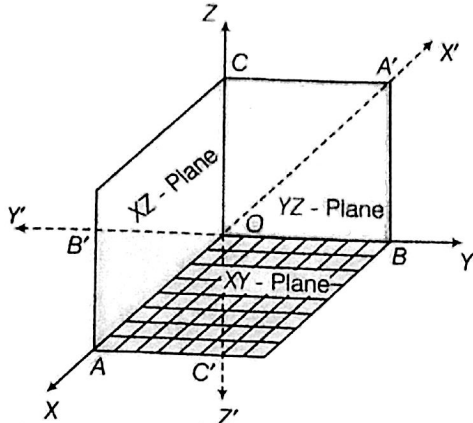


Introduction to Three-Dimensional Geometry

Coordinate Axes and Coordinate Planes

Let XOX' , YOY' and ZOZ' be the three mutually perpendicular lines, intersecting at O . The point O is called the **origin** and these lines are called **rectangular coordinate axes**. These three lines define three mutually perpendicular planes XOY , YOZ and ZOX which are called **coordinate planes** and divide the space into eight parts called **octants**.



In the given figure, XOY is called the XY -plane. YOZ is called the YZ -plane. ZOX is called the ZX -plane. $XOYZ$, $X'OYZ$, $XOY'Z$, $X'OY'Z$, $XOYZ'$, $X'OYZ'$, $XOY'Z'$ and $X'OY'Z'$ are called octants and denoted by I, II, III, ..., VIII, respectively.

2. Coordinates of a Point in Space

The coordinates of a point in the space are the perpendicular distances from P on three mutually perpendicular coordinate planes YZ , ZX and XY , respectively. The coordinates of a point P are written in the form of triplet like (x, y, z) .

The coordinates of any point on

- (i) X -axis are of the form $(x, 0, 0)$.

- (ii) Y -axis are of the form $(0, y, 0)$.
- (iii) Z -axis are of the form $(0, 0, z)$.
- (iv) XY -plane are of the form $(x, y, 0)$.
- (v) YZ -plane are of the form $(0, y, z)$.
- (vi) ZX -plane are of the form $(x, 0, z)$.

3. Distance Formula

- (i) The distance between two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

- (ii) The distance of a point $P(x, y, z)$ from the origin $O(0,0,0)$ is given by $OP = \sqrt{x^2 + y^2 + z^2}$.

4. Section Formulae

- (i) The coordinates of the point R which divides the line segment joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ internally and externally in the ratio $m : n$ are given by

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n} \right) \text{ and}$$

$$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n} \right), \text{ respectively.}$$

- (ii) The coordinates of the mid-point of the line segment joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right).$$

Note (i) The coordinates of the centroid of the triangle, whose vertices are (x_1, y_1, z_1) , (x_2, y_2, z_2) and (x_3, y_3, z_3) are

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3} \right).$$

Points A , B and C are said to be **collinear**, if $AB + BC = AC$.