Alternating Current

[TOPIC 1] Introduction to Alternating Current

1.1 Alternating Current (AC)

An alternating current is the current whose magnitude changes continuously with time between zero and a maximum value and whose direction reverses periodically.

$$\boxed{I = I_0 \sin \omega t} \Rightarrow I = I_0 \sin 2\pi\nu t = I_0 \sin \frac{2\pi}{T} t$$

where, ω = angular frequency in rad/s

I_0 = peak value or maximum value of AC.

Average or Mean Value of AC

It is defined as the value of AC which would send same amount of charge through a circuit in half-cycle that is sent by steady current in the same time. (i.e. $T/2$) $I_{av} = \frac{2I_0}{\pi} = 0.637 I_0$

The 63.7% of peak value of AC gives average or mean value of AC.

Mean value of AC(I_m) is 63% of peak value of AC(I_0) over any half-cycle. In a complete cycle of AC, the mean value of AC will be zero.

Effective Value or rms Value of AC

It is defined as the value of AC over a complete cycle which would generate same amount of heat in a given resistors that is generated by steady current in the same resistor and in the same time during a complete cycle.

$$I_{rms} = \frac{I_0}{\sqrt{2}} = 0.707 I_0 = 70.7\% \text{ of } I_0$$

The 70.7% of peak value of current gives effective or rms value of AC.

1.2 Alternating emf or Voltage

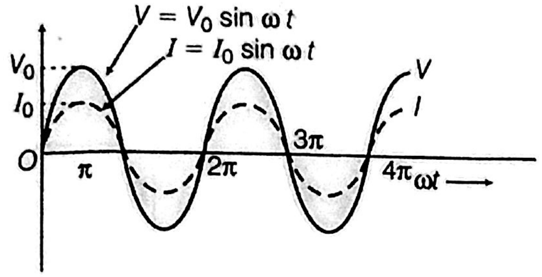
It is the emf or voltage which varies in both magnitude as well as direction alternatively and periodically. The instantaneous alternating emf is given by

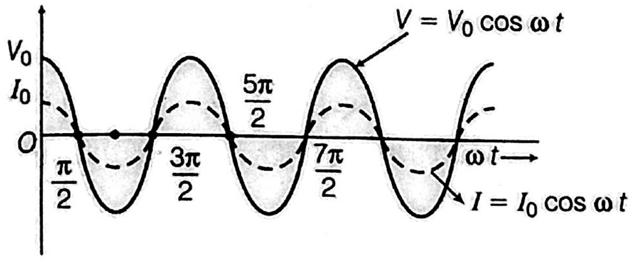
$$V = V_0 \sin \omega t \quad \text{or} \quad V = V_0 \cos \omega t$$

$$V_{rms} = \frac{V_0}{\sqrt{2}} = 0.707 V_0 \quad \text{or} \quad V_{rms} = 70.7\% \text{ of } V_0$$

$$V_{av} = \frac{2V_0}{\pi} = 0.637 V_0 \quad \text{or} \quad V_{av} = 63.7\% \text{ of } V_0$$

Both AC voltage and AC current are represented by diagrams as shown below





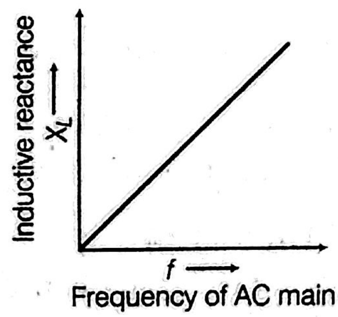
Inductive Reactance (X_L)

The effective resistance or opposition offered by the inductor to the flow of current is called inductive reactance and is denoted by X_L.

$$X_L = \omega L = 2\pi f L$$

Also for a given inductor, $\boxed{X_L = (2\pi L) f}$

$$\Rightarrow \quad X_L \propto f$$



[∵ $2\pi L$ = constant]

where, L = self-inductance.

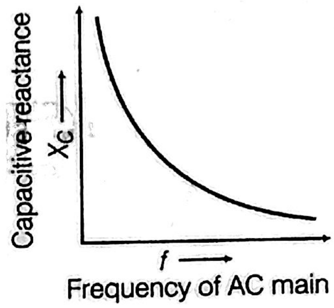
Capacitive Reactance (X_C)

The opposing nature of capacitor to the flow of alternating current is called capacitive reactance.

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

For a given capacitor,

$$X_C = \left(\frac{1}{2\pi C}\right) \cdot \frac{1}{f} \Rightarrow X_C \propto \frac{1}{f}$$



[∵ $1/2\pi C$ = constant]

where, C = capacitance for AC.

1.3 AC Circuits

In an AC circuit, both emf and current change continuously w.r.t. time, so in circuit, we have to calculate average power in complete cycle ($0 \rightarrow T$).

$P_{av} = V_{rms} I_{rms} \cos \phi$

where, $\cos \phi$ = power factor.

Average power consumption in pure inductive and pure capacitive circuit is equal to zero because Phase difference, $\phi = \pi/2$

$$\Rightarrow \quad \text{Power factor} = \cos \frac{\pi}{2} = 0$$

$$\therefore \quad \boxed{P_{av} = 0}$$

Wattless Current

The current in a purely inductive or capacitive AC circuit when average power consumption in AC circuit is zero, is referred as wattless current or **idle current**. In other words, in an AC circuit $R = 0 \Rightarrow \cos \phi = 0$. i.e. in resistance less circuit, the power consumed is zero.

Phasor Diagram

The representation of AC current and voltage (of same frequency) by rotating vectors is called phasor and the diagram representing these phasors is known as phasor diagram.

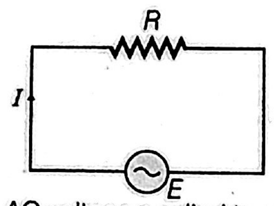
The length of the vector represents the maximum or peak value. The projection of the vectors on fixed axis gives the instantaneous value of alternating current and voltage.

Types of AC Circuits

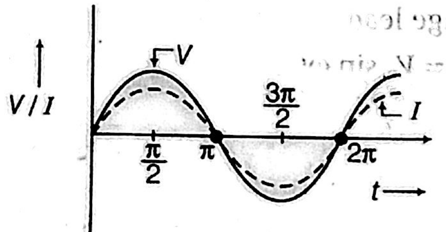
There are some basic types of AC circuits

AC Through Resistor

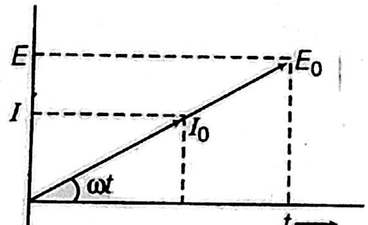
Suppose a resistor of resistance R is connected to an AC source of emf with instantaneous value (E) is given by $E = E_0 \sin\omega t$



An AC voltage applied to a resistor



V-*I* graph



Phasor diagram

Then,

(*i*) Voltage and current are in same phase.

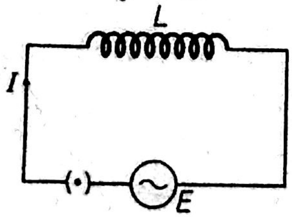
(*ii*) Maximum current, $I_0 = \dfrac{V_0}{R}$

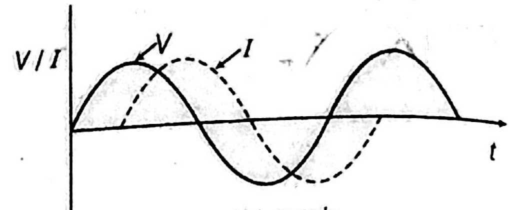
(*iii*) $I_{rms} = \dfrac{V_{rms}}{R}$

(*iv*) If $V = V_0 \sin\omega t$, then $I = I_0 \sin\omega t$.

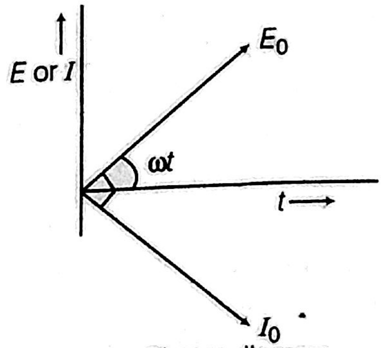
AC Through Inductor

Suppose an inductor with self-inductance L is connected to AC source with instantaneous emf E is given by $E = E_0 \sin\omega t$





V-I graph



Phasor diagram

Then,

(*i*) Inductive reactance, $X_L = \omega L = 2\pi f L$

(*ii*) $I_0 = \dfrac{V_0}{X_L} = \dfrac{V_0}{\omega L}$

(*iii*) $I_{rms} = \dfrac{V_{rms}}{X_L} = \dfrac{V_{rms}}{\omega L} = \dfrac{V_0}{\omega L\sqrt{2}}$

(*iv*) Voltage leads the current by phase $\dfrac{\pi}{2}$

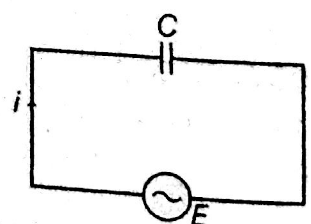
(*v*) If $V = V_0 \sin\omega t$, then $I = I_0 \sin\left(\omega t - \dfrac{\pi}{2}\right)$

(*vi*) Power factor, $\cos\phi = \cos\dfrac{\pi}{2} = 0$

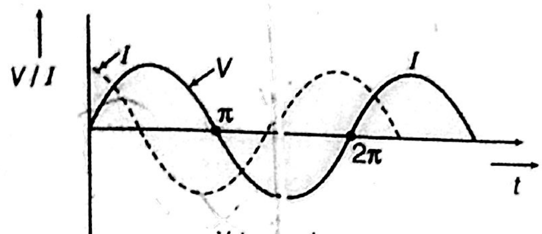
(*vii*) Average power consumption,
$P_{av} = V_{rms} I_{rms} \cos\phi = 0$

AC Through Capacitor

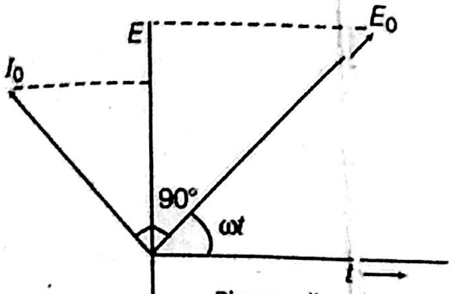
Suppose a capacitor with capacitance C is connected to an AC source with emf having instantaneous value E is given by $E = E_0 \sin\omega t$



An AC source connected to a capacitor



V-I graph



Phasor diagram

Then,

(*i*) Capacitive reactance, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$ i.e.

$X_C \propto \frac{1}{f}$

(*ii*) Capacitor offers infinite reactance in DC circuit as $f = 0$.

$\therefore \quad X_C = \infty$

(*iii*) $I_0 = \frac{V_0}{X_C} = \frac{V_0}{(1/\omega C)} = V_0 \omega C$

(*iv*) Voltage lags behind the current by phase $\frac{\pi}{2}$.

(*v*) If $V = V_0 \sin \omega t$, then $I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$.

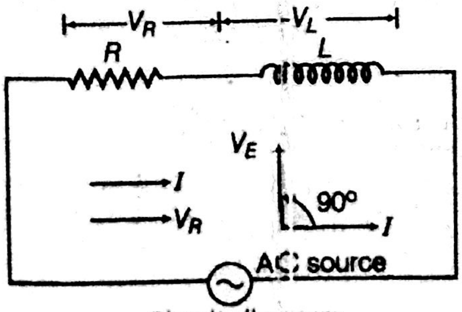
$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{\varepsilon_0}{1/\omega C \cdot \sqrt{2}} = \frac{\varepsilon_{rms}}{1/\omega C} = \frac{\varepsilon_{rms}}{X_C}$$

(*vi*) Power factor is minimum and equal to zero.

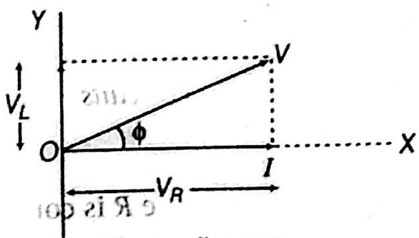
(*vii*) Average power consumption (during a complete cycle), $P_{av} = V_{rms} I_{rms} \cos \phi = 0$

L-R Series AC Circuit

v - iR



Circuit diagram



Phasor diagram

Then,

(*i*) Impedance, $Z = \sqrt{R^2 + X_L^2}$

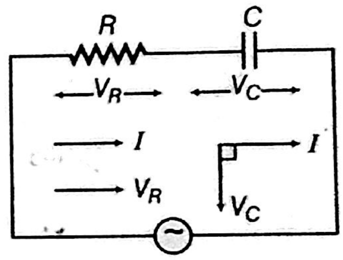
$= \sqrt{R^2 + \omega^2 L^2} \quad [\because X_L = \omega L]$

$= V_{rms} / I_{rms}$

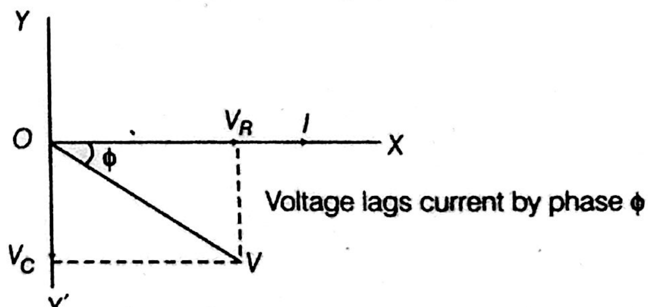
(*ii*) For the phase angle, $\tan \phi = \frac{X_L}{R} = \frac{\omega L}{R}$, voltage leads current by phase ϕ.

(*iii*) If $V = V_0 \sin \omega t$, then $I = I_0 \sin (\omega t - \phi)$

R-C Series AC Circuit



Circuit diagram



Phasor diagram

Then,

(*i*) Impedance, $Z = \frac{V_{rms}}{I_{rms}} = \sqrt{R^2 + X_C^2}$

$= \sqrt{R^2 + \frac{1}{\omega^2 C^2}} \quad \left[\because X_C = \frac{1}{\omega C}\right]$

(*ii*) For the phase angle, $\tan \phi = \frac{X_C}{R} = \frac{1}{\omega C R}$

(*iii*) If $V = V_0 \sin \omega t$, then $I = I_0 \sin (\omega t + \phi)$

(*iv*) Power factor, $\cos \phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + X_C^2}}$

L- C Series AC Circuit

(*i*) Impedance, $Z = \dfrac{V_{rms}}{I_{rms}} = X_L - X_C$

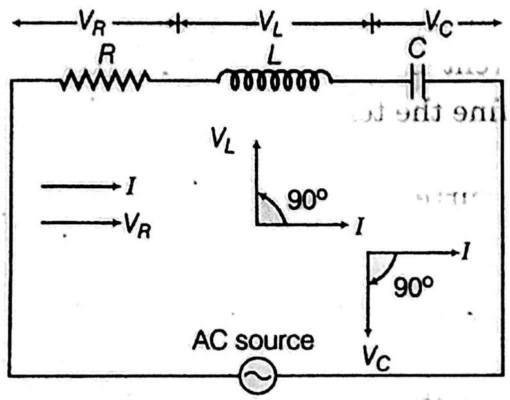
(*ii*) Applied voltage $= V_L - V_C$

(*iii*) Phase difference between voltage and current is $\pi/2$.

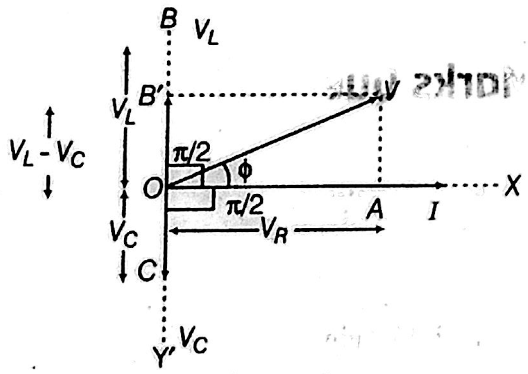
(*iv*) Power factor, $\cos\phi = 0$.

(*v*) Current, $I = I_0 \sin\left(\omega t \pm \dfrac{\pi}{2}\right)$

L- C- R Series AC Circuit



Circuit diagram



Phasor diagram

Then,

(*i*) Impedance,

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \frac{V_{rms}}{I_{rms}}$$

$$= \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

(*ii*) If $X_L > X_C$, then V leads I by ϕ and if $X_L < X_C$, then V lags behind I by ϕ.

where, $\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{V_L - V_C}{V_R}$

$$= \frac{\omega L - \dfrac{1}{\omega C}}{R} = \frac{2\pi\nu L - \dfrac{1}{2\pi\nu C}}{R}$$

(*iii*) If net reactance is inductive, circuit behaves as *L-R* circuit.

(*iv*) If net reactance is capacitive, circuit behaves as *C-R* circuit.

Resonant L-C-R Series AC Circuit

(*i*) $X_L = X_C$

(*ii*) Impedance, $Z = Z_{min} = R$ i.e. circuit behaves as resistive circuit.

(*iii*) The phase difference between V and I is 0°.

(*iv*) Resonant angular frequency,

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow \quad \nu_r = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

(*v*) Average power consumption P_{av} becomes maximum.

(*vi*) Current becomes maximum and

$$I_{max} = \frac{V_{rms}}{R}.$$

Quality Factor

The characteristic of a series resonant circuit is determined by *Q*-factor. It indicates the sharpness of resonance in an *L -C -R* series AC circuit.

$$\text{Quality factor} = \frac{V_L}{V_R} = \frac{V_C}{V_R} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$Q = \frac{\omega_0}{\omega_2 - \omega_1},$$

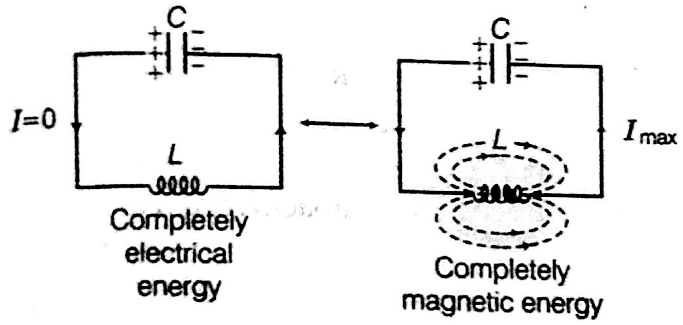
where, ω_1 and ω_2 are the frequencies when current decreases to 0.707 times of the peak value of current.

Quality factor is also defined as

$$\boxed{Q = 2\pi\left(\frac{\text{Maximum energy stored}}{\text{Energy dissipated per cycle}}\right)}$$

L-C Oscillations

Consider the L-C circuit shown as below:



When the charged capacitor is connected with the inductor, current flows through the inductor and energy is stored in the inductor in the form of magnetic field and capacitor discharges and *vice-versa*. In this way, energy oscillates between capacitor and inductor. If the circuit is not ideal oscillations finally die away.

The frequency of oscillation is

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow \boxed{f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}}$$

TOPIC 2] AC Devices

2.1 Choke Coil

A choke coil is an electrical device which is used for controlling current in AC circuits without wasting electrical energy.

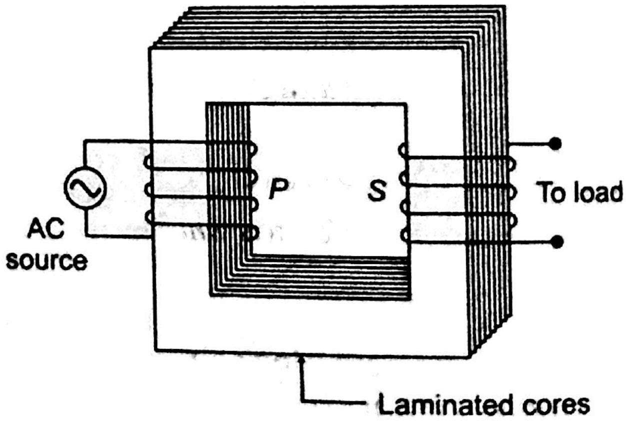
There are two types of choke coil

(*i*) To reduce low frequency alternating currents, choke coils with laminated soft iron cores are used. These are called **AF choke coils.**

(*ii*) To reduce high frequency alternating currents, choke coils with air cores are used. These are called RF **choke coils.**

2.2 Transformer

It is a device which converts high voltage AC into low voltage AC and *vice-versa*. It is based upon the principle of mutual induction. When a variable current is passed through one of the two inductively coupled coils, an induced emf is set up in other coil.



Working

When an alternating current is passed through the primary coil, the magnetic flux through the iron core changes, which does two things, produces emf in the primary coil and an induced emf is set up in the secondary coil . If we assume that the resistance of primary coil is negligible, then the back emf will be equal to the voltage applied to the primary coil.

$\therefore \quad \boxed{V_1 = -N_1 \frac{d\phi}{dt}}$ and $\boxed{V_2 = -N_2 \frac{d\phi}{dt}}$

where, N_1 and N_2 are number of turns in the primary and the secondary coil respectively, while V_1 and V_2 are their voltages, respectively.

$$\therefore \quad \frac{\text{Output emf}}{\text{Input emf}} = \frac{V_2}{V_1} = \frac{N_2}{N_1}$$

Energy Losses in a Transformer

There are some energy losses in a transforms

(i) **Eddy Current Loss** Eddy current in iron core of transformer facilitate the loss of energy in the form of heat.

(ii) **Flux Leakage** Total fluxes linked with primary do not completely pass through the secondary which denotes the loss in the flux or flux leakage.

(iii) **Copper Loss** Due to heating, energy loss takes place in copper wires of primary and secondary coils.

(iv) **Hysteresis Loss** The energy loss takes place in magnetising and demagnetising the iron core over every cycle.

(v) **Humming Loss** The magnetostriction effect leads to set the core in vibration which in turn produced the sound. This loss is referred as humming loss.

Types of Transformer

There are two types of transformers which are given as below:

(i) **Step-up transformer** $(N_2 > N_1)$ It converts low alternating voltage into high alternating voltage.

(ii) **Step-down transformer** $(N_2 < N_1)$ It converts high alternating voltage into low alternating voltage.

Important Points Related to Transformer

- For an ideal transformer,
  Input power = Output power
  $$V_1 I_1 = V_2 I_2 \Rightarrow V_1/V_2 = I_2/I_1$$
- Transformation ratio (r)
  $$\boxed{r = N_S/N_P = V_S/V_P = I_P/I_S}$$

where, N_P and N_S are the number of turns in primary and secondary coils. V_P and V_S are alternative voltage in primary and secondary coils. I_P and I_S are AC in primary and secondary coils.

- Long distance power transmission takes place at high alternating voltage so as to minimise losses in the form of heat, therefore, step-up transformers are used at power stations and step-down transformers at receiving ends.
- Efficiency of transformer,
  $$\boxed{\eta = \frac{\text{Output power}}{\text{Input power}} \times 100}$$
- The flux linked with the coil at any instant t is given by $\phi = NBA \cos \omega t$
  $$\Rightarrow \quad \phi_{max} = NBA \quad [\because (\cos \omega t)_{max} = 1]$$
- Instantaneous value of alternating voltage is
- given by $\boxed{V = NBA\omega \sin \omega t}$
- Peak value of alternating voltage is given by
  $$\boxed{V_0 = NBA\omega}$$
- Instantaneous AC is given by $\boxed{I = I_0 \sin \omega t}$

where, $I_0 = NBA\omega/R$

where, N = number of turns, B = magnetic field, A = area and ω = frequency.