

SOLVED EXAMPLES

Ex. 1 Evaluate: $\lim_{x \rightarrow 1} \frac{4 - \sqrt{15x+1}}{2 - \sqrt{3x+1}}$

Sol.
$$\lim_{x \rightarrow 1} \frac{4 - \sqrt{15x+1}}{2 - \sqrt{3x+1}} = \lim_{x \rightarrow 1} \frac{(4 - \sqrt{15x+1})(2 + \sqrt{3x+1})(4 + \sqrt{15x+1})}{(2 - \sqrt{3x+1})(4 + \sqrt{15x+1})(2 + \sqrt{3x+1})}$$

$$= \lim_{x \rightarrow 1} \frac{(15 - 15x)}{(3 - 3x)} \times \frac{2 + \sqrt{3x+1}}{4 + \sqrt{15x+1}} = \frac{5}{2}$$

Ex. 2 If $\lim_{x \rightarrow \infty} \left(\frac{x^3 + 1}{x^2 + 1} - (ax + b) \right) = 2$, then find value of a and b.

Sol.
$$\lim_{x \rightarrow \infty} \left(\frac{x^3 + 1}{x^2 + 1} - (ax + b) \right) = 2 \quad \Rightarrow \quad \lim_{x \rightarrow \infty} \frac{x^3(1-a) - bx^2 - ax + (1-b)}{x^2 + 1} = 2$$

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{x(1-a) - b - \frac{a}{x} + \frac{(1-b)}{x^2}}{1 + \frac{1}{x^2}} = 2 \quad \Rightarrow \quad 1-a=0, -b=2 \quad \Rightarrow \quad a=1, b=-2$$

Ex. 3 Evaluate: $\lim_{x \rightarrow 1} \frac{x^{P+1} - (P+1)x + P}{(x-1)^2}$

Sol.
$$\lim_{x \rightarrow 1} \frac{x^{P+1} - (P+1)x + P}{(x-1)^2} \quad \left(\frac{0}{0} \text{ form} \right)$$

$$= \lim_{x \rightarrow 1} \frac{x^{P+1} - Px - x + P}{(x-1)^2} = \lim_{x \rightarrow 1} \frac{x(x^P - 1) - P(x-1)}{(x-1)^2}$$

Dividing numerator and denominator by $(x-1)$, we get

$$= \lim_{x \rightarrow 1} \frac{\frac{x(x^P - 1)}{x-1} - P}{(x-1)} = \lim_{x \rightarrow 1} \frac{(x + x^2 + x^3 + \dots + x^P) - P}{(x-1)}$$

$$= \lim_{x \rightarrow 1} \frac{(x + x^2 + x^3 + \dots + x^P) - (1 + 1 + 1 + \dots \text{upto } P \text{ times})}{(x-1)}$$

$$= \lim_{x \rightarrow 1} \left\{ \frac{(x-1)}{(x-1)} + \frac{(x^2-1)}{(x-1)} + \frac{(x^3-1)}{(x-1)} + \dots + \frac{(x^P-1)}{(x-1)} \right\}$$

$$= 1 + 2(1)^{2-1} + 3(1)^{3-1} + \dots + P(1)^{P-1} = 1 + 2 + 3 + \dots + P = \frac{P(P+1)}{2}$$

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Ex. 4 Evaluate : $\lim_{x \rightarrow 2} \left[\frac{1}{x-2} - \frac{2(2x-3)}{x^3-3x^2+2x} \right]$

Sol. We have

$$\begin{aligned} \lim_{x \rightarrow 2} \left[\frac{1}{x-2} - \frac{2(2x-3)}{x^3-3x^2+2x} \right] &= \lim_{x \rightarrow 2} \left[\frac{1}{x-2} - \frac{2(2x-3)}{x(x-1)(x-2)} \right] = \lim_{x \rightarrow 2} \left[\frac{x(x-1)-2(2x-3)}{x(x-1)(x-2)} \right] \\ &= \lim_{x \rightarrow 2} \left[\frac{x^2-5x+6}{x(x-1)(x-2)} \right] = \lim_{x \rightarrow 2} \left[\frac{(x-2)(x-3)}{x(x-1)(x-2)} \right] = \lim_{x \rightarrow 2} \left[\frac{x-3}{x(x-1)} \right] = -\frac{1}{2} \end{aligned}$$

Ex. 5 Evaluate $\lim_{x \rightarrow \infty} x \cos \left(\frac{\pi}{4x} \right) \sin \left(\frac{\pi}{4x} \right)$.

Sol.

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{x}{2} \left(2 \sin \frac{\pi}{4x} \cos \frac{\pi}{4x} \right) &= \lim_{x \rightarrow \infty} \frac{x}{2} \sin \left(\frac{\pi}{2x} \right) \\ &= \lim_{x \rightarrow \infty} \frac{\sin \left(\frac{\pi}{2x} \right)}{\frac{\pi}{2x}} \cdot \frac{\pi}{4} = \frac{\pi}{4} \lim_{y \rightarrow 0} \frac{\sin y}{y} = \frac{\pi}{4}, \text{ where } y = \frac{\pi}{2x} \end{aligned}$$

Ex. 6 Evaluate $\lim_{x \rightarrow 1} (\log_3 3x)^{\log_x 3}$

Sol.

$$\begin{aligned} \lim_{x \rightarrow 1} (\log_3 3x)^{\log_x 3} &= \lim_{x \rightarrow 1} (\log_3 3 + \log_3 x)^{\log_x 3} \\ &= \lim_{x \rightarrow 1} (1 + \log_3 x)^{1/\log_3 x} = e \\ \therefore \log_b a &= \frac{1}{\log_a b} \end{aligned}$$

Ex. 7 Evaluate : $\lim_{x \rightarrow 0} \frac{x^3 \cot x}{1 - \cos x}$

Sol.

$$\lim_{x \rightarrow 0} \frac{x^3 \cos x}{\sin x (1 - \cos x)} = \lim_{x \rightarrow 0} \frac{x^3 \cos x (1 + \cos x)}{\sin x \cdot \sin^2 x} = \lim_{x \rightarrow 0} \frac{x^3}{\sin^3 x} \cdot \cos x (1 + \cos x) = 2$$

Ex. 8 Evaluate $\lim_{x \rightarrow 1} \frac{(7+x)^{\frac{1}{3}} - 2}{x-1}$

Sol. Put $x = 1 + h$

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{(8+h)^{\frac{1}{3}} - 2}{h} &= \lim_{h \rightarrow 0} \frac{2 \cdot \left(1 + \frac{h}{8} \right)^{\frac{1}{3}} - 2}{h} = \lim_{h \rightarrow 0} \frac{2 \left\{ 1 + \frac{1}{3} \cdot \frac{h}{8} + \frac{1}{3} \left(\frac{1}{3} - 1 \right) \left(\frac{h}{8} \right)^2 + \dots - 1 \right\}}{h} \\ &= \lim_{h \rightarrow 0} 2 \times \frac{1}{24} = \frac{1}{12} \end{aligned}$$

Ex. 9 Evaluate $\lim_{x \rightarrow \infty} \frac{1}{1+n^2} + \frac{1}{2+n^2} + \dots + \frac{n}{n+n^2}$.

Sol. $P_n = \frac{1}{1+n^2} + \frac{2}{2+n^2} + \dots + \frac{n}{n+n^2}$

Now, $P_n < \frac{1}{1+n^2} + \frac{2}{1+n^2} + \dots + \frac{n}{1+n^2}$

$$= \frac{1}{1+n^2} (1 + 2 + 3 + \dots + n)$$

$$= \frac{n(n+1)}{2(1+n^2)}$$

Also, $P_n > \frac{1}{n+n^2} + \frac{2}{n+n^2} + \frac{3}{n+n^3} + \dots + \frac{n}{n+n^2}$

$$= \frac{n(n+1)}{2(n+n^2)}$$

Thus, $\frac{n(n+1)}{2(n+n^2)} < P_n < \frac{n(n+1)}{2(1+n^2)}$

or $\lim_{n \rightarrow \infty} \frac{n(n+1)}{2(n+n^2)} < \lim_{n \rightarrow \infty} P_n < \lim_{n \rightarrow \infty} \frac{n(n+1)}{2(1+n^2)}$

or $\lim_{n \rightarrow \infty} \frac{1\left(1 + \frac{1}{n}\right)}{2\left(\frac{1}{n} + 1\right)} < \lim_{n \rightarrow \infty} P_n < \lim_{n \rightarrow \infty} \frac{1\left(1 + \frac{1}{n}\right)}{2\left(\frac{1}{n^2} + 1\right)}$

or $\frac{1}{2} < \lim_{n \rightarrow \infty} P_n < \frac{1}{2}$

or $\lim_{n \rightarrow \infty} P_n = \frac{1}{2}$

Ex. 10 Evaluate $\lim_{x \rightarrow 0} \left(\frac{a^x + b^x + c^x}{3} \right)^{1/x}$

Sol. $\lim_{x \rightarrow 0} \left(\frac{a^x + b^x + c^x}{3} \right)^{1/x} = \lim_{x \rightarrow 0} \left(1 + \frac{a^x + b^x + c^x - 3}{3} \right)^{1/x}$

$$= \lim_{x \rightarrow 0} \left[\left(1 + \frac{(a^x - 1)}{3} + \frac{(b^x - 1)}{3} + \frac{(c^x - 1)}{3} \right)^{\frac{3}{(a^x - 1) + (b^x - 1) + (c^x - 1)}} \right]^{\frac{a^x - 1 + b^x - 1 + c^x - 1}{3x}}$$

$$= e^{1/3} \lim_{x \rightarrow 0} \left[\frac{a^x - 1}{x} + \frac{b^x - 1}{x} + \frac{c^x - 1}{x} \right] = e^{1/3} (\log a + \log b + \log c) = e^{\log(abc)/3} = (abc)^{1/3}$$

Ex. 11 Evaluate $\lim_{x \rightarrow a} \left(2 - \frac{a}{x}\right)^{\tan \frac{\pi x}{2a}}$.

Sol. $\lim_{x \rightarrow a} \left(2 - \frac{a}{x}\right)^{\tan \frac{\pi x}{2a}}$ put $x = a + h$

$$= \lim_{h \rightarrow 0} \left(1 + \frac{h}{(a+h)}\right)^{\tan\left(\frac{\pi}{2} + \frac{\pi h}{2a}\right)} = \lim_{h \rightarrow 0} \left(1 + \frac{h}{a+h}\right)^{-\cot\left(\frac{\pi h}{2a}\right)} = e^{\lim_{h \rightarrow 0} -\cot \frac{\pi h}{2a} \cdot \left(1 + \frac{h}{a+h} - 1\right)}$$

$$= e^{\lim_{h \rightarrow 0} -\left(\frac{\pi h}{2a}\right) \cdot \frac{2a}{a+h}} = e^{-\frac{2}{\pi}}$$

Ex. 12 Evaluate $\lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \tan x}{1 - \sqrt{2} \sin x}$

Sol. Put $x = \frac{\pi}{4} + h$

$\therefore x \rightarrow \frac{\pi}{4}$

$\Rightarrow h \rightarrow 0$

$$\lim_{h \rightarrow 0} \frac{1 - \tan\left(\frac{\pi}{4} + h\right)}{1 - \sqrt{2} \sin\left(\frac{\pi}{4} + h\right)} = \lim_{h \rightarrow 0} \frac{1 - \frac{1 + \tan h}{1 - \tan h}}{1 - \sin h - \cos h} = \lim_{h \rightarrow 0} \frac{\frac{-2 \tan h}{1 - \tan h}}{2 \sin^2 \frac{h}{2} - 2 \sin \frac{h}{2} \cos \frac{h}{2}}$$

$$= \lim_{h \rightarrow 0} \frac{-2 \tan h}{2 \sin \frac{h}{2} \left[\sin \frac{h}{2} - \cos \frac{h}{2} \right]} \cdot \frac{1}{(1 - \tanh)} = \lim_{h \rightarrow 0} \frac{-2 \frac{\tanh}{h}}{\frac{\sin \frac{h}{2}}{\frac{h}{2}} \left[\sin \frac{h}{2} - \cos \frac{h}{2} \right]} \cdot \frac{1}{(1 - \tanh)} = \frac{-2}{-1} = 2.$$

Ex. 13 If $\lim_{x \rightarrow 0} \frac{\cos 4x + a \cos 2x + b}{x^4}$ is finite, find a and b using expansion formula.

Sol. $\lim_{x \rightarrow 0} \frac{\cos 4x + a \cos 2x + b}{x^4} = \text{Finite}$

Using expansion formula for $\cos 4x$ and $\cos 2x$, we get

$$\lim_{x \rightarrow 0} \frac{\left(1 - \frac{(4x)^2}{2!} + \frac{(4x)^4}{4!}\right) + a\left(1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!}\right) + b}{x^4} = \text{finite}$$

or $\lim_{x \rightarrow 0} \frac{(1 + a + b) + (-8 - 2a)x^2 + \left(\frac{32}{3} + \frac{2}{3}a\right)x^4 + \dots}{x^4}$

or $1 + a + b = 0$...**(i)**

$-8 - 2a = 0$...**(ii)**

Solving **(i)** and **(ii)** for a and b, we get

$a = -4$ and $b = 3$

Also, $L = \frac{32}{3} + \frac{2}{3}a = \frac{32-8}{3} = 8$

Ex. 14 Evaluate $\lim_{n \rightarrow \infty} \frac{n^p \sin^2(n!)}{n+1}$.

Sol.
$$\lim_{n \rightarrow \infty} \frac{n^p \sin^2(n!)}{n+1} = \lim_{n \rightarrow \infty} \frac{\sin^2(n!)}{n^{1-p} \left(1 + \frac{1}{n}\right)}$$

$$= \frac{\text{some number between 0 and 1}}{\infty} = 0$$

Ex. 15 Evaluate $\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^{\left(\frac{\sin x}{x - \sin x} \right)}$

Sol. Since $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ and $\lim_{x \rightarrow 0} \frac{\sin x}{x - \sin x} = \lim_{x \rightarrow 0} \frac{1}{\left(\frac{x}{\sin x} - 1 \right)}$

$$= \frac{1}{1-1} = \infty$$

we have,
$$\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^{\left(\frac{\sin x}{x - \sin x} \right)} = e^{\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} - 1 \right) \left(\frac{\sin x}{x - \sin x} \right)}$$

$$= e^{\lim_{x \rightarrow 0} -\frac{\sin x}{x}} = e^{-1} = \frac{1}{e}$$

Exercise # 1

[Single Correct Choice Type Questions]

- $\lim_{x \rightarrow \infty} x \left(\tan^{-1} \frac{x+1}{x+2} - \cot^{-1} \frac{x+2}{x} \right)$ is

(A) -1 (B) $\frac{1}{2}$ (C) $-\frac{1}{2}$ (D) non existent
- Limit $\lim_{x \rightarrow 0} \left(\frac{3}{1 + \sqrt{4+x}} \right)^{\operatorname{cosec} x}$ has the value equal to :

(A) $e^{-1/12}$ (B) $e^{-1/6}$ (C) $e^{-1/4}$ (D) $e^{-1/3}$
- Limit $\lim_{x \rightarrow \infty} \frac{\cot^{-1}(\sqrt{x+1} - \sqrt{x})}{\sec^{-1} \left\{ \left(\frac{2x+1}{x-1} \right)^x \right\}}$ is equal to

(A) 1 (B) 0 (C) $\pi/2$ (D) non existent
- $\lim_{n \rightarrow \infty} \left(\frac{1}{1.3} + \frac{1}{3.5} + \dots + \text{to } n \text{ terms} \right)$ is equal to -

(A) $1/4$ (B) $1/2$ (C) 1 (D) 2
- $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2+1} - \sqrt[3]{x^2+1}}{\sqrt[4]{x^4+1} - \sqrt[5]{x^4-1}}$ is equal to -

(A) 1 (B) -1 (C) 0 (D) none of these
- If $\lim_{x \rightarrow a} [f(x) + g(x)] = 2$ and $\lim_{x \rightarrow a} [f(x) - g(x)] = 1$, then $\lim_{x \rightarrow a} f(x)g(x) -$

(A) need not exist (B) exist and is $\frac{3}{4}$ (C) exists and is $-\frac{3}{4}$ (D) exists and is $\frac{4}{3}$
- Let α and β be the distinct roots of $ax^2 + bx + c = 0$ then $\lim_{x \rightarrow \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - \alpha)^2}$ is equal to -

(A) $\frac{1}{2}(\alpha - \beta)^2$ (B) $\frac{-a^2}{2}(\alpha - \beta)^2$ (C) 0 (D) $\frac{a^2}{2}(\alpha - \beta)^2$
- If $\lim_{x \rightarrow 0} (x^{-3} \sin 3x + ax^{-2} + b)$ exists and is equal to zero then :

(A) $a = -3$ & $b = 9/2$ (B) $a = 3$ & $b = 9/2$ (C) $a = -3$ & $b = -9/2$ (D) $a = 3$ & $b = -9/2$
- If α, β are the roots of the quadratic equation $ax^2 + bx + c = 0$ then $\lim_{x \rightarrow \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - \alpha)^2}$ equals

(A) 0 (B) $\frac{1}{2}(\alpha - \beta)^2$ (C) $\frac{a^2}{2}(\alpha - \beta)^2$ (D) $-\frac{a^2}{2}(\alpha - \beta)^2$

10. Let $l = \lim_{x \rightarrow \infty} \left(\frac{x+1}{x-1} \right)^x$ then $\{l\}$ where $\{x\}$ denotes the fractional part function is
 (A) $8 - e^2$ (B) $7 - e^2$ (C) $e^2 - 6$ (D) $e^2 - 7$
11. $\lim_{x \rightarrow 1} (1-x) \tan \left(\frac{\pi x}{2} \right)$ is equal to -
 (A) $2/\pi$ (B) $\pi/2$ (C) π (D) none of these
12. The value of $\lim_{x \rightarrow 0} \frac{(\tan(\{x\}-1)) \sin \{x\}}{\{x\}(\{x\}-1)}$ where $\{x\}$ denotes the fractional part function -
 (A) is 1 (B) is $\tan 1$ (C) is $\sin 1$ (D) is not existent
13. $\lim_{x \rightarrow \infty} (\sin \sqrt{x+1} - \sin \sqrt{x})$ is equal to -
 (A) 1 (B) -1 (C) 0 (D) none of these
14. $\lim_{x \rightarrow \pi/2} \frac{[1 - \tan x / 2][1 - \sin x]}{[1 + \tan x / 2][\pi - 2x]^3}$ is equal to -
 (A) 0 (B) $1/32$ (C) ∞ (D) $1/8$
15. If $\lim_{x \rightarrow \infty} \left(1 + \frac{a}{x} + \frac{b}{x^2} \right)^{2x} = e^2$, then the values of a and b are -
 (A) $a \in \mathbb{R}, b = 2$ (B) $a = 1, b \in \mathbb{R}$ (C) $a \in \mathbb{R}, b \in \mathbb{R}$ (D) $a = 1$ and $b = 2$
16. $\lim_{x \rightarrow 0} \frac{\sqrt[3]{1+3x} - 1 - x}{(1+x)^{101} - 1 - 101x}$ has the value equal to -
 (A) $-\frac{3}{5050}$ (B) $-\frac{1}{5050}$ (C) $\frac{1}{5051}$ (D) $\frac{1}{4950}$
17. $\lim_{x \rightarrow 0} \frac{e^{x^3} - \tan x + \sin x - 1}{x^n}$ exists and is non-zero, then the value of n is -
 (A) 1 (B) 3 (C) 2 (D) 0
18. $\lim_{x \rightarrow 0} \frac{\left(\sqrt{(1-\cos x) + \sqrt{(1-\cos x) + \sqrt{(1-\cos x) + \dots \dots \dots \infty}}} \right) - 1}{x^2}$ equals
 (A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2
19. $\lim_{x \rightarrow \infty} \left(\sqrt[3]{(x+a)(x+b)(x+c)} - x \right) =$
 (A) \sqrt{abc} (B) $\frac{a+b+c}{3}$ (C) abc (D) $(abc)^{1/3}$
20. $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{r}{n^2 + n + r}$ equals
 (A) 0 (B) $1/3$ (C) $1/2$ (D) 1

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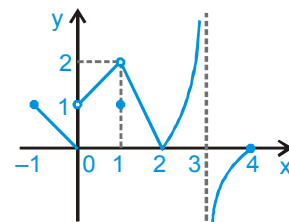
21. $\lim_{n \rightarrow \infty} \left(\sqrt{n^2 + n + 1} - \left[\sqrt{n^2 + n + 1} \right] \right)$ ($n \in \mathbb{I}$) where $[\]$ denotes the greatest integer function is
 (A) 0 (B) 1/2 (C) 2/3 (D) 1/4
22. $\lim_{x \rightarrow 0} \frac{\sin^{-1} x - \tan^{-1} x}{x^3}$ is equal to
 (A) 0 (B) $\frac{1}{2}$ (C) 1 (D) none of these
23. If α and β be the roots of equation $ax^2 + bx + c = 0$, then $\lim_{x \rightarrow \alpha} (1 + ax^2 + bx + c)^{\frac{1}{x - \alpha}}$ is equal to
 (A) $a(\alpha - \beta)$ (B) $\ln |a(\alpha - \beta)|$ (C) $e^{a(\alpha - \beta)}$ (D) $e^{a|\alpha - \beta|}$
24. $\lim_{x \rightarrow 0} \left[(1 - e^x) \frac{\sin x}{|x|} \right]$, where $[\cdot]$ represents greatest integer function, is equal to
 (A) -1 (B) 1 (C) 0 (D) does not exist
25. $\lim_{n \rightarrow \infty} \left(\frac{1}{\sqrt{n^2}} + \frac{1}{\sqrt{n^2 + 1}} + \frac{1}{\sqrt{n^2 + 2}} + \dots + \frac{1}{\sqrt{n^2 + 2n}} \right)$ is equal to
 (A) 1 (B) $\frac{1}{2}$ (C) 0 (D) 2
26. $\lim_{y \rightarrow 0} \left(\lim_{x \rightarrow \infty} \frac{\exp \left(x \ln \left(1 + \frac{ay}{x} \right) \right) - \exp \left(x \ln \left(1 + \frac{by}{x} \right) \right)}{y} \right)$ is equal to
 (A) $a + b$ (B) $a - b$ (C) $b - a$ (D) $-(a + b)$
27. Let $f(x) = \begin{cases} x \sin \left(\frac{1}{x} \right) + \sin \left(\frac{1}{x^2} \right), & x \neq 0 \\ 0, & x = 0 \end{cases}$, then $\lim_{x \rightarrow \infty} f(x)$ is equal to
 (A) 0 (B) $-\frac{1}{2}$ (C) 1 (D) none of these.
28. If $\lim_{x \rightarrow 0} \frac{x^3}{\sqrt{a + x} (bx - \sin x)} = 1$, then constants 'a' and 'b' are (where $a > 0$)
 (A) $b = 1, a = 36$ (B) $a = 1, b = 6$ (C) $a = 1, b = 36$ (D) $b = 1, a = 6$
29. $\lim_{x \rightarrow \infty} \sec^{-1} \left(\frac{x}{x + 1} \right)$ is equal to
 (A) 0 (B) π (C) $\frac{\pi}{2}$ (D) does not exist
30. The value of $\lim_{x \rightarrow 0} (\cos ax)^{\operatorname{cosec}^2 bx}$ is -
 (A) $e^{\left(\frac{8b^2}{a^2} \right)}$ (B) $e^{\left(-\frac{8a^2}{b^2} \right)}$ (C) $e^{\left(\frac{a^2}{2b^2} \right)}$ (D) $e^{\left(-\frac{b^2}{2a^2} \right)}$

Exercise # 2

Part # I

[Multiple Correct Choice Type Questions]

1. Let $f(x) = \lim_{n \rightarrow \infty} \frac{2x^{2n} \sin \frac{1}{x} + x}{1 + x^{2n}}$ then which of the following alternative(s) is/are correct?
- (A) $\lim_{x \rightarrow \infty} x f(x) = 2$ (B) $\lim_{x \rightarrow 1} f(x)$ does not exist.
 (C) $\lim_{x \rightarrow 0} f(x)$ does not exist. (D) $\lim_{x \rightarrow -\infty} f(x)$ is equal to zero.
2. If $\ell = \lim_{x \rightarrow \infty} \left(\frac{x+1}{x-1} \right)^x$, then $\{\ell\}$ and $[\ell]$ (where $\{\cdot\}$ & $[\cdot]$ denotes the fractional part function & greatest integer function respectively), is/are -
- (A) 7 (B) $7 - e^2$ (C) -7 (D) $e^2 - 7$
3. If $\lim_{x \rightarrow 0} \frac{\sin 2x + a \sin x}{x^3} = p$ (finite), then
- (A) $a = -2$ (B) $a = -1$ (C) $p = -2$ (D) $p = -1$
4. For $\lim_{x \rightarrow 0} \frac{\cot^{-1}\left(\frac{1}{x}\right)}{x}$
- (A) R.H.L exists
 (B) L.H.L does not exist
 (C) limit does not exist as R.H.L is 1 and L.H.L is -1
 (D) limit does not exist as R.H.L and L.H.L both are non-existent.
5. Let $f(x) = \frac{|x + \pi|}{\sin x}$, then
- (A) $f(-\pi^+) = -1$ (B) $f(-\pi^-) = 1$
 (C) $\lim_{x \rightarrow -\pi} f(x)$ does not exist (D) $\lim_{x \rightarrow \pi} f(x)$ does not exist
6. The graph of the function $y = f(x)$ is shown in the adjacent figure, then correct statement is -
- (A) $\lim_{x \rightarrow 0^+} f(x) = 1$ (B) $\lim_{x \rightarrow 1} f(x) = 2$
 (C) $\lim_{x \rightarrow 3} f(x)$ does not exist. (D) $\lim_{x \rightarrow 4} f(x) = 0$
7. If $\lim_{x \rightarrow 0} (1 + ax + bx^2)^{\frac{2}{x}} = e^3$, then possible values of a and b is/are :
- (A) $a = 3, b = 0$ (B) $a = \frac{3}{2}, b = \frac{1}{2}$ (C) $a = \frac{3}{2}, b = \frac{3}{2}$ (D) $a = \frac{3}{2}, b = 0$



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8. Which of the following limits is/are unity ?

(A) $\lim_{x \rightarrow \pi} \frac{\sin(\pi - x)}{(\pi - x)}$ (B) $\lim_{x \rightarrow 0} \frac{\tan(\sin x)}{\tan x}$ (C) $\lim_{x \rightarrow \infty} \frac{x^2 + x}{x^2 - x}$ (D) $\lim_{x \rightarrow 1} \frac{\tan(\pi x)}{\pi(x - 1)}$

9. Assume that $\lim_{\theta \rightarrow -1} f(\theta)$ exists and $\frac{\theta^2 + \theta - 2}{\theta + 3} \leq \frac{f(\theta)}{\theta^2} \leq \frac{\theta^2 + 2\theta - 1}{\theta + 3}$ holds for certain interval containing the point

$\theta = -1$ then $\lim_{\theta \rightarrow -1} f(\theta)$

(A) is equal to $f(-1)$ (B) is equal to 1 (C) is non existent (D) is equal to -1

10. Which of the following limits vanishes ?

(A) $\lim_{x \rightarrow \infty} x^{\frac{1}{4}} \sin \frac{1}{\sqrt{x}}$ (B) $\lim_{x \rightarrow \pi/2} (1 - \sin x) \cdot \tan x$

(C) $\lim_{x \rightarrow \infty} \frac{2x^2 + 3}{x^2 + x - 5} \cdot \text{sgn}(x)$ (D) $\lim_{x \rightarrow 3^+} \frac{[x]^2 - 9}{x^2 - 9}$

where $[\]$ denotes greatest integer function

11. $\lim_{x \rightarrow c} f(x)$ does not exist when -

(A) $f(x) = [[x]] - [2x - 1], c = 3$ (B) $f(x) = [x] - x, c = 1$
 (C) $f(x) = \{x\}^2 - \{-x\}^2, c = 0$ (D) $f(x) = \frac{\tan(\text{sgn } x)}{\text{sgn } x}, c = 0$

where $[x]$ denotes step up function & $\{x\}$ fractional part function.

12. For a > 0 , let $\ell = \lim_{x \rightarrow \frac{\pi}{2}} \frac{a^{\cot x} - a^{\cos x}}{\cot x - \cos x}$ and $m = \lim_{x \rightarrow -\infty} \left(\sqrt{x^2 + ax} - \sqrt{x^2 - ax} \right)$ then -

(A) ' ℓ ' is always greater than ' m ' for all values of $a > 0$
 (B) ' ℓ ' is always greater than ' m ' only when $a \geq 1$
 (C) ' ℓ ' is always greater than ' m ' for all values of ' a ' satisfying $a > e^{-a}$
 (D) $e^\ell + m = 0$

13. Which of the following limits is/are unity ?

(A) $\lim_{t \rightarrow 0} \frac{\sin(\tan t)}{\sin t}$ (B) $\lim_{x \rightarrow \pi/2} \frac{\sin(\cos x)}{\cos x}$

(C) $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}$ (D) $\lim_{x \rightarrow 0} \frac{\sqrt{x^2}}{x}$

14. If $f(x) = \begin{cases} \frac{\sin[x]}{[x]}, & [x] \neq 0 \\ 0 & , [x] = 0 \end{cases}$ where $[x]$ denotes the greatest integer less than or equal to x , then
- (A) $\lim_{x \rightarrow 0^-} f(x) = \sin 1$ (B) $\lim_{x \rightarrow 0^+} f(x) = 0$
 (C) limit does not exist at $x = 0$ (D) limit exist at $x = 0$
15. Let $f(x) = \frac{x \cdot 2^x - x}{1 - \cos x}$ & $g(x) = 2^x \sin\left(\frac{\ln 2}{2^x}\right)$ then -
- (A) $\lim_{x \rightarrow 0} f(x) = \ln 2$ (B) $\lim_{x \rightarrow \infty} g(x) = \ln 4$ (C) $\lim_{x \rightarrow 0} f(x) = \ln 4$ (D) $\lim_{x \rightarrow \infty} g(x) = \ln 2$
16. Which of the following limits vanishes ?
- (A) $\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{\tan x}\right)$ (B) $\lim_{x \rightarrow \infty} \left(\frac{3x^2 + 1}{2x^2 - 1}\right)^{\frac{x^3}{1-x}}$
 (C) $\lim_{x \rightarrow \frac{\pi^+}{4}} \left[\tan\left(x + \frac{\pi}{8}\right)\right]^{\tan 2x}$ (D) $\lim_{x \rightarrow 1} \frac{x^4 - 2x^2 + 1}{x^3 - 1}$
17. If $\lim_{x \rightarrow 3} \frac{x^3 + Cx^2 + 5x + 12}{x^2 - 7x + 12} = \ell$ (finite real number) then -
- (A) $\ell = 4$ (B) $C = -6$ (C) $C = 4$ (D) $C \in \mathbb{R}$
18. Consider the function $f(x) = \left(\frac{ax+1}{bx+2}\right)^x$ where $a, b > 0$ then $\lim_{x \rightarrow \infty} f(x)$ -
- (A) exists for all values of a and b (B) is zero for $a < b$
 (C) is non existent for $a > b$ (D) is $e^{-\left(\frac{1}{a}\right)}$ or $e^{-\left(\frac{1}{b}\right)}$ if $a = b$
19. Identify the true statement(s) -
- (A) $\lim_{n \rightarrow \infty} \left[\sum_{r=1}^n \frac{1}{2^r}\right] = 1$, where $[.]$ denotes the greatest integer function.
 (B) If $f(x) = (x - 1) \{x\}$, then limit of $f(x)$ does not exist at all integers.
 (C) $\lim_{x \rightarrow 0^+} \left[\frac{\tan x}{x}\right] = 1$, where $[.]$ denotes the greatest integer function.
 (D) $\left[\lim_{x \rightarrow 0^+} \frac{\tan x}{x}\right] = 1$, where $[.]$ denotes the greatest integer function.

These questions contains, Statement I (assertion) and Statement II (reason).

- (A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.
 (B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.
 (C) Statement-I is true, Statement-II is false.
 (D) Statement-I is false, Statement-II is true.

1. **Statement - I** If $\lim_{x \rightarrow 0} f(x)$ and $\lim_{x \rightarrow 0} g(x)$ exists finitely, then $\lim_{x \rightarrow 0} f(x).g(x)$ exists finitely.

Statement - II If $\lim_{x \rightarrow 0} f(x).g(x)$ exists finitely then $\lim_{x \rightarrow 0} f(x).g(x) = \lim_{x \rightarrow 0} f(x) \cdot \lim_{x \rightarrow 0} g(x)$

2. **Statement - I** $\lim_{x \rightarrow \infty} \frac{x^n + nx^{n-1} + 1}{[x^n]} = 0$, $n \in I$ (where $[.]$ represents greatest integer function).

Statement - II $x - 1 < [x] \leq x$, (where $[.]$ represents greatest integer function).

3. **Statement - I** $\lim_{x \rightarrow 0} \frac{\sqrt{1 - \cos 2x}}{x}$ does not exist.

Statement - II $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin x - (\sin x)^{\sin x}}{1 - \sin x + \ln \sin x} = 2$

4. **Statement - I** $\lim_{x \rightarrow \infty} \frac{2x^4 + 3x^3 + 7x}{3x^4 + 2x^2 + 3x} = \frac{2}{3}$.

Statement - II If $P(x)$ and $Q(x)$ are two polynomials with rational coefficients, then

$$\lim_{x \rightarrow \infty} \frac{P(x)}{Q(x)} = \frac{\text{coefficient of highest power of } x \text{ in } P(x)}{\text{coefficient of highest power of } x \text{ in } Q(x)}$$

5. **Statement - I** $\lim_{x \rightarrow 0} \left\{ 1^{1/\sin^2 x} + 2^{1/\sin^2 x} + 3^{1/\sin^2 x} + \dots + n^{1/\sin^2 x} \right\}^{\sin^2 x} = n$

Statement - II For $0 < a < 1$, $\lim_{x \rightarrow \infty} a^x = 0$

Exercise # 3

Part # I

[Matrix Match Type Questions]

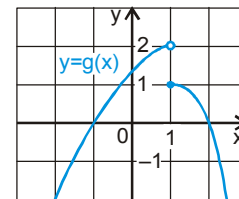
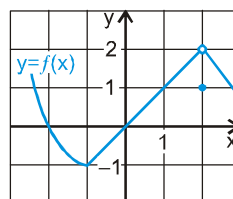
Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with one or more statement(s) in **Column-II**.

1. Let $\phi(x) = \frac{a_0x^m + a_1x^{m+1} + \dots + a_kx^{m+k}}{b_0x^n + b_1x^{n+1} + \dots + b_\ell x^{n+\ell}}$, where $a_0 \neq 0, b_0 \neq 0$ and $m, n \in \mathbb{N}$, then $\lim_{x \rightarrow 0} \phi(x)$ is equal to

Column - I	Column - II
(A) $m > n$	(p) ∞
(B) $m = n$	(q) $-\infty$
(C) $m < n$ and $n - m$ is even, $\frac{a_0}{b_0} > 0$	(r) $\frac{a_0}{b_0}$
(D) $m < n$ and $n - m$ is even, $\frac{a_0}{b_0} < 0$	(s) 0

2. The graphs of f and g are given. Use them to evaluate each limit.

Column-I	Column-II
(A) $\lim_{x \rightarrow 1} f(g(x))$	(p) 1
(B) $\lim_{x \rightarrow 2} \sqrt{3f(x) - 2}$	(q) does not exist
(C) $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)} + f(x)g(x)$	(r) 0
(D) $\lim_{x \rightarrow 1^+} \frac{3f(x) - g(x)}{f(x) + g(x)}$	(s) 2



3. $\lim_{x \rightarrow 0} f(x)$, where $f(x)$ is as in column - I, is

Column - I	Column - II
(A) $f(x) = \frac{\tan[e^2]x^2 - \tan[-e^2]x^2}{\sin^2 x}$ where $[.]$ represents greatest integer function.	(p) $\frac{\sqrt{2}}{8}$
(B) $f(x) = \left[\left(\min(t^2 + 4t + 6) \right) \frac{\sin x}{x} \right]$ where $[.]$ represents greatest integer function.	(q) 15
(C) $f(x) = \frac{\sqrt[3]{1+x^2} - \sqrt[4]{1-2x}}{x+x^2}$	(r) 1
(D) $f(x) = \frac{\sqrt{2} - \sqrt{1+\cos x}}{\sin^2 x}$	(s) $\frac{1}{2}$

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4.	Column-I	Column-II
(A)	If $L = \lim_{x \rightarrow \pi/2^+} \frac{\cos(\tan^{-1}(\tan x))}{x - \pi/2}$ then $\cos(2\pi L)$ is	(p) 1
(B)	Number of solutions of the equation $\operatorname{cosec}\theta = k$ in $[0, \pi]$ where $k = \lim_{n \rightarrow \infty} \prod_{r=2}^n \frac{r^3 - 1}{r^3 + 1}$	(q) -1
(C)	If c satisfies the equation $\lim_{x \rightarrow \infty} \left(\frac{x+c}{x-c} \right)^x = 4$ then $-\frac{e^c}{2}$ is	(r) 0
(D)	If $\lim_{x \rightarrow -\infty} \frac{(3x^4 + 2x^2)\sin(1/x) + x ^3 + 5}{ x ^3 + x ^2 + x + 1} = k$, then $\frac{k}{2}$ is	(s) 2

5.	Column I	Column II
(A)	$\lim_{x \rightarrow \infty} \left(\frac{x}{1+x} \right)^x$ equals	(p) e^2
(B)	$\lim_{x \rightarrow \infty} \left(\sin \frac{1}{x} + \cos \frac{1}{x} \right)^x$	(q) $e^{-1/2}$
(C)	$\lim_{x \rightarrow 0} (\cos x)^{\cot^2 x}$	(r) e
(D)	$\lim_{x \rightarrow 0} \left(\tan \left(\frac{\pi}{4} + x \right) \right)^{1/x}$	(s) e^{-1}

6.	Column-I	Column-II
(A)	$\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{e^{x^2} - e^x + x}$ equals	(p) 1
(B)	If the value of $\lim_{x \rightarrow 0^+} \left(\frac{(3/x)+1}{(3/x)-1} \right)^{1/x}$ can be expressed in the form of $e^{p/q}$, where p and q are relative prime then $(p + q)$ is equal to	(q) 2
(C)	$\lim_{x \rightarrow 0} \frac{\tan^3 x - \tan x^3}{x^5}$ equals	(r) 4
(D)	$\lim_{x \rightarrow 0} \frac{x + 2\sin x}{\sqrt{x^2 + 2\sin x + 1} - \sqrt{\sin^2 x - x + 1}}$	(s) 5

Comprehension # 1

Consider $f(x) = \frac{\sin x + ae^x + be^{-x} + c \ln(1+x)}{x^3}$, where a, b, c are real numbers.

- If $\lim_{x \rightarrow 0^+} f(x)$ is finite, then the value of $a + b + c$ is
 (A) 0 (B) 1 (C) 2 (D) -2
- If $\lim_{x \rightarrow 0^+} f(x) = \ell$ (finite), then the value of ℓ is
 (A) -2 (B) $-\frac{1}{2}$ (C) -1 (D) $-\frac{1}{3}$
- Using the values of a, b, c as found in Q.No. 1 or Q. No. 2 above, the value of $\lim_{x \rightarrow 0^+} x f(x)$ is
 (A) 0 (B) $\frac{1}{2}$ (C) $-\frac{1}{2}$ (D) 2

Comprehension # 2

Consider two functions $f(x) = \lim_{n \rightarrow \infty} \left(\cos \frac{x}{\sqrt{n}} \right)^n$ and $g(x) = -x^{4b}$, where $b = \lim_{x \rightarrow \infty} \left(\sqrt{x^2 + x + 1} - \sqrt{x^2 + 1} \right)$.

- $f(x)$ is -
 (A) e^{-x^2} (B) $e^{\frac{-x^2}{2}}$ (C) e^{x^2} (D) $e^{\frac{x^2}{2}}$
- $g(x)$ is -
 (A) $-x^2$ (B) x^2 (C) x^4 (D) $-x^4$
- Number of solutions of $f(x) + g(x) = 0$ is -
 (A) 2 (B) 4 (C) 0 (D) 1

Comprehension # 3

Consider two functions $f(x) = \lim_{n \rightarrow \infty} \left(\cos \frac{x}{\sqrt{n}} \right)^n$ and $g(x) = -x^{4b}$, where $b = \lim_{x \rightarrow \infty} \left(\sqrt{x^2 + x + 1} - \sqrt{x^2 + 1} \right)$, then

- $f(x)$ is
 (A) e^{-x^2} (B) $e^{\frac{-x^2}{2}}$ (C) e^{x^2} (D) $e^{\frac{x^2}{2}}$
- $g(x)$ is
 (A) $-x^2$ (B) x^2 (C) x^4 (D) $-x^4$
- Number of solutions of equation $f(x) + g(x) = 0$ is
 (A) 2 (B) 4 (C) 0 (D) 1

Exercise # 4

[Subjective Type Questions]

1. Evaluate $\lim_{x \rightarrow \frac{3\pi}{4}} \frac{1 + \sqrt[3]{\tan x}}{1 - 2 \cos^2 x}$
2. Evaluate $\lim_{x \rightarrow \infty} \left(\frac{a_1^{1/x} + a_2^{1/x} + a_3^{1/x} + \dots + a_n^{1/x}}{n} \right)^{nx}$ where $a_1, a_2, a_3, \dots, a_n > 0$
3. Evaluate $\lim_{x \rightarrow \infty} x^2 \sin \left(\ln \sqrt{\cos \frac{\pi}{x}} \right)$
4. Evaluate $\lim_{x \rightarrow 1} \frac{(\ln(1+x) - \ln 2)(3 \cdot 4^{x-1} - 3x)}{\left[(7+x)^{\frac{1}{3}} - (1+3)^{\frac{1}{2}} \right] \cdot \sin(x-1)}$
5. Using Sandwich theorem, evaluate $\lim_{n \rightarrow \infty} \left(\frac{1}{\sqrt{n^2}} + \frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \dots + \frac{1}{\sqrt{n^2+2n}} \right)$
6. Find the values of a, b & c so that $\lim_{x \rightarrow 0} \frac{ae^x - b \cos x + ce^{-x}}{x \cdot \sin x} = 2$.
7. Evaluate $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{1 - \sqrt{\sin 2x}}}{\pi - 4x}$
8. Evaluate $\lim_{n \rightarrow \infty} \frac{[1 \cdot x] + [2 \cdot x] + [3 \cdot x] + \dots + [n \cdot x]}{n^2}$, where [.] denotes the greatest integer function.
9. Using Sandwich theorem, evaluate $\lim_{n \rightarrow \infty} \frac{1}{1+n^2} + \frac{2}{2+n^2} + \dots + \frac{n}{n+n^2}$
10. Evaluate $\lim_{x \rightarrow 0} \frac{27^x - 9^x - 3^x + 1}{\sqrt{2} - \sqrt{1 + \cos x}}$
11. Verify the following limits
 - (i) $\lim_{x \rightarrow 0} \left(\frac{(1+x)^{\frac{1}{x}}}{e} \right)^{\frac{1}{x}} = e^{-\frac{1}{2}}$
 - (ii) $\lim_{x \rightarrow 0} \left[\sin^2 \left(\frac{\pi}{2-ax} \right) \right]^{\sec^2 \left(\frac{\pi}{2-bx} \right)} = e^{-\frac{a^2}{b^2}}$
12. Evaluate $\lim_{x \rightarrow 1} \frac{\left[\sum_{k=1}^{100} x^k \right] - 100}{x-1}$
13. Let $f(x) = \frac{\sin^{-1}(1-\{x\}) \cdot \cos^{-1}(1-\{x\})}{\sqrt{2\{x\}}(1-\{x\})}$, then find $\lim_{x \rightarrow 0^+} f(x)$ and $\lim_{x \rightarrow 0^-} f(x)$.
(where {.} denotes the fractional part function)

Exercise # 5

Part # I

[Previous Year Questions] [AIEEE/JEE-MAIN]

1. If $f(1) = 1$, $f'(1) = 2$, then $\lim_{x \rightarrow 1} \frac{\sqrt{f(x)} - 1}{\sqrt{x} - 1} =$ [AIEEE - 2002]
 (1) 2 (2) 1 (3) 3 (4) 4
2. The values of $\lim_{x \rightarrow 0} \frac{(1 - \cos 2x) \sin 5x}{x^2 \sin 3x}$ is- [AIEEE - 2002]
 (1) 10/3 (2) 3/10 (3) 6/5 (4) 5/6
3. $\lim_{x \rightarrow \infty} \left(\frac{x^2 + 5x + 3}{x^2 + x + 3} \right)^x =$ [AIEEE - 2002]
 (1) e^4 (2) e^2 (3) e^3 (4) e
4. $\lim_{x \rightarrow \infty} \frac{\log x^n - [x]}{[x]}$, $n \in \mathbb{N}$, (where $[x]$ denotes greatest integer less than or equal to x)- [AIEEE - 2002]
 (1) Has value -1 (2) Has values 0 (3) Has value 1 (4) Does not exist
5. If $\lim_{x \rightarrow 0} \frac{\log(3+x) - \log(3-x)}{x} = k$, the value of k is- [AIEEE - 2003]
 (1) $-\frac{2}{3}$ (2) 0 (3) $-\frac{1}{3}$ (4) $\frac{2}{3}$
6. Let $f(a) = g(a) = k$ and their n^{th} derivatives $f^n(a)$, $g^n(a)$ exist and are not equal for some n . Further if $\lim_{x \rightarrow a} \frac{f(a)g(x) - f(x)g(a)}{g(x) - f(x)} = 4$ then the value of k is- [AIEEE - 2003]
 (1) 0 (2) 4 (3) 2 (4) 1
7. $\lim_{x \rightarrow \pi/2} \frac{\left[1 - \tan\left(\frac{x}{2}\right) \right] [1 - \sin x]}{\left[1 + \tan\left(\frac{x}{2}\right) \right] [\pi - 2x]^3}$ is- [AIEEE - 2003]
 (1) ∞ (2) $\frac{1}{8}$ (3) 0 (4) $\frac{1}{32}$
8. If $\lim_{x \rightarrow \infty} \left(1 + \frac{a}{x} + \frac{b}{x^2} \right)^{2x} = e^2$, then the values of a and b , are- [AIEEE - 2004]
 (1) $a \in \mathbb{R}, b \in \mathbb{R}$ (2) $a = 1, b \in \mathbb{R}$ (3) $a \in \mathbb{R}, b = 2$ (4) $a = 1$ and $b = 2$
9. Let α and β be the distinct roots of $ax^2 + bx + c = 0$, then $\lim_{x \rightarrow \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - \alpha)^2}$ is equal to- [AIEEE - 2005]
 (1) $\frac{a^2}{2} (\alpha - \beta)^2$ (2) 0 (3) $-\frac{a^2}{2} (\alpha - \beta)^2$ (4) $\frac{1}{2} (\alpha - \beta)^2$

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10. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a positive increasing function with $\lim_{x \rightarrow \infty} \frac{f(3x)}{f(x)} = 1$. Then $\lim_{x \rightarrow \infty} \frac{f(2x)}{f(x)} =$ [AIEEE-2010]
 (1) 1 (2) $\frac{2}{3}$ (3) $\frac{3}{2}$ (4) 3
11. $\lim_{x \rightarrow 2} \left(\frac{\sqrt{1 - \cos \{2(x-2)\}}}{x-2} \right)$ [AIEEE-2011]
 (1) equals $-\sqrt{2}$ (2) equals $\frac{1}{\sqrt{2}}$ (3) does not exist (4) equals $\sqrt{2}$
12. Let $f : \mathbb{R} \rightarrow [0, \infty)$ be such that $\lim_{x \rightarrow 5} f(x)$ exists and $\lim_{x \rightarrow 5} \frac{(f(x))^2 - 9}{\sqrt{|x-5|}} = 0$. Then $\lim_{x \rightarrow 0} f(x)$ equal - [AIEEE-2011]
 (1) 3 (2) 0 (3) 1 (4) 2
13. If function $f(x)$ is differentiable at $x = a$ then $\lim_{x \rightarrow a} \frac{x^2 f(a) - a^2 f(x)}{x-a}$ [AIEEE-2011]
 (1) $2a f(a) + a^2 f'(a)$ (2) $-a^2 f'(a)$ (3) $af(a) - a^2 f'(a)$ (4) $2af(a) - a^2 f'(a)$
14. $\lim_{x \rightarrow 0} \frac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$ is equal to : [JEE(Main)-2013]
 (1) $-\frac{1}{4}$ (2) $\frac{1}{2}$ (3) 1 (4) 2
15. $\lim_{x \rightarrow 0} \frac{\sin(\pi \cos^2 x)}{x^2}$ is equal to : [JEE(Main)-2014]
 (1) $\frac{\pi}{2}$ (2) 1 (3) $-\pi$ (4) π
16. $\lim_{x \rightarrow 0} \frac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$ is equal to [JEE(Main)-2015]
 (1) 2 (2) $\frac{1}{2}$ (3) 4 (4) 3
17. Let $f(x)$ be a polynomial of degree four having extreme values at $x = 1$ and $x = 2$.
 If $\lim_{x \rightarrow 0} \left[1 + \frac{f(x)}{x^2} \right] = 3$, then $f(2)$ is equal to : [JEE(Main)-2015]
 (1) 0 (2) 4 (3) -8 (4) -4
18. Let $p = \lim_{x \rightarrow 0^+} (1 + \tan^2 \sqrt{x})^{\frac{1}{2x}}$ then $\log p$ is equal to : [JEE(Main)-2016]
 (1) 1 (2) $\frac{1}{2}$ (3) $\frac{1}{4}$ (4) 2
19. $\lim_{n \rightarrow \infty} \left(\frac{(n+1)(n+2)\dots 3n}{n^{2n}} \right)^{\frac{1}{n}}$ is equal to : [JEE(Main)-2016]
 (1) $\frac{27}{e^2}$ (2) $\frac{9}{e^2}$ (3) $3 \log 3 - 2$ (4) $\frac{18}{e^4}$

1. $\lim_{x \rightarrow 0} \frac{\sin(\pi \cos^2 x)}{x^2}$ equals [JEE 2001]
 (A) $-\pi$ (B) π (C) $\frac{\pi}{2}$ (D) 1
2. Evaluate $\lim_{x \rightarrow 0} \frac{a^{\tan x} - a^{\sin x}}{\tan x - \sin x}$, $a > 0$. [JEE 2001]
3. The integer n for which $\lim_{x \rightarrow 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n}$ is a finite non-zero number is :-
 (A) 1 (B) 2 (C) 3 (D) 4 [JEE 2002]
4. If $\lim_{x \rightarrow 0} \frac{\sin(nx)[(a-n)nx - \tan x]}{x^2} = 0$ ($n > 0$) then the value of 'a' is equal to :-
 (A) $\frac{1}{n}$ (B) $n^2 + 1$ (C) $\frac{n^2 + 1}{n}$ (D) None [JEE 2003]
5. Find the value of $\lim_{n \rightarrow \infty} \left[\frac{2}{\pi}(n+1) \cos^{-1} \left(\frac{1}{n} \right) - n \right]$. [JEE 2004]
6. Let $L = \lim_{x \rightarrow 0} \frac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}$, $a > 0$. If L is finite, then :-
 (A) $a = 2$ (B) $a = 1$ (C) $L = \frac{1}{64}$ (D) $L = \frac{1}{32}$ [JEE 2009]
7. If $\lim_{x \rightarrow \infty} \left(\frac{x^2 + x + 1}{x + 1} - ax - b \right) = 4$, then - [JEE 2012]
 (A) $a = 1, b = 4$ (B) $a = 1, b = -4$ (C) $a = 2, b = -3$ (D) $a = 2, b = 3$
8. Let $\alpha(A)$ and $\beta(A)$ be the roots of the equation $(\sqrt[3]{1+a} - 1)x^2 + (\sqrt{1+a} - 1)x + (\sqrt[5]{1+a} - 1) = 0$ where $a > -1$.
 Then $\lim_{a \rightarrow 0^+} \alpha(a)$ and $\lim_{a \rightarrow 0^+} \beta(a)$ are [JEE 2012]
 (A) $-\frac{5}{2}$ and 1 (B) $-\frac{1}{2}$ and -1 (C) $-\frac{7}{2}$ and 2 (D) $-\frac{9}{2}$ and 3
9. The largest value of the nonnegative integer a for which [JEE Ad. 2014]
 $\lim_{x \rightarrow 1} \left\{ \frac{-ax + \sin(x-1) + a}{x + \sin(x-1) - 1} \right\}^{\frac{1-x}{1-\sqrt{x}}} = \frac{1}{4}$
10. Let $\alpha, \beta \in \mathbb{R}$ be such that $\lim_{x \rightarrow 0} \frac{x^2 \sin(\beta x)}{ax - \sin x} = 1$. Then $6(\alpha + \beta)$ is equals [JEE Ad. 2016]
11. Let $f(x) = \lim_{n \rightarrow \infty} \left(\frac{n^n(x+n)(x+\frac{n}{2}) \dots (x+\frac{n}{n})}{n!(x^2+n^2)(x^2+\frac{n^2}{n^2})} \right)^{\frac{1}{n}}$, for all $x > 0$. Then
 (A) $f\left(\frac{1}{2}\right) \geq f(1)$ (B) $f\left(\frac{1}{3}\right) \leq f\left(\frac{2}{3}\right)$ (C) $f'(2) \leq 0$ (D) $\frac{f'(3)}{f(3)} \geq \frac{f'(2)}{f(2)}$

MOCK TEST

SECTION - I : STRAIGHT OBJECTIVE TYPE

- If $\lim_{x \rightarrow 0} \frac{e^{-nx} + e^{nx} - 2 \cos \frac{nx}{2} - kx^2}{(\sin x - \tan x)}$ exists and finite, then possible values of 'n' and 'k' is :

(A) $k=3, n=2$ (B) $k=3, n=-2$ (C) $k=5, n=2$ (D) $k=-5, n=2$
- $\lim_{x \rightarrow 1} \left(\sec \frac{\pi}{2^x} \right) (\ln x)$ is equal to

(A) $-\frac{\pi}{2 \ln 2}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{2 \ln 2}$ (D) $\frac{2}{\pi \ln 2}$
- $\lim_{x \rightarrow 0} |x(x-1)|^{\lfloor \cos 2x \rfloor}$, where $\lfloor \cdot \rfloor$ denotes greatest integer function, is equal to

(A) 1 (B) 0 (C) e (D) Does not exist
- The value of $\lim_{x \rightarrow 1} \left(\frac{p}{1-x^p} - \frac{q}{1-x^q} \right)$; $p, q \in \mathbb{N}$ equals

(A) $\frac{p+q}{2}$ (B) $\frac{pq}{2}$ (C) $\frac{p-q}{2}$ (D) $\sqrt{\frac{p}{q}}$
- The value of $\lim_{x \rightarrow \pi} \frac{1}{(x-\pi)} \left(\sqrt{\frac{4 \cos^2 x}{2 + \cos x}} - 2 \right)$ is

(A) 0 (B) 2 (C) -2 (D) does not exist
- The value of $\lim_{x \rightarrow \frac{1}{\sqrt{2}}} \frac{\cos^{-1}(2x\sqrt{1-x^2})}{x - \frac{1}{\sqrt{2}}}$ equals

(A) $2\sqrt{2}$ (B) $-2\sqrt{2}$ (C) 0 (D) does not exist
- $\lim_{x \rightarrow 0} \frac{\sin x^4 - x^4 \cos x^4 + x^{20}}{x^4(e^{2x^4} - 1 - 2x^4)}$ is equal to

(A) 0 (B) $-\frac{1}{6}$ (C) $\frac{1}{6}$ (D) does not exist
- $\lim_{n \rightarrow \infty} \frac{1^2 n + 2^2 (n-1) + 3^2 (n-2) + \dots + n^2 \cdot 1}{1^3 + 2^3 + 3^3 + \dots + n^3}$ is equal to:

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{1}{2}$ (D) $\frac{1}{6}$

9. $\lim_{x \rightarrow 0^+} (x)^{\ln \sin x}$ is equal to
 (A) 1 (B) 0 (C) e (D) does not exist

10. S_1 : If $\lim_{x \rightarrow a} f(x) - \lim_{x \rightarrow a} g(x)$ exists, then it is not necessary that $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ will exist

S_2 : $\lim_{x \rightarrow 0^+} \sqrt{x} (\log x)^2 = 0$

S_3 : $\lim_{x \rightarrow 0} \left\{ \frac{\sin x}{x} \right\} = 1$, (where $\{.\}$ denotes fractional part function)

S_4 : $\lim_{x \rightarrow \infty} \frac{\ln \cos^2 x}{x^2} = \frac{1}{2}$

- (A) T T T F (B) F T T F (C) T T F F (D) F F T F

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

11. $f(x) = \begin{cases} x + \frac{1}{2}, & x < 0 \\ 2x + \frac{1}{3}, & x \geq 0 \end{cases}$, identify the correct statement(s)

([] denotes greatest integer function)

- (A) $\lim_{x \rightarrow 0} [f(x)] = 0$ (B) $\lim_{x \rightarrow 0} f(x)$ does not exist
 (C) $\left[\lim_{x \rightarrow 0} f(x) \right]$ exists (D) $\lim_{x \rightarrow 0} \frac{[f(x)]}{x}$ does not exist

12. Which of the following limits tend to unity ?

(A) $\lim_{t \rightarrow 0} \frac{\sin(\tan t)}{\sin t}$ (B) $\lim_{t \rightarrow \pi/2} \frac{\sin(\cos x)}{\cos x}$ (C) $\lim_{x \rightarrow 0} \frac{\sqrt{x^2}}{x}$ (D) $\lim_{x \rightarrow \pi/2} \left(\frac{1 - \cos x}{x^2} \right)$

13. Which of the following is/are true

- (A) If $\lim_{x \rightarrow a} \{f(x) + g(x)\}$ exists, then both $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ exist
 (B) If $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ exist, then $\lim_{x \rightarrow a} \{f(x) + g(x)\}$ exists
 (C) If $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ exist, then $\lim_{x \rightarrow a} f(x) g(x)$ exists
 (D) If $\lim_{x \rightarrow a} \{f(x)g(x)\}$ exists, then both $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ exist

14. $f(x) = \left(\frac{x}{2+x}\right)^{2x}$, then
- (A) $\lim_{x \rightarrow \infty} f(x) = -4$ (B) $\lim_{x \rightarrow \infty} f(x) = 2$ (C) $\lim_{x \rightarrow \infty} f(x) = e^{-4}$ (D) $\lim_{x \rightarrow 1} f(x) = \frac{1}{9}$
15. If x is a real number in $[0, 1]$, then the value of $\lim_{m \rightarrow \infty} \lim_{n \rightarrow \infty} [1 + \cos^{2m}(n! \pi x)]$ is -
- (A) 1 if $x \notin \mathbb{Q}$ (B) 2 if $x \notin \mathbb{Q}$ (C) 1 if $x \in \mathbb{Q}$ (D) 2 if $x \in \mathbb{Q}$

SECTION - III : ASSERTION AND REASON TYPE

16. **Statement-I:** $\lim_{x \rightarrow 0} \sin^{-1} \{x\}$ does not exist
- Statement-II:** $\{x\}$ is discontinuous at $x=0$ (where $\{.\}$ denotes fractional part function).
- (A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.
 (B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
 (C) Statement-I is True, Statement-II is False
 (D) Statement-I is False, Statement-II is True
17. **Statement-I:** $\lim_{x \rightarrow \infty} \left(\frac{1}{x^2} + \frac{2}{x^2} + \frac{3}{x^2} + \dots + \frac{x}{x^2}\right) = \lim_{x \rightarrow \infty} \frac{1}{x^2} + \lim_{x \rightarrow \infty} \frac{2}{x^2} + \dots + \lim_{x \rightarrow \infty} \frac{x}{x^2} = 0$
- Statement-II:** $\lim_{x \rightarrow a} (f_1(x) + f_2(x) + \dots + f_n(x)) = \lim_{x \rightarrow a} f_1(x) + \lim_{x \rightarrow a} f_2(x) + \dots + \lim_{x \rightarrow a} f_n(x)$, $n \in \mathbb{N}$.
- (A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.
 (B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
 (C) Statement-I is True, Statement-II is False
 (D) Statement-I is False, Statement-II is True
18. **Statement-I:** If $a_1, a_2, a_3, \dots, a_n > 0$, then $\lim_{x \rightarrow \infty} \left\{ \frac{a_1^{1/x} + a_2^{1/x} + a_3^{1/x} + \dots + a_n^{1/x}}{n} \right\}^{nx} = \prod_{i=1}^n a_i$
- Statement-II:** If $\lim_{x \rightarrow a} f(x) \rightarrow 1$, $\lim_{x \rightarrow a} g(x) \rightarrow \infty$, then $\lim_{x \rightarrow a} \{f(x)\}^{g(x)} = e^{\lim_{x \rightarrow a} (f(x)-1)g(x)}$
- (A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.
 (B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
 (C) Statement-I is True, Statement-II is False
 (D) Statement-I is False, Statement-II is True

19. **Statement-I :** If $f(x) = \frac{2}{\pi} \cot^{-1} \left(\frac{3x^2 + 1}{(x-1)(x-2)} \right)$, then $\lim_{x \rightarrow 1^-} f(x) = 0$ and $\lim_{x \rightarrow 2^-} f(x) = 2$

Statement-II : $\lim_{x \rightarrow \infty} \cot^{-1} x = 0$ and $\lim_{x \rightarrow -\infty} \cot^{-1} x = \pi$

- (A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.
- (B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
- (C) Statement-I is True, Statement-II is False
- (D) Statement-I is False, Statement-II is True

20. **Statement-I :** $\lim_{x \rightarrow \infty} \left(\frac{1^2}{x^3} + \frac{2^2}{x^3} + \frac{3^2}{x^3} + \dots + \frac{x^2}{x^3} \right) = \frac{1}{3}$

Statement-II : $\lim_{x \rightarrow a} (f_1(x) + f_2(x) + \dots + f_n(x)) = \lim_{x \rightarrow a} f_1(x) + \lim_{x \rightarrow a} f_2(x) + \dots + \lim_{x \rightarrow a} f_n(x)$ where $n \in \mathbb{N}$

- (A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.
- (B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
- (C) Statement-I is True, Statement-II is False
- (D) Statement-I is False, Statement-II is True

SECTION - IV : MATRIX - MATCH TYPE

21.

Column – I

Column – II

(A) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function and (p) 0

$f(1) = 1, f'(1) = 3$. Then the value of $\lim_{x \rightarrow 1} \int_1^{x^2} \frac{(f(t) - t)}{(x-1)^2} dt$ is

(B) $\lim_{n \rightarrow \infty} \left(\frac{1 + \sqrt[n]{4}}{2} \right)^n$ is equal to (q) - 1

(C) If $f(x) = \lim_{n \rightarrow \infty} \frac{2x}{\pi} \cdot \tan^{-1}(nx), x > 0$, then $\lim_{x \rightarrow 0^+} [f(x) - 1]$ is (r) 2
 {where $[.]$ represents greatest integer function}

(D) $\lim_{n \rightarrow \infty} \left[\sum_{r=1}^n \frac{1}{2^r} \right] =$ (s) 1

(where $[.]$ denotes the greatest integer function) (t) 4

22.

Column – I

(A) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be such that $f(A) = 1$, $f'(A) = 2$ and

$$\lim_{x \rightarrow 0} \left(\frac{f^2(a+x)}{f(a)} \right)^{1/x} = e^k, \text{ then } k =$$

(B) $\lim_{x \rightarrow \frac{\pi^+}{2}} \frac{\cos(\tan^{-1}(\tan x))}{x - \frac{\pi}{2}} =$

(C) $\lim_{x \rightarrow \pi} \frac{\sin(\cos x + 1)}{\cos\left(\frac{x}{2}\right)} =$

(D) $\lim_{x \rightarrow 0} \frac{x e^{\sin x} - e^x \sin^{-1}(\sin x)}{\sin^2 x - x \sin x} =$

Column – II

(p) 0

(q) 1

(r) 4

(s) 3

(t) does not exist

SECTION - V : COMPREHENSION TYPE

23. Read the following comprehension carefully and answer the questions.

If $\lim_{x \rightarrow 0^+} f(x) = \text{finite}$ where $f(x) = \frac{\sin x + ae^x + be^{-x} + c \ln(1+x)}{x^3}$ and a, b, c are real numbers.

1. The value of a is

- (A) $-\frac{1}{2}$ (B) 0 (C) $\frac{1}{2}$ (D) 1

2. The value of b is

- (A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) 0 (D) 1

3. The value of c is

- (A) $-1/2$ (B) $1/2$ (C) 0 (D) 2

24. Read the following comprehension carefully and answer the questions.

Let $f(x) = \lim_{n \rightarrow \infty} \left(\cos \sqrt{\frac{x}{n}} \right)^n$, $g(x) = \lim_{n \rightarrow \infty} \left(1 - x + x \sqrt[n]{e} \right)^n$. Now, consider the function $y = h(x)$, where

$$h(x) = \tan^{-1} (g^{-1} f^{-1}(x)).$$

1. $\lim_{x \rightarrow 0^+} \frac{\ln(f(x))}{\ln(g(x))}$ is equal to

- (A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) 0 (D) 1

2. Domain of the function $y = h(x)$ is
 (A) $(0, \infty)$ (B) \mathbb{R} (C) $(0, 1)$ (D) $[0, 1]$
3. Range of the function $y = h(x)$ is
 (A) $\left(0, \frac{\pi}{2}\right)$ (B) $\left(-\frac{\pi}{2}, 0\right)$ (C) \mathbb{R} (D) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

25. Read the following comprehension carefully and answer the questions.

Let $f(x) = \max \{a, b, c\}$, where

$$a = \lim_{n \rightarrow \infty} \lim_{\alpha \rightarrow 1^+} \frac{\alpha^n |\sin x| + \alpha^{-n} |\cos x|}{\alpha^n + \alpha^{-n}}$$

$$b = \lim_{n \rightarrow \infty} \lim_{\alpha \rightarrow 1^-} \frac{\alpha^n |\sin x| + \alpha^{-n} |\cos x|}{\alpha^n + \alpha^{-n}}$$

$$c = \lim_{n \rightarrow \infty} \frac{\pi}{4n} \left[1 + \cos \frac{\pi}{2n} + \cos \frac{2\pi}{2n} + \dots + \cos \frac{(n-1)\pi}{2n} \right]. \text{ Then}$$

1. The value of a is
 (A) $2|\sin x|$ (B) $|\cos x|$ (C) $|\sin x|$ (D) $\frac{1}{2}$
2. The value of $b + c - \frac{1}{2}$ is
 (A) $|\cos x|$ (B) $2|\cos x| - 1$ (C) $|\sin x| + 1$ (D) $|\sin x| + |\cos x|$
3. Range of $f(x)$ is
 (A) $[0, 1]$ (B) $\left[\frac{1}{2}, 1\right]$ (C) $\left[\frac{1}{\sqrt{2}}, 1\right]$ (D) $\left[\frac{1}{2}, 2\right]$

SECTION - VI : INTEGER TYPE

26. If $\lim_{x \rightarrow \infty} (\sqrt{x^4 + ax^3 + 3x^2 + bx + 2} - \sqrt{x^4 + 2x^3 - cx^2 + 3x - d}) = 4$, then value of $(a + c)$.

27. If $\lim_{x \rightarrow 0} (x^{-3} \sin 3x + ax^{-2} + b)$ exists and is equal to zero then value of $(a + 2b)$.

28. Let $f(x) = \frac{\tan x}{x}$ and $\lim_{x \rightarrow 0} ([f(x)] + x^2)^{\frac{1}{f(x)}} = e^\lambda$, then find λ

(where $[.]$ and $\{.\}$ denotes greatest integer and fractional part function respectively)

29. The value of $\lim_{x \rightarrow \infty} \frac{\cot^{-1}(x^{-a} \log_a x)}{\sec^{-1}(a^x \log_x a)}$ ($a > 1$).

30. Let $P = \frac{\left(1^4 + \frac{1}{4}\right)\left(3^4 + \frac{1}{4}\right)\left(5^4 + \frac{1}{4}\right)\dots\dots\left((2n-1)^4 + \frac{1}{4}\right)}{\left(2^4 + \frac{1}{4}\right)\left(4^4 + \frac{1}{4}\right)\left(6^4 + \frac{1}{4}\right)\dots\dots\left((2n)^4 + \frac{1}{4}\right)}$ and $\lim_{n \rightarrow \infty} (n^a P)$ exists, then find α

ANSWER KEY

EXERCISE - 1

1. B 2. A 3. A 4. B 5. A 6. B 7. D 8. A 9. C 10. D 11. A 12. D 13. C
 14. B 15. B 16. B 17. B 18. B 19. B 20. C 21. B 22. B 23. C 24. A 25. D 26. B
 27. C 28. A 29. D 30. C

EXERCISE - 2 : PART # I

1. ABD 2. AD 3. AD 4. AB 5. ABCD 6. ABCD 7. BCD 8. ABCD 9. AD
 10. ABD 11. BC 12. CD 13. ABC 14. ABC 15. CD 16. ABCD 17. AB 18. BCD
 19. CD

PART - II

1. C 2. D 3. B 4. C 5. A

EXERCISE - 3 : PART # I

1. $A \rightarrow s$ $B \rightarrow r$ $C \rightarrow p$ $D \rightarrow q$ 2. $A \rightarrow q$ $B \rightarrow s$ $C \rightarrow r$ $D \rightarrow p$
 3. $A \rightarrow q$ $B \rightarrow r$ $C \rightarrow s$ $D \rightarrow p$ 4. $A \rightarrow p$ $B \rightarrow r$ $C \rightarrow q$ $D \rightarrow q$
 5. $A \rightarrow s$ $B \rightarrow r$ $C \rightarrow q$ $D \rightarrow p$ 6. $A \rightarrow r$ $B \rightarrow s$ $C \rightarrow p$ $D \rightarrow q$

PART - II

- Comprehension #1: 1. A 2. D 3. A Comprehension #2: 1. B 2. A 3. A
 Comprehension #3: 1. B 2. A 3. A

EXERCISE - 5 : PART # I

1. 1 2. 1 3. 1 4. 1 5. 4 6. 2 7. 4 8. 2 9. 1 10. 1 11. 3 12. 1 13. 4
 14. 4 15. 4 16. 1 17. 1 18. 2 19. 1

PART - II

1. B 2. $\ln a$ 3. C 4. C 5. $1 - \frac{2}{\pi}$ 6. A,C 7. B 8. B 9. 0 10. 7 11. B,C

MOCK TEST

1. C 2. D 3. A 4. C 5. A 6. B 7. C 8. A 9. C 10. A 11. A,B 12. A,B
13. B,C 14. C 15. A,D 16. B 17. D 18. A 19. A 20. C
21. $A \rightarrow t$ $B \rightarrow r$ $C \rightarrow q$ $D \rightarrow p$ 22. $A \rightarrow r$ $B \rightarrow q$ $C \rightarrow p$ $D \rightarrow q$ 23. 1. A 2. B 3. C 24. 1. B
2. C 3. D 25. 1. C 2. A 3. C 26. 7 27. 6 28. 3 29. 1 30. 2