DCAM classes
Dynamic Classes for Academic Mastery

Work, Energy and Power

Work

When a force acts on an object such that it displaces through some distance in the direction of applied force, then the work is said to be done by the force.

Work done by the force is equal to the product of the force and the displacement of the object in the direction of force.

If under a constant force **F** the object is displaced through a distance **s,** then work done by the force

$$W = \mathbf{F} \cdot \mathbf{s} = Fs \cos \theta$$

where, θ is the smaller angle between **F** and **s**.

Work is a scalar quantity. Its SI unit is joule and CGS unit is erg.

$\therefore$ $$1 \text{ joule} = 10^7 \text{ erg}$$

Its dimensional formula is $[ML^2T^{-2}]$.

Work done by a force is zero, if

(a) body is not displaced actually, *i.e.* $\mathbf{s} = 0$.

(b) body is displaced perpendicular to the direction of force, *i.e.* $\theta = 90°$.

Work done by a force is **positive**, if angle between **F** and **s** is acute angle.

Work done by a force is **negative**, if angle between **F** and **s** is obtuse angle.

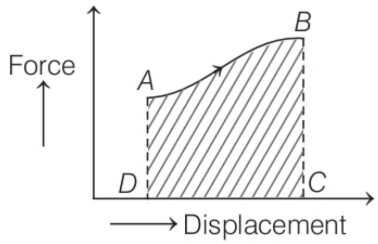
Work done by a constant force depends only on the initial and final positions of the object and not on the actual path followed between initial and final positions.

Work done in different conditions

(i) Work done by a variable force is given by

$$W = \int \mathbf{F} \cdot \mathbf{ds}$$

It is equal to the area under the force-displacement graph, along with proper sign.



Work done = Area $ABCDA$

(ii) Work done in displacing any body under the action of a number of forces is equal to the work done by the resultant force.

(iii) In equilibrium (static or dynamic), the resultant force is zero, therefore resultant work done is zero.

(iv) If work done by a force during a rough trip of a system is zero, then the force is **conservative,** otherwise it is called **non-conservative** force.

- Gravitational force, electrostatic force, magnetic force etc are conservative forces. All the central forces are conservative forces.
- Frictional force, viscous force etc are non-conservative forces.

(v) Work done by the force of gravity on a particle of mass m is given by $W = mgh$

where, g is acceleration due to gravity and h is height through which the particle is displaced.

(vi) Work done in compressing or stretching a spring is given by

$$W = -\frac{1}{2}kx^2$$

where, k is spring constant and x is displacement from mean position.

(vii) When on end of a spring is attached to a fixed vertical support and a block attached to the free end moves on a horizontal table from $x = x_1$ to $x = x_2$, then $W = \frac{1}{2}k(x_2^2 - x_1^2)$.

(viii) Work done by the couple for an angular displacement θ is given by $W = \tau \cdot \theta$,

where τ is the torque of the couple.

Energy

Energy of a body is its capacity of doing work. It is a scalar quantity.

Its SI unit is joule and CGS unit is erg. Its dimensional formula is $[ML^2T^{-2}]$.

There are several types of energies, such as mechanical energy (kinetic energy and potential energy), chemical energy, light energy, heat energy, sound energy, nuclear energy and electric energy etc.

Mechanical Energy

The sum of kinetic and potential energy is known as mechanical energy.

Mechanical energy is of two types

1. Kinetic Energy

The energy possessed by any object by virtue of its motion is called its kinetic energy.

Kinetic energy of an object is given by $K = \frac{1}{2} mv^2 = \frac{p^2}{2m}$

where, m = mass of the object, v = velocity of the object
and $p = mv$ = momentum of the object.

2. Potential Energy

The energy possessed by any object by virtue of its position or configuration is called its potential energy.

In one dimensional motion, potential energy $U(x)$ is defined if force $F(x)$ can be written as

$$F(x) = -\frac{dU}{dx}$$

or $$F(x) \cdot dx = -dU$$

or $$\int_{x_i}^{x_f} F(x) \cdot dx = -\int_{U_i}^{U_f} dU = U_i - U_f$$

Potential energy is defined only for conservative forces. It does not exist for non-conservative forces.

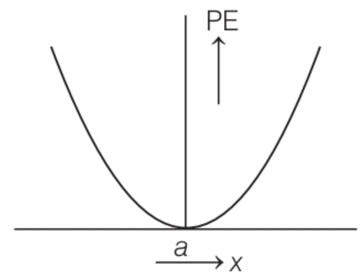
Potential energy depends upon frame of reference.

There are three important types of potential energies

(i) **Gravitational Potential Energy** If a body of mass m is raised through a height h against gravity, then its gravitational potential energy = mgh.

(ii) **Elastic Potential Energy** If a spring of spring constant k is stretched through a distance x, then elastic potential energy of the spring $= \frac{1}{2} kx^2$.

The variation of potential energy with distance is shown in figure.



(iii) **Electric Potential Energy** The electric potential energy of two point charges q_1 and q_2 separated by a distance r in vacuum is given by

$$U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r}$$

Here, $\frac{1}{4\pi\varepsilon_0} = 9.0 \times 10^9 \frac{\text{N-m}^2}{\text{C}^2} = \text{constant}$

Equilibrium

If the forces acting an the object are conservative and it is in equilibrium, then

$$F_{\text{net}} = 0 \Rightarrow \frac{-dU}{dr} = 0 \text{ or } \frac{dU}{dr} = 0$$

Equilibrium of an object or system can be divided into three types

(i) **Stable equilibrium** An object is said to be in stable equilibrium, if on slight displacement from equilibrium position, it has the tendency to come back.

Here, $\frac{d^2U}{dr^2} = \text{positive}$

(ii) **Unstable equilibrium** An object is said to be in unstable equilibrium, if on slight displacement from equilibrium position, it moves in the direction of displacement.

Here, $\frac{d^2U}{dr^2} = \text{negative}$

(iii) **Neutral equilibrium** An object is said to be in neutral equilibrium, if on displacement from its equilibrium position, it has neither the tendency to move in direction of displacement nor to come back to equilibrium position.

Here, $\frac{d^2U}{dr^2} = 0$

Work-Energy Theorem

Work done by a force in displacing a body is equal to change in its kinetic energy.

$$W = \int_{v_1}^{v_2} F \cdot ds = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 = K_f - K_i = \Delta \text{KE}$$

where, K_i = initial kinetic energy

and K_f = final kinetic energy.

Regarding the work-energy theorem, it is worth noting that

(i) If W_{net} is positive, then $K_f - K_i$ = positive, *i.e.* $K_f > K_i$ or kinetic energy will increase and *vice-versa*.

(ii) This theorem can be applied to non-inertial frames also. In a non-inertial frame it can be written as

Work done by all the forces (including the Pseudo force) = Change in kinetic energy in non-inertial frame.

Other Forms of Energy

Heat Energy

A body possess heat energy due to the disorderly motion of its molecules. Heat energy is also related to the internal energy of the body.

Chemical Energy

Chemical energy is stored in the chemical bonds of atoms and molecules.

If the total energy of the reactant is more than the product of the reaction, then heat is released and the reaction is said to be an **exothermic reaction.** If the reverse is true, then heat is absorbed and the reaction is **endothermic**.

Electrical Energy

It is the energy which is associated with the flow of electric current or with charging or discharging of a body.

Nuclear Energy

It is the binding energy of the nucleus of an atom. It is used in nuclear reactors, nuclear fission etc.

Mass-Energy Equivalence

According to Einstein, the mass can be transformed into energy and *vice-versa*.

When Δm mass disappears, then produced energy, $E = \Delta mc^2$
where, c is the speed of light in vacuum.

Principle of Conservation of Energy

Energy can neither be created nor be destroyed, it can only be transferred from one form to another form.

Principle of Conservation of Mechanical Energy

For conservative forces, the total mechanical energy (sum of kinetic and potential energies) of any object remains constant.

Power

The rate at which work is done by a body or energy is transferred is called its power.

$$\text{Power} = \text{Rate of doing work} = \frac{\text{Work done}}{\text{Time taken}}$$

If under a constant force $\mathbf{F}$ a body is displaced through a distance $\mathbf{s}$ in time t, then the power $P = \frac{W}{t} = \frac{\mathbf{F} \cdot \mathbf{s}}{t}$

But $\frac{\mathbf{s}}{t} = \mathbf{v}$, uniform velocity with which body is displaced.

$$\therefore \qquad P = \mathbf{F} \cdot \mathbf{v} = F\, v \cos \theta$$

where, θ is the smaller angle between $\mathbf{F}$ and $\mathbf{v}$.

Power is a scalar quantity. Its SI unit is watt and its dimensional formula is $[\mathrm{ML^2 T^{-3}}]$.

Its other units are kilowatt and horse power,

$$1 \text{ kilowatt} = 1000 \text{ watt}$$
$$1 \text{ horse power} = 746 \text{ watt}$$
$$1 \text{ kWh} = 3.6 \times 10^6 \text{ J}$$

Collisions

Collision between two or more particles is the interaction for a short interval of time in which they apply relatively strong forces on each other.

In a collision, physical contact of two bodies is not necessary.

There are two types of collisions

1. Elastic Collision

The collision in which both the momentum and the kinetic energy of the system remains conserved are called elastic collisions.

In an elastic collision, all the involved forces are conservative forces and total energy remains conserved.

2. **Inelastic Collision**

The collision in which only the momentum remains conserved but kinetic energy does not remain conserved are called inelastic collisions.

The collision in which two particles move together after the collision is called a completely inelastic collision.

In an inelastic collision, some or all the involved forces are non-conservative forces. Total energy of the system remains conserved. If after the collision two bodies stick to each other, then the collision is said to be perfectly inelastic.

Coefficient of Restitution or Resilience (e)

The ratio of relative velocity of separation after collision to the relative velocity of approach before collision is called coefficient of restitution or resilience. It is represented by e and it depends upon the material of the colliding bodies.

For a perfectly elastic collision, $e = 1$

For a perfectly inelastic collision, $e = 0$

For all other collisions, $0 < e < 1$

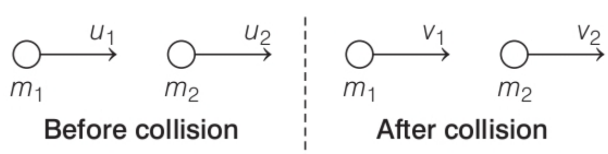
One Dimensional or Head-on Collision

If the initial and final velocities of colliding bodies lie along the same line, then the collision is called one dimensional or head-on collision.

Perfectly Elastic One Dimensional Collision

Applying Newton's experimental law, we have

$$v_2 - v_1 = u_1 - u_2$$



Velocities after collision

$$v_1 = \frac{(m_1 - m_2)\, u_1 + 2m_2u_2}{(m_1 + m_2)} \text{ and } v_2 = \frac{(m_2 - m_1)\, u_2 + 2m_1u_1}{(m_1 + m_2)}$$

Important Points Related to Perfectly Elastic one Dimensional Collision

- When masses of two colliding bodies are equal, then after the collision, the bodies exchange their velocities.

$$v_1 = u_2 \quad \text{and} \quad v_2 = u_1$$

- If second body of same mass ($m_1 = m_2$) is at rest, then after collision first body comes to rest and second body starts moving with the initial velocity of first body.

$$v_1 = 0 \quad \text{and} \quad v_2 = u_1$$

- If a light body of mass m_1 collides with a very heavy body of mass m_2 at rest, then after collision

$$v_1 = -u_1 \quad \text{and} \quad v_2 = 0$$

It means light body will rebound with its own velocity and heavy body will continue to be at rest.

- If a very heavy body of mass m_1 collides with a light body of mass $m_2 (m_1 >> m_2)$ at rest, then after collision

$$v_1 = u_1 \quad \text{and} \quad v_2 = 2u_1$$

In Inelastic One Dimensional Collision

Loss of kinetic energy

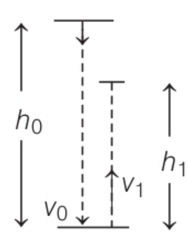
$$\Delta K = \frac{m_1 m_2}{2(m_1 + m_2)} (u_1 - u_2)^2 (1 - e^2)$$

In Perfectly Inelastic One Dimensional Collision

Velocity of separation after collision = 0.

$$\text{Loss of kinetic energy} = \frac{m_1 m_2 (u_1 - u_2)^2}{2(m_1 + m_2)}$$

If a body is dropped from a height h_0 and it strikes the ground with velocity v_0 and after inelastic collision it rebounds with velocity v_1 and rises to a height h_1, then



$$e = \frac{v_1}{v_0} = \sqrt{\frac{2gh_1}{2gh_0}} = \sqrt{\frac{h_1}{h_0}}$$

If after n collisions with the ground, the body rebounds with a velocity v_n and rises to a height h_n, then

$$e^n = \frac{v_n}{v_0} = \sqrt{\frac{h_n}{h_0}}$$

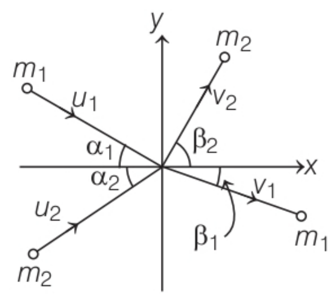
Height covered by the body after nth rebound, $h_n = e^{2n} h_o$

Two Dimensional or Oblique Collision

If the initial and final velocities of colliding bodies do not lie along the same line, then the collision is called two dimensional or oblique collision.

In horizontal direction,

$$m_1 u_1 \cos \alpha_1 + m_2 u_2 \cos \alpha_2 = m_1 v_1 \cos \beta_1 + m_2 v_2 \cos \beta_2$$



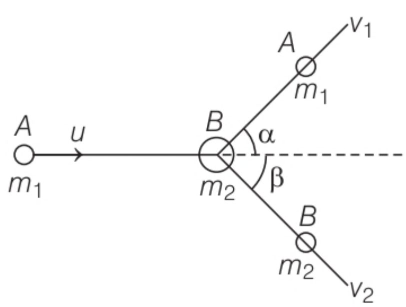
In vertical direction,

$$m_1 u_1 \sin \alpha_1 - m_2 u_2 \sin \alpha_2 = m_1 v_1 \sin \beta_1 - m_2 v_2 \sin \beta_2$$

If $m_1 = m_2$ and $\alpha_1 + \alpha_2 = 90°$

then $\beta_1 + \beta_2 = 90°$

If a particle A of mass m_1 is moving along X-axis with a speed u and makes an elastic collision with another stationary body B of mass m_2, then



From conservation law of momentum,

$$m_1 u = m_1 v_1 \cos \alpha + m_2 v_2 \cos \beta$$

$$0 = m_1 v_1 \sin \alpha - m_2 v_2 \sin \beta$$