

Nuclei

[TOPIC 1] Nucleus, Radioactivity and Decay Law

1.1 Nucleus

In every atom, the positive charge and mass are densely concentrated at the central core of the atom forming its nucleus. More than 99.9% mass of the atom is concentrated in the nucleus.

Atomic Mass Unit (amu)

The unit of mass used to express mass of an atom is called atomic mass unit. Atomic mass unit is defined as 1/12th of the mass of carbon (${}^{12}_6\text{C}$) atom.

i.e. 1 amu or 1 u = 1.660539×10^{-27} kg

- (i) Mass of proton (m_p) = 1.00727 u
- (ii) Mass of neutron (m_n) = 1.00866 u
- (iii) Mass of electron (m_e) = 0.000549 u

Relation between amu and MeV

1 amu = 931 MeV

Composition of Nucleus

The composition of a nucleus can be described by using the following terms and symbols.

- (i) **Atomic Number (Z)** Atomic number of an element is the number of protons present inside the nucleus of an atom of the element.

Atomic number (Z) = Number of protons = Number of electrons (in a neutral atom)

- (ii) **Mass Number (A)** Mass number of an element is the total number of protons and neutrons inside the atomic nucleus of the element.

Mass number (A) = Number of protons + Number of neutrons
= Number of electrons + Number of neutrons

i.e.,

$$A = Z + N$$

Size of Nucleus

According to the scattering experiments, nuclear sizes of different elements are assumed to be spherical, so the volume of a nucleus is directly proportional to its mass number. If R is the radius of the nucleus having mass number A , then

$$\frac{4}{3}\pi R^3 \propto A \Rightarrow R \propto A^{1/3}$$

\Rightarrow

$$R = R_0 A^{1/3}$$

where, $R_0 = 1.2 \times 10^{-15}$ m is the range of nuclear size. It is also known as nuclear unit radius.

Nuclear Density

Density of nuclear matter is the ratio of mass of nucleus and its volume.

i.e.

$$\rho = \frac{m}{\frac{4}{3}\pi R_0^3} = \frac{3m}{4\pi R_0^3}$$

$$= 2.38 \times 10^{17} \text{ kg/m}^3$$

where, m = average mass of one nucleon and
 $R_0 = 1.2 \text{ fm} = 1.2 \times 10^{-15} \text{ m}$

\Rightarrow The nuclear density (ρ) does not depend on A (mass number).

1.2 Radioactivity

It is the phenomenon of spontaneous disintegration of the nucleus of an unstable atom of heavy elements with emission of one or more penetrating radiations like α -particle or β -particle and electromagnetic radiations like γ -rays.

Law of Radioactivity Decay

According to this law, the rate of decay of radioactive atoms at any instant is proportional to the number of atoms present at that instant.

$$(i) \frac{-dN}{dt} \propto N,$$

or

$$\frac{dN}{dt} = -\lambda N$$

where, λ = decay constant and
 N = number of undisintegrated nucleus present in the sample at any instant t .

On integrating both sides, we get

$$\Rightarrow \int \frac{dN}{N} = -\lambda \int dt$$

$$\Rightarrow \log_e N = -\lambda t + C$$

C = constant of integration

(ii) $N = N_0 e^{-\lambda t}$, where N_0 = original amount of atom.

The SI unit of radioactivity is Becquerel (Bq).

1 Becquerel (Bq) = 1 disintegration/second

Some other units are as follows

(i) **Curie** The activity of a radioactive sample is said to be one curie.

$$1 \text{ curie (Ci)} = 3.7 \times 10^{10} \text{ decays/second}$$

$$= 3.7 \times 10^{10} \text{ Bq}$$

(ii) **Rutherford** The activity of a radioactive sample is said to be one Rutherford.

$$1 \text{ Rutherford (Rd)} = 10^6 \text{ decays/second}$$

$$= 10^6 \text{ Bq}$$

Decay Constant

The radioactive decay constant may be defined as the reciprocal of the time interval during which the number of atoms in a radioactive substance reduces to 36.8% of their initial number.

$$\lambda = \frac{2.303}{t} \log_{10} \left(\frac{N_0}{N} \right)$$

Value of λ depends on the nature of radioactive substance.

Half-life

Time interval in which the mass of a radioactive substance or the number of its atom reduces to half of its initial value, is called half-life of the substance and is denoted by $T_{1/2}$.

i.e. if $N = N_0/2$, then $t = T_{1/2}$

Hence, from $N = N_0 e^{-\lambda t}$

$$\Rightarrow N_0/2 = N_0 e^{-\lambda(T_{1/2})}$$

$$\Rightarrow T_{1/2} = \log_e 2 / \lambda = 0.693 / \lambda$$

Mean Life/Average Life

Average life of a radioactive element can be obtained by calculating the total life time of all the atoms of the radioactive element and dividing it by the total number of atoms present initially in the sample of the element. It is denoted by τ .

From half-life, $T_{1/2} = 0.693 / \lambda$

$$\Rightarrow \frac{1}{\lambda} = \tau = \frac{1}{0.693} T_{1/2} = 1.44(T_{1/2})$$

Time required to decay from N_0 to N

$$t = \frac{2.303}{\lambda} \log_{10} (N_0/N)$$

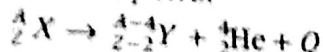
1.3 Radioactive Displacement Law

The law of radioactive displacement is also known as **Rajan's and Soddy law**. This law describes which chemical element and isotope is created during the particular type of radioactive decay.

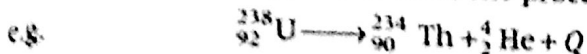
Some of the radioactive decays are as follows

α -Decay

In α -decay, the mass number of the product nucleus is four less than that of decaying nucleus while the atomic number decreases by two.

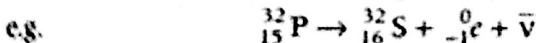
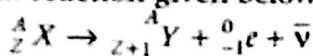


where, Q is the energy released in the process.

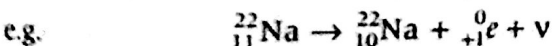
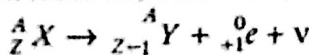


β -Decay

In β -decay, the mass number of product nucleus remains same but atomic number increases or decreases by one. In beta-minus decay (β^-), an electron and an anti-neutrino are created and emitted from the nucleus *via* reaction given below

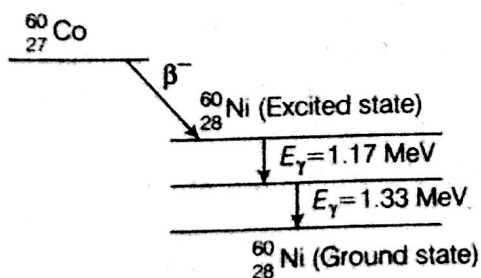
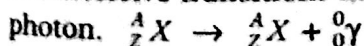


In beta-plus decay (β^+), a positron and a neutrino are created and emitted from the nucleus *via* the reaction given below



γ -Decay

A γ -ray is emitted when α or β -decay results in a daughter nucleus in an excited state. Atom then returns to ground state by a single photon transition or successive transitions involving more than one photon.



Energy-level diagram showing the emission of γ -rays by a ${}^{60}_{28} \text{Ni}$ nucleus

[TOPIC 2] Mass Defect and Binding Energy

2.1 Mass Defect

The sum of the masses of neutrons and protons forming a nucleus is more than the actual mass of the nucleus. This difference of masses is known as mass defect.

$$\Delta m = Zm_p + (A - Z)m_n - M$$

where, Z = atomic number, A = mass number, m_p = mass of one proton, m_n = mass of one neutron and M = mass of nucleus.

Mass-Energy Relation

Einstein showed that mass is another form of energy and one can convert mass-energy into other forms of energy. Einstein's mass-energy equivalence equation is given by

$$E = mc^2$$

where, E is the energy and c is the speed of light $= 3 \times 10^8$ m/s and m = mass of nucleus.

2.2 Binding Energy

The binding energy of a nucleus is defined as the minimum energy required to separate its nucleons and place them at rest at infinite distance apart.

Using Einstein's mass-energy relation, $\Delta E = (\Delta m)c^2$, the binding energy of the nucleus is

$$\Delta E_b = [Zm_p + (A - Z)m_n - M] c^2$$

Note It is also called separation energy.

Average Binding Energy Per Nucleon of a Nucleus

It is the average energy required to extract a nucleon from the nucleus to infinite distance. It is

given by total binding energy divided by the mass number of the nucleus.

Binding energy per nucleon

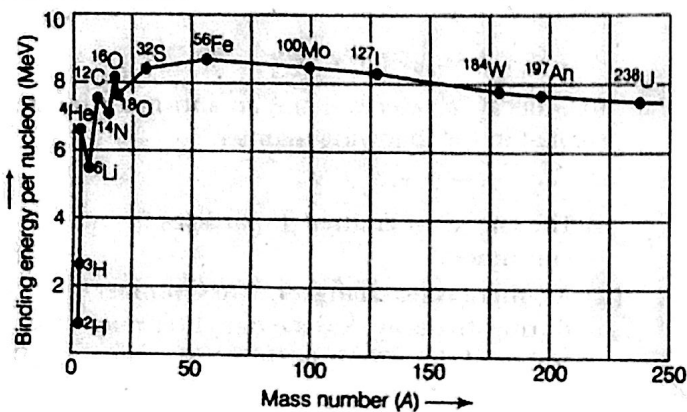
$$= \frac{\text{Total binding energy}}{\text{Number of nucleons}}$$

⇒

$$E_{bn} = \frac{E_b}{A}$$

Binding Energy Curve

It is a plot of the binding energy per nucleon *versus* the mass number A for a large number of nuclei as shown below:



Binding energy per nucleon as a function of mass number

It is used to explain phenomena of nuclear fission and fusion.

Nuclear Stability

The stability of a nucleus is determined by the value of its binding energy per nucleon. The constancy of the binding energy in the range $30 < A < 170$ is a consequence of the fact that the nuclear force is short-ranged.

Nuclear Forces

Short ranged ($d \approx 2-3$ fm) strong attractive forces which hold protons and neutrons together in against of coulombian repulsive forces between positively charged particle. The nuclear force between neutron-neutron, proton-neutron and proton-proton is approximately same. The nuclear force does not depend on the electric charge.

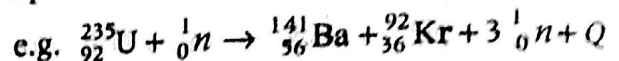
Nuclear Energy

Transmutation of less stable nuclei into more tightly bound nuclei provides an excellent possibility of releasing nuclear energy.

Two distinct ways of obtaining energy from nucleus are given below

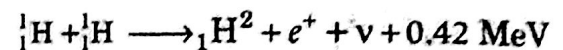
Nuclear Fission

The phenomenon of splitting of heavy nuclei (usually $A > 230$) into lighter nuclei of nearly equal masses is known as nuclear fission,



Nuclear Fusion

The phenomenon of fusing or combining of two lighter nuclei into a single heavy nucleus is called nuclear fusion, e.g.



NOTE The energy released during nuclear fusion is known as thermonuclear energy.

Nuclear Chain Reaction

The fact that more neutrons are produced in fission than are consumed gives the possibility of a chain reaction with each neutron that is produced triggering or initiating another fission.

- (i) **Controlled Chain Reaction** The chain reaction can be controlled and maintained steadily by absorbing a suitable number of neutrons at each stage, so that on an average one neutron remains available for exciting further fission such a reaction is called controlled chain reaction. e.g. Nuclear reactor
- (ii) **Uncontrolled Chain Reaction** During fission reaction, neutron released again absorbed by the fissile isotopes, the cycle repeats to give a reaction that is self sustaining leading to large amount of radiation. Such a reaction is called uncontrolled chain reaction. e.g. Atom bomb.