HINTS & SOLUTIONS

EXERCISE - 1

Single Choice

2. $f(x) = \sin^{-1}\left(\dfrac{1+x^3}{2x^{3/2}}\right) + \sqrt{\sin(\sin x)} + \log_{(3\{x\}+1)}(x^2+1)$

Domain : $3\{x\} + 1 \neq 1$ or $0 \Rightarrow \quad x \notin I$

and $\quad -1 \leq \dfrac{1+x^3}{2x^{3/2}} \leq 1$

$-2x^{3/2} \leq 1 + x^3 \leq 2x^{3/2}$

$1 + x^3 + 2x^{3/2} \geq 0$

$(1 + x^{3/2})^2 \geq 0 \quad \Rightarrow \quad x \in R$

$1 + x^3 - 2x^{3/2} \leq 0 \quad$ or $\quad (1 - x^{3/2})^2 \leq 0$

or $\quad 1 - x^{3/2} = 0 \quad$ or $\quad x = 1$

Hence domain $x \in \phi$

6. $f(x) = \sqrt{\log\dfrac{(5x - x^2)}{6}}$

$\log\dfrac{5x - x^2}{6} \geq 0$

$\Rightarrow \quad \dfrac{5x - x^2}{6} \geq 1 \quad \Rightarrow \quad x^2 - 5x + 6 \leq 0$

$\Rightarrow \quad (x-2)(x-3) \leq 0 \quad \Rightarrow \quad 2 \leq x \leq 3$

So domain $\in [2, 3]$

7. (i) $f(x) = \sqrt{x-1} + 2\sqrt{3-x}$

D : $x - 1 \geq 0$ & $3 - x \geq 0 \quad \Rightarrow \quad x \in [1, 3]$

Range : $f'(x) = \dfrac{1}{2\sqrt{x-1}} - \dfrac{1}{\sqrt{3-x}} = 0$

or $\quad f'(x) = 0$ at $x = \dfrac{7}{5}$

$f'\left(\dfrac{7^-}{5}\right) > 0 \quad \& \quad f'\left(\dfrac{7^+}{5}\right) < 0$

$\Rightarrow \quad$ maxima at $x = \dfrac{7}{5}$

Range : $\left[\sqrt{2}, \sqrt{10}\right]$

9. $f(x) = (\sin^{-1}x + \cos^{-1}x)^3 - 3\sin^{-1}x\cos^{-1}x(\sin^{-1}x + \cos^{-1}x)$

$= \dfrac{\pi^3}{8} - 3\sin^{-1}x\left(\dfrac{\pi}{2} - \cos^{-1}x\right)\dfrac{\pi}{2}$

$= \dfrac{\pi^3}{8} - \dfrac{3\pi^2}{4}\sin^{-1}x + 3\dfrac{\pi}{2}(\sin^{-1}x)^2$

$= \dfrac{\pi^3}{8} + \dfrac{3\pi}{2}\left[(\sin^{-1}x)^2 - \dfrac{\pi}{2}\sin^{-1}x + \dfrac{\pi^2}{16}\right] - \dfrac{3\pi^3}{32}$

$= \dfrac{\pi^3}{32} + \dfrac{3\pi}{2}\left(\sin^{-1}x - \dfrac{\pi}{4}\right)^2$

maximum value of f(x) at $x = -1$

$f_{maximum} = \dfrac{\pi^3}{32} + \dfrac{3\pi}{2} \times \dfrac{9\pi^3}{16} = \dfrac{7\pi^3}{8}$

11. $f(x) = {}^{7-x}P_{x-3}$

For domain

$7 - x \geq 0, \quad \& \quad x - 3 \geq 0 \quad \& \quad 7 - x \geq x - 3$

$x \leq 7, \quad \& \quad x \geq 3 \quad \& \quad 2x \leq 10 \Rightarrow x \leq 5$

$x \in \{3, 4, 5\}$

Range $\in \{f(3), f(4), f(5)\}$

Range $\in \{1, 3, 2\}$

14. $f(xy + 1) = f(yx + 1)$

$f(x)f(y) - f(y) - x + 2 = f(y)f(x) - f(x) - y + 2$

$f(x) - f(y) = x - y$

Putting $\quad y = 0$

$f(x) - 1 = x - 0$

$f(x) = x + 1$

16. (A) $\quad f(x) = e^{1/2\,\ell n\,x} = \sqrt{x}, \quad D : x > 0$

$g(x) = \sqrt{x}$, $D : x \geq 0$

(B) $\quad \tan^{-1}(\tan x) = x \quad D : x \neq \pm(2n+1)\dfrac{\pi}{2}$

$\cot^{-1}(\cot x) = x \quad D : x \neq \pm n\pi$

(C) $f(x) = \cos^2x + \sin^4x = \cos^2x + (1 - \cos^2x)^2$

$= 1 - \cos^2x + \cos^4x = \sin^2x + \cos^4x$

$g(x) = \sin^2x + \cos^4x$

(D) $f(x) = \dfrac{|x|}{x}$, $D : x \neq 0$

$g(x) = \text{sgn}(x)$, $D : x \in R$

19. $f(x+1) - f(x) = 8x + 3$

$f(0+1) - f(0) = 3$ (put $x = 0$)

$\Rightarrow$ $(b + c + d) - d = 3$

$\Rightarrow$ $b + c = 3$ ....**(i)**

Also $f(-1+1) - f(-1) = -8 + 3$ (put $x = -1$)

$\Rightarrow$ $f(0) - f(-1) = -5 \Rightarrow d - (b - c + d) = -5$

$\Rightarrow$ $-b + c = -5$ ....**(ii)**

from **(i)** and **(ii)**

$b = 4$, $c = -1$

20. $f(x) = \dfrac{4a-7}{3}x^3 + (a-3)x^2 + x + 5$

$f'(x) = (4a - 7)x^2 + 2(a - 3)x + 1$

$D \leq 0$ for all $x \in R$

$4(a-3)^2 - 4(4a-7) \leq 0$

$a^2 + 9 - 6a - 4a + 7 \leq 0$

$a^2 - 10a + 16 \leq 0$

$(a - 8)(a - 2) \leq 0$ or $a \in [2, 8]$

$f'(x)$ is always +ve for $a \in [2, 8]$

22. $f(x) = x - [x] + (x+1) - [x+1] + ...$

$(x + 99) - [x + 99]$

$= x - [x] + x - [x] + ...+ x - [x]$

$= 100(x - [x]) = 100\{x\}$

$f(\sqrt{2}) = 100\{\sqrt{2}\} = 41$

25. $f\left(x+\frac{1}{3}\right) = \left[x+\frac{1}{3}\right] + \left[x+\frac{2}{3}\right] + [x+1] - 3\left(x+\frac{1}{3}\right) + 15$

$= \left[x+\frac{1}{3}\right] + \left[x+\frac{2}{3}\right] + [x] - 3x + 15$

$= f(x)$

$\therefore$ fundamental period is 1/3

26. $f(x) = |x - 1|$ $f : R^+ \to R$

$g(x) = e^x$, $g : [-1, \infty) \to R$

$fog(x) = f[g(x)] = |e^x - 1|$

$D : [-1, \infty)$

$R : [0, \infty)$

27. **Hint :** $f(x) = \dfrac{e^{|x|} - e^{-x}}{e^x + e^{-x}} = \begin{cases} \dfrac{e^x - e^{-x}}{e^x + e^{-x}} & x > 0 \\ 0 & x \leq 0 \end{cases}$

and $\dfrac{e^x - e^{-x}}{e^x + e^{-x}} > 0 \ \forall\, x > 0$

29. $f : R \to R$, $f(x) = x^3 + ax^2 + bx + c$

$f'(x) = 3x^2 + 2ax + b$

$D \leq 0$ or $4a^2 - 12b \leq 0$

or $a^2 \leq 3b$

31. $f(x) = \sin\sqrt{[a]}\ x$

period of $\sin x = 2\pi$

$\Rightarrow$ period of $f(x) = \dfrac{2\pi}{\sqrt{[a]}} = \pi$

$\Rightarrow \sqrt{[a]} = 2 \Rightarrow [a] = 4 \Rightarrow a \in [4, 5)$

33. Put $y = -x$, we get $f(x) = -x$ also $f(0) = 0$

$f(x + y) = f(x) + f(y)$ is an odd function so it is symmetric about origin.

34. $f(x+1) + f(x+3) = K \ \forall x$

put $x = -1$

$f(0) + f(2) = K$ .....**(i)**

put $x = 1$ $f(2) + f(4) = K$ .....**(ii)**

from **(i) & (ii)**

$f(4) = f(0) = 0 \Rightarrow$ period $= 4$

36. $f(x) = 2^{x(x-1)}$

It is one-one onto function

$\log_2 y = x(x-1)$

$\Rightarrow$ $x^2 - x - \log_2 y = 0$

$x = \dfrac{1 \pm \sqrt{1 + 4\log_2 y}}{2}$

$f^{-1}(x) = \dfrac{1 + \sqrt{1 + 4\log_2 x}}{2}$

EXERCISE - 2

Part # I : Multiple Choice

3. $f(x) = \sin \log\left(\dfrac{\sqrt{4-x^2}}{1-x}\right)$

$4 - x^2 > 0$ or $x \in (-2, 2)$

and $\dfrac{\sqrt{4-x^2}}{1-x} > 0$

D : $(-2, 1)$

R : $[-1, 1]$

8. $f(x) = \sin\left[\log\left(\dfrac{\sqrt{4-x^2}}{1-x}\right)\right]$

f(x) will be defined if $\dfrac{\sqrt{4-x^2}}{1-x} > 0$ & $4 - x^2 > 0$

$\Rightarrow$ $-2 < x < 1$ & $-\infty < \log\dfrac{\sqrt{4-x^2}}{1-x} < \infty$

$-1 \le \sin\left[\log\left(\dfrac{\sqrt{4-x^2}}{1-x}\right)\right] < 1$

So range of f(x) is $[-1, 1]$

13. put $x = 1$

$2f(1) + 1f(1) - 2f(|\sqrt{2}\sin\frac{5\pi}{4}|) = -1$

$\Rightarrow$ $3f(1) - 2f(1) = -1$ $\Rightarrow$ $f(1) = -1$

Now put x = 2

$2f(2) + 2f\left(\frac{1}{2}\right) - 2f(1) = 4\cos^2\pi + 2\cos\frac{\pi}{2}$

$\Rightarrow$ $2f(2) + 2f\left(\frac{1}{2}\right) - 2f(1) = 4$

$\Rightarrow$ $f(2) + f\left(\frac{1}{2}\right) = 1$ ....(i)

Now put x = 1/2 we get

$4f\left(\frac{1}{2}\right) + f(2) = 1$ ....(ii)

from (i) and (ii)

$f\left(\frac{1}{2}\right) = 0$ & $f(2) = 1$

14. $\log_{x^2}(x) \ge 0$ & $x > 0,\ x \ne \pm 1$

$\therefore$ $x \in (0, 1) \cup (1, \infty)$

16. **(A)** $f(x) = e^{\ell n(\sec^{-1} x)} = \sec^{-1}x$,

$x \in (-\infty, -1] \cup (1, \infty)$

$g(x) = \sec^{-1}x$, $x \in (-\infty, -1] \cup [1, \infty)$

non-identical functions

(B) $f(x) = \tan(\tan^{-1}x) = x,\ x \in R$

$g(x) = \cot(\cot^{-1}x) = x,\ x \in R$

identical functions

(C) $f(x) = \text{sgn}(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$

$\Rightarrow$ $g(x) = \text{sgn}(\text{sgn}\, x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$

Identical functions

(D) $f(x) = \cot^2 x \cdot \cos^2 x$,

$x \in R - \{n\pi\}$, $n \in I$

$g(x) = \cot^2 x - \cos^2 x = \cot^2 x(1 - \sin^2 x)$

$= \cot^2 x \cdot \cos^2 x$

$x \in R - \{n\pi\}$, $n \in I$

Identical functions

17. $f(x) = \dfrac{1-x}{1+x}$, $0 \le x \le 1$

$g(x) = 4x(1-x), 0 \le x \le 1$

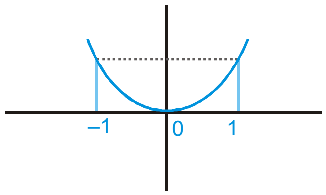
$fog(x) = \dfrac{1-g(x)}{1+g(x)} = \dfrac{1-4x(1-x)}{1+4x(1-x)} = \dfrac{1-4x+4x^2}{1+4x-4x^2}$

$gof(x) = 4f(x) \cdot (1 - f(x))$

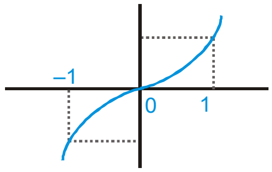
$= 4\left(\dfrac{1-x}{1+x}\right)\left(1 - \left(\dfrac{1-x}{1+x}\right)\right) = \dfrac{8x(1-x)}{(1+x)^2}$

20. Domain $D \in [-1, 1]$

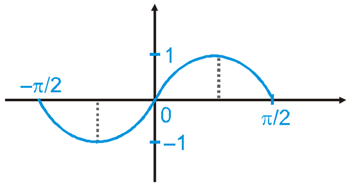
(A) $f(x) = x^2$ many -one



(B) $g(x) = x^3$ one - one



(C) $h(x) = \sin 2x$ many - one



(D) $k(x) = \sin\left(\frac{\pi x}{2}\right)$ one-one function

23. $f : R \to [-1, 1]$

$$f(x) = \sin\left(\frac{\pi}{2}[x]\right) = \begin{cases} -1 & , \quad -1 \le x < 0 \\ 0 & , \quad 0 \le x < 1 \\ 1 & , \quad 1 \le x \le 2 \end{cases}.$$

Many - one function into function

Also $f(x+4) = \sin\left(\frac{\pi}{2}[x+4]\right)$

$$= \sin\left(2\pi + \frac{\pi}{2}[x]\right) = \sin\left(\frac{\pi}{2}[x]\right)$$

$= f(x)$ and hence periodic

EXERCISE - 3

Part # II : Comprehension

Comprehension # 3

$f(2-x) = f(2+x)$

& $f(20-x) = f(x)$

$\Rightarrow$ $f(2-(2-x)) = f(4-x)$

& $f(20-(x+16)) = f(x+16)$

$\Rightarrow$ $f(x) = f(4-x)$

& $f(4-x) = f(x+16)$

$\Rightarrow$ $f(x) = f(x+16)$

1. $f(0) = f(4) = f(16)$

no. of values of $x = 22$

2. If graph is symmetric about $x = a$ then

$f(a+x) = f(a-x)$

$f(16) = f(20)$ $\Rightarrow$ symmetric about $x = 18$

$f(4) = f(32)$

3. $f(0) = f(1) = f(2) = f(3) = f(4) = f(5) = f(6)$

Hence period can't be one.

EXERCISE - 4
Subjective Type

1. **(i)** $[-1, 1]$ **(ii)** R

(iii) $\bigcup_{n\in I} \left[n\pi, n\pi + \frac{\pi}{4}\right]$

(iv) $(2, 3)$

(v) $\frac{f}{g}(x) = \frac{\sqrt{x^2 - 5x + 4}}{x+3}$

$\Rightarrow$ $x^2 - 5x + 4 \geq 0$

$(x-4)(x-1) \geq 0$

also $x \neq -3$

So $x \in (-\infty, -3) \cup (-3, 1] \cup [4, \infty)$

(vi) $\frac{1}{[x]}$ $\Rightarrow$ $x \notin [0, 1)$

and $\log_{1-\{x\}}(x^2 - 3x + 10)$

$x^2 - 3x + 10 > 0$ $\Rightarrow$ $x \in R$

$1 - \{x\} > 0$ $\Rightarrow$ $x \in R$

$1 - \{x\} \neq 1$ $\Rightarrow$ $x \notin I$

and $2 - |x| > 0$ $\Rightarrow$ $|x| - 2 < 0$

$\Rightarrow$ $x \in (-2, 2)$

and $\sec(\sin x) > 0$

$\Rightarrow$ $-1 \leq \sin x \leq 1$ $\Rightarrow$ $x \in R$

$x \in (-2, -1) \cup (-1, 0) \cup (1, 2)$

2. **(i)** $(-\infty, 1]$ **(ii)** R^+

(iii) $\left[\frac{1}{3}, 1\right]$ **(iv)** $\left(-\infty, -\frac{1}{4}\right] \cup \left[-\frac{1}{20}, \infty\right)$

(v) $\left[\frac{1}{3}, 3\right]$ **(vi)** $\left[0, \frac{3}{\sqrt{2}}\right]$

(vii) $[4, \infty)$ **(viii)** $[-11, 16]$

(ix) $\left[\frac{3}{4}, 1\right]$

3. $f(3) = 1$

$f(3x) = x + f(3x - 3)$

put $x = 1$

$f(3) = 1 + f(0)$

$f(0) = 0$

$f(6) = 2 + f(3) = 3$

$f(9) = 3 + f(6) = 3 + 3 = 6$

$f(12) = 4 + 6 = 10$

Hence $f(300) = 1 + 3 + 6 + 1 + \ldots\ldots$ 100^{th} term

$S = 1 + 3 + 6 + 10 \ldots\ldots\ldots + T_n$

$S = 1 + 3 + 6 + \ldots\ldots + T_n$

$T_n = 1 + 2 + 3 + 4 \ldots\ldots\ldots\ldots$ up 100 term

$= \frac{100}{2} \times 101 = 5050$

4. $f(x) = \frac{9^x}{9^x + 3}$

$f(1-x) = \frac{9^{1-x}}{9^{1-x} + 3} = \frac{3}{3 + 9^x}$

$f(x) + f(1-x) = 1$

$f\left(\frac{1}{2008}\right) + f\left(\frac{2007}{2008}\right) = 1$ ...... **(i)**

$f\left(\frac{2}{2008}\right) + f\left(\frac{2006}{2008}\right) = 1$ ...... **(ii)**

....

....

$f\left(\frac{1003}{2008}\right) + f\left(\frac{1005}{2008}\right) = 1$ ...... **(iii)**

& $f\left(\frac{1004}{2008}\right) + f\left(\frac{1004}{2008}\right) = 1$

$\Rightarrow$ $f\left(\frac{1004}{2008}\right) = \frac{1}{2}$ ...... **(iv)**

add all we get

$f\left(\frac{1}{2008}\right) + f\left(\frac{2}{2008}\right) + f\left(\frac{3}{2008}\right) + \ldots + f\left(\frac{2007}{2008}\right)$

$= 1003.5$

5. **(i)** neither even nor odd **(ii)** even
(iii) odd **(iv)** even
(v) odd

6. **(i)** π **(ii)** 2 **(iii)** $\frac{2\pi}{3}$ **(iv)** 2π
(v) $2^n \pi$ **(vi)** π

7. (9)

9. $f(x) = (a - x^n)^{1/n}$

$f(f(x)) = (a-(f(x))^n)^{1/n}$

$= [a-\{(a-x^n)^{1/n}\}^n]^{1/n} = (a - a + x^n)^{1/n} = x$

So $fof(x) = x$

$\Rightarrow f^{-1}(x) = f(x) = (a - x^n)^{1/n}$

10. $f^{-1}(x) = x + (-1)^{x-1}, x \in N$

EXERCISE - 5

Part # I : AIEEE/JEE-MAIN

5. $y = \sin^{-1}[\log_3(x/3)] \Rightarrow -1 \le \log_3(x/3) \le 1$

$\Rightarrow \frac{1}{3} \le \frac{x}{3} \le 3 \Rightarrow 1 \le x \le 9 \Rightarrow x \in [1, 9]$

6. $f(x) = \log(x + \sqrt{x^2+1})$

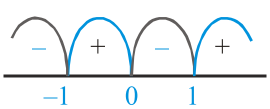
and $f(-x) = -\log(x + \sqrt{x^2+1}) = -f(x)$

$f(x)$ is odd function.

7. $f(x) = \frac{3}{4-x^2} + \log_{10}(x^3 - x)$. So, $4 - x^2 \neq 0$

$\Rightarrow x \neq \pm\sqrt{4}$

and $x^3 - x > 0 \Rightarrow x(x^2 - 1) > 0 \Rightarrow x>0, x>1$

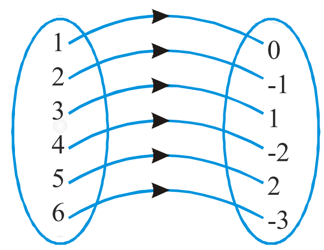


$\therefore D=(-1,0)\cup(1,\infty)-\{\sqrt{4}\}$ i.e., $D=(-1,0)\cup(1,2)\cup(2,\infty)$

9. $f : N \to I$

$f(1) = 0, f(2) = -1, f(3) = 1, f(4) = -2, f(5) = 2$

and $f(6) = -3$ so on.



In this type of function every element of set A has unique image in set B and there is no element left in set B. Hence f is one-one and onto function.

10. To define $f(x)$, $9 - x^2 > 0 \Rightarrow -3 < x < 3$ ... (i)

$-1 \le (x - 3) \le 1 \Rightarrow 2 \le x \le 4$ ... (ii)

From **(i)** and **(ii)**, $2 \le x < 3$ i.e., $[2, 3)$.

11. $f(3) = {}^{7-3}P_0 = 1$, $f(4) = {}^3P_1 = 3$ and $f(5) = {}^2P_2 = 2$

Hence, range of $f = \{1, 2, 3\}$.

12. Using $-\sqrt{a^2+b^2} \le (a\sin x + b\cos x) \le \sqrt{a^2+b^2}$

$-\sqrt{1+(-\sqrt{3})^2} \le (\sin x - \sqrt{3}\cos x) \le \sqrt{1+(-\sqrt{3})^2}$

$-2 \le (\sin x - \sqrt{3}\cos x) \le 2$

$-2 + 1 \le (\sin x - \sqrt{3}\cos x + 1) \le 2 + 1$

$-1 \le (\sin x - \sqrt{3}\cos x + 1) \le 3$ i.e., range $= [-1,3]$

$\therefore$ For f to be onto $S = [-1, 3]$.

13. For $-1 < x < 1$, $\tan^{-1}\frac{2x}{1-x^2} = 2\tan^{-1} x$

Range of $f(x) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\therefore$ Co-domain of function = B = $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

14. $f(a-(x-a)) = f(A)f(x-a) - f(0)f(x)$ ... (i)

Put x=0, y = 0 ; $f(0) = (f(0))^2 - [f(A)]^2 \Rightarrow f(A) = 0$

[$\because f(0) = 1$]. From (i), $f(2a - x) = -f(x)$.

15. Let $y = \frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$

$\Rightarrow 3(y-1)x^2 + 9(y-1)x + 7y - 17 = 0$

Since x is real, we have

$\{9(y-1)\}^2 - 4.3(y-1)(7y-17) \geq 0$

$\Rightarrow \quad -3y^2 + 126y - 123 \geq 0$

$\Rightarrow \quad (y-41)(y-1) \leq 0$

$\Rightarrow \quad 1 \leq y \leq 41$

So, maximum value of y is 41.

16. f(x) is defined if $-1 \leq \frac{x}{2} - 1 \leq 1$ and $\cos x > 0$

or $\quad 0 \leq x \leq 4 \quad$ and $\quad -\frac{\pi}{2} < x < \frac{\pi}{2}$

$\therefore \quad x \in \left[0, \frac{\pi}{2}\right)$

19. For real x, $f(x) = x^3 + 5x + 1$

$\lim_{x\to\infty} f(x) = +\infty$

and $\lim_{x\to\infty} f(x) = -\infty$

$\therefore$ Range is R – f(x) is on to

Now $f'(x) = 3x^2 + 5 > 0$

$\therefore$ f(x) is one-one

f(x) is one-one onto.

20. $f(x) = (x+1)^2 - 1 \ ; \ x \geq -1$

$f'(x) = 2(x+1) \geq 0$ for $x \geq -1$

$\therefore$ f(x) is bijection

Statement (2) is correct

Now $f^{-1}(x) = f(x)$

To solve put y = x in f(x)

$x = (x+1)^2 - 1$

$x + 1 = (x+1)^2$

$x = -1, \ x = 0$

$x = \{0, -1\}$ Statement (1) is also correct

21. $f(x) = \frac{1}{\sqrt{|x| - x}}$

For domain of real function

$|x| - x > 0$

$|x| > x$

$x \in (-\infty, 0)$

22. $f(x) = (x-1)^2 + 1 \ ; (x \geq 1)$

and $f'(x) = 2(x-1) \geq 0$ for $x \geq 1$

$\therefore \quad f(x)$ is one-one and onto

$\Rightarrow \quad f(x)$ is Bijection

and $f^{-1}(x) = 1 + \sqrt{x-1}$

Statement-2 is true

Now $\quad f(x) = f^{-1}(x)$

$\Rightarrow (x-1)^2 + 1 = \sqrt{x-1} + 1$

$\Rightarrow x = 1,2$

$\therefore \quad$ Statement-1 is true

23. [x] is contincous at R – I

$\therefore$ f(x) is continuous at R – I

Now At x = I

LHL = $\lim_{h\to 0} [I - h] \cos\frac{(2(I-h)-1)}{2}\pi$

$\lim_{h\to 0} (I-1)\cos[2I - 2h - 1]\frac{\pi}{2}$

$= (I - 1) \cos (2I - 1)\frac{\pi}{2} = 0$

similarly,

RHL = 0

and f(i) = 0

$\therefore$ Function is continous everywhere

25. $f(x) + 2f\left(\frac{1}{x}\right) = 3x, x \neq 0$

$f\left(\frac{1}{x}\right) + 2f(x) = \frac{3}{x}$

$3f(x) = \frac{6}{x} - 3x$

$f(x) = \frac{2}{x} - x$

$f(x) = f(-x) \Rightarrow \frac{2}{x} - x = \frac{-2}{x} + x$

$\frac{4}{x} = 2x$

$\Rightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2}$

26. $g(x) = f(f(x))$

$\Rightarrow g'(x) = f'(f(x)) f'(x)$

$\Rightarrow g'(0) = f'(f(0)) f'(0)$

For $x \to 0, \log 2 > \sin x$

$\therefore f(x) = \log 2 - \sin x$

$\therefore f'(x) = -\cos x$

$\Rightarrow f'(0) = -1$

Also $x \to \log 2, \log 2 > \sin x$

$\therefore f(x) = \log 2 - \sin x$

$\therefore f(x) = -\cos x$

$\Rightarrow f'(\log 2) = -\cos (\log 2)$

$\therefore g'(0) = (-\cos (\log 2))(-1) = \cos (\log 2)$

Part # II : IIT-JEE ADVANCED

3. $g(x) = 1 + \{x\}$

$\Rightarrow 0 + 1 \le g(x) \le 1 + 1$

$\Rightarrow 1 \le g(x) < 2$

$f(g(x)) = 1 \quad (\because g(x) > 0)$

6. n (into + onto) = 2^4

n (into) = 2

n (onto) = 16 – 2 = 14

7. $f(x) = \frac{\alpha x}{x+1}, x \neq -1$

Now $f(f(x)) = x \Rightarrow f(x) = f^{-1}(x)$

Let $y = \frac{\alpha x}{x+1} \Rightarrow xy + y = \alpha x$

$\Rightarrow x(y - \alpha) = -y \Rightarrow x = \frac{-y}{y-\alpha}$

$f^{-1}(x) = \frac{-x}{x-\alpha}$

Now $\frac{\alpha x}{x+1} = \frac{-x}{x-\alpha}$

on solving we get $\alpha = -1$

13. $\phi(x) = f(x) - g(x)$

$= \begin{cases} -x & x \in Q \\ x & x \notin Q \end{cases}$

It is one-one onto function

14. Given $f(x) = x^2$; $g(x) = \sin x$

$fogogof(x) = \sin^2(\sin x^2)$

and $gogof(x) = \sin(\sin x^2)$

given $fogogof(x) = gogof(x)$

$\Rightarrow \sin^2(\sin x^2) = \sin(\sin x^2)$

$\Rightarrow \sin(\sin x^2) = 0$ or 1 (rejected)

$\sin(\sin x^2) = 0$

$\Rightarrow x^2 = n\pi$

$\Rightarrow x = \pm\sqrt{n\pi};\ x \in \{0, 1, 2, 3, ...\}$

15. $f(x) = 2x^3 - 15x^2 + 36x + 1$

$\Rightarrow$ $f'(x) = 6(x^2 - 5x + 6)$

$= 6(x - 2)(x - 3)$

$\because$ $f(x)$ is non monotonic in $x \in [0,3]$

$\Rightarrow$ $f(x)$ is not one-one

$f(x)$ is increasing in $x \in [0,2)$ and decreasing in $x \in (2,3]$

$f(0) = 1,\ f(2) = 29\ \&\ f(3) = 28$

$\therefore$ Range of $f(x)$ is $[1,29]$

$\Rightarrow$ $f(x)$ is onto.

16. $\because$ $\theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right) \Rightarrow 2\theta \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right)$

Now $f(\cos 4\theta) = \dfrac{2}{2 - \sec^2\theta}$

$= \dfrac{1 + \cos 2\theta}{\cos 2\theta} = 1 + \dfrac{1}{\cos 2\theta}$ ....(i)

Let $\cos 4\theta = \frac{1}{3}$ $\Rightarrow$ $2\cos^2 2\theta - 1 = \frac{1}{3}$

$\Rightarrow$ $\cos 2\theta = \pm\sqrt{\frac{2}{3}}$

$\Rightarrow$ From (i), $f\left(\frac{1}{3}\right) = 1 \pm \sqrt{\frac{3}{2}}$

$\Rightarrow$ (A, B) are correct

21. $f(x) = x^3 + 3x + 2$, $f(1) = 6, g(6) = 1$

$g(f(x)) = x$ $\Rightarrow$ $g'(f(x)) \times f'(x) = 1$

put $x = 0$, $g'(f(0)).\ f'(0) = 1$

$g'(2)\ \dfrac{1}{f'(0)} = \dfrac{1}{3}$

$f(3) = 38$

$\therefore$ $g(38) = 3$

$\therefore$ $h(g(3)) = h(g(g38))) = 38$

$f(2) = 16$

$\Rightarrow$ $g(16) = 2$

$\therefore$ $h(g(g(16)) = h(g(2)) = h(0)$

$\therefore$ $16 = h(g(g(16)) = h(0)$

$\therefore$ **(C)** is correct.

$f'(x) = 3x^2 + 3$

$f'(6) = 111,\quad f'(1) = 6$ $\Rightarrow$ $g'(6) = \frac{1}{6}$

$h(g(g(x))) = x$

$\Rightarrow$ $h'(g(g(x))) \times g'(g(x)) \times g'(x) = 1$

Put $x = 236$, $h'(g(g(236))) \times g'(g(236)) \times g'(236) = 1$

$\Rightarrow$ $h'(g(6))\ g'(6) \times \dfrac{1}{f'(6)} = 1$

$\Rightarrow$ $h'(1) = 666$ But $g(1) \neq 1$

MOCK TEST

1. (B)

Put $x = y = 1$, $\quad (f(1))^2 = 3\,f(1) - 2$

$\Rightarrow$ $f(1) = 1$ or 2

Let $f(1) = 1$, then put $y = 1$

$f(x)\,.\,f(1) = f(x) + f(1) + f(x) - 2$

$\Rightarrow$ $f(x) = 1$

constant function but f(x) is not constant function

$\therefore$ $f(1) \neq 1$, hence $f(1) = 2$

2. $f(x) = \sqrt{-\log_{\frac{x+4}{2}}\left(\log_2 \dfrac{2x-1}{3+x}\right)}$

For domain : $\log_{\frac{x+4}{2}}\left(\log_2 \dfrac{2x-1}{3+x}\right) \le 0$

Case-I

$0 < \dfrac{x+4}{2} < 1 \quad \Rightarrow \quad -4 < x < -2$ ....A

then $\log_{\frac{x+4}{2}}\left(\log_2 \dfrac{2x-1}{3+x}\right) \le 0$

$\Rightarrow \log_2 \dfrac{2x-1}{3+x} \ge 1 \quad \Rightarrow \quad \dfrac{2x-1}{3+x} \ge 2$

$\Rightarrow$ $x < -3$ ....B

$\Rightarrow$ on $A \cap B$ $\quad x \in (-4, -3)$ ....(i)

Case-II

$\dfrac{x+4}{2} > 1$ or $x > -2$ ....A

$\log_{\frac{x+4}{2}}\left(\log_2 \dfrac{2x-1}{3+x}\right) \le 0$

$\Rightarrow 0 < \log_2 \dfrac{2x-1}{3+x} \le 1 \quad \Rightarrow \quad 1 < \dfrac{2x-1}{3+x} \le 2$

$\Rightarrow$ $x \in (4, \infty)$ ....(ii)

$\therefore$ (i) $\cup$ (ii) $\quad$ Domain $x \in (-4, -3) \cup (4, \infty)$

3. $f(x) = ax^2 + bx + c$

$f(0) = c \quad \Rightarrow \quad c \in I$

$f(1) = a + b + c \quad \Rightarrow \quad (a + b + c) \in I$

$\Rightarrow$ $(a + b) \in I$

4. $y = \dfrac{\sin^2 x + 4\sin x + 4}{2\sin^2 x + 8\sin x + 8} + \dfrac{1}{2\sin^2 x + 8\sin x + 8}$

$= \dfrac{1}{2} + \dfrac{1}{2(\sin x + 2)^2}$

$y_{max} = \dfrac{1}{2} + \dfrac{1}{2(-1+2)^2} = 1$

$\Rightarrow$ $y_{min} = \dfrac{1}{2} + \dfrac{1}{2(1+2)^2} = \dfrac{5}{9}$

$\therefore$ range $= \left[\dfrac{5}{9}, 1\right]$

5. (C)

$f(x) = x + \tan x$

$f(f^{-1}(x)) = f^{-1}(x) + \tan(f^{-1}(x))$

$x = g(x) + \tan(g(x))$

$1 = g'(x) + \sec^2(g(x))\,g'(x)$

$g'(x) = \dfrac{1}{2 + \tan^2(g(x))}$

$g'(x) = \dfrac{1}{2 + (x - g(x))^2}$

6. (A)

$g(f(x)) = \tan\left(x - \dfrac{\pi}{4}\right) = \dfrac{\tan x - 1}{\tan x + 1}$

$\Rightarrow$ $g(x) = \dfrac{x-1}{x+1}$

$f(g(x)) = \tan\left(\dfrac{x-1}{x+1}\right)$.

7. $f(x) = \sin^{-1}\left[x^2 + \frac{1}{2}\right] + \cos^{-1}\left[x^2 - \frac{1}{2}\right]$

Domain : $-1 \le \left[x^2 - \frac{1}{2}\right] \le 1$

$\Rightarrow$ $x \in \left(-\sqrt{\frac{5}{2}}, \sqrt{\frac{5}{2}}\right)$ and $-1 \le \left[x^2 + \frac{1}{2}\right] \le 1$

$\Rightarrow$ $x \in \left(-\sqrt{\frac{3}{2}}, \sqrt{\frac{3}{2}}\right)$

$\Rightarrow$ domain is

$x \in \left(-\sqrt{\frac{3}{2}}, \sqrt{\frac{3}{2}}\right)$ or $x^2 \in \left[0, \frac{3}{2}\right)$

if (i) $x^2 \in \left[0, \frac{1}{2}\right)$, then $f(x) = \pi$

if (ii) $x^2 \in \left[\frac{1}{2}, 1\right)$, then $f(x) = \pi$

if (iii) $x^2 \in \left[1, \frac{3}{2}\right)$, then $f(x) = \pi$

$\Rightarrow$ range = $\{\pi\}$

8. $f(x) + 5 \le f(x+5) \le f(x+4) + 1 \le f(x+3) + 2 \le f(x+2) + 3 \le f(x+1) + 4 \le f(x) + 5$

$\Rightarrow$ In all steps there is equality only

$\Rightarrow$ $f(x+1) = f(x) + 1$

Now $f(1) = 1$

$\Rightarrow$ $f(2) = 2$

$f(3) = 3$

$f(4) = 4$

⋮

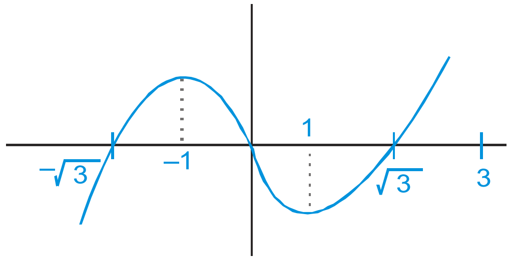
⋮

$f(2013) = 2013$

$\Rightarrow$ $g(2013) = 2013 + 1 - 2013 = 1$

9. **(D)**

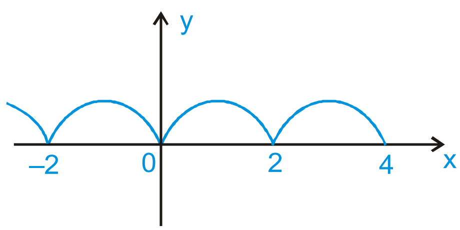
$f(x) = 4x(x^2 - 3)$



$f'(x) = 12x^2 - 12 = 0$

or $x = \pm 1$

$f(x) \in [f(1), \max(f(-1), f(3))] = [-8, 72]$

10. 

1.

$y = f(x+2)$ is drawn by shifting the graph by 2 units horizontally.

11. $f[g(x)] = a(bx + a) + b = abx + a^2 + b$ ....**(i)**

$g[f(x)] = b(ax + b) + a$

$= abx + b^2 + a$ ....**(ii)**

(i) – (ii) $f[g(50)] - g[f(50)] = a^2 - b^2 + b - a$

$\therefore$ $(a^2 - b^2) + (b - a) = 28$

$(a - b)(a + b - 1) = 28 = (1 \times 28)$ or (2×14) or (4×7)

let $a - b = 1$ and $a + b - 1 = 28$

and $2a - 2 = 28$ $\Rightarrow$ $a = 15;$ $b = 14$

$\therefore$ $ab = 210$

if $a - b = 2$ and $a + b - 1 = 14$ (not possible)

if $a - b = 4$ and $a + b - 1 = 7$

$2a - 1 = 11$ $\Rightarrow$ $a = 6$ and $b = 2$

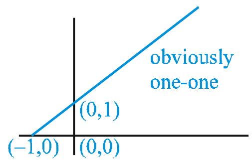
$\therefore$ $ab = 12$

12. $f(-x) = \begin{cases} 0 & x = 0 \\ -x^2 \sin\left(\frac{\pi}{x}\right) & x \in (-1,1) - \{0\} \\ -x \quad |x| & |x| > | \end{cases} = -f(x)$

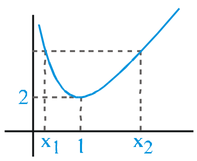
odd function

13. $f(x) = x + 1$

$f(x) = x+1 \quad \text{for } x > -1$



$g(x) = x + \frac{1}{x} \quad \text{for } x > 0$

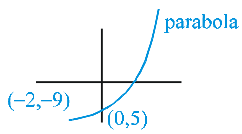


Many-one function

$h(x) = x^2 + 4x - 5 \quad \text{for } x > 0$

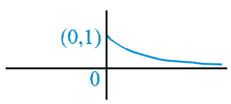
Hence One - one

$= (x+2)^2 - 9$



$f(x) = e^{-x} \quad \text{for } x \geq 0$

Obviously one-one



14. $y = f(x) \Rightarrow x = f^{-1}(x)$ now $y = ax + b$

$x = \frac{y}{a} - \frac{b}{a}$

$f^{-1}(y) = \frac{y}{a} - \frac{b}{a}$

$f^{-1}(x) = \frac{x}{a} - \frac{b}{a}$ ....**(i)**

and $f(x) = ax + b$ ....**(ii)**

now in order that **(i)** and **(ii)** coincide

$a = \frac{1}{a}$ ....**(i)**

$\frac{b}{a} = -b$ ....**(ii)**

from (1), $a^2 = 1 \Rightarrow a = 1$ or -1

if $a = -1,\ b = b \Rightarrow b \in R$

if $a = +1$, then $2b = 0 \Rightarrow b = 0$

hence $(-1, R), (1, 0)$

15. Let, $2x + y = 3x - y$

$\Rightarrow 2y = x \Rightarrow y = \frac{x}{2}$

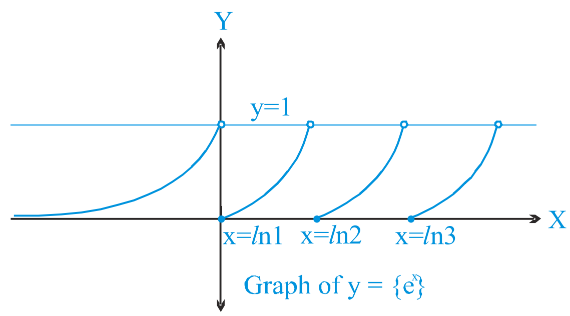
$\therefore$ put $y = \frac{x}{2}$

$f(x) + f\left(\frac{5x}{2}\right) + \frac{5x^2}{2} = f\left(\frac{5x}{2}\right) + 2x^2 + 1$

$f(x) = 1 - \frac{x^2}{2} \Rightarrow$ A, B

16. **(D)**

$$y = \{e^x\} = \begin{cases} e^x & ; \quad -\infty < x < 0 \\ e^x - 1 & ; \quad 0 \leq x < \ln 2 \\ e^x - 2 & ; \quad \ln 2 \leq x < \ln 3 \end{cases}$$



Graph of $y = \{e^x\}$

and so on

Clearly f(x) is aperiodic on R.

17. **(C)**

Statement-I : L.C.M. of $(2\pi, \pi) = 2\pi$

Statement-II : $f(x) = |\sin x| + |\cos x|$, fundamental period is $\frac{\pi}{2}$

18. **(A)**

(i) $y = f(x)$ is symmetric about $y = x$

$\Rightarrow x = f(y)$

$\therefore f(f(x)) = f(y) = x$

$\therefore$ statement 1 is true

(ii) $f(x) = \begin{cases} x & , \quad x \text{ is rational} \\ 1 - x & , \quad x \text{ is irrational} \end{cases}$ is

symmetric about $y = x$

$\therefore f(f(x)) = x$

19. (A)

$$f\left(g\left(h(1)\right)\right) = f\left(g(3)\right) = f\left(-g(-3)\right) = f(-2) = 1$$

$$g\left(h\left(f(3)\right)\right) = g\left(h(-5)\right) = g\left(-h(5)\right) = g(-1) = -g(1) = -1$$

$$h\left(f\left(g(-1)\right)\right) = h\left(f\left(-g(1)\right)\right) = h\left(f(-1)\right) = h\left(f(1)\right) = h(0)$$

as h is odd $\Rightarrow$ $h(x) + h(-x) = 0$

$h(0) + h(0) = 0$ $\Rightarrow$ $h(0) = 0$

$\Rightarrow$ sum of composite functions is zero.

20. (D)

Statement -I : Every function can be written as the sum of even and odd function

Statement -II : $f(x) = e^x$

$f(-x) = e^{-x}$

Here neither $f(x) = f(-x)$ nor $f(-x) = -f(x)$

So e^x is neither even nor odd function.

21. (A) → (p, r), (B) → (p, s), (C) → (q, s), (D) → (q, s)

(A) $f'(x) = \dfrac{1}{2\sqrt{\sin x}} \cos x$

$f'(x)$ is positive if $x \in \left[0, \dfrac{\pi}{3}\right]$

f is one to one function

Since $0 \le x \le \dfrac{\pi}{3}$

$0 \le \sin x \le \dfrac{\sqrt{3}}{2}$

$0 \le \sqrt{\sin x} \le \sqrt{\dfrac{\sqrt{3}}{2}} < 1$

f is into function

(B) $f(x) = \dfrac{x+3}{x-1}$

$f'(x) = \dfrac{(x-1).1-(x+3).1}{(x-1)^2}$

$f'(x) = \dfrac{-4}{(x-1)^2}$

$f'(x) < 0$ Hence f(x) is one to one

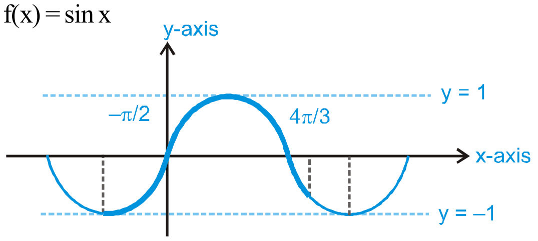
Since $x > 1$

$\therefore$ Range of $y = \dfrac{x+3}{x-1}$ is $(1, \infty)$

f is onto function

(C) $-\dfrac{\pi}{2} \le x \le \dfrac{4\pi}{3}$

$f(x) = \sin x$



from graph f(x) is many-one and onto

(D) $f(x) = \dfrac{x^2}{x-2}$

$f'(x) = \dfrac{(x-2).2x - x^2}{(x-2)^2}$

$f'(x) = \dfrac{x^2 - 4x}{(x-2)^2}$

$\therefore$ $f'(x) < 0$ if $2 < x < 4$

$f'(x) > 0$ if $x > 4$

f(x) is many-one

$f(4) = 8$ (is the least value of f(x))

$\therefore$ range $= [8, \infty)$

$\therefore$ f(x) is onto.

22. (A) → (s) ; **(B)** → (q) ; **(C)** → (r) ; **(D)** → (p)

$fog = \dfrac{2x^2+6x+5}{x^2+3x+2}$; $gof = \dfrac{x^2+x+1}{(x+1)^2}$;

$fof = \dfrac{x^4+3x^2+1}{x(x^2+1)}$; $gog = \dfrac{2x+3}{3x+5}$

23.

1. $f(x) = (x-1)^2 - 2 \quad a = 1, b = -2$

$a + b = -1$

2. $g(x) = x^2 + x - 2 = \left(x + \frac{1}{2}\right)^2 - \frac{9}{4}$

$\Rightarrow \quad g|x| = \left(|x| + \frac{1}{2}\right)^2 - \frac{9}{4}$

$g_{min} = g(0) = -2$

3. $f: [1, \infty) \to [-2, \infty)$, then $f^{-1}: [-2, \infty) \to [1, \infty)$;

$f(x) = y \quad \Rightarrow \quad x^2 - 2x - (1 + y)$

$x = \frac{2 \pm \sqrt{4 + 4(1+y)}}{2}$; $\quad x = 1 \pm \sqrt{2 + y}$;

$f^{-1}(y) = 1 + \sqrt{2 + y}$; $\quad f^{-1}(x) = 1 + \sqrt{2 + x}$

24.

1. (A)

Since period of $f(x)$ is $2(10 - 2) = 16$

$\therefore \quad f(0) = f(16) = f(32) = ... = f(160) = 5$

$\therefore$ there are atleast 11 values of x for which $f(x) = 5$

$f(0) = f(4) = f(16)$

due to symmetry in one period length $f(x) = 5$

has solution other then 0, 16, 32, ____

$\therefore$ minimum possible number of values of x is

$10 + 11 = 21$

2. (A) Obvious by definition

3. (C)

If 1 is a period, then $f(x) = f(x+1), \forall x \in R$

$\Rightarrow \quad f(2) = f(3) = f(4) = f(5) = f(6)$

which contradicts the given hypothesis that $f(2) \neq f(6)$

$\therefore$ 1 cannot be period of $f(x)$

25.

1. (A)

$f: (0, \infty) \to (0, \infty)$

$f(x\,f(y)) = x^2 y^a \; (a \in R)$

Put $x = 1$, we get $f(f(y)) = y^a$

Put $f(y) = \frac{1}{x}$, we get $f(1) = \frac{1}{(f(y))^2} \cdot y^a$ and put $y = 1$ we

get $(f(1))^3 = 1$

$\therefore \quad f(1) = 1$

for $y = 1$, we have $f(x(f(1))) = x^2$

$\therefore \quad f(x) = x^2$

thus $a = 4$

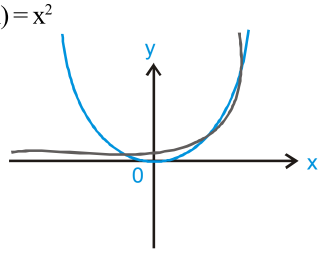
2. (C)

$$\sum_{r=1}^{n} f(r)\,{}^nC_r = \sum_{r=1}^{n} r^2\,{}^nC_r = \sum_{r=1}^{n} (r(r-1) + r)\,{}^nC_r$$

$$= n(n-1)\,2^{n-2} + n.2^{n-1}$$

3. (C)

Since $f(x) = x^2$



$\therefore \quad 2x^2 = e^x$

26. Let $x = y = 1$

$f(x) + f(y) + f(xy) = 2 + f(x) \,.\, f(y)$

$3f(1) = 2 + (f(1))^2 \quad \Rightarrow \quad f(1) = 1, 2$. But given that

$f(1) \neq 1$ so $f(1) = 2$

Now put $y = \frac{1}{x}$

$f(x) + f\left(\frac{1}{x}\right) + f(1) = 2 + f(x) \,.\, f\left(\frac{1}{x}\right)$

$\Rightarrow \quad f(x) + f\left(\frac{1}{x}\right) = f(x) \,.\, f\left(\frac{1}{x}\right)$

so $f(x) = \pm x^n + 1$

Now $\quad f(4) = 17$

$\Rightarrow \quad \pm(4)^n + 1 = 17 \quad \Rightarrow \quad n = 2$

$f(x) = +(x)^2 + 1$.

$\therefore \quad f(5) = 5^2 + 1 = 26$

27. (9)

Let $x = y = 1$

$f(x) + f(y) + f(xy) = 2 + f(x) . f(y)$

$3f(1) = 2 + (f(1))^2 \Rightarrow f(1) = 1, 2$. But given that

$f(1) \neq 1$ so $f(1) = 2$

Now put $y = \frac{1}{x}$

$$f(x) + f\left(\frac{1}{x}\right) + f(1) = 2 + f(x) . f\left(\frac{1}{x}\right)$$

so $f(x) = \pm x^n + 1$

Now $f(4) = 17$

$\Rightarrow \pm (4)^n + 1 = 17 \Rightarrow n = 2$

$f(x) = +(x)^2 + 1$

$\Rightarrow f(5) = 126$

28. Put $y = -x$

$f(x) + f(-x) = f(0) = 0$

$f(-x) = -f(x) \Rightarrow$ f(x) is an odd function.

Now put $y = x \Rightarrow 2f(x) = f(2x\sqrt{1-x^2})$

Now put $y = 2x\sqrt{1-x^2}$ in

$$f(x) + f(y) = f(x\sqrt{1-y^2} + y\sqrt{1-x^2})$$

$$f(x) + f(2x\sqrt{1-x^2}) = f(x\sqrt{1-4x^2+4x^4} + 2x(1-x^2))$$

$f(x) + 2f(x) = f(x(1-2x^2) + 2x - 2x^3)$

$3f(x) = f(3x - 4x^3) = -f(4x^3 - 3x)$

$\Rightarrow f(4x^3 - 3x) + 3f(x) = 0$

29. (5)

Domain of $\sin^{-1}(\sin x)$ is whole of R

$$-\log_{\frac{x+4}{2}} \log_2\left(\frac{2x-1}{3+x}\right) > 0$$

i.e. $$\log_{\frac{x+4}{2}}\left(\log_2\left(\frac{2x-1}{3+x}\right)\right) < 0$$

Case-I If $0 < \frac{x+4}{2} < 1$ i.e. $-4 < x < -2$

then $\log_2 \frac{2x-1}{3+x} > 1$ i.e. $\frac{2x-1}{3+x} > 2$

i.e. $\frac{2x-1-6-2x}{3+x} > 0$

i.e. $x + 3 < 0$ i.e. $x < -3$

$\therefore -4 < x < -3$ ....**(i)**

Case-II If $\frac{x+4}{2} > 1$ i.e. $x > -2$

then $0 < \log_2 \frac{2x-1}{3+x} < 1$ i.e. $1 < \frac{2x-1}{3+x} < 2$

i.e. $\frac{2x-1-3-x}{x+3} > 0$ and $\frac{2x-1-6-2x}{x+3} < 0$

i.e. $\frac{x-4}{x+3} > 0$ and $\frac{-7}{x+3} < 0$

i.e. $\{x < -3 \text{ or } x > 4\}$ and $x > -3$

i.e. $x > 4$ ....**(iii)**

from **(i)** and **(ii)** $x \in (-4, -3) \cup (4, \infty)$

$\therefore a = -4, b = -3, c = 4$ and so $a + b + 3c = 5$

30. (1)

$$f(x) + f\left(\frac{1}{1-x}\right) = \frac{2}{x} - \frac{2}{1-x}$$

Put $x = \frac{1}{1-t}$ $1 - x = 1 - \frac{1}{1-t} = -\frac{t}{1-t}$

$$\therefore f\left(\frac{1}{1-t}\right) + f\left(\frac{t-1}{t}\right) = 2(1-t) - \frac{2(1-t)}{-t}$$

$$\therefore f\left(\frac{1}{1-x}\right) + f\left(\frac{x-1}{x}\right) = 2(1-x) + 2\left(\frac{1-x}{x}\right)$$

Put $\frac{x-1}{x} = t$

$$x - 1 = xt\, x = \frac{1}{1-t}$$

$$1-x=1-\frac{1}{1-t}=\frac{-t}{1-t}$$

$$\frac{1}{1-x}=\frac{t-1}{t}$$

$$\therefore\ f\left(\frac{t-1}{t}\right)+f(t)=\frac{2t}{t-1}+\left(\frac{2t}{t-1}\right)\left(\frac{1-t}{1}\right)$$

$$\therefore\ f\left(\frac{x-1}{x}\right)+f(x)=\frac{2x}{x-1}-2x$$

$$\therefore\ 2f(x)=\frac{2}{x}-\frac{2}{1-x}-2(1-x)-\frac{2(1-x)}{x}+\frac{2x}{x-1}-2x$$

$$=\frac{2}{x}-\frac{2}{1-x}-2+2x-\frac{2}{x}+2+\frac{2x}{x-1}-2x$$

$$=\frac{2x}{x-1}+\frac{2}{x-1}=2\left(\frac{x+1}{x-1}\right)$$

$$\therefore\ f(x)=\frac{x+1}{x-1}$$