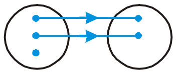
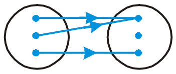
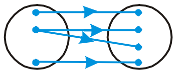
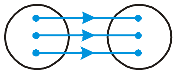
SOLVED EXAMPLES

Ex. 1 Which of the following pictorial diagrams represent the function

(A)  **(B)**  **(C)**  **(D)** 

Sol. B and D. In **(A)** one element of domain has no image, while in **(C)** one element of 1st set has two images in 2nd set

Ex. 2 Find the Domain of the following function :

(i) $y = \log_{(x-4)}(x^2 - 11x + 24)$ **(ii)** $f(x) = \sqrt{x^2 - 5}$

(iii) $\sin^{-1}(2x - 1)$ **(iv)** $f(x) = \sqrt{\sin x} - \sqrt{16 - x^2}$

Sol. (i) $y = \log_{(x-4)}(x^2 - 11x + 24)$

Here 'y' would assume real value if,

$x - 4 > 0$ and $\neq 1$, $x^2 - 11x + 24 > 0$ $\Rightarrow$ $x > 4$ and $\neq 5$, $(x-3)(x-8) > 0$

$\Rightarrow$ $x > 4$ and $\neq 5$, $x < 3$ or $x > 8$ $\Rightarrow$ $x > 8$

$\Rightarrow$ Domain $(y) = (8, \infty)$

(ii) $\sqrt{x^2-5}$ $f(x) =$ is real iff $x^2 - 5 \geq 0$

$\Rightarrow$ $|x| \geq \sqrt{5}$ $\Rightarrow$ $x \leq -\sqrt{5}$ or $x \geq \sqrt{5}$

$\therefore$ the domain of f is $(-\infty, -\sqrt{5}\,] \cup [\,\sqrt{5}, \infty)$

(iii) $\sin^{-1}(2x - 1)$ is real iff $-1 \leq 2x - 1 \leq +1$

$\therefore$ domain is $x \in [0, 1]$

(iv) $\sqrt{\sin x}$ is real iff $\sin x \geq 0$ $\Leftrightarrow$ $x \in [2n\pi, 2n\pi + \pi]$, $n \in I$.

$\sqrt{16 - x^2}$ is real iff $16 - x^2 \geq 0$ $\Leftrightarrow$ $-4 \leq x \leq 4$.

Thus the domain of the given function is $\{x : x \in [2n\pi, 2n\pi + \pi], n \in I\} \cap [-4, 4] = [-4, -\pi] \cup [0, \pi]$.

Ex. 3 Find the range of following functions :

(i) $f(x) = \dfrac{1}{8 - 3\sin x}$ **(ii)** $f(x) = \dfrac{x^2 - 4}{x - 2}$

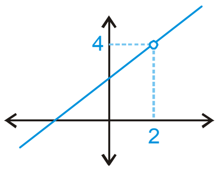
Sol. (i) $f(x) = \dfrac{1}{8 - 3\sin x}$

$-1 \leq \sin x \leq 1$

$\therefore$ Range of $f = \left[\dfrac{1}{11}, \dfrac{1}{5}\right]$

(ii) $f(x) = \dfrac{x^2 - 4}{x - 2} = x + 2;\ x \neq 2$

$\therefore$ graph of f(x) would be



Thus the range of $f(x)$ is $R - \{4\}$

Ex. 4 Find the range of following functions :

(i) $y = \ell n\,(2x - x^2)$ **(ii)** $y = \sec^{-1}(x^2 + 3x + 1)$

Sol. (i) **Step – 1**

We have $2x - x^2 \in (-\infty, 1]$

Step – 2

Let $t = 2x - x^2$

For ℓnt to be defined accepted values are (0, 1]

Now, using monotonocity of $\ell n\, t$,

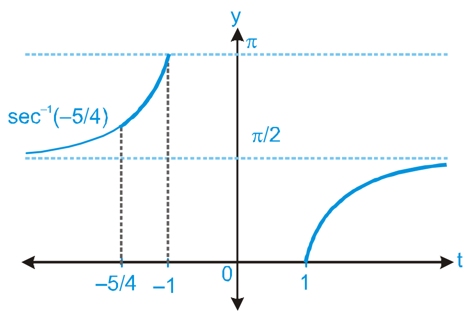
$\ell n\,(2x - x^2) \in (-\infty, 0]$

$\therefore$ range is $(-\infty, 0]$

(ii) $y = \sec^{-1}(x^2 + 3x + 1)$

Let $t = x^2 + 3x + 1$ for $x \in R$, then $t \in \left[-\frac{5}{4}, \infty\right)$

but $y = \sec^{-1}(t)$ $\Rightarrow$ $t \in \left[-\frac{5}{4}, -1\right] \cup [1, \infty)$



from graph the range is $\left[0, \frac{\pi}{2}\right) \cup \left[\sec^{-1}\left(-\frac{5}{4}\right), \pi\right]$

Ex. 5 **(i)** Let {x} and [x] denote the fractional and integral part of a real number x respectively. Solve $4\{x\} = x + [x]$

(ii) Draw graph of $f(x) = \text{sgn}\,(\ell n\, x)$

Sol. **(i)** As $x = [x] + \{x\}$

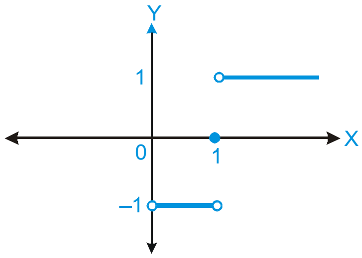
$\therefore$ Given equation $\Rightarrow$ $4\{x\} = [x] + \{x\} + [x]$ $\Rightarrow$ $\{x\} = \frac{2[x]}{3}$

As [x] is always an integer and $\{x\} \in [0, 1)$, possible values are

[x]	{x}	x = [x] + {x}
0	0	0
1	$\frac{2}{3}$	$\frac{5}{3}$

$\therefore$ There are two Solution of given equation $x = 0$ and $x = \frac{5}{3}$

(ii)



Ex. 6 Find the domain $f(x) = \dfrac{1}{\sqrt{|[|x|-5]|-11}}$ where [.] denotes greatest integer function.

Sol. $|[|x|-5]| > 11$

So $[|x|-5] > 11$ or $[|x|-5] < -11$

$[|x|] > 16$ $[|x|] < -6$

$|x| \geq 17$ or $[|x|] < -6$ (Not Possible)

$\Rightarrow$ $x \leq -17$ or $x \geq 17$

So $x \in (-\infty, -17] \cup [17, \infty)$

Ex. 6 Examine whether following pair of functions are identical or not ?

(i) $f(x) = \dfrac{x^2-1}{x-1}$ and $g(x) = x + 1$

(ii) $f(x) = \sin^2x + \cos^2x$ and $g(x) = \sec^2x - \tan^2x$

Sol. **(i)** No, as domain of f(x) is $R - \{1\}$
while domain of g(x) is R

(ii) No, as domain are not same. Domain of f(x) is R

while that of g(x) is $R - \left\{(2n+1)\dfrac{\pi}{2};\ n \in I\right\}$

Ex. 7 Find the value of $\left[\dfrac{1}{2}\right] + \left[\dfrac{1}{2} + \dfrac{1}{1000}\right] + \ldots\ldots \left[\dfrac{1}{2} + \dfrac{2946}{1000}\right]$ where [.] denotes greatest integer function ?

Sol. $\left[\dfrac{1}{2}\right] + \left[\dfrac{1}{2} + \dfrac{1}{1000}\right] + \ldots\ldots \left[\dfrac{1}{2} + \dfrac{499}{1000}\right] + \left[\dfrac{1}{2} + \dfrac{500}{1000}\right] + \ldots\ldots \left[\dfrac{1}{2} + \dfrac{1499}{1000}\right] + \left[\dfrac{1}{2} + \dfrac{1500}{1000}\right] + \ldots\ldots$

$+ \left[\dfrac{1}{2} + \dfrac{2499}{1000}\right] + \left[\dfrac{1}{2} + \dfrac{2500}{1000}\right] + \ldots\ldots \left[\dfrac{1}{2} + \dfrac{2946}{1000}\right]$

$= 0 + 1 \times 1000 + 2 \times 1000 + 3 \times 447 = 3000 + 1341 = 4341$

Ex. 8 Find the range of $f(x) = \dfrac{x-[x]}{1+x-[x]}$, where [.] denotes greatest integer function.

Sol. $y = \dfrac{x-[x]}{1+x-[x]} = \dfrac{\{x\}}{1+\{x\}}$

$\therefore \quad \dfrac{1}{y} = \dfrac{1}{\{x\}} + 1 \quad \Rightarrow \quad \dfrac{1}{\{x\}} = \dfrac{1-y}{y} \quad \Rightarrow \quad \{x\} = \dfrac{y}{1-y}$

$0 \le \{x\} < 1 \quad \Rightarrow \quad 0 \le \dfrac{y}{1-y} < 1$

Range = [0, 1/2)

Ex. 9 Let $f(x) = e^x$; $R^+ \to R$ and $g(x) = \sin^{-1} x$; $[-1, 1] \to \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$. Find domain and range of fog(x)

Sol. Domain of f(x) : $(0, \infty)$ Range of g(x) : $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

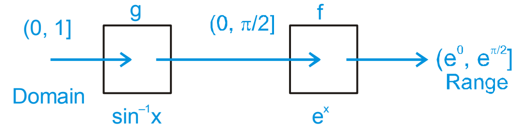
values in range of g(x) which are accepted by f(x) are $\left(0, \dfrac{\pi}{2}\right]$

$\Rightarrow \quad 0 < g(x) \le \dfrac{\pi}{2} \quad \Rightarrow \quad 0 < \sin^{-1}x \le \dfrac{\pi}{2} \quad \Rightarrow \quad 0 < x \le 1$

Hence domain of fog(x) is $x \in (0, 1]$

Therefore Domain : (0, 1]

Range : $(1, e^{\pi/2}]$



Ex. 10 Let $A = \{x : -1 \le x \le 1\} = B$ be a mapping $f : A \to B$. For each of the following functions from A to B, find whether it is surjective or bijective.

(A) $f(x) = |x|$ **(B)** $f(x) = x|x|$ **(C)** $f(x) = x^3$

(D) $f(x) = [x]$ **(E)** $f(x) = \sin \dfrac{\pi x}{2}$

Sol. **(A)** $f(x) = |x|$

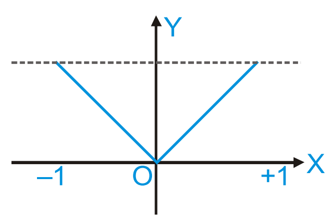
Graphically ;

Which shows many one, as the straight line is parallel to x-axis and cuts at two points. Here range for $f(x) \in [0, 1]$

Which is clearly subset of co-domain i.e., $[0, 1] \subseteq [-1, 1]$ Thus, into.

Hence, function is many-one-into

$\therefore$ Neither injective nor surjective

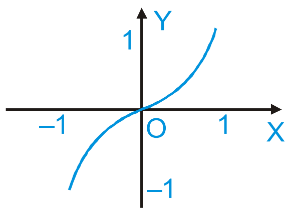


(B) $f(x)=x\,|x|=\begin{Bmatrix} -x^2, & -1<x<0 \\ x^2, & 0\ \ x<1 \end{Bmatrix},$

Graphically,

The graph shows f(x) is one-one, as the straight line parallel to x-axis cuts only at one point.

Here, range



$f(x) \in [-1, 1]$

Thus, range = co-domain

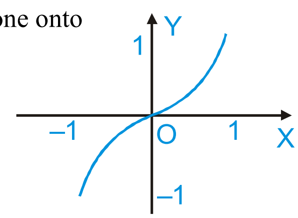
Hence, onto.

Therefore, f(x) is one-one onto or (Bijective).

(C) $f(x)=x^3$,

Graphically;

Graph shows f(x) is one-one onto (i.e. Bijective)

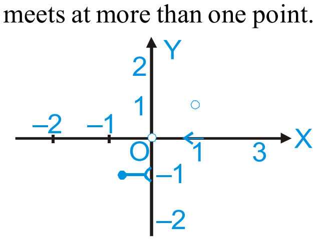


[as explained in above example]

(D) $f(x)=[x]$,

Graphically;

Which shows f(x) is many-one, as the straight line parallel to x-axis meets at more than one point.



Here, range

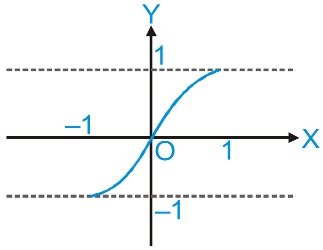
$f(x) \in \{-1, 0, 1\}$

which shows into as range $\subseteq$ co-domain

Hence, many-one-into

(E) f(x) = sin

Graphically;



Which shows f(x) is one-one and onto as range = co-domain.

Therefore, f(x) is bijective.

Ex. 11 Composition of piecewise defined functions :

If $f(x) = ||x - 3| - 2|$ $\quad 0 \le x \le 4$

$g(x) = 4 - |2 - x|$ $\quad -1 \le x \le 3$

then find fog(x) and draw rough sketch of fog(x).

Sol. $f(x) = ||x - 3| - 2| \; 0 \le x \le 4$

$$= \begin{cases} |x-1| & 0 \le x < 3 \\ |x-5| & 3 \le x \le 4 \end{cases} = \begin{cases} 1-x & 0 \le x < 1 \\ x-1 & 1 \le x < 3 \\ 5-x & 3 \le x \le 4 \end{cases}$$

$g(x) = 4 - |2 - x| \qquad -1 \le x \le 3$

$$= \begin{cases} 4-(2-x) & -1 \le x < 2 \\ 4-(x-2) & 2 \le x \le 3 \end{cases} = \begin{cases} 2+x & -1 \le x < 2 \\ 6-x & 2 \le x \le 3 \end{cases}$$

$$\therefore \quad fog(x) = \begin{cases} 1-g(x) & 0 \le g(x) < 1 \\ g(x)-1 & 1 \le g(x) < 3 \\ 5-g(x) & 3 \le g(x) \le 4 \end{cases} = \begin{cases} 1-(2+x) & 0 \le 2+x < 1 & \text{and} & -1 \le x < 2 \\ 2+x-1 & 1 \le 2+x < 3 & \text{and} & -1 \le x < 2 \\ 5-(2+x) & 3 \le 2+x \le 4 & \text{and} & -1 \le x < 2 \\ 1-6+x & 0 \le 6-x < 1 & \text{and} & 2 \le x \le 3 \\ 6-x-1 & 1 \le 6-x < 3 & \text{and} & 2 \le x \le 3 \\ 5-6+x & 3 \le 6-x \le 4 & \text{and} & 2 \le x \le 3 \end{cases}$$

$$= \begin{cases} -1-x & -2 \le x < -1 & \text{and} & -1 \le x < 2 \\ 1+x & -1 \le x < 1 & \text{and} & -1 \le x < 2 \\ 3-x & 1 \le x \le 2 & \text{and} & -1 \le x < 2 \\ x-5 & -6 \le -x < -5 & \text{and} & 2 \le x \le 3 \\ 5-x & -5 \le -x < -3 & \text{and} & 2 \le x \le 3 \\ x-1 & -3 \le -x \le -2 & \text{and} & 2 \le x \le 3 \end{cases} = \begin{cases} -1-x & -2 \le x < -1 & \text{and} & -1 \le x < 2 \\ 1+x & -1 \le x < 1 & \text{and} & -1 \le x < 2 \\ 3-x & 1 \le x \le 2 & \text{and} & -1 \le x < 2 \\ x-5 & 5 < x \le 6 & \text{and} & 2 \le x \le 3 \\ 5-x & 3 < x \le 5 & \text{and} & 2 \le x \le 3 \\ x-1 & 2 \le x \le 3 & \text{and} & 2 \le x \le 3 \end{cases}$$

$$= \begin{cases} 1+x & -1 \le x < 1 \\ 3-x & 1 \le x < 2 \\ x-1 & 2 \le x \le 3 \end{cases}$$

Ex. 12 **(i)** Find whether $f(x) = x + \cos x$ is one-one.

(ii) Identify whether the function $f(x) = -x^3 + 3x^2 - 2x + 4$ for $f : R \to R$ is ONTO or INTO

(iii) $f(x) = x^2 - 2x + 3; [0, 3] \to A$. Find whether f(x) is injective or not. Also find the set A, if f(x) is surjective.

Sol. (i) The domain of f(x) is R. $f'(x) = 1 - \sin x$.

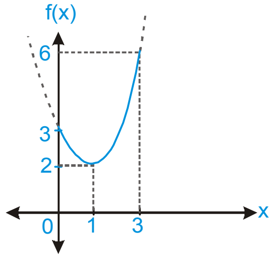
$\therefore$ $f'(x) \geq 0 \ \forall \ x \in$ complete domain and equality holds at discrete points only

$\therefore$ f(x) is strictly increasing on R. Hence f(x) is one-one.

(ii) As range $\equiv$ codomain, therefore given function is ONTO

(iii) $f'(x) = 2(x-1); 0 \leq x \leq 3$

$$\therefore \quad f'(x) = \begin{cases} -ve & ; \quad 0 \leq x < 1 \\ +ve & ; \quad 1 < x < 3 \end{cases}$$



$\therefore$ f(x) is non monotonic. Hence it is not injective.

For f(x) to be surjective, A should be equal to its range. By graph range is [2, 6]

$\therefore$ $A \equiv [2, 6]$

Ex. 13 If f be the greatest integer function and g be the modulus function, then $(gof)\left(-\frac{5}{3}\right) - (fog)\left(-\frac{5}{3}\right) =$

(A) 1 **(B)** –1 **(C)** 2 **(D)** 4

Sol. Given $(gof)\left(\frac{-5}{3}\right) - (fog)\left(\frac{-5}{3}\right) = g\left\{f\left(\frac{-5}{3}\right)\right\} - f\left\{g\left(\frac{-5}{3}\right)\right\} = g(-2) - f\left(\frac{5}{3}\right) = 2 - 1 = 1$ **Ans.(A)**

Ex. 14 Show that $\log\left(x + \sqrt{x^2+1}\right)$ is an odd function.

Sol. Let $f(x) = \log\left(x + \sqrt{x^2+1}\right)$.

Then $f(-x) = \log\left(-x + \sqrt{(-x)^2+1}\right)$

$$= \log\left(\frac{\left(\sqrt{x^2+1} - x\right)\left(\sqrt{x^2+1} + x\right)}{\sqrt{x^2+1} + x}\right) = \log \frac{1}{\sqrt{x^2+1}+x} = -\log\left(x + \sqrt{x^2+1}\right) = -f(x)$$

or $f(x) + f(-x) = 0$

Hence f(x) is an odd function.

Ex. 15 Show that $\cos^{-1} x$ is neither odd nor even.

Sol. Let $f(x) = \cos^{-1}x$. Then $f(-x) = \cos^{-1}(-x) = \pi - \cos^{-1} x$ which is neither equal to f(x) nor equal to – f(x).

Hence $\cos^{-1} x$ is neither odd nor even

Ex. 161 Which of the following functions is (are) even, odd or neither :

(i) $f(x) = x^2 \sin x$ **(ii)** $f(x) = \sqrt{1+x+x^2} - \sqrt{1-x+x^2}$

(iii) $f(x) = \log\left(\frac{1-x}{1+x}\right)$ **(iv)** $f(x) = \sin x - \cos x$

(v) $f(x) = \frac{e^x + e^{-x}}{2}$

Sol. (i) $f(-x) = (-x)^2 \sin(-x) = -x^2 \sin x = -f(x).$ Hence f(x) is odd.

(ii) $f(-x) = \sqrt{1+(-x)+(-x)^2} - \sqrt{1-(-x)+(-x)^2}$

$= \sqrt{1-x+x^2} - \sqrt{1+x+x^2} = -f(x).$ Hence f(x) is odd.

(iii) $f(-x) = \log\left(\frac{1-(-x)}{1+(-x)}\right) = \log\left(\frac{1+x}{1-x}\right) = -f(x).$

Hence f(x) is odd

(iv) $f(-x) = \sin(-x) - \cos(-x) = -\sin x - \cos x.$

Hence f(x) is neither even nor odd.

(v) $f(-x) = \frac{e^{-x} + e^{-(-x)}}{2} = \frac{e^{-x} + e^{x}}{2} = f(x).$ Hence f(x) is even

Ex. 17 Let $f : R \to R$ be defined by $f(x) = (e^x - e^{-x})/2$. Is f(x) invertible ? If so, find its inverse.

Sol. Let us check for invertibility of f(x) :

(A) One-One :

Let $x_1, x_2 \in R$ and $x_1 < x_2$

$\Rightarrow$ $e^{x_1} < e^{x_2}$ (Because base e > 1) .....**(i)**

Also $x_1 < x_2$ $\Rightarrow$ $-x_2 < -x_1$

$\Rightarrow$ $e^{-x_2} < e^{-x_1}$ (Because base e > 1) .....**(ii)**

(i) + (ii) $\Rightarrow$ $e^{x_1} + e^{-x_2} < e^{x_2} + e^{-x_1}$

$\Rightarrow$ $\frac{1}{2}\left(e^{x_1} - e^{-x_1}\right) < \frac{1}{2}\left(e^{x_2} - e^{-x_2}\right)$ $\Rightarrow$ $f(x_1) < f(x_2)$ i.e. f is one-one.

(B) Onto :

As x tends to larger and larger values so does f(x) and

when $x \to \infty$, $f(x) \to \infty$.

Similarly as $x \to -\infty$, $f(x) \to -\infty$ i.e. $-\infty < f(x) < \infty$ so long as $x \in (-\infty, \infty)$

Hence the range of f is same as the set R. Therefore f(x) is onto.

Since f(x) is both one-one and onto, f(x) is invertible.

(C) To find f^{-1} :

Let f^{-1} be the inverse function of f, then by rule of identity $fof^{-1}(x) = x$

$$\frac{e^{f^{-1}(x)} - e^{-f^{-1}(x)}}{2} = x \quad \Rightarrow \quad e^{2f^{-1}(x)} - 2xe^{f^{-1}(x)} - 1 = 0$$

$$\Rightarrow \quad e^{f^{-1}(x)} = \frac{2x \pm \sqrt{4x^2+4}}{2} \quad \Rightarrow \quad e^{f^{-1}(x)} = x \pm \sqrt{1+x^2}$$

Since $e^{f^{-1}(x)} > 0$, hence negative sign is ruled out and

Hence $e^{f^{-1}(x)} = x + \sqrt{1+x^2}$

Taking logarithm, we have $f^{-1}(x) = \ell n(x + \sqrt{1+x^2})$.

Ex. 18 Find the periods (if periodic) of the following functions, where [.] denotes the greatest integer function

(i) $f(x) = e^{\ell n(\sin x)} + \tan^3 x - \text{cosec}(3x - 5)$

(ii) $f(x) = x - [x - b], b \in R$

(iii) $f(x) = \dfrac{|\sin x + \cos x|}{|\sin x| + |\cos x|}$

(iv) $f(x) = \tan \dfrac{\pi}{2}[x]$

(v) $f(x) = \cos(\sin x) + \cos(\cos x)$

(vi) $f(x) = \dfrac{(1+\sin x)(1+\sec x)}{(1+\cos x)(1+\text{cosec}\, x)}$

(vii) $f(x) = e^{x - [x] + |\cos \pi x| + |\cos 2\pi x| + \ldots\ldots + |\cos n\pi x|}$

Sol.(i) $f(x) = e^{\ell n(\sin x)} + \tan^3 x - \text{cosec}(3x - 5)$

Period of $e^{\ell n \sin x} = 2\pi$, $\tan^3 x = \pi$

$\text{cosec}\,(3x - 5) = \dfrac{2\pi}{3}$

$\therefore$ Period $= 2\pi$

(ii) $f(x) = x - [x - b] = b + \{x - b\}$

$\therefore$ Period $= 1$

(iii) $f(x) = \dfrac{|\sin x + \cos x|}{|\sin x| + |\cos x|}$

Since period of $|\sin x + \cos x| = \pi$ and period of $|\sin x| + |\cos x|$ is $\dfrac{\pi}{2}$. Hence f(x) is periodic with π as its period

(iv) $f(x) = \tan \dfrac{\pi}{2}[x]$

$$\tan \frac{\pi}{2}[x+T] = \tan \frac{\pi}{2}[x] \quad \Rightarrow \quad \frac{\pi}{2}[x+T] = n\pi + \frac{\pi}{2}[x]$$

$\therefore$ T = 2

$\therefore$ Period = 2

(v) Let f(x) is periodic then f(x + T) = f(x)

$\Rightarrow$ cos(sin(x + T))+ cos(cos(x + T)) = cos(sinx) + cos(cosx)

If x = 0 then cos(sinT) + cos(cosT) = cos(0) + cos(1) = $\cos\left(\cos\frac{\pi}{2}\right) + \cos\left(\sin\frac{\pi}{2}\right)$

On comparing $T = \frac{\pi}{2}$

(vi) $f(x) = \frac{(1+\sin x)(1+\sec x)}{(1+\cos x)(1+\operatorname{cosec} x)} = \frac{(1+\sin x)(1+\sec x)}{(1+\cos x)(1+\operatorname{cosec} x)}$

$\Rightarrow$ f(x) = tanx

Hence f(x) has period π.

(vii) $f(x) = e^{x-[x]+|\cos \pi x|+|\cos 2\pi x|+........+|\cos n\pi x|}$

Period of x – [x] = 1

Period of |cosπx| = 1

Period of |cos2πx| = $\frac{1}{2}$

....

Period of |cosnπx| = $\frac{1}{2}$

So period of f(x) will be L.C.M. of all period = 1

Ex. 19 Find the periods (if periodic) of the following functions, where [.] denotes the greatest integer function

(i) $f(x) = e^{x-[x]} + \sin x$ **(ii)** $f(x) = \sin\frac{\pi x}{\sqrt{2}} + \cos\frac{\pi x}{\sqrt{3}}$

(iii) $f(x) = \sin\frac{\pi x}{\sqrt{3}} + \cos\frac{\pi x}{2\sqrt{3}}$

Sol.(i) Period of $e^{x-[x]} = 1$

period of sinx = 2π

$\because$ L.C.M. of rational and an irrational number does not exist.

$\therefore$ not periodic.

(ii) Period of $= \sin\frac{\pi x}{\sqrt{2}} = \frac{2\pi}{\pi/\sqrt{2}} = 2\sqrt{2}$

Period of $= \cos\frac{\pi x}{\sqrt{3}} = \frac{2\pi}{\pi/\sqrt{3}} = 2\sqrt{3}$

$\because$ L.C.M. of two different kinds of irrational number does not exist.

$\therefore$ not periodic.

(iii) Period of $\sin\frac{\pi x}{\sqrt{3}} = \frac{2\pi}{\pi/\sqrt{3}} = 2\sqrt{3}$

Period of $\cos\frac{\pi x}{2\sqrt{3}} = \frac{2\pi}{\pi/2\sqrt{3}} = 4\sqrt{3}$

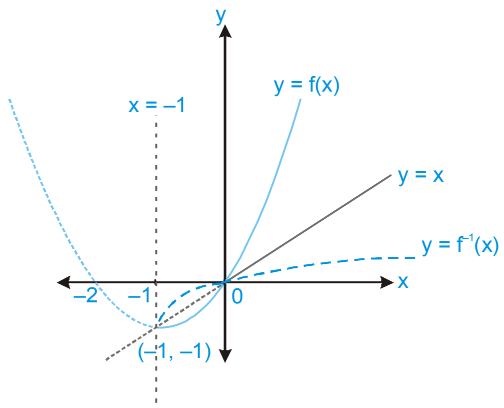
$\because$ L.C.M. of two similar irrational number exist.

$\therefore$ Periodic with period = $4\sqrt{3}$ **Ans.**

20.(i) Let $f(x) = x^2 + 2x$; $x \geq -1$. Draw graph of $f^{-1}(x)$ also find the number of solutions of the equation, $f(x) = f^{-1}(x)$

(ii) If $y = f(x) = x^2 - 3x + 1$, $x \geq 2$. Find the value of $g'(1)$ where g is inverse of f

Sol. (i)



$f(x) = f^{-1}(x)$ is equivalent to $f(x) = x$

$\Rightarrow$ $x^2 + 2x = x$ $\Rightarrow$ $x(x + 1) = 0$ $\Rightarrow$ $x = 0, -1$

Hence two solution for $f(x) = f^{-1}(x)$

(iv) $y = 1$

$\Rightarrow$ $x^2 - 3x + 1 = 1$

$\Rightarrow$ $x(x - 3) = 0$ $\Rightarrow$ $x = 0, 3$

But $x \geq 2$ $\therefore$ $x = 3$

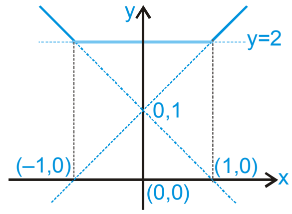
Now $g(f(x)) = x$

Differentiating both sides w.r.t. x

$\Rightarrow$ $g'(f(x)).\, f'(x) = 1$ $\Rightarrow$ $g'(f(x)) = \frac{1}{f'(x)}$

$\Rightarrow$ $g'(f(3)) = \frac{1}{f'(3)}$ $\Rightarrow$ $g'(1) = = \frac{1}{6-3} = \frac{1}{3}$ (As $f'(x) = 2x - 3$)

Ex. 21 Find f(x) = max {1 + x, 1 – x, 2}.

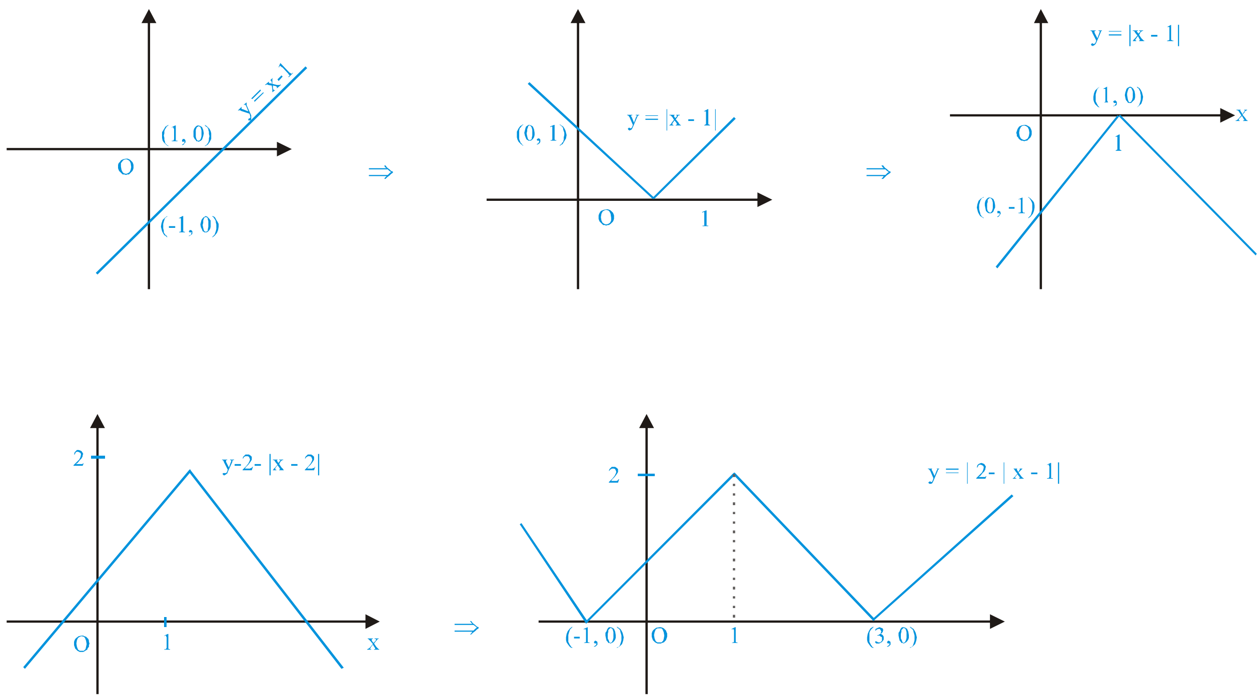


Sol. From the graph it is clear that

$$f(x) = \begin{cases} 1-x & ; \quad x < -1 \\ 2 & ; \quad -1 \le x \le 1 \\ 1+x & ; \quad x > 1 \end{cases}$$

Ex. 22 Draw the graph of y = |2 –|x –1||.

Sol.

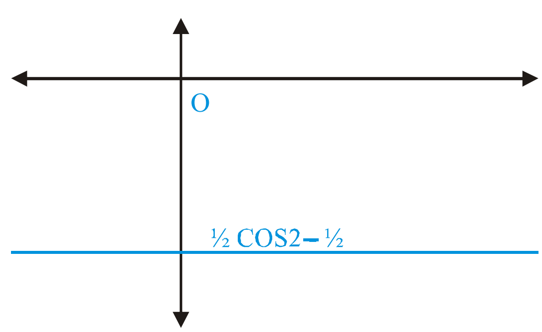


Ex. 23 Draw the graph of f(x) = cosx cos(x + 2) – cos²(x + 1).

Sol. $f(x) = \cos x \cos(x+2) - \cos^2(x+1)$

$$= \frac{1}{2}[\cos(2x+2) + \cos 2] - \frac{1}{2}[\cos(2x+2) + 1]$$

$$= \frac{1}{2}\cos 2 - \frac{1}{2} < 0$$



Exercise # 1

[Single Correct Choice Type Questions]

1. The domain of $f(x)=\sqrt{\dfrac{1-|x|}{2-|x|}}$, is -

(A) $(-\infty,\infty)-[-2,2]$ **(B)** $(-\infty,\infty)-[-1,1]$

(C) $[-1,1]\cup(-\infty,-2)\cup(2,\infty)$ **(D)** none

2. The domain of the function $f(x)=\sin^{-1}\left(\dfrac{1+x^3}{2x^{3/2}}\right)+\sqrt{\sin(\sin x)}+\log_{(3\{x\}+1)}(x^2+1)$,

where {.} represents fractional part function, is:

(A) $x\in\{1\}$ **(B)** $x\in R-\{1,-1\}$ **(C)** $x>3, x\neq I$ **(D)** none of these

3. The domain of the function $f(x)=\dfrac{1}{\log_{10}(1-x)}+\sqrt{x+2}$, is -

(A) $[-2,0)\cup(0,1)$ **(B)** $(-2,0)\cup(0,1]$ **(C)** $(-2,0)\cup(0,1]$ **(D)** $(-2,0)\cup[0,1]$

4. If $q^2-4pr=0$, $p>0$, then the domain of the function $f(x)=\log(px^3+(p+q)x^2+(q+r)x+r)$ is:

(A) $R-\left\{-\dfrac{q}{2p}\right\}$ **(B)** $R-\left[(-\infty,-1]\cup\left\{-\dfrac{q}{2p}\right\}\right]$

(C) $R-\left[(-\infty,-1)\cap\left\{-\dfrac{q}{2p}\right\}\right]$ **(D)** none of these

5. If f(x) is a polynomial function satisfying the condition $f(x).f(1/x)=f(x)+f(1/x)$ and $f(2)=9$ then -

(A) $2f(4)=3f(6)$ **(B)** $14f(1)=f(3)$ **(C)** $9f(3)=f(5)$ **(D)** $f(10)=f(11)$

6. Domain to function $\sqrt{\log\left\{(5x-x^2)/6\right\}}$ is -

(A) (2, 3) **(B)** [2, 3] **(C)** [1, 2] **(D)** [1, 3]

7. Domain and range of $f(x)=\sqrt{x-1}+2\sqrt{3-x}$ is

(A) $D:[1,3];R:\left[\sqrt{2},\sqrt{10}\right]$ **(B)** $D:[1,5];R:\left[\sqrt{2},\sqrt{10}\right]$

(C) $D:(-\infty,1]\cup[3,\infty), R:\left[1,\sqrt{3}\right]$ **(D)** $D:[1,5], R:\left[1,\sqrt{3}\right]$

8. If $A=\{-2,-1,0,1,2\}$ & $f:A\to Z$; $f(x)=x^2+1$, then the range of f is

(A) {0, 1, 2, 5} **(B)** {1, 2, 5} **(C)** {–5, –2, 1, 2, 3} **(D)** A

9. The greatest value of the function $f(x) = (\sin^{-1} x)^3 + (\cos^{-1} x)^3$ is:

(A) $\frac{\pi^3}{32}$ **(B)** $\frac{\pi^3}{8}$ **(C)** $\frac{3\pi^3}{8}$ **(D)** $\frac{7\pi^3}{8}$

10. The range of the function $f(x) = e^{x} - e^{-x}$, is -

(A) $[0, \infty)$ **(B)** $(-\infty, 0)$ **(C)** $(-\infty, \infty)$ **(D)** none

11. The range of the function $f(x) = {}^{7-x}P_{x-3}$, is -

(A) $\{1, 2, 3\}$ **(B)** $\{1, 2, 3, 4, 5, 6\}$ **(C)** $\{1, 2, 3, 4\}$ **(D)** $\{1, 2, 3, 4, 5\}$

12. If $f(x) = 2[x] + \cos x$, then $f: R \to R$ is: (where [ ] denotes greatest integer function)

(A) one–one and onto **(B)** one–one and into

(C) many–one and into **(D)** many–one and onto

13. $f: [-1, 1] \to [-1, 2]$, $f(x) = x + |x|$, is -

(A) one-one onto **(B)** one-one into **(C)** many one onto **(D)** many one into

14. Let $f: R \to R$ be a function such that $f(0) = 1$ and for any $x, y \in R$, $f(xy + 1) = f(x) f(y) - f(y) - x + 2$. Then f is

(A) one-one and onto **(B)** one-one but not onto **(C)** many one but onto **(D)** many one and into

15. Let f: R R be a function defined by $f(x) = \frac{x^2 - 3x + 4}{x^2 + 3x + 4}$ then f is -

(A) one – one but not onto **(B)** onto but not one – one

(C) onto as well as one – one **(D)** neither onto nor one – one

16. Which one of the following pair of functions are identical ?

(A) $e^{(\ell nx)/2}$ and $\sqrt{x}$

(B) $\tan^{-1}(\tan x)$ and $\cot^{-1}(\cot x)$

(C) $\cos^2 x + \sin^4 x$ and $\sin^2 x + \cos^4 x$

(D) a $\frac{|x|}{x}$ and sgn (x), where sgn(x) stands for signum function.

17. If $f(x) = \cos\left[\frac{1}{2}\pi^2\right] x + \sin x\left[\frac{1}{2}\pi^2\right]$, [x] denoting the greatest integer function, then -

(A) $f(0) = 0$ **(B)** $f\left(\frac{\pi}{3}\right) = \frac{1}{4}$ **(C)** $f\left(\frac{\pi}{2}\right) = 1$ **(D)** $f(\pi) = 0$

18. If $f(x) = \cos(\log x)$, then $f(x) f(y) - \frac{1}{2}[f(x/y) + f(xy)]$ is equal to -

(A) -1 **(B)** $1/2$ **(C)** -2 **(D)** 0

19. The value of b and c for which the identity $f(x + 1) - f(x) = 8x + 3$ is satisfied, where $f(x) = bx^2 + cx + d$, are –

(A) $b = 2, c = 1$ **(B)** $b = 4, c = -1$ **(C)** $b = -1, c = 4$ **(D)** $b = -1, c = 1$

20. If $f(x) = \dfrac{4a-7}{3}x^3 + (a-3)x^2 + x + 5$ is a one-one function, then

(A) $2 \le a \le 8$ **(B)** $1 \le a \le 2$ **(C)** $0 \le a \le 1$ **(D)** None of these

21. Let $f : R \to R$ be a function defined by then -

(A) f is a bijection **(B)** f is an injection only

(C) f is a surjection **(D)** f is neither injection nor a surjection

22. If $f(x) = \{x\} + \{x+1\} + \{x+2\}........\{x+99\}$, then the value of $[f(\sqrt{2})]$ is, where {.} denotes fractional part function & [.] denotes the greatest integer function

(A) 5050 **(B)** 4950 **(C)** 41 **(D)** 14

23. The minimum value of $f(x) = |3-x| + |2+x| + |5-x|$ is -

(A) 0 **(B)** 7 **(C)** 8 **(D)** 10

24. If the function $f : R \to A$ given by $f(x) = \dfrac{x^2}{x^2+1}$ is a surjection, then A =

(A) R **(B)** [0, 1] **(C)** (0, 1] **(D)** [0, 1)

25. The fundamental period of function $f(x) = [x] + \left[x + \dfrac{1}{3}\right] + \left[x + \dfrac{2}{3}\right] - 3x + 15$, where [.] denotes greatest integer function, is :

(A) $\dfrac{1}{3}$ **(B)** $\dfrac{2}{3}$ **(C)** 1 **(D)** non-periodic

26. $f(x) = |x-1|$, f: $R^+ \to R$, $g(x) = e^x$, g: $[-1, \infty) \to R$. If the function fog (x) is defined, then its domain and range respectively are:

(A) $(0, \infty)$ and $[0, \infty)$ **(B)** $[-1, \infty)$ and $[0, \infty)$

(C) $[-1, \infty)$ and $\left[1 - \dfrac{1}{e}, \infty\right)$ **(D)** $[-1, \infty)$ and $\left[\dfrac{1}{e} - 1, \infty\right)$

27. Let $f : R \to R$ be a function defined by $f(x) = \dfrac{e^{|x|} - e^{-x}}{e^x + e^{-x}}$ then -

(A) f is a bijection **(B)** f is an injection only

(C) f is a surjection **(D)** f is neither injection nor a surjection

28. Let $f : (2, 4) \to (1, 3)$ be a function defined by $f(x) = x - \left[\dfrac{x}{2}\right]$ (where [.] denotes the greatest integer function), then $f^{-1}(x)$ is equal to :

(A) 2x **(B)** $x + \left[\dfrac{x}{2}\right]$ **(C)** x + 1 **(D)** x – 1

29. The mapping $f : R \to R$ given by $f(x) = x^3 + ax^2 + bx + c$ is a bijection if

(A) $b^2 \le 3a$ **(B)** $a^2 \le 3b$ **(C)** $a^2 \ge 3b$ **(D)** $b^2 \ge 3a$

30. The period of the function $f(x) = \sin\left(\cos\frac{x}{2}\right) + \cos(\sin x)$ equal -

(A) $\frac{\pi}{2}$ **(B)** 2π **(C)** π **(D)** 4π

31. Let $f(x) = \sin\sqrt{[a]}\; x$ (where [ ] denotes the greatest integer function). If f is periodic with fundamental period π, then a belongs to -

(A) $[2, 3)$ **(B)** $\{4, 5\}$ **(C)** $[4, 5]$ **(D)** $[4, 5)$

32. Which of the following function has a period of 2π ?

(A) $f(x) = \sin\left(2\pi x + \frac{\pi}{3}\right) + 2\sin\left(3\pi x + \frac{\pi}{4}\right) + 3\sin 5\pi x$ **(B)** $f(x) = \sin\frac{\pi x}{3} + \sin\frac{\pi x}{4}$

(C) $f(x) = \sin x + \cos 2x$ **(D)** none

33. A function whose graph is symmetrical about the origin is given by -

(A) $f(x) = e^x + e^{-x}$ **(B)** $f(x) = \sin(\sin(\cos(\sin x)))$

(C) $f(x + y) = f(x) + f(y)$ **(D)** $\sin x + \sin|x|$

34. If $f : R \to R$ is a function satisfying the property $f(x+1) + f(x+3) =$ then the period of $f(x)$ is -

(A) 4 **(B)** K **(C)** 1 **(D)** π

35. If $f(x) = 3x - 5$, then $f^{-1}(x)$ -

(A) is given by $\frac{1}{3x-5}$ **(B)** is given by $\frac{x+5}{3}$

(C) does not exist because f is not one–one **(D)** does not exist because f is not onto

36. If the function $f : [1, \infty)\; [1, \infty)$ is defined by $f(x) = 2^{x(x-1)}$, then $f^{-1}(x)$ is -

(A) $\left(\frac{1}{2}\right)^{x(x-1)}$ **(B)** $\frac{1}{2}\left(1 + \sqrt{1 + 4\log_2 x}\right)$ **(C)** $\frac{1}{2}\left(1 - \sqrt{1 + 4\log_2 x}\right)$ **(D)** Not defined

Exercise # 2 — Part # I [Multiple Correct Choice Type Questions]

1. Which of the functions defined below are NOT one-one function(s) ?

(A) $f(x) = 5(x^2 + 4), (x \ R)$ **(B)** $g(x) = 2x+(1/x)$

(C) $h(x) = \ell n(x^2+x+1), (x \ R)$ **(D)** $f(x) = e^{-x}$

2. Which of the following functions from Z to itself are NOT bijections ?

(A) $f(x) = x^3$ **(B)** $f(x) = x + 2$ **(C)** $f(x) = 2x + 1$ **(D)** $f(x) = x^2 + x$

3. If $f(x) = \sin \ell n \left(\frac{\sqrt{4-x^2}}{1-x}\right)$, then

(A) domain of f(x) is $(-2, 1)$ **(B)** domain of f(x) is $[-1, 1]$

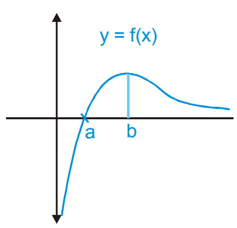
(C) range of f(x) is $[-1, 1]$ **(D)** range of f(x) is $[-1, 1)$

4. The function cot(sinx) -

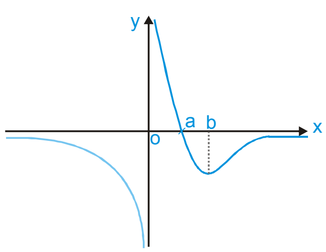
(A) is not defined for $x = (4n + 1)\frac{\pi}{2}$ **(B)** is not defined for $x = n\pi$

(C) lies between $-\cot 1$ and $\cot 1$ **(D)** can't lie between $-\cot 1$ and $\cot 1$

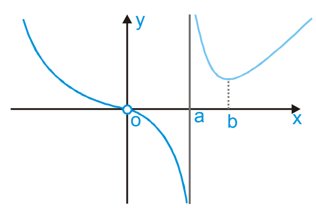
5. The graph of function f(x) is as shown, adjacently. Then the graph of $\frac{1}{f(|x|)}$ is -



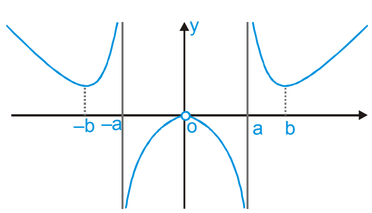
(A)



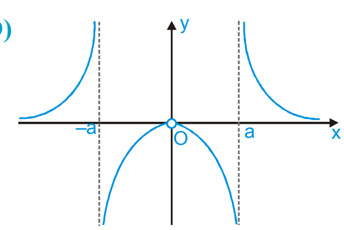
(B)



(C)



(D)



6. Which of the following function(s) is/are periodic ?

(A) $f(x) = 3x - [3x]$ **(B)** $g(x) = \sin(1/x)$, $x\ \ 0$ & $g(0) = 0$

(C) $h(x) = x \cos x$ **(D)** $w(x) = \sin(\sin(\sin x))$

7. The fundamental period of $\dfrac{|\sin x| + |\cos x|}{|\sin x - \cos x| + |\sin x + \cos x|}$ is -

(A) π **(B)** $\dfrac{\pi}{2}$ **(C)** 2π **(D)** $\dfrac{2\pi}{3}$

8. The range of the function $f(x) = \sin\left[\log\left(\dfrac{\sqrt{4-x^2}}{1-x}\right)\right]$ is -

(A) $[-1, 1]$ **(B)** $(-1, 1)$ **(C)** $[-1, 1)$ **(D)** cannot be determined

9. If $F(x) = \dfrac{\sin \pi [x]}{\{x\}}$, then F (x) is: (where { . } denotes fractional part function and [.] denotes greatest integer function and sgn (x) is a signum function)

(A) periodic with fundamental period 1 **(B)** even

(C) range is singleton **(D)** identical to $\text{sgn}\left(\text{sgn}\dfrac{\{x\}}{\sqrt{\{x\}}}\right) - 1$

10. In the following functions defined from [–1, 1] to [–1, 1] , then functions which are not bijective are

(A) $\sin(\sin^{-1}x)$ **(B)** $\dfrac{2}{\pi}\sin^{-1}(\sin x)$ **(C)** $(\text{sgn}\ x)\ \ell n\ e^x$ **(D)** $x^3\ \text{sgn}\ x$

11. Let $f : [-1, 1] \to [0, 2]$ be a linear function which is onto, then f(x) is/are

(A) $1 - x$ **(B)** $1 + x$ **(C)** $x - 1$ **(D)** $x + 2$

12. Which of the following functions are not homogeneous ?

(A) $x + y\cos\dfrac{y}{x}$ **(B)** $\dfrac{xy}{x + y^2}$ **(C)** $\dfrac{x - y\cos x}{y\sin x + y}$ **(D)** $\dfrac{x}{y}\ \ell n\left(\dfrac{y}{x}\right) + \dfrac{y}{x}\ \ell n\left(\dfrac{x}{y}\right)$

13. Given the function f(x) $2f(x) + xf\left(\dfrac{1}{x}\right) - 2f\left(\left|\sqrt{2}\sin\ \pi\left(x + \dfrac{1}{4}\right)\right|\right) = 4\cos^2\dfrac{\pi x}{2} + x\cos\dfrac{\pi}{x}$ such that , then which one of the following is correct ?

(A) $f(2) + f(1/2) = 1$ **(B)** $f(1) = -1$, but the values of $f(2)$, $f(1/2)$ cannot be determined

(C) $f(2) + f(1) = f(1/2)$ **(D)** $f(2) + f(1) = 0$

14. The function $f(x) = \sqrt{\log_{x^2}(x)}$ is defined for x belonging to -

(A) $(-\infty, 0)$ **(B)** $(0, 1)$ **(C)** $(1, \infty)$ **(D)** $(0, \infty)$

15. If $f(x + ay, x - ay) = axy$ then $f(x, y)$ is equal to -

(A) $\frac{x^2 - y^2}{4}$ **(B)** $\frac{x^2 + y^2}{4}$ **(C)** 4xy **(D)** none

16. Which of following pairs of functions are identical.

(A) $f(x) = e^{\ell n \sec^{-1} x}$ and $g(x) = \sec^{-1} x$

(B) $f(x) = \tan(\tan^{-1} x)$ and $g(x) = \cot(\cot^{-1} x)$

(C) $f(x) = \text{sgn}(x)$ and $g(x) = \text{sgn}(\text{sgn}(x))$

(D) $f(x) = \cot^2 x . \cos^2 x$ and $g(x) = \cot^2 x - \cos^2 x$

17. Let $f(x) = \left(\frac{1-x}{1+x}\right), 0 \le x \le 1$ and $g(x) = 4x(1-x), 0 \le x \le 1$, then

(A) $fog = \frac{1-4x+4x^2}{1+4x-4x^2}, 0 \le x \le 1$

(B) $fog = \frac{1-4x-4x^2}{1+4x-4x^2}, \frac{1}{2} \le x \le 1$

(C) $gof = \frac{8x(1-x)}{(1+x)^2}, 0 \le x \le 1$

(D) $gof = \frac{8x(1+x)}{(1+x)^2}, 0 \le x \le 1$

18. Function $f(x) = \sin x + \tan x + \text{sgn}(x^2 - 6x + 10)$ is

(A) periodic with period 2π

(B) periodic with period π

(C) Non-periodic

(D) periodic with period 4π

19. Which of the functions are even -

(A) $\log\left(\frac{1+x^2}{1-x^2}\right)$ **(B)** $\sin^2 x + \cos^2 x$ **(C)** $\log\left(\frac{1+x^3}{1-x^3}\right)$ **(D)** $\frac{(1+2^x)^2}{2^x}$

20. Let $D \equiv [-1, 1]$ is the domain of the following functions, state which of them are injective.

(A) $f(x) = x^2$ **(B)** $g(x) = x^3$ **(C)** $h(x) = \sin 2x$ **(D)** $k(x) = \sin(\pi x/2)$

21. The period of the function $f(x) = \sin^4 3x + \cos^4 3x$ is:

(A) $\pi/6$ **(B)** $\pi/3$ **(C)** $\pi/2$ **(D)** $\pi/12$

22. Which of the following functions are aperiodic (where [.] denotes greatest integer function)

(A) $y = [x + 1]$ **(B)** $y = \sin x^2$ **(C)** $y = \sin^2 x$ **(D)** $y = \sin^{-1} x$

23. If $f: R \to [-1, 1]$, where $f(x) = \sin\left(\frac{\pi}{2}[x]\right)$, (where [.] denotes the greatest integer function), then

(A) f (x) is onto **(B)** f (x) is into **(C)** f (x) is periodic **(D)** f (x) is many one

24. Identify the statement(s) which is/are incorrect ?

(A) the function $f(x) = \cos(\cos^{-1} x)$ is neither odd nor even

(B) the fundamental period of $f(x) = \cos(\sin x) + \cos(\cos x)$ is π

(C) the range of the function $f(x) = \cos(3 \sin x)$ is $[-1, 1]$

(D) none of these

Part # II [Assertion & Reason Type Questions]

These questions contains, Statement I (assertion) and Statement II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.

(C) Statement-I is true, Statement-II is false.

(D) Statement-I is false, Statement-II is true.

1. **Statement-I :** Fundamental period of cosx + cotx is 2π.

Statement-II : If the period of f(x) is T_1 and the period of g(x) is T_2, then the fundamental period of f(x) + g(x) is the L.C.M. of T_1 and T_2.

2. **Statement - I** If y = f (x) is increasing in $[\alpha, \beta]$, then its range is $[f(\alpha), f(\beta)]$

Statement - II Every increasing function need not to be continuous.

3. **Statement-I :** Function f(x) = sin(x + 3sinx) is periodic.

Statement-II : If g(x) is periodic, then f(g(x)) may or may not be periodic.

4. **Statement : I :** All points of intersection of y = f (x) and $y = f^{-1}(x)$ lies on y = x only.

Statement : II : If point $P(\alpha, \beta)$ lies on y = f (x), then $Q(\beta, \alpha)$ lies on $y = f^{-1}(x)$.

5. Let function $f : R \to R$ is such that $f(x)\, f(y) - f(xy) = x + y$ for all $x, y \in R$

Statement-I : $f(x)$ is a Bijective function.

Statement-II : $f(x)$ is a linear function.

Exercise # 3 Part # I [Matrix Match Type Questions]

Following questions contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with one statement in **Column-II**.

1. Let $f(x) = \sin^{-1} x$, $g(x) = \cos^{-1} x$ and $h(x) = \tan^{-1} x$. For what complete interval of variation of x the following are true.

Column – I	Column – II
(A) $f\left(\sqrt{x}\right) + g\left(\sqrt{x}\right) = \pi/2$	**(p)** $[0, \infty)$
(B) $f(x) + g\left(\sqrt{1-x^2}\right) = 0$	**(q)** $[0, 1]$
(C) $g\left(\dfrac{1-x^2}{1+x^2}\right) = 2h(x)$	**(r)** $(-\infty, 1)$
(D) $h(x) + h(1) = h\left(\dfrac{1+x}{1-x}\right)$	**(s)** $[-1, 0]$

2.

Column - I	Column - II
(A) Total number of solution $x^2 - 4 - [x] = 0$ where [ ] denotes greatest integer function.	**(p)** 0
(B) Minimum period of $e^{\cos^4 \pi x + \cos^2 \pi x + x - [x]}$	**(q)** 1
(C) If $A = \{(x, y); y = \dfrac{1}{x}, x \in R_0\}$ and $B = \{(x, y) : y = x, x \in R\}$ then number of elements in $A \cap B$ is (are)	**(r)** 2
(D) Number of integers in the domain of $\sqrt{2^x - 3^x} + \log_3 \log_{1/2} x$	**(s)** 3

3.

Column – I	Column – II
(A) The period of the function $y = \sin(2\pi t + \pi/3) + 2\sin(3\pi t + \pi/4) + 3\sin 5\pi t$ is	**(p)** 1/2
(B) $y = \{\sin(\pi x)\}$ is a many one function for $x \in (0, a)$, where {x} denotes fractional part of x, then a may be	**(q)** 8
(C) The fundamental period of the function $y = \dfrac{1}{2}\left(\dfrac{\lvert\sin(\pi/4)x\rvert}{\cos(\pi/4)x} + \dfrac{\sin(\pi/4)x}{\lvert\cos(\pi/4)x\rvert}\right)$ is	**(r)** 2
(D) If $f : [0, 2] \to [0, 2]$ is bijective function defined by $f(x) = ax^2 + bx + c$, where a, b, c are non-zero real numbers, then f(2) is equal to	**(s)** 0

4. **Column - I** | **Column - II**

	Column - I		Column - II
(A)	$f : R \to R$ $f(x) = (x-1)(x-2)........(x-11)$	**(p)**	one one
(B)	$f : R - \{-4/3\} \to R$ $f(x) = \frac{2x+1}{3x+4}$	**(q)**	onto
(C)	$f : R \to R$ $f(x) = e^{\sin x} + e^{-\sin x}$	**(r)**	many one
(D)	$f : R \to R$ $f(x) = \log(x^2 + 2x + 3)$	**(s)**	into

Part # II [Comprehension Type Questions]

Comprehension # 1

Given a function $f : A \to B$; where $A = \{1, 2, 3, 4, 5\}$ and $B = \{6, 7, 8\}$

1. Find number of all such functions y = f(x) which are one-one ?
(A) 0 **(B)** 3^5 **(C)** ${}^5P_3$ **(D)** 5^3

2. Find number of all such functions y = f(x) which are onto
(A) 243 **(B)** 93 **(C)** 150 **(D)** none of these

3. The number of mappings of $g(x) : B \to A$ such that $g(i) \le g(j)$ whenever $i < j$ is
(A) 60 **(B)** 140 **(C)** 10 **(D)** 35

Comprehension # 2

If $f(x) = \begin{cases} x+1, & \text{if } x \le 1 \\ 5-x^2, & \text{if } x > 1 \end{cases}$ & $g(x) = \begin{cases} x, & \text{if } x \le 1 \\ 2-x, & \text{if } x > 1 \end{cases}$

On the basis of above information, answer the following questions :

1. The range of $f(x)$ is -
(A) $(-\infty, 4)$ **(B)** $(-\infty, 5)$ **(C)** R **(D)** $(-\infty, 4]$

2. If $x \in (1, 2)$, then $g(f(x))$ is equal to -
(A) $x^2 + 3$ **(B)** $x^2 - 3$ **(C)** $5 - x^2$ **(D)** $1 - x$

3. Number of negative integral solutions of $g(f(x)) + 2 = 0$ are -
(A) 0 **(B)** 3 **(C)** 1 **(D)** 2

Comprehension # 3

Let $f : R \to R$ is a function satisfying $f(2 - x) = f(2 + x)$ and $f(20 - x) = f(x)$, $\forall\ x \in R$.

On the basis of above information, answer the following questions :

1. If $f(0) = 5$, then minimum possible number of values of x satisfying $f(x) = 5$, for $x \in [0, 170]$ is-

(A) 21 **(B)** 12 **(C)** 11 **(D)** 22

2. Graph of $y = f(x)$ is -

(A) symmetrical about $x = 18$ **(B)** symmetrical about $x = 5$

(C) symmetrical about $x = 8$ **(D)** symmetrical about $x = 20$

3. If $f(2) \neq f(6)$, then

(A) fundamental period of f(x) is 1 **(B)** fundamental period of f(x) may be 1

(C) period of f(x) can't be 1 **(D)** fundamental period of f(x) is 8

Comprehension # 4

Let $f(x) = \frac{x^3}{3} + \frac{x^2}{2} + ax + b \quad \forall\ x \in R$

1. Least value of 'a' for which f(x) is injective function, is

(A) $\frac{1}{4}$ **(B)** 1 **(C)** $\frac{1}{2}$ **(D)** $\frac{1}{8}$

2. If $a = -1$, then f(x) is

(A) bijective **(B)** many–one and onto **(C)** one–one and into **(D)** many– one and into

3. f(x) is invertible iff

(A) $a \in \left[\frac{1}{4}, \infty\right), b \in R$ **(B)** $a \in \left[\frac{1}{8}, \infty\right), b \in R$

(C) $a \in \left(-\infty, \frac{1}{4}\right], b \in R$ **(D)** $a \in \left(-\infty, \frac{1}{4}\right), b \in R$

Exercise # 4 [Subjective Type Questions]

1. Find the domain of definitions of the following functions :

(i) $f(x) = \sqrt{3 - 2^x - 2^{1-x}}$

(ii) $f(x) = (x^2 + x + 1)^{-3/2}$

(iii) $f(x) = \sqrt{\tan x - \tan^2 x}$

(iv) $f(x) = \ell og_{10}(1 - \ell og_{10}(x^2 - 5x + 16))$

(v) If $f(x) = \sqrt{x^2 - 5x + 4}$ & $g(x) = x + 3$, then find the domain of $\frac{f}{g}(x)$

(vi) $f(x) = \frac{1}{[x]} + \log_{1-\{x\}}(x^2 - 3x + 10) + \frac{1}{\sqrt{2 - |x|}} + \frac{1}{\sqrt{\sec(\sin x)}}$

2. Find the range of the following functions :

(i) $f(x) = 1 - |x - 2|$

(ii) $f(x) = \frac{1}{\sqrt{x - 5}}$

(iii) $f(x) = \frac{1}{2 - \cos 3x}$

(iv) $f(x) = \frac{x + 2}{x^2 - 8x - 4}$

(v) $f(x) = \frac{x^2 - 2x + 4}{x^2 + 2x + 4}$

(vi) $f(x) = 3 \sin \sqrt{\frac{\pi^2}{16} - x^2}$

(vii) $f(x) = x^4 - 2x^2 + 5$

(viii) $f(x) = x^3 - 12x$, where $x \in [-3, 1]$

(ix) $f(x) = \sin^2 x + \cos^4 x$

3. Let f be a function such that f(3) = 1 and f(3x) = x + f(3x – 3) for all x. Then find the value of f(300).

4. Let $f(x) = \frac{9^x}{9^x + 3}$ then find the value of the sum $f\left(\frac{1}{2008}\right) + f\left(\frac{2}{2008}\right) + f\left(\frac{3}{2008}\right) + \ldots\ldots. f\left(\frac{2007}{2008}\right)$

5. Examine whether the following functions are even or odd or neither even nor odd, where [ ] denotes greatest integer function.

(i) $f(x) = \frac{(1 + 2^x)^7}{2^x}$

(ii) $f(x) = \frac{\sec x + x^2 - 9}{x \sin x}$

(iii) $f(x) = \sqrt{1 + x + x^2} - \sqrt{1 - x + x^2}$

(iv) $f(x) = \begin{cases} x|x|, & x \le -1 \\ [1 + x] + [1 - x], & -1 < x < 1 \\ -x|x|, & x \ge 1 \end{cases}$

(v) $f(x) = \frac{2x(\sin x + \tan x)}{2\left[\frac{x + 2\pi}{\pi}\right] - 3}$

6. Find the fundamental period of the following functions :

(i) $f(x) = 1 - \dfrac{\sin^2 x}{1 + \cot x} - \dfrac{\cos^2 x}{1 + \tan x}$

(ii) $f(x) = \tan \dfrac{\pi}{2} [x]$, where [.] denotes greatest integer function.

(iii) $f(x) = \ell og\,(2 + \cos 3x)$

(iv) $f(x) = e^{\ln \sin x} + \tan^3 x - \text{cosec}\,(3x - 5)$

(v) $f(x) = \sin x + \tan \dfrac{x}{2} + \sin \dfrac{x}{2^2} + \tan \dfrac{x}{2^3} + \ldots\ldots\ldots + \sin \dfrac{x}{2^{n-1}} + \tan \dfrac{x}{2^n}$

(vi) $f(x) = \dfrac{\sin x + \sin 3x}{\cos x + \cos 3x}$

7. Let $f(x) = \begin{cases} 1 + x, & 0 \le x \le 2 \\ 3 - x, & 2 < x \le 3 \end{cases}$, then find (fof)(x).

8. Let $f : R \to R$ is a function satisfying $f(10 - x) = f(x)$ and $f(2 - x) = f(2 + x)$ $\forall$ $x \in R$. If $f(0) = 101$, then the minimum possible number of values of x satisfying $f(x) = 101$ $\forall$ $x \in [0,25]$ is

9. Show if $f(x) = \sqrt[n]{a - x^n}$, $x > 0$ $n \ge 2$, $n \in N$, then (fof) (x) = x. Find also the inverse of f(x).

10. Let $f : N \to N$, where $f(x) = x + (-1)^{x-1}$, then find the inverse of f.

Exercise # 5 Part # I [Previous Year Questions] [AIEEE/JEE-MAIN]

1. Which of the following is not a periodic function- **[AIEEE 2002]**

(1) $\sin 2x + \cos x$ (2) $\cos\sqrt{x}$ (3) $\tan 4x$ (4) $\log\cos 2x$

2. The period of $\sin^2 x$ is- **[AIEEE 2002]**

(1) $\pi/2$ (2) π (3) $3\pi/2$ (4) 2π

3. The function $f : R \to R$ defined by $f(x) = \sin x$ is- **[AIEEE 2002]**

(1) into (2) onto (3) one-one (4) many-one

4. The range of the function $f(x) = \frac{2+x}{2-x}$, $x \neq 2$ is- **[AIEEE 2002]**

(1) R (2) $R - \{-1\}$ (3) $R - \{1\}$ (4) $R - \{2\}$

5. The domain of $\sin^{-1}\left[\log_3\left(\frac{x}{3}\right)\right]$ **[AIEEE 2002]**

(1) [1, 9] (2) [-1, 9] (3) [-9, 1] (4) [-9, -1]

6. The function $f(x) = \log(x + \sqrt{x^2+1})$, is- **[AIEEE 2003]**

(1) neither an even nor an odd function (2) an even function

(3) an odd function (4) a periodic function

7. Domain of definition of the function $f(x) = \frac{3}{4-x^2} + \log_{10}(x^3 - x)$, is- **[AIEEE 2003]**

(1) $(-1, 0) \cup (1, 2) \cup (2, \infty)$ (2) $(1, 2)$

(3) $(-1, 0) \cup (1, 2)$ (4) $(1, 2) \cup (2, \infty)$

8. If $f : R \to R$ satisfies $f(x + y) = f(x) + f(y)$, for all $x, y \in R$ and $f(1) = 7$, then $\sum_{r=1}^{n} f(r)$ is - **[AIEEE 2003]**

(1) $\frac{7n(n+1)}{2}$ (2) $\frac{7n}{2}$ (3) $\frac{7(n+1)}{2}$ (4) $7n(n + 1)$

9. A function f from the set of natural numbers to integers defined by $f(n) = \begin{cases} \frac{n-1}{2}, & \text{when n is odd} \\ -\frac{n}{2}, & \text{when n is even} \end{cases}$ is - **[AIEEE 2003]**

(1) neither one-one nor onto (2) one-one but not onto

(3) onto but not one-one (4) one-one and onto both

10. The domain of the function $f(x) = \dfrac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is **[AIEEE 2004]**

(1) $[1, 2)$ **(2)** $[2, 3)$ **(3)** $[1, 2]$ **(4)** $[2, 3]$

11. The range of the function $f(x) = {}^{7-x}P_{x-3}$ is- **[AIEEE 2004]**

(1) $\{1, 2, 3, 4, 5\}$ **(2)** $\{1, 2, 3, 4, 5, 6\}$ **(3)** $\{1, 2, 3\}$ **(4)** $\{1, 2, 3, 4\}$

12. If $f : R \to S$ defined by $f(x) = \sin x - \sqrt{3}\cos x + 1$ is onto, then the interval of S is-

[AIEEE 2004]

(1) $[-1, 3]$ **(2)** $[-1, 1]$ **(3)** $[0, 1]$ **(4)** $[0, -1]$

13. Let $f : (-1, 1) \to B$, be a function defined by $f(x) = \tan^{-1}\dfrac{2x}{1-x^2}$, then f is both one-one and onto when B is the interval- **[AIEEE 2005]**

(1) $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$ **(2)** $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$ **(3)** $\left(0, \dfrac{\pi}{2}\right)$ **(4)** $\left[0, \dfrac{\pi}{2}\right)$

14. A real valued function $f(x)$ satisfies the function equation $f(x-y) = f(x)f(y) - f(a-x)f(a+y)$ where a is a given constant and $f(0) = 1$, $f(2a-x)$ is equal to **[AIEEE 2005]**

(1) $f(1) + f(a-x)$ **(2)** $f(-x)$ **(3)** $-f(x)$ **(4)** $f(x)$

15. If x is real, the maximum value of $\dfrac{3x^2+9x+17}{3x^2+9x+7}$ is- **[AIEEE 2006]**

(1) 41 **(2)** 1 **(3)** $\dfrac{17}{7}$ **(4)** $\dfrac{1}{4}$

16. The largest internal lying in $\left(\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right)$ for which the function is defined, $\left[f(x) = 4^{-x^2} + \cos^{-1}\left(\dfrac{x}{2}-1\right) + \log(\cos x)\right]$ is **[AIEEE 2007]**

(1) $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$ **(2)** $\left[-\dfrac{\pi}{4}, \dfrac{\pi}{2}\right)$ **(3)** $\left[0, \dfrac{\pi}{2}\right)$ **(4)** $[0, \pi]$

17. Let $f : R \to R$ be a function defined by $f(x) = \text{Min}\,\{x+1, |x|+1\}$. Then which of the following is true ? **[AIEEE 2007]**

(1) $f(x)$ is not differentiable at $x = 1$ **(2)** $f(x)$ is differentiable everywhere

(3) $f(x)$ is not differentiable at $x = 0$ **(4)** $f(x) \geq 1$ for all $x \in R$

18. Let f : N Y be a function defined as $f(x) = 4x + 3$ where **[AIEEE 2008]**

$Y = \{y \in N : y = 4x + 3 \text{ for some } x \in N\}$. So that f is invertible and its inverse is

(1) $g(y) = \dfrac{3y+4}{3}$ **(2)** $g(y) = 4 + \dfrac{y+3}{4}$ **(3)** $g(y) = \dfrac{y+3}{4}$ **(4)** $g(y) = \dfrac{y-3}{4}$

19. For real x, let $f(x) = x^3 + 5x + 1$, then :- **[AIEEE 2009]**

(1) f is one-one and onto R **(2)** f is neither one-one nor onto R

(3) f is one-one but not onto R **(4)** f is onto R but not one-one

20. Let $f(x) = (x + 1)^2 – 1$, x –1. **[AIEEE 2009]**

Statement–1 : The set $\{x : f(x) = f^{-1}(x)\} = \{0, –1\}$.

Statement–2 : f is a bijection.

(1) Statement–1 is true, Statement–2 is false.

(2) Statement–1 is false, Statement–2 is true.

(3) Statement–1 is true, Statement–2 is true ; Statement–2 is a correct explanation for Statement–1.

(4) Statement–1 is true, Statement–2 is true ; Statement–2 is not a correct explanation for statement–1.

21. The domain of the function $f(x) = \dfrac{1}{\sqrt{|x| - x}}$ is :- **[AIEEE 2011]**

(1) $(-\infty, 0)$ **(2)** $(-\infty, \infty) - \{0\}$ **(3)** $(-\infty, \infty)$ **(4)** $(0, \infty)$

22. Let f be a function defined by $f(x) = (x – 1)^2 + 1, (x \geq 1)$ **[AIEEE 2011]**

Statement - 1 : The set $\{x : f(x) = f^{-1}(x)\} = \{1, 2\}$

Statement - 2 : f is bijection and $f^{-1}(x) = 1 + \sqrt{x-1}$, $x \geq 1$.

(1) Statement–1 is true, Statement–2 is false.

(2) Statement–1 is false, Statement–2 is true.

(3) Statement–1 is true, Statement–2 is true ; Statement–2 is a correct explanation for Statement–1.

(4) Statement–1 is true, Statement–2 is true ; Statement–2 is not a correct explanation for statement–1.

23. If $f : R \to R$ is a function defined by $f(x) = [x]\cos\pi\left(\frac{2x-1}{2}\right)$, where [x] denotes the greatest integer function, then f is : **[AIEEE 2012]**

(1) continuous only at x = 0.

(2) continuous for every real x.

(3) discontinuous only at x = 0.

(4) discontinuous only at non-zero integral values of x.

24. If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n-1) : n \in N\}$, where N is the set of natural numbers, then $X \cup Y$ is equal to :

(1) N **(2)** Y – X **(3)** X **(4)** Y **[Main 2014]**

25. If $f(x) + 2f\left(\frac{1}{x}\right) = 3x$, $x \neq 0$ and $S = \{x \in R : f(x) = f(-x)\}$; then S : **[Main 2016]**

(1) contains exactly one element. **(2)** contains exactly two elements.

(3) contains more than two elements. **(4)** is an empty set.

26. For $x \in R$, $f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then: **[Main 2016]**

(1) $g'(0) = \cos(\log 2)$ **(2)** $g'(0) = -\cos(\log 2)$

(3) g is differentiable at x = 0 and $g'(0) = -\sin(\log 2)$ **(4)** g is not differentiable at x = 0

Part # II [Previous Year Questions][IIT-JEE ADVANCED]

1. The domain of definition of the function, $y(x)$ given by the equation, $2^x + 2^y = 2$ is :

(A) $0 < x \le 1$ **(B)** $0 \le x \le 1$ **(C)** $-\infty < x \le 0$ **(D)** $-\infty < x < 1$ **[JEE 2000]**

2. Given $x = \{1, 2, 3, 4\}$, find all one-one, onto mappings, $f : X \to X$ such that, $f(1) = 1$, $f(2) \neq 2$ and $f(4) \neq 4$. **[JEE 2000]**

3. Let $g(x) = 1 + x - [x]$ & $f(x) = \begin{cases} -1 & , x < 0 \\ 0 & , x = 0 \\ 1 & , x > 0 \end{cases}$. Then for all x, $f(g(x))$ is equal to **[JEE 2001]**

(A) x **(B)** 1 **(C)** f(x) **(D)** g(x)

where [ ] denotes the greatest integer function.

4. If $f : [1, \infty) \to [2, \infty)$ is given by, $f(x) = x + \frac{1}{x}$, then $f^{-1}(x)$ equals : **[JEE 2001]**

(A) $\frac{x + \sqrt{x^2 - 4}}{2}$ **(B)** $\frac{x}{1 + x^2}$ **(C)** $\frac{x - \sqrt{x^2 - 4}}{2}$ **(D)** $1 - \sqrt{x^2 - 4}$

5. The domain of definition of $f(x) = \dfrac{\log_2(x+3)}{x^2+3x+2}$ is : **[JEE 2001]**

(A) $R\backslash\{-1,-2\}$ (B) $(-2,\infty)$ (C) $R\backslash\{-1,-2,-3\}$ (D) $(-3,\infty)\backslash\{-1,-2\}$

6. Let $E = \{1, 2, 3, 4\}$ & $F = \{1, 2\}$. Then the number of onto functions from E to F is **[JEE 2001]**

(A) 14 (B) 16 (C) 12 (D) 8

7. Let $f(x) = \dfrac{\alpha x}{x+1}$, $x \neq -1$. Then for what value of α is $f(f(x)) = x$?

(A) $\sqrt{2}$ (B) $-\sqrt{2}$ (C) 1 (D) -1 **[JEE 2001]**

8. Suppose $f(x) = (x+1)^2$ for $x \geq -1$. If g(x) is the function whose graph is the reflection of the graph of f(x) with respect to the line y = x, then g(x) equals -

(A) $-\sqrt{x}-1, x \geq 0$ (B) $\dfrac{1}{(1+x)^2}, x \geq -1$ (C) $\sqrt{x+1}, x \geq -1$ (D) $\sqrt{x}-1, x \geq 0$

9. Let function $f: R \to R$ be defined by $f(x) = 2x + \sin x$ for $x \in R$. Then f is -

(A) one to one and onto (B) one to one but not onto

(C) onto but not one to one (D) neither one to one nor onto **[JEE 2002]**

10. Range of the function $f(x)=\dfrac{x^2+x+2}{x^2+x+1}$ is -

(A) [1, 2] (B) $[1, \infty)$ (C) $\left[2, \dfrac{7}{3}\right]$ (D) $\left(1, \dfrac{7}{3}\right]$ **[JEE 2003]**

11. Let $f(x) = \dfrac{x}{1+x}$ defined from $(0,\infty) \to [0,\infty)$, then by f(x) is -

(A) one-one but not onto (B) one-one and onto

(C) Many one but not onto (D) Many one and onto

12. Let $f(x) = \sin x + \cos x$, $g(x) = x^2-1$. Thus g(f(x)) is invertible for $x \in$ **[JEE 2004]**

(A) $\left[-\dfrac{\pi}{2}, 0\right]$ (B) $\left[-\dfrac{\pi}{2}, \pi\right]$ (C) $\left[-\dfrac{\pi}{4}, \dfrac{\pi}{4}\right]$ (D) $\left[0, \dfrac{\pi}{2}\right]$

13. If functions f (x) and g (x) are defined on $R \to R$ such that $f(x) = \begin{cases} 0, & x \in \text{rational} \\ x, & x \in \text{irrational} \end{cases}$, $g(x) = \begin{cases} 0, & x \in \text{irrational} \\ x, & x \in \text{rational} \end{cases}$,

then (f – g) (x) is -

(A) one-one and onto (B) neither one-one nor onto

(C) one-one but not onto (D) onto but not one-one **[JEE 2005]**

14. Let f(x) = x^2 and g(x) = sinx for all x∈R. Then the set of all x satisfying

$(f \circ g \circ g \circ f)(x) = (g \circ g \circ f)(x)$, where $(f \circ g)(x) = f(g(x))$, is- **[JEE 2011]**

(A) $\pm\sqrt{n\pi}, n \in \{0,1,2,....\}$

(B) $\pm\sqrt{n\pi}, n \in \{1,2,...\}$

(C) $\frac{\pi}{2} + 2n\pi, n \in \{...,-2,-1,0,1,2,.....\}$

(D) $2n\pi, n \in \{...,-2,-1,0,1,2,.....\}$

15. The function $f : [0,3] \to [1,29]$, defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$, is : **[JEE 2012]**

(A) one-one and onto

(B) onto but not one-one

(C) one-one but not onto

(D) neither one-one nor onto

16. Let $f : (-1,1) \to \mathbb{R}$ be such that $f(\cos 4\theta) = \frac{2}{2 - \sec^2\theta}$ for $\theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$. Then the value(s) of $f\left(\frac{1}{3}\right)$ is (are)- **[JEE 2012]**

(A) $1 - \sqrt{\frac{3}{2}}$

(B) $1 + \sqrt{\frac{3}{2}}$

(C) $1 - \sqrt{\frac{2}{3}}$

(D) $1 + \sqrt{\frac{2}{3}}$

17. For every pair of continuous functions f, g : [0, 1] → R such that max {f(x) : x ∈ [0, 1]} = max {g(x) : x ∈ [0, 1]}, the correct statement(s) is (are) : **[JEE Ad. 2014]**

(A) $(f(c))^2 + 3f(c) = (g(c))^2 + 3g(c)$ for some c ∈ [0, 1]

(B) $(f(c))^2 + f(c) = (g(c))^2 + 3g(c)$ for some c ∈ [0, 1]

(C) $(f(c))^2 + 3f(c) = (g(c))^2 + g(c)$ for some c ∈ [0, 1]

(D) $(f(c))^2 = (g(c))^2$ for some c ∈ [0, 1]

18. Let $f : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to R$ be given by **[JEE Ad. 2014]**

$f(x) = (\log(\sec x + \tan x))^3$

Then

(A) f(x) is an odd function

(B) f(x) is a one-one function

(C) f(x) is an onto function

(D) f(x) is an even function

19. Let $f_1 : R \to R$, $f_2 : [0, \infty) \to R$, $f_3 : R \to R$ and $f_4 : R \to [0, \infty)$ be defined by **[JEE Ad. 2014]**

$$f_1(x) = \begin{cases} |x| & \text{if } x < 0 \\ e^x & \text{if } x \ge 0 \end{cases}; \ f_2(x) = x^2; \ f_3(x) = \begin{cases} \sin x & \text{if } x < 0 \\ x & \text{if } x \ge 0 \end{cases}; \ f_4(x) = \begin{cases} f_2(f_1(x)) & \text{if } x < 0 \\ f_2(f_1(x)) - 1 & \text{if } x \ge 0 \end{cases}$$

	List - I		**List - II**
(p)	f_4 is	**(1)**	onto but not one-one
(q)	f_3 is	**(2)**	neither continuous nor one-one
(r)	$f_2 \circ f_1$ is	**(3)**	differentiable but not one-one
(s)	f_2 is	**(4)**	continuous and one-one

Codes :

	p	q	r	s
(A)	3	1	4	2
(B)	1	3	4	2
(C)	3	1	2	4
(D)	1	3	2	4

20. Let $f(x) = \sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$ for all $x \in R$ and $g(x) = \frac{\pi}{2}\sin x$ for all $x \in R$. Let (fog) (x) denote f(g(x)) and (gof) (x) denote g(f(x)). Then which of the following is (are) true ? **[JEE Ad. 2015]**

(A) Range of f is $\left[-\frac{1}{2},\frac{1}{2}\right]$

(B) Range of fog is $\left[-\frac{1}{2},\frac{1}{2}\right]$

(C) $\lim\limits_{x\to 0}\frac{f(x)}{g(x)} = \frac{\pi}{6}$

(D) There is an $x \in R$ such that (gof) (x) = 1

21. Let $f : R \to R$, $g : R \to R$ and $h : R \to R$ be differentiable functions such that $f(x) = x^3 + 3x + 2$ $(12\alpha + 20)\frac{K^2}{2} = K^3$, $g(f(x)) = x$ and $h(g(g(x))) = x$ for all $x \in R$. Then **[JEE Ad. 2016]**

(A) $g'(2) = \frac{1}{15}$

(B) $h'(1) = 666$

(C) $h(0) = 16$

(D) $h(g(3)) = 36$

MOCK TEST

SECTION - I : STRAIGHT OBJECTIVE TYPE

1. If f(x) . f(y) = f(x) + f(y) + f(xy) – 2 $\forall$ x, y $\in$ R and if f(x) is not a constant function, then the value of f(1) is equal to

(A) 1 **(B)** 2 **(C)** 0 **(D)** –1

2. The domain of the function $f(x) = \sqrt{-\log_{\frac{x+4}{2}}\left(\log_2 \dfrac{2x-1}{3+x}\right)}$ is

(A) $(-4,-3)\cup(4,\infty)$ **(B)** $(-\infty,-3)\cup(4,\infty)$ **(C)** $(-\infty,-4)\cup(3,\infty)$ **(D)** None

3. Let $f(x) = ax^2 + bx + c$, where a , b , c are rational and $f : Z \to Z$, where Z is the set of integers. Then a + b is :

(A) a negative integer **(B)** an integer

(C) non-integral rational number **(D)** none of these

4. If $f(x) = \dfrac{\sin^2 x + 4\sin x + 5}{2\sin^2 x + 8\sin x + 8}$, then range of f(x) is

(A) $\left(\dfrac{1}{2}, \infty\right)$ **(B)** $\left(\dfrac{5}{9}, 1\right)$ **(C)** $\left[\dfrac{5}{9}, 1\right]$ **(D)** $\left[\dfrac{5}{9}, \infty\right)$

5. If f(x) = x + tan x and g(x) is the inverse of f(x) then g′(x) is equal to

(A) $\dfrac{1}{1+(g(x)-x)^2}$ **(B)** $\dfrac{1}{2+(g(x)-x)^2}$ **(C)** $\dfrac{1}{2+(g(x)-x)^2}$ **(D)** none of these

6. Let f(x) = tanx, $g(f(x)) = f\left(x - \dfrac{\pi}{4}\right)$, where f(x) and g(x) are real valued functions. For all possible values of x, f(g(x)) =

(A) $\tan\left(\dfrac{x-1}{x+1}\right)$ **(B)** $\tan(x-1) - \tan(x+1)$ **(C)** $\dfrac{f(x)+1}{f(x)-1}$ **(D)** $\dfrac{x-\pi/4}{x+\pi/4}$

7. The range of the function $f(x) = \sin^{-1}\left[x^2 + \dfrac{1}{2}\right] + \cos^{-1}\left[x^2 - \dfrac{1}{2}\right]$, where [ ] is the greatest integer function, is:

(A) $\left\{\dfrac{\pi}{2}, \pi\right\}$ **(B)** $\left\{0, \dfrac{\pi}{2}\right\}$ **(C)** $\{\pi\}$ **(D)** $\left(0, \dfrac{\pi}{2}\right)$

8. It is given that f(x) is a function defined on R, satisfying f(1) = 1 and for any $x \in R$

$$f(x+5) \geq f(x) + 5$$

and $$f(x+1) \leq f(x) + 1$$

If g(x) = f(x) + 1 – x, then g(2013) equals

(A) 2014 **(B)** 2013 **(C)** 1 **(D)** 0

9. The image of the interval [– 1, 3] under the mapping specified by the function $f(x) = 4x^3 - 12x$ is :

(A) [f(+1), f(– 1)] **(B)** [f(– 1), f(3)] **(C)** [– 8, 16] **(D)** [– 8, 72]

10. Let $f(x) = x(2-x)$, $0 \leq x \leq 2$. If the definition of 'f' is extended over the set, $R - [0, 2]$ by $f(x-2) = f(x)$, then 'f' is a :

(A) periodic function of period 1 **(B)** non-periodic function

(C) periodic function of period 2 **(D)** periodic function of period 1/2

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

11. Suppose f (x) = ax + b and g (x) = bx + a, where a and b are positive integers.
If $f(g(50)) - g(f(50)) = 28$ then the product (ab) can have the value equal to

(A) 12 **(B)** 48 **(C)** 180 **(D)** 210

12. Let $f(x) = \begin{cases} 0 & \text{for } x = 0 \\ x^2 \sin\left(\frac{\pi}{x}\right) & \text{for } -1 < x < 1 \ (x \neq 0) \\ x\ |x| & \text{for } x > 1 \text{ or } x < -1 \end{cases}$, then:

(A) f(x) is an odd function **(B)** f(x) is an even function

(C) f(x) is neither odd nor even **(D)** $f'(x)$ is an even function

13. Which of the functions defined below are one-one function**(s)** ?

(A) $f(x) = (x+1)$, $(x \geq -1)$ **(B)** $g(x) = x + (1/x)$ $(x > 0)$

(C) $h(x) = x^2 + 4x - 5$, $(x > 0)$ **(D)** $f(x) = e^{-x}$, $(x \geq 0)$

14. If the function $f(x) = ax + b$ has its own inverse then the ordered pair (a, b) can be

(A) (1, 0) **(B)** (–1, 0) **(C)** (–1, 1) **(D)** (1, 1)

15. A continuous function f (x) on $R \to R$ satisfies the relation

$$f(x) + f(2x+y) + 5xy = f(3x-y) + 2x^2 + 1 \quad \text{for } \forall \ x, y \in R$$

then which of the following hold**(s)** good ?

(A) f is many one **(B)** f has no minima

(C) f is neither odd nor even **(D)** f is bounded

SECTION - III : ASSERTION AND REASON TYPE

16. Let $g: R \to R$ defined by $g(x) = \{e^x\}$, where $\{x\}$ denotes fractional part function.

Statement-I : $g(x)$ is periodic function.

Statement-II : $\{x\}$ is periodic function.

(A) Statement-I is true, statement-II is true and statement-II is correct explanation for statement-I.

(B) Statement-I is true, statement-II is true and statement-II is NOT the correct explanation for statement-I.

(C) Statement-I is true, statement-II is false.

(D) Statement-I is false, statement-II is true

17. **Statement-I :** Fundamental period of $\sin x + \tan x$ is 2π

Statement-II : If the period of f(x) is T_1 and the period of g(x) is T_2, then the fundamental period of f(x) + g(x) is the L.C.M. of T_1 and T_2

(A) Statement-I is true, statement-II is true and statement-II is correct explanation for statement-I.

(B) Statement-I is true, statement-II is true and statement-II is NOT the correct explanation for statement-I.

(C) Statement-I is true, statement-II is false.

(D) Statement-I is false, statement-II is true

18. **Statement-I :** If a function $y = f(x)$ is symmetric about $y = x$, then $f(f(x)) = x$

Statement-II : If $f(x) = \begin{cases} x & : & x \text{ is rational} \\ 1-x & : & x \text{ is irrational} \end{cases}$, then $f(f(x)) = x$

(A) Statement-I is true, statement-II is true and statement-II is correct explanation for statement-I.

(B) Statement-I is true, statement-II is true and statement-II is NOT the correct explanation for statement-I.

(C) Statement-I is true, statement-II is false.

(D) Statement-I is false, statement-II is true

19. **Statement-1:** f is an even function, g and h are odd functions, all 3 are polynomials. Given $f(1) = 0$, $f(2) = 1$, $f(3) = -5$, $g(1) = 1$, $g(-3) = 2$, $g(5) = 3$, $h(1) = 3$, $h(3) = 5$ and $h(5) = 1$.

The value of $f\left(g\left(h(1)\right)\right) + g\left(h\left(f(3)\right)\right) + h\left(f\left(g(-1)\right)\right)$ is equal to zero.

Statement-2: If a polynomial function P(x) is odd then $P(0) = 0$.

(A) Statement-I is true, statement-II is true and statement-II is correct explanation for statement-I.

(B) Statement-I is true, statement-II is true and statement-II is NOT the correct explanation for statement-I.

(C) Statement-I is true, statement-II is false.

(D) Statement-I is false, statement-II is true

20. **Statement -1** : e^x can not be expressed as the sum of even and odd function.

Statement -2 : e^x is neither even nor odd function

(A) Statement-I is true, statement-II is true and statement-II is correct explanation for statement-I.

(B) Statement-I is true, statement-II is true and statement-II is NOT the correct explanation for statement-I.

(C) Statement-I is true, statement-II is false.

(D) Statement-I is false, statement-II is true

SECTION - IV : MATRIX - MATCH TYPE

21. **Column – I** | **Column – II**

(A) Function $f:\left[0,\frac{\pi}{3}\right]\to[0,1]$ defined by $f(x)=\sqrt{\sin x}$ is — **(p)** one to one function

(B) Function $f:(1,\infty)\to(1,\infty)$ defined by $f(x)=\frac{x+3}{x-1}$ is — **(q)** many – one function

(C) Function $f:\left[-\frac{\pi}{2},\frac{4\pi}{3}\right]\to[-1,1]$ defined by $f(x)=\sin x$ is — **(r)** into function

(D) Function $f:(2,\infty)\to[8,\infty)$ defined by $f(x)=\frac{x^2}{x-2}$ is — **(s)** onto function

22. Let $f(x)=x+\frac{1}{x}$ and $g(x)=\frac{x+1}{x+2}$.

Match the composite function given in Column-I with their respective domains given in Column-II.

Column-I	Column-II
(A) fog	**(p)** $R-\{-2,-5/3\}$
(B) gof	**(q)** $R-\{-1,0\}$
(C) fof	**(r)** $R-\{0\}$
(D) gog	**(s)** $R-\{-2,-1\}$

SECTION - V : COMPREHENSION TYPE

23. **Read the following comprehension carefully and answer the questions.**

Let $f(x)=x^2-2x-1\ \forall\ x\in R$. Let $f:(-\infty,a]\to[b,\infty)$, where 'a' is the largest real number for which f (x) is bijective.

1. The value of $(a+b)$ is equal to

(A) -2 **(B)** -1 **(C)** 0 **(D)** 1

2. Let $f:R\to R$, $g(x)=f(x)+3x-1$, then the least value of function $y=g(|x|)$ is

(A) $-9/4$ **(B)** $-5/4$ **(C)** -2 **(D)** -1

3. Let $f:[a,\infty)\to[b,\infty)$, then $f^{-1}(x)$ is given by

(A) $1+\sqrt{x+2}$ **(B)** $1-\sqrt{x+3}$ **(C)** $1-\sqrt{x+2}$ **(D)** $1+\sqrt{x+3}$

4. Let $f:R\to R$, then range of values of k for which equation $f(|x|)=k$ has 4 distinct real roots is

(A) $(-2,-1)$ **(B)** $(-2,0)$ **(C)** $(-1,0)$ **(D)** $(0,1)$

24. **Read the following comprehension carefully and answer the questions.**

Let $f(x) = \begin{cases} 2x + a & : & x \geq -1 \\ bx^2 + 3 & : & x < -1 \end{cases}$

and $g(x) = \begin{cases} x + 4 & : & 0 \leq x \leq 4 \\ -3x - 2 & : & -2 < x < 0 \end{cases}$

1. g(f(x)) is not defined if

(A) $a \in (6, \infty), b \in (5, \infty)$ **(B)** $a \in (4, 6), b \in (5, \infty)$ **(C)** $a \in (6, \infty), b \in (0, 1)$ **(D)** $a \in (4, 6), b \in (1, 5)$

2. If domain of g(f(x)) is [–1, 2], then

(A) $a = 1, b > 5$ **(B)** $a = 2, b > 7$ **(C)** $a = 2, b > 10$ **(D)** $a = 0, b \in R$

3. If a = 2 and b = 3 then range of g(f(x)) is

(A) (–2, 8] **(B)** (0, 8] **(C)** [4, 8] **(D)** [–1, 8]

25. **Read the following comprehension carefully and answer the questions.**

Let $f : R \rightarrow R$ is a function satisfying $f(2 - x) = f(2 + x)$ and $f(20 - x) = f(x), \forall x \in R$. For this function f answer the following.

1. If f(0) = 5, then minimum possible number of values of x satisfying f(x) = 5, for $x \in [0, 170]$, is

(A) 21 **(B)** 12 **(C)** 11 **(D)** 22

2. Graph of y = f(x) is

(A) symmetrical about x = 18 **(B)** symmetrical about x = 5

(C) symmetrical about x = 8 **(D)** symmetrical about x = 20

3. If $f(2) \neq f(6)$, then

(A) fundamental period of f(x) is 1 **(B)** fundamental period of f(x) may be 1

(C) period of f(x) can't be 1 **(D)** fundamental period of f(x) is 8

SECTION - VI : INTEGER TYPE

26. If $f(x) + f(y) + f(xy) = 2 + f(x) . f(y)$, for all real values of x and y and f(x) is a polynomial function with $f(4) = 17$ and $f(1) \neq 1$, then find the value of f(5).

27. If $f(x) + f(y) + f(xy) = 2 + f(x) . f(y)$, for all real values of x & y and f(x) is a polynomial function with $f(4) = 17$, then find the value of f(5)/14, where $f(1) \neq 1$.

28. If f is a function satisfying the condition $f(x) + f(y) = f(x\sqrt{1-y^2} + y\sqrt{1-x^2})$ for all x and y in domain of f, then find value of $f(4x^3 - 3x) + 3 f(x)$.

29. If domain of $f(x) = \dfrac{\sin^{-1}(\sin x)}{\sqrt{-\log_{\left(\frac{x+4}{2}\right)} \log_2\left(\frac{2x-1}{3+x}\right)}}$ is $(a, b) \cup (c, \infty)$, then find the value of a + b + 3c.

30. The functional relation $f(x) + f\left(\dfrac{1}{1 - x}\right) = \dfrac{2(1 - 2x)}{x(1 - x)}$ is satisfying by the function $f(x) = \dfrac{x+1}{\lambda(x-1)}$, then find value of λ

ANSWER KEY

EXERCISE - 1

1. C **2.** D **3.** A **4.** B **5.** B **6.** B **7.** A **8.** B **9.** D **10.** C **11.** A **12.** C **13.** D
14. A **15.** D **16.** C **17.** C **18.** D **19.** B **20.** A **21.** D **22.** C **23.** B **24.** D **25.** A **26.** B
27. D **28.** C **29.** B **30.** D **31.** D **32.** C **33.** C **34.** A **35.** B **36.** B

EXERCISE - 2 : PART # I

1. ABC **2.** ACD **3.** AC **4.** BD **5.** AD **6.** AD **7.** B **8.** A **9.** ABCD
10. BCD **11.** AB **12.** BC **13.** ACD **14.** BC **15.** B **16.** BCD **17.** AC **18.** AD
19. ABD **20.** BD **21.** ABC **22.** ABD **23.** BCD **24.** ABC

PART - II

1. C **2.** D **3.** C **4.** D **5.** A

EXERCISE - 3 : PART # I

1. $A \rightarrow q$ $B \rightarrow s$ $C \rightarrow p$ $D \rightarrow r$ **2.** $A \rightarrow q$ $B \rightarrow r$ $C \rightarrow p$ $D \rightarrow s$
3. $A \rightarrow q,r$ $B \rightarrow q,r$ $C \rightarrow q$ $D \rightarrow s$ **4.** $A \rightarrow r$ $B \rightarrow p$ $C \rightarrow s$ $D \rightarrow q$

PART - II

Comprehension # 1 : 1. A 2. C 3. D **Comprehension # 2 : 1. A 2. B 3. C**
Comprehension # 3 : 1. D 2. A 3. C **Comprehension # 4 : 1. A 2. B 3. A**

EXERCISE - 5 : PART # I

1. 2 **2.** 2 **3.** 1,4 **4.** 2 **5.** 1 **6.** 3 **7.** 1 **8.** 1 **9.** 4 **10.** 2 **11.** 3 **12.** 1 **13.** 2
14. 3 **15.** 1 **16.** 3 **17.** 2 **18.** 4 **19.** 1 **20.** 4 **21.** 1 **22.** 2 **23.** 2 **24.** 4 **25.** 2 **26.** 1

PART - II

1. D **2.** $\{(1,1),(2,3),(3,4),(4,2)\}$; $\{(1,1),(2,4),(3,2),(4,3)\}$ and $\{(1,1),(2,4),(3,3),(4,2)\}$ **3.** B **4.** A
5. D **6.** A **7.** D **8.** D **9.** A **10.** D **11.** A **12.** C **13.** A **14.** A **15.** B
16. (zero marks to all) **17.** AD **18.** ABC **19.** D **20.** ABC **21.** BC

MOCK TEST

1. B **2.** A **3.** B **4.** C **5.** C **6.** A **7.** C **8.** C **9.** D **10.** C **11.** A, D **12.** A, D

13. A, C, D **14.** A, B, C **15.** A, B **16.** D **17.** C **18.** A **19.** A **20.** D

21. A $\rightarrow$ p,r B $\rightarrow$ p,s C $\rightarrow$ q,s D $\rightarrow$ q,s **22.** A $\rightarrow$ s B $\rightarrow$ q C $\rightarrow$ r D $\rightarrow$ p

23. **1.** B **2.** C **3.** A **4.** A **24.** **1.** A **2.** A **3.** C **25.** **1.** A **2.** A **3.** C

26. 8 **27.** 9 **28.** 0 **29.** 5 **30.** 1