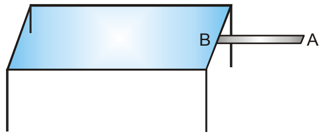
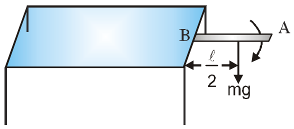
SOLVED EXAMPLES

Ex.1 A uniform thin stick of length ℓ and mass m is held horizontally with its end B hinged at a point B on the edge of a table. Point A is suddenly released. The acceleration of the centre of mass of the stick at the time of release, is :-



(A) $\frac{3}{4}$ g **(B)** $\frac{3}{7}$ g **(C)** $\frac{2}{7}$ r g **(D)** $\frac{1}{7}$ g

Sol.

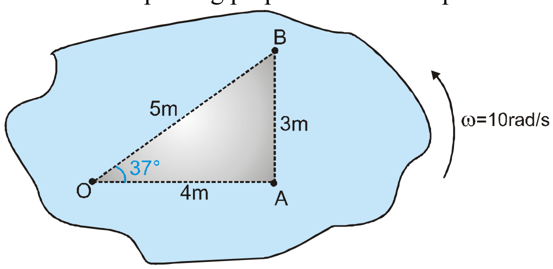


For angular motion of the stick $\frac{1}{100} = mg\left(\frac{\ell}{2}\right) = I\,\alpha$

moment of intertia of stick about B is $I = \frac{m\ell^2}{3} \Rightarrow mg\left(\frac{\ell}{2}\right) = \left(\frac{m\ell^2}{3}\right)\alpha \Rightarrow \alpha = \frac{3g}{2\ell}$

Acceleration of centre of mass $= \alpha\left(\frac{\ell}{2}\right) = \left(\frac{3g}{2\ell}\right)\left(\frac{\ell}{2}\right) = \frac{3}{4}g$

Ex. 2 A rigid lamina is rotating about an axis passing perpendicular to its plane through point O as shown in figure.



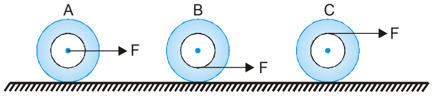
The angular velocity of point B w.r.t. A is

(A) 10 rad/s **(B)** 8 rad/s **(C)** 6 rad/s **(D)** 0

Sol. **(A)**

In a rigid body, angular velocity of any point on the rigid body w.r.t. any other point on the rigid body is constant and equal to angular velocity of rigid body.

Ex. 3 Three spools A, B and C are placed on rough ground and acted by equal force F. Then which of the following statement is incorrect ?



(A) Frictional force on spool A is in backward direction
(B) Frictional force on spool B is in backward direction
(C) Frictional force on spool C is in backward direction
(D) Frictional force on spool C is in forward direction

Sol. **(D)**

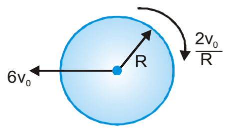
For spool A, sliding tendency of point of contact is forward $\Rightarrow$ frictional force is in backward direction
For spool B, sliding tendency of point of contact is forward $\Rightarrow$ frictional force is in backward direction
For spool C, slidin7g tendency of point of contact is forward as without friction

$$a = \frac{F}{m}, \alpha = \frac{\tau}{I} = \frac{Fr}{\frac{m}{2}(R^2 + r^2)} \Rightarrow \alpha R = \frac{2FRr}{m(R^2 + r^2)}$$

Here $a > \alpha R$ so acceleration of point of contact will be in forward direction.

$\Rightarrow$ frictional force is in backward direction.

Ex. 4 A disc of radius R = 2m moves as shown in the figure, with a velocity of translation of $6v_0$ of its centre of mass and an angular velocity of $\frac{2v_0}{R}$. The distance (in m) of instantaneous axis of rotation from its centre of mass is

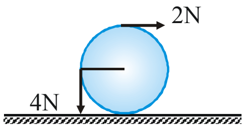


(A) 3 **(B)** 4 **(C)** 5 **(D)** 6

Sol. **(D)**

Instantaneous axis of rotation lies above the centre of mass where $v - \omega r = 0 \Rightarrow r = \frac{v}{\omega} = \frac{6v_0}{\frac{2v_0}{R}} = 3R$

Ex. 5 A uniform solid disc of mass 1 kg and radius 1m is kept on a rough horizontal surface. Two forces of magnitude 2 N and 4 N have been applied on the disc as shown in the figure. Linear acceleration of the centre of mass of the disc is if there is no slipping.

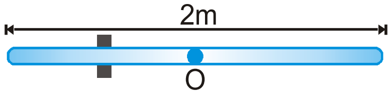


(A) 4 m/s^2 **(B)** 2 m/s^2 **(C)** 1 m/s^2 **(D)** zero

Sol. **(D)**

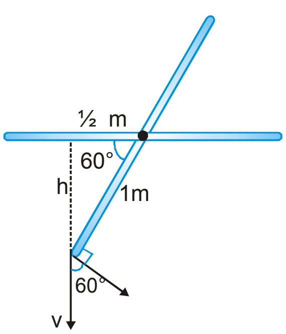
Taking torque about contact point, $\tau = 4 \times R - 2 \times 2R = 0$, $F_{net} = 0$

Ex. 6 A 2m long rod of negligible mass is free to rotate about its centre. An object of mass 5 kg is threaded into the rod at a distance of 50 cm from its end in such a way that the object can move without friction. The rod is then released from its horizontal position. The speed of the rod's end in the rod's vertical position is (in m/s)



(A) $\sqrt{\frac{5\sqrt{3}}{2}}$ **(B)** $\sqrt{\frac{4\sqrt{3}}{2}}$ **(C)** $\sqrt{\frac{3\sqrt{5}}{2}}$ **(D)** $\sqrt{\frac{3\sqrt{3}}{2}}$

Sol. Since friction and the rod's mass is negligible, the only force acting on the object is gravitational force, therefore the object undergoes free-fall.



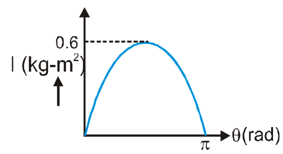
The object-moves a distance of $h = \sqrt{1^2 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2}$ m until it drops off from the rod.

Its velocity at this moment $v = \sqrt{2gh}$

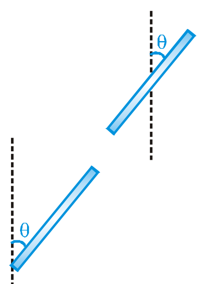
The object's velocity perpendicular to the rod equals to the velocity of the rod's end at the moment when the object leaves the rod. After this moment the rod's end maintains its speed, so in vertical position its

$$\text{speed} = v\cos 60^\circ = \left(\sqrt{2gh}\right)\left(\frac{1}{2}\right) = \sqrt{\frac{gh}{2}} = \sqrt{\frac{10 \times \sqrt{3}/2}{2}} = \sqrt{\frac{5\sqrt{3}}{2}} \text{ m/s}$$

Ex. 7 Figure shows the variation of the moment of inertia of a uniform rod, about an axis passing through its centre and inclined at an angle θ to the length. The moment of inertia of the rod about an axis passing through one of its ends and making an angle $\theta = \frac{\pi}{3}$ will be



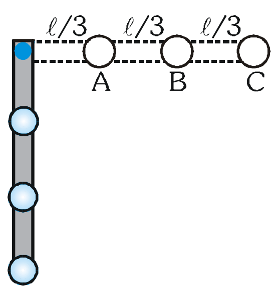
(A) 0.45 $kg - m^2$ **(B)** 1.8 $kg - m^2$ **(C)** 2.4 $kg - m^2$ **(D)** 1.5 $kg - m^2$



Sol. $I = \frac{ML^2}{12}\sin^2\theta \Rightarrow 0.6 = \frac{ML^2}{12}\sin\left(\frac{\pi}{2}\right) \Rightarrow ML^2 = 7.2$

$I = \frac{ML^2}{3}\sin^2\theta$, at $\theta = \frac{\pi}{3}, I = \frac{ML^2}{3}\left(\frac{3}{4}\right) = \frac{ML^2}{4} = \frac{7.2}{4} = 1.8$ kg-m²

Ex. 8 A light rod carries three equal masses A, B and C as shown in figure. What will be velocity of B in vertical position of rod, if it is released from horizontal position as shown in figure ?



(A) $\sqrt{\frac{8g\ell}{7}}$ **(B)** $\sqrt{\frac{4g\ell}{7}}$ **(C)** $\sqrt{\frac{2g\ell}{7}}$ **(D)** $\sqrt{\frac{10g\ell}{7}}$

Sol. Loss in P.E. = Gain in K.E. $mg\frac{\ell}{3} + mg\left(\frac{2\ell}{3}\right) + mg\ell = \frac{1}{2}\left(m\left(\frac{\ell}{3}\right)^2 + m\left(\frac{2\ell}{3}\right)^2 + m\ell^2\right)\omega^2$

$\Rightarrow \omega = \sqrt{\frac{36g}{14\ell}} \Rightarrow v_B = \omega\ell_B = \frac{2\ell}{3}\sqrt{\frac{36g}{14\ell}} = \sqrt{\frac{8g\ell}{7}}$

Ex. 9 A child's top is spun with angular acceleration $\alpha = 4t^3 - 3t^2 + 2t$ where t is in seconds and α is in radian per second-squared. At t =0, the top has angular velocity $\omega_0 = 2$ rad/s and a reference line on it is at angular position $\theta_0 = 1$ rad.

Statement I : Expression for angular velocity $\omega = \left(2 + t^2 - t^3 + t^4\right)$ rad/s

Statement II : Expression for angular position $\theta = \left(1 + 2t - 3t^2 + 4t^3\right)$ rad

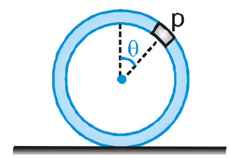
(A) Only statement-I is true **(B)** Only statement-II is true

(C) Both of them are true **(D)** None of them are true

Sol. $\int_2^{\omega} d\omega = \int_0^t \alpha dt \Rightarrow \omega - 2 = \left(t^4 - t^3 + t^2\right)_0^t \Rightarrow \omega = 2 + t^2 - t^3 + t^4$

$\int_1^{\theta} d\theta = \int_0^t \omega dt \Rightarrow \theta - 1 = \left(2t + \frac{t^3}{3} - \frac{t^4}{4} + \frac{t^5}{5}\right)_0^t \Rightarrow \theta = 1 + 2t + \frac{t^3}{3} - \frac{t^4}{4} + \frac{t^5}{5}$

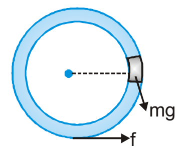
Ex. 10 A small block of mass 'm' is rigidly attached at 'P' to a ring of mass '3m' and radius 'r'. The system is released from rest at $\theta = 90°$ and rolls without sliding. The angular acceleration of hoop just after release is–



(A) $\frac{g}{4r}$ **(B)** $\frac{g}{8r}$ **(C)** $\frac{g}{3r}$ **(D)** $\frac{g}{2r}$

Sol. f = 4 ma .....**(i)** $(mg - f)\, r = (3mr^2 + mr^2)\, \alpha$

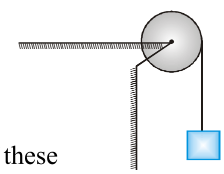
mg – f = 4 ma .....**(ii)**



from (i) and (ii)

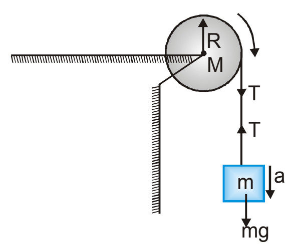
$\Rightarrow$ 8 ma = mg $\Rightarrow a = \frac{g}{8} \Rightarrow \alpha = \frac{g}{8r}$

Ex. 11 Figure shows a uniform disk, with mass M = 2.4 kg and radius R = 20 cm, mounted on a fixed horizontal axle. A block of mass m = 1.2 kg hangs from a massless cord that is wrapped around the rim of the disk. The tension in cord is



(A) 12 N **(B)** 20 N **(C)** 24 N **(D)** None of these

Sol. For block : mg – T = ma ....**(i)**

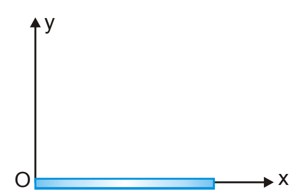


For disk (pulley) $TR = I\alpha = \frac{MR^2}{2}\alpha$

But $\alpha = \frac{a}{R}$ so $T = \frac{Ma}{2}$ ....**(ii)**

Therefore $\frac{mg - T}{T} = \frac{2m}{M} \Rightarrow \frac{mg}{T} = \frac{2m}{M} + 1 \Rightarrow t = \frac{mg}{\left(\frac{2m}{M} + 1\right)} = \frac{(1.2 \times 10)}{\left(\frac{211.2}{2.4} + 1\right)} = 6\text{N}$

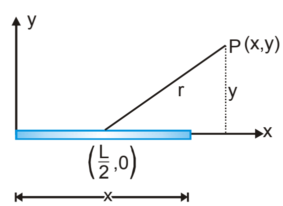
Ex. 12 The figure shows a uniform rod lying along the x-axis. The locus of all the points lying on the xy-plane, about which the moment of inertia of the rod is same as that about O is :



(A) an ellipse **(B)** a circle

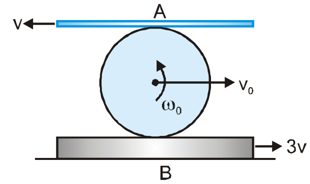
(C) a parabola **(D)** a straight line

Sol. $I_P = I_{CM} + Mr^2 = \frac{ML^2}{12} + M\left[\left(x - \frac{L}{2}\right)^2 + y^2\right]$



$I_0 = \frac{ML^2}{3} \Rightarrow \left(x - \frac{L}{2}\right)^2 + y^2 = \left(\frac{L}{2}\right)^2 \Rightarrow$ Locus is a circle

Ex. 13 The disc of radius r is confined to roll without slipping at A and B. If the plates have the velocities shown, then



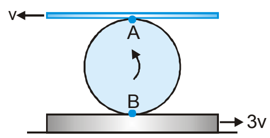
(A) linear velocity $v_0 = v$

(B) angular velocity of disc is $\frac{3v}{2r}$

(C) angular velocity of disc is $\frac{2v}{r}$

(D) None of these

Sol.



$v_A = \omega_0 r - v_0 = v$

$\omega_0 r - v_0 = v$ .....**(i)**

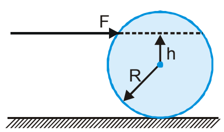
$v_B = \omega_0 r + v_0 = 3v$ .....**(ii)**

from equation (i) & (ii) $2\omega_0 r = 4V$

$\Rightarrow \quad \omega_0 r = 2v$

$\omega_0 = \frac{2v}{r}$ from equation (i) $v_0 = v$

Ex. 14 An impulsive force F acts horizontally on a solid sphere of radius R placed on a horizontal surface. The line of action of the impulsive force is at a height h above the centre of the sphere. If the rotational and translational kinetic energies of the sphere just after the impulse are equal, then the value of h will be-



(A) $\frac{2}{5}R$ **(B)** $\sqrt{\frac{2}{3}}R$ **(C)** $\sqrt{\frac{2}{5}}R$ **(D)** None of these

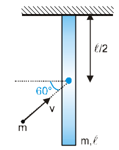
Sol. **(C)**

$F\Delta t = Mv \quad ; \quad Fh\Delta t = I\omega$

$\Rightarrow \quad Mvh = I\,\omega = \frac{2}{5}MR^2\omega$ Also. $\frac{1}{2}Mv^2 = \frac{1}{2}I\omega^2 = \frac{1}{2} \times \frac{2}{5}MR^2\omega^2$

$\Rightarrow \quad h = \sqrt{\frac{2}{5}}R$

Ex. 15 A thin rod of mass m and length ℓ is hinged to a ceiling and it is free to rotate in a vertical plane. A particle of mass m, moving with speed v strikes it as shown in the figure and gets stick with the rod. The value of v , for which the rod becomes horizontal after collision is



(A) The value of v, for which rod becomes horizontal after collision is $\sqrt{\frac{168}{9}g\ell}$

(B) The value of v, for which rod becomes horizontal after collision is $\sqrt{\frac{53}{3}g\ell}$

(C) Angular momentum of (rod + particle) system will remain constant about hinge just before and after collision

(D) Angular momentum of (rod + particle) system will remain same about centre of mass just before and after collision

Sol. $m.\frac{v}{2}.\frac{\ell}{2} = \left(m\frac{\ell^2}{3} + m\frac{\ell^2}{4}\right)\omega$; $\frac{v}{4} = \frac{7}{12}\ell\omega \Rightarrow \omega = \frac{3}{7}\frac{v}{\ell}$ .....**(i)**

$\frac{1}{2}\left(\frac{7}{12}m\ell^2\right)\omega^2 = 2mg\frac{\ell}{2} \Rightarrow \omega = \sqrt{\frac{24}{7}\frac{g}{\ell}}$ .....**(ii)**

from equation **(i)** and **(ii)** $v = \sqrt{\frac{168}{9}g\ell}$

Ex. 16 A thin uniform rod of mass m and length ℓ is free to rotate about its upper end. When it is at rest, it receives an impulse J at its lowest point, normal to its length. Immediately after impact

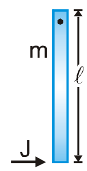
(A) the angular momentum of the rod is $J\ell$.

(B) the angular velocity of the rod is $\frac{3J}{m\ell}$

(C) the kinetic energy of the rod is $\frac{3J^2}{2m}$

(D) the linear velocity of the midpoint of the rod is $\frac{3J}{2m}$

Sol. By impulse momentum theorem $J\ell = \frac{m\ell^2}{3}\omega \Rightarrow \omega = \frac{3J}{m\ell}$



KE of rod $= \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{m\ell^2}{3}\right)\left(\frac{3J}{m\ell}\right)^2 = \frac{3J^2}{2m}$

Linear velocity of midpoints $= \omega\left(\frac{\ell}{2}\right) = \frac{3J}{2m}$

Ex. 17 A bicycle is in motion. The force of friction exerted by the ground on its wheel is such that it acts:

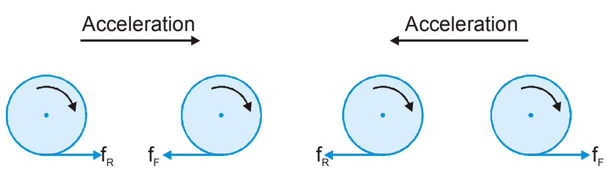
(A) in backward direction on front wheel and in forward direction on rear wheel when it is accelerating

(B) in forward direction on front wheel and in backward direction on rear wheel when brakes are applied on rear wheel only

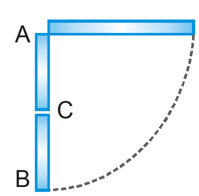
(C) in backward direction on front wheel and in forward direction on rear wheel when brakes are applied on rear wheel only

(D) in backward direction on both the wheels when brakes are applied on front wheel

Sol.



Ex. 18 A uniform rod AB of length ℓ is free to rotate about a horizontal axis passing through A. The rod is released from rest from horizontal position. If the rod gets broken at midpoint C when it becomes vertical, then just after breaking of the rod :



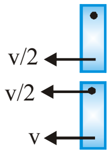
(A) Angular velocity of upper part starts to decrease while that of lower part remains constant.

(B) Angular velocity of upper part starts to decrease while that of lower part starts to increase

(C) Angular velocity of both the parts is identical

(D) Angular velocity of lower part becomes equal to zero

Sol.

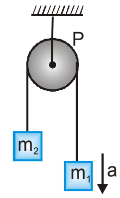


— On upper part torque of mg about A will decrease the angular velocity.

— Lower part of rod will not experience any couple hence its angular velocity can't change.

Initially both parts are having same angular velocities.

Ex. 19 In the figure, the blocks have unequal masses m_1 and m_2 ($m_1 > m_2$). m_1 has a downward acceleration a. The pulley P has a radius r, and some mass. The string does not slip on the pulley–

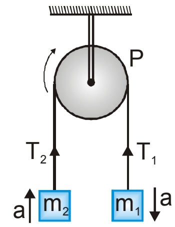


(A) The two sections of the string have unequal tensions.

(B) The two blocks have accelerations of equal magnitude.

(C) The angular acceleration of P is $\frac{a}{r}$

(D) $a < \left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$



Sol. In this situation $T_1 \neq T_2 \Rightarrow \alpha = \frac{a}{r} \Rightarrow a = \frac{(m_1 - m_2)g}{m_1 + m_2 + \frac{I}{r^2}}$

Ex. 20 to 22

A mouse, searching for food, jumped onto the rim of a stationary circular disk mounted on a vertical axle. The disk is free to rotate without friction. The velocity of the mouse was tangent to the edge of the disk before it landed. When the mouse landed, it gripped the surface, remained fixed on the outer edge of the disk at a distance R from the center, and set it into rotation. The sketch indicates the situation.

Before After

The mass of the mouse is m = 0.10 kg, the radius of the disk is R = 0.20 m, and the rotational inertia of the disk is I = 0.0080 kg·m². The speed of the mouse, just before it landed on the disk is v_o = 1.5 m/s.

20. Magnitude of the angular velocity of the disk plus mouse, after it landed becomes

(A) 0.25 rad/s **(B)** 2.5 rad/s **(C)** 0.375 rad/s **(D)** 3.75 rad/s

21. Find the magnitude of the impulse received by the mouse as it landed on the disk.

(A) 0.01 kg.m/s opposite to direction of motion

(B) 0.01 kg.m/s in the direction of motion

(C) 0.10 kg.m/s opposite to direction of motion

(D) 0.10 kg.m/s in the direction of motion

22. The mouse, still searching for food, crept to the center of the disk (where r = 0). Find angular velocity of the disk plus mouse, when the mouse was at the center of the disk.

(A) 0.25 rad/s **(B)** 2.5 rad/s **(C)** 0.375 rad/s **(D)** 3.75 rad/s

Sol.

20. By conservation of angular momentum $mv_0R = (I + mR^2)\omega$

$$\Rightarrow \omega = \frac{mv_0R}{I + mR^2} = \frac{(0.1)(1.5)(0.2)}{0.008 + 0.004} = 2.5 \text{ rad/s}$$

21. Impulse received by mouse = change in momentum

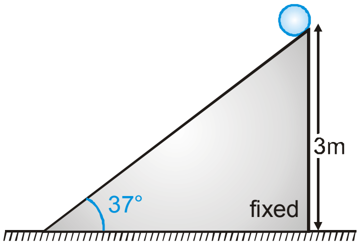
$= 0.1\ (2.5 \times 0.2 - 1.5) = -0.1$ kg m/s

22. By conservation of angular momentum : $mv_0R = I\omega$

$$\Rightarrow \omega = \frac{(0.1)(1.5)(0.2)}{0.008} = 3.75 \text{ rad/s}$$

Ex. 23 to 25

A uniform hollow sphere is released from the top of a fixed inclined plane of inclination 37° and height 3m. It rolls without sliding.



23. The acceleration of the centre of mass of the hollow sphere is

(A) $\frac{30}{7}$ m/s² **(B)** $\frac{18}{5}$ m/s² **(C)** $\frac{9}{5}$ m/s² **(D)** $\frac{15}{7}$ m/s²

24. The speed of the point of contact of the sphere with the inclined plane when the sphere reaches half–way of the incline is

(A) $\sqrt{42}$ m/s **(B)** $\sqrt{21}$ m/s **(C)** $\sqrt{84}$ m/s **(D)** zero

25. The time taken by the sphere to reach the bottom is

(A) $\frac{3}{5}$s **(B)** $\frac{5}{3}$s **(C)** $\frac{5}{4}$s **(D)** None of these

Sol.

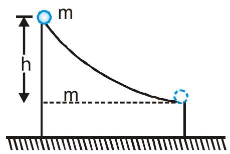
23. $$a = \frac{g\sin\theta}{1 + \frac{K^2}{R^2}} = \frac{(10)(3/5)}{1 + \frac{2}{3}} = \frac{18}{5} ms^{-2}$$

24. Speed of point of contact in pure rolling is always zero

25. $$s = ut + \frac{1}{2}at^2 \Rightarrow \frac{3}{\sin 37°} = \frac{1}{2}\left(\frac{18}{5}\right)(t^2) \Rightarrow t = \frac{5}{3}s$$

Ex. 26 to 27

A hollow sphere is released from the top of a wedge, friction is sufficient for pure rolling of sphere on the wedge. There is no friction between the wedge and the ground. Radius of sphere is R. At the instant it leaves the wedge horizontally.



26. Angular velocity of sphere ω is-

(A) $\sqrt{\frac{12gh}{7R^2}}$ (B) $\sqrt{\frac{27}{7}\frac{gh}{R^2}}$ (C) $\sqrt{\frac{20gh}{7R^2}}$ (D) $\sqrt{\frac{44gh}{7R^2}}$

27. Velocity of centre of mass of sphere w.r.t. ground is-

(A) $\sqrt{\frac{5}{7}gh}$ (B) $\sqrt{2gh}$ (C) $\sqrt{\frac{3}{7}gh}$ (D) $\sqrt{\frac{11}{7}gh}$

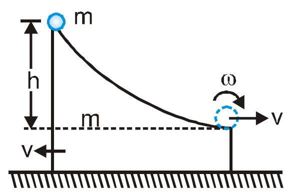
Sol.

26. $\omega = \frac{2v}{R} = \sqrt{\frac{4}{R^2}\times\frac{3}{7}gh} = \sqrt{\frac{12gh}{7R^2}}$

27. $R\omega - v = v \Rightarrow \omega = \frac{2v}{R}$

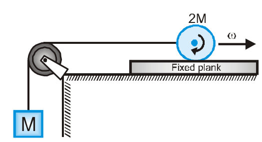
Energy conservation

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{3}mR^2\right)\left(\frac{4v^2}{R^2}\right) \Rightarrow v = \sqrt{\frac{3}{7}gh}$$



Ex. 28 to 30

A disc of mass 2 M and radius R is placed on a fixed plank (rough) of length L. The coefficient of friction between the plank and disc is $\mu = 0.5$. String (light) is connected to centre of disc and passing over a smooth light pulley and connected to a block of mass M as shown in the figure. Now the disc is given an angular velocity ω_0 in clockwise direction and is gently placed on the plank. Consider this instant as t=0. Based on above information, answer the following questions :



28. Time time t_{01} at which the disc will cross the other end of the plank is-

(A) $\sqrt{\frac{8L}{g}}$ (B) $\frac{\omega_0 R}{g} + \sqrt{\frac{8L}{g}}$ (C) $\sqrt{\frac{8L}{g}} + \frac{4\omega_0 R}{g}$ (D) $\sqrt{\frac{\omega_0 R}{g}} + \sqrt{\frac{4L}{g}}$

29. Time t_0 upto which the block remains stationary is

(A) $\frac{\omega_0 R}{g}$ (B) $\frac{4\omega_0 R}{g}$ (C) Zero (D) Question is irrelevant

30. Mark the correct statement w.r.t. motion of block and disc.

(A) The block remains at rest for some time, $t > 0$.

(B) The block starts accelerating just after placing of disc on plank.

(C) The disc is performing pure rotational motion for some time $t > 0$

(D) Both (A) and (C) are correct.

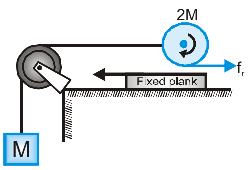
Sol.

28. $Mg - T = Ma$ ...**(i)** $\qquad T - f_r = 2Ma$ ...**(ii)**

By adding **(i)** and **(ii)** $\quad Mg - f_r = 3\,Ma$ ...**(iii)** $\quad f_r R = I\alpha = \frac{2MR^2}{2}\frac{a}{R} \Rightarrow f_r = Ma$ ...**(iv)**

From equation **(iii)** and **(iv)** $a = g/4 \Rightarrow L = \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{8L}{g}}$

Total time $= \frac{\omega_0 R}{g} + \sqrt{\frac{8L}{g}}$



29. Frictional force $= \mu Mg$

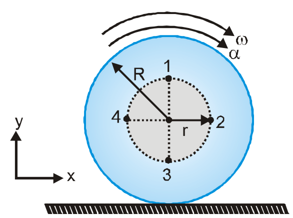
Friction will act up to the instant when the velocity of contact point becomes zero

$\omega_f = \omega_i + \alpha t$

$\Rightarrow t = \frac{\omega_i}{\alpha}$ .....**(i)** $\quad \Rightarrow \quad \alpha = \frac{f_r R}{I}$ .....**(ii)** By solving **(i)** and **(ii)** time $= \frac{\omega_0 R}{g}$

30. **(D)**

Ex. 31 A disc of radius R is rolling without slipping with an angular acceleration α, on a horizontal plane. Four points are marked at the end of horizontal and vertical diameter of a circle of radius r (<R) on the disc. If horizontal and vertical direction are chosen as x and y axis as shown in the figure, then acceleration of points 1, 2, 3 and 4 are $\vec{a}_1, \vec{a}_2, \vec{a}_3$ and $\vec{a}_4$ respectively, at the moment when angular velocity of the disc is ω. Match the following



Column-I		**Column-II**	
(A)	$\vec{a}_1$	**(P)**	$(R\alpha - r\alpha)\hat{i} + (r\omega^2)\hat{j}$
(B)	$\vec{a}_2$	**(Q)**	$(R\alpha + r\alpha)\hat{i} - (r\omega^2)\hat{j}$
(C)	$\vec{a}_3$	**(R)**	$(R\alpha - r\omega^2)\hat{i} - (r\alpha)\hat{j}$
(D)	$\vec{a}_4$	**(S)**	$(R\alpha + r\omega^2)\hat{i} + (r\alpha)\hat{j}$
		(T)	None of these

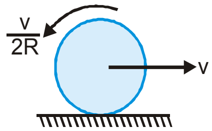
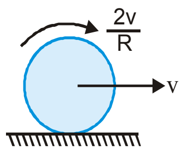
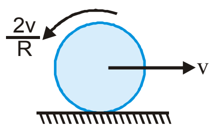
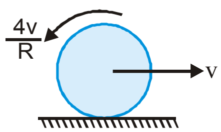
Sol. **For (A):** Acceleration of 1 w.r.t. centre of mass $= r\alpha\hat{i} - \omega^2 r\hat{j} \Rightarrow \vec{a}_1 = r\alpha\hat{i} - \omega^2 r\hat{j} + R\alpha\hat{i} = (R+r)\alpha\hat{i} - \omega^2 r\hat{j}$

For (B): $\vec{a}_2 = -r\alpha\hat{j} - \omega^2 r\hat{i} + R\alpha\hat{i} = (R\alpha - \omega^2 r)\hat{i} - r\alpha\hat{j}$

For (C): $\vec{a}_3 = -r\alpha\hat{i} + \omega^2 r\hat{j} + R\alpha\hat{i} = (R\alpha - r\alpha)\hat{i} + \omega^2 r\hat{j}$

For (D): $\vec{a}_4 = r\alpha\hat{j} + \omega^2 r\hat{i} + R\alpha\hat{i} = (R\alpha + \omega^2 r)\hat{i} + r\alpha\hat{j}$

Ex. 32 Four different situations of a moving disc are shown in column I and predictions about its final motion and forces acting on it are given in column - II.

Column I	Column II
(A) 	**(P)** finally disc will roll along the initial direction of velocity (v)
(B) 	**(Q)** finally, disc will roll in direction opposite to the initial direction of velocity (v)
(C) 	**(R)** finally, disc stops
(D) 	**(S)** Initially friction force acts in the direction opposite to that of initial velocity
	(T) None of these

Sol. Final direction of pure rolling will be in direction of initial angular momentum about point of contact $L = mv_{cm}r + I_{cm}\,\omega$

For (A) : $mvr - \frac{MR^2}{2}\left(\frac{v}{2R}\right) = \frac{3mvR}{4}$: hence in clockwise direction.

For (B) : $mvR + \frac{mR^2}{2}\left(\frac{2v}{R}\right) = 2mvR$: hence in clockwise direction.

For (C) : $mvR - \frac{mR^2}{2}\left(\frac{2v}{R}\right) = 0$

For (D) : $mvR - \frac{mR^2}{2}\left(\frac{4v}{R}\right) = -mRv$: hence in anticlockwise direction.

Direction of friction force

For (A) : Velocity of point of contact $\bullet \xrightarrow[v/2]{} \rightarrow v$ $\Rightarrow$ Friction will be opposite to velocity

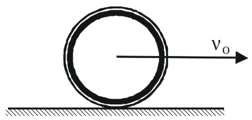
For (B) : Velocity of point of contact $2v \leftarrow \bullet \rightarrow v$ $\Rightarrow$ Friction will be in the direction of velocity

For (C) : Velocity of point of contact $\bullet \xrightarrow{v} \rightarrow 2v$ $\Rightarrow$ Friction will be opposite to the velocity

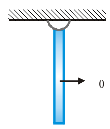
For (D) : Velocity of point of contact $\bullet \xrightarrow{v} \rightarrow 4v$ $\Rightarrow$ Friction will be opposite to the velocity

Ex. 33 **Column I** **Column II**

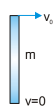
(A) A ring of mass m is projected on rough horizontal plane with velocity v_0. The magnitude of work done by frictional force to start rolling — **(P)** $\frac{1}{3}mv_0^2$



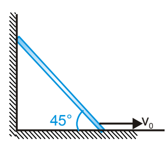
(B) Kinetic energy of pivoted rod of mass m, velocity of centre of mass is v_0. — **(Q)** $\frac{1}{8}mv_0^2$



(C) Kinetic energy of translation of a smooth rod of mass m, where velocity of one end is v_0. — **(R)** $\frac{1}{4}mv_0^2$



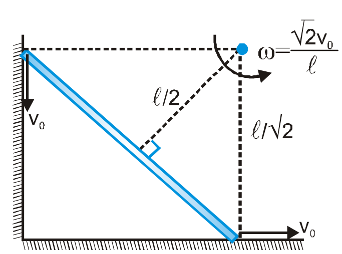
(D) Kinetic energy of a rod of mass m, as shown in figure. — **(S)** $\frac{2}{3}mv_0^2$



(T) $\frac{1}{9}mv_0^2$

Sol. **For (A) :** Final velocity $v = \frac{v_0}{2}$ $\left[\because mv_0R = mR^2\left(\frac{v}{R}\right) + mvR\right]$

So work done by friction $= \frac{1}{2}mv_0^2 - \left[\frac{1}{2}m\left(\frac{v_0}{2}\right)^2 + \frac{1}{2}m\left(\frac{v_0}{2}\right)^2\right] = \frac{1}{4}mv_0^2$

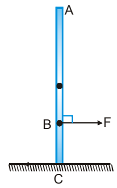


For (B) : $\omega = \frac{v_0}{\ell/2} = \frac{2v_0}{\ell}$ so $KE = \frac{1}{2}\left(\frac{m\ell^2}{3}\right)\left(\frac{2v_0}{\ell}\right)^2 = \frac{2}{3}mv_0^2$

For (C) : $v_{cm} = \frac{v_0}{2}$ so $KE = \frac{1}{2}mv_{cm}^2 = \frac{1}{8}mv_0^2$

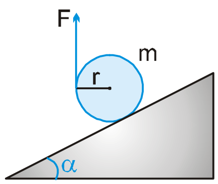
For (D) : KE of rod $= \frac{1}{2}I_C\omega^2 = \frac{1}{2}\left(\frac{m\ell^2}{12} + \frac{m\ell^2}{4}\right)\left(\frac{\sqrt{2}v_0}{\ell}\right) = \frac{1}{3}mv_0^2$

Ex.34 A uniform rod ABC of mass M and length ℓ is placed vertically on a rough horizontal surface. The coefficient of friction between the rod and surface is μ. A force F = 1.2 μmg is applied on the rod at point B at a distance $\ell/3$ below centre of rod horizontally as shown in figure. If the initially acceleration of point A is kμ then find value of k. (Friction is sufficient to prevent slipping)

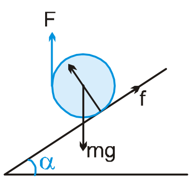


Sol. Taking torque about C $F\frac{\ell}{6} = \left(\frac{m\ell^2}{3}\right)\alpha \Rightarrow \alpha = \frac{3}{5}\frac{\mu g}{\ell}$ So, $a_A = \alpha\ell = \frac{3}{5}\mu g = 6\mu$

Ex. 35 A solid uniform cylinder of mass $m = 6$ kg and radius $r = 0.1$ m is kept in balance on a slope of inclination $\alpha = 37°$ with the help of a thread fastened to its jacket. The cylinder does not slip on the slope. The minimum required coefficient of friction to keep the cylinder in balance when the thread is held vertically is given as μ. Find the value of 4μ.



Sol. $Fr - fr = 0$ and $mg\sin\alpha - F\sin\alpha - f = 0$



$$F = f = \frac{mg\sin\alpha}{1+\sin\alpha}$$

$$F\cos\alpha + N - mg\cos\alpha = 0$$

$$N = \left(mg - \frac{mg\sin\alpha}{1+\sin\alpha}\right)\cos\alpha = \frac{mg\cos\alpha}{1+\sin\alpha};$$

$$f_{max} = \mu N; \frac{mg\sin\alpha}{1+\sin\alpha} = \frac{\mu mg\cos\alpha}{1+\sin\alpha} \Rightarrow \mu = \tan\alpha = 0.75$$

Exercise # 1 [Single Correct Choice Type Questions]

1. A fly wheel originally at rest is to reach an angular velocity of 36 radian/s in 6 second. The total angle it turns through in the 6 second is
(A) 54 radian **(B)** 108 radian **(C)** 6 radian **(D)** 216 radian

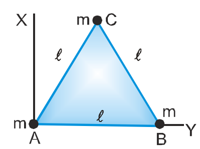
2. On account of the earth rotating about its axis :-
(A) the linear velocity of objects at equator is greater than at other places
(B) the angular velocity of objects at equator is more than that of objects at poles
(C) the linear velocity of objects at all places at the earth is equal, but angular velocity is different
(D) at all places the angular velocity and linear velocity are uniform

3. The rotating rod starts from rest and acquires a rotational speed n = 600 revolution/minute in 2 seconds with constant angular acceleration. The angular acceleration of the rod is
(A) 10π rad/s^2 **(B)** 5π rad/s^2 **(C)** 15π rad/s^2 **(D)** None of these

4. Two gear wheels which are meshed together have radii of 0.50 cm and 0.15 cm. The number of revolutions does the smaller turns when the larger turns through 3 revolution is
(A) 5 revolution **(B)** 20 revolution **(C)** 1 revolution **(D)** 10 revolution

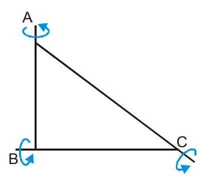
5. The number of revolutions must the 60 cm diameter wheel of a car turn as the car travels 2.5 km is
(A) 8000 revolution **(B)** 1000 revolution **(C)** 1330 revolution **(D)** 500 revolution

6. Three particles, each of mass m are situated at the vertices of an equilateral triangle ABC of side ℓ cm (as shown in the figure). The moment of inertia of the system about a line AX perpendicular to AB and in the plane of ABC, in gram cm^2 units will be :–



(A) $2\,m\ell^2$ **(B)** $\frac{5}{4}m\ell^2$ **(C)** $\frac{3}{2}m\ell^2$ **(D)** $\frac{3}{4}m\ell^2$

7. In the adjoining figure along which axis the moment of inertia of the triangular lamina will be maximum- [Given that AB < BC < AC]



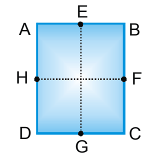
(A) AB
(B) BC
(C) CA
(D) For all axis

8. The radius of a wheel of a car is 0.4m. The car is accelerated from rest by an angular acceleration of 1.5 rad/s^2 for 20s. The linear velocity of the wheel is
(A) 10 m/s **(B)** 3 m/s **(C)** 12 m/s **(D)** 2 m/s

9. A circular disc is to be made by using iron and aluminium so that it acquired maximum moment of inertia about geometrical axis. It is possible with :–
(A) aluminium at interior and iron surrounded to it.
(B) iron at interior and aluminium surrounded to it.
(C) using iron and aluminium layers in alternate order.
(D) sheet of iron is used at both external surface and aluminium sheet as internal layer.

10. Off two eggs which have identical sizes, shapes and weights, one is raw and the other is half-boiled. The ratio between the moment of inertia of the raw egg and that of the half-boiled egg about a central axis is :–
(A) one **(B)** greater than one **(C)** less than one **(D)** incomparable

11. We have a rectangular slab of same thickness. E, F, G, H are the middle point of AB, BC, CD and AD respectively then which of the following axis the moment of inertia will be minimum :–



(A) AD **(B)** EG **(C)** BD **(D)** HF

12. Two disc one of density 7.2 g/cm^3 and the other of density 8.9 g/cm^3 are of same mass and thickness. Their moments of inertia are in the ratio :–

(A) $\frac{8.9}{7.2}$ **(B)** $\frac{7.2}{8.9}$ **(C)** $(8.9 \times 7.2) : 1$ **(D)** $1 : (8.9 \times 7.2)$

13. The moment of inertia of a rod about an axis through its centre and perpendicular to it is $\frac{1}{12}$ ML^2 (where M is the mass and L is the length of the rod). The rod is bent in the middle so that the two half make an angle of 60°. The moment of inertia of the bent rod about the same axis would be :–

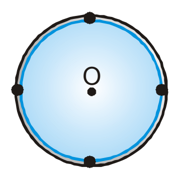
(A) $\frac{1}{48}ML^2$ **(B)** $\frac{1}{12}ML^2$ **(C)** $\frac{1}{24}ML^2$ **(D)** $\frac{ML^3}{8\sqrt{3}}$

14. The moment of inertia of a thin uniform rod of mass M and length ℓ about an axis perpendicular to the rod, through its centre is I. The moment of inertia of the rod about an axis perpendicular to the rod through its end point is :–
(A) I/4 **(B)** I/2 **(C)** 2I **(D)** 4I

15. Two rings of the same radius and mass are placed such that their centres are at a common point and their planes are perpendicular to each other. The moment of inertia of the system about an axis passing through the centre and perpendicular to the plane of one of the rings is (mass of the ring = m, radius = r) :–

(A) $\frac{1}{2}mr^2$ **(B)** mr^2 **(C)** $\frac{3}{2}mr^2$ **(D)** $2mr^2$

16. Four similar point masses (each of mass m) are placed on the circumference of a disc of mass M and radius R. The M.I. of the system about the normal axis through the centre O will be:-



(A) $MR^2 + 4mR^2$ **(B)** $\frac{1}{2}MR^2 + 4mR^2$ **(C)** $MR^2 + \frac{8}{5}mR^2$ **(D)** None of these

17. Three point masses, each of m, are placed at the corners of an equilateral triangle of side ℓ. Then the moment of inertia of this system about an axis along one side of the triangle is :–

(A) $3m\ell^2$ **(B)** $m\ell^2$ **(C)** $\frac{3}{4}m\ell^2$ **(D)** $\frac{3}{2}m\ell^2$

18. If the mass of hydrogen atom is 1.7×10^{-24} g and interatomic distance in a molecule of hydrogen is 4×10^{-8} cm, then the moment of inertia [in kg-m^2] of a molecule of hydrogen about the axis passing through the centre of mass and perpendicular to the line joining the atoms will be:–
(A) 6.8×10^{-32} **(B)** 1.7×10^{-24} **(C)** 13.6×10^{-27} **(D)** 13.6×10^{-47}

19. Two rods each of mass m and length ℓ are joined at the centre to form a cross. The moment of inertia of this cross about an axis passing through the common centre of the rods and perpendicular to the plane formed by them, is :–

(A) $\frac{m\ell^2}{12}$ **(B)** $\frac{m\ell^2}{6}$ **(C)** $\frac{m\ell^2}{3}$ **(D)** $\frac{m\ell^2}{2}$

20. For the same total mass which of the following will have the largest moment of inertia about an axis passing through its centre of mass and perpendicular to the plane of the body

(A) a disc of radius a **(B)** a ring of radius a
(C) a square lamina of side 2a **(D)** four rods forming a square of side 2a

21. If a body completes one revolution in π sec then the moment of inertia would be:–

(A) Equal to rotational kinetic energy **(B)** Double of rotational kinetic energy
(C) Half of rotational kinetic energy **(D)** Four times of the rotational kinetic energy

22. A rigid body can be hinged about any point on the x-axis. when it is hinged such that the hinge is at x, the moment of inertia is given by $I = x^2 - 2x + 99$. The x-coordinate of centre of mass is :–

(A) x=2 **(B)** x=0 **(C)** x=1 **(D)** x=3

23. Two rods of equal mass m and length ℓ lie along the x axis and y axis with their centres origin. What is the moment of inertia of both about the line x=y :

(A) $\frac{m\ell^2}{3}$ **(B)** $\frac{m\ell^2}{4}$ **(C)** $\frac{m\ell^2}{12}$ **(D)** $\frac{m\ell^2}{6}$

24. A wheel is rotating about an axis through its centre at 720 rpm. It is acted on by a constant torque opposing its motion for 8 second to bring it to rest finally. The value of torque in Nm is :– (given $I = \frac{24}{\pi} kg - m^2$)

(A) 48 **(B)** 72 **(C)** 96 **(D)** 120

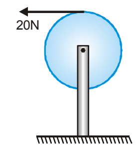
25. The axis X and Z in the plane of a disc are mutually perpendicular and Y-axis is perpendicular to the plane of the disc. If the moment of inertia of the body about X and Y axes is respectively 30 kg m^2 and 40 kgm^2 then M.I. about Z-axis in kg m^2 will be:–

(A) 70 **(B)** 50 **(C)** 10 **(D)** Zero

26. A rod of mass M and length L is placed in a horizontal plane with one end hinged about the vertical axis. A horizontal force of $F=\frac{Mg}{2}$ is applied at a distance $\frac{5L}{6}$ from the hinged end. The angular acceleration of the rod will be :-

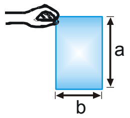
(A) $\frac{4g}{5L}$ **(B)** $\frac{5g}{4L}$ **(C)** $\frac{3g}{4L}$ **(D)** $\frac{4g}{3L}$

27. A string is wrapped around the rim of a wheel of moment of inertia 0.20 kg-m^2 and radius 20 cm. The wheel is free to rotate about its axis and initially the wheel is rest. The string is now pulled by a force of 20N. The angular velocity of the string after 5 seconds will be:–



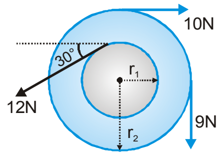
(A) 90 rad/s **(B)** 70 rad/s **(C)** 95 rad/s **(D)** 100 rad/s

28. A person supports a book between finger and thumb as shown (the point of grip is assumed to be at the corner of the book). If the book has a weight of W then the person is producing a torque on the book of



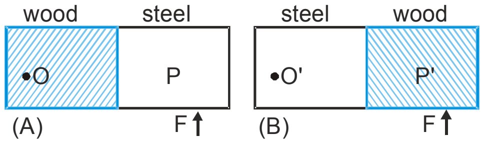
(A) $W\frac{a}{2}$ anticlockwise **(B)** $W\frac{b}{2}$ anticlockwise **(C)** Wa anticlockwise **(D)** Wa clockwise

29. In the following figure r_1 and r_2 are 5 cm and 30 cm respectively. If the moment of inertia of the wheel is 5100 kg-m^2 then its angular acceleration will be :-



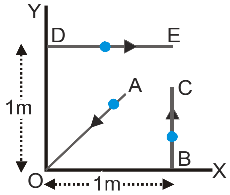
(A) 10^{-4} rad/sec^2 **(B)** 10^{-3} rad/sec^2
(C) 10^{-2} rad/sec^2 **(D)** 10^{-1} rad/sec^2

30. In the figure **(A)** half of the meter scale is made of wood while the other half of steel. The wooden part is pivoted at O. A force F is applied at the end of steel part. In figure **(B)** the steel part is pivoted at O' and the same force is applied at the wooden end:–



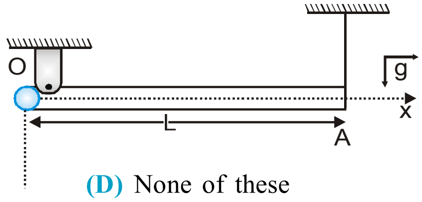
(A) More angular acceleration will be produced in (A)
(B) More angular acceleration will be produced in (B)
(C) Same angular acceleration will be produced in both conditions
(D) Information is incomplete

31. A particle of mass m moves with a constant velocity. Which of the following statements is not correct about its angular momentum :



(A) it is zero when it is at A and moving along OA
(B) the same at all points along the line DE
(C) of the same magnitude but oppositely directed at B and D
(D) increases as it moves along the line BC

32. A non uniform rod OA of liner mass density $\lambda = \lambda_0 x$ $(\lambda_0 = \text{const.})$ is suspended from ceiling with hinge joint O & light string as shown in figure. Find the angular acceleration of rod just after the string is cut



(A) $\frac{2g}{L}$ **(B)** $\frac{g}{L}$ **(C)** $\frac{4g}{3L}$ **(D)** None of these

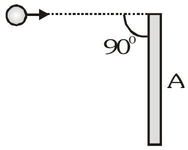
33. If the earth is a point mass of 6 × 10^{24} kg revolving around the sun at a distance of 1.5 × 10^8 km and in time T= 3.14 × 10^7 second, then the angular momentum of the earth around the sun is :–

(A) 1.2 × 10^{18} kg m^2/s **(B)** 1.8 × 10^{20} kg m^2/s **(C)** 1.5 × 10^{37} kg m^2/s **(D)** 2.7 × 10^{40} kg m^2/s

34. If the earth were to suddenly contract to $\frac{1}{n}$th of its present radius without any change in its mass then the duration of the new day will be nearly :–

(A) $\frac{24}{n}$ hour **(B)** 24n hour **(C)** $\frac{24}{n^2}$ hour **(D)** $24n^2$ hour

35. A thin rod of mass M and length L is struck at one end by a ball of clay of mass m, moving with speed v as shown in figure. The ball sticks to the rod. After the collision, the angular momentum of the clay-rod system about A, the midpoint of the rod, is



(A) $\left(m + \frac{M}{3}\right)\left(\frac{vL}{2}\right)$ **(B)** $\left(m + \frac{M}{12}\right)\left(\frac{vL}{2}\right)$ **(C)** $\frac{mvL}{2}$ **(D)** MVL

36. A circular turn table has a block of ice placed at its centre. The system rotates with an angular speed ω about an axis passing through the centre of the table. If the ice melts on its own without any evaporation, the speed of rotation of the system :–

(A) becomes zero
(B) remains constant at the same value of ω
(C) increases to value greater than ω
(D) decreases to a value less than ω

37. The angular velocity of a body changes from ω_1 to ω_2 without applying torque. The ratio of initial radius of gyration to the final radius of gyration is :–

(A) $\sqrt{\omega_2} : \sqrt{\omega_1}$ **(B)** $\sqrt{\omega_1} : \sqrt{\omega_2}$ **(C)** $\omega_2 : \omega_1$ **(D)** $\omega_1 : \omega_2$

38. A person is standing on the edge of a circular platform, which is moving with constant angular speed about an axis passing through its centre and perpendicular to the plane of platform. If person is moving along any radius towards axis of rotation then the angular velocity will :–

(A) decrease **(B)** remain unchanged **(C)** increase **(D)** data is insufficient

39. A thin circular ring of mass M and radius 'r' is rotating about its axis with a constant angular velocity ω. Four objects each of mass m, are kept gently to the opposite ends of two perpendicular diameters of the ring. The new angular velocity of the ring will be :–

(A) $\frac{M\omega}{4m}$ **(B)** $\frac{M\omega}{M+4m}$ **(C)** $\frac{(M+4m)\omega}{M}$ **(D)** $\frac{(M+4m)\omega}{M+4m}$

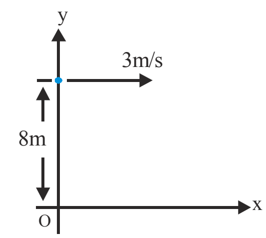
40. A boy stands over the centre of a horizontal platform which is rotating freely with a speed of 2 revolutions/s about a vertical axis through the centre of the platform and straight up through the boy. He holds 2 kg masses in each of his hands close to his body. The combined moment of inertia of the system is 1 kg-m.^{2}. The boy now stretches his arms so as to hold the masses far from his body. In this situation the moment of inertia of the system increases to 2 kg-m.^{2}. The kinetic energy of the system in the latter case as compared with that in the previous case will-

(A) Remain unchanged **(B)** Decrease **(C)** Increase **(D)** Remain uncertain

41. An ant is sitting at the edge of a rotating disc. If the ant reaches the other end, after moving along the diameter, the angular velocity of the disc will :-

(A) remain constant
(B) first decreases and then increases
(C) first increases, then decrease
(D) Increase continuously

42. A particle starts from the point (0m, 8m) and moves with uniform velocity of $3\hat{i}$ m/s. After 5 seconds, the angular velocity of the particle about the origin will be



(A) $\frac{8}{289}$ rad/s
(B) $\frac{3}{8}$ rad/s
(C) $\frac{24}{289}$ rad/s
(D) $\frac{8}{17}$ rad/s

43. A horizontal platform is rotating with uniform angular velocity around the vertical axis passing through its centre. At some instant of time a viscous fluid of mass "m" is dropped at the centre and is allowed to spread out and finally fall. The angular velocity during this period :–

(A) Decreases continuously
(B) Decreases initially and increases again
(C) Remains unaltered
(D) Increases continuously

44. A thin rod of length L is suspended from one end and rotated with n rotations per second. The rotational kinetic energy of the rod will be:–

(A) $2mL^2\pi^2n^2$ **(B)** $\frac{1}{2}mL^2\pi^2n^2$ **(C)** $\frac{2}{3}mL^2\pi^2n^2$ **(D)** $\frac{1}{6}mL^2\pi^2n^2$

45. Two rotating bodies have same angular momentum but their moments of inertia are I_1 and I_2 respectively ($I_1 > I_2$). Which body will have higher kinetic energy of rotation:–

(A) First **(B)** Second
(C) Both will have same kinetic energy **(D)** Not possible to predict

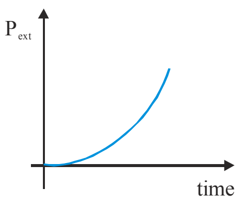
46. A weightless rod is acted on by upward parallel forces of 2N and 4N at ends A and B respectively. The total length of the rod is AB =3 m. To keep the rod in equilibrium a force of 6N should act in the following manner:–

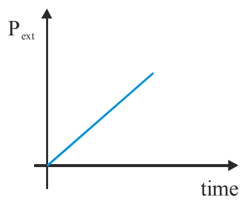
(A) Downwards at any point between A and B **(B)** Downwards at mid point of AB
(C) Downwards at a point C such that AC =1m **(D)** Downwards at a point D such that BD =1m

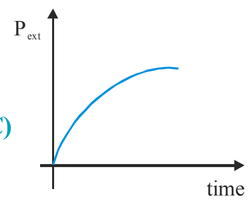
47. A rigid body of mass m rotates with angular velocity ω about an axis at a distance d from the centre of mass G. The radius of gyration about a parallel axis through G is K. The kinetic energy of rotation of the body is :–

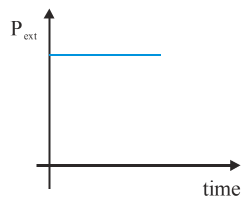
(A) $\frac{1}{2}mk^2\omega^2$ **(B)** $\frac{1}{2}md^2\omega^2$ **(C)** $\frac{1}{2}m(d^2+k^2)\omega^2$ **(D)** $\frac{1}{2}m(d+k)^2\omega^2$

48. A rod is hinged at its centre and rotated by applying a constant torque starting from rest. The power developed by the external torque as a function of time is :–

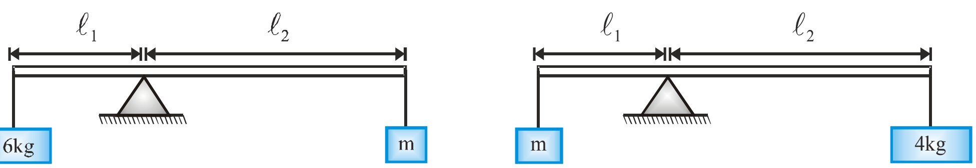
(A) 

(B) 

(C) 

(D) 

49. In an experiment with a beam balance an unknown mass m is balanced by two known masses of 16kg and 4 kg as shown in figure. The value of the unknown mass m is :–

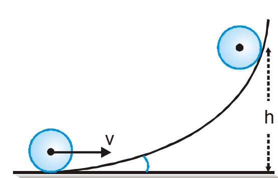


(A) 10 kg **(B)** 6 kg **(C)** 8 kg **(D)** 12 kg

50. A body is rolling without slipping on a horizontal surface and its rotational kinetic energy is equal to the translational kinetic energy. The body is :–
(A) disc **(B)** sphere **(C)** cylinder **(D)** ring

51. If a ring, a disc, a solid sphere and a cylinder of same radius rolls down on inclined plane, the first one to reach the bottom will be :–
(A) disc **(B)** ring **(C)** solid sphere **(D)** cylinder

52. A disc of mass M and radius R rolls on a horizontal surface and then rolls up an inclined plane as shown in the figure. If the velocity of the disc is v, the height to which the disc will rise will be :–

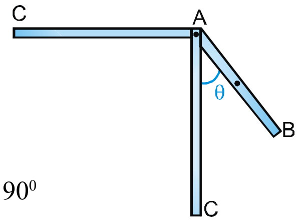


(A) $\frac{3v^2}{2g}$ **(B)** $\frac{3v^2}{4g}$ **(C)** $\frac{v^2}{4g}$ **(D)** $\frac{v^2}{2g}$

53. A solid cylinder of mass M and radius R rolls without slipping down an inclined plane of length L and height h. What is the speed of its centre of mass when the cylinder reaches its bottom :–

(A) $\sqrt{2gh}$ **(B)** $\sqrt{\frac{3}{4}gh}$ **(C)** $\sqrt{\frac{4}{3}gh}$ **(D)** $\sqrt{4gh}$

54. A rod hinged at one end is released from the horizontal position as shown in the figure. When it becomes vertical its lower half separates without exerting any reaction at the breaking point. Then the maximum angle 'θ' made by the hinged upper half with the vertical is



(A) 30^0 **(B)** 45^0 **(C)** 60^0 **(D)** 90^0

55. A solid sphere is rolling on a frictionless surface, shown in figure with a translational velocity v m/s. If it is to climb the inclined surface then v should be:–

(A) $\geq \sqrt{10/7\,gh}$ **(B)** $> \sqrt{2gh}$ **(C)** 2gh **(D)** 10/7gh

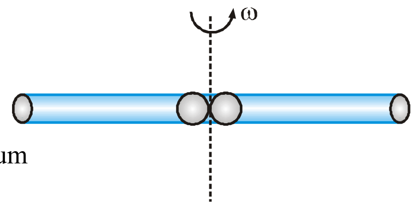
56. There is rod of length ℓ. The velocities of its two ends are v_1 and v_2 in opposite directions normal to the rod. The distance of the instantaneous axis of rotation from v_1 is :–

(A) Zero **(B)** $\frac{v_2}{v_1+v_2}\ell$ **(C)** $\frac{v_1\ell}{v_1+v_2}$ **(D)** 2ℓ

Exercise # 2

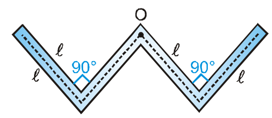
Part # I [Multiple Correct Choice Type Questions]

1. A smooth tube of certain mass is rotated in gravity free space and released. The two balls shown in the figure move towards ends of the tube. For the whole system which of the following quantity is not conserved :-



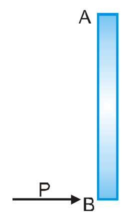
(A) Angular momentum **(B)** Linear momentum
(C) Kinetic energy **(D)** Angular speed

2. A thin rod of length 4ℓ, mass 4m is bent at the points as shown in the fig. What is the moment of inertia of the rod about the axis passing point O & perpendicular to the plane of the paper



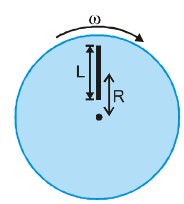
(A) $\frac{m\ell^2}{3}$ **(B)** $\frac{10m\ell^2}{3}$ **(C)** $\frac{m\ell^2}{12}$ **(D)** $\frac{m\ell^2}{24}$

3. A uniform rod AB of mass m and length ℓ at rest on a smooth horizontal surface. An impulse P is applied to the end B. The time taken by the rod to turn through a right angle is :-



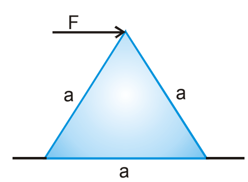
(A) $\frac{2\pi m\ell}{P}$ **(B)** $\frac{\pi m\ell}{3P}$
(C) $\frac{\pi m\ell}{12P}$ **(D)** $\frac{2\pi m\ell}{3P}$

4. A uniform rod of mass M and length L lies radially on a disc rotating with angular speed ω in a horizontal plane about its axis. The rod does not slip on the disc and the centre of the rod is at a distance R from the centre of the disc. Then the kinetic energy of the rod is-



(A) $\frac{1}{2}m\omega^2\left(R^2+\frac{L^2}{12}\right)$ **(B)** $\frac{1}{2}m\omega^2R^2$ **(C)** $\frac{1}{24}m\omega^2L^2$ **(D)** None of these

5. An equilateral prism of mass m rests on a rough horizontal surface with coefficient of friction μ. A horizontal force F is applied on the prism as shown in the figure. If the coefficient of friction is sufficiently high so that the prism does not slide before toppling, then the minimum force required to topple the prism is-



(A) $\frac{mg}{\sqrt{3}}$ **(B)** $\frac{mg}{4}$ **(C)** $\frac{\mu mg}{\sqrt{3}}$ **(D)** $\frac{\mu mg}{4}$

6. Two point masses of 0.3 kg and 0.7 kg are fixed at the ends of a rod of length 1.4 m and of negligible mass. The rod is set rotating about an axis perpendicular to its length with a uniform angular speed. The point on the rod through which the axis should pass in order that the work required for rotation of the rod is minimum, is located at a distance of :-

(A) 0.42 m from mass of 0.3 kg **(B)** 0.70 m from mass of 0.7 kg
(C) 0.98 m from mass of 0.3 kg **(D)** 0.98 m from mass of 0.7 kg

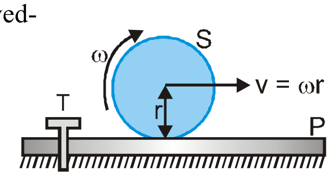
7. A tube of length L is filled completely with an incompressible liquid of mass M and closed at both the ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω. The force exerted by the liquid at the other end is :-

(A) $\frac{M\omega^2 L}{2}$ **(B)** $M\omega^2 L$ **(C)** $\frac{M\omega^2 L}{4}$ **(D)** $\frac{M\omega^2 L^2}{2}$

8. A particle of mass m is projected with a velocity v making an angle of 45° with the horizontal. The magnitude of the angular momentum of the projectile about the point of projection when the particle is at its maximum height h is :-

(A) zero **(B)** $\frac{mv^3}{(4\sqrt{2}g)}$ **(C)** $\frac{mv^3}{\sqrt{2}g}$ **(D)** $m\sqrt{2gh^3}$

9. A sphere S rolls without slipping, moving with a constant speed on a plank P. The friction between the upper surface of P and the sphere is sufficient to prevent slipping, while the lower surface of P is smooth and rests on the ground. Initially, P is fixed to the ground by a pin T. If T is suddenly removed-



(A) S will begin to slip on P.

(B) P will begin to move backwards.

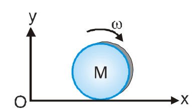
(C) the speed of S will decrease and its angular velocity will increase.

(D) there will be no change in the motion of S and P will still be at rest.

10. A cord is wound over a cylinder of radius r and moment of inertia I. A mass m is attached to the free end of the cord. The cylinder is free to rotate about its own horizontal axis. If mass m is released from rest, then the velocity of the mass after it had fallen through a distance h will be-

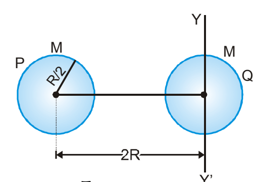
(A) $(2gh)^{1/2}$ **(B)** $\left(\frac{2mghr}{I}\right)^{1/2}$ **(C)** $\left(\frac{2mghr}{I+mr}\right)^{1/2}$ **(D)** $\left(\frac{mghr}{I+2mr}\right)^{1/2}$

11. A disc of mass M and radius R is rolling with angular speed ω on a horizontal plane as shown. The magnitude of angular momentum of the disc about the origin O is :–



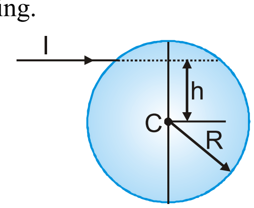
(A) $\frac{1}{2}MR^2\omega$ **(B)** $MR^2\omega$ **(C)** $\frac{3}{2}MR^2\omega$ **(D)** $2MR^2\omega$

12. Two spheres each of mass M and radius R/2 are connected with a mass less rod of length 2R as shown in the-figure. What will be the moment of inertia of the system about an axis passing through the centre of one of the spheres and perpendicular to the rod



(A) $\frac{21}{5}MR^2$ **(B)** $\frac{2}{5}MR^2$ **(C)** $\frac{5}{2}MR^2$ **(D)** $\frac{5}{21}MR^2$

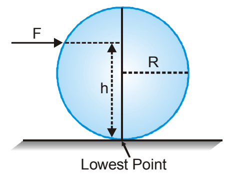
13. A solid sphere is placed on a horizontal plane. A horizontal impulse I is applied at a distance h above the central line as shown in the figure. Soon after giving the impulse the sphere starts rolling.
The ratio h/R would be-



(A) $\frac{1}{2}$ **(B)** $\frac{2}{5}$

(C) $\frac{1}{4}$ **(D)** $\frac{1}{5}$

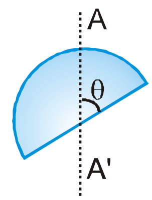
14. A solid sphere of radius R is placed on smooth horizontal surface. A horizontal force 'F' is applied at height 'h' from the lowest point. For the maximum acceleration of centre of mass, which is correct-



(A) $h = R$ (B) $h = 2R$

(C) $h = 0$ (D) No relation between h and R

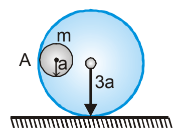
15. The moment of inertia of semicircular plate of radius R and mass M about axis AA' in its plane passing through its centre is



(A) $\frac{MR^2}{2}$ (B) $\frac{MR^2}{4}\cos^2\theta$

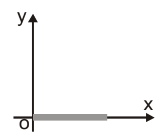
(C) $\frac{MR^2}{4}\sin^2\theta$ (D) $\frac{MR^2}{4}$

16. A ring of radius 3a is fixed rigidly on a table. A small ring whose mass is m and radius a, rolls without slipping inside it as shown in the figure. The small ring is released from position A. When it reaches at the lowest point, the speed of the centre of the ring at that time would be-



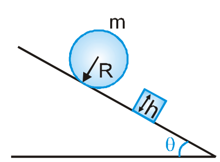
(A) $\sqrt{2ga}$ (B) $\sqrt{3ga}$ (C) $\sqrt{6ga}$ (D) $\sqrt{4ga}$

17. The figure shows a uniform rod lying along the x-axis. The locus of all the points lying on the xy-plane, about which the moment of inertia of the rod is same as that about O is



(A) an ellipse (B) a circle (C) a parabola (D) a straight line

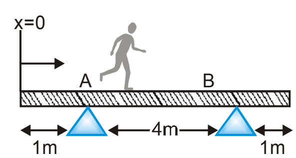
18. Find minimum height of obstacle so that the sphere can stay in equilibrium

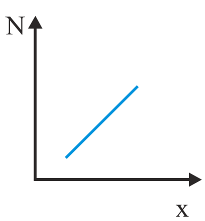


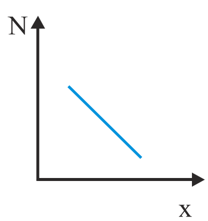
(A) $\frac{R}{1+\cos\theta}$ (B) $\frac{R}{1+\sin\theta}$

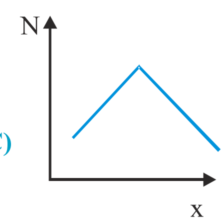
(C) $R(1-\sin\theta)$ (D) $R(1-\cos\theta)$

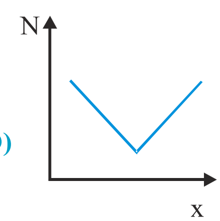
19. A man can move on a horizontal plank supported symmetrically as shown. The variation of normal reaction on support A with distance x of the man from the end of the plank is best represented by



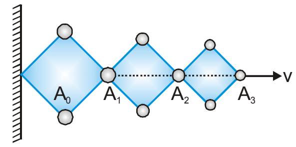
(A) 

(B) 

(C) 

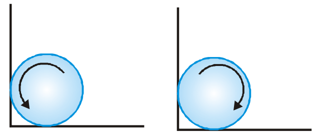
(D) 

20. A hinged construction consists of three rhombus with the ratio of sides 5:3:2. Vertex A_3 moves in the horizontal direction at a velocity v. Velocity of A_2 is



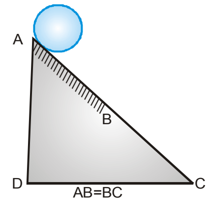
(A) 2.5 v (B) 1.5 v (C) $\frac{2}{3}$ v (D) 0.8 v

21. A sphere is placed rotating with its centre initially at rest in a corner **(A)** **(B)** as shown in figure **(A)** & **(B)**. Coefficient of friction between all surfaces and the sphere is $\frac{1}{3}$. Find the ratio of the frictional force $\frac{f_a}{f_b}$ by ground in situations **(A)** & **(B)**



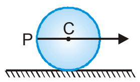
(A) 1 **(B)** 9/10 **(C)** 10/9 **(D)** None

22. Portion AB of the wedge shown in figure is rough and BC is smooth. A solid cylinder rolls without slipping from A to B. The ratio of translational kinetic energy to rotational kinetic energy, when the cylinder reaches point C is



(A) 3/4 **(B)** 5 **(C)** 7/5 **(D)** 8/3

23. A disc of radius R is rolling purely on a flat horizontal surface, with a constant angular velocity. The angle between the velocity and acceleration vectors of point P is



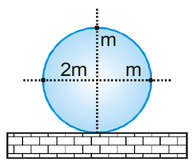
(A) Zero **(B)** 45^0 **(C)** 135^0 **(D)** $\tan^{-1}(1/2)$

24. A slender uniform rod of length ℓ is balanced vertically at a point P on a horizontal surface having some friction. If the top of the rod is displaced slightly to the right, the position of its centre of mass at the time when the rod becomes horizontal

(A) lies at some point to the right of P **(B)** lies at some point to the left of P

(C) must be $\frac{\ell}{2}$ to the right of P **(D)** lies at P

25. A ring of mass m and radius R has three particles attached to the ring as shown in the figure. The centre of the ring has a speed v_0. The kinetic energy of the system is : (slipping is absent)



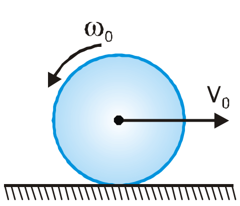
(A) $6\ mv_0^2$ **(B)** $12\ mv_0^2$ **(C)** $4\ mv_0^2$ **(D)** $8\ mv_0^2$

26. A solid sphere with a velocity (of centre of mass) v and angular velocity ω is gently placed on a rough horizontal surface. The frictional force on the sphere

(A) must be forward (in direction of v) **(B)** must be backward (opposite to v)
(C) cannot be zero **(D)** none of the above

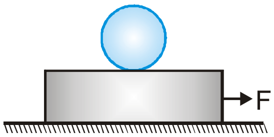
27. A body is in equilibrium under the influence of a number of forces. Each force has a different line of action. The minimum number of forces required is
(A) 2, if their lines of action pass through the centre of mass of the body
(B) 3, if their lines of action are not parallel
(C) 3, if their lines of action are parallel
(D) 4, if their lines of action are parallel and all the forces have the same magnitude

28. A uniform circular disc placed on a rough horizontal surface has initially velocity v_0 and an angular velocity ω_0 as shown in the figure. The disc comes to rest after moving some distance in the direction of motion. Then $\frac{v_0}{r\omega_0}$ is



(A) 1/2 **(B)** 1 **(C)** 3/2 **(D)** 2

29. A plank with a uniform sphere placed on it, rests on a smooth horizontal plane. Plank is pulled to right by a constant force F. If the sphere does not slip over the plank

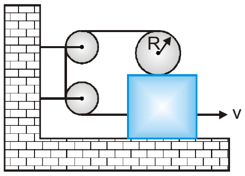


(A) Acceleration of centre of sphere is less than that of the plank
(B) Acceleration of centre of sphere is greater than the plank because friction acts rightward on the sphere
(C) Acceleration of the centre of sphere may be towards left
(D) Acceleration of the centre of sphere relative to plank may be greater than that of the plank relative to floor

30. A particle falls freely near the surface of the earth. Consider a fixed point O (not vertically below the particle) on the ground
(A) Angular momentum of the particle about O is increasing
(B) Torque of the gravitational force on the particle about O is decreasing
(C) The moment of inertia of the particle about O is decreasing
(D) The angular velocity of the particle about O is increasing

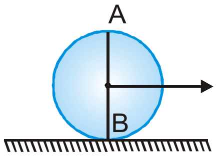
31. If a cylinder is rolling down the incline with sliding
(A) after some time it may start pure rolling
(B) after some time it will start pure rolling
(C) it may be possible that it will never start pure rolling
(D) None of these

32. In the figure shown, the plank is being pulled to the right with a constant speed v. If the cylinder does not slip then



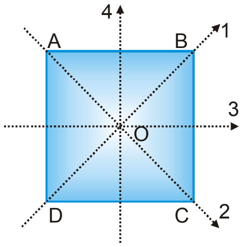
(A) the speed of the centre of mass of the cylinder is 2v
(B) the speed of the centre of mass of the cylinder is zero
(C) the angular velocity of the cylinder is v/R
(D) the angular velocity of the cylinder is zero

33. A uniform disc is rolling on a horizontal surface. At a certain instant B is the point of contact and A is at height 2R from ground, where R is radius of disc



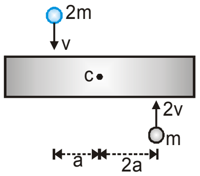
(A) The magnitude of the angular momentum of the disc about B is thrice that about A
(B) The angular momentum of the disc about A is anticlockwise
(C) The angular momentum of the disc about B is clockwise
(D) The angular momentum of the disc about A is equal to that of about B

34. The moment of inertia of a thin square plate ABCD, of uniform thickness about an axis passing through the centre O and perpendicular to the plane of the plate is (where I_1, I_2, I_3 and I_4 are respectively moments of inertia about axis 1, 2, 3 and 4 which are in the plane of the plate)



(A) $I_1 + I_2$ **(B)** $I_3 + I_4$ **(C)** $I_1 + I_3$ **(D)** $I_1 + I_2 + I_3 + I_4$

35. A uniform bar of length 6a and mass 8m lies on a smooth horizontal table. Two point masses m and 2m moving in the same horizontal plane with speed 2v and v respectively, strike the bar (as shown in the fig.) and stick to the bar after collision. Denoting angular velocity (about the centre of mass), total energy and centre of mass velocity by ω, E and v_C respectively, we have after collision:



(A) $v_C = 0$ **(B)** $\omega = \frac{3v}{5a}$ **(C)** $\omega = \frac{v}{5a}$ **(D)** $E = 3\frac{mv^2}{5}$

Part # II [Assertion & Reason Type Questions]

In each of the following questions, a Statement of Assertion (A) is given followed by a corresponding Statement of Reason (R) just below it . Of the Statements mark the correct answer as

(A) Statement–1 is True, Statement–2 is True ; Statement–2 is a correct explanation for Statement–1
(B) Statement–1 is True, Statement–2 is True ; Statement–2 is not a correct explanation for Statement–1
(C) Statement–1 is True, Statement–2 is False.
(D) Statement–1 is False, Statement–2 is True.

1. Statement –1 : The torque can be applied only about two points.
(i) centre of mass and (ii) point about which the body is rolling.
Statement –2 : The equation $a = r\alpha$ can always be applied in case of rolling.

2. Statement –1 : A rigid disc rolls without slipping on a fixed rough horizontal surface with uniform angular velocity. Then the acceleration of lowest point on the disc is zero.
Statement –2 : For a rigid disc rolling without slipping on a fixed rough horizontal surface, the velocity of the lowest point on the disc is always zero.

3. Statement –1 : In case of rolling friction force can in forward and backward direction both.
Statement –2 : The angular momentum of a system will be conserved only about that point about which external angular impulse is zero.

4. Statement –1 : If a body (ball) is rolling on a surface without slipping, no frictional force acts on it.
Statement –2 : In the case of rolling without slipping point of contacts are relatively at rest.

5. Statement –1 : For the purpose of calculation of moment of inertia, a body's mass can be thought to be concentrated at its centre of mass.
Statement –2 : Moment of inertia is a measure of how the mass is distributed about a certain axis.

6. Statement –1 : A sphere rolling on a rough horizontal surface with constant velocity then it start going up on a smooth inclined plane. Rotational KE of sphere decreases continuously on horizontal and inclined surface.
Statement –2 : Rotational KE decreases if torque due to friction opposes angular velocity of sphere.

7. Statement –1 : The moment of inertia of a rigid body is not unique, about a given axis.
Statement –2 : The moment of inertia of a rigid body depends on axis about which it has to be calculated.

8. Statement –1 : Torque $(\vec{\tau})$ acting on a rigid body is defined as $\vec{\tau} = \vec{A} \times \vec{L}, \vec{A}$ is a constant vector and $\vec{L}$ is the angular momentum of the body. The magnitude of the angular momentum of the body remains same.
Statement –2 : $\vec{\tau}$ is perpendicular to $\vec{L}$ and also perpendicular to $\vec{\omega}$, hence torque does not deliver any power to the body.

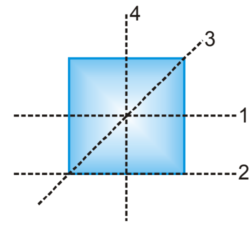
9. Statement –1 : A disc is rolling on an inclined plane without slipping. The velocity of centre of mass is v. These others points on the disc lies on a circular arc having same speed as centre of mass.
Statement –2 : When a disc is rolling on an inclined plane. The magnitude of velocities of all the point from the contact point is same, having distance equal to radius r.

10. Statement –1 : A sphere is performing pure rolling on a rough horizontal surface with constant angular velocity. Frictional force acting on the sphere is zero.
Statement –2 : Velocity of contact point is zero.

11. Statement –1 : A non-uniform sphere is placed such that its centre is origin of coordinate system. If I_x and I_y be moment of inertia about x axis and y axis respectively then moment of inertia about z axis is $I_x + I_y$.
Statement –2 : According to perpendicular axis theory $I_z = I_x + I_y$ when object is lying in x-y plane.

Exercise # 3 Part # I [Matrix Match Type Questions]

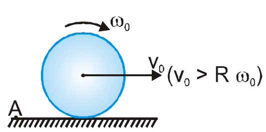
1. Four rods of equal length l and mass m each form a square as shown in figure. Moment of inertia about four axes 1, 2,3 and 4 are say I_1, I_2, I_3 and I_4.

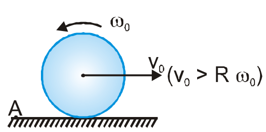


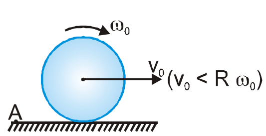
	Column I		**Column II**
(A)	I_1	**(P)**	$\frac{4}{3}m\ell^2$
(B)	I_2	**(Q)**	$\frac{2}{3}m\ell^2$
(C)	I_3	**(R)**	$\frac{1}{2}m\ell^2$
(D)	I_4	**(S)**	None

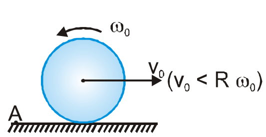
2. In each situation of column-I, a uniform disc of mass m and radius R rolls on a rough fixed horizontal surface as shown. At t = 0 (initially) the angular velocity of disc is ω_0 and velocity of centre of mass of disc is v_0 (in horizontal direction). The relation between v_0 and ω_0 for each situation and also initial sense of rotation is given for each situation in column-I. Then match the column the Statement in column-I with the corresponding results in column-II.

Column I

(A) 

(B) 

(C) 

(D) 

Column II

(P) The angular momentum of disc about point A (as shown in figure) remains conserved

(Q) The kinetic energy of disc after it starts rolling without slipping is less than its initial kinetic energy

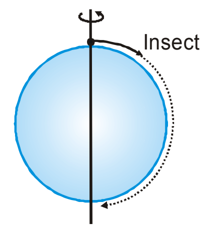
(R) In the duration disc rolls with slipping, the friction acts on disc towards left.

(S) In the duration disc rolls with slipping, the friction acts on disc for some time to right and for some time to left.

3.

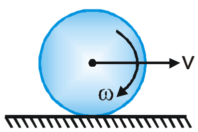
	Column I		Column II
(A)	In pure rolling work done by friction	**(P)**	is always zero
		(Q)	may be zero
(B)	In forward slipping work done by friction	**(R)**	is negative
		(S)	is positive
(C)	In backward slipping work done by friction	**(T)**	may be negative
		(U)	may be positive

4. A solid sphere is rotating about an axis as shown in figure. An insect follows the dotted path on the circumference of sphere as shown.



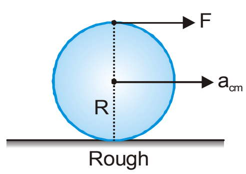
	Column I		Column II
(A)	Moment of inertia	**(P)**	will remain constant
(B)	Angular velocity	**(Q)**	will first increase then decrease
(C)	Angular momentum	**(R)**	will first decrease then increase
(D)	Rotational kinetic energy	**(S)**	will continuously decrease
		(T)	will continuously increase
		(U)	data is insufficient

5. A disc with linear velocity v and angular velocity ω is placed on rough ground. Suppose a and α be the magnitudes of linear and angular acceleration due to friction. Then :-



	Column I		Column II
(A)	When $v = R\omega$	**(P)**	$a = R\alpha \ (a \neq 0)$
(B)	When $v = \dfrac{R\omega}{2}$	**(Q)**	$a > R\alpha$
(C)	When $v = 2R\omega$	**(R)**	$a < R\alpha$
		(S)	None

6. In the adjacent figure a uniform rigid body of mass m and radius R is kept at rest on a rough horizontal surface. A constant horizontal force F is applied at the top most point of the body. The body starts rolling without slipping. Different shapes of bodies are given in the column I and based on this problem some physical quantities related to them are given in column II.

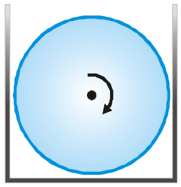


	Column I		Column II
(A)	Solid sphere	**(P)**	Friction force is zero
(B)	Ring	**(Q)**	Magnitude of friction force is maximum
(C)	Hollow sphere	**(R)**	Acceleration of C. O. M. is 4F/3 m
(D)	Disc	**(S)**	Magnitude of friction force is F/5

Part # II [Comprehension Type Questions]

Comprehension # 1

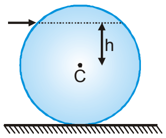
In rotational motion if angular acceleration (or retardation) is constant we can apply equations of motion $\omega = \omega_0 \pm \alpha t$ etc. Here $\alpha = \frac{\tau}{I}$.



1. A solid sphere of mass 5 kg and radius 1 m after rotating with angular speed ω_0 = 40 rad/s is placed between two smooth walls on a rough ground. Distance between the walls is slightly greater than the diameter of the sphere. Coefficient of friction between the sphere and the ground is μ = 0.1. Sphere will stop rotating after time t = ... s :-

(A) 8 **(B)** 12 **(C)** 20 **(D)** 16

Comprehension # 2



A solid sphere is kept over a smooth surface as shown is figure. It is hit by a cue at height h above the centre C.

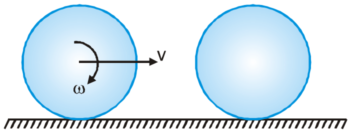
1. In case 1, $h = \frac{R}{4}$ and in case 2, $h = \frac{R}{2}$. Suppose in case 1 the sphere acquires a total kinetic energy K_1 and in case 2 total kinetic energy is K_2. Then :-
(**Note:** That in both the cases, sphere is hit by the same impulse)

(A) $K_1 = K_2$ **(B)** $K_1 > K_2$ **(C)** $K_1 < K_2$ **(D)** data is insufficient

2. If the surface is rough, then after hitting the sphere, in which case the force of friction is in forward direction:-

(A) in case 1 **(B)** in case 2 **(C)** in both the cases **(D)** in none of the case

Comprehension # 3



A solid sphere is rolling without slipping on rough ground as shown in figure. It collides elastically with an identical another sphere at rest. There is no friction betweenthe two spheres. Radius of each sphere is R and mass is m.

1. Linear velocity of first sphere after it again starts rolling without slipping is :-

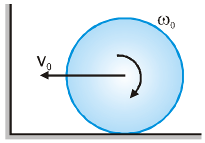
(A) $\frac{2}{5}\omega R$ **(B)** $\frac{2}{7}\omega R$ **(C)** $\frac{7}{10}\omega R$ **(D)** $\frac{7}{5}\omega R$

2. What is the net angular impulse imparted to second sphere by the external forces ?

(A) $\frac{2}{7}mRv$ **(B)** $\frac{5}{7}mRv$ **(C)** $\frac{2}{5}mRv$ **(D)** $\frac{7}{10}mRv$

Comprehension # 4

A solid sphere has linear velocity v_0 = 4 m/s and angular velocity ω_0 =9 rad/s as shown. Ground on which it is moving, is smooth. It collides elastically with a rough wall of coefficient of friction μ. Radius of the sphere is 1 m and mass is 2 kg.



1. If the sphere after colliding with the wall roll without slipping in opposite direction, then coefficient of friction μ is :-

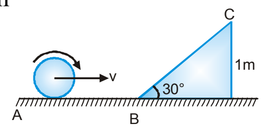
(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$

2. What is net linear impulse imparted by the wall on the sphere during impact :-

(A) $\sqrt{32}$ N-s (B) $4\sqrt{17}$ N-s (C) $4\sqrt{5}$ N-s (D) $15\sqrt{2}$ N-s

Comprehension # 5

A small sphere of mass 1 kg is rolling without slipping on a stationary base with linear speed v = $\sqrt{\frac{200}{7}}$ m/s. It leaves the inclined plane at point C.



1. Find its linear speed at point C :-

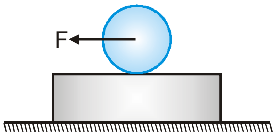
(A) $\sqrt{\frac{100}{7}}$ m/s (B) $\sqrt{\frac{50}{7}}$ m/s (C) $\sqrt{\frac{100}{35}}$ m/s (D) $\sqrt{\frac{200}{35}}$ m/s

2. Find ratio of rotational and translational kinetic energy of the sphere when it strikes the ground after leaving from point C :-

(A) $\frac{2}{5}$ (B) $\frac{2}{3}$ (C) $\frac{1}{6}$ (D) $\frac{1}{2}$

Comprehension # 6

A disc of mass m and radius R is placed over a plank of same mass m. There is sufficient friction between disc and plank to prevent slipping. A force F is applied at the centre of the disc.



1. Acceleration of the plank is :-

(A) $\frac{F}{2m}$ (B) $\frac{3F}{4m}$ (C) $\frac{F}{4m}$ (D) $\frac{3F}{2m}$

2. Force of friction between the disc and the plank is :-

(A) $\frac{F}{2}$ (B) $\frac{F}{4}$ (C) $\frac{F}{3}$ (D) $\frac{2F}{3}$

Comprehension # 7

A rod of mass m and length ℓ in placed on a smooth table. An another particle of same mass m strikes the rod with velocity v_0 in a direction perpendicular to the rod at distance x $\left(<\frac{\ell}{2}\right)$ from its centre. Particle sticks to the rod. Let ω be the angular speed of system after collision, then :

1. As x is increased from 0 to $\frac{\ell}{2}$, the angular speed ω :-

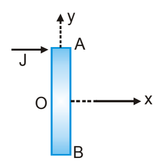
(A) will continuously increase
(B) will continuously decrease
(C) will first increase and then decrease
(D) will first decrease and then increase

2. Find maximum possible value of impulse (by varying x) that can be imparted to the particle during collision. Particle still sticks to the rod :-

(A) $\frac{mv_0}{2}$ **(B)** $\frac{2mv_0}{3}$ **(C)** $\frac{3mv_0}{4}$ **(D)** $\frac{4mv_0}{5}$

Comprehension # 8

A rod AB of length 2 m and mass 2 kg is lying on smooth horizontal x- y plane with its centre at origin O as shown figure. An impulse J of magnitude 10 N-s is applied perpendicular to AB at A.



1. The distance of point P from centre of the rod which is at rest just after the impact is :-

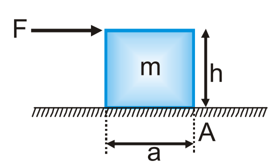
(A) $\frac{2}{3}$ m **(B)** $\frac{1}{3}$ m **(C)** $\frac{1}{2}$ m **(D)** $\frac{1}{4}$ m

2. Co-ordinates of point A of the rod after time $t = \frac{\pi}{45}$ s will be :-

(A) $\left[\left(\frac{\pi}{9}+\frac{\sqrt{3}}{2}\right)m, \frac{1}{2}m\right]$ **(B)** $\left[\left(\frac{3}{4}m, \frac{3}{2}m\right)\right]$ **(C)** $\left[\left(\frac{\pi}{6}+\frac{1}{2}\right)m, \frac{1}{2}m\right]$ **(D)** $\left[\frac{1}{2}m, \frac{1}{2}m\right]$

Comprehension # 9

When a force F is applied on a block of mass m resting on a horizontal surface then there are two possibilities, either block moves by translation or it moves by toppling. If the surface is smooth then the block always translates but on a rough surface it topples only when the torque of the applied force F is greater than the torque of mg about a point in contact with the ground.



When the force F is applied the body may topple about A or it may translate.

1. When the block topples about A, the normal force :-

(A) passes through centre of mass
(B) is zero
(C) shifts to the right and passes through rightmost edge containing A
(D) is zero if the surface is smooth

2. If the block be a cube of edge a and $\mu = 0.2$ then :-

(A) the body will translate **(B)** the body will topple

(C) the body may translate or topple **(D)** none of the above

3. If the block is a cube of edge a and $\mu = 0.6$ then :-

(A) the body will translate **(B)** the body will topple

(C) the body first translates and then topples **(D)** none of the above

Comprehension # 10

A cylinder and a ring of same mass M and radius R are placed on the top of a rough inclined plane of inclination θ. Both are released simultaneously from the same height h

1. Choose the correct statement(s) related to the motion of each body

(A) The friction force acting on each body opposes the motion of its centre of mass

(B) The friction force provides the necessary torque to rotate the body about its centre of mass

(C) Without friction none of the two bodies can roll

(D) The friction force ensures that the point of contact must remain stationary

2. Identify the correct statement(s)

(A) The friction force acting on the cylinder may be more than that acting on the ring

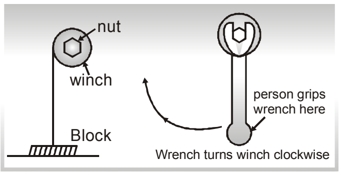
(B) The friction force acting on the ring may be more than that acting on the cylinder

(C) The velocity of centre of mass of the ring is $\sqrt{gh}$

(D) The velocity of centre of mass of each body is $\sqrt{2gh}$

Comprehension # 11

In figure, the winch is mounted on an axle, and the 6-sided nut is welded to the winch. By turning the nut with a wrench, a person can rotate the winch. For instance, turning the nut clockwise lifts the block off the ground, because more and more rope gets wrapped around the winch.



Three students agree that using a longer wrench makes it easier to turn the winch. But they disagree about why. All three students are talking about the case where the winch is used, over a 10 s time interval, to lift the block one metre off the ground.

Student 1 By using a longer wrench, the person decreases the average force he must exert on the wrench, in order to lift the block one metre in 10 s.

Student 2 : Using a longer wrench reduces the work done by the person as he uses the winch to lift the block 1m in 10s.

Student 3 : Using a longer wrench reduces the power that the person must exert to lift the block 1m in 10s.

1. Student 1 is :-

(A) correct, because the torque that the wrench must exert to lift the block doesn't depend on the wrench's length

(B) correct, because using a longer wrench decreases the torque it must exert on the winch

(C) incorrect, because the torque that the wrench must exert to lift the block doesn't depend on the wrench's length

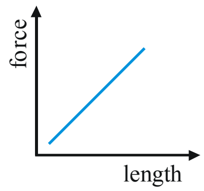
(D) Incorrect, because using a longer wrench decreases the torque it must exert on the winch.

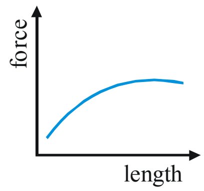
2. Which of the following is true about student 2 and 3 :-

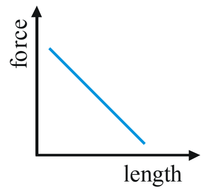
(A) Student 2 and 3 are both correct **(B)** Student 2 is correct, but student 3 is incorrect

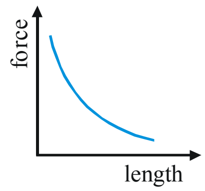
(C) Student 3 is correct, but student 2 is incorrect **(D)** Student 2 and 3 are both incorrect

3. If several wrenches all apply the same torque to a nut, which graph best expresses the relationship between the force the person must apply to the wrench, and the length of the wrench :-

(1) 

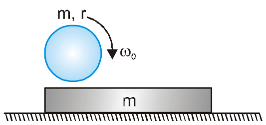
(2) 

(3) 

(4) 

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4

Comprehension # 12



In the figure shown a plank of mass m is lying at rest on a smooth horizontal surface. A disc of same mass m and radius r is rotated to an angular speed ω_0 and then gently placed on the plank. If we consider the plank and the disc as a system then frictional force between them is an internal force. Momentum of the system changes due to external force only. It is found that finally slipping cease, and 50% of total kinetic energy of the system is lost. Assume that plank is long enough. μ is coefficient of friction between disc and plank.

1. Final velocity of the plank is

(A) $\frac{r\omega_0}{4}$ **(B)** $\frac{r\omega_0}{\sqrt{10}}$ **(C)** $\frac{r\omega_0}{2}$ **(D)** $\frac{r\omega_0}{2\sqrt{10}}$

2. Time when slipping ceases

(A) $\frac{r\omega_0}{2\mu g}$ **(B)** $\frac{r\omega_0}{\sqrt{10}\mu g}$ **(C)** $\frac{r\omega_0}{4\mu g}$ **(D)** $\frac{r\omega_0}{2\sqrt{10}\mu g}$

3. Magnitude of the change in angular momentum of disc about centre of mass of disc

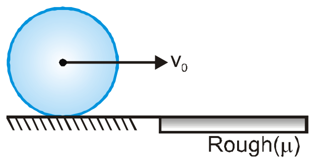
(A) $\frac{3}{4}mr^2\omega_0$ **(B)** $\frac{1}{4}mr^2\omega_0$ **(C)** zero **(D)** $\frac{1}{2}mr^2\omega_0$

4. Distance moved by the plank from the placing of disc on the plank till the slipping ceases between disc and plank

(A) $\frac{r^2\omega_0^2}{16\mu g}$ **(B)** $\frac{r^2\omega_0^2}{8\mu g}$ **(C)** $\frac{r^2\omega_0^2}{32\mu g}$ **(D)** $\frac{r^2\omega_0^2}{200\mu g}$

Comprehension # 13

A ring of mass M and radius R sliding with a velocity v_0 suddenly enters into rough surface where the coefficient of friction is μ, as shown in figure.



1. Choose the correct statement(s)

(A) As the ring enters on the rough surface, the limiting friction force acts on it

(B) The direction of friction is opposite to the direction of motion

(C) The friction force accelerates the ring in the clockwise sense about its centre of mass

(D) As the ring enters on the rough surface it starts rolling

2. Choose the correct statement(s)

(A) The momentum of the ring is conserved

(B) The angular momentum of the ring is conserved about its centre of mass

(C) The angular momentum of the ring conserved about any point on the horizontal surface

(D) The mechanical energy of the ring is conserved

3. Choose the correct statement(s) :–

(A) The ring starts its rolling motion when the centre of mass stationary

(B) The ring starts rolling motion when the point of contact becomes stationary

(C) The time after which the ring starts rolling is $\frac{v_0}{2\mu g}$

(D) The rolling velocity is $\frac{v_0}{2}$

4. Choose the correct alternative(s)

(A) The linear distance moved by the centre of mass before the ring starts rolling is $\frac{3v_0^2}{8\mu g}$

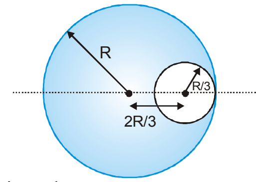
(B) The net work done by friction force is $-\frac{3}{8}mv_0^2$

(C) The loss in kinetic energy of the ring is $\frac{mv_0^2}{4}$

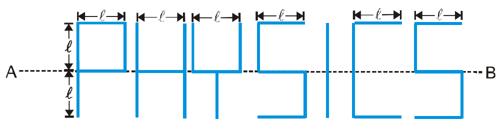
(D) The gain in rotational kinetic energy is $+\frac{mv_0^2}{8}$

Exercise # 4 [Subjective Type Questions]

1. A disc of certain radius is cut from a disk of mass 9M and radius R. Find its moment of inertia about an axis passing through its centre and perpendicular to its plane.

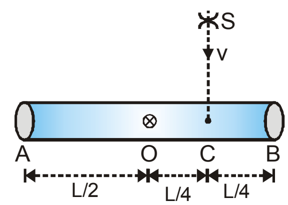


2. Find out the moment of inertia of the following structure (written as **PHYSICS**) about axis AB and made of thin uniform rods of the mass per unit length λ.



3. Calculate the moment of inertia of a wheel about its axis which having rim of mass 24M and twenty four spokes each of mass M and length ℓ.

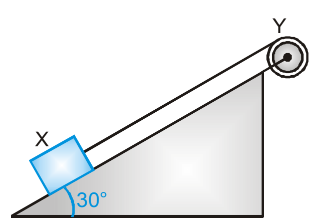
4. A homogeneous rod AB of length L = 1.8 m and mass M is pivoted at the centre O in such a way that it can rotate freely in the vertical plane (fig.). The rod is initially in the horizontal position. An insect S of the same mass M falls vertically with speed v on the point C, midway between the points O and B. Immediately after falling, the insect moves towards the end B such that the rod rotates with a constant angular velocity ω.



(i) Determine the angular velocity ω in terms of v and L.
(ii) If the insect reaches the end B when the rod has turned through an angle of 90°, determine v.

5. A carpet of mass M made of inextensible material is rolled along its length in the form of a cylinder of radius R and is kept on a rough floor. The carpet starts unrolling without sliding on the floor when a negligibly small push is given to it. Calculate the horizontal velocity of the axis of the cylindrical part of the carpet when its radius reduces to $\frac{R}{2}$.

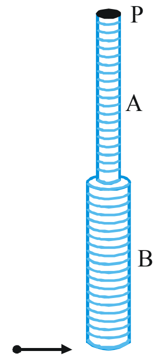
6. A block X of mass 0.5 kg is held by a long massless string on a frictionless inclined plane of inclination 30° to the horizontal. The string is wound on a uniform solid cylindrical drum Y of mass 2 kg and of radius 0.2 m as shown in figure. The drum is given an initial angular velocity such that the block X starts moving up the plane.



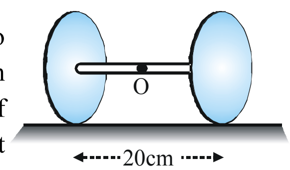
(i) Find the tension in the string during the motion.
(ii) At a certain instant of time the magnitude of the angular velocity of Y is 10 rad s^{-1}. Calculate the distance travelled by X from that instant of time until it comes to rest.

7. Determine the minimum co-efficient of friction between a thin rod and a floor at which a person can slowly lift the rod from the floor without slipping, to the vertical position applying to its end a force always perpendicular to its length.

8. Two uniform rods A and B of length 0.6 m each and of masses 0.01 kg and 0.02 kg respectively are rigidly joined end to end. The combination is pivoted at the lighter end, P as shown in figure. Such that it can freely rotate about point P in a vertical plane. A small object of mass 0.05 kg, moving horizontally, hits the lower end of the combination and sticks to it. What should be the velocity of the object, so that the system could just be raised to the horizontal position.



9. Two thin circular disc of mass 2 kg and radius 10 cm each are joined by a rigid massless rod of length 20 cm. The axis of the rod is along the perpendicular to the planes of the disc through their centres. This object is kept on a truck in such a way that the axis of the object is horizontal and perpendicular to the direction of motion of the truck. Its friction with the floor of the truck is large enough, so that the object can roll on the truck without slipping.



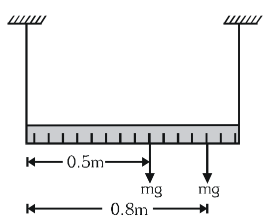
Take x-axis as the direction of motion of the truck and z-axis as the vertically upwards direction. If the truck has an acceleration 9 m/s^2, calculate :

(i) the force of friction on each disc and

(ii) the magnitude and direction of the frictional torque acting on each disc about the centre of mass O of the object. Express the torque in the vector form in terms of unit vector $\tilde{i}$, $\tilde{j}$ and $\tilde{k}$ in x, y and z-directions.

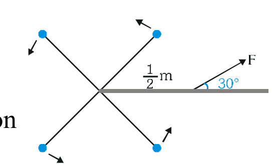
10. A uniform metre scale of mass m is suspended by two vertical string attached to its two ends as shown in figure.

A body of mass m is placed on the 80 cm mark. Calculate the ratio of tension is string.

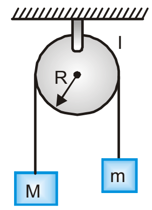


11. Why a force is applied at right angles to the heavy door at its outer edges while closing or opening it ?

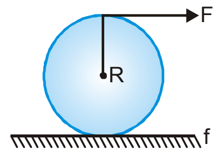
12. Four 2kg masses are connected by $\frac{1}{4}$ m long spokes to an axle as in shown figure. A force F of 24N acts on a lever $\frac{1}{2}$ m long to produce an angular acceleration α. Determine the magnitude of α.



13. The pulley shown in fig. has a moment of inertia I about its axis and its radius is R. Find the magnitude of the acceleration of the two blocks. Assume that the string is light and does not slip on the pulley ?



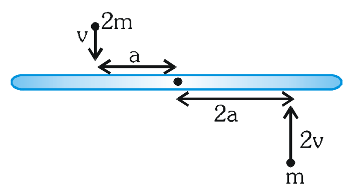
14. A tangential force F acts at the top of a thin spherical shell of mass m and radius R. Find the acceleration of the shell if it rolls without slipping.



15. A cylinder of mass 5 kg and radius 30 cm, and free to rotate about its axis, receives an angular impulse of $3\ kg\ m^2\ s^{-1}$ initially followed by a similar impulse after every 4s. What is the angular speed of the cylinder after 30s of the initial impulse ? The cylinder is at rest initially.

16. A moving particle in X - Y plane has its angular momentum in Z-direction only. Prove it.

17. A uniform rod of mass 8m and length 6a is lying on a horizontal table. Two point masses m and 2m moving with speed 2v and v respectively strike the rod and stick to it as shown in figure then-



(i) Calculate the speed of centre of mass of rod after the collision.

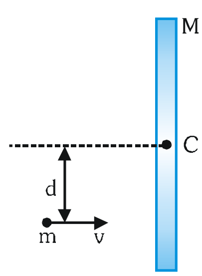
(ii) Calculate angular velocity of the rod about an axis passing through its centre of mass.

(iii) Kinetic energy of system after collision.

18. A rod of length ℓ and mass M held vertically is let go down, without slipping at the point of contact. What is the velocity of the top end at the time of touching the ground ?

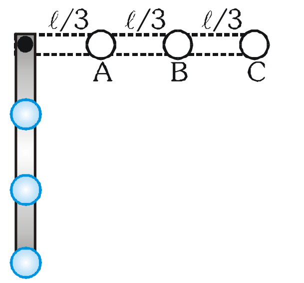
19. A stick of length L and mass M lies on a frictionless horizontal surface on which it is free to move in anyway. A ball of mass m moving with speed v as shown in fig.

What must be the mass of the ball so that it remains at rest immediately after collision.

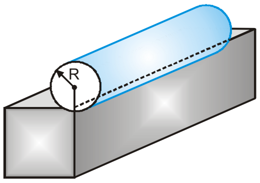


20. An initial momentum is imparted to a homogeneous cylinder as a result of which it begins to roll without slipping up an inclined plane at speed $v_0 = 4ms^{-1}$. The plane makes an angle of 30° with the horizontal. What time does the cylinder take before stopping.

21. A light rod carries three equal masses A, B and C as shown in figure. What will be velocity of B in vertical position of rod, if it is released from horizontal position as shown in figure ?



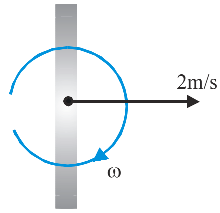
22. A rectangular rigid fixed block has a long horizontal edge. A solid homogeneous cylinder of radius R is placed horizontally at rest with its length parallel to the edge such that the axis of the cylinder and the edge of the block are in the same vertical plane as shown in figure. There is sufficient friction present at the edge, so that a very small displacement causes the cylinder to roll of the edge without slipping. Determine :



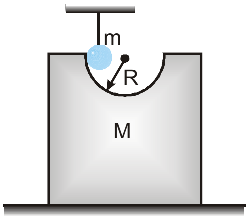
(i) The angle θ_C through which the cylinder rotates before it leaves contact with the edge.
(ii) The speed of the centre of mass of the cylinder before leaving contact with the edge and
(iii) The ratio of the translational to rotational kinetic energies of the cylinder when its centre of mass is in horizontal line with the edge.

23. As shown in the figure, a rod moves with v=2 m/sec and rotates with $\omega = 2\pi$ rad/sec.

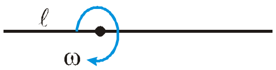
Find the point on the rod whose velocity is zero in this frame.



24. A semi circular track of radius R = 62.5 cm is cut in a block. Mass of block, having track, is M = 1 kg and rests over a smooth horizontal floor. A cylinder of radius r = 10 cm and mass m = 0.5 kg is hanging by thread such that axis of cylinder and track are in same level and surface of cylinder is in contact with the track as shown in figure. When the thread is burnt, cylinder starts to move down the track. Sufficient friction exists between surface of cylinder and track, so that cylinder does not slip. Calculate velocity of axis of cylinder and velocity of the block when it reaches bottom of the track. Also find force applied by block on the floor at the moment. ($g = 10\ m/s^2$)

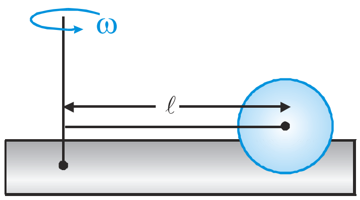


25. A uniform rod of length 4ℓ and mass m is free to rotate about a horizontal axis passing through a point distance ℓ from its one end. When the rod is horizontal, its angular velocity is ω as shown in figure. Calculate

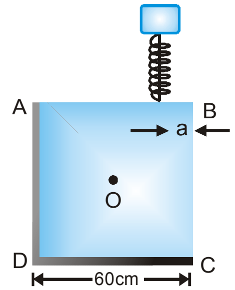


(i) reaction of axis at this instant,
(ii) acceleration of centre of mass of the rod at this instant,
(iii) reaction of axis and acceleration of centre of mass of the rod when rod becomes vertical for the first time.
(iv) minimum value of ω so that centre of rod can complete circular motion.

26. A thin rod is passing through the centre of a sphere. The rod is fixed to a vertical axis and the sphere is made to roll on a surface with friction. The radius of the sphere is r, the mass is m and the length of the rod is ℓ. The rod is rotating with an angular velocity ω_0. Find the energy of the sphere in terms of ω_0, m, ℓ and r. Assume the rod to be of negligible mass.



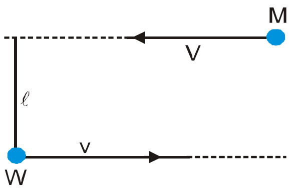
27. A square frame is formed by four rods, each of length $\ell = 60$ cm. Mass of two rods AB and BC is m = 25/18 kg each while that of rods AD and CD is 2kg each. The frame is free to rotate about a fixed horizontal axis passing through its geometric centre O shown in figure. A spring is placed on the rod AB at a distance a = 15 cm from B. The spring is held vertical and a block is placed on upper end of the spring so that rod AB is horizontal.



(i) Calculate mass M of the block,

(ii) If the spring is initially compressed by connecting a thread between its ends and energy stored in it is 76.5 joule, calculate velocity with which block bounces up when the thread is burnt.

28. A man and a woman skate towards each other on smooth ice, but in parallel lines. The distance between the lines is ℓ. The mass of the man is M and that of the woman is m. The velocity of the man is given by V and that of the woman by v. The woman holds a stick of length ℓ and negligible mass. The stick is directed normal to the direction of motion as shown in the figure. When the couple passes each other, the man grasps the stick and the couple move together, each of them holding different ends of the stick.



(i) What is the angular velocity of the rod after the couple begin moving together ?

(ii) The couple start moving towards each other by pulling the stick until the distance between them is $\ell_0 (\ell_0 < \ell)$. What is the velocity of the centre of mass now ?

(iii) What is the angular velocity of the couple now ?

(iv) What is the work done by the couple as they move from ℓ to ℓ_0 ?

29. A small ring of mass m is threaded on a horizontal smooth rod which is rotating about its end with constant angular velocity ω. The ring is initially located at the axis of rotation. When the distance of the ring from the axis becomes r, then find the power required to rotate the system with same angular velocity.

Exercise # 5 Part # I [Previous Year Questions] [AIEEE/JEE-MAIN]

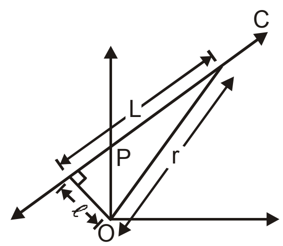
1. Initial angular velocity of a circular disc of mass M is ω_1. Then two small spheres of mass m are attached gently to two diametrically opposite points on the edge of the disc. What is the final angular velocity of the disc ? **[AIEEE - 2002]**

(1) $\left(\frac{M+m}{M}\right)\omega_1$ **(2)** $\left(\frac{M+m}{m}\right)\omega_1$ **(3)** $\left(\frac{M}{M+4m}\right)\omega_1$ **(4)** $\left(\frac{M}{M+2m}\right)\omega_1$

2. Moment of inertia of a circular wire of mass M and radius R about its diameter is- **[AIEEE - 2002]**

(1) $MR^2/2$ **(2)** MR^2 **(3)** $2MR^2$ **(4)** $MR^2/4$

3. A particle of mass m moves along line PC with velocity v as shown.What is the angular momentum of the particle about O ? **[AIEEE - 2002]**



(1) mvL **(2)** $mv\ell$ **(3)** mvr **(4)** zero

4. A circular disc X of radius R is made from an iron plate of thickness t and another disc Y of radius 4R is made from an iron plate of thickness t/4. Then the relation between the moment of inertia I_X and I_Y is- **[AIEEE - 2003]**

(1) $I_Y = 32\,I_X$ **(2)** $I_Y = 16I_X$ **(3)** $I_Y = I_X$ **(4)** $I_Y = 64\,I_X$

5. A particle performing uniform circular motion has angular momentum L. If its angular frequency is doubled and its kinetic energy halved, then the new angular momentum is- **[AIEEE - 2003]**

(1) $\frac{L}{4}$ **(2)** 2L **(3)** 4L **(4)** $\frac{L}{2}$

6. Let $\vec{F}$ be the force acting on particle having position vector $\vec{r}$ and $\vec{\tau}$ be the torque of this force about the origin. Then- **[AIEEE - 2003]**

(1) $\vec{r}\,.\,\vec{\tau} = 0$ and $\vec{F}\,.\,\vec{\tau} \neq 0$ **(2)** $\vec{r}\,.\,\vec{\tau} \neq 0$ and $\vec{F}\,.\,\vec{\tau} = 0$

(3) $\vec{r}\,.\,\vec{\tau} \neq 0$ and $\vec{F}\,.\,\vec{\tau} \neq 0$ **(4)** $\vec{r}\,.\,\vec{\tau} = 0$ and $\vec{F}\,.\,\vec{\tau} = 0$

7. Which of the following statements is false for a particle moving in a circle with a constant angular speed ? **[AIEEE - 2004]**

(1) The velocity vector is tangent to the circle
(2) The acceleration vector is tangent to the circle
(3) the acceleration vector points to the centre of the circle
(4) The velocity and acceleration vectors are perpendicular to each other

8. A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass same which one of the following will not be affected ? **[AIEEE - 2004]**

(1) moment of inertia **(2)** Angular momentum **(3)** Angular velocity **(4)** Rotational kinetic anergy

9. One solid sphere A and another hollow sphere B are of same mass and same outer radii. Their moment of inertia about their diameters are respectively I_A and I_B such that- **[AIEEE - 2004]**

(1) $I_A = I_B$ **(2)** $I_A > I_B$ **(3)** $I_A < I_B$ **(4)** $\frac{I_A}{I_B} = \frac{d_A}{d_B}$

where d_A and d_B are their densities.

10. An annular ring with inner and outer radii R_1 and R_2 is rolling without slipping with a uniform angular speed. The ratio of the forces experienced by the two particles situated on the inner and outer parts of the ring $\frac{F_1}{F_2}$ is- **[AIEEE - 2005]**

(1) $\frac{R_2}{R_1}$ (2) $\left(\frac{R_1}{R_2}\right)^2$ (3) 1 (4) $\frac{R_1}{R_2}$

11. The moment of inertia of uniform semicircular disc of mass M and radius r about a line perpendicular to the plane of the disc through the centre is- **[AIEEE - 2005]**

(1) $\frac{1}{4}Mr^2$ (2) $\frac{2}{5}Mr^2$ (3) Mr^2 (4) $\frac{1}{2}Mr^2$

12. A spherical ball of mass 20 kg is stationary at the top of a hill of height 100 m. It rolls down a smooth surface to the ground, then climbs up another hill of height 30 m and finally rolls down to a horizontal base at a height of 20 m above the ground. The velocity attained by the ball is- **[AIEEE - 2005]**

(1) $40\sqrt{\frac{5}{7}}$ m/s (2) 20 m/s (3) 10 m/s (4) $10\sqrt{30}$ m/s

13. A coin is placed on a horizontal platform which undergoes vertical simple harmonic motion of angular frequency ω. The amplitude of oscillation is gradually increased. The coin will leave contact with the platform for the first time : **[AIEEE - 2006]**

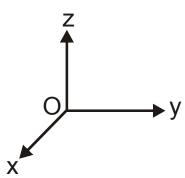
(1) at the mean position of the platform (2) for an amplitude of g/ω^2
(3) for an amplitude of g^2/ω^2 (4) at the highest position of the platform

14. Four point masses, each of value m, are placed at the corners of a square ABCD of side ℓ. The moment of inertia of this system about an axis passing through A and parallel to BD is **[AIEEE - 2006]**

(1) $2m\ell^2$ (2) $\sqrt{3}\,m\ell^2$ (3) $3m\ell^2$ (4) $m\ell^2$

15. A force of $-F\hat{k}$ acts on O, the origin of the co-ordinate system. The torque about the point (1, –1) is : **[AIEEE - 2006]**

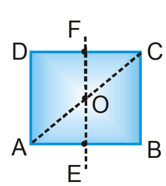
(1) $F(\hat{i}-\hat{j})$
(2) $-F(\hat{i}+\hat{j})$
(3) $F(\hat{i}+\hat{j})$
(4) $-F(\hat{i}-\hat{j})$



16. A thin circular ring of mass m and radius R is rotating about its axis with a constant angular velocity ω. Two objects each of mass M are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity ω' = : **[AIEEE - 2006]**

(1) $\frac{\omega(m+2M)}{m}$ (2) $\frac{\omega(m-2M)}{(m+2M)}$ (3) $\frac{\omega m}{(m+M)}$ (4) $\frac{\omega m}{(m+2M)}$

17. For the given uniform square lamina ABCD, whose centre is O : **[AIEEE - 2007]**



(1) $\sqrt{2}\,I_{AC}=I_{EF}$ (2) $I_{AD}=3I_{EF}$ (3) $I_{AC}=I_{EF}$ (4) $I_{AC}=\sqrt{2}\,I_{EF}$

18. A circular disc of radius R is removed from a bigger circular disc of radius 2R, such that the circumference of the discs coincide. The centre of mass of the new disc is $\frac{\alpha}{R}$ from the centre of the bigger disc. The value of α is- **[AIEEE - 2007]**

(1) $\frac{1}{3}$ **(2)** $\frac{1}{2}$ **(3)** $\frac{1}{6}$ **(4)** $\frac{1}{4}$

19. A round uniform body of radius R, mass M and moment of inertia I, rolls down (without slipping) an inclined plane making an angle θ with the horizontal. Then its acceleration is **[AIEEE - 2007]**

(1) $\frac{g\sin\theta}{1+I/MR^2}$ **(2)** $\frac{g\sin\theta}{1+MR^2/I}$ **(3)** $\frac{g\sin\theta}{1-I/MR^2}$ **(4)** $\frac{g\sin\theta}{1-MR^2/I}$

20. Angular momentum of the particle rotating with a central force is constant due to- **[AIEEE - 2007]**

(1) constant force **(2)** constant linear momentum

(3) zero torque **(4)** constant torque

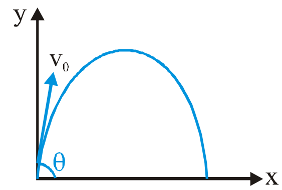
21. Consider a uniform square plat of side 'a' and mass 'm' the moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is **[AIEEE - 2008]**

(1) $\frac{5}{6}ma^2$ **(2)** $\frac{1}{12}ma^2$ **(3)** $\frac{7}{12}ma^2$ **(4)** $\frac{2}{3}ma^2$

22. A thin uniform rod of length *l* and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its centre of mass rises to a maximum height of: **[AIEEE - 2009]**

(1) $\frac{1}{2}\frac{l^2\omega^2}{g}$ **(2)** $\frac{1}{6}\frac{l^2\omega^2}{g}$ **(3)** $\frac{1}{3}\frac{l^2\omega^2}{g}$ **(4)** $\frac{1}{6}\frac{l\omega}{g}$

23. A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity v_0 in the x-y plane as shown in the figure. At a time $t < \frac{v_0\sin\theta}{g}$, the angular momentum of the particle is: **[AIEEE - 2010]**



(1) $\frac{1}{2}mg\,v_0\,t^2\cos\theta\,\hat{i}$ **(2)** $-mg\,v_0\,t^2\cos\theta\,\hat{j}$ **(3)** $mg\,v_0\,t\cos\theta\,\hat{k}$ **(4)** $-\frac{1}{2}mg\,v_0\,t^2\cos\theta\,\hat{k}$

Where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axis respectively.

24. A pulley of radius 2 m is rotated about its axis by a force $F = (20t - 5t^2)$ newton (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kg m^2, the number of rotations made by the pulley before its direction of motion it reversed, is :- **[AIEEE - 2011]**

(1) more than 6 but less than 9 **(2)** more than 9

(3) less than 3 **(4)** more than 3 but less than 6

25. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. Euring the fjourney of the insect, then angular speed of the disc :- **[AIEEE - 2011]**

(1) continuously increases **(2)** first increases and then decreases

(3) remains unchanged **(4)** continuously decreses

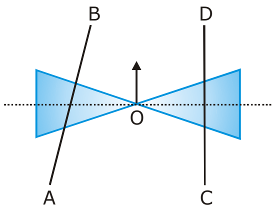
26. A particle of mass 'm' is projected with a velocity v making an angle of 30° with the horizontal. The magnitude of angular momentum of the projectile about the point of projection when the particle is at its maximum height 'h' is :-

[AIEEE - 2011]

(1) $\frac{\sqrt{3}}{2}\frac{mv^2}{g}$ **(2)** zero **(3)** $\frac{mv^3}{\sqrt{2}g}$ **(4)** $\frac{\sqrt{3}}{16}\frac{mv^3}{g}$

27. A roller is made by joining together two cones at their vertices O. It is kept on two rails AB and CD which are placed asymmetrically (see figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD (see figure). It is given a light push so that it starts rolling with its centre O moving

parallel to CD in the direction shown. As it moves, the roller will tend to :



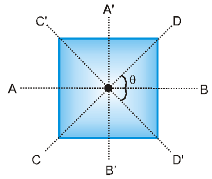
[JEE (Main) -2016]

(1) turn right **(2)** go straight **(3)** turn left and right altemately. **(4)** turn left.

Part # II [Previous Year Questions][IIT-JEE ADVANCED]

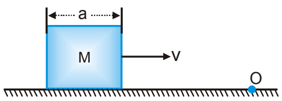
MCQ's with one correct answers

1. Let I be the moment of inertia of a uniform square plate about an axis AB that passes through its centre and is parallel to two of its sides. CD is a line in the plane of the plate that passes through the centre of the plate and makes an angle θ with AB. The moment of inertia of the plate about the axis CD is then equal to:- **[IIT-JEE 1998]**



(A) I **(B)** $I \sin^2\theta$ **(C)** $I \cos^2\theta$ **(D)** $I \cos^2\left(\frac{\theta}{2}\right)$

2. A cubical block of side 'a' moving with velocity v on a horizontal smooth plane as shown. It hits a ridge at point O. The angular speed of the block after it hits O is :- **[IIT-JEE 1999]**

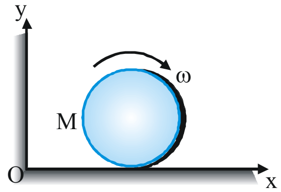


(A) $\frac{3v}{4a}$ **(B)** $\frac{3v}{2a}$ **(C)** $\frac{\sqrt{3}}{\sqrt{2}a}$ **(D)** zero

3. A smooth sphere A is moving on a frictionless horizontal plane with angular velocity ω and centre of mass velocity v. It collides elastically and head on with an identical sphere B at rest. Neglect friction everywhere. After the collision their angular speed are ω_A and ω_B respectively. Then :- **[IIT-JEE 1999]**

(A) $\omega_A < \omega_B$ **(B)** $\omega_A = \omega_B$ **(C)** $\omega_A = \omega$ **(D)** $\omega_B = \omega$

4. A disc of mass M and radius R is rolling with angular speed ω on a horizontal plane as shown. The magnitude of angular momentum of the disc about the origin O is :- **[IIT-JEE 1999]**



(A) $\left(\frac{1}{2}\right)MR^2\omega$ **(B)** $MR^2\omega$ **(C)** $\left(\frac{3}{2}\right)MR^2\omega$ **(D)** $2MR^2\omega$

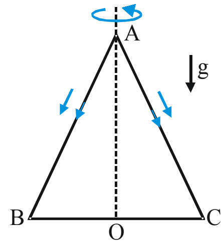
5. An equilateral triangle ABC formed from a uniform wire has two small identical beads initially located at A. The triangle is set rotating about the vertical axis AO. Then the beads are released from rest simultaneously and allowed to slide down, one along AB and other along AC as shown. Neglecting frictional effects, the quantities that are conserved as beads slides down are **[IIT-JEE 2000]**

(A) angular velocity and total energy (kinetic and potential)

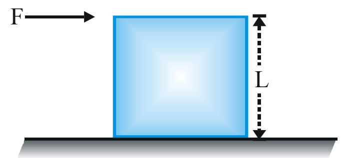
(B) total angular momentum and total energy

(C) angular velocity and moment of inertia about the axis of rotation

(D) total angular momentum and moment of inertia about the axis of rotation

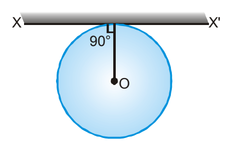


6. A cubical block of side L rests on a rough horizontal surface with coefficient of friction μ. A horizontal force F is applied on the block as shown. If the coefficient of friction is sufficiently high, so that the block does not slide before toppling, the minimum force required to topple the block is :- **[IIT-JEE 2000]**



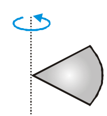
(A) infinitesimal **(B)** $\frac{mg}{4}$ **(C)** $\frac{mg}{2}$ **(D)** $mg(1-\mu)$

7. A thin wire of length L and uniform linear mass density ρ is bent into a circular loop with centre at O as shown. The moment of inertia of the loop about the axis XX' is :- **[IIT-JEE 2000]**



(A) $\frac{\rho L^3}{8\pi^2}$ **(B)** $\frac{\rho L^3}{16\pi^2}$ **(C)** $\frac{5\rho L^3}{16\pi^2}$ **(D)** $\frac{3\rho L^3}{8\pi^2}$

8. One quarter section is cut from a uniform circular disc of radius R. This section has a mass M. It is made to rotate about a line perpendicular to its plane and passing through the centre of the original disc. Its moment of inertia about the axis of rotation is :- **[IIT-JEE 2001]**

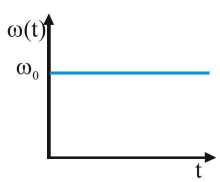


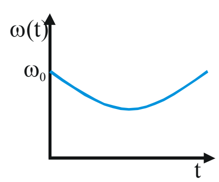
(A) $\frac{1}{2}MR^2$ **(B)** $\frac{1}{4}MR^2$ **(C)** $\frac{1}{8}MR^2$ **(D)** $\sqrt{2}MR^2$

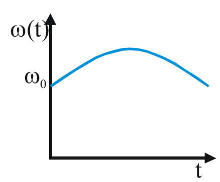
9. A cylinder rolls up an inclined plane and reaches some height and then rolls down (without slipping throughout these motions). The directions of the frictional force acting on the cylinder are :- **[IIT-JEE 2002]**

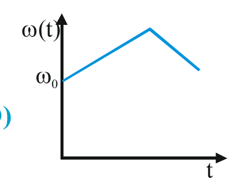
(A) up the incline while ascending and down the incline while descending
(B) up the incline while ascending as well as descending
(C) down the incline while ascending and up the incline while descending
(D) down the incline while ascending as well as descending.

10. A circular platform is free to rotate in a horizontal plane about a vertical axis passing through its centre. A tortoise is sitting at the edge of the platform. Now the platform is given an angular velocity ω_0. When the tortoise move along a chord of the platform with a constant velocity (with respect to the platform). The angular velocity of the platform $\omega(t)$ will vary with time t as :- **[IIT-JEE 2002]**

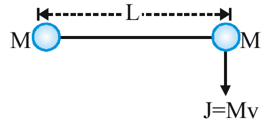
(A)


(B)


(C)


(D)


11. Consider a body, shown in figure, consisting of two identical balls, each of mass M connected by a light rigid rod. If an impulse J = Mv is imparted to the body at one of its end, what would be its angular velocity :-



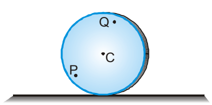
(A) $\frac{v}{L}$ **(B)** $\frac{2v}{L}$ **(C)** $\frac{v}{3L}$ **(D)** $\frac{v}{4L}$ **[IIT-JEE 2003]**

12. A particle undergoes uniform circular motion. About which point on the plane of the circle, will the angular momentum of the particle remain conserved ? **[IIT-JEE 2003]**

(A) Centre of circle **(B)** On the circumference of the circle

(C) Inside the circle **(D)** Outside the circle

13. A disc is rolling (with slipping) on a horizontal surface. C is its centre and Q and P are two points equidistant from C. Let v_P, v_Q and v_C be the magnitude of velocities of points P, Q and C respectively, then :-**[IIT-JEE 2004]**



(A) $v_Q > v_C > v_P$ **(B)** $v_Q < v_C < v_P$ **(C)** $v_Q = v_P$, $v_C = \frac{1}{2}v_P$ **(D)** $v_Q < v_C > v_P$

14. A child is standing with folded hands at the centre of a platform rotating about its central axis. The kinetic energy of the system is K and moment of inertia is I. The child now stretches his arms so that the moment of inertia of the system doubles. The kinetic energy of the system now is :- **[IIT-JEE 2004]**

(A) 2K **(B)** $\frac{K}{2}$ **(C)** $\frac{K}{4}$ **(D)** 4K

15. A particle moves in a circular path with decreasing speed. Choose the correct Statement :

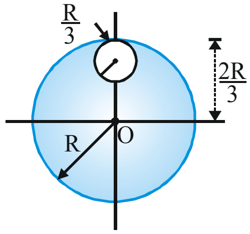
(A) Angular momentum remains constant **[IIT-JEE 2005]**

(B) Acceleration ($\vec{a}$) is towards the centre

(C) Particle moves in a spiral path with decreasing radius

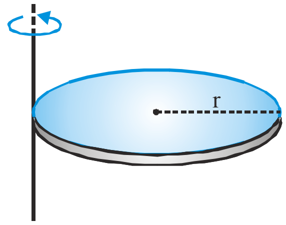
(D) The direction of angular momentum remains constant

16. From a circular disc of radius R and mass 9M, a small disc of radius $\frac{R}{3}$ is removed from the disc. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through O is :- **[IIT-JEE 2005]**



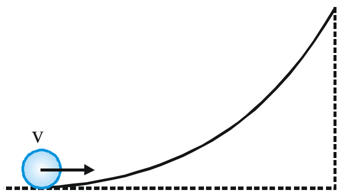
(A) $4MR^2$ **(B)** $\frac{40}{9}MR^2$ **(C)** $10MR^2$ **(D)** $\frac{37}{9}MR^2$

17. A solid sphere of radius R has moment of inertia I about its geometrical axis. If it is melted into a disc of radius r and thickness t. If it's moment of inertia about the tangential axis (which is perpendicular to plane of the disc), is also equal to I, then the value of r is equal to :- **[IIT-JEE 2006]**



(A) $\frac{2}{\sqrt{15}}R$ **(B)** $\frac{2}{\sqrt{5}}R$ **(C)** $\frac{3}{\sqrt{15}}R$ **(D)** $\frac{\sqrt{3}}{\sqrt{15}}R$

18. A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches up to a maximum height of $\frac{3v^2}{4g}$ with respect to the initial position. The object is :- **[IIT-JEE 2007]**



(A) ring **(B)** solid sphere **(C)** hollow sphere **(D)** disc

19. A block of base 10 cm × 10 cm and height 15 cm is kept on an inclined plane. The coefficient of friction between them is $\sqrt{3}$. The inclination θ of this inclined plane from the horizontal plane is gradually increased from 0°. Then :- **[IIT-JEE 2009]**

(A) at θ =30°, the block will start sliding down the plane
(B) the block will remain at rest on the plane up to certain θ and then it will topple
(C) at θ=60°, the block will start sliding down the plane and continue to do so at higher angles
(D) at θ=60°, the block will start sliding down the plane and on further increasing θ, it will topple at certain θ

20. If the resultant of the external forces acting on a system of particles is zero, then from an inertial frame, one can surely say that **[IIT-JEE 2009]**

(A) linear momentum of the system does not change in time
(B) kinetic energy of the system does not change in time
(C) angular momentum of the system does not change in time
(D) potential energy of the system does not change in time

MCQ's with one or more than one correct answers

1. The torque $\vec{\tau}$ on a body about a given point is found to be equal to $\vec{A} \times \vec{L}$, where $\vec{A}$ is a constant vector and $\vec{L}$ is the angular momentum of the body about that point. From this it follows that :-**[IIT-JEE 1998]**

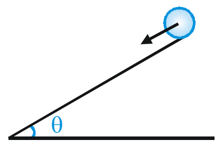
(A) $\frac{d\vec{L}}{dt}$ is perpendicular to $\vec{L}$ at all instants of time

(B) the component of $\vec{L}$ in the direction of $\vec{A}$ does not change with time

(C) the magnitude of $\vec{L}$ does not change with time

(D) $\vec{L}$ does not change with time

2. A solid sphere is in pure rolling motion on an inclined surface having inclination θ :- **[IIT-JEE 2006]**



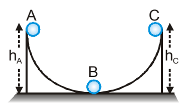
(A) frictional force acting on sphere is $f = \mu\ mg\cos\theta$

(B) f is disspative force

(C) friction will increase its angular velocity and decreases its linear velocity

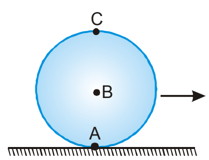
(D) if θ decrease, friction will decrease

3. A ball moves over a fixed track as shown in the figure. From A to B the ball rolls without slipping. If surface BC is frictionless and K_A, K_B and K_C are kinetic energy of the ball at A, B and C respectively, then:- **[IIT-JEE 2006]**



(A) $h_A > h_C$; $K_B > K_C$ **(B)** $h_A > h_C$; $K_C > K_A$ **(C)** $h_A = h_C$; $K_B = K_C$ **(D)** $h_A < h_C$; $K_B > K_C$

4. A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure, A is the point of contact, B is the centre of the sphere and C is its topmost point. Then **[IIT-JEE 2009]**



(A) $\vec{v}_C - \vec{v}_A = 2(\vec{v}_B - \vec{v}_C)$ **(B)** $\vec{v}_C - \vec{v}_B = \vec{v}_B - \vec{v}_A$

(C) $|\vec{v}_C - \vec{v}_A| = 2|\vec{v}_B - \vec{v}_C|$ **(D)** $|\vec{v}_C - \vec{v}_A| = 4|\vec{v}_B|$

Assertion-Reason

1. **Statement–I :** Two cylinders, one hollow (metal) and the other solid (wood) with the same mass and identical dimensions are simultaneously allowed to roll without slipping down an inclined plane from the same height. The hollow cylinder will reach the bottom of the inclined plane first.

Statement–2 : By the principle of conservation of energy, the total kinetic energies of both the cylinders are identical when they reach the bottom of the incline.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 is **NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

Comprehension Based Questions

Comprehension #1

Two discs A and B are mounted coaxially on a vertical axle. The discs have moments of inertia I and 2I respectively about the common axis. Disc A is imparted an initial angular velocity 2ω using the entire potential energy of a spring compressed by a distance x_1. Disc B is imparted an angular velocity ω by a spring having the same spring constant and compressed by a distance x_2. Both the discs rotate in the clockwise direction. **[IIT-JEE 2007]**

1. The ratio $\frac{x_1}{x_2}$ is :-

(A) 2 **(B)** $\frac{1}{2}$ **(C)** $\sqrt{2}$ **(D)** $\frac{1}{\sqrt{2}}$

2. When disc B is brought in contact with disc A, they acquire a common angular velocity in time t. The average frictional torque on one disc by the other during this period is :-

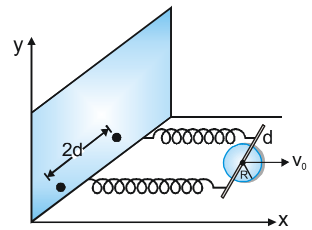
(A) $\frac{2I\omega}{3t}$ **(B)** $\frac{9I\omega}{2t}$ **(C)** $\frac{9I\omega}{4t}$ **(D)** $\frac{3I\omega}{2t}$

3. The loss of kinetic energy during the above process is :-

(A) $\frac{I\omega^2}{2}$ **(B)** $\frac{I\omega^2}{3}$ **(C)** $\frac{I\omega^2}{4}$ **(D)** $\frac{I\omega^2}{6}$

Comprehension #2

A uniform thin cylindrical disk of mass M and radius R is attached to two identical massless springs of spring constant k which are fixed to the wall as shown in the figure. The springs are attached to the axle of the disk symmetrically on either side at a distance d from its centre. The axle is massless and both the springs and the axle are in a horizontal plane. The unstretched length of each spring is L. The disk is initially at its equilibrium position with its centre of mass (CM) at a distance L from the wall. The disk rolls without slipping with velocity $\vec{v}_0 = v_0\hat{i}$. The coefficient of friction is μ. **[IIT-JEE 2008]**



1. The net external force acting on the disk when its centre of mass is at displacement x with respect to its equilibrium position is

(A) $-kx$ **(B)** $-2kx$ **(C)** $-\frac{2kx}{3}$ **(D)** $-\frac{4kx}{3}$

2. The centre of mass of the disk undergoes simple harmonic motion with angular frequency ω equal to

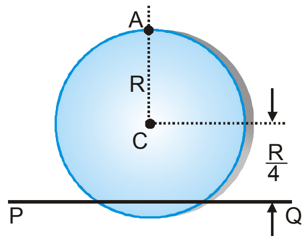
(A) $\sqrt{\frac{k}{M}}$ **(B)** $\sqrt{\frac{2k}{M}}$ **(C)** $\sqrt{\frac{2k}{3M}}$ **(D)** $\sqrt{\frac{4k}{3M}}$

3. The maximum value of v_0 for which the disk will roll without slipping is

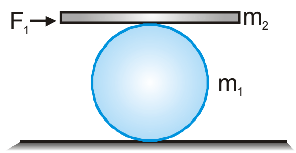
(A) $\mu g\sqrt{\frac{M}{k}}$ **(B)** $\mu g\sqrt{\frac{M}{2k}}$ **(C)** $\mu g\sqrt{\frac{3M}{k}}$ **(D)** $\mu g\sqrt{\frac{5M}{2k}}$

Subjective Questions

1. A uniform circular disc has radius R and mass m. A particle, also of mass m, is fixed at a point A on the edge of the disc as shown in the figure. The disc can rotate freely about a horizontal chord PQ that is at a distance $\frac{R}{4}$ from the centre C of the disc. The line AC is perpendicular to PQ. Initially the disc is held vertical with the point A at its highest position. It is then allowed to fall, so that it starts rotation about PQ. Find the linear speed of the particle as it reaches its lower position. **[IIT-JEE 1998]**



2. A man pushes a cylinder of mass m_1 with the help of a plank of mass m_2 as shown. There is no slipping at any contact. The horizontal component of the force applied by the man is F, Find :



(i) the acceleration of the plank and the centre of mass of the cylinder and

(ii) the magnitude and directions of frictional forces at contact points. **[IIT-JEE 1999]**

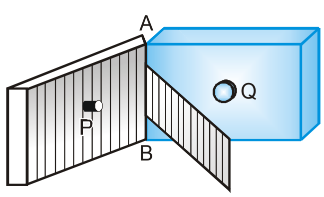
3. A rod AB of mass M and length L is lying on a horizontal frictionless surface. A particle of mass m travelling along the surface hits the end A of the rod with a velocity v_0 in a direction perpendicular to AB. The collision is elastic. After the collision the particle comes to rest. **[IIT-JEE 2000]**

(i) Find the ratio $\frac{m}{M}$

(ii) A point P on the rod is at rest immediately after collision. Find the distance AP.

(iii) Find the linear speed of the point P after a time $\frac{\pi L}{3V_0}$ after the collision.

4. Two heavy metallic plates are joined together at 90° to each other. A laminar sheet of mass 30 kg is hinged at the line AB joining the two heavy metallic plates. The hinges are frictionless. The moment of inertia of the laminar sheet about an axis parallel to AB and passing through its centre of mass is 1.2 kg-m^2. Two rubber obstacles P and Q are fixed, one on each metallic plate at a distance 0.5 m from the line AB. This distance is chosen, so that the reaction due to the hinges on the laminar sheet is zero during the impact. Initially the laminar sheet hits one of the obstacles with an angular velocity 1 rad/s and turns back. If the impulse on the sheet due to each obstacle is 6 N-s. **[IIT-JEE 2001]**

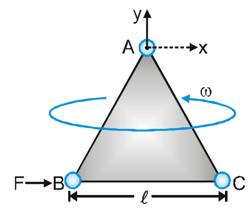


(i) Find the location of the centre of mass of the laminar sheet from AB.

(ii) At what angular velocity does the laminar sheet come back after the first impact.

(iii) After how many impact, does the laminar sheet come to rest.

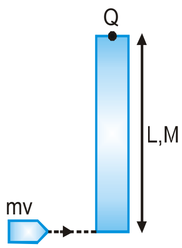
5. Three particles A, B and C each of mass m, are connected to each other by three massless rigid rods to form a rigid, equilateral triangular body of side ℓ. This body is placed on a horizontal frictionless table (x-y plane) and is hinged to it at the point A, so that it can move without friction about the vertical axis through A (see figure). The body is set into rotational motion on the table about A with a constant angular velocity ω. **[IIT-JEE 2002]**



(i) Find the magnitude of the horizontal force exerted by the hinge on the body.

(ii) At time T, when the side BC is parallel to the x-axis, a force F is applied on B along BC (as shown). Obtain the x-component and the y-component of the force exerted by the hinge on the body, immediately after time T.

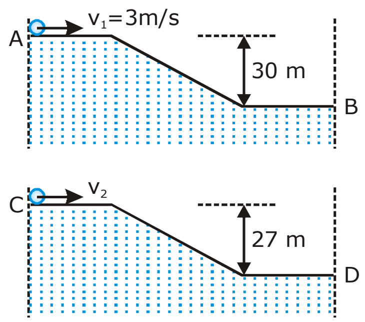
6. A rod of length L and mass M is hinged at point O. A small bullet of mass m hits the rod as shown in the figure. The bullet gets embedded in the rod. Find angular velocity of the system just after impact. **[IIT-JEE 2005]**



7. A solid cylinder rolls without slipping on an inclined plane inclined at an angle θ. Find the linear acceleration of the cylinder. Mass of the cylinder is M. **[IIT-JEE 2005]**

8. A uniform circular disc of mass 50 kg and radius 0.4 m is rotating with an angular velocity of 10 rad s^{-1} about its own axis, which is vertical. Two uniform circular rings, each of mass 6.25 kg and radius 0.2 m, are gently placed symmetrically on the disc in such a manner that they are touching each other along the axis of the disc and are horizontal. Assume that the friction is large enough such that the rings are at rest relative to the disc and the system rotates about the original axis. The new angular velocity (in rad s^{-1}) of the system is **[IIT-JEE 2013]**

9. Two identical uniform discs roll without slipping on two different surface AB and CD (see figure) starting at A and C with linear speeds v_1 and v_2, respectively, and always remain in contact with the surfaces. If they reach B and D with the same linear speed and $v_1 = 3$ m/s, then v_2 in m/s is ($g = 10$ m/s^2) **[IIT-JEE 2015]**



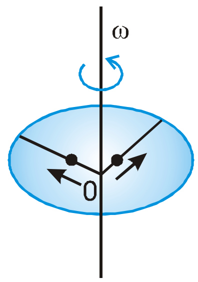
10. A ring of mass M and radius R is rotating with angular speed ω about a fixed vertical axis passing through its centre O with two point masses each of mass $\frac{M}{8}$ at rest at O. These masses can move radially outwards along two massless rods fixed on the ring as shown in the figure. At some instant the angular speed of the system is $\frac{8}{9}\omega$ and one of the masses is at a distance of $\frac{3}{5}$R from O. At this instant the distance of the other mass from O is. **[IIT-JEE 2015]**

(A) $\frac{2}{3}R$

(B) $\frac{1}{3}R$

(C) $\frac{3}{5}R$

(D) $\frac{4}{5}R$



11. The densities of two solid spheres A and B of the same radii R vary with radial distance r as $\rho_A(r) = k\left(\frac{r}{R}\right)$ and $\rho_B(r) = k\left(\frac{r}{R}\right)^5$, respectively, where k is a constant. The moments of inertia of the individual spheres about axes passing through their centres are I_A and I_B, respectively. If $\frac{I_B}{I_A} = \frac{n}{10}$, the value of n is - **[IIT-JEE 2015]**

12. Two thin circular discs of mass ma and 4m, having radii of a and 2a, respectively, are rigidly fixed be a massless,, rigid rod of length $l = \sqrt{24}a$ through their center. This assembly is laid on a firm and flat surface, and set rolling without slipping on the surface so that the angular speed about the axis of the rod is ω. The angular momentum of the entire assembly about the point 'O' is $\vec{L}$ (see the figure). Which of the following statement(s) is(are) true?

[IIT-JEE 2016]

(A) The centre of mass of the assembly rotates about the z-axis with an angular speed of ω/5

(B) The magnitude of angular momentum of center of mass of the assembly about the point O is 81 $ma^2\omega$

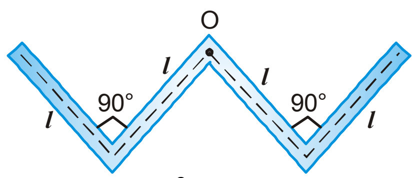
(C) The magnitude of angular momentum of the assembly about its center of mass is 17 $ma^2\omega/2$

(D) The magnitude of the z-component of $\vec{L}$ is 55 $ma^2\omega$.

MOCK TEST

SECTION - I : STRAIGHT OBJECTIVE TYPE

1. A uniform thin rod of length 4 l, mass 4m is bent at the points as shown in the figure. What is the moment of inertia of the rod about the axis passing through point O & perpendicular to the plane of the paper.

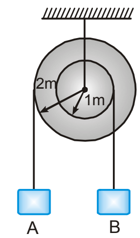


(A) $\frac{m\ell^2}{3}$ (B) $\frac{10m\ell^2}{3}$ (C) $\frac{m\ell^2}{12}$ (D) $\frac{m\ell^2}{24}$

2. Two points of a rod move with velocities 3 v & v perpendicular to the rod and in the same direction, separated by a distance 'r'. Then the angular velocity of the rod is:

(A) $\frac{3v}{r}$ (B) $\frac{4v}{r}$ (C) $\frac{5v}{r}$ (D) $\frac{2v}{r}$

3. In the pulley system shown, if radii of the bigger and smaller pulley are 2 m and 1 m respectively and the acceleration of block A is 5 m/s^2 in the downward direction, then the acceleration of block B will be:



(A) 0 m/s^2 (B) 5 m/s^2
(C) 10 m/s^2 (D) 5/2 m/s^2

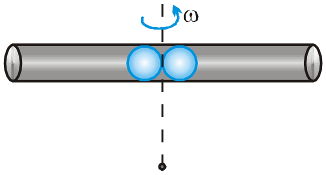
4. A uniform thin rod of mass 'm' and length L is held horizontally by two vertical strings attached to the two ends. One of the string is cut. Find the angular acceleration soon after it is cut :

(A) $\frac{g}{2L}$ (B) $\frac{g}{L}$ (C) $\frac{3g}{2L}$ (D) $\frac{2g}{L}$

5. A sphere is released on a smooth inclined plane from the top. When it moves down its angular momentum is:

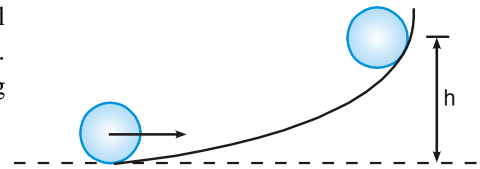
(A) conserved about every point
(B) conserved about the point of contact only
(C) conserved about the centre of the sphere only
(D) conserved about any point on a line parallel to the inclined plane and passing through the centre of the ball.

6. A smooth tube of certain mass is rotated in gravity free space and released. The two balls shown in the figure move towards ends of the tube. For the whole system which of the following quantity is not conserved?



(A) Angular momentum (B) Linear momentum
(C) Kinetic energy (D) Angular speed

7. In the figure shown a ball rolls without sliding. On a horizontal surface. It ascends a curved track upto height h and returns. Value of h is h_1 for sufficiently rough curved track to avoid sliding and h_2 for smooth curved track, then:



(A) $h_1 = h_2$ **(B)** $h_1 < h_2$
(C) $h_1 > h_2$ **(D)** $h_2 = 2\,h_1$

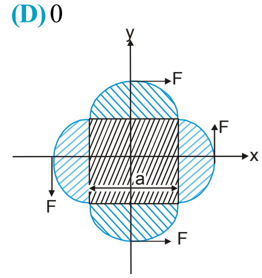
8. A uniform ring of radius R rolls without sliding with a constant velocity. The radius of curvature of the path followed by any particle of the ring at the highest point of its path will be :

(A) R **(B)** 2 R **(C)** 4 R **(D)** none of these

9. A uniform ring of radius R is given a back spin of angular velocity $V_0/2R$ and thrown on a horizontal rough surface with velocity of center to be V_0. The velocity of the centre of the ring when it starts pure rolling will be

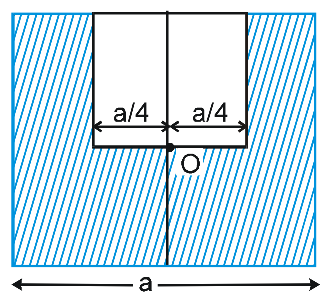
(A) $V_0/2$ **(B)** $V_0/4$ **(C)** $3V_0/4$ **(D)** 0

10. A planar object made up of a uniform square plate and four semicircular discs of the same thickness and material is being acted upon by four forces of equal magnitude as shown in figure. The coordinates of point of application of forces is given by



(A) (0, a) **(B)** (0, –a) **(C)** (a, 0) **(D)** (–a, 0)

11. A square plate of edge a/2 is cut out from a uniform square plate of edge 'a' as shown in figure. The mass of the remaining portion is M. The moment of inertia of the shaded portion about an axis passing through 'O' (centre of the square of side a) and perpendicular to plane of the plate is :

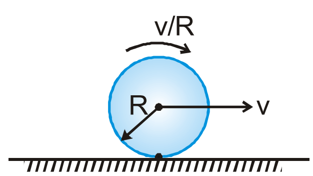


(A) $\dfrac{9}{64}Ma^2$ **(B)** $\dfrac{3}{16}Ma^2$

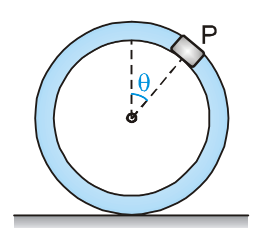
(C) $\dfrac{5}{12}Ma^2$ **(D)** $\dfrac{Ma^2}{6}$

12. A uniform disc is performing pure rolling on a smooth stationary surface with constant angular velocity as shown in figure. At any instant, for the lowermost point of the disc

(A) Velocity is v, acceleration is zero
(B) Velocity is zero, acceleration is zero
(C) velocity is v, acceleration is $\dfrac{v^2}{R}$.
(D) velocity is zero, acceleration is $\dfrac{v^2}{R}$

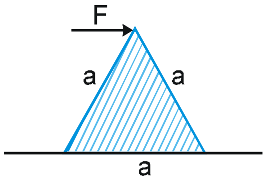


13. A small block of mass ‘m’ is rigidly attached at ‘P’ to a ring of mass ‘3m’ and radius ‘r’. The system is released from rest at θ = 90° and rolls without sliding. The angular acceleration of hoop just after release is –



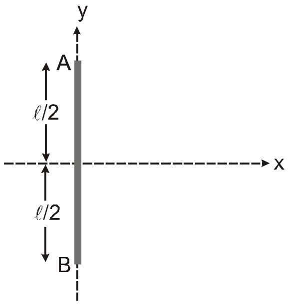
(A) $\dfrac{g}{4r}$ **(B)** $\dfrac{g}{8r}$ **(C)** $\dfrac{g}{3r}$ **(D)** $\dfrac{g}{2r}$

14. A uniform equilateral prism of mass m rests on a rough horizontal surface with coefficient of friction μ. A horizontal force F is applied on the prism as shown in the figure. If the coefficient of friction is sufficiently high so that the prism does not slide before toppling, then the minimum force required to topple the prism is :



(A) $\frac{mg}{\sqrt{3}}$ **(B)** $\frac{mg}{4}$ **(C)** $\frac{\mu mg}{\sqrt{3}}$ **(D)** $\frac{\mu mg}{4}$

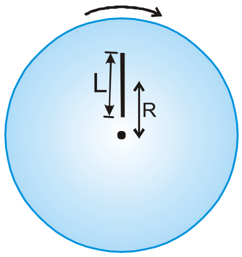
15. A uniform rod of mass m, length ℓ is placed over a smooth horizontal surface along y-axis and is at rest as shown in figure. An impulsive force F is applied for a small time Δt along x-direction at point A. The x–coordinate of end A of the rod when the rod becomes parallel to x–axis for the first time is (initially the coordinate of centre of mass of the rod is (0, 0)) :



(A) $\frac{\pi\ell}{12}$ **(B)** $\frac{\ell}{2}\left(1+\frac{\pi}{12}\right)$

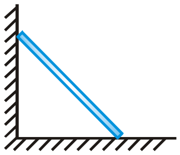
(C) $\frac{\ell}{2}\left(1-\frac{\pi}{6}\right)$ **(D)** $\frac{\ell}{2}\left(1+\frac{\pi}{6}\right)$

16. A uniform rod of mass M and length L lies radially on a disc rotating with angular speed ω in a horizontal plane about its axis. The rod does not slip on the disc and the centre of the rod is at a distance R from the centre of the disc. Then the kinetic energy of the rod is :



(A) $\frac{1}{2}m\omega^2\left(R^2+\frac{L^2}{12}\right)$ **(B)** $\frac{1}{2}m\omega^2 R^2$ **(C)** $\frac{1}{24}m\omega^2 L^2$ **(D)** None of these

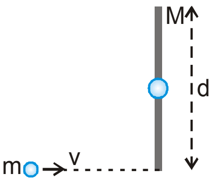
17. A uniform rod of length ℓ is sliding such that one of its ends is always in contact with a vertical wall and its other end is always in contact with horizontal surface. Just after the rod is released from rest, the magnitude of acceleration of end points of the rod are a and b respectively. The angular acceleration of rod at this instant will be



(A) $\frac{a+b}{\ell}$ **(B)** $\frac{\sqrt{|a^2-b^2|}}{\ell}$

(C) $\frac{\sqrt{a^2+b^2}}{\ell}$ **(D)** None of these

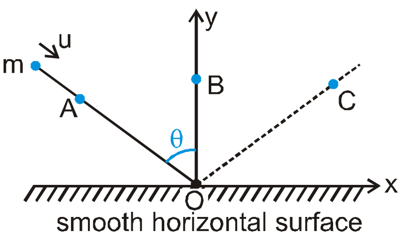
18. A particle of mass m is moving at speed v perpendicular to a rod of length d and mass M = 6m which pivots around a frictionless axle running through its centre. It strikes and sticks to the end of the rod. The moment of inertia of the rod about its centre is $Md^2/12$. Then the angular speed of the system just after the collision is



(A) 2v/3d. **(B)** 2v/d. **(C)** v/d. **(D)** 3v/2d.

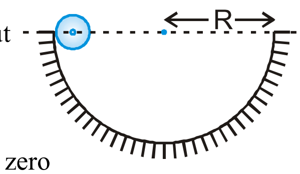
19. A particle falls freely near the surface of the earth. Consider a fixed point O (not vertically below the particle) on the ground. Then pickup the incorrect alternative
(A) Angular momentum of the particle about O is increasing.
(B) Torque of the gravitational force on the particle about O is decreasing.
(C) The moment of inertia of the particle about O is decreasing.
(D) The angular velocity of the particle about O is increasing.

20. A ball of mass m moving with constant velocity u collides with a smooth horizontal surface at O as shown. Neglect gravity and friction. The y-axis is drawn normal to the horizontal surface at the point of impact O and x-axis is horizontal as shown in the figure. About which point will the angular momentum of ball be conserved.



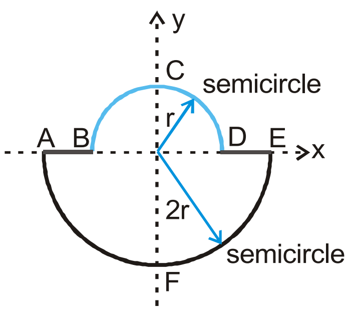
(A) point A
(B) point B
(C) point C
(D) None of these

21. In the figure shown, a small solid spherical ball of mass 'm' can move without sliding in a fixed semicircular track of radius R in vertical plane. It is released from the top. The resultant force on the ball at the lowest point of the track is



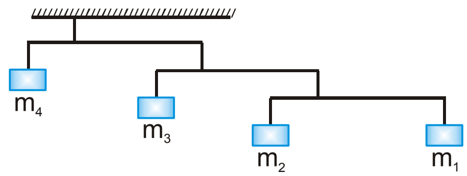
(A) $\frac{10mg}{7}$ **(B)** $\frac{17mg}{7}$ **(C)** $\frac{3mg}{7}$ **(D)** zero

22. A uniform thin rod is bent in the form of closed loop ABCDEFA as shown in the figure.



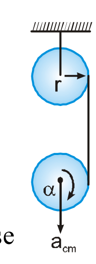
The ratio of moment of inertia of the loop about x-axis to that about y-axis is.
(A) > 1 **(B)** < 1 **(C)** = 1 **(D)** = 1/2

23. Figure shows an arrangement of masses hanging from a ceiling. In equilibrium, each rod is horizontal, has negligible mass and extends three times as far to the right of the wire supporting it as to the left. If mass m_4 is 48 kg then mass m_1 is equal to



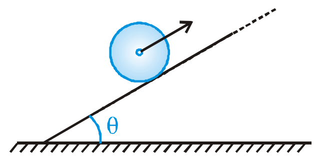
(A) 1 kg **(B)** 2 kg **(C)** 3 kg **(D)** 4 kg

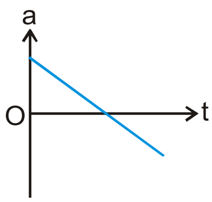
24. Two identical uniform discs of mass m and radius r are arranged as shown in the figure. If α is the angular acceleration of the lower disc and a_{cm} is acceleration of centre of mass of the lower disc, then relation among a_{cm}, α & r is :



(A) $a_{cm} = \frac{\alpha}{r}$ **(B)** $a_{cm} = 2\alpha r$ **(C)** $a_{cm} = \alpha r$ **(D)** none of these

25. A uniform solid sphere rolls up (without slipping) the rough fixed inclined plane, and then back down. Which is the correct graph of acceleration 'a' of centre of mass of solid sphere as function of time t (for the duration sphere is on the incline) ? Assume that the sphere rolling up has a positive velocity.



(A)


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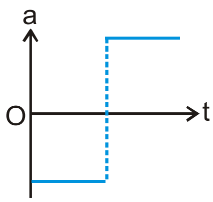

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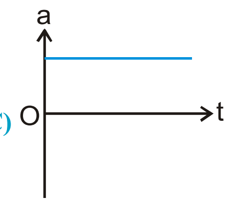

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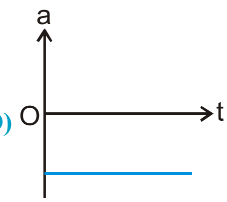
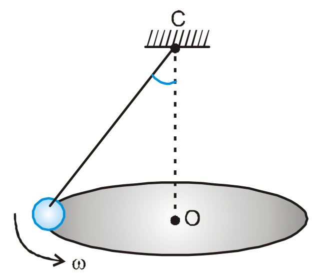
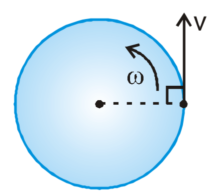


26. A conical pendulum consists of a simple pendulum moving in a horizontal circle as shown in the figure. C is the pivot, O the centre of the circle in which the pendulum bob moves and ω the constant angular velocity of the bob. If $\vec{L}$ is the angular momentum about point C, then



(A) $\vec{L}$ is constant
(B) only direction of $\vec{L}$ is constant
(C) only magnitude of $\vec{L}$ is constant
(D) none of the above.

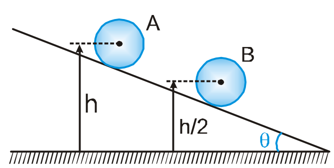
27. A child with mass m is standing at the edge of a playground merry-go-round (A large uniform disc which rotates in horizontal plane about a fixed vertical axis in parks) with moment of inertia I, radius R, and initial angular velocity ω as shown in figure. The child jumps off the edge of the merry-go-round with a velocity v with respect to the ground in direction tangent to periphery of the disc as shown. The new angular velocity of the merry-go-round is:



(A) $\sqrt{\dfrac{I\omega^2 - mv^2}{I}}$
(B) $\sqrt{\dfrac{(I + mR^2)\omega^2 - mv^2}{I}}$

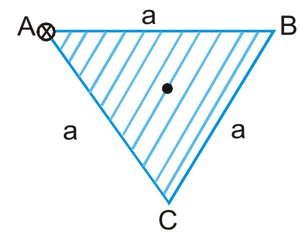
(C) $\dfrac{I\omega - mvR}{I}$
(D) $\dfrac{(I + mR^2)\omega - mvR}{I}$

28. Two identical uniform solid spherical balls A & B of mass m each are placed on a fixed wedge as shown in figure. Ball B is kept at rest and it is released just before two balls collides. Ball A rolls down without slipping on inclined plane & collide elastically with ball B. The kinetic energy of ball A just after the collision with ball B is :



(A) $\dfrac{mgh}{7}$
(B) $\dfrac{mgh}{2}$
(C) $\dfrac{2mgh}{5}$
(D) $\dfrac{7mgh}{5}$

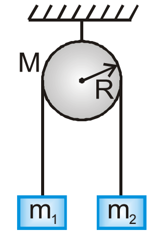
29. A uniform triangular plate ABC of moment of mass m and inertia I (about an axis passing through A and perpendicular to plane of the plate) can rotate freely in the vertical plane about point 'A' as shown in figure. The plate is released from the position shown in the figure. Line AB is horizontal. The acceleration of centre of mass just after the release of plate is :



(A) $\frac{mga^2}{\sqrt{3}\,I}$ **(B)** $\frac{mga^2}{4I}$ **(C)** $\frac{mga^2}{2\sqrt{3}\,I}$ **(D)** $\frac{mga^2}{3I}$

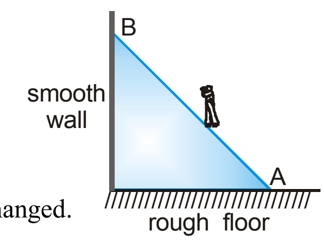
SECTION - II : MULTIPLE CORRECT ANSWER TYPE

30. In the shown figure, the pulley of mass M and radius R can rotate about its fixed horizontal axis (axle) without friction. Friction between light inextensible string and pulley is sufficient to prevent slipping of string over pulley. The masses of blocks are m_1 and m_2 such that $m_2 > m_1$. The system is initially released from rest as shown. Before the block of mass m_1 touches the pulley, pick up the correct statements.



(A) The magnitude of acceleration of any small length $d\ell$ of string is constant throughout the motion.
(B) Magnitude of force exerted by string on mass m_2 is larger as compared to that exerted by string on mass m_1.
(C) Accelerations of both blocks are same.
(D) The acceleration of small length $d\ell$ of string in contact with block of mass m_2 remains constant.

31. A ladder AB is supported by a smooth vertical wall and rough horizontal floor as shown. A boy starts moving from A to B slowly. The ladder remains at rest, then pick up the correct statement(s) :

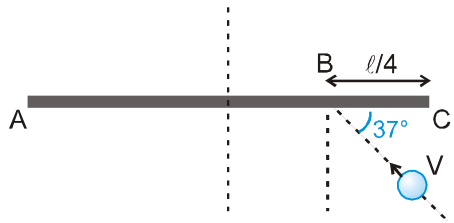


(A) Magnitude of normal reaction by wall on ladder at point B will increase.
(B) Magnitude of normal reaction by wall on ladder at point B will decrease.
(C) Magnitude of normal reaction by floor on ladder at point A will remain unchanged.
(D) Magnitude of friction force by floor on ladder at point A will increase.

32. A rigid body is in pure rotation, that is, undergoing fixed axis rotation. Then which of the following statement(s) are true?
(A) You can find two points in the body in a plane perpendicular to the axis of rotation having same velocity.
(B) You can find two points in the body in a plane perpendicular to the axis of rotation having same acceleration.
(C) Speed of all the particles lying on the curved surface of a cylinder whose axis coincides with the axis of rotation is same.
(D) Angular speed of the body is same as seen from any point in the body.

33. A uniform disc of mass 2kg and radius 1m is mounted on an axle supported on fixed frictionless bearings. A light cord is wrapped around the rim of the disc and mass of 1kg is tied to the free end. If it is released from rest,
(A) the tension in the cord is 5N
(B) in first 4 seconds the angular displacement of the disc is 40 rad.
(C) the work done by the torque on the disc in first 4 sec. is 200J
(D) the increase in the kinetic energy of the disc in the first 4 seconds is 200J.

34. A uniform rod AC of length ℓ and mass m is kept on a horizontal smooth plane. It is free to rotate and move. A particle of same mass m moving on the plane with velocity v strikes the rod at point B making an angle 37^0 with the rod. The collision is elastic. After collision :



(A) The angular velocity of the rod will be $\frac{72}{55}\frac{v}{\ell}$

(B) The centre of the rod will travel a distance $\frac{\pi\ell}{3}$ in the time in which it makes half rotation

(C) Impulse of the impact force is $\frac{24mv}{55}$

(D) None of these

SECTION - III : ASSERTION AND REASON TYPE

35. **Statement-1 :** A uniform rigid disc rolls without slipping on a fixed rough horizontal surface with uniform angular velocity. Then the acceleration of lowest point on the disc is zero.

Statement-2 : For a uniform rigid disc rolling without slipping on a fixed rough horizontal surface, the velocity of the lowest point on the disc is always zero.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 is **NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True.

36. **Statement-1 :** A uniform thin rod of length L is hinged about one of its end and is free to rotate about the hinge without friction. Neglect the effect of gravity. A force F is applied at a distance x from the hinge on the rod such that force is always perpendicular to the rod. As the value of x is increased from zero to L, the component of reaction by hinge on the rod perpendicular to length of rod increases.

Statement-2 : Under the conditions given in statement-1 as x is increased from zero to L, the angular acceleration of rod increases.

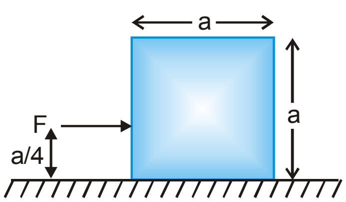
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 is **NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True.

37. **Statement-1 :** A uniform cubical block(of side a) undergoes translational motion of a smooth horizontal surface under action of horizontal force F as shown. Under the given condition, the horizontal surface exerts normal reaction non-uniformly on lower surface of the block.



Statement-2 : For the cubical block given in statement-1, the horizontal force F has tendency to rotate the cube about its centre in clockwise sense. Hence, the lower right edge of cube presses the horizontal surface harder in comparison to the force exerted by lower left edge of cube on horizontal surface.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True.

38. **STATEMENT–1 :** A uniform solid sphere and a uniform hollow sphere of same radius and same material are released (at rest) from the top of a fixed inclined plane at the same time. They will reach the bottom simultaneously, if they roll with sliding.

STATEMENT–2: In the situation of statement-1, the centres of both spheres have the same acceleration and they travel the same distance. Hence time taken is same.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

39. **Statement-1 :** The net momentum of a system of two moving particles is zero. Then at a particular instant of time, the net angular momentum of system of given two particle is same about any point.

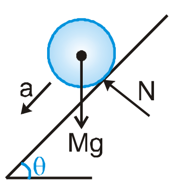
Statement-2 : If net momentum of a system of two moving particle is zero, then angular momentum of system of given two particles is zero about any point.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

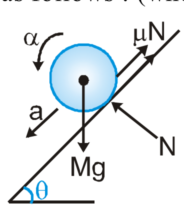
SECTION - IV : COMPREHENSION TYPE

Comprehension # 1

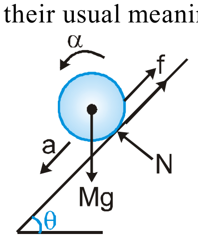
In this passage a brief idea is given of the motion of the rolling bodies on an inclined plane.
We will consider three cases : Objects are released on an incline plane
Case A : which is smooth. ;
Case B : where friction is insufficient to provide pure rolling.
Case C : where friction is sufficient to provide pure rolling.
Force diagram for three cases are as follows : (where symbols have their usual meanings)



Case (A)
$\alpha = 0$



Case (B)
$a \neq \alpha R$



Case (C)
$a = \alpha R$

Equations for case (C) :

$Mg\sin\theta - f = Ma$

$fR = (Mk^2)\alpha$; where k = radius of gyration and f is force of friction.

$a = \alpha R$

on solving the above equations we will get

$$a = \frac{g\sin\theta}{\left(1+\frac{k^2}{R^2}\right)}$$

Object	Ring	Disc	Hollow sphere	Solid sphere
k	R	$\frac{R}{\sqrt{2}}$	$\sqrt{\frac{2}{3}}R$	$\sqrt{\frac{2}{5}}R$

To decide the minimum friction coefficient to provide pure rolling put

$$f = \mu\, Mg\cos\theta$$

And we will get $\mu_{min} = \dfrac{\tan\theta\left(\dfrac{k^2}{R^2}\right)}{\left(1+\dfrac{k^2}{R^2}\right)}$

Equations for case (B) :

$Mg\sin\theta - \mu N = Ma$

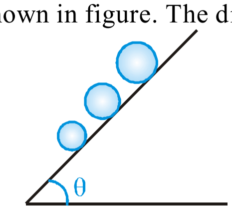
$\mu NR = Mk^2\alpha$

$N = Mg\cos\theta$

The K.E. of a rolling body can be expressed as :

$$K.E. = \frac{1}{2}MV_{CM}^2 + \frac{1}{2}I_{CM}\omega^2$$

40. Three solid uniform spheres are released on an inclined plane as shown in figure. The distance between the spheres remains constant during motion in :



(A) all three cases
(B) case 'A' & 'B'
(C) only case 'C'
(D) depends on the mass of the spheres.

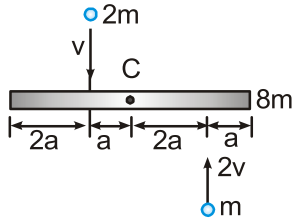
41. We have four objects : a solid sphere, a hollow sphere, a ring & a disc, all of same radius. When these are released on an inclined plane, it may happen that all of them do not perform pure rolling. But from the information of pure rolling, if one object can be confirmed to be purely rolling then it can be said that rest all will perform pure rolling. This object whose pure rolling confirms pure rolling of all other objects is :
(A) hollow sphere **(B)** solid sphere **(C)** ring **(D)** disc

42. If the four objects given in the above question are of same mass, same radius having the same friction coefficient & are released from the same height, then at the bottom the object which will have least kinetic energy for case 'B' will be then :-
(A) hollow sphere **(B)** solid sphere **(C)** ring **(D)** disc

43. Two children 'A' & 'B' use bicycles, having wheels of ring type and disc type respectively. During a race, bicycles are given the same velocity from the bottom of the inclined bridge to ascend the bridge without pedalling, then (assuming pure rolling) :
(A) both bicycles with reach upto same height
(B) bicycle of child 'A' will reach a greater height
(C) bicycle of child B will reach a greater height
(D) depends on the masses of bicycles and the child

Comprehension # 2

A uniform bar of length 6 a & mass 8 m lies on a smooth horizontal table. Two point masses m & 2 m moving in the same horizontal plane with speeds 2 v and v respectively strike the bar as shown in the figure & stick to the bar after collision.



44. Velocity of the centre of mass of the system is

(A) $\frac{v}{2}$ (B) v (C) $\frac{2v}{3}$ (D) Zero

45. Angular velocity of the rod about centre of mass of the system is

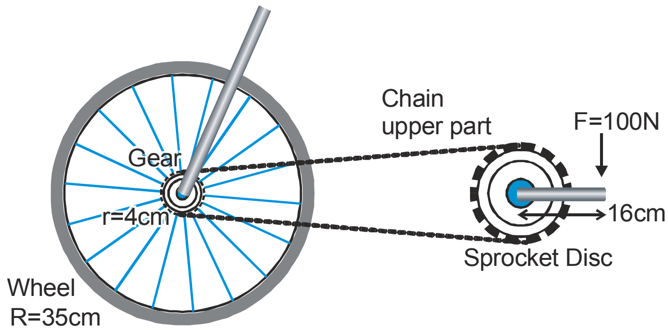
(A) $\frac{v}{5a}$ (B) $\frac{v}{15a}$ (C) $\frac{v}{3a}$ (D) $\frac{v}{10a}$

46. Total kinetic energy of the system, just after the collision is

(A) $\frac{3}{5}mv^2$ (B) $\frac{3}{25}mv^2$ (C) $\frac{3}{15}mv^2$ (D) $3\ mv^2$

Comprehension # 3

A bicycle has pedal rods of length 16 cm connected to a sprouted disc of radius 10 cm. The bicycle wheels are 70 cm in diameter and the chain runs over a gear of radius 4 cm. The speed of the cycle is constant and the cyclist applies 100 N force that is always perpendicular to the pedal rod, as shown in the figure. Assume tension in the lower part of chain is negligible. The cyclist is peddling at a constant rate of two revolutions per second. Assume that the force applied by other foot is zero when one foot is exerting 100 N force. Neglect friction within cycle parts & the rolling friction.



47. The tension in the upper portion of the chain is equal to

(A) 100 N (B) 120 N (C) 160 N (D) 240 N

48. Net torque on the rear wheel of the bicycle is equal to

(A) zero (B) 16 N-m (C) 6.4 N-m (D) 4.8 N-m

49. The power delivered by the cyclist is equal to

(A) 280 W (B) 100 W (C) 64 πW (D) 32 W

50. The speed of the bicycle is

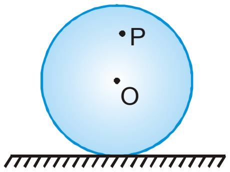
(A) 6.4 π m/s (B) 3.5 π m/s (C) 2.8 π m/s (D) 5.6 π m/s

51. The net force of the friction on the rear wheel due to the road is :

(A) 100 N (B) 62 N (C) 32.6 N (D) 18.3 N

SECTION - V : MATRIX - MATCH TYPE

52. A uniform disc rolls without slipping on a rough horizontal surface with uniform angular velocity. Point O is the centre of disc and P is a point on disc as shown in figure. In each situation of column I a statement is given and the corresponding results are given in column-II. Match the statements in column-I with the results in column-II.



Column I

(A) The velocity of point P on disc

(B) The acceleration of point P on disc

(C) The tangential acceleration of point P on disc

(D) The acceleration of point on disc which is in contact with rough horizontal surface

Column II

(P) Changes in magnitude with time

(Q) is always directed from that point (the point on disc given in column-I) towards centre of disc.

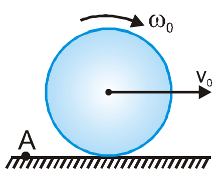
(R) is always zero

(S) is non-zero and remains constant in magnitude

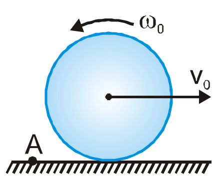
(T) Changes in direction with time.

53. In each situation of column-I, a uniform disc of mass m and radius R rolls on a rough fixed horizontal surface as shown. At t=0 (initially) the angular velocity of disc is ω_o and velocity of centre of mass of disc is v_o(in horizontal direction). The relation between v_o and ω_o for each situation and also initial sense of rotation is given for each situation in column-I . Then match the statements in column-I with the corresponding results in column-II .

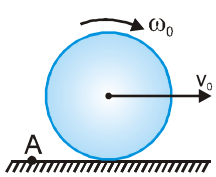
Column-I

(A) 

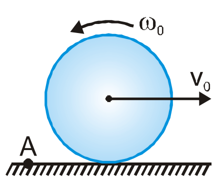
($v_o > R\,\omega_o$)

(B) 

($v_o > R\,\omega_o$)

(C) 

($v_o < R\,\omega_o$)

(D) 

($v_o < R\,\omega_o$)

Column-II

(P) The angular momentum of disc about point A (as shown in figure) remains conserved.

(Q) The kinetic energy of disc after it starts rolling without slipping is less than its initial kinetic energy.

(R) In the duration disc rolls with slipping, the friction acts on disc towards left.

(S) Before rolling starts acceleration of the disc remains constant in magnitude and direction.

(T) Final angular velocity is independent of friction coefficient between disc and the surface.

54. Consider a system of particles (it may be rigid or non rigid). In the column-I some condition on force and torque is given. Column-II contains the effects on the system. (Letters have usual meaning)

Column-I	Column-II
(A) $\vec{F}_{res} = 0$	(P) $\vec{P}_{system}$ will be constant
(B) $\vec{\tau}_{res} = 0$	(Q) $\vec{L}_{system}$ will be constant
(C) External force is absent	(R) total work done by all forces will be zero
(D) No non conservative force acts.	(S) total mechanical energy will be constant.
	(T) total work done by all the forces may be non zero.

SECTION - VI : INTEGER TYPE

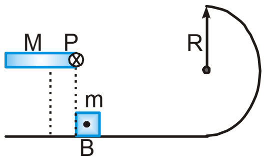
55. A uniform rod of length 75 cm is hinged at one of its ends and is free to rotate in vertical plane. It is released from rest when rod is horizontal. When the rod becomes vertical, it is broken at mid–point and lower part now moves freely. The distance of centre of lower part from hinge, when it again becomes vertical for the first time is r. Find the approximate value of 2r.

56. A solid homogeneous cylinder of height h=1m and base radius r=1m is kept vertically on a conveyer belt moving horizontally with an increasing velocity $v = 1 + t^2$. If the cylinder is not allowed to slip find the time when the cylinder is about to topple.

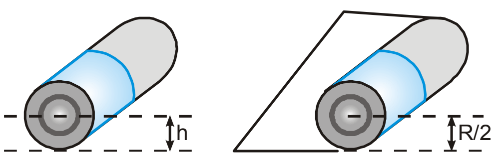
57. A billiard ball at rest is struck horizontally one tenth of the diameter below the top. If P be the linear impulse of the blow find the initial kinetic energy of the ball is $\frac{xp^2}{10M}$ then x is given by the mass of the ball is being M.

58. In the figure shown a block B of mass m can slide on a fixed horizontal smooth plane. A uniform solid sphere A of radius r of the same mass rolls without sliding on the block B. Find the angular acceleration of the sphere $\frac{xg}{r}$ then x is.

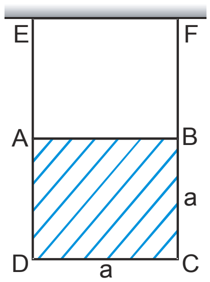
59. A uniform rod of length R and mass M is free to rotate about a horizontal axis passing through hinge P as in figure. First it is taken aside such that it becomes horizontal and then released. At the lowest point the rod hits the block B of mass m and stops. if ratio of mass of rod M to the mass of block m such that the block B completes the circle $\sqrt{x}$ then x is. Neglect any friction.



60. A carpet of mass M made of inextensible material is rolled along its length in the form of a cylinder of radius R and is kept on a rough floor. The carpet starts unrolling without sliding on the floor when a negligibly small push is given to it. the horizontal velocity of the axis of the cylindrical part of the carpet when its radius reduces to R/2.is $\sqrt{\frac{xgR}{3}}$ ms^{-1} then x is



61. A thin uniform square plate ABCD of side 'a' and mass m is suspended in vertical plane as shown in the figure. AE and BF are two massless inextensible strings. The line AB is horizontal. Find the tension in the string AE just after BF is cut is $\frac{2mg}{x}$ then x is.



ANSWER KEY

EXERCISE - 1

1. B **2.** A **3.** A **4.** D **5.** C **6.** B **7.** A **8.** C **9.** A **10.** B **11.** B **12.** A **13.** B
14. D **15.** C **16.** B **17.** C **18.** D **19.** B **20.** D **21.** C **22.** C **23.** C **24.** B **25.** C **26.** B
27. D **28.** B **29.** B **30.** B **31.** D **32.** B **33.** D **34.** C **35.** C **36.** D **37.** A **38.** C **39.** B
40. B **41.** C **42.** B **43.** A **44.** C **45.** B **46.** D **47.** C **48.** B **49.** C **50.** D **51.** C **52.** B
53. C **54.** C **55.** B **56.** C

EXERCISE - 2 : PART # I

1. B **2.** D **3.** C **4.** A **5.** A **6.** C **7.** A **8.** B **9.** D **10.** C **11.** C **12.** A **13.** B
14. D **15.** D **16.** A **17.** B **18.** D **19.** B **20.** D **21.** B **22.** B **23.** B **24.** A **25.** A **26.** D
27. B,C,D **28.** A **29.** A **30.** A,C,D **31.** A,C **32.** B,C **33.** A,B,C **34.** A,B,C **35.** A,C,D

PART # II

1. D **2.** B **3.** B **4.** D **5.** D **6.** D **7.** D **8.** A **9.** A **10.** A **11.** D

EXERCISE - 3 : PART # I

1. $A \to Q$; $B \to S$; $C \to Q$; $D \to Q$
2. $A \to P,Q,R$; $B \to P,Q,R$; $C \to P,Q$; $D \to P,Q,R$
3. $A \to Q,T,U$; $B \to Q,T,U$; $C \to Q,T,U$
4. $A \to Q$; $B \to R$; $C \to P$; $D \to R$
5. $A \to S$; $B \to R$; $C \to R$
6. $A \to Q$; $B \to P$; $C \to S$; $D \to R$

PART # II

Comprehension # 1 : **1.** D
Comprehension # 2 : **1.** C **2.** B
Comprehension # 3 : **1.** B **2.** A
Comprehension # 4 : **1.** D **2.** B
Comprehension # 5 : **1.** A **2.** C
Comprehension # 6 : **1.** C **2.** B
Comprehension # 7 : **1.** C **2.** A
Comprehension # 8 : **1.** B **2.** A
Comprehension # 9 : **1.** C **2.** A **3.** B
Comprehension # 10 : **1.** A,B,C,D **2.** B,C
Comprehension # 11 : **1.** A **2.** D **3.** D
Comprehension # 12 : **1.** A **2.** C **3.** B **4. C**
Comprehension # 13 : **1.** A,B,C **2.** C **3.** B,C,D **4.** A,C,D

EXERCISE - 4

1. $4MR^2$ **2.** $13\lambda\ell^3$ **3.** $32M\ell^2$ **4. (i)** $\frac{12v}{7\ell}$ **(ii)** 3.5 m/s **5.** $\sqrt{\frac{14Rg}{3}}$ **6. (i)** 1.63 N **(ii)** 1.22 m

7. $\frac{1}{2\sqrt{2}}$ **8.** 6.3 m/s **9. (i)** $6\,\hat{i}$ N **(ii)** $0.6\left(-\hat{j} \pm \hat{k}\right)$, 0.85 Nm **10.** $\frac{7}{13}$ **11.** To maximize the torque

12. 12 rad/s^2 **13.** $\frac{(M-m)g}{(M+m)+\frac{I}{R^2}}$ **14.** $\frac{6F}{5m}$ **15.** 106.7 rad/s

17. (i) 0 **(ii)** $\frac{v}{5a}$ anticlockwise **(iii)** $E = \frac{3mv^2}{5}$ **18.** $\sqrt{3g\ell}$ **19.** $\frac{ML^2}{\left(L^2 + 12d^2\right)}$ **20.** 1.2 second

21. $\sqrt{\frac{8g\ell}{7}}$ **22. (i)** $\theta = \cos^{-1}\left(\frac{4}{7}\right)$ **(ii)** $\sqrt{\frac{4gR}{7}}$ **(iii)** 6 **23.** $\left(\frac{1}{\pi}\right)$ m down to the centre of mass

24. 2 m/s, 1.5 m/s, 16.67 N

25. **(i)** $\frac{4}{7}mg\sqrt{1+\left(\frac{7\ell\omega^2}{4g}\right)^2}$ **(ii)** $\sqrt{\left(\frac{3g}{7}\right)^2+\left(\ell^2\omega\right)^2}$ **(iii)** $\left(\frac{6g}{7}\right)+\ell\omega^2,\left(\frac{13}{7}mg+m\ell\omega^2\right)$ **(iv)** $\sqrt{\frac{6g}{7\ell}}$

26. $\frac{1}{2}\left(\frac{2}{5}mr^2+m\ell^2\right)\omega_0^2+\frac{1}{2}\left(\frac{2}{5}mr^2\right)\omega^2$ where $\omega=\frac{v}{r}=\frac{\ell\omega_0}{r}$ **27.** **(i)** $\frac{11}{9}$ kg **(ii)** 11 m/s

28. **(i)** $\frac{V+v}{\ell}$ **(ii)** $\frac{Mv-mv}{M+m}$ **(iii)** $\left(\frac{V+v}{\ell}\right)\left(\frac{\ell}{\ell_0}\right)^2$ **(iv)** $\frac{1}{2}\frac{Mm}{M+m}(V+v)^2\left[1-\frac{\ell}{\ell_0}\right]$ **29.** $2m\omega^3r^2$

EXERCISE - 5 : PART # I

1. 3 **2.** 1 **3.** 2 **4.** 4 **5.** 1 **6.** 4 **7.** 2 **8.** 2 **9.** 3 **10.** 4 **11.** 4 **12.** 3 **13.** 2
14. 3 **15.** 3 **16.** 4 **17.** 3 **18.** 1 **19.** 1 **20.** 3 **21.** 4 **22.** 4 **23.** 4 **24.** 4 **25.** 2 **26.** 4
27. 4

PART # II

MCQ's with one correct answer

1. A **2.** A **3.** C **4.** C **5.** B **6.** C **7** . D **8.** A **9.** B **10.** C **11.** A **12.** A **13.** A
14. B **15.** D **16.** A **17.** A **18.** D **19.** B **20.** A

MCQ's with one or more than one correct answer

1. A,B,C **2.** C,D **3.** A,B **4.** B,C

Assertion-Reasoning

1. D

Comprehension Based questions

Comprehension # 1 : **1.** C **2.** A **3.** B **Comprehension # 2 :** **1.** D **2.** D **3.** C

Subjective Questions

1. $\sqrt{5gR}$ **2.** **(i)** $a_{cm}=\frac{4F}{3m_1+8m_2}$, $a_{plank}=\frac{8F}{3m_1+8m_2}$ **(ii)** $\frac{3m_1F}{(3m_1+8m_2)}$, $\frac{m_1F}{(3m_1+8m_2)}$

3. **(i)** $\frac{1}{4}$ **(ii)** $\frac{2}{3}L$ **(iii)** $\frac{v_0}{2\sqrt{2}}$ **4.** **(i)** 0.1 m **(ii)** 1 rad/s **(iii)** Laminar sheet will never come to rest

5. **(i)** $\sqrt{3}m\omega^2\ell$ **(ii)** $F_x=\frac{F}{4}$, $F_y=\sqrt{3}\ m\omega^2\ell$ **6.** $\frac{3mv}{(3m+M)L}$ **7.** $\frac{2g\sin\theta}{3}$ **8.** 8 **9.** 7 **10.** D **11.** 6

12. D

MOCK TEST

1.	B	**2.**	D	**3.**	D	**4.**	C	**5.**	D	**6.**	D	**7.**	C
8.	C	**9.**	B	**10.**	B	**11.**	B	**12.**	D	**13**	B	**14.**	A
15.	D	**16.**	A	**17.**	C	**18.**	A	**19.**	B	**20.**	B	**21.**	A
22.	B	**23.**	A	**24.**	B	**25**	D	**26.**	C	**27.**	D	**28.**	A
29.	C	**30.**	B,D	**31.**	A,C,D	**32.**	C,D	**33.**	A,B,C,D	**34.**	A, B, C	**35.**	D
36.	D	**37.**	C	**38.**	A	**39.**	C	**40.**	A	**41.**	C	**42.**	C
43.	B	**44.**	D	**45.**	A	**46.**	A	**47.**	C	**48.**	A	**49.**	C
50.	B	**51.**	D										

52. A → P,T ; B → Q,S,T ; C → P, T ; D → Q,S

53. A → P,Q,R, S, T ; B → P,Q,R,S, T ; C → P,Q, S,T ; D → P,Q,R, S, T

54. A → P ; B → Q; C → P,Q ; D → S

55.	5	**56.**	1	**57.**	13	**58.**	10	**59.**	15	**60.**	14	**61.**	5