2}mv^2 + \frac{1}{2}ma^2 \times \frac{v^2}{a^2}$$

$$v^2 = 2ag \quad \Rightarrow \quad v = \sqrt{2ag}$$

17. Let consider a point (x,y). Calculate moment of inertia about point P.



$$I_P = M\left(\sqrt{\left(x - \frac{L}{2}\right)^2 + y^2}\right)^2 + \frac{ML^2}{12}$$

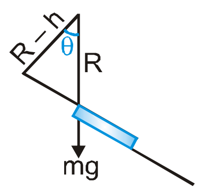
It must be equal to $\frac{ML^2}{3}$

So $M\left(\sqrt{\left(x - \frac{L}{2}\right)^2 + y^2}\right)^2 + \frac{ML^2}{12} = \frac{ML^2}{3}$

$$\Rightarrow \quad \left(x - \frac{L}{2}\right)^2 + y^2 = \frac{L^2}{4}$$

This is the equation of circle

18. Sphere is on verge of toppling when line of action of weight passes through edge.



$$\cos\theta = \frac{R - h}{R} \Rightarrow h = R - R\cos\theta$$

19. For the translatory & rotatory equilibrium let the normal at point 'A' & 'B' be N_A & N_B

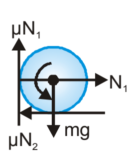
$\therefore$ $N_A + N_B = mg$ (m $\rightarrow$ mass of man) & Torque about the centre of plank $- N_A \times 2 + N_B \times 2 + mg(3 - x) = 0$

As x increases ($N_A \equiv N$) decreases.

$(N_B - N_A) \times 2 = mg(3 - x)$

20. In figure (b) $N_1 = 0, N_2 = mg$ ...(ii)

$$f_b = \mu N_2 = \frac{mg}{3} \quad \therefore \quad \frac{f_a}{f_b} = \frac{9}{10}$$



21. In figure (a) $\mu N_1 + N_2 = mg$ ... (i)

$$N_1 = \mu N_2, N_2 = \frac{mg}{1 + \mu^2} \qquad \therefore \quad f_a = \frac{3}{10}mg$$

22. For motion from A to B from energy considerations

$$\frac{mgh}{2} = \frac{1}{2}mv^2\left(1 + \frac{1}{2}\right) \text{ or } v^2 = \frac{2gh}{3}$$

Now, for motion from B to C

$$\frac{mgh}{2} = \frac{1}{2}I\omega^2 + \frac{1}{2}m\left(\frac{2gh}{3}\right)$$

$$\Rightarrow \quad \frac{mgh}{2} - \frac{mgh}{3} = \frac{1}{2}I\omega^2 \Rightarrow \frac{1}{2}I\omega^2 = \frac{mgh}{6} \qquad .....(i)$$

thus remaining translational KE

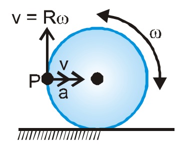
$$KE_t = mgh - \frac{mgh}{6} = \frac{5mgh}{6}$$

Hence required ratio

$$\frac{KE_t}{KE_r} = \frac{5mgh}{6} \times \frac{6}{mgh} = 5$$

23. $\therefore$ For purely rolling

$v = R\omega$

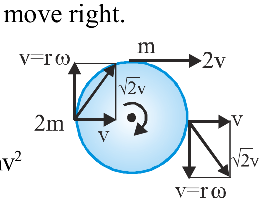


So at point 'P' the resultant velocity makes 45° with horizontal 'x' axis and it is also the angle between 'v' and 'a'.

24. As we displace to the right, lower point of rod is having tendency to move towards left. So friction is in direction right so center of mass will move right.

25. $KE_{system} = \frac{1}{2}mv^2(1+1) + \frac{1}{2}$

$$(m+2m)2v^2 + \frac{1}{2}m(2v)^2 = 6mv^2$$



26. Frictional force always acts in such a way to prevent the sliding or slipping.

28. As the disc comes to rest,

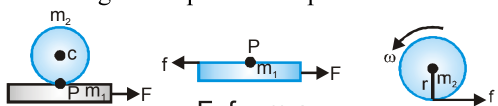
So $0 = v_0 - \mu g t \quad \Rightarrow \quad t = \frac{v_0}{\mu g}$

Also $0 = \omega_0 - \alpha t \quad \Rightarrow \quad t = \frac{\omega_0}{\alpha}$

Thus $\frac{v_0}{\mu \times g} = \frac{\omega_0}{\alpha} \quad \Rightarrow \quad \frac{v_0}{\omega_0} = \frac{\mu g}{\alpha}$

$$\Rightarrow \quad \frac{v_0}{r\omega_0} = \frac{\mu g}{\alpha}\frac{mr}{mr^2} = \frac{\tau}{\alpha mr^2} = \frac{I}{mr^2} = \frac{mr^2/2}{mr^2} = \frac{1}{2}$$

29. Force diagram of plank and sphere



$F - f = m_1 a$

$f = m_2 a_c \quad \Rightarrow \quad f \times r = \frac{2}{5} m_2 r^2 \alpha$ also $a_p = a$

$r\alpha + a_c = a \quad \therefore \quad a_c = \frac{2}{7} a$

30. Angular momentum $= \vec{r} \times \vec{p}$

As $\vec{v}$ increases when falls down then $\vec{p}$ also and so angular momentum is also increases. As particle falls down the length of position vector decrease so MI about 'O' also decreases.

$\omega = \frac{v}{r} \quad \Rightarrow \quad r\downarrow = \omega\uparrow$

31. Cylinder is rolling down the incline with sliding there is friction. Cylinder can start pure rolling or if friction is not sufficient then pure rolling may not be possible.

32. Angular velocity is $\omega = \frac{v}{R}$. As the top point of cylinder is moving with velocity v so centre of mass have zero velocity.

34. Form the perpendicular axis theorem, MI about an axis passing through 'O' & perpendicular to the plane is the sum of any two MI about an axis passing through 'O' & lie on the plane of plate.

35. Two point masses and bar taken together as a system the angular momentum about centre of bar is ($J = I\omega$)

$$2mva + 2mv \times 2a = \left[\frac{8m \times (6a)^2}{12} + 2ma^2 + m4a^2\right]\omega$$

$$= 30a\omega = 6v \quad \Rightarrow \quad \omega = \frac{v}{5a}$$

& applied force on bar

$2mv - m \times 2v = 0 \Rightarrow v_c = 0$

$$E = \frac{1}{2} I\omega^2 = \frac{1}{2} \times 30ma^2 \times \left(\frac{v}{5a}\right)^2 \Rightarrow E = \frac{3mv^2}{5}$$

Part # II : Assertion & Reason

1. D **2.** B **3.** B **4.** D **5.** D **6.** D
7. D **8.** A **9.** A **10.** A **11.** D

EXERCISE - 3

Part # I : Matrix Match Type

1. $I_1 = \frac{M\ell^2}{12} + \frac{M\ell^2}{4} + \frac{M\ell^2}{12} + \frac{M\ell^2}{4} = \frac{2}{3}M\ell^2$

$I_2 = 0 + \frac{M\ell^2}{3} + M\ell^2 + \frac{M\ell^2}{3} = \frac{5}{3}M\ell^2$

$I_3 = 4\left(\frac{1}{3}M\ell^2 \sin^2\theta\right) = \frac{2}{3}M\ell^2 \; [\theta = 45°]$

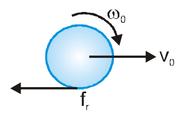
$I_4 = \frac{M\ell^2}{12} + \frac{M\ell^2}{4} + \frac{M\ell^2}{12} + \frac{M\ell^2}{4} = \frac{2}{3}M\ell^2$

2. Net torque about point 'A' is zero therefore angular momentum in all the four configuration is conserved.

(A) $(v_0 > \omega_0 R)$

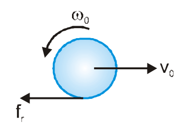
$KE_f < KE_i$

ω_{fr} (–ve)



(B) $KE_f < KE_i$

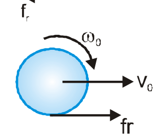
ω_{fr} (– ive)



(C) $(\omega_0 R > v_0)$

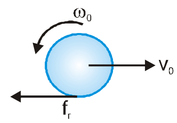
$KE_f < KE_i$

ω_{fr} (– ive)



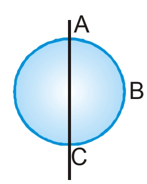
(D) $KE_f < KE_i$

ω_{fr} (– ive)



3. Work done on a particle is frame dependent therefore it may be zero, positive, negative.

4. **(A)** When insects move from A to B moment of inertia increases and when if move from B to C moment of inertia decreases



(B) No torque acts on a particle angular momentum of the particle remain constant

$I\omega$ = constant

$\therefore$ ω first decreases & then increases

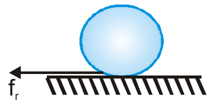
(C) Remain constant

(D) $KI = \frac{L^2}{2I} \quad \therefore$ first decreases & then increases

5. When $v = R\omega$ no friction acts on the disc

$\therefore \quad a_{cm} = 0; \alpha R = 0$

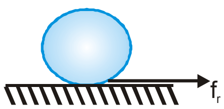
(B) $f_r = Ma_{cm} \Rightarrow a_{cm} = \dfrac{f_r}{M}$



$$f_r R = \frac{MR^2}{2}\alpha \Rightarrow \alpha R = \frac{2f_r}{M} \Rightarrow \alpha R > a_{cm}$$

(C) $f_r = Ma_{cm}; a_{cm} = \dfrac{f_r}{M}$

$$f_r R = \frac{MR^2}{2}\alpha$$



$$\alpha = \frac{2f_r}{MR^2} \Rightarrow \alpha R = \frac{2f_r}{M}$$

6. For translatory motion

$F - f_r = ma_{cm}$ .....(i)

For rotatory motion

$FR + f_r R = I\alpha$ .....(ii)

By solving equation (i) and (ii)

$$\frac{2FR^2}{I + mR^2} = a_{cm} \Rightarrow a_{cm} = \frac{2F}{m}\left[\frac{1}{1 + \frac{K^2}{R^2}}\right]$$

(A) For $\dfrac{K^2}{R^2}$ minimum a_{cm} is maximum in sphere

(B) For $\dfrac{K^2}{R^2} = 1 \Rightarrow a_{cm} = \dfrac{F}{m}$ that is $f_r = 0$

(C) For $\dfrac{K^2}{R^2} = \dfrac{2}{3} \Rightarrow a_{cm} = \dfrac{6F}{5m}$ that is $f_r = \dfrac{F}{5}$

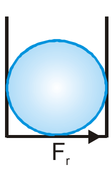
(D) For $\dfrac{K^2}{R^2} = \dfrac{1}{2} \Rightarrow a_{cm} = \dfrac{4F}{3m}$

Part # II : Comprehension

Comprehension # 1

1. $f_r = \mu mg(R) = I\alpha = (0.1)5 \times 10 \times 1$

$$= \frac{2}{5}MR^2\alpha \Rightarrow \alpha = 2.5 \text{ rad/sec}^2$$



$\omega = \omega_0 - \alpha t$

$0 = 40 - (2.5)\,t \Rightarrow t = 16$ sec

Comprehension # 2

1. **Case I** | **Case II**

Case I: $\int N dt = mv_1 - 0$; $\dfrac{R}{4}\int N dt = I\omega_1 - 0$

Case II: $\int N dt = mv_2$; $\dfrac{R}{2}\int N dt = I\omega_2$

as $\omega_2 > \omega_1 \Rightarrow KE_2 > KE_1$

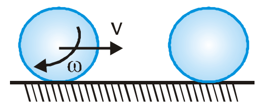
2. Point of percussion at which ($f_r = 0$)

$$= \frac{I}{MR} \Rightarrow h_0 = 0.4R$$

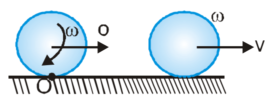
For $h > h_0$ friction act in forward direction

Comprehension # 3

1. By applying conservation of angular momentum about O.



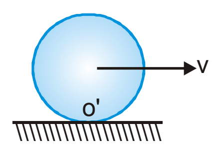
$$MvR + \frac{2}{5}mR^2\omega = mv'R + \frac{2}{5}\frac{mR^2v'}{R} + MVR$$



$$\frac{2}{5}mR^2\omega = \frac{7}{5}mv'R \Rightarrow v' = \frac{2}{7}\omega R$$

2. By applying angular momentum conservation law

$$mvR = \frac{7}{5}\frac{mv'R^2}{R} \Rightarrow v' = \frac{5v}{7}$$

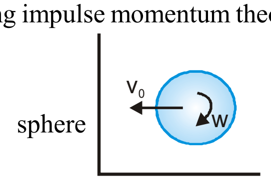


Change in angular momentum gives angular impulse

$$R\int N dt = I\omega = 0 = \frac{2}{5}mR^2\left(\frac{5v}{7R}\right) = \frac{2}{7}mvR$$

Comprehension # 4

1. By applying impulse momentum theorem on the



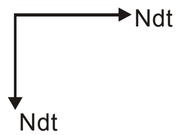
$\int N dt = mv_0 - (mv_0) = 2\,mv_0$

$= [2(2\,(4\text{ m/s})] = 16$ N–sec

$$\int \mu NR\, dt = I\omega - I\omega_0 \Rightarrow 16\mu = \frac{2}{5}MR^2\left[\frac{4}{R} - 9\right]$$

$$16\mu = -\left[\frac{2}{5} \times 2 \times 1(5)\right] \Rightarrow \mu = \frac{1}{4}$$

2. Net impulse



$= \sqrt{(Ndt)^2 + (\mu Ndt)^2} = 4\sqrt{17}\ N-s$

Comprehension # 5

1. Considering energy conservation we have

$$\frac{1}{2}mv^2\left(1+\frac{k^2}{R^2}\right) = mgh + \frac{1}{2}mv_1^2\left(1+\frac{k^2}{R^2}\right)$$

$$\frac{1}{2}\times\frac{200}{7}\left(\frac{7}{5}\right) = 10\times 1 + \frac{1}{2}\times v_1^2\left(\frac{7}{5}\right) \Rightarrow v_1 = \sqrt{\frac{100}{7}}$$

2. Energy at point C

$$\frac{1}{2}\times 1\times\frac{200}{7}\times\frac{7}{5}$$

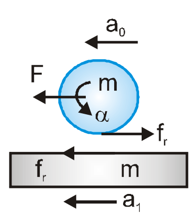
$$= 10 + \frac{1}{2}\times 1\times\frac{100}{7} + KE_r \text{ or } KE_r = \frac{40}{14}$$

Now for motion from point C to the ground

$$10 + \frac{1}{2}\times 1\times\frac{100}{7}\times\frac{7}{5} = \frac{1}{2}mv_2^2 + \frac{40}{14}$$

or $\frac{1}{2}mv_2^2 = \frac{240}{14}$ thus $\frac{KE_r}{KE_t} = \frac{40/14}{240/14} = \frac{1}{6}$

Comprehension # 6



1,2 For translatory equilibrium

$F - f_r = ma_0$ ....(i)

For rotatory motion

$f_r R = I\alpha$ ....(ii)

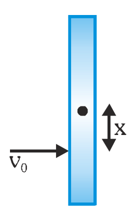
For plank $f_r = ma_1$ ...(iii)

By solving equation (i) (ii) and (iii)

$$a_0 - \alpha R = a_1 \Rightarrow \left(\frac{F-f_r}{m}\right) - \left(\frac{2f_r}{m}\right) = \frac{f_r}{m}$$

$$\Rightarrow f_r = \frac{F}{4}\ ;\ a_p = \frac{F}{4m}$$

Comprehension # 7



1. Angular momentum is constant

So $mvx = I\omega \Rightarrow mvx = \left(\frac{mL^2}{12} + mx^2\right)\omega$

So $\omega = \dfrac{mv_0x}{\frac{mL^2}{12} + mx^2}$

Let $x << \frac{L}{2}$ so x can neglect so $\omega = \dfrac{mv_0x}{\frac{mL^2}{12}}$

So for small x when x increase ω also increases and for other x as x increases ω will decreases.

2. For x = 0 particle sticks to centre and the final velocity of particle is minimum is v/2

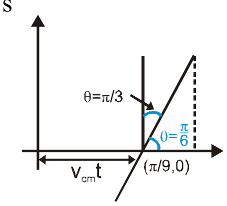
Comprehension # 8

1. $J = mv_{cm} - 0$ ...(i) $\Rightarrow J\frac{L}{2} = Iw$ ...(ii)

$$mv_{cm}\frac{L}{2} = \frac{ML^2}{12}\omega \Rightarrow \omega = \frac{6v}{L} \Rightarrow v_p = v_{cm} + \omega x$$

$$0 = v + \frac{6v}{L}(x) \Rightarrow x = \frac{L}{6} = \frac{1}{3}m$$

2. $J = mv_{cm} \Rightarrow v_{cm} = 5$ m/s



$$\omega = \frac{6v}{L} = 15 \text{ rad/s}$$

$$x = \left(\frac{\pi}{9} + 1\cos 30\right) = \frac{\pi}{9} + \frac{\sqrt{3}}{2}$$

$$y = 1\sin 60 = \frac{1}{2}$$

Comprehension # 9

1. Normal of the cube shift towards the point 'A' to balance torque of friction.

2. For toppling before translation torque of F and f_r about centre of mass.

$$\tau_{net} = f\left(\frac{a}{2}\right) + f_r\left(\frac{a}{2}\right) = \mu_r mga = 0.2mga$$

and torque of normal about centre of mass

mg a/2 = 0.5 mga

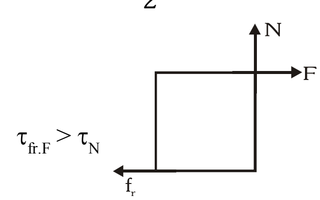
Torque of normal > Torque of mg and fr

∴ Body will translate before toppling

3. Torque of f_r and F is

$= \mu_r mga = 0.6$ mga

Torque of normal $= \frac{mga}{2} \Rightarrow 0.5$ mg;



∴ body will topple before translate

Comprehension#10

2. From energy conservation for ring

$$mgh = \frac{1}{2}mv^2\left(1+\frac{k^2}{R^2}\right) \Rightarrow 2gh = v^2(2) \Rightarrow v = \sqrt{gh}$$

Comprehension # 11

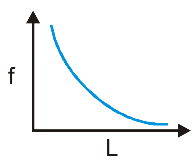
1. Torque needed to lift the block = mgR

Torque due to wrench on = FL as L increases, force decreases

2. By applying work energy theorem

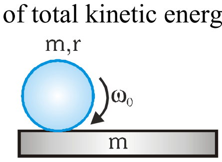
$0 = -mg(1) + w_{wrench} \quad \Rightarrow \quad w_{wrench} = mg$

work done and power will remain same

3. 

Comprehension # 12

After some time both block and disc move with same velocity due to momentum conservation. As given in question 50% of total kinetic energy of system is lost.



$$\frac{1}{2}\left(\frac{1}{2}\times\frac{1}{2}mr^2\omega_0^2\right) = mv^2 + \frac{1}{2}\times\frac{1}{2}mr^2\omega^2 \qquad .....(i)$$

By impulse equation due to friction.

$$\frac{1}{2}mr^2\omega_0 - \frac{1}{2}mr^2\omega = \mu mgr\Delta t \qquad .....(ii)$$

Block will achive velocity v in time Δt

$$\text{so } \Delta t = \frac{v}{\mu g} \qquad .....(iii)$$

So by solving equation (i), (ii),(iii)

1. $v = \frac{r\omega_0}{4}$

2. By eq^n (iii) Time $= \frac{r\omega_0}{4\mu g}$

3. By equation (ii) & (iii) $\omega_0 r - \omega r = 2v \Rightarrow \omega = \frac{\omega_0}{2}$

So change in angular momentum

$$\Delta L = I\omega_0 - I\omega = \frac{I\omega_0}{2} = \frac{1}{2}mr^2\frac{\omega_0}{2} \Rightarrow \Delta L = \frac{1}{4}mr^2\omega_0$$

4. Distance traveled by $s = ut + \frac{1}{2}at^2$

$$\Rightarrow \quad s = \frac{1}{2}\times\frac{\mu g r^2\omega_0^2}{16\mu^2 g^2} = \frac{r^2\omega_0^2}{2\times 16\mu g} = \frac{r^2\omega_0^2}{32\mu g}$$

Comprehension#13

3. For centre of mass $v_i = v_0$ & $v_f = \frac{v_0}{2}$ (rolling velocity) as initial translatory motion get converted in to rolling.

Thus $\frac{v_0}{2} = v_0 - \mu g t$ or $t = \frac{v_0}{2\mu g}$

4. From $s = ut + \frac{1}{2}at^2$

$$\Rightarrow \quad s = v_0\times\frac{v_0}{2\mu g} - \frac{1}{2}\mu g\frac{v_0^2}{4\mu^2 g^2}$$

$$\Rightarrow \quad s = \frac{v_0^2}{2\mu g}\left[1-\frac{1}{4}\right] = \frac{3}{8}\frac{v_0^2}{\mu g}$$

Also the loss in K.E.

$$= \frac{1}{2}mv_0^2 - \frac{1}{2}m\left(\frac{v_0}{2}\right)^2(2) = \frac{1}{2}mv_0^2 - \frac{1}{4}mv_0^2$$

Loss in KE $= \frac{1}{4}mv_0^2$

Work done by frictin $w = \Delta KE = \frac{3}{4}mv^2$

The gain in rotational KE is

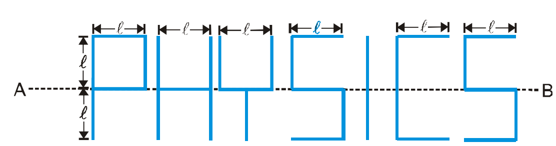
$$\frac{1}{2}\times mr^2\times\left(\frac{v_0}{2r}\right)^2 = \frac{mv_0^2}{8}\left(\text{as } \frac{v_0}{2} = r\omega\right)$$

EXERCISE - 4

Subjective Type

1. Moment of inertia can be calculated by principle of superposition

$$MI = \frac{1}{2}\times 9MR^2 - \left(\frac{MR^2}{18} + M\times\frac{4R^2}{9}\right) = \frac{72MR^2}{18} = 4MR^2$$

2. 

Let the mass of the 'ℓ' length rod is $M = \lambda\ell$.

Moment of inertia of structure about axis AB is

$$= \left(\frac{M\ell^2}{3}\times 3 + M\ell^2\right) + \left(\frac{M\ell^2}{3}\times 4\right) + \left(\frac{M\ell^2}{3}\right)\times 3$$

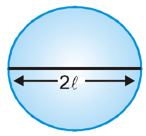
$$+\left(\frac{M\ell^2}{3}\times 2+M\ell^2\times 2\right)+\left(\frac{M\ell^2}{3}\times 2\right)$$

$$+\left(\frac{M\ell^2}{3}\times 2+2\times M\ell^2\right)+\left(\frac{M\ell^2}{3}\times 2+2M\ell^2\right)$$

$$= 13\,M\ell^2 = 13\lambda\ell^3 \quad (\therefore M=\lambda\ell)$$

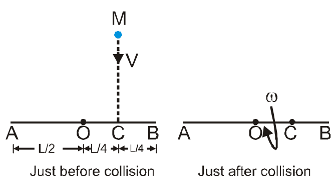
3. Length of each spoke is 'ℓ'. So the radius of wheel is 'ℓ' mass 24M and also having 24 spokes of mass 'M' of each.

MI of a wheel about axis is



$$24M\ell^2+\frac{M\ell^2}{3}\times 24 = 32\,M\ell^2$$

4. (i) In this problem we will write K for the angular momentum because L has been used for length of the rod.



Angular momentum of the system (rod + insect) about the centre of the rod O will remain conserved just before collision and after collision i.e., $K_i = K_f$.

$$\Rightarrow \quad Mv\frac{L}{4} = I\omega = \left[\frac{ML^2}{12}+M\left(\frac{L}{4}\right)^2\right]\omega$$

$$\Rightarrow \quad Mv\frac{L}{4} = \frac{7}{48}ML^2\omega \text{ i.e., } \omega = \frac{12}{7}\frac{v}{L}$$

(ii) Since the weight of the insect will exert a torque the angular momentum of the system will not be conserved. Let at any time t, the insect be at a distance x from o on the rod and by then the rod has rotated through on angle 'θ'. Then angular momentum of the system will be

$$J=\left(\frac{ML^2}{12}+Mx^2\right)\omega \quad \Rightarrow \quad \frac{dJ}{dt}=2Mx\frac{dx}{dt}\omega$$

and $\tau = Mgx\cos\theta = Mgx\cos\omega t \quad [\theta = \omega t]$

So $Mgx\cos\omega t = 2Mx\frac{dx}{dt}\omega \Rightarrow dx = \left(\frac{g}{2\omega}\right)\cos\omega t\, dt$

According to given condition i.e. for

$x = L/4,\ t = 0$ and for $x=\frac{L}{2}, t=\frac{\pi}{2\omega}$ [as $\omega t = \frac{\pi}{2}$],

the above equation becomes

$$\int_{L/4}^{L/2} dx = \frac{g}{2\omega}\int_0^{\pi/2\omega}\cos\omega t\, dt$$

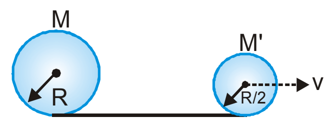
$$\Rightarrow [x]_{L/4}^{L/2}\frac{g}{2\omega^2}(\sin\omega t)_0^{\pi/2\omega} \Rightarrow \frac{L}{4}=\frac{g}{2\omega^2} \text{ i.e. } \omega=\sqrt{\frac{2g}{L}}$$

Substituting this value in $\omega = \frac{12v}{7L}$

$$\sqrt{\frac{2g}{L}}=\frac{12v}{7L} \text{ i.e. } v=\frac{7}{12}\sqrt{2\times 10\times 1.8} = 3.5 \text{ m/s.}$$

5. Let M' be the mass of unwound carpet.

Then $M' = \left(\frac{M}{\pi R^2}\right)\pi\left(\frac{R}{2}\right)^2 = \frac{M}{4}$



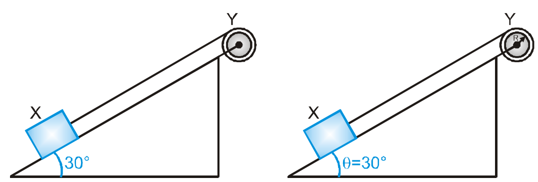
From conservation of mechanical energy :

$$MgR - M'g\frac{R}{2} = \frac{1}{2}\left(\frac{M}{4}\right)v^2+\frac{1}{2}I\omega^2$$

$$\Rightarrow MgR-\left(\frac{M}{4}\right)g\left(\frac{R}{2}\right)=\frac{Mv^2}{8}+\frac{1}{2}\left(\frac{1}{2}\times\frac{M}{4}\times\frac{R^2}{4}\right)\left(\frac{v}{R/2}\right)^2$$

$$\Rightarrow \frac{7}{8}MgR=\frac{3Mv^2}{16} \quad \therefore v=\sqrt{\frac{14Rg}{3}}$$

6. Given mass of block X
m = 0.5kg



mass of drum Y M = 2kg
Radius of drum R = 0.2 m
Angle of inclined plane
θ = 30°

Let 'a' be the linear retardation of block X and α the angular retardation of drum Y.

Then, $a = R\alpha;\ g\sin 30° - T = ma$ .....(i)

$$\Rightarrow \frac{mg}{2}-T=ma \qquad .....(ii)$$

$$\alpha=\frac{\tau}{I}=\frac{TR}{\frac{1}{2}MR^2} \Rightarrow \alpha=\frac{2T}{MR} \qquad .....(iii)$$

Solving equation (i), (ii) and (iii) for T,

We get $T=\frac{1}{2}\frac{Mmg}{M+2m}$

Substituting the value,

We get $T = \left(\frac{1}{2}\right)\left\{\frac{(2)(0.5)(9.8)}{2+(0.5)(2)}\right\} = 1.63\text{N}$

$\Rightarrow$ T = 1.63 N

(ii) From equation (iii), angular retardation of drum

$$\alpha = \frac{2T}{MR} = \frac{(2)(1.63)}{(2)(0.2)} = 8.15 \text{ rad}/\text{s}^2$$

$\Rightarrow$ linear retardation of block

$a = R\alpha = (0.2)(8.15) = 1.63 \text{m/s}^2$

At the moment when angular velocity of drum is

$\omega_0 = 10$ rad/s

The linear velocity of block will be

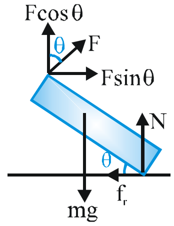
$v_0 = \omega_0 R = (10)(0.2) = 2$m/s

Now, the distance (s) travelled by the block until it comes to rest will be given by

$$s = \frac{v_0^2}{2a} \qquad \text{[Using } v^2 = v_0^2 - 2as \text{ with } v = 0]$$

$$= \frac{(2)^2}{2(1.63)}\text{m} \quad \Rightarrow \quad s = 1.22 \text{ m}$$

7. As shown in diagram



$F\sin\theta = \mu N$ ...(i)

$F\cos\theta + N = mg$ ...(ii)

As person slowly lift then α=0.

So net torque is zero

$$\Rightarrow F\ell = \frac{mg\ell\cos\theta}{2} \Rightarrow F = \frac{mg\cos\theta}{2} \quad .....(iii)$$

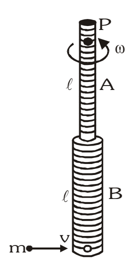
$$\Rightarrow \frac{mg}{2}\cos\theta\sin\theta = \mu\left(mg - \frac{mg\cos^2\theta}{2}\right)$$

$$\Rightarrow \frac{\cos\theta\sin\theta}{2-\cos^2\theta} = \mu \quad \Rightarrow \quad \mu = \frac{\sin 2\theta}{3-\cos 2\theta}$$

For μ minimum $\frac{d\mu}{d\theta} = 0$

So $\cos 2\theta = 0.56 \quad \Rightarrow \quad \mu = 0.34$

8. System is free to rotate but not free to translate. During collision, net torque on the system (rod A + rod B + mass m) about point P is zero. Therefore, angular momentum of system before collision= Angular momentum of system just after collision. (About P).



Let ω be the angular velocity of system just after collision, then

$L_i = L_f \quad \Rightarrow \quad mv(2\ell) = I\omega$

Here, I = moment of inertia of system about P

$$= m(2\ell)^2 + m_A(\ell^2/3) + m_B\left[\frac{\ell^2}{12} + \left(\frac{\ell}{2}+\ell\right)^2\right]$$

Given : $\ell = 0.6$m, m = 0.05kg

$m_A = 0.01$ kg and $m_B = 0.02$ kg

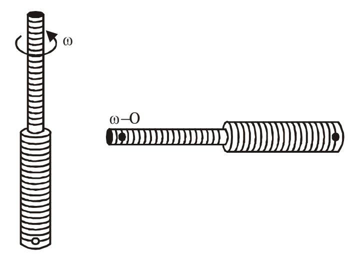
Substituting the values, we get

$I = 0.09$ kg–m²

Therefore, from Equation (i)

$$\Rightarrow \omega = \frac{2mv\ell}{I} = \frac{(2)(0.05)(v)(0.6)}{0.09} \Rightarrow \omega = 0.67\text{ v} .....(ii)$$

Now, after collision, mechanical energy will be conserved.



Therefore, decrease in rotational KE= increase in gravitational PE

$$\Rightarrow \frac{1}{2}I\omega^2 = mg(2\ell) + m_A g\left(\frac{\ell}{2}\right) + m_B g\left(\ell + \frac{\ell}{2}\right)$$

$$\Rightarrow \omega^2 = \frac{g\ell(4m + m_A + 3m_B)}{I}$$

$$= \frac{(9.8)(0.6)(4\times0.05 + 0.01 + 3\times0.02)}{0.09}$$

$= 17.64\ (\text{rad/s})^2 \quad \therefore \omega = 4.2$ rad/s .....(iii)

Equating equation (ii) and (iii), we get

$$v = \frac{4.2}{0.67}\text{m/s} \Rightarrow v = 6.3 \text{ m/s}$$

9. Given mass of disc m = 2kg and radius R =0.1 m

(i) FBD of any one disc is

Frictional force on the disc should be in forward direction. Let a_0 be the linear acceleration of COM of disc and α the angular acceleration about its COM. Then,

$$a_0 = \frac{f}{m} = \frac{f}{2} \quad ....(i)$$

$$\alpha = \frac{\tau}{I} = \frac{fR}{\frac{1}{2}mR^2} = \frac{2f}{mR} = \frac{2f}{2\times0.1} = 10f \quad .....(ii)$$

Since, there is no slipping between disc and truck.

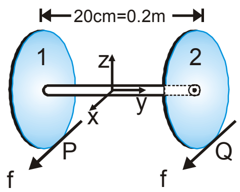
Therefore. Acceleration of point P = Acceleration of point Q

$\therefore \quad a_0 + R\,\alpha = a \quad \Rightarrow \left(\frac{f}{2}\right) + (0.1)(10f) = a$

$\Rightarrow \quad \frac{3}{2}f = a \;\Rightarrow\; f = \frac{2a}{3} = \frac{2\times 9.0}{3}N \quad \therefore\; f = 6N$

Since, this force is acting in positive x – direction.

Therefore, in vector from $\vec{f} = (6\hat{i})N$



(ii) $\vec{\tau} = \vec{r} \times \vec{f}$ Here, $\vec{f} = (6\hat{i})N$ (for both the discs)

$\vec{r}_P = \vec{r}_1 = -0.1\hat{j} - 0.1\hat{k}$ and $\vec{r}_Q = \vec{r}_2 = 0.1\hat{j} - 0.1\hat{k}$

Therefore, frictional torque on disk 1 about point O (centre of mass).

$\vec{\tau}_1 = \vec{r}_1 \times \vec{f} = (-0.1\hat{j} - 0.1\hat{k}) \times (6\hat{i})N - m$

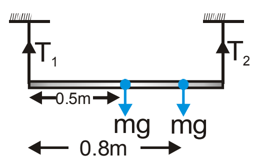
$= (0.6\hat{k} - 0.6\hat{j}) \;\Rightarrow\; \vec{\tau}_1 = 0.6(\hat{k} - \hat{j})$ N–m

and $|\vec{\tau}_1| = \sqrt{(0.6)^2 + (0.6)^2} = 0.85$ N–m

Similarly, $\vec{\tau}_2 = \vec{r}_2 \times \vec{f} = 0.6(-\hat{j} - \hat{k})$

and $\quad |\vec{\tau}_1| = |\vec{\tau}_2| = 0.85$ N – m

10. For translational equilibrium $T_1 + T_2 = 2mg$



For rotatory equilibrium

Take torque about extreme left point

$= mg[0.5 + 0.8] = T_2 \times 1$

$T_2 = mg \times 1.3$

$\therefore \quad T_1 = 2mg - T_2 = 0.7\,mg$

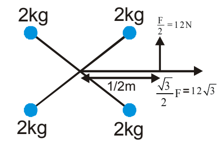
$\therefore \quad \text{Ratio } \frac{T_1}{T_2} = \frac{0.7mg}{1.3mg} = \frac{7}{13}$

11. $\therefore\; \tau = I\alpha$ and $\tau = |\vec{r} \times \vec{F}| = rf\sin\theta$

for maximum torque the angle between r and f must be 90° i.e. perpendicular to the heavy door.

12. Moment of inertia about an axis passing through intersection point

$= mr^2 \times 4 = 2 \times \left(\frac{1}{4}\right)^2 \times 4 = \frac{1}{2}$



Torque of force about this point = $12 \times \frac{1}{2} = 6$

$\therefore$ Angular acceleration

$\alpha = \frac{\tau}{I} = 6 \times 2 = 12 \text{rad/s}^2$

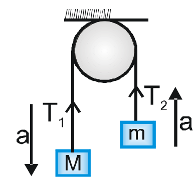
13. Let the acceleration be 'a'

For translatory motion

$Mg - T_1 = ma$ ...(i)

$-mg + T_2 = ma$ ...(ii)

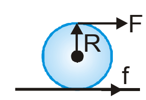
& for rotatory motion



$(T_1 - T_2)R = I \times \frac{a}{R}$ ...(iii)

From equation (i), (ii) & (iii) $a = \frac{(M - m)g}{(M + m) + I/R^2}$

14. Let the mass of sphere be 'm' & the linear acceleration of sphere be = a



For translatory motion

$F + f = ma$ .....(i)

For rotatory motion (for no slipping) $\Rightarrow \tau = I\alpha$

$(F - f)\,R = \frac{2}{3}mR^2 \times \frac{a}{R}$ .....(ii)

From equation (i) & (ii) $F + f = ma$

$F - f = \frac{2}{3}ma \quad \Rightarrow \quad a = \frac{6F}{5m}$

15. Using angular impulse = $\Delta J = I\omega$

$\Rightarrow \quad 3 \times 8 = 5 \times \left(\frac{0.3}{2}\right)^2 \times \omega$

$\Rightarrow \quad \omega = \frac{24 \times 2}{5 \times 0.09} = \frac{48}{0.45} = 106.66$ rad/sec

16. The plane of the particle is x – y & its velocity i.e. momentum & position vector is also in x – y plane

$\therefore$ Angular momentum $\vec{L} = \vec{r} \times \vec{P}$

The direction of $\vec{\tau}$ is perpendicular to the plane where $\vec{r}$ & $\vec{P}$ lie.

17. Two point masses and bar taken together as a system the angular momentum about centre of bar is

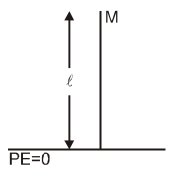
$$2mva + 2mv \times 2a = \left[\frac{8m \times (6a)^2}{12} + 2ma^2 + m4a^2\right]\omega$$

$$\Rightarrow \quad 30\,a\omega = 6v \Rightarrow \omega = \frac{v}{5a}$$

By COLM $2mv - m \times 2v = 0 \quad \Rightarrow v_c = 0$

$$E = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 30ma^2 \times \left(\frac{v}{5a}\right)^2 \Rightarrow E = \frac{3mv^2}{5}$$

18. When the top end touching the ground its P.E. changes into the rotational kinetic energy.



Let the angular velocity about the point of contact be ω at ground there is no slipping.

$$\therefore \; Mg\frac{\ell}{2} = \frac{1}{2} \times \frac{M\ell^2}{3} \times \omega^2; \qquad \omega = \sqrt{\frac{3g}{\ell}}$$

$\therefore$ Velocity of the top end

$$= v = \ell\omega = \ell \times \sqrt{\frac{3g}{\ell}} \Rightarrow v = \sqrt{3g\ell}$$

19. Linear momentum, angular momentum and kinetic energy are conserved in the process.

From conservation of linear momentum

$Mv' = mv \Rightarrow v' = mv/M$

Conservation of angular momentum gives.

$$\Rightarrow \quad mvd = \frac{M\ell^2}{12}\omega \quad \Rightarrow \quad \omega = \frac{12\,mvd}{M\ell^2}$$

Collision is elastic hence

Relative speed of approach

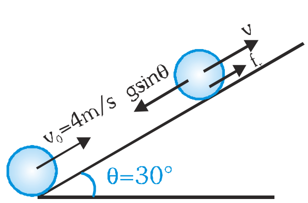
= Relative speed of separation

$v = v' + d\omega$

Substituting the values,we have

$$v = \frac{mv}{M} + \frac{12mvd^2}{M\ell^2} \Rightarrow m = \frac{M\ell^2}{12d^2 + \ell^2}$$

20. We have FBD as shown



So cylinder stop at when translatory and rotatory both stops. So constant acceleration is there

$$a = g\sin\theta - \frac{f_r}{m}$$

$\because \; f_r = \mu mg\cos\theta \Rightarrow a = g\sin\theta - \mu g\cos\theta$

So we have equation $v = u + at$ (as $v = 0$)

$$\Rightarrow \quad 4 = (g\sin\theta - \mu g\cos\theta) \Rightarrow \quad t = \frac{4}{g\sin\theta - \mu g\cos\theta}$$

By $\tau = I\alpha$

Torque by friction $\tau = f_r R \Rightarrow f_r R = \frac{mR^2\alpha}{2}$

$$\Rightarrow \quad \mu mg\cos\theta = \frac{1}{2}mR\alpha \qquad \Rightarrow \alpha = \frac{2\mu g\cos\theta}{R}$$

We know $a = \alpha R$

$$\Rightarrow \quad g\sin\theta - \mu g\cos\theta = 2\mu g\cos\theta \;\; (\text{as } \theta = 30)$$

$$\Rightarrow \quad \mu = \frac{1}{3\sqrt{3}} \quad \text{so } t = \frac{4}{\frac{g}{2}\left(1 - \frac{\sqrt{3}}{3\sqrt{3}}\right)}$$

$\Rightarrow \quad t = 1.2$ sec

21. Centre of mass from hinged point

$$CM = \frac{m\left[\frac{\ell}{3} + \frac{2\ell}{3} + \ell\right]}{3m} = \frac{2\ell}{3}$$

when the rod becomes vertical, its potential energy changes into rotational kinetic energy of system.

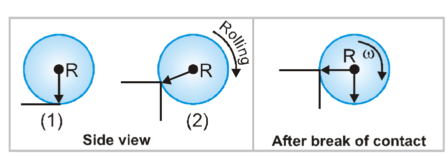
$$\frac{1}{2}\left[m\left[\frac{\ell^2}{9} + \frac{4\ell^2}{9} + \ell^2\right]\right]\omega^2 = 3mg \times \frac{2\ell}{3}$$

Angular velocity $\omega = \sqrt{\frac{18g}{7\ell}}$

$\therefore$ Linear velocity of B in vertical position

$$= \frac{2\ell}{3} \times \sqrt{\frac{18g}{7\ell}} = \sqrt{\frac{8g\ell}{7}}$$

22. **(i)** As the cylinder rolls without slipping about an axis passing through C.M, hence mechanical energy of the cylinder will be conserved i.e.



$\therefore \quad (U+KE)_1 = (U+KE)_2$

$$mgR + 0 = mgR\cos\theta + \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

but $\omega = \frac{v}{R}$ and $I = \frac{1}{2}mR^2$.

Therefore $mgR = mgR\cos\theta + \frac{1}{2}\left(\frac{1}{2}mR^2\right)$

$$\frac{v^2}{R^2} + \frac{1}{2}mv^2\frac{v^2}{R^2} = \frac{4}{3}g(1-\cos\theta) \qquad .....(i)$$

When the cylinder leaves the contact, normal reaction $N = 0$ and $\theta = \theta_c$.

Hence $mg\cos\theta_c = \frac{mv^2}{R} \Rightarrow \frac{v^2}{R} = g\cos\theta_c \qquad .....(ii)$

(i) & (ii) $= \frac{4}{3}g(1-\cos\theta) = g\cos\theta_c$

$$\cos\theta_c = \frac{4}{7} = \theta_c = \cos^{-1}\left(\frac{4}{7}\right)$$

At the time it leaves the contact $\cos\theta = \cos\theta_c = \frac{4}{7}$

(ii) On substituting it in equation (i),

$$v = \sqrt{\frac{4}{3}gR\left(1-\frac{4}{7}\right)} = \sqrt{\frac{4}{7}gR}$$

(iii) At the moment, when the cylinder leaves the contact

$$v = \sqrt{\frac{4}{7}gR}$$

Therefore rotational kinetic energy $K_R = \frac{1}{2}I\omega^2$

$$= \frac{1}{2}\left(\frac{1}{2}mR^2\right)\frac{v^2}{R^2} = \frac{1}{4}mv^2 = \frac{1}{4}m\left[\frac{4}{7}gR\right]$$

$$= K_R = \frac{mgR}{7} \qquad ....(iii)$$

When the cylinder loses its contact i.e. the frictional force vanishes, also torque due to gravitational force it zero. Hence its angular velocity is constant and rotational kinetic becomes constant, while its translation kinetic energy increases.

Applying conservation of energy at (i) and (ii)

$$K_T = mgR - \frac{mgR}{7} = \frac{6}{7}mgR \qquad .....(iv)$$

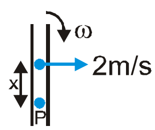
From (iii) & (iv) : $\frac{K_T}{K_R} = \frac{6mgR/7}{mgR/7} = 6$

23. Rod rotates about the C.M. let the point 'P' at a distance 'x' from C.M. having a zero velocity.

So 

$\therefore$ For zero velocity

$\omega x - 2 = 0$



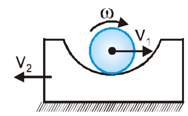
$\Rightarrow x = \frac{2}{\omega} = \frac{2}{2\pi} = \frac{1}{\pi}m$

$\Rightarrow x = \frac{1}{\pi}m$

(down to the centre of mass)

24. Form conservation of linear momentum $mv_1 = Mv_2$

$\therefore$ velocity of cylinder axis relative to block



Applying conservation of mechanical energy

$$Mg(R-r) = \frac{1}{2}mv_1^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}Mv_2^2$$

near $I = \frac{1}{2}mr^2$ and $\omega = \frac{V_r}{r}$

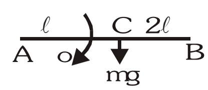
Solving above equation, we get

$v_1 = 2m/s$, $v_2 = 1.5$ m/s

Further $N - mg = \frac{mv_r^2}{R-r} = 16.67\,N$

25. (i) Angular acceleration about 'O'

$$\alpha = \frac{mg\ell}{\frac{m(4\ell)^2}{12} + m\ell^2} = \frac{3g}{7\ell}$$



$(a_c)_v = \alpha\ell = \frac{3}{7}g$ (downward)

Let N_v be the vertical reaction (upwards) at axis, then

$$mg - N_v = ma_c = \frac{3mg}{7} \Rightarrow N_v = \frac{4}{7}mg$$

If N_n horizontal reaction (towards (o) at axis, then

$N_H = m\omega^2\ell \quad \therefore$ Total reaction

$$N = \sqrt{N_H^2 + N_v^2} = \frac{4}{7}mg\sqrt{1+\left(\frac{7\ell\omega^2}{4g}\right)^2}$$

(ii) $a_c = \sqrt{(a_c)_v^2 + (\ell\omega^2)^2} = \sqrt{\left(\frac{3}{7}g\right)^2 + (\ell\omega^2)^2}$

(iii) Let ω' be the angular speed of the rod when it becomes vertical for the first time then from conservation of mechanical energy.

$$\frac{1}{2}I(\omega'^2 - \omega^2) = mg\ell$$

$$\omega'^2 = \omega^2 + \frac{2mg\ell}{I} = \omega^2 + \frac{2mg\ell}{\frac{7}{3}m\ell^2} = \omega^2 + \frac{6g}{7\ell}$$

Acceleration of C.M

$$a_c = \ell\omega'^2 = \ell\omega^2 + \frac{6g}{7}$$

Let 'N' be reaction (upward) at axis at this instant

$$N - mg = ma_c = m\ell\omega^2 + \frac{6mg}{7}$$

$$\therefore \quad N = \frac{13}{7}mg + m\ell\omega^2$$

(iv) From conservation of mechanical energy

$$mg\ell = \frac{1}{2} I\,\omega_{min}^2$$

$$\Rightarrow \omega_{min} = \sqrt{\frac{2mg\ell}{I}} = \sqrt{\frac{2mg\ell}{7m\ell^2/3}} = \sqrt{\frac{6g}{7\ell}}$$

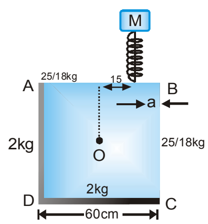
26. By using parallel axis theorem, the MI of the sphere about the rod (vertical axis) is

$$I = \frac{2}{5}mr^2 + m\ell^2$$

$\therefore$ Rotational kinetic energy

$$= \frac{1}{2}\left[\frac{2}{5}mr^2 + m\ell^2\right]\omega_0^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{\omega_0\ell}{r}\right)^2$$

27. (i) To calculate mass M we have to balance torque about point O.



So $\frac{25}{18} \times 30 + M \times 15 = 2 \times 30$; $M = \frac{11}{9}$ Kg

Torque due to rod AB and CD is zero.

(ii) When thread is burnt then energy stored in spring is distributed in term of velocity of block and to rotate the fream.

By angular momentum conservation.

Initial momentum = Final momentum

$\Rightarrow 0 = mvr - I\omega \quad \Rightarrow \quad I\omega = mvr$ .....(i)

By energy conservation

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \quad ...(ii)$$

Value of Moment of inertia about point O is

$$I = 2\left[\frac{25}{18} \times \frac{(0.6)^2}{12} + \frac{25}{18} \times (0.3)^2\right] + 2\left[2 \times \frac{(0.6)^2}{12} + 2 \times (0.3)^2\right]$$

So $I = \frac{61}{75}$. So by equation (i) We have

$$\frac{61}{75}\omega = \frac{11}{9} \times v \times 0.15 \Rightarrow \frac{61}{75}\omega = \frac{11}{9} \times v \times \frac{15}{100}$$

$$\Rightarrow \omega = \frac{5 \times 11}{61 \times 4}v \quad \Rightarrow \quad \omega = \frac{55}{244}v$$

Put this value in equation (ii)

$$\Rightarrow 76.5 = \frac{1}{2} \times \frac{61}{75} \times \left(\frac{5 \times 11}{61 \times 4}\right)^2 v^2 + \frac{1}{2} \times \frac{11}{9} \times v^2$$

$$\Rightarrow 76.5 = \frac{1}{2} \times \frac{61}{75} \times \frac{25 \times 121}{61 \times 61 \times 16}v^2 + \frac{1}{2} \times \frac{11}{9} \times v^2$$

$$\Rightarrow 76.5 = v^2\left[\frac{121}{96 \times 61} + \frac{11}{18}\right]$$

$$\Rightarrow 76.5 = v^2\left[\frac{2178 + 64416}{105408}\right]$$

$$\Rightarrow 76.5 = v^2 \times \frac{66594}{105408}$$

$$\Rightarrow v^2 = 121.08 \Rightarrow v \approx 11 \text{ m/s}$$

28. (i) The whole (Man + Woman + Rod) system rotates about the C.M of the system.

$\therefore$ $CM = \frac{m\ell}{M+m}$ from Man or $\frac{M\ell}{m+M}$ from woman

Now angular momentum $= I_{sy}\omega$

$$MV \times \frac{m\ell}{M+m} + mv \times \frac{M\ell}{m+M}$$

$$= \left[M \times \left(\frac{m\ell}{M+m}\right)^2 + m\left(\frac{M\ell}{M+m}\right)^2\right]\omega \quad \therefore \omega = \frac{V+v}{\ell}$$

(ii) Velocity of CM $= \frac{MV - mv}{M+m}$ $\because V_{cm} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$

Velocity of C.M remains constant as the

$\vec{f}_{ext} = 0$ for system

(iii) From the angular momentum conservation

$$I_1\omega_1 = I_2\omega_2$$

$$\Rightarrow \omega_2 = \frac{I_1\omega_1}{I_2} \quad \Rightarrow \quad \omega_2 = \frac{\frac{Mm\ell^2}{(M+m)} \times \left(\frac{V+v}{\ell}\right)}{\frac{Mm\ell_0^2}{(M+m)}}$$

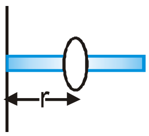
$$\therefore \quad \omega_2 = \frac{V+v}{\ell}\left(\frac{\ell^2}{\ell_0^2}\right)$$

(iv) Work done by couple

= change in rotational kinetic energy

$$= \frac{1}{2}I_2\omega_2^2 - \frac{1}{2}I_1\omega_1^2 = \frac{1}{2}\frac{Mm}{M+m}(V+v)^2\left[1-\frac{\ell}{\ell_0}\right]$$

(By putting I_1, I_2, ω_1 & ω_2) we get the answer

29. 

Power

$$= \frac{dw}{dt} = \frac{\tau d\theta}{dt} = \frac{\frac{dL}{dt}\times d\theta}{dt} = \frac{dL}{dt}\times\omega = 2\,mr^2\omega^2\times\omega$$

$$(\because L = mr^2\omega;\ \tau = \frac{dL}{dt} = 2\,mr\omega\frac{dr}{dt})$$

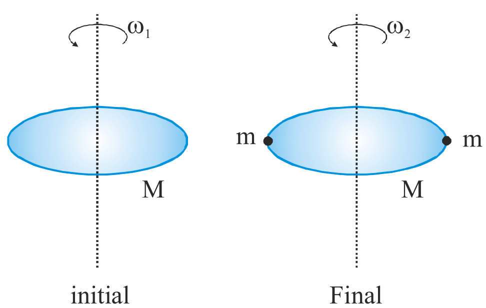
$$\frac{dL}{dt} = 2\,mr\omega\times r\omega = 2\,mr^2\omega^2\ [\frac{dr}{dt} = V = r\omega]$$

Power $= 2mr^2\omega^3$

EXERCISE - 5

Part # I : AIEEE/JEE-MAIN

1. Angular momentum of the disc +sphere will remain conserved.

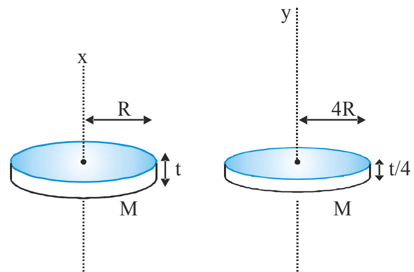


$$\frac{MR^2}{2}\omega_1 = \left(\frac{MR^2}{2} + mR^2 + mR^2\right)\omega_2$$

$$\omega_2 = \left(\frac{M}{M+4m}\right)\omega_1$$

3. The angular momentum about origin is $L = (r_\perp)p$

where $r_\perp$ is the perpendicular position of the linear momentum which in this case is ℓ; hence $L = mv\ell$.

4. 

$$I_{Disc} = \frac{MR^2}{2} \text{ where M = density (volume)}$$

$M_X = \rho(\pi R^2)t$

$M_Y = \rho(\rho\pi 16R^2)t/4 = 4M_X$

$$\Rightarrow \frac{I_X}{I_Y} = \frac{M_XR_X^2}{M_YR_Y^2} = \frac{1}{4}\left(\frac{R}{4R}\right)^2 = \frac{1}{64} \Rightarrow I_Y = 64I_X$$

5. $L = I\omega$ ...(i)

$K = \frac{1}{2}I\omega^2$ ...(ii)

On replacing I from eq. (i) we get

$$K = \frac{1}{2}\left(\frac{L}{\omega}\right)\omega^2 \Rightarrow L = \frac{2K}{\omega}$$

$$\frac{L_2}{L_1} = \left(\frac{K_2}{\omega_2}\right)\left(\frac{\omega_1}{K_1}\right) = \left(\frac{K_2}{K_1}\right)\left(\frac{\omega_1}{\omega_2}\right) = \frac{1}{4} \Rightarrow L_2 = \frac{L_1}{4}$$

6. As $\vec{\tau}\perp\vec{r}$ and $\vec{\tau}\perp\vec{F}$ so $\vec{\tau}\cdot\vec{r} = 0$ and $\vec{\tau}\cdot\vec{F} = 0$

($\because$ if 2 vectors are perpendicular their dot product is zero).

7. The acceleration vector is towards the centre of circle.

8. The angular momentum of the sphere will remain unchanged even on changing the radius of the sphere.

9. Momentum of inertia of solid sphere A

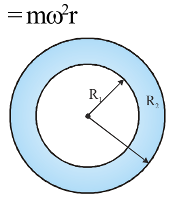
$$= \frac{2}{5}MR^2 = I_A$$

Moment of inertia of hollow sphere B

$$= \frac{2}{3}MR^2 = I_B;\ \frac{I_A}{I_B} = \frac{2}{5}\times\frac{3}{2} = \frac{3}{5} < 1$$

$I_A < I_B$

10. Force experienced by a particle on the surface of a ring $= m\omega^2 r$



$$\frac{F_1}{F_2} = \frac{m\omega^2R_1}{m\omega^2R_2} = \frac{R_1}{R_2}$$

11. Moment of inertia of a sector about an axis passing through its centre and perpendicular to its plane is

$I = \frac{1}{2}$ (Mass of sector) (Radius)^2

12. As surface is smooth

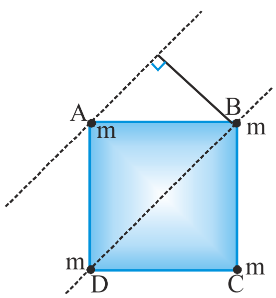
$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 30} = 40\ m/s$

if surface is rough $v = \sqrt{\frac{2gh}{\left(1+\frac{2}{5}\right)}} = 40\sqrt{\frac{5}{7}}\ m/s$

13. Coin will leave the contact if N = 0

so $mg = m(\omega^2 A) \Rightarrow A = \frac{g}{\omega^2}$

14. $I_{abt.A} = I_{due\ to\ A} + I_{due\ to\ B} + I_{due\ to\ C} + I_{due\ to\ D}$



$$I_{abt.A} = 0 + m(\ell\cos45°)^2 + m(\ell\cos45°)^2 + (\sqrt{2}\ell)^2$$

$$I_{abt.A} = \frac{m\ell^2}{2} + \frac{m\ell^2}{2} + 2m\ell^2 = 3m\ell^2$$

15. $\vec{F} = F(-\tilde{k})$

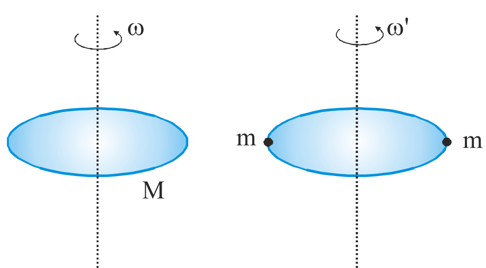
Position vector of O w.r. to given point $\vec{r} = -\tilde{i} + \tilde{j}$

Torque about $P = \vec{r} \times \vec{F}$

$= (-\tilde{i} + \tilde{j}) \times F(-\tilde{k})$

$= F(-\tilde{j} - \tilde{i}) = -F(\tilde{i} + \tilde{j})$

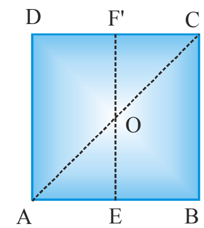
16. Conserving angular momentum, we get



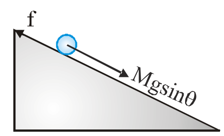
$$mR^2\omega = (mR^2 + MR^2 + MR^2)\omega'$$

$$\Rightarrow \omega' = \left(\frac{m}{m+2M}\right)\omega$$

17. The moment of inertia of a uniform square lamina about any axis passing through its centre and in the plane of the lamina is same hence $I_{AC} = I_{EF}$



19.



The force equation

$Mg\sin\theta - f = Ma$ .....(i)

The torque equation

$fR = I\alpha$ .....(ii)

for pure rolling motion $\alpha = \frac{a}{R}$; $fR = \frac{Ia}{R}$; $f = \frac{Ia}{R^2}$

From equations (i) and (iii), we get

$$\Rightarrow Mg\sin\theta - \frac{Ia}{R^2} = Ma \Rightarrow Mg\sin\theta = a\left[M + \frac{I}{R^2}\right]$$

$$\Rightarrow Mg\sin\theta = Ma\left[1 + \frac{I}{MR^2}\right] \Rightarrow a = \frac{g\sin\theta}{1 + \frac{I}{MR^2}}$$

20. A central force cannot apply the torque about the centre, hence the angular momentum of the body under the central force will be a constant.

22. $\frac{1}{2}I\omega^2 = mgh$ where $I = \frac{m\ell^2}{3}$

$$\Rightarrow \frac{1}{2}\frac{m\ell^2}{3}\omega^2 = mgh \Rightarrow h = \frac{\ell^2\omega^2}{6g}$$

23. Angular momentum

$\vec{L} = \vec{r} \times \vec{p}$

where $\vec{r} = u\cos\theta t\ \tilde{i} + \left(u\sin\theta\ t - \frac{1}{2}gt^2\right)\tilde{j}$

$\vec{p} = m[u\cos\theta\tilde{i} + (u\sin\theta - gt)\tilde{j}]$

$\vec{L} = \vec{r} \times \vec{p} = -\frac{1}{2}mgv_0t^2\cos\theta\,\hat{k}$

24. As $\tau = rF$

So $\tau = 2(20t - 5t^2) = 40t - 10t^2$

but $\tau = I\alpha = 40t - 10t^2$

$\Rightarrow 10 \times \alpha = 40t - 10t^2 \Rightarrow \alpha = 4t - t^2$

$\therefore \frac{d^2\theta}{dt^2} = 4t - t^2$

$\omega = \frac{d\theta}{dt} = \frac{4t^2}{2} - \frac{t^3}{3}$ .....(i)

for $\omega = 0$; $\frac{4t^2}{2} - \frac{t^3}{3} = 0$

ω will be zero at $t = 6$ sec.

from eq. (i) $\frac{d\theta}{dt} = 2t^2 - \frac{t^3}{3}$

$\theta = \frac{2t^3}{3} - \frac{t^4}{3 \times 4}$

$\theta = \frac{2}{3} \times 216 - \frac{36 \times 36}{12} = 36$ rad.

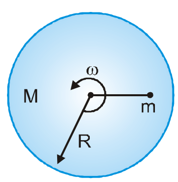
Number of turns

$N = \frac{\theta}{2\pi} = \frac{18}{\pi} = 5.732$

So more than 3 but less than 6

25. $MI = \frac{MR^2}{2} + mx^2$

As the insect moves MI. First decreases, then increases. So by conservation of angular momentum, angular velocity (ω) first increases then decreases.



26. $L = m(v\cos 30°)(H_{max}) = \frac{\sqrt{3}}{16}\frac{mv^3}{g}$

27. The radius towards the left side of O is smaller. Thus system turns towards left.

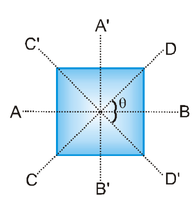
Part # II : IIT-JEE ADVANCED

1. $A'B' \perp AB$ and $C'D' \perp CD$

From symmetry

$I_{AB} = I_{A'B'}$ and $I_{CD} = I_{C'D'}$

From theorem of perpendicular axes.



$I_{ZZ} = I_{A'B} + I_{A'B'} = I_{CD} + I_{C'D'} = 2I_{AB} = 2I_{CD} \Rightarrow I_{AB} = I_{CD}$

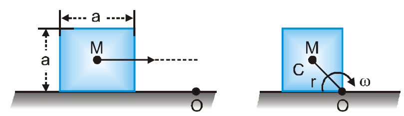
Alternate Solution

The relation between I_{AB} and I_{CD} should be true for all values of θ :

and at $\theta = 0, I_{CD} = I_{AB}$

Similarly at $\theta = \pi/2, I_{CD} = I_{AB}$ (by symmetry)

2. $r = \sqrt{2}\frac{a}{2} \Rightarrow r^2 = \frac{a^2}{2}$



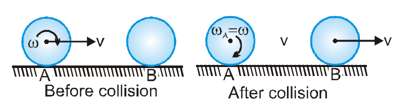
Net torque about O is zero. Therefore, angular momentum (L) about O will be conserved

$Mv\left(\frac{a}{2}\right) = I_0\omega = (I_{CM} + Mr^2)\,\omega$

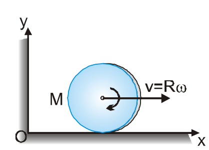
$= \left\{\left(\frac{Ma^2}{6}\right) + M\left(\frac{a^2}{2}\right)\right\}\omega = \frac{2}{3}Ma^2\omega \Rightarrow \omega = \frac{3v}{4a}$

3. Since, it is head on elastic collision between two identical balls, they will exchange their linear velocities i.e., A comes to rest and B starts moving with linear velocity v. As there is no friction anywhere, torque on both the spheres about their centre of mass is zero and their angular velocities remain unchanged.

Therefore, $\omega_A = \omega$ and $\omega_B = 0$.



4. From the theorem



$\vec{L}_0 = \vec{L}_{CM} + M(\vec{r} \times \vec{v})$ .....(i)

We may write

Angular momentum about O = Angular momentum about CM + Angular momentum of CM about origin.

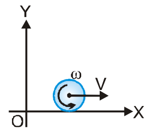
$\therefore L_0 = I\omega + MRv = \frac{1}{2}MR^2\omega + MR(R\omega) = \frac{3}{2}MR^2\omega$

Note that in this case both the terms in Equation (i) i.e. $\vec{L}_{CM}$ and $M(\vec{r} \times \vec{v})$ have the same direction ⊗.

That is why we have used $L_0 = I\omega + MRv$

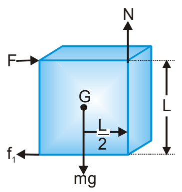
We will use $L_0 = I\omega \sim MRv$

if they are in opposite direction as shown in figure.



5. Net external torque on the system is zero. Therefore, angular momentum is conserved. Force acting on the system are only conservative. Therefore, total mechanical energy of the system is also conserved.

6. At the critical condition, normal reaction N will pass through point P.



In this condition $\tau_N = 0 = \tau_{fr}$ (about P)

the block will topple when

$$\tau_F > \tau_{mg} \Rightarrow FL > (mg)\frac{L}{2} \quad \therefore \quad F > \frac{mg}{2}$$

Therefore, the minimum force required to topple the block is $F = \frac{mg}{2}$

7. Mass of the ring $M = \rho L$

Let R be the radius of the ring, then

$$= 2\pi R \quad \Rightarrow \quad R = \frac{L}{2\pi}$$

Moment of inertia about an axis passing through O and parallel to XX' will be

$$I_0 = \frac{1}{2}MR^2$$

Therefore, moment of inertia about XX' (from parallel axis theorem) will be given by

$$I_{XX'} = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

Substituting values of M and R

$$I_{XX'} = \frac{3}{2}(\rho L)\left(\frac{L^2}{4\pi^2}\right) = \frac{3\rho L^3}{8\pi^2}$$

8. Mass of the whole disc = 4 M

Moment of inertia of the disc about the given axis

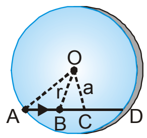
$$= \frac{1}{2}(4M)R^2 = 2MR^2$$

$\therefore$ Moment of inertia of quarter section of the disc

$$= \frac{1}{4}(2MR^2) = \frac{1}{2}MR^2$$

9. mg sinθ component is always down the plane whether it is rolling up or rolling down. Therefore, for no slipping, sense of angular acceleration should also be same in both the cases. Therefore, force of friction f always act upwards.

10. Since, there is no external torque, angular momentum will remain conserved. The moment of inertia will first decrease till the tortoise moves from A to C and then increase as it moves from C and D. Therefore, ω will initially increase and then decrease.



Let R be the radius of platform, m the mass of tortoise and M is the mass of platform.

Moment of inertia when the tortoise is at A

$$I_1 = mR^2 + \frac{MR^2}{2}$$

and moment of inertia when the tortoise is at B

$$I_2 = mr^2 + \frac{MR^2}{2}$$

Here, $r^2 = a^2 + [\sqrt{R^2 - a^2} - vt]^2$

From conservation of angular momentum

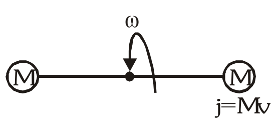
$$\omega_0 I_1 = \omega(t)\, I_2$$

Substituting the values we can see that variation of ω(t) is non–linear.

11. Let ω be the angular velocity of the rod.

Applying, angular impulse = change in angular momentum about centre of mass of the system.

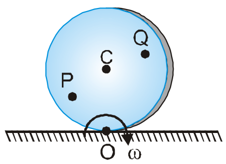
$$J.\frac{L}{2} = I_c\,\omega$$



$$\therefore \quad (Mv)\left(\frac{L}{2}\right) = 2\left(\frac{ML^2}{4}\right)\omega \quad \therefore \quad \omega = \frac{v}{L}$$

12. In uniform circular motion the only force acting on the particle is centripetal (towards centre). Torque of this force about the centre is zero. Hence, angular momentum about centre remain conserved.

13. In case of pure rolling bottom most point is the instantaneous centre of zero velocity. Velocity of any point on the disc, $v = r\omega$, where r is the distance of point from O.



$r_Q > r_C > r_P \therefore v_Q > v_C > v_P$

14. From conservation of angular momentum ($I\omega$ = constant), angular velocity will remain half.

$$K = \frac{1}{2}I\omega^2$$

The rotational kinetic energy will become half.

15. $\vec{L} = m(\vec{r} \times \vec{v})$

Direction of $(\vec{r} \times \vec{v})$, hence the direction of angular momentum remains the same.

16. $I_{remaining} = I_{whole} - I_{removed}$

$$\Rightarrow \quad I = \frac{1}{2}(9M)(R)^2 - \left[\frac{1}{2}m\left(\frac{R}{3}\right)^2 + 1m\left(\frac{2R}{3}\right)^2\right] \quad \text{....(i)}$$

Here, $m = \frac{9M}{\pi R^2} \times \pi\left(\frac{R}{3}\right)^2 = M$

Substituting in Equation (1), we have $I = 4MR^2$

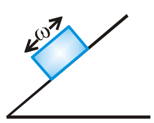
17. $\frac{2}{5}MR^2 = \frac{1}{2}Mr^2 + Mr^2 \quad \Rightarrow \quad \frac{2}{5}MR^2 = \frac{3}{2}Mr^2$

$\therefore \quad r = \frac{2}{\sqrt{15}}R$

18. $\frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2 = mg\left(\frac{3v^2}{4g}\right) \quad \therefore I = \frac{1}{2}mR^2$

$\therefore$ Body is disc.

19. Condition for no toppling



$$N\left(\frac{W}{2}\right) > \mu N\left(\frac{h}{2}\right)$$

$$\frac{\omega}{h} > \mu \quad \Rightarrow \quad \frac{2}{3} > \mu \quad \Rightarrow \quad \mu < \frac{2}{3}$$

but as the coefficient of friction is greater than 1 block will topple at some angle

20. **(A)** $\frac{dp}{dt} = F_{ext}$ as $F_{ext} = 0 \quad \Rightarrow \quad dp = 0,$

change in momentum is zero

(B) Kinetic energy of particle is scaler quantity therefore. It may change for a system, if external force is zero

(C) $F_{ext} = 0$ does not indicate that torque is zero therefore there may be change in angular momentum.

(D) Internal force on the system may decrease due to the potential energy

MCQ

1. $\vec{\tau} = \vec{A} \times \vec{L}$ i.e. $\frac{d\vec{L}}{dt} = \vec{A} \times \vec{L}$

This relation implies that $\frac{d\vec{L}}{dt}$ is perpendicular to both $\vec{A}$ and $\vec{L}$.

Therefore option (a) is correct.

For (B)

Component of $\vec{L}$ in the direction of $\vec{A} = \vec{L} \cdot \hat{A}$

$$\text{As} \quad \frac{d}{dt}\left(\vec{L} \cdot \hat{A}\right) = \vec{L} \cdot \frac{d\hat{A}}{dt} + \frac{d\vec{L}}{dt} \cdot \hat{A} = 0$$

The component of $\vec{L}$ in the direction of $\vec{A}$ does not change with time.

For (C)

Here, $\vec{L}.\vec{L} = L^2$

Differentiating w.r.t. time, we get

$$\vec{L}.\frac{d\vec{L}}{dt} + \frac{d\vec{L}}{dt}.\vec{L} = 2L\frac{dL}{dt} \quad \Rightarrow \quad 2\vec{L}.\frac{d\vec{L}}{dt} = 2L\frac{dL}{dt}$$

But since, $\vec{L} \perp \frac{d\vec{L}}{dt} \quad \therefore \quad \vec{L}.\frac{d\vec{L}}{dt} = 0$

Therefore, from Equation (i) $\frac{dL}{dt} = 0$

or magnitude of $\vec{L}$ i.e. L does not change with time.

2. In case of pure rolling

$$f = \frac{mg\sin\theta}{1 + \frac{mR^2}{I}} \text{ (upwards)} \quad \therefore \quad f \propto \sin\theta$$

Therefore, as θ decreases force of friction will also decrease.

3. On smooth part BC, due to zero torque, angular velocity and hence the rotational kinetic energy remains constant. While moving from B to C translational kinetic energy converts into gravitational potential energy.

4. Since it is performing pure rolling therefore velocity of C will be twice of velocity of B and velocity of A is zero.

Comprehension–1

1. $\frac{1}{2}I(2\omega)^2 = \frac{1}{2}kx_1^2$ .....**(i)**

$\frac{1}{2}(2I)(\omega)^2 = \frac{1}{2}kx_2^2$ .....**(ii)**

From equations (i) and (ii), we have $\frac{x_1}{x_2} = \sqrt{2}$

2. Let ω' be the common velocity. Then from conservation of angular momentum, we have

$$(I + 2I)\omega' = I(2\omega) + 2I(\omega) \quad \Rightarrow \quad \omega' = \frac{4}{3}\omega$$

From the equation, angular impulse = change in angular momentum, for any of the disc, we have

$$\tau.t = I(2\omega) - I\left(\frac{4}{3}\omega\right) = \frac{2I\omega}{3} \qquad \therefore \quad \tau = \frac{2I\omega}{3t}$$

3. Loss of kinetic energy $= K_i - K_f$

$$= \left\{\frac{1}{2}I(2\omega)^2 + \frac{1}{2}(2I)(\omega)^2\right\} - \frac{1}{2}(3I)\left(\frac{4}{3}\omega\right)^2 = \frac{1}{3}I\omega^2$$

Comprehension–2

1. For linear motion of disc

$F_{net} = Ma = -2kx + f$

where f = frictional force.

For rolling motion

$$f_R = -\left(\frac{MR^2}{2}\right)(\alpha) = -\left(\frac{Ma}{2}\right)R \Rightarrow f = -\frac{Ma}{2} = -\frac{F_{ext}}{2}$$

Therefore $F_{ext} = -2kx - \frac{F_{ext}}{2} \Rightarrow -\frac{4kx}{3}$

2. Total energy of system

$$E = \frac{1}{2}Mv^2\left(1 + \frac{1}{2}\right) + 2 \times \frac{1}{2}kx^2 = \frac{3}{4}Mv^2 + kx^2$$

$$\frac{dE}{dt} = 0 \Rightarrow \frac{3}{4}M(2v)\frac{dv}{dt} + 2k \times \left(\frac{dx}{dt}\right) = 0$$

$$\Rightarrow \frac{dv}{dt} + \left(\frac{4k}{3M}\right)x = 0 \quad \Rightarrow \quad \omega = \sqrt{\frac{4k}{3M}}$$

3. Using energy conservation law

$$\frac{1}{2}Mv_0^2\left(1 + \frac{1}{2}\right) = 2 \times \frac{1}{2}kx_1^2$$

$$2kx_1 - f_{max} = Ma \;\&\; f_{max}R = \left(\frac{MR^2}{2}\right)\alpha$$

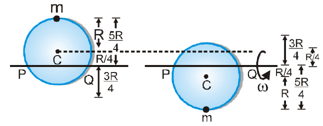
But $f_{max} = \mu Mg \quad \Rightarrow \quad x_1 = \frac{3\mu Mg}{2K}$

$$\Rightarrow \frac{3}{4}Mv_0^2 = Kx_1^2 = \frac{1}{K}\left(\frac{9\mu^2M^2g^2}{4}\right)$$

$$\Rightarrow v_0 = \mu g\sqrt{\frac{3M}{K}}$$

Subjective Questions

1. Initial and final positions are shown.



Decrease in potential energy of mass

$$m = mg\left\{2 \times \frac{5R}{4}\right\} = \frac{5mgR}{2}$$

Decrease in potential energy of disc

$$= mg\left\{2 \times \frac{R}{4}\right\} = \frac{mgR}{2}$$

Therefore, total decrease in potential energy of system

$$= \frac{5mgR}{2} + \frac{mgR}{2} = 3mgR$$

Gain in kinetic energy of system $= \frac{1}{2}I\omega^2$

where I = moment of inertia of system (disc + mass) about axis PQ = moment of inertia of disc + moment of inertia of mass

$$= \left\{\frac{mR^2}{4} + m\left(\frac{R}{4}\right)^2\right\} + m\left(\frac{5R}{4}\right)^2 \Rightarrow I = \frac{15mR^2}{8}$$

From conservation of mechanical energy,

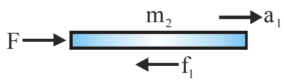
Decrease in potential energy = Gain in kinetic energy

$$\therefore \quad 3mgR = \frac{1}{2}\left(\frac{15mR^2}{8}\right)\omega^2 \quad \Rightarrow \quad \omega = \sqrt{\frac{16g}{5R}}$$

Therefore, linear speed of particle at its lowest point

$$v = \left(\frac{5R}{4}\right)\omega = \frac{5R}{4}\sqrt{\frac{16g}{5R}} \quad \Rightarrow \quad v = \sqrt{5gR}$$

2. We can choose any arbitrary directions of frictional forces at different contacts.



In the final answer the negative values will show the opposite directions.

Let

f_1 = friction between plank and cylinder

f_2 = friction between cylinder and ground

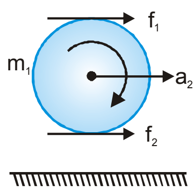
a_1 = acceleration of plank

a_2 = acceleration of centre of mass of cylinder

and

α = angular acceleration of cylinder about its CM.

Directions of f_1 and f_2 are as shown here



Since, there is no slipping anywhere

$\therefore \quad a_1 = 2a_2$ .....(i)

(Acceleration of plank = acceleration of top point of cylinder

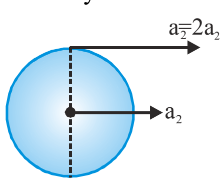
$$a_1 = \frac{F - f_1}{m_2} \quad .....(ii)$$

$$a_2 = \frac{f_1 + f_2}{m_1} \quad .....(iii)$$

$$\alpha = \frac{(f_1 - f_2)R}{I}$$

(I = moment of inertia of cylinder above CM)

$$= \frac{(f_1 - f_2)R}{\frac{1}{2}m_1R^2}$$



$$\alpha = \frac{2(f_1 - f_2)}{m_1R} \quad .....(iv)$$

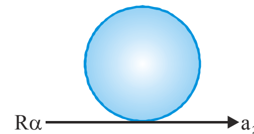
$$a_2 = R\alpha = \frac{2(f_1 - f_2)}{m_1} \quad .....(v)$$

(Acceleration of bottom most point of cylinder = 0)

(a) Solving Equation (i), (ii), (iii) and (v),

we get $a_1 = \dfrac{8F}{3m_1 + 8m_2}$

and $a_2 = \dfrac{4F}{3m_1 + 8m_2}$

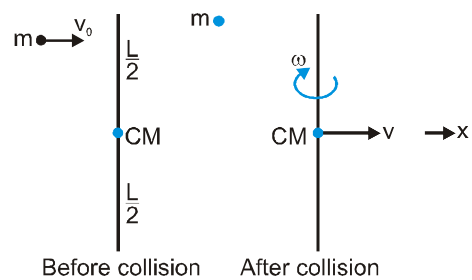


(b) $f_1 = \dfrac{3m_1F}{3m_1 + 8m_2}$; $f_2 = \dfrac{m_1F}{3m_1 + 8m_2}$

Since all quantities are positive.

3. **(i)** Let just after collision, velocity of CM of rod is v and angular velocity about CM is ω.

Applying following three laws :



(a) External force on the system (rod + mass) in horizontal plane along x–axis is zero.

$\therefore$ Applying conservation of linear momentum in x–direction.

$mv_0 = Mv$ ...(i)

(b) Net torque on the system about CM of rod is zero.

$\therefore$ Applying conservation of angular momentum about CM of rod

$$mv_0\left(\frac{L}{2}\right) = I\omega \Rightarrow mv_0\frac{L}{2} = \frac{ML^2}{12}\omega$$

$$\Rightarrow mv_0 = \frac{ML\omega}{6}$$

(c) Since,the collision is elastic, kinetic energy is also conserved.

$$\therefore \frac{1}{2}mv_0^2 = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

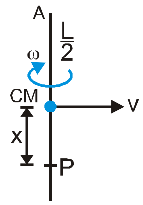
$$\Rightarrow mv_0 = Mv^2 + \frac{ML^2}{12}\omega^2$$

From equation (i), (ii) and (iii),

we get the following results $\dfrac{m}{M} = \dfrac{1}{4}$

$$v = \frac{mv_0}{M} \text{ and } \omega = \frac{6mv_0}{ML}$$

(ii) Point P will be at rest if $x\,\omega = v$



$$\Rightarrow \quad x = \frac{v}{\omega} = \frac{mv_0/M}{6mv_0/ML}$$

$$\Rightarrow \quad x = L/6$$

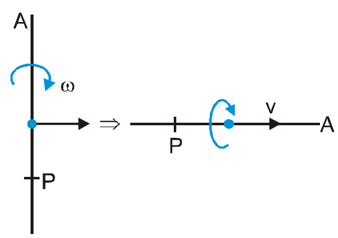
$$\therefore \quad AP = \frac{L}{2} + \frac{L}{6} \Rightarrow AP = \frac{2}{3}L$$

(iii) After time $t = \frac{\pi L}{3v_0}$ angle rotated by rod,

$$\theta = \omega t = \frac{6mv_0}{ML} \cdot \frac{\pi L}{3v_0} = 2\pi\left(\frac{m}{M}\right) = 2\pi\left(\frac{1}{4}\right) = \frac{1}{4}$$

$$\therefore \quad \theta = \frac{\pi}{2}$$

Therefore, situation will be as shown below :



$\therefore$ Resultant velocity of point P will be

$$|\vec{v}_p| = \sqrt{2V} = \sqrt{2}\left(\frac{m}{M}\right)v_0 = \frac{\sqrt{2}}{4}v_0 = \frac{v_0}{2\sqrt{2}}$$

4. Let r be the perpendicular distance of CM from the line AB and ω the angular velocity of the sheet just after colliding with rubber obstacle for the first time.

Obviously the linear velocity of CM before and after collision will be

$v_i = (r)\,(1 \text{ rad/s}) = r$ and $v_f = r\omega$

$\vec{v}_i$ and $\vec{v}_f$ will be in opposite directions.

Now, linear impulse on CM

= change in linear momentum of CM

$$\Rightarrow \quad 6 = m(v_f + v_i) = 30\,(r + r\omega) \quad \Rightarrow \quad r\,(1+\omega) = \frac{1}{5}$$

Similarly, angular impulse about AB

= change in angular momentum about AB

Angular impulse = Linear impulse ×

perpendicular distance of impulse from AB

Hence, $6\,(0.5 \text{ m}) = I_{AB}\,(\omega + I)$

[Initial angular velocity = 1 rad/s]

$$\Rightarrow \quad 3 = [I_{CM} + Mr^2]\,(1+\omega)$$

$$\Rightarrow \quad 3 = [1.2 + 30r^2]\,(1+\omega) \quad \text{....(ii)}$$

Solving Equation (i) and (ii) for r,

we get r = 0.4 m and r = 0.1 m

But at r = 0.4m, ω comes out to be negative (–0.5 rad/s) which is not acceptable.

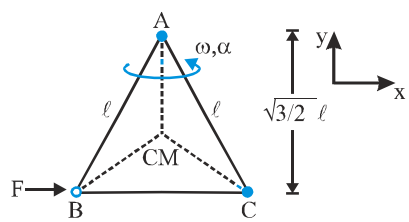
(i) r = distance of CM from AB = 0.1 m

(ii) Substituting r = 0.1m in Equation (i), we get ω=1 rad/s i.e., the angular velocity with which sheet comes back after the first impact is 1 rad/s.

(iii) Since, the sheet returns with same angular velocity of 1 rad/s, the sheet will never come to rest.

5. **(i)** The distance of centre of mass (CM) of the system about point A will be: $r = \frac{\ell}{\sqrt{3}}$

Therefore, the magnitude of horizontal force exerted by the hinge on the body is



F = centripetal force $\quad \Rightarrow \quad F = (3m)\,r\omega^2$

$$\Rightarrow \quad F = (3m)\left(\frac{\ell}{\sqrt{3}}\right)\omega^2 \quad \Rightarrow \quad F = \sqrt{3}\,m\ell\omega^2$$

(ii) Angular acceleration of system about point A is

$$\alpha = \frac{\tau_A}{I_A} = \frac{F\left(\frac{\sqrt{3}}{2}\ell\right)}{2m\ell^2} = \frac{\sqrt{3}F}{4m\ell}$$

Now, acceleration of CM along x–axis is

$$a_x = r\alpha = \left(\frac{\ell}{\sqrt{3}}\right)\left(\frac{\sqrt{3}F}{4m\ell}\right) \Rightarrow a_x = \frac{F}{4m}$$

Now, let F_x be the force applied by the hinge along x–axis. Then,

$$F_x + F = (3m)a_x \quad \Rightarrow \quad F_x + F = (3m)\left(\frac{F}{4m}\right)$$

$$\Rightarrow \quad F_x + F = \frac{3}{4}F \quad \Rightarrow \quad F_x = -\frac{F}{4}$$

Now, let F_y be the force applied by the hinge along y–axis. Then,

F_y = centripetal force

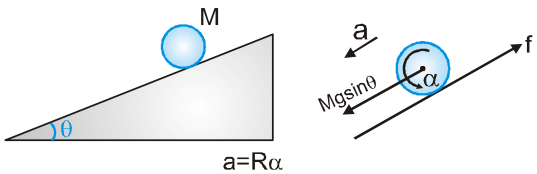
$$\Rightarrow \quad F_y = \sqrt{3}\,m\ell\omega^2$$

6. Angular momentum of the system about point O will remain conserved.

$L_i = L_f$

$\therefore \quad mvL = I\omega = \left[mL^2 + \frac{ML^2}{3}\right]\omega \quad \therefore \quad \omega = \frac{3mv}{L(3m+M)}$

7. For rolling without slipping, we have



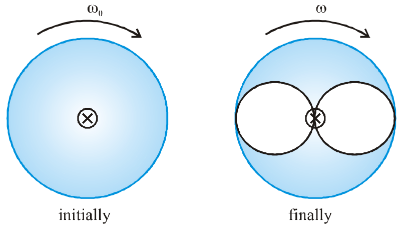
$\Rightarrow \frac{Mg\sin\theta - f}{M} = R\left(\frac{fR}{\frac{1}{2}MR^2}\right) \Rightarrow \frac{Mg\sin\theta - f}{M} = \frac{2f}{M}$

$\therefore \quad f = \frac{Mg\sin\theta}{3}$

Therefore, linear acceleration of cylinder,

$a = \frac{Mg\sin\theta - f}{M} = \frac{2}{3}g\sin\theta$

8. by angular momentum conservation



$I_{disc}\,\omega_0 = \left(I_{disc} + 2I_{ring}\right)\omega$

$\Rightarrow \frac{50(0.4)^2}{2}\times 10 = \left[\frac{50(0.4)^2}{2} + 2\left(2\times 6.25\times(0.2)^2\right)\right]\omega$

$\Rightarrow \omega = 8$ rad/sec.

9. $\frac{1}{2}mv^2 + \frac{1}{2}\frac{MR^2}{2}\times\frac{v^2}{R_2}$

$\frac{3}{4}m\times 3^2 + m\times 30\times 20 = \frac{3}{4}m.v^2$

$\frac{3}{4}mu_2^{\,2} + m\times 27\times 10 = \frac{3}{4}m\upsilon^2$

$= \frac{3}{4}m\times 3^2 + 300\,m = \frac{27}{4} + 30$

$\frac{3}{4}v_2^{\,2} = \frac{147}{4} \;;\; v_2 = 7$

10. $L_i = L_f$

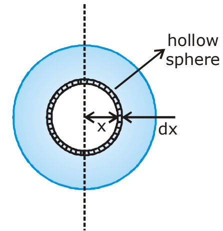
$mR^2\omega = \frac{8}{9}\,\omega\,(I_1 + I_2 + I_2 + I_{ring})$

$mR^2\omega = \frac{8}{9}\,\omega\left(\frac{m}{8}\left(\frac{3}{5}R\right)^2 + \frac{m}{8}x^2 + mR^2\right)$

on solving we get

$x = \frac{4R}{5}$

11. $dI = \frac{2}{3}dm\cdot x^2$



$dm = \rho\times 4\pi x^2\cdot dx$

$dI_A = \frac{2}{3}\left[k\cdot\frac{x}{R}\right]4\pi x^4 dx$

For B

$dI_B = \frac{2}{3}\times\frac{kx^5}{R^5}4\pi x^4 dx \qquad = \frac{2k}{3R}\cdot 4\pi\int_0^R x^5 dx$

$= \frac{8k\pi}{3R^5}\int x^9\cdot dx$

$= \frac{8k\pi}{3R}\cdot\frac{R^6}{6} = \frac{4k\pi R^5}{9}$

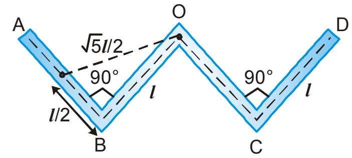
$I_B = \frac{8k\pi}{3R^5}\times\frac{R^{10}}{10} = \frac{8k\pi}{3}\cdot\frac{R^5}{10}\times\frac{9R}{4k\pi R^5}$

$n = 6$

MOCK TEST

1. The given structure can be broken into 4 parts

For AB : $I = I_{CM} + m\times d^2 = \frac{m\ell^2}{12} + \frac{5m}{4}\ell^2 \;;\; I_{AB} = \frac{4}{3}ml^2$



For BO : $I = \frac{m\ell^2}{3}$

$\therefore$ For composite frame : (by symmetry)

$I = 2[I_{AB} + I_{OB}] = 2\left[\frac{4m\ell^2}{3} + \frac{m\ell^2}{3}\right] = \frac{10}{3}m\ell^2.$

2. $\omega_{rod} = \omega_{point} = \left(\frac{v_{rel.}}{r}\right)$; $v_{rel.}$ bring the velocity of one point w.r.t. other.

$= \frac{3v - v}{r}$ and 'r' being the distance between them.

$= \frac{2v}{r}$

3. Given $a_A = 2\alpha = 5\ m/s^2 \Rightarrow \alpha = 5/2\ rad/s^2$

$\Rightarrow a_B = 1.(\alpha) = 5/2\ m/s^2$

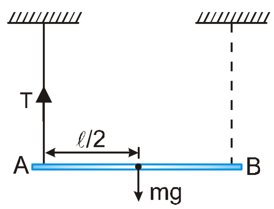
4. Immediately after string connected to end B is cut, the rod has tendency to rotate about point A.
Torque on rod AB about axis passing through A and normal to plane of paper is

$$\frac{m\ell^2}{3}\alpha = mg\frac{\ell}{2} \Rightarrow \alpha = \frac{3g}{2\ell}$$

Alternative

Applying Newton's law on center of mass

$mg - T = ma$ .....(i)



Writing $\tau = I\alpha$ about center of mass

$T\frac{\ell}{2} = \frac{m\ell^2}{12}\alpha$ ....(ii) Also $a = \frac{\ell}{2}\alpha$ ....(iii)

From (i), (ii) and (iii)

$$\alpha = \frac{3g}{2\ell}$$

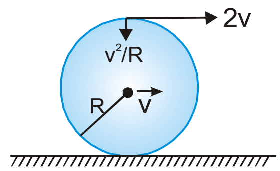
5. As the inclined plane is smooth, the sphere can never roll rather it will just slip down.
Hence, the angular momentum remains conserved about any point on a line parallel to the inclined plane and passing through the centre of the ball.

6. As $\Sigma\tau = 0$; Angular momentum, linear momentum remains conserved.
As the two balls will move radially out, I changes.In order to keep the angular momentum ($L = I\omega$) conserved, angular speed (ω) should change.
Hence (D).

7. If the track is smooth (case A), only translational kinetic energy changes to the gravitational potential energy.
But, if the track is rough (case B), both translational and rotational kinetic energy changes to potential energy.
Therefore, potential energy (=mgh) will be more in case B than in case A.
Hence $h_1 > h_2$.

8. Radius of Curvature $= \frac{(\text{velocity})^2}{\text{Normal Acceleration}}$

$$= \frac{(2v)^2}{v^2/R} = 4R$$



9. Here, $u = V_0, \omega_0 = -\frac{V_0}{2R}$

At pure rolling ;

$$V = V_0 - \left(\frac{F_f}{m}\right)t \quad \& \quad \frac{V}{R} = -\frac{V_0}{2R} + \left(\frac{F_f}{m.R}\right)t$$

(In pure rolling $V = R\omega$) $\left(\alpha = \frac{\tau}{I} = \frac{F_f.R}{mR^2}\right)$

$\Rightarrow V_0 - V = V + \frac{V_0}{2} \Rightarrow 2V = \frac{V_0}{2} \Rightarrow V = \frac{V_0}{4}$ **Ans.**

10. The two forces along y-direction balance each other. Hence, the resultant force is 2F along x-direction

Let the point of application of force be at (0, y). (By symmetry x-coordinate will be zero).

For rotational equilibrium :

$F(a) + F(a) + F(a + y) - F(a - y) = 0 \Rightarrow y = -a$

Hence (B).

Alternate :

Torque will only be produced by the two forces along y–direction in anti-clockwise direction. To balance this torque we should apply a force 2F in order to produce a torque in the clockwise direction, which is only possible if we apply a force at a point below the x-axis.

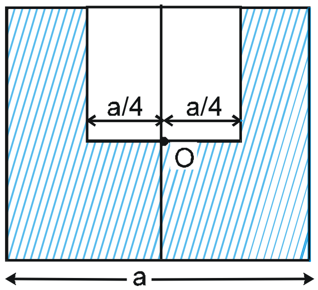
Then, $\tau = F(a) + F(a) - 2F \times y = 0 \Rightarrow y = a$

Hence (B).

11. Let m_1 = mass of the square plate of side 'a' and m_2 = mass of the square of side 'a/2'

Then $m_1 = \sigma\left(\frac{a}{2}\right)^2$; $m_2 = \sigma(a)^2$; (σ being the areal density)

and $m_2 - m_1 = M$.



$$\Rightarrow I = \frac{m_2a^2}{6} - \left\{\frac{m_1(a/2)^2}{6} + m_1\left(\frac{a}{4}\right)^2\right\}$$

$$= \frac{\sigma a^4}{6} - \left\{\frac{\sigma(a/2)^4}{6} + \sigma\left(\frac{a}{2}\right)^2 \cdot \left(\frac{a}{4}\right)^2\right\}$$

$$= \sigma a^4 \left\{\frac{1}{6} - \frac{1}{16\times 6} - \frac{1}{4\times 16}\right\} = \sigma a^4 \left\{\frac{(2\times 16) - 2 - 3}{16\times 12}\right\}$$

$$I = \sigma a^4 \left\{\frac{27}{12\times 16}\right\}$$

Also; $M = \sigma\left(1 - \frac{1}{4}\right)a^2 \quad \Rightarrow \quad \sigma = \frac{4}{3}\frac{M}{a^2}$

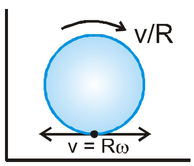
$$\Rightarrow I = \left(\frac{4}{3}\frac{M}{a^2}\right).a^4\left\{\frac{27}{12\times 16}\right\} \quad \Rightarrow \quad I = \frac{3Ma^2}{16}$$

12. As the disc is in combined rotation and translation, each point has a tangential velocity and a linear velocity in the forward direction.

From figure

v_{net} (for lowest point) $= v - R\omega = v - v = 0$.

and Acceleration $= \frac{v^2}{R} + 0 = \frac{v^2}{R}$



(Since linear speed is constant)

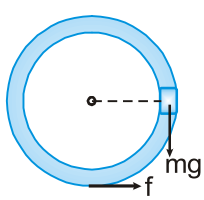
Hence (D).

13 $f = 4\,ma$ .... (1)

$(mg - f)r = (3\,mr^2 + mr^2)\,\alpha$

$mg - f = 4\,ma$ .... (2)

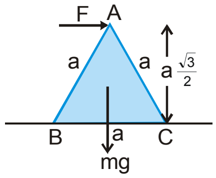
from (1) and (2)



$$\Rightarrow 8\,ma = mg \Rightarrow a = \frac{g}{8} \Rightarrow \alpha = \frac{g}{8r}$$

14. The tendency of rotating will be about the pont C. For minimum force, the torque of F about C has to be equal to the torque of mg about C.

$$\therefore \; F\left(a\frac{\sqrt{3}}{2}\right) = mg\left(\frac{a}{2}\right) \Rightarrow F = \frac{mg}{\sqrt{3}}$$



15. As torque = change in angular momentum

$\therefore$ $F\,.\,\Delta t = mv$ (Linear) ..... (1)

and $\left(F\cdot\frac{\ell}{2}\right)\Delta t = \frac{m\ell^2}{12}\cdot\omega$ (Angular) ..... (2)

Dividing : (1) and (2)

$$2 = \frac{12v}{\omega\ell} \Rightarrow \omega = \frac{6v}{\ell}$$

Using : S = ut :

Displacement of COM is : $\frac{\pi}{2} = \omega t = \left(\frac{6v}{\ell}\right)t$

and $x = vt$

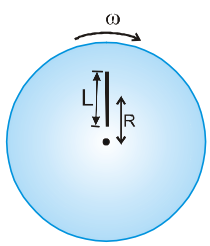
Dividing : $\frac{2x}{\pi} = \frac{\ell}{6} \Rightarrow x = \frac{\pi\ell}{12}$

$\Rightarrow$ Coordinate of A will be $\left[\frac{\pi\ell}{12} + \frac{\ell}{2}, 0\right]$

Hence (D)

16. Moment of inertia of the rod w.r.t. the axis through centre of the disc is : (by parallel axis theorem).

$$I = \frac{mL^2}{12} + mR^2$$



& K.E. of rod w.r.t. disc

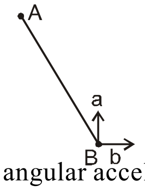
$$= \frac{1}{2}m\omega^2\left[R^2 + \frac{L^2}{12}\right]$$ **Ans.**

17. At the initial moment, angular velocity of rod is zero. Acceleration of end B of rod with respect to end A is shown in figure.

Centripetal acceleration of point B with respect to A is zero ($\because \omega^2\ell = 0$)

So at the initial moment , acceleration of end B with respect to end A is perpendicular to the rod which is equal to $\sqrt{a^2+b^2}$

$a_{rel} = \ell\alpha$



$\frac{\sqrt{a^2+b^2}}{\ell} = \alpha$ where α is angular acceleration

18. By conservation of angular momentum about pivot

$L = I\omega$

$$mv\frac{d}{2} = \left[\frac{Md^2}{12} + m\left(\frac{d}{2}\right)^2\right]\omega$$

$$\Rightarrow \frac{mvd}{2} = \left(\frac{md^2}{2} + \frac{md^2}{4}\right)\omega$$

$$\frac{mvd}{2} = \frac{3}{4}md^2\omega \qquad \frac{2}{3}\frac{v}{d} = \omega$$

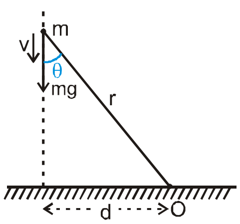
19. The magnitude of angular momentum of particle about O = mvd

Since speed v of particle increases, its angular momentum about O increases.

Magnitude of torque of gravitational force about O = mgd $\Rightarrow$ constant.

Moment of inertia of particle about O = mr^2

Hence MI of particle about O decreases.



angular velocity of particle about O = $\frac{v\sin\theta}{r}$

$\because$ v and $\sin\theta$ increase and r decreases

$\therefore$ angular velocity of particle about O increases.

20. As the normal force exerted by horizontal surface passes through point B, external torque on the ball is zero about point B. So angular momentum of ball is conserved about point B ($\because \tau = \frac{dL}{dt}$)

21. From conservation of energy, the kinetic energy of ball at lowest portion is (v_c = speed of centre of ball)

$$\frac{1}{2}mv_c^2 + \frac{1}{2}\times\frac{2}{5}mv_c^2 = mgR \text{ or } \frac{7}{10}mv_c^2 = mgR$$

Since net tangential force on sphere at lowest point is zero, net force on sphere at lowest position is

$$= \frac{mv_c^2}{R} = \frac{10}{7}mg$$ **upwards.**

22. Moment of inertia of semicircular portions about x and y axes are same. But moment of inertia of straight portions about x-axis is zero.

$\therefore I_x < I_y$ or $\frac{I_x}{I_y} < I$

23. $m_2g.\ 1 = m_1g.3 \rightarrow m_2 = 3m_1$

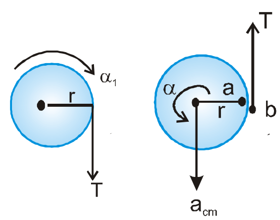
$\rightarrow 4m_1g\ .\ 3 = m_3\ g \rightarrow m_3 = 12\ m_1$

$\rightarrow 16\ m_1g\ .\ 3 = m_4\ g \rightarrow m_1 = \frac{48}{48} = 1$ kg.

24. $T\,r = \frac{mr^2}{2}\alpha_1$ ....(1)

$Tr = \frac{mr^2}{2}\alpha$ .... (2)

$\alpha_1 = \alpha$ .... (3)



From (1) & (2)

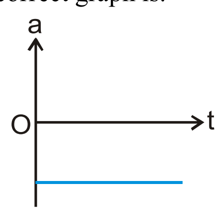
accⁿ . of point b = accⁿ of point a

$r\alpha$ **1** $= a_{cm} - r\alpha$ .... (4)

Hence $2r\alpha = a_{cm}$ **Ans. (B)**

25 As the sphere rolls up its speed is decreasing and while rolling down its speed is increasing. Hence the acceleration of its centre of mass is down the incline and is thus always negative.

Therefore the correct graph is.



26. The direction of $\vec{L}$ is perpendicular to the line joining the bob to point C. Since this line keeps changing its orientation in space, direction of $\vec{L}$ keeps changing how-ever as ω is constant, magnitude of $\vec{L}$ remain constant.

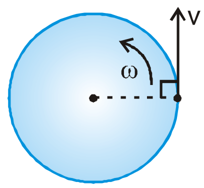
Aliter : The torque about point is perpendicular to the angular momentum vector about point C. Hence it can only change the direction of L, and not its magnitude.

27. Let the angular velocity of disc after child jumps off, be ω'

$\therefore$ From conservation of angular momentum

$(I + mR^2)\omega = mvR + I\omega'$

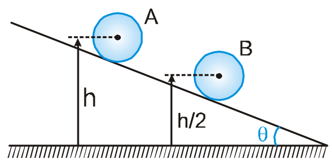
$\therefore \ \omega' = \dfrac{(I + mR^2)\omega - mvR}{I}$



28. Just before collision Between two Balls

potential energy lost by Ball A = kinetic energy gained by Ball A.

$$mg\frac{h}{2} = \frac{1}{2}I_{cm}\omega^2 + \frac{1}{2}mv_{cm}^2$$



$$= \frac{1}{2} \times \frac{2}{5}mR^2 \times \left(\frac{v_{cm}}{R}\right)^2 + \frac{1}{2}mv_{cm}^2$$

$$= \frac{1}{5}\ mv_{cm}^2 + \frac{1}{2}\ mv_{cm}^2$$

$$\Rightarrow \frac{5}{7}mgh = mv_{cm}^2 \Rightarrow \frac{mgh}{7} = \frac{1}{5}\ mv_{cm}^2$$

After collision only translational kinetic energy is transferred to ball B

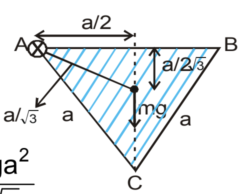
So just after collision rotational kinetic energy of Ball A =

$$\frac{1}{5}\ mv_{cm}^2 = \frac{mgh}{7}$$

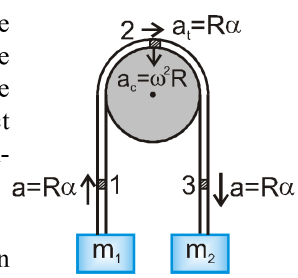
29. Torque about A :

$$mg\frac{a}{2} = I\alpha \Rightarrow \alpha = \frac{mga}{2I}$$

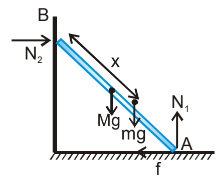
$$\Rightarrow \text{a cceleration} = \frac{a}{\sqrt{3}}\alpha = \frac{mga^2}{2\sqrt{3}I}$$



30. The acceleration of three sections of rope 1, 2, 3 are as shown. Hence for the section of rope in contact with pulley, acceleration increases till the section looses contact with pulley. Due to friction between string and pulley, the tension in right portion of string is larger in comparison to tension in the left portion of string.



31. Let m and M be mass of man and ladder. From FBD normal reaction at A is $N_1 = (m + M)g$ which remains constant. Net torque on man + ladder is zero about B. If x decreases then torque of mg about B will decrease. Hence f must increase.



32. All points in the body, in plane perpendicular to the axis of rotation revolve in concentric circles. All points lying on circle of same radius have same speed (and also same magnitude of acceleration) but different directions of velocity (also different directions of acceleration)

Hence there cannot be two points in the given plane with same velocity or with same acceleration.

As mentioned above, points lying on circle of same radius have same speed.

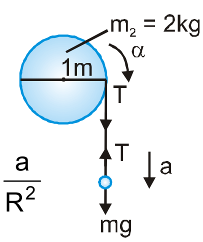
Angular speed of body at any instant w.r.t. any point on body is same by definition.

33. By FBD of particle

$mg - T = ma$

$10 - T = a$ ....(i)

By FBD of disc



$$TR = I\alpha = I.\frac{a}{R} \Rightarrow \quad T = \frac{mR^2}{2}.\frac{a}{R^2}$$

$$T = \frac{ma}{2} = a \quad \text{..............(ii)}$$

By eq. (i) and (ii)

(A) $a = 5\ m/s^2$ and $T = 5N$ and $\alpha = \dfrac{a}{R} = 5\ rad/s^2$ **Ans.**

(B) For angular displacement of disc : $\theta = \omega t + \dfrac{1}{2}\alpha t^2$

$\theta = \dfrac{1}{2} \times 5 \times 4^2 = 40$ rad. **Ans.**

(C) Work done by torque $= \int \tau d\theta = \tau\int d\theta = 5 \times 40$

$= 200\ J$ **Ans.**

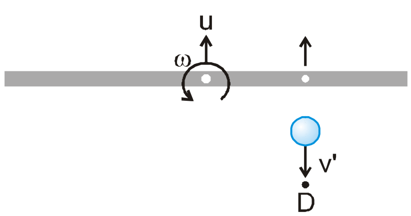
(D) $\Delta K.E. = \Delta w = 200\ J$

$k_2 - k_1 = 200\ J.$ **Ans.**

34. The ball has V', component of its velocity perpendicular to the length of rod immediately after the collision. u is velocity of COM of the rod and ω is angular velocity of the rod, just after collision. The ball strikes the rod with speed vcos53° in perpendicular direction and its component along the length of the rod after the collision is unchanged.

Using for the point of collision.

Velocity of separation = Velocity of approach



$$\Rightarrow \frac{3v}{5} = \left(\frac{\omega\ell}{4} + u\right) + V' \quad \text{.... (1)}$$

Conserving linear momentum (of rod + particle), in the direction ⊥ to the rod.

$$mv.\frac{3}{5} = mu - mV' \quad \text{....(2)}$$

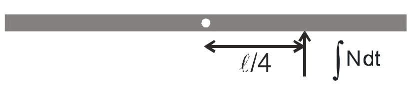
Conserving angular moment about point 'D' as shown in the figure

$$0 = 0 + \left[mu\frac{\ell}{4} - \frac{m\ell^2}{12}\omega\right] \Rightarrow u = \frac{\omega\ell}{3} \quad \text{.... (3)}$$

By solving

$$u = \frac{24v}{55}, \; w = \frac{72v}{55\ell}$$

Time taken to rotate by π angle $t = \frac{\pi}{\omega}$



In the same time, distance travelled $= u_2.t = \frac{\pi\ell}{3}$

Using angular impulse-angular momentum equation.

$$\int N.dt.\frac{\ell}{4} = \frac{1}{3}\frac{m\ell^2}{4}.\frac{72v}{55\ell} \Rightarrow \int N.dt = \frac{24mv}{55}$$

or $\left\{\begin{array}{l}\text{using impulse} - \text{momentum equation on Rod} \\ \int Ndt = mu = \frac{24mv}{55}\end{array}\right.$

35. For a disc rolling without slipping on a horizontal rough surface with uniform angular velocity, the acceleration of lowest point of disc is directed vertically upwards and is not zero(Due to translation part of rolling, acceleration of lowest point is zero. Due to rotational part of rolling, the tangential acceleration of lowest point is zero and centripetal acceleration is non-zero and upwards). Hence statement 1 is false.

36. As x increases, the required component of reaction decreases to zero and then increases (with direction reversed). Hence statement-1 is false.

37. The applied horizontal force F has tendency to rotate the cube in anticlockwise sense about centre of cube. Hence statement-2 is false.

38. The acceleration of centres of both spheres is μg up the incline. Since initial velocity of centres of both spheres is zero, they shall travel same distance in same time interval. Hence Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.

39. $\vec{L} = \vec{r}_1 \times \vec{p}_1 + \vec{r}_2 \times \vec{p}_2 \quad \because \vec{p}_1 + \vec{p}_2 = 0$

$= \vec{r}_1 \times (-\vec{p}_2) + r_2 \times \vec{p}_2 = (\vec{r}_2 - \vec{r}_1) \times \vec{p}_2$

$\vec{L} = \vec{r}_{rel} \times \vec{p}_2$. Hence Statement-1 is True, Statement-2 is False

40. Since acceleration is same for all the three spheres, they cover equal distances in equal intervals of time in all the cases (A), (B) and (C). Hence (A).

41. From passage, for case (C) ;

$$\mu_{min.} = \frac{\tan\theta\left(\frac{k^2}{R^2}\right)}{\left(1+\frac{k^2}{R^2}\right)} \rightarrow \text{(pure rolling)}$$

Putting the values of 'k' for different objects given in the table (in passage) we get ;

$$\mu_{min\ (Ring)} = \frac{\tan\theta}{2}$$

$$\mu_{min\ (Disc)} = \frac{\tan\theta}{3}, \; \mu_{min\ (Solid\ sphere)}$$

$$= \frac{2}{5}\tan\theta, \; \mu_{min\ (hollow\ sphere)} = \frac{2}{7}\tan\theta$$

⇒ $\mu_{min\ (Ring)}$ is greater than either of $\mu_{min\ (Disc)}$, $\mu_{min(Solid\ sphere)}$, $\mu_{min\ (hollow\ sphere)}$.

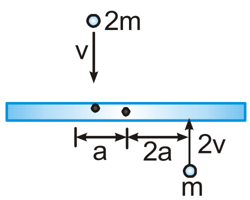
Therefore, the pure rolling of ring will confirm pure rolling of all other bodies.

42. As given in the equation of case (B) ;
$\mu NR = Mk^2\alpha$
and $N = Mg\cos\theta$
As ; θ, M, R, μ are same for all, 'α' will be least for that object for which 'k' and hence I is maximum.
Therefore 'α' for ring (k = R) and hence ω for ring at the bottom is minimum.
Also, $Mg\sin\theta - \mu N = Ma$
Since M, μ, θ , N are same for all objects, they have same linear acceleration and hence same linear velocity and hence same $\frac{1}{2}Mv_{cm}^2$.

$\therefore$ K.E. $\left(\frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2\right)$ is least for the ring.

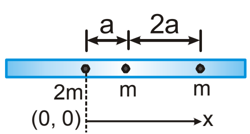
43. For ring $a = g\sin\theta/2$ (for pure rolling) is less than that of disc.
Hence (B).

44. Cons. linear momentum
$-2m.v + 2v.m = 0 = MV_{cm}$
$V_{cm} = 0$



45. As ball sticks to Rod
Conserving angular momentum about C

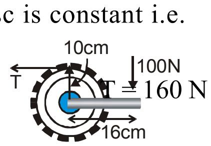
$$2v.m.\,2a + 2mva = I\omega = \left(\frac{8m.\,36a^2}{12} + 2m.\,a^2 + m.4a^2\right)$$



$6mv.a = 30\,ma^2.\omega \quad \Rightarrow \quad \omega = \frac{v}{5a}$

46. $KE = \frac{1}{2}\,I\omega^2 = \frac{1}{2}\,.\,30\,ma^2 \times \frac{v^2}{25a^2} = \frac{3mv^2}{5}$.

47. As angular velocity of the disc is constant i.e.
$\Sigma\tau = 0$
$100N \times 16\text{ cm} = T \times 10\text{ cm}$



T = 160 N

48. As angular acceleration of the rear wheel is zero therefore net torque on the wheel is zero.

49. Power delivered $= \vec{F}.\vec{v}$
where $\vec{v}$ is velocity of the point of application of the force.
$v = 16\text{ cm} \times 2\pi.2\ (= R\omega) = 0.64\,\pi$ m/s
$P = 100 \times 0.64\,\pi = 64\,\pi W$.
ALT : $P = \tau\omega$

50. $RN = rn \quad \Rightarrow \quad n = \frac{10cm \times 2}{4cm} = 5$ cy/s

So rear wheel rotates 5 cycles/second.

Hence $V = \frac{35}{100} \times 2\pi \times 5 = 3.5\,\pi$ m/s

51. As $\Sigma\tau = 0$
$160\text{ N} \times 4\text{ cm} = f \times 35\text{ cm}$

$$f = \frac{160 \times 4\,cm}{35\,cm} = 18.3\text{ N}$$

52. **(A)** Speed of point P changes with time

(B) Acceleration of point P is equal to $\omega^2 x$ (ω = angular speed of disc and x = OP). The acceleration is di rected from P towards O.

(C) The angle between acceleration of P (constant in magnitude) and velocity of P changes with time. Therefore, tangential acceleration of P changes with time.

(D) The acceleration of lowest point is directed towards centre of disc and remains constant with time

53. Since all forces on disc pass through point of contact with horizontal surface, the angular momentum of disc about point on ground in contact with disc is conserved. Also the angular momentum of disc in all cases is conserved about any point on the line passing through point of contact and parallel to velocity of centre of mass.

The K.E. of disc is decreased in all cases due to work done by friction.

From calculation of velocity of lowest point on disc, the direction of friction in case A, B and D is towards left and in case C is towards right.

The direction of frictional force cannot change in any given case.

54. **(A)** If resultant force is zero, $\vec{P}_{system}$ will be constant.

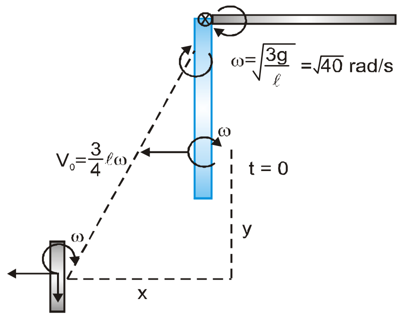
(B) If resultant torque is zero, $\vec{L}_{system}$ will be constant.

(C) If external forces are absent, both $\vec{P}_{system}$ and $\vec{L}_{system}$ will be constant.

(D) If no non conservative force acts, total mechanical energy of system will be constant.

55. $t = \frac{\pi}{\omega} = \frac{\pi}{\sqrt{40}}$ sec.

$x = v_0 t$



$y = \frac{1}{2} gt^2$

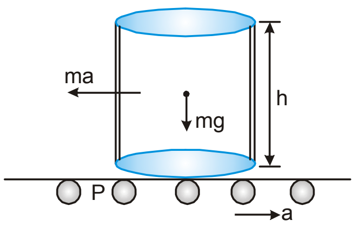
$r = \sqrt{x^2 + \left(y + \frac{3\ell}{4}\right)^2} \simeq 2.5\text{m}$.

56. WRT of belt, pseudo force ma acts on cylinder at COM as shown about to cylinder will be just about to topple when torque to weight w.r.t. P.

$\frac{dv}{dt} = 2t$

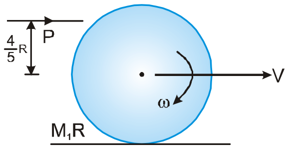
$m.2t . \frac{h}{2} = mg.r$

$t = \frac{rg}{bh}$



57. Using imp - momentum equation.

$P = M.v \Rightarrow v = \frac{P}{M}$ ....(1)



using angular impulse–momentum equation. wrt. centre.

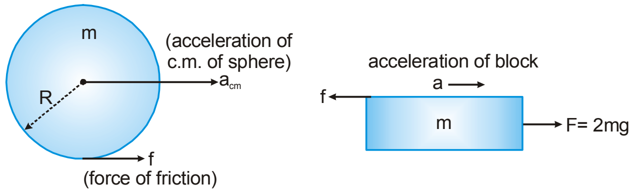
$P \frac{4}{5} R = \frac{2}{5} MR^2 \omega. \Rightarrow \omega = \frac{2P}{MR}$

Total K.E. = Trans KE + Rotational KE

$= \frac{1}{2} Mv^2 + \frac{1}{2} I\omega^2 = \frac{1}{2} M. \times \frac{P^2}{M} + \frac{1}{2} . \frac{2}{5} MR^2 . \frac{4P^2}{M^2R^2}$

$= \frac{13P^2}{10M}$.

58. The free body diagrams of sphere A and Block B are as shown



Applying Newton's law to block and sphere

$F - f = m a$ ....(1)

$f = m a_{cm}$ ....(2)

$fr = \frac{2}{5} mr^2 \alpha$ ....(3)

Since the sphere does not slip over the block, therefore from constraint

$a = a_{cm} + r\alpha$ ....(4)

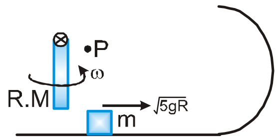
solving equation (1), (2), (3) and (4)

we get the angular acceleration of sphere

59. Minimum velocity required by block 'm' to complete the motion in $\sqrt{5gR}$

conserving mech. energy

$\frac{1}{2} I\omega^2 = Mg \times \frac{R}{2} \Rightarrow \omega = \sqrt{\frac{MgR}{I}}$



Cons. angular momentum wrt P before & after collision.

$I\omega = m.R\sqrt{5gR}$

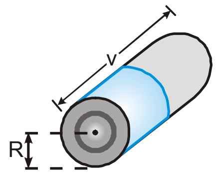
$I. \sqrt{\frac{MgR}{I}} = mR\sqrt{5gR} \Rightarrow MgRI = m^2R^2 5gR$

putting $I = \frac{ML^2}{3} = \frac{MR^2}{3}$ (since L = R)

$\Rightarrow \frac{M}{m} = \sqrt{15}$

60. $M_{initial} = \pi R^2. L.\rho.$

ρ = density of carpet material

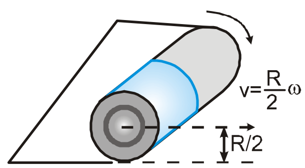


$$M_{final} = \pi.\left(\frac{R}{2}\right)^2 . L . \rho = \frac{M_i}{4}.$$

Initial PE of carpet = Mg.R

$$\text{Final PE of carpet} = \frac{M}{4}.g.\frac{R}{2} = \frac{MgR}{8}$$

$$\Delta PE \text{ (decrease)} = \frac{7}{8} MgR$$



It's equal to gain in KE

$$= K_{trans} + K_{Rot} = \frac{1}{2} Mv^2 + \frac{1}{2} I\omega^2$$

$$\text{using mass} = \frac{M}{4}, v = \frac{R}{2}.\omega \Rightarrow I = \frac{1}{2}\left(\frac{M}{4}\right)\left(\frac{R}{2}\right)^2$$

$$K = \frac{1}{2}\left(\frac{M}{4}\right)v^2 + \frac{1}{2}\left(\frac{MR^2}{32}\right)\left(\frac{2v}{R}\right)^2 = \frac{3}{16}Mv^2$$

$$\text{Equating } \frac{7}{8} Mg R = \frac{3}{16} Mv^2 \Rightarrow v = \sqrt{\frac{14gR}{3}}$$

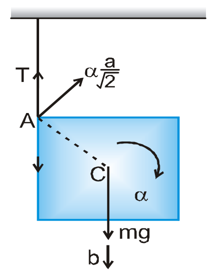
61. Let b and a are linear acc. of centre of mass and angular acc. of the plane, just after BF is cut.

$mg - T = mb$ .... (1)

Taking torques about CoM

$$\frac{Ta}{2} = \frac{ma^2}{6}.\alpha \quad .... (2)$$

$$mg = mb + \frac{ma\alpha}{3} \Rightarrow g = b + \frac{a\alpha}{3}$$



and $b = \alpha \frac{a}{2}$

$$\therefore g = b + \frac{2b}{3} = \frac{5b}{3} \Rightarrow b = \frac{3g}{5}$$

$$\therefore T = mg - \frac{m.3g}{5} = \frac{2mg}{5}$$ **Ans.**