SOLVED EXAMPLES

Ex. 1 If $y = \log_e(\tan^{-1}\sqrt{1+x^2})$, find $\frac{dy}{dx}$.

Sol. $y = \log_e\ (\tan^{-1}\sqrt{1+x^2})$

On differentiating we get,

$$= \frac{1}{\tan^{-1}\sqrt{1+x^2}}.\frac{1}{1+(\sqrt{1+x^2})^2}.\frac{1}{2\sqrt{1+x^2}}.2x$$

$$= \frac{x}{\left(\tan^{-1}\sqrt{1+x^2}\right)\left\{1+\left(\sqrt{1+x^2}\right)^2\right\}\sqrt{1+x^2}} = \frac{x}{\left(\tan^{-1}\sqrt{1+x^2}\right)(2+x^2)\sqrt{1+x^2}}$$

Ex. 2 Prove that the function represented parametrically by the equations. $x = \frac{1+t}{t^3}$; $y = \frac{3}{2t^2}+\frac{2}{t}$ satisfies the relationship

$x(y')^3 = 1 + y'$ (where $y' = \frac{dy}{dx}$)

Sol. Here $x = \frac{1+t}{t^3} = \frac{1}{t^3}+\frac{1}{t^2}$

Differentiating w.r. to t

$$\frac{dx}{dt} = -\frac{3}{t^4}-\frac{2}{t^3}, \qquad y = \frac{3}{2t^2}+\frac{2}{t}$$

Differentiating w.r. to t

$$\frac{dy}{dt} = -\frac{3}{t^3}-\frac{2}{t^2}, \qquad \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = t = y'$$

Since $x = \frac{1+t}{t^3}$ $\Rightarrow$ $x = \frac{1+y'}{(y')^3}$ or $x(y')^3 = 1+y'$

Ex. 3 If $x^y + y^x = 2$, then find $\frac{dy}{dx}$.

Sol. Let $u = x^y$ and $v = y^x$

$u + v = 2$ $\Rightarrow$ $\frac{du}{dx}+\frac{dv}{dx} = 0$

Now $u = x^y$ and $v = y^x$

$\Rightarrow$ $\ell n\ u = y\ \ell nx$ and $\ell n\ v = x\ \ell n\ y$

$\Rightarrow$ $\frac{1}{u}\frac{du}{dx} = \frac{y}{x} + \ell nx\frac{dy}{dx}$ and $\frac{1}{v}\frac{dv}{dx} = \ell n\ y + \frac{x}{y}\frac{dy}{dx}$

$\Rightarrow$ $\frac{du}{dx} = x^y\left(\frac{y}{x}+\ell nx\frac{dy}{dx}\right)$ and $\frac{dv}{dx} = y^x\left(\ell n\, y+\frac{x}{y}\frac{dy}{dx}\right)$

$$\Rightarrow \quad x^y\left(\frac{y}{x}+\ell n\, x\frac{dy}{dx}\right)+y^x\left(\ell ny+\frac{x}{y}\frac{dy}{dx}\right) = 0 \quad \Rightarrow \quad \frac{dy}{dx} = -\frac{\left(y^x\ell ny + x^y.\frac{y}{x}\right)}{\left(x^y\ell nx + y^x.\frac{x}{y}\right)}$$

Ex. 4 Find $\frac{dy}{dx}$, where $y = \tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$.

Sol. Let $x = \cos\theta$, where $\theta \in [0, \pi]$

$\Rightarrow$ $y = \tan^{-1}\left(\frac{\sqrt{1+\cos\theta}-\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}+\sqrt{1-\cos\theta}}\right)$

$\Rightarrow$ $y = \tan^{-1}\left(\frac{\sqrt{2}\cos\frac{\theta}{2}-\sqrt{2}\sin\frac{\theta}{2}}{\sqrt{2}\cos\frac{\theta}{2}+\sqrt{2}\sin\frac{\theta}{2}}\right)$

$\left[\because \sqrt{1+\cos\theta} = \left|\sqrt{2}\cos\frac{\theta}{2}\right| \text{ but for } \frac{\theta}{2} \in \left(0, \frac{\pi}{2}\right), \left|\sqrt{2}\cos\frac{\theta}{2}\right| = \sqrt{2}\cos\frac{\theta}{2}\right]$

$\Rightarrow$ $y = \tan^{-1}\left(\frac{1-\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}}\right)$ $\Rightarrow$ $y = \frac{\pi}{4} - \frac{\theta}{2}$ as $-\frac{\pi}{4} \le \frac{\pi}{4} - \frac{\theta}{2} \le \frac{\pi}{4}$

$\Rightarrow$ $y = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$ $\Rightarrow$ $\frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}}$

Ex. 5 If $y = \cfrac{\sin x}{1+\cfrac{\cos x}{1+\cfrac{\sin x}{1+\cos x.....}}}$, prove that $\frac{dy}{dx} = \frac{(1+y)\cos x + y\sin x}{1+2y+\cos x - \sin x}$.

Sol. Given function is $y = \cfrac{\sin x}{1+\cfrac{\cos x}{1+y}} = \frac{(1+y)\sin x}{1+y+\cos x}$

or $y + y^2 + y\cos x = (1+y)\sin x$

Differentiate both sides with respect to x,

$$\frac{dy}{dx} + 2y\frac{dy}{dx} + \frac{dy}{dx}\cos x - y\sin x = (1+y)\cos x + \frac{dy}{dx}\sin x$$

$$\frac{dy}{dx}(1+2y+\cos x - \sin x) = (1+y)\cos x + y\sin x$$

or $\frac{dy}{dx} = \frac{(1+y)\cos x + y\sin x}{1+2y+\cos x - \sin x}$

Ex. 6 y = f(x) and x = g(y) are inverse functions of each other then express g'(y) and g"(y) in terms of derivative of f(x).

Sol. $\frac{dy}{dx} = f'(x)$ and $\frac{dx}{dy} = g'(y)$

$\Rightarrow \quad g'(y) = \frac{1}{f'(x)}$ .....(i)

Again differentiating w.r.t. to y

$$g''(y) = \frac{d}{dy}\left(\frac{1}{f'(x)}\right) = \frac{d}{dx}\left(\frac{1}{f'(x)}\right).\frac{dx}{dy} = -\frac{f''(x)}{(f'(x))^2}.\left(\frac{1}{f'(x)}\right)$$

$\Rightarrow \quad g''(y) = -\frac{f''(x)}{(f'(x))^3}$ .....(ii)

Which can also be remembered as $\frac{d^2x}{dy^2} = -\frac{\frac{d^2y}{dx^2}}{\left(\frac{dy}{dx}\right)^3}$

Ex. 7 Evaluate $\lim_{n\to\infty}\left(\frac{e^n}{\pi}\right)^{1/n}$.

Sol. Here, $A = \lim_{n\to\infty}\left(\frac{e^n}{\pi}\right)^{1/n}$ (∞^0 form)

$\therefore \quad \log A = \lim_{n\to\infty}\frac{1}{n}\log\left(\frac{e^n}{\pi}\right) = \lim_{n\to\infty}\frac{n\log e - \log\pi}{n}$ $\left(\frac{\infty}{\infty}\text{form}\right)$

$= \lim_{n\to\infty}\frac{\log e - 0}{1}$ {applying L'Hospital's rule}

$\log A = 1 \Rightarrow A = e^1$ or $\lim_{n\to\infty}\left(\frac{e^n}{\pi}\right)^{1/n} = e$

Ex. 8 If $y = \left(\frac{1}{x}\right)^x$, then find $y''(1)$

Sol. Now taking $\log_e$ both sides, we get

$\ell n\, y = -x\, \ell n\, x$ when x = 1, then y = 1

$\ell n\, y = -x\, \ell n\, x$

$\Rightarrow \quad \frac{y'}{y} = -(1 + \ell n\, x) \quad \Rightarrow \quad y' = -y\,(1 + \ell n\, x)$ ......(i)

again diff. w.r.t. to x,

$y'' = -y'(1 + \ell n\, x) - y\,.\,\frac{1}{x} \quad \Rightarrow \quad y'' = y\,(1 + \ell n\, x)^2 - \frac{y}{x}$ (using (i))

$\Rightarrow \quad y''(1) = 0$

Ex. 9 If x = a (t + sin t) and y = a(1 – cos t), find $\frac{d^2y}{dx^2}$.

Sol. Here x = a (t + sin t) and y = a (1–cos t)

Differentiating both sides w.r.t. t, we get :

$\frac{dx}{dt}$ = a(1 + cos t) and $\frac{dy}{dt}$ = a (sin t)

$$\therefore \quad \frac{dy}{dx} = \frac{a\sin t}{a(1+\cos t)} = \frac{2\sin\frac{t}{2}.\cos\frac{t}{2}}{2\cos^2\frac{t}{2}} = \tan\left(\frac{t}{2}\right)$$

Again differentiating both sides, we get,

$$\frac{d^2y}{dx^2} = \sec^2\left(\frac{t}{2}\right)\cdot\frac{1}{2}\cdot\frac{dt}{dx} = \frac{1}{2}\sec^2(t/2)\cdot\frac{1}{a(1+\cos t)} = \frac{1}{2a}\cdot\frac{\sec^2\left(\frac{t}{2}\right)}{2\left(\cos^2\frac{t}{2}\right)}$$

Hence, $\frac{d^2y}{dx^2} = \frac{1}{4a}\cdot\sec^4\left(\frac{t}{2}\right)$

Ex. 10 If $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ then find **(i)** f'(2) **(ii)** $f'\left(\frac{1}{2}\right)$ **(iii)** f'(1)

Sol. x = tanθ, where $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ ⇒ $y = \sin^{-1}(\sin 2\theta)$

$$y = \begin{cases} \pi - 2\theta & \frac{\pi}{2} < 2\theta < \pi \\ 2\theta & \frac{-\pi}{2} \le 2\theta \le \frac{\pi}{2} \\ -(\pi + 2\theta) & -\pi < 2\theta < -\frac{\pi}{2} \end{cases} \Rightarrow f(x) = \begin{cases} \pi - 2\tan^{-1}x & x > 1 \\ 2\tan^{-1}x & -1 \le x \le 1 \\ -(\pi + 2\tan^{-1}x) & x < -1 \end{cases}$$

$$\Rightarrow \quad f'(x) = \begin{cases} -\frac{2}{1+x^2} & x > 1 \\ \frac{2}{1+x^2} & -1 < x < 1 \\ \frac{-2}{1+x^2} & x < -1 \end{cases}$$

(i) $f'(2) = -\frac{2}{5}$ **(ii)** $f'\left(\frac{1}{2}\right) = \frac{8}{5}$ **(iii)** $f'(1^+) = -1$ and $f'(1^-) = +1$ ⇒ f '(1) does not exist

Ex. 11 Solve $\lim_{x\to 0^+} \log_{\sin x} \sin 2x$.

Sol. Here $\lim_{x\to 0^+} \log_{\sin x} \sin 2x$

$$= \lim_{x\to 0^+} \frac{\log \sin 2x}{\log \sin x} \qquad \left(\frac{-\infty}{-\infty}\text{form}\right)$$

$$= \lim_{x\to 0^+} \frac{\frac{1}{\sin 2x}\cdot 2\cos 2x}{\frac{1}{\sin x}\cdot \cos x} \qquad \text{\{applying L'Hospital's rule\}}$$

$$= \lim_{x\to 0^+} \frac{\left(\frac{(2x)}{\sin(2x)}\right)\cos 2x}{\left(\frac{x}{\sin x}\right)\cos x} = \lim_{x\to 0^+} \frac{\cos 2x}{\cos x} = 1$$

Ex. 12 y = sin (sinx) then prove that $y'' + (\tan x)\, y' + y\cos^2 x = 0$

Sol. Such expression can be easily proved using implicit differentiation

$\Rightarrow$ $y' = \cos(\sin x)\cos x$

$\Rightarrow$ $\sec x . y' = \cos(\sin x)$

again differentiating w.r.t x, we get

$\sec x\, y'' + y' \sec x \tan x = -\sin(\sin x)\cos x$

$\Rightarrow$ $y'' + y' \tan x = -y\cos^2 x$

$\Rightarrow$ $y'' + (\tan x)\, y' + y\cos^2 x = 0$

Ex. 13 If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$ -

Sol. Put $x = \sin\alpha \Rightarrow \alpha = \sin^{-1}(x)$

$y = \sin\beta \Rightarrow \beta = \sin^{-1}(y)$

$\Rightarrow$ $\cos\alpha + \cos\beta = a(\sin\alpha - \sin\beta)$

$\Rightarrow$ $2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = 2a\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

$\Rightarrow$ $\cot\left(\frac{\alpha-\beta}{2}\right) = a$

$\Rightarrow$ $\alpha - \beta = 2\cot^{-1}(a)$

$\Rightarrow$ $\sin^{-1}x - \sin^{-1}y = 2\cot^{-1}(a)$

differentiating w.r.t. to x.

$$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0 \quad \Rightarrow \quad \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$$ Hence proved

Ex. 14 If $y = (\tan^{-1}x)^2$ then prove that $(1 + x^2)^2 \frac{d^2y}{dx^2} + 2x(1 + x^2)\frac{dy}{dx} = 2$

Sol. $y = (\tan^{-1}x)^2$

Differentiating w.r.t. x

$$\frac{dy}{dx} = \frac{2\tan^{-1}x}{1+x^2}$$

$$\Rightarrow \quad (1+x^2)\frac{dy}{dx} = 2\tan^{-1}(x)$$

Again differentiating w.r.t. x

$$\Rightarrow \quad (1+x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = \frac{2}{(1+x^2)} \quad \Rightarrow \quad (1+x^2)^2\frac{d^2y}{dx^2} + 2x(1+x^2)\frac{dy}{dx} = 2$$

Ex. 15 Obtain differential coefficient of $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$ with respect to $\cos^{-1}\sqrt{\frac{1+\sqrt{1+x^2}}{2\sqrt{1+x^2}}}$

Sol. Assume $u = \tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$, $v = \cos^{-1}\sqrt{\frac{1+\sqrt{1+x^2}}{2\sqrt{1+x^2}}}$

The function needs simplification before differentiation Let $x = \tan\theta$

$$\therefore \quad u = \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right) = \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right) = \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{\theta}{2}$$

$$v = \cos^{-1}\sqrt{\frac{1+\sec\theta}{2\sec\theta}} = \cos^{-1}\sqrt{\frac{1+\cos\theta}{2}} = \cos^{-1}\left(\cos\frac{\theta}{2}\right) = \frac{\theta}{2} \quad \Rightarrow \quad u = v$$

$$\therefore \quad \frac{du}{dv} = 1.$$

Exercise # 1

[Single Correct Choice Type Questions]

1. Let $u(x)$ and $v(x)$ are differentiable functions such that $\frac{u(x)}{v(x)} = 7$. If $\frac{u'(x)}{v'(x)} = p$ and $\left(\frac{u(x)}{v(x)}\right)' = q$, then $\frac{p+q}{p-q}$ has the value equal to

(A) 1 **(B)** 0 **(C)** 7 **(D)** – 7

2. If $y = f\left(\frac{3x+4}{5x+6}\right)$ & $f'(x) = \tan x^2$ then $\frac{dy}{dx} =$

(A) $\tan x^3$

(B) $-2\tan\left[\frac{3x+4}{5x+6}\right]^2 \cdot \frac{1}{(5x+6)^2}$

(C) $f\left(\frac{3\tan x^2+4}{5\tan x^2+6}\right)\tan x^2$

(D) none

3. If $y = \frac{\sec x - \tan x}{\sec x + \tan x}$ then $\frac{dy}{dx}$ equals -

(A) $2\sec x(\sec x - \tan x)$

(B) $-2\sec x(\sec x - \tan x)^2$

(C) $2\sec x(\sec x + \tan x)^2$

(D) $-2\sec x(\sec x + \tan x)^2$

4. If $y = \frac{1}{1+x^{n-m}+x^{p-m}} + \frac{1}{1+x^{m-n}+x^{p-n}} + \frac{1}{1+x^{m-p}+x^{n-p}}$ then $\frac{dy}{dx}$ at $e^{m^{n^p}}$ is equal to:

(A) e^{mnp} **(B)** $e^{mn/p}$ **(C)** $e^{np/m}$ **(D)** none

5. A differentiable function satisfies $3f^2(x)f'(x) = 2x$. Given $f(2) = 1$ then the value of $f(3)$ is

(A) $\sqrt[3]{24}$ **(B)** $\sqrt[3]{6}$ **(C)** 6 **(D)** 2

6. Let g is the inverse function of f & $f'(x) = \frac{x^{10}}{(1+x^2)}$. If $g(2) = a$ then $g'(2)$ is equal to

(A) $\frac{5}{2^{10}}$ **(B)** $\frac{1+a^2}{a^{10}}$ **(C)** $\frac{a^{10}}{1+a^2}$ **(D)** $\frac{1+a^{10}}{a^2}$

7. Let $f(x) = x + 3\,ln(x-2)$ & $g(x) = x + 5\,ln(x-1)$, then the set of x satisfying the inequality $f'(x) < g'(x)$ is -

(A) $\left(2, \frac{7}{2}\right)$ **(B)** $(1, 2) \cup \left(\frac{7}{2}, \infty\right)$ **(C)** $(2, \infty)$ **(D)** $\left(\frac{7}{2}, \infty\right)$

8. Suppose $\begin{vmatrix} f'(x) & f(x) \\ f''(x) & f'(x) \end{vmatrix} = 0$ where $f(x)$ is continuously differentiable function with $f'(x) \neq 0$ and satisfies $f(0) = 1$ and $f'(0) = 2$ then $f(x)$ is

(A) $x^2 + 2x + 1$ **(B)** $2e^x - 1$ **(C)** e^{2x} **(D)** $4e^{x/2} - 3$

9. Suppose that $f(0) = 0$ and $f'(0) = 2$, and let $g(x) = f(-x + f(f(x)))$. The value of $g'(0)$ is equal to

(A) 0 **(B)** 1 **(C)** 6 **(D)** 8

10. If $f(x)=\prod_{n=1}^{100}(x-n)^{n(101-n)}$; then $\dfrac{f(101)}{f'(101)}=$

(A) 5050 (B) $\dfrac{1}{5050}$ (C) 10010 (D) $\dfrac{1}{10010}$

11. Let $f(x)=\begin{cases} g(x)\,.\cos\frac{1}{x} & \text{if } x\neq 0 \\ 0 & \text{if } x=0 \end{cases}$ where g(x) is an even function differentiable at x = 0, passing through the origin.

Then $f'(0)$

(A) is equal to 1 (B) is equal to 0 (C) is equal to 2 (D) does not exist

12. If f is twice differentiable such that $f''(x)=-f(x),\ f'(x)=g(x)$

$$h'(x)=[f(x)]^2+[g(x)]^2 \text{ and}$$
$$h(0)=2,\ h(1)=4$$

then the equation $y=h(x)$ represents :

(A) a curve of degree 2 (B) a curve passing through the origin

(C) a straight line with slope 2 (D) a straight line with y intercept equal to -2.

13. Differential coefficient of $\left(x^{\frac{\ell+m}{m-n}}\right)^{\frac{1}{n-\ell}}.\left(x^{\frac{m+n}{n-\ell}}\right)^{\frac{1}{\ell-m}}.\left(x^{\frac{n+\ell}{\ell-m}}\right)^{\frac{1}{m-n}}$ w.r.t. x is -

(A) 1 (B) 0 (C) –1 (D) x^{lmn}

14. A function $y=f(x)$ satisfies $f''(x)=-\dfrac{1}{x^2}-\pi^2\sin(\pi x)$; $f'(2)=\pi+\dfrac{1}{2}$ and $f(1)=0$. The value of $f\left(\dfrac{1}{2}\right)$ is

(A) $\ln 2$ (B) 1 (C) $\dfrac{\pi}{2}-\ln 2$ (D) $1-\ln 2$

15. The derivative of the function,

$f(x)=\cos^{-1}\left\{\dfrac{1}{\sqrt{13}}(2\cos x-3\sin x)\right\}+\sin^{-1}\left\{\dfrac{1}{\sqrt{13}}(2\cos x+3\sin x)\right\}$ w.r.t. $\sqrt{1+x^2}$ at $x=\dfrac{3}{4}$ is

(A) $\dfrac{3}{2}$ (B) $\dfrac{5}{2}$ (C) $\dfrac{10}{3}$ (D) 0

16. If $\sin(xy)+\cos(xy)=0$ then $\dfrac{dy}{dx}$ is equal to

(A) $\dfrac{y}{x}$ (B) $-\dfrac{y}{x}$ (C) $-\dfrac{x}{y}$ (D) $\dfrac{x}{y}$

17. A function $y=f(x)$ has second order derivative $f''(x)=6(x-1)$. If its graph passes through the point (2, 1) and at that point the tangent to the graph is $y=3x-5$, then the function is -

(A) $(x+1)^3$ (B) $(x+1)^2$ (C) $(x-1)^2$ (D) $(x-1)^3$

18. People living at Mars, instead of the usual definition of derivative D f(x), define a new kind of derivative, D*f(x) by the formula

$D^*f(x) = \underset{h\to 0}{\text{Limit}} \dfrac{f^2(x+h) - f^2(x)}{h}$ where $f^2(x)$ means $[f(x)]^2$. If $f(x) = x\, l\text{n}x$ then

$D^*f(x)\big|_{x=e}$ has the value

(A) e **(B)** 2e **(C)** 4e **(D)** 8e

19. If $f(x)$ is a twice differentiable function, then between two consecutive roots of the equation $f'(x) = 0$, there exists :

(A) atleast one root of $f(x) = 0$
(B) atmost one root of $f(x) = 0$
(C) exactly one root of $f(x) = 0$
(D) atmost one root of $f''(x) = 0$

20. If $x = e^{y + e^{y + \ldots\ldots\ldots \text{to}\,\infty}}$, $x > 0$ then $\dfrac{dy}{dx}$ is -

(A) $\dfrac{x}{1+x}$ **(B)** $\dfrac{1+x}{x}$ **(C)** $\dfrac{1-x}{x}$ **(D)** $\dfrac{1}{x}$

21. If $f(x) = \sqrt{x + 2\sqrt{2x-4}} + \sqrt{x - 2\sqrt{2x-4}}$, then the value of $10\, f'(102^+)$

(A) is – 1 **(B)** is 0 **(C)** is 1 **(D)** does not exist

22. Let f (x) be a polynomial function of second degree. If f (1) = f (–1) and a, b, c are in A.P., then f '(a), f '(b) and f '(c) are in

(A) G.P. **(B)** H.P. **(C)** A.G.P. **(D)** A.P.

23. If $x^2 + y^2 = R^2$ $(R > 0)$ then $k = \dfrac{y''}{\sqrt{(1 + y'^2)^3}}$ where k in terms of R alone is equal to

(A) $-\dfrac{1}{R^2}$ **(B)** $-\dfrac{1}{R}$ **(C)** $\dfrac{2}{R}$ **(D)** $-\dfrac{2}{R^2}$

24. If $y = \dfrac{x}{a+}\dfrac{x}{b+}\dfrac{x}{a+}\dfrac{x}{b+}\dfrac{x}{a+}\dfrac{x}{b+}\ldots\ldots\ldots\infty$ then $\dfrac{dy}{dx}$ -

(A) $\dfrac{a}{ab+2ay}$ **(B)** $\dfrac{b}{ab+2by}$ **(C)** $\dfrac{a}{ab+2by}$ **(D)** $\dfrac{b}{ab+2ay}$

25. Let $f(x) = x^n$, n being a non-negative integer. The number of values of n for which $f'(p+q) = f'(p) + f'(q)$ is valid for all $p, q > 0$ is :

(A) 0 **(B)** 1 **(C)** 2 **(D)** none of these

26. A non zero polynomial with real coefficients has the property that f (x) = f ' (x) · f "(x). The leading coefficient of f (x) is

(A) $\dfrac{1}{6}$ **(B)** $\dfrac{1}{9}$ **(C)** $\dfrac{1}{12}$ **(D)** $\dfrac{1}{18}$

27. If $f(x) = \sqrt{x^2+1}$, $g(x) = \dfrac{x+1}{x^2+1}$ and $h(x) = 2x - 3$, then $f'(h'(g'(x)) =$

(A) 0 **(B)** $\dfrac{1}{\sqrt{x^2+1}}$ **(C)** $\dfrac{2}{\sqrt{5}}$ **(D)** $\dfrac{x}{\sqrt{x^2+1}}$

28. The ends A and B of a rod of length $\sqrt{5}$ are sliding along the curve $y = 2x^2$. Let x_A and x_B be the x-coordinate of the ends. At the moment when A is at (0, 0) and B is at (1, 2) the derivative $\dfrac{dx_B}{dx_A}$ has the value(s) equal to

(A) 1/3 **(B)** 1/5 **(C)** 1/8 **(D)** 1/9

29. If $\dfrac{d^2x}{dy^2}\left(\dfrac{dy}{dx}\right)^3 + \dfrac{d^2y}{dx^2} = K$ then the value of K is equal to

(A) 1 **(B)** –1 **(C)** 2 **(D)** 0

30. If $y = at^2 + 2bt + c$ and $t = ax^2 + 2bx + c$, then $\dfrac{d^3y}{dx^3}$ equals

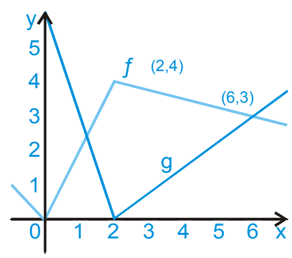
(A) $24a^2(at+b)$ **(B)** $24a(ax+b)^2$ **(C)** $24a(at+b)^2$ **(D)** $24a^2(ax+b)$

31. Suppose the function $f(x) - f(2x)$ has the derivative 5 at $x = 1$ and derivative 7 at $x = 2$. The derivative of the function $f(x) - f(4x)$ at $x = 1$, has the value equal to

(A) 19 **(B)** 9 **(C)** 17 **(D)** 14

32. If f & g are the functions whose graphs are as shown, let $u(x) = f(g(x))$; $w(x) = g(g(x))$,

then the value of $u'(1) + w'(1)$ is -



(A) $-\dfrac{1}{2}$ **(B)** $-\dfrac{3}{2}$

(C) $-\dfrac{5}{4}$ **(D)** does not exist

33. If $y = x - x^2$, then the derivative of y^2 w.r.t. x^2 is

(A) $2x^2 + 3x - 1$ **(B)** $2x^2 - 3x + 1$ **(C)** $2x^2 + 3x + 1$ **(D)** none of these

34. If $x + y = 3e^2$ then $D(x^y)$ vanishes when x equals to

(A) e **(B)** e^2 **(C)** e^e **(D)** $2e^2$

35. If $f(x) = x^n$, then the value of $f(1) - \dfrac{f'(1)}{1!} + \dfrac{f''(1)}{2!} - \dfrac{f'''(1)}{3!} + \ldots\ldots + \dfrac{(-1)^n f^{''''''''\ldots\ldots(n\ \text{times})}(1)}{n!}$

(A) 2^{n-1} **(B)** 0 **(C)** 1 **(D)** 2^n

Exercise # 2

Part # I [Multiple Correct Choice Type Questions]

1. The slope(s) of common tangent(s) to the curves $y = e^{-x}$ & $y = e^{-x}\sin x$ can be -

(A) $-e^{-\pi/2}$ **(B)** $-e^{-\pi}$ **(C)** $\frac{\pi}{2}$ **(D)** 1

2. If $x = \cos t$, $y = \log_e t$ then

(A) $\frac{dy}{dx} = -\frac{2}{\pi}$ at $t = \frac{\pi}{2}$ **(B)** $\frac{dy}{dx} = \frac{4}{\pi^2}$ at $t = \frac{\pi}{2}$ **(C)** $\frac{dy}{dx} = \frac{144}{\pi^2}$ at $t = \frac{\pi}{6}$ **(D)** $\frac{dy}{dx} = -\frac{12}{\pi}$ at $t = \frac{\pi}{6}$

3. If $\sqrt{y+x} + \sqrt{y-x} = c$ (where c ¹ 0), then $\frac{dy}{dx}$ has the value equal to

(A) $\frac{2x}{c^2}$ **(B)** $\frac{x}{y+\sqrt{y^2-x^2}}$ **(C)** $\frac{y-\sqrt{y^2-x^2}}{x}$ **(D)** $\frac{c^2}{2y}$

4. If $y + \ln(1 + x) = 0$, which of the following is true ?

(A) $e^y = xy' + 1$ **(B)** $y' = -\frac{1}{(x+1)}$ **(C)** $y' + e^y = 0$ **(D)** $y' = e^y$

5. If $\sqrt{x^2+y^2} = e^t$ where $t = \sin^{-1}\left(\frac{y}{\sqrt{x^2+y^2}}\right)$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{x-y}{x+y}$ **(B)** $\frac{x+y}{x-y}$ **(C)** $\frac{y-x}{y+x}$ **(D)** $\frac{x-y}{2x+y}$

6. If $y = \tan x \tan 2x \tan 3x$ then $\frac{dy}{dx}$ has the value equal to

(A) $3\sec^2 3x \tan x \tan 2x + \sec^2 x \tan 2x \tan 3x + 2\sec^2 2x \tan 3x \tan x$

(B) $2y(\operatorname{cosec} 2x + 2\operatorname{cosec} 4x + 3\operatorname{cosec} 6x)$

(C) $3\sec^2 3x - 2\sec^2 2x - \sec^2 x$

(D) $\sec^2 x + 2\sec^2 2x + 3\sec^2 3x$

7. If $f(x) = \begin{vmatrix} \cos(x+x^2) & \sin(x+x^2) & -\cos(x+x^2) \\ \sin(x-x^2) & \cos(x-x^2) & \sin(x-x^2) \\ \sin 2x & 0 & \sin 2x^2 \end{vmatrix}$, then

(A) $f(-2) = 0$ **(B)** $f'(-1/2) = 0$ **(C)** $f'(-1) = -2$ **(D)** $f''(0) = 4$

8. If $f_n(x) = e^{f_{n-1}(x)}$ for all $n \in N$ and $f_o(x) = x$, then $\frac{d}{dx}\{f_n(x)\}$ is equal to

(A) $f_n(x).\frac{d}{dx}\{f_{n-1}(x)\}$ **(B)** $f_n(x).\, f_{n-1}(x)$

(C) $f_n(x).\, f_{n-1}(x)........ f_2(x).\, f_1(x)$ **(D)** none of these

9. If g is inverse of f and $f(x) = x^2 + 3x - 3$ $(x > 0)$ then g'(1) equals-

(A) $\frac{1}{2g(1)+3}$ **(B)** -1 **(C)** $\frac{1}{5}$ **(D)** $\frac{-f'(1)}{(f(1))^2}$

10. If $2^x + 2^y = 2^{x+y}$ then $\frac{dy}{dx}$ has the value equal to

(A) $-\frac{2^y}{2^x}$ **(B)** $\frac{1}{1-2^x}$ **(C)** $1-2^y$ **(D)** $\frac{2^x(1-2^y)}{2^y(2^x-1)}$

11. The functions $u = e^x \sin x$; $v = e^x \cos x$ satisfy the equation

(A) $v\frac{du}{dx} - u\frac{dv}{dx} = u^2 + v^2$ **(B)** $\frac{d^2u}{dx^2} = 2v$

(C) $\frac{d^2v}{dx^2} = -2u$ **(D)** $\frac{du}{dx} + \frac{dv}{dx} = 2v$

12. For the function $y = f(x) = (x^2 + bx + c)e^x$, which of the following holds ?

(A) if $f(x) > 0$ for all real $x \not\Rightarrow f'(x) > 0$ **(B)** if $f(x) > 0$ for all real $x \Rightarrow f'(x) > 0$

(C) if $f'(x) > 0$ for all real $x \Rightarrow f(x) > 0$ **(D)** if $f'(x) > 0$ for all real $x \not\Rightarrow f(x) > 0$

13. Let $y = \sqrt{x + \sqrt{x + \sqrt{x + \ldots\ldots\infty}}}$ then $\frac{dy}{dx}$ -

(A) $\frac{1}{2y-1}$ **(B)** $\frac{x}{x-2y}$ **(C)** $\frac{1}{\sqrt{1+4x}}$ **(D)** $\frac{y}{2x+y}$

14. If f is twice differentiable such that $f''(x) = -f(x)$ and $f'(x) = g(x)$. If h(x) is a twice differentiable function such that $h'(x) = (f(x))^2 + (g(x))^2$. If $h(0) = 2$, $h(1) = 4$, then the equation $y = h(x)$ represents

(A) a curve of degree 2 **(B)** a curve passing through the origin

(C) a straight line with slope 2 **(D)** a straight line with y intercept equal to 2.

15. Let $f(x) = \frac{\sqrt{x - 2\sqrt{x-1}}}{\sqrt{x-1} - 1} . x$ then

(A) $f'(10) = 1$ **(B)** $f'(3/2) = -1$

(C) domain of $f(x)$ is $x \geq 1$ **(D)** none

16. If $f(x) = (ax + b)\sin x + (cx + d)\cos x$, then the values of a, b, c and d such that $f'(x) = x\cos x$ for all x are

(A) $a = d = 1$ **(B)** $b = 0$ **(C)** $c = 0$ **(D)** $b = c$

17. Let $f'(x) = 3x^2 \sin\frac{1}{x} - x\cos\frac{1}{x}$, if $x \neq 0$; $f(0) = 0$ and $f(1/\pi) = 0$ then :

(A) $f(x)$ is continuous at $x = 0$ **(B)** $f(x)$ is non derivable at $x = 0$

(C) $f'(x)$ is continuous at $x = 0$ **(D)** $f'(x)$ is non derivable at $x = 0$

18. $y = \cos^{-1}\sqrt{\dfrac{\sqrt{1+x^2}+1}{2\sqrt{1+x^2}}}$, then $\dfrac{dy}{dx}$ is equal to

(A) $\dfrac{1}{2(1+x^2)}, x \in R$ **(B)** $\dfrac{1}{2\,(1+x^2)},\ x > 0$ **(C)** $\dfrac{-1}{2\,(1+x^2)}, x < 0$ **(D)** $\dfrac{1}{2(1+x^2)}, x < 0$

19. If $y = x^{(\ell n\, x)^{\ell n\,(\ell n\, x)}}$, then $\dfrac{dy}{dx}$ is equal to :

(A) $\dfrac{y}{x}\left(\ell n\, x^{\ell n\, x - 1} + 2\,\ell n\, x\ \ell n\left(\ell n\, x\right)\right)$

(B) $\dfrac{y}{x}\,(\ln x)^{\ln(\ln x)}\,(2\ln(\ln x) + 1)$

(C) $\dfrac{y}{x\ \ell n\, x}\,((\ln x)^2 + 2\ln(\ln x))$

(D) $\dfrac{y\ \ell n\, y}{x\ \ell n\, x}\,(2\ln(\ln x) + 1)$

20. Two functions f & g have first & second derivatives at x = 0 & satisfy the relations,

$f(0) = \dfrac{2}{g(0)}$, $f'(0) = 2\,g'(0) = 4g\,(0),\ g''(0) = 5\,f''(0) = 6\,f(0) = 3$ then -

(A) if $h(x) = \dfrac{f(x)}{g(x)}$ then $h'(0) = \dfrac{15}{4}$

(B) if $k(x) = f(x)\,.\,g(x)\,\sin x$ then $k'(0) = 2$

(C) $\lim\limits_{x\to 0}\dfrac{g'(x)}{f'(x)} = \dfrac{1}{2}$

(D) none of these

Part # II [Assertion & Reason Type Questions]

These questions contain Statement I (assertion) and Statement II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.

(C) Statement-I is true, Statement-II is false.

(D) Statement-I is false, Statement-II is true.

1. **Statement-I** Let f(x) is a continuous function defined from R to Q and f(5) = 3 then differential coefficient of f(x) w.r.t. x will be 0.

Statement-II Differentiation of constant function is always zero.

2. **Statement-I** Let $f : [0, \infty) \to [0, \infty)$ be a function defined by $y = f(x) = x^2$, then $\left(\dfrac{d^2y}{dx^2}\right)\left(\dfrac{d^2x}{dy^2}\right) = 1.$

Statement-II $\dfrac{d^2y}{dx^2} = -\dfrac{d^2x}{dy^2}\cdot\left(\dfrac{dy}{dx}\right)^3$

3. Consider $f(x) = \frac{x}{x^2 - 1}$ & $g(x) = f''(x)$.

Statement-I Graph of g(x) is concave up for x > 1.

Statement-II $\frac{d^n}{dx^n}(f(x)) = \frac{(-1)^n n!}{2}\left\{\frac{1}{(x+1)^{n+1}} + \frac{1}{(x-1)^{n+1}}\right\}$, $n \in N$

4. Consider the following statements

Statement-I $f(x) = x e^x$ and $g(x) = e^x(x+1)$ are both aperiodic function.

Statement-II Derivative of a differentiable aperiodic function is an aperiodic function.

5. **Statement - I** $\frac{d}{dx}\{\tan^{-1}(\sec x + \tan x)\} = \frac{d}{dx}\{\cot^{-1}(\operatorname{cosec} x + \cot x)\}$, $x \in \left(0, \frac{\pi}{4}\right)$.

Statement - II $\sec^2 x - \tan^2 x = 1 = \operatorname{cosec}^2 x - \cot^2 x$.

6. **Statement - I** Derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is 1 for $0 < x < 1$.

Statement - II $\sin^{-1}\left(\frac{2x}{1+x^2}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ for $-1 \le x \le 1$.

7. **Statement - I** For $x < 0$, $\frac{d}{dx}(\ell n|x|) = \frac{1}{x}$.

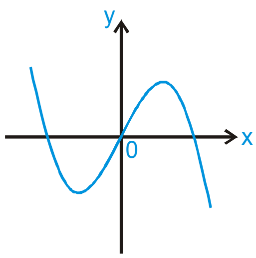
Statement - II For $x < 0, |x| = -x \Rightarrow \frac{d}{dx}|x| = -1$

Exercise # 3 Part # I [Matrix Match Type Questions]

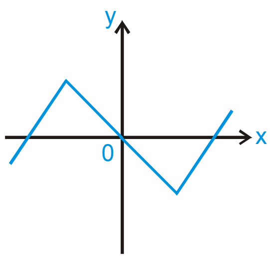
Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with one or more statement(s) in **Column-II**.

1. **Column-I** — **Graph of f(x)**

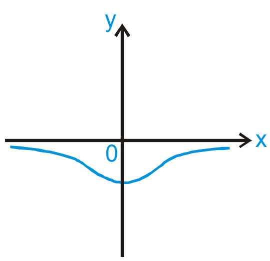
(A)



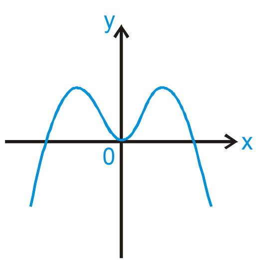
(B)



(C)

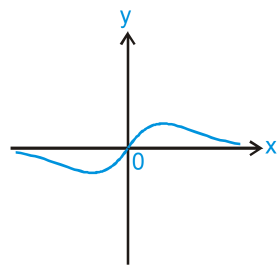


(D)

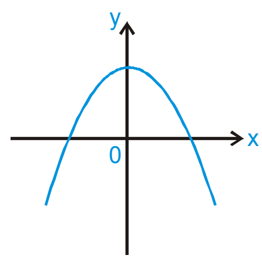


Column-II — **Graph of f'(x)**

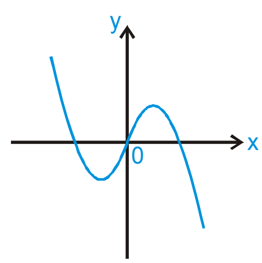
(p)



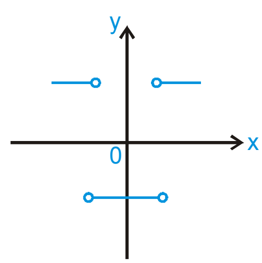
(q)



(r)



(s)



2. Match the column

Column – I		**Column – II**	
(A)	If $y = \cos^{-1}(\cos x)$, then y' at $x = 5$ is equal to	**(p)**	-1
(B)	For the function $f(x) = \ell n \tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$ if $\frac{dy}{dx} = \sec x + p$, then p is equal to	**(q)**	0

(C)	The derivative of $\tan^{-1}\left(\frac{1+x}{1-x}\right)$ at $x = -1$ is	(r)	$\frac{1}{2}$
(D)	The derivative of $\frac{\ell n\lvert x\rvert}{x}$ at $x = -1$ is	(s)	1

3.

Column-I		Column-II	
(A)	If $f(x) = x^3 + x + 1$, then $f'(x^2 + 1)$ at $x = 0$ is	(p)	1
(B)	If $f(x) = \log_{x^2}(\log x)$, then $f'(e^e)$ is equal to	(q)	0
(C)	For the function $y = \ell n \tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$ if $\frac{dy}{dx} = \sec x + p$, then p is equal to	(r)	28
(D)	If $f(x) = \lvert x^3 - x^2 + x - 1\rvert \sin x$, then $4f'(28f(f(\pi)))$ is equal to	(s)	4

4.

Column – I		Column – II	
(A)	If $f'(x) = \sqrt{3x^2+6}$ and $y = f(x^3)$, then at $x = 1$, $\frac{dy}{dx} =$	(p)	– 2
(B)	If f be a differentiable function such that $f(xy) = f(x) + f(y)$; $x, y \in R$, then $f(e) + f(1/e) =$	(q)	–1
(C)	If f be a twice differentiable function such that $f''(x) = -f(x)$ and $f'(x) = g(x)$, If $h(x) = (f(x))^2 + (g(x))^2$ & $h(5) = 9$, then $h(10) =$	(r)	0
(D)	$y = \tan^{-1}(\cot x) + \cot^{-1}(\tan x)$, $\frac{\pi}{2} < x < \pi$, then $\frac{dy}{dx} =$	(s)	9

Part # II [Comprehension Type Questions]

Comprehension # 1

Let $\frac{f(x+y)-f(x)}{2} = \frac{f(y)-1}{2} + xy$,, $xy \in R$. f(x) is differentiable and $f'(0) = 1$. Let g(x) be a derivable function at $x = 0$ and follows the functional rule $g\left(\frac{x+y}{k}\right) = \frac{g(x)+g(y)}{k}$ ($k \in R$, $k \neq 0, 2$)

Let $g'(0) = \lambda \neq 0$

On the basis of above information, answer the following questions

1. Domain of $\ell n(f(x))$ is-

(A) R^+ **(B)** $R - \{0\}$ **(C)** R **(D)** R^-

2. Range of $y = \log_{3/4}(f(x))$

(A) $(-\infty, 1]$ **(B)** $\left[\frac{3}{4}, \infty\right)$ **(C)** $(-\infty, \infty)$ **(D)** R

3. If the graphs of y = f(x) and y = g(x) intersect in coincident points the λ can take values-

(A) 3 **(B)** 1 **(C)** –1 **(D)** 4

Comprehension # 2

Let the derivative of f(x) be defined as $D * f(x) = \underset{h \to 0}{\text{Lim}} \frac{f^2(x+h) - f^2(x)}{h}$, where $f^2(x) = \{f(x)\}^2$.

1. If u = f(x), v = g(x), then the value of D* (u . v) is

(A) (D* u) v + (D* v) u **(B)** $u^2 D^* v + v^2 D^* u$ **(C)** D*u + D* v **(D)** uvD* (u + v)

2. If u = f(x), v = g(x), then the value of $D^* \left\{\frac{u}{v}\right\}$ is

(A) $\frac{u^2 D^* v - v^2 D^* u}{v^4}$ **(B)** $\frac{uD^* v - v D^* u}{v^2}$ **(C)** $\frac{v^2 D^* u - u^2 D^* v}{v^4}$ **(D)** $\frac{v D^* u - u D^* v}{v^2}$

3. The value of D* c, where c is constant, is

(A) non-zero constant **(B)** 2 **(C)** does not exist **(D)** zero

Comprehension # 3

Left hand derivative and right hand derivative of a function f(x) at a point x = a are defined as

$$f'(a^-) = \lim_{h \to 0^+} \frac{f(a) - f(a-h)}{h} = \lim_{h \to 0^-} \frac{f(a+h) - f(a)}{h} \text{ and}$$

$$f'(a^+) = \lim_{h \to 0^+} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0^-} \frac{f(a) - f(a-h)}{h} = \lim_{x \to a^+} \frac{f(a) - f(x)}{a - x} \text{ respectively}$$

Let f be a twice differentiable function. We also know that derivative of an even function is odd function and derivative of an odd function is even function.

On the basis of above information, answer the following questions

1. If f is odd, which of the following is Left hand derivative of f at x = –a

(A) $\lim_{h \to 0^-} \frac{f(a-h) - f(a)}{-h}$ **(B)** $\lim_{h \to 0^-} \frac{f(h-a) - f(a)}{h}$

(C) $\lim_{h \to 0^+} \frac{f(a) + f(a-h)}{-h}$ **(D)** $\lim_{h \to 0^-} \frac{f(-a) - f(-a-h)}{-h}$

2. If f is even, which of the following is Right hand derivative of f' at x = a

(A) $\lim_{h\to 0^-} \frac{f'(a)+f'(-a+h)}{h}$ **(B)** $\lim_{h\to 0^+} \frac{f'(a)+f'(-a-h)}{h}$

(C) $\lim_{h\to 0_-} \frac{-f'(-a)+f'(-a-h)}{-h}$ **(D)** $\lim_{h\to 0^+} \frac{f'(a)+f'(-a+h)}{-h}$

3. The statement $\lim_{h\to 0} \frac{f(-x)-f(-x-h)}{h} = \lim_{h\to 0} \frac{f(x)-f(x-h)}{-h}$ implies that for all $x \in R$

(A) f is odd **(B)** f is even

(C) f is neither odd nor even **(D)** nothing can be concluded

Comprehension # 4

A curve is represented parametrically by the equations $x = e^t \cos t$ and $y = e^t \sin t$ where t is a parameter. Then The relation between the parameter 't' and the angle α between the tangent to the given curve and the x-axis is given by, 't' equals

(A) $\frac{\pi}{2}-\alpha$ **(B)** $\frac{\pi}{4}+\alpha$ **(C)** $\alpha-\frac{\pi}{4}$ **(D)** $\frac{\pi}{4}-\alpha$

1. The value of $\frac{d^2y}{dx^2}$ at the point where t = 0 is

(A) 1 **(B)** 2 **(C)** –2 **(D)** 3

2. If $F(t) = \int (x+y)\,dt$ then the value of $F\left(\frac{\pi}{2}\right) - F(0)$ is

(A) 1 **(B)** – 1 **(C)** $e^{\pi/2}$ **(D)** 0

Comprehension # 5

Consider the implicit equation $x^2 + 5xy + y^2 - 2x + y - 6 = 0$ ......**(i)**

1. The value of $\frac{dy}{dx}$ at (1, 1) is

(A) $\frac{5}{8}$ **(B)** $-\frac{5}{8}$ **(C)** $\frac{8}{5}$ **(D)** $-\frac{8}{5}$

2. The value of $\frac{d^2y}{dx^2}$ at (1, 1) is

(A) $\frac{111}{256}$ **(B)** $-\frac{111}{256}$ **(C)** $\frac{256}{111}$ **(D)** $-\frac{256}{111}$

3. The equation of normal to the conic **(i)** at (1, 1) is

(A) $5x-8y-3=0$ **(B)** $8y-5x-3=0$ **(C)** $8x-5y-3=0$ **(D)** $8x-5y+3=0$

Comprehension # 6

Limits that lead to the indeterminate forms 1^{∞}, 0^0, ∞^0 can sometimes be solved taking logarithm first and then using L'Hõpital's rule

Let $\lim\limits_{x \to a}(f(x))^{g(x)}$ is in the form of ∞^0, it can be written as $e^{\lim\limits_{x \to a} g(x)\ell n f(x)} = e^L$

where $L = \lim\limits_{x \to a} \frac{\ell n f(x)}{1/g(x)}$ is $\frac{\infty}{\infty}$ form and can be solved using L'Hõpital's rule.

On the basis of above information, answer the following questions :

1. $\lim\limits_{x \to 1^+} x^{1/(1-x)}$ -

(A) –1 **(B)** e^{-1} **(C)** –2 **(D)** e^{-2}

2. $\lim\limits_{x \to \infty}\left[(\ell n x)^{1/2x} + x^{1/x^n}\right] \forall\, n \in N$ -

(A) 2 **(B)** 0 **(C)** $e^{1/2}$ **(D)** e

3. $\lim\limits_{x \to 0^+}(\sin x)^{2\sin x}$

(A) 1 **(B)** 0 **(C)** 2 **(D)** does not exist

Comprehension # 7

An operator Δ is defined to operate on differentiable functions defined as follows.

If f (x) is a differentiable function then $\Delta(f(x)) = \lim\limits_{h \to 0} \frac{f^3(x+h) - f^3(x)}{h}$

g(x) is a differentiable function such that the slope of the tangent to the curve y = g(x) at any point (a, g(a)) is equal to $2e^a$ (a+1) also g(0)=0.

On the basis of above information, answer the following questions :

1. $\Delta(g(x))$ at x=ℓn2 is –

(A) $24\ell n2\ \{2\ell n2+2\}$ **(B)** $\ell n(4e^2)\ell n^2 2$ **(C)** $96\ell n(4e^2)\ell n^2 2$ **(D)** $192\ell n(4e)\ \ell n^2 2$

2. $\Delta\ (\Delta\ (x + 2))_{x=0}$

(A) $2^5 \cdot 3^9$ **(B)** $2^9 \cdot 3^5$ **(C)** $2^4 \cdot 3^5$ **(D)** $2^6 \cdot 3^4$

3. $\lim\limits_{x \to 0} \frac{\Delta g(x)}{\ell n(\cos 2x)}$

(A) –12 **(B)** 12 **(C)** 24 **(D)** –24

Exercise # 4 [Subjective Type Questions]

1. If $y = \frac{a + bx^{3/2}}{x^{5/4}}$ and $\frac{dy}{dx}$ vanishes at x = 5 then find $\frac{a}{b}$.

2. It is known for $x \neq 1$ that $1 + x + x^2 + ... + x^{n-1} = \frac{1-x^n}{1-x}$, hence find the sum of the series $S = 1 + 2x + 3x^2 + ... + (n+1)x^n$.

3. If $f'(x) = \sqrt{2x^2 - 1}$ and $y = f(x^2)$ then find $\frac{dy}{dx}$ at x = 1.

4. If $y = a^{x^{a^{x^{\cdot^{\cdot^{\infty}}}}}}$, then prove that $\frac{dy}{dx} = \frac{y^2 \log_e y}{x(1 - y\log_e x \ \log_e y)}$.

5. If $f_n(x) = e^{f_{n-1}(x)}$ for all $n \in N$ and $f_0(x) = x$ then show that $\frac{d}{dx}\{f_n(x)\} = f_1(x).f_2(x).........f_n(x)$.

6. **(i)** If $e^y(x+1) = 1$, show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$.

(ii) If $y = \sin(2\sin^{-1}x)$, show that $(1 - x^2)\frac{d^2y}{dx^2} = x\frac{dy}{dx} - 4y$.

7. If $x = \frac{1}{z}$ and $y = f(x)$, show that : $\frac{d^2f}{dx^2} = 2z^3\frac{dy}{dz} + z^4\frac{d^2y}{dz^2}$

8. If P_n is the sum of a GP upto n terms. Show that $(1 - r)\frac{dP_n}{dr} = n \cdot P_{n-1} - (n-1)P_n$.

9. Prove that if $|a_1 \sin x + a_2 \sin 2x + ... + a_n \sin nx| \le |\sin x|$ for $x \in R$, then $|a_1 + 2a_2 + 3a_3 + ... + na_n| \le 1$

10. If $f(x) = x^n$, then find the value of $f(1) + \frac{f^1(1)}{1!} + \frac{f^2(1)}{2!} + ... + \frac{f^n(1)}{n!}$, where $f^r(x)$ denotes the r^{th} derivative of f(x) w.r.t. x

11. If $y = 1 + \frac{c_1}{x - c_1} + \frac{c_2 x}{(x - c_1)(x - c_2)} + \frac{c_3 x^2}{(x - c_1)(x - c_2)(x - c_3)}$, then show that

$$\frac{dy}{dx} = \frac{y}{x}\left\{\frac{c_1}{c_1 - x} + \frac{c_2}{c_2 - x} + \frac{c_3}{c_3 - x}\right\}$$

12. Let $f(x) = x + \cfrac{1}{2x + \cfrac{1}{2x + \cfrac{1}{2x + ...\infty}}}$

Compute the value of f(100).f'(100).

13. Find a polynomial function f(x) such that f(2x) = f'(x) f''(x).

14. **B–4.** If $y = \cfrac{x}{a + \cfrac{x}{b + \cfrac{x}{a + \ldots\ldots\ldots\ldots\infty}}}$, then find $\frac{dy}{dx}$

15. If $\sqrt{x^2+y^2} = e^{\arcsin\frac{y}{\sqrt{x^2+y^2}}}$. Prove that $\frac{d^2y}{dx^2} = \frac{2(x^2+y^2)}{(x-y)^3}$, $x > 0$.

16. If $\cos\frac{x}{2} \cdot \cos\frac{x}{2^2} \cdot \cos\frac{x}{2^3} \ldots \infty = \frac{\sin x}{x}$, then find the value of $\frac{1}{2^2}\sec^2\frac{x}{2} + \frac{1}{2^4}\sec^2\frac{x}{2^2} + \frac{1}{2^6}\sec^2\frac{x}{2^3} + \ldots\infty$

17. Show that the substitution $z = \ell n\left(\tan\frac{x}{2}\right)$ changes the equation $\frac{d^2y}{dx^2} + \cot x\frac{dy}{dx} + 4y\,\text{cosec}^2\,x = 0$ to $(d^2y/dz^2) + 4y = 0$

18. Solve using L'Hospital's rule or series expansion.

(i) $\lim_{x\to 0}\frac{x\cos x - \ell n(1+x)}{x^2}$ **(ii)** If $\lim_{x\to a}\frac{a^x - x^a}{x^x - a^a} = -1$ find 'a'.

(iii) $\lim_{x\to 0}\log_{\tan^2 x}(\tan^2 2x)$

19. If $H'(1) = 1$, $g'(1) = 2$; $H(1) = 1$, $g(1) = 2$, then find $\underset{x\to 1}{\text{Limit}}\frac{H(x)\,.\,g(1) - g(x)\,.\,H(1)}{\sin(x-1)}$

20. If Y = sX and Z = tX, where all the letters denotes the function of x and suffixes denotes the differentiation w.r.t. x then prove that $\begin{vmatrix} X & Y & Z \\ X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \end{vmatrix} = X^3\begin{vmatrix} s_1 & t_1 \\ s_2 & t_2 \end{vmatrix}$

21. If $x = at^3$ and $y = bt^2$, where t is a parameter, then prove that $\frac{d^3y}{dx^3} = \frac{8\,.\,b}{27a^3\,.t^7}$

22. Differentiate

(i) $\tan^{-1}\left(\frac{1+2x}{1-2x}\right)$ w.r.t. $\sqrt{1+4x^2}$ **(ii)** $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}(x)$

23. If $x = \sec\theta - \cos\theta$; $y = \sec^n\theta - \cos^n\theta$, then show that $(x^2+4)\left(\frac{dy}{dx}\right)^2 - n^2(y^2+4) = 0$.

24. If α be a repeated root of a quadratic equation f(x) = 0 & A(x), B(x), C(x) be the polynomials of degree 3, 4 & 5 respectively, then show that $\begin{vmatrix} A(x) & B(x) & C(x) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix}$ is divisible by f(x), where dash denotes the derivative.

25. If $y = A e^{-kt} \cos(pt + c)$, then prove that $\frac{d^2 y}{dt^2} + 2k\frac{dy}{dt} + n^2 y = 0$, where $n^2 = p^2 + k^2$.

26. If $y = \tan^{-1}\frac{1}{x^2+x+1} + \tan^{-1}\frac{1}{x^2+3x+3} + \tan^{-1}\frac{1}{x^2+5x+7} + \tan^{-1}\frac{1}{x^2+7x+13} +$......... upto n terms.

Find dy/dx, expressing your answer in 2 terms.

27. **(A)** Differentiate $y = \cos^{-1}\frac{1-x^2}{1+x^2}$ w. r. t. $\tan^{-1}x$, stating clearly where function is not differentiable.

(B) If $y = \sin^{-1}(3x - 4x^3)$ find dy/dx stating clearly where the function is not derivable in $(-1,1)$.

28. If $e^{x+y} = xy$, then show that $\frac{d^2y}{dx^2} = \frac{-y\left((x-1)^2 + (y-1)^2\right)}{x^2 (y-1)^3}$.

29. If $\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix}$ are given, then prove that $\frac{d}{dx}\Delta_1 = 3\Delta_2$

30. If $f(x) = \begin{vmatrix} (x-a)^4 & (x-a)^3 & 1 \\ (x-b)^4 & (x-b)^3 & 1 \\ (x-c)^4 & (x-c)^3 & 1 \end{vmatrix}$ then $f'(x) = \lambda.\begin{vmatrix} (x-a)^4 & (x-a)^2 & 1 \\ (x-b)^4 & (x-b)^2 & 1 \\ (x-c)^4 & (x-c)^2 & 1 \end{vmatrix}$ Find the value of λ.

31. If $y = x\,\ell n((ax)^{-1} + a^{-1})$, prove that $x(x+1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = y - 1$.

32. Let $\frac{f(x+y)-f(x)}{2} = \frac{f(y)-a}{2} + xy$ for all real x and y. If f(x) is differentiable and f'(0) exists for all real permissible values of 'a' and is equal to $\sqrt{5a-1-a^2}$. Prove that f(x) is positive for all real x.

33. If $F(x) = f(x).\,g(x)$ and $f'(x).\,g'(x) = c$, prove that $\frac{F''}{F} = \frac{f''}{f} + \frac{g''}{g} + \frac{2c}{fg}$ and $\frac{F'''}{F} = \frac{f'''}{f} + \frac{g'''}{g}$.

34. Let g(x) be a polynomial, of degree one & f(x) be defined by $f(x) = \begin{cases} g(x) & , \; x \le 0 \\ \left(\frac{1+x}{2+x}\right)^{1/x} & , \; x > 0 \end{cases}$

Find the continuous function f(x) satisfying $f'(1) = f(-1)$

35. If $\lim_{x\to 0}\frac{a\sin x - bx + cx^2 + x^3}{2x^2.\ell n(1+x) - 2x^3 + x^4}$ exists & is finite, find the value of a, b, c & the limit.

Exercise # 5

Part # I [Previous Year Questions] [AIEEE/JEE-MAIN]

1. If $f(1) = 1, f'(1) = 2$, then $\lim\limits_{x\to 1}\frac{\sqrt{f(x)}-1}{\sqrt{x}-1} =$ **[AIEEE - 2002]**

(1) 2 **(2)** 1 **(3)** 3 **(4)** 4

2. $\lim\limits_{x\to\infty}\frac{\log x^n - [x]}{[x]}$, $n \in N$, (where [x] denotes greatest integer less than or equal to x)- **[AIEEE - 2002]**

(1) Has value -1 **(2)** Has values 0 **(3)** Has value 1 **(4)** Does not exist

3. If $y = \log_y x$, then $\frac{dy}{dx} =$ **[AIEEE-2002]**

(1) $\frac{1}{x+\log y}$ **(2)** $\frac{1}{\log x(1+y)}$ **(3)** $\frac{1}{x(1+\log y)}$ **(4)** $\frac{1}{y+\log x}$

4. If $x = 3\cos\theta - 2\cos^3\theta$ and $y = 3\sin\theta - 2\sin^3\theta$, then $\frac{dy}{dx} =$ **[AIEEE-2002]**

(1) $\sin\theta$ **(2)** $\cos\theta$ **(3)** $\tan\theta$ **(4)** $\cot\theta$

5. If $y = \left(x+\sqrt{1+x^2}\right)^n$ then $(1 + x^2)y_2 + xy_1 =$ **[AIEEE-2002]**

(1) ny^2 **(2)** n^2y **(3)** n^2y^2 **(4)** None of these

6. If $f(x) = x^n$, then the values of $f(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} - \frac{f'''(1)}{3!} + \ldots + \frac{(-1)^n f^n(1)}{n!}$ is- **[AIEEE-2003]**

(1) 1 **(2)** 2^n **(3)** 2^{n-1} **(4)** 0

7. Let f(x) be a polynomial function of second degree. If $f(1) = f(-1)$ and a, b, c are in A.P. then f'(a), f'(b) and f'(c) are in- **[AIEEE-2003]**

(1) Arithmetic-Geometric Progression **(2)** Arithmetic progression (A.P.)
(3) Geometric progression (G.P.) **(4)** Harmonic progression (H.P.)

8. If $x = e^{y+e^{y+\ldots\ldots to\infty}}$, $x > 0$, then $\frac{dy}{dx}$ is - **[AIEEE-2004]**

(1) $\frac{x}{1+x}$ **(2)** $\frac{1}{x}$ **(3)** $\frac{1-x}{x}$ **(4)** $\frac{1+x}{x}$

9. If $x^m.y^n = (x + y)^{m+n}$, then $\frac{dy}{dx}$ is- **[AIEEE-2006]**

(1) $\frac{x+y}{xy}$ **(2)** xy **(3)** $\frac{x}{y}$ **(4)** $\frac{y}{x}$

10. Let y be an implicit function of x defined by $x^{2x} - 2x^x \cot y - 1 = 0$. Then y'(1) equals :- **[AIEEE-2009]**

(1) log 2 **(2)** –log2 **(3)** –1 **(4)** 1

11. Let f : (–1, 1) → R be a differentiable function with f(0) = – 1 and f'(0) = 1. Let $g(x) = [f(2f(x) + 2)]^2$. Then g'(0) :- **[AIEEE-2010]**

(1) 4 **(2)** –4 **(3)** 0 **(4)** –2

12. $\frac{d^2x}{dy^2}$ equals :- **[AIEEE-2011]**

(1) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$ **(2)** $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$ **(3)** $\left(\frac{d^2y}{dx^2}\right)^{-1}$ **(4)** $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$

13. If $y = \sec(\tan^{-1}x)$, then $\frac{dy}{dx}$ at x = 1 is equal to : **[JEE-(Main)-2013]**

(1) $\frac{1}{\sqrt{2}}$ **(2)** $\frac{1}{2}$ **(3)** 1 **(4)** $\sqrt{2}$

14. If g is the inverse of a function f and $f'(x) = \frac{1}{1+x^5}$, then g' (x) is equal to : **[Main 2014]**

(1) $1 + x^5$ **(2)** $5x^4$ **(3)** $\frac{1}{1+\{g(x)\}^5}$ **(4)** $1 + \{g(x)\}^5$

Part # II [Previous Year Questions][IIT-JEE ADVANCED]

1. **(A)** If ln(x + y) = 2xy, then y'(0) = **[JEE 2004 (Scr.)]**

(A) 1 **(B)** –1 **(C)** 2 **(D)** 0

(B)
$$f(x) = \begin{cases} b\sin^{-1}\left(\frac{x+c}{2}\right), & -\frac{1}{2} < x < 0 \\ \frac{1}{2}, & \text{at } x = 0 \\ \frac{e^{ax/2}-1}{x}, & 0 < x < \frac{1}{2} \end{cases}$$

If f(x) is differentiable at x = 0 and | c | < 1/2 then find the value of 'a' and prove that $64b^2 = 4 - c^2$. **[JEE 2004]**

2.(A) If y = y(x) and it follows the relation x cos y + y cos x = π, then y"(0) :-

(A) 1 **(B)** –1 **(C)** π **(D)** –π

(B) If P(x) is a polynomial of degree less than or equal to 2 and S is the set of all such polynomials so that P(1) =1, P(0) = 0 and P'(x) > 0 ∀ x ∈ [0, 1], then :-

(A) S = ϕ **(B)** $S = (1 - a)x^2 + ax, 0 < a < 2$

(C) $(1 - a)x^2 + ax, a \in (0, \infty)$ **(D)** $S = (1 - a)x^2 + ax, 0 < a < 1$

(C) If f(x) is a continuous and differentiable function and $f(1/n) = 0$, $\forall\ n \geq 1$ and $n \in I$, then :-

(A) $f(x) = 0, x \in (0, 1]$ **(B)** $f(0) = 0, f'(0) = 0$

(C) $f'(x) = 0 = f''(x), x \in (0, 1]$ **(D)** $f(0) = 0$ and $f'(0)$ need not to be zero

[JEE 2005]

(D) If $f(x - y) = f(x) \cdot g(y) - f(y) \cdot g(x)$ and $g(x - y) = g(x) \cdot g(y) + f(x) \cdot f(y)$ for all $x, y \in R$. If right hand derivative at $x = 0$ exists for f(x). Find derivative of g(x) at $x = 0$. **[JEE 2004]**

3. For $x > 0$, $\lim_{x\to 0}((\sin x)^{1/x} + (1/x)^{\sin x})$ is :- **[JEE 2006]**

(A) 0 **(B)** –1 **(C)** 1 **(D)** 2

4. $\dfrac{d^2x}{dy^2}$ equals :- **[JEE 2007]**

(A) $\left(\dfrac{d^2y}{dx^2}\right)^{-1}$ **(B)** $-\left(\dfrac{d^2y}{dx^2}\right)^{-1}\left(\dfrac{dy}{dx}\right)^{-3}$ **(C)** $\left(\dfrac{d^2y}{dx^2}\right)\left(\dfrac{dy}{dx}\right)^{-2}$ **(D)** $-\left(\dfrac{d^2y}{dx^2}\right)\left(\dfrac{dy}{dx}\right)^{-3}$

5. (A) Let $g(x) = \ell n\, f(x)$ where f(x) is a twice differentiable positive function on $(0, \infty)$ such that $f(x + 1) = x\, f(x)$. Then for $N = 1, 2, 3$

$$g''\left(N + \frac{1}{2}\right) - g''\left(\frac{1}{2}\right) =$$

(A) $-4\left\{1 + \dfrac{1}{9} + \dfrac{1}{25} + \ldots\ldots \dfrac{1}{(2N-1)^2}\right\}$ **(B)** $4\left\{1 + \dfrac{1}{9} + \dfrac{1}{25} + \ldots\ldots \dfrac{1}{(2N-1)^2}\right\}$

(C) $-4\left\{1 + \dfrac{1}{9} + \dfrac{1}{25} + \ldots\ldots \dfrac{1}{(2N+1)^2}\right\}$ **(D)** $4\left\{1 + \dfrac{1}{9} + \dfrac{1}{25} + \ldots\ldots \dfrac{1}{(2N+1)^2}\right\}$

(B) Let f and g be real valued functions defined on interval (–1, 1) such that g''(x) is continuous, $g(0) \neq 0$, $g'(0) = 0$, $g''(0) \neq 0$, and $f(x) = g(x) \sin x$.

Statement-1 : $\lim_{x\to 0} [g(x) \cot x - g(0) \operatorname{cosec} x] = f''(0)$

Statement-2 : $f'(0) = g(0)$

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation of statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true. **[JEE 2008]**

6. If the function $f(x) = x^3 + e^{\frac{x}{2}}$ and $g(x) = f^{-1}(x)$, then the value of $g'(1)$ is **[JEE 2009]**

7. Let $f(\theta) = \sin\left(\tan^{-1}\left(\dfrac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$, where $-\dfrac{\pi}{4} < \theta < \dfrac{\pi}{4}$. Then the value of $\dfrac{d}{d(\tan\theta)}(f(\theta))$ is **[JEE 2011]**

MOCK TEST

SECTION - I : STRAIGHT OBJECTIVE TYPE

1. If $y = x^{(\ell nx)^{\ell n\, \ell n\, x}}$, then $\frac{dy}{dx}$ is equal to :

(A) $\frac{y\ell ny}{x\ell nx}(2\ell n\, \ell n\, x + 1)$ **(B)** $\frac{x\ell nx}{y\ell ny}(2\ell n\, \ell nx + 1)$ **(C)** $\frac{2y\, \ell ny}{x\ell n\, x}(\ell n\ell n\, x + 1)$ **(D)** None of these

2. Let $y = e^{2x}$. Then $\left(\frac{d^2y}{dx^2}\right)\left(\frac{d^2x}{dy^2}\right)$ is:

(A) 1 **(B)** e^{-2x} **(C)** $2e^{-2x}$ **(D)** $-2e^{-2x}$

3. If $x = \text{cosec}\theta - \sin\theta$; $y = \text{cosec}^n\theta - \sin^n\theta$, then $(x^2 + 4)\left(\frac{dy}{dx}\right)^2 - n^2y^2$ equals to

(A) n^2 **(B)** $2n^2$ **(C)** $3n^2$ **(D)** $4n^2$

4. If $y^2 = P(x)$, where P(x) is a polynomial of degree 3, then $2\left(\frac{d}{dx}\right)\left(y^3 . \frac{d^2y}{dx^2}\right)$ equals:

(A) $P'''(x) + P'(x)$ **(B)** $P''(x). P'''(x)$ **(C)** $P(x). P'''(x)$ **(D)** constant

5. Let g(x) be the inverse of an invertible function f(x) which is differentiable for all real x, then $g''(f(x))$ equals to

(A) $\frac{-f''(x)}{(f'(x))^3}$ **(B)** $\frac{f'(x)f''(x) - (f'(x))^3}{f'(x)}$ **(C)** $\frac{f'(x)f''(x) - (f'(x))^2}{(f'(x))^2}$ **(D)** None of these

6. If $f''(x) = -f(x)$ and $g(x) = f'(x)$ and $F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2$ and given that $F(5) = 5$, then $F(10)$ is equal to

(A) 5 **(B)** 10 **(C)** 0 **(D)** 15

7. If $x^y . y^x = 1$, then $\frac{dy}{dx}$ equals to

(A) $\frac{y(x\ell n\, y - 1)}{x(y\ell n\, x - y)}$ **(B)** $\frac{y(x\ell n\, y - y)}{x(y\ell n\, x + x)}$ **(C)** $\frac{y(x\ell n\, y + y)}{x(y\ell n\, x - x)}$ **(D)** $\frac{-y(x\, \ell n\, y + y)}{x(y\ell n\, x + x)}$

8. If $f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$ (where p is constant), then at $x = 0$, $\frac{d^3f(x)}{dx^3}$ is equal to–

(A) p **(B)** p^2 **(C)** $-p$ **(D)** 0

9. If $y = \ell n\left(\frac{x}{a+bx}\right)^x$, then $x^3 \frac{d^2y}{dx^2}$ is equals to

(A) $\left(x\frac{dy}{dx} - y\right)^2$ **(B)** $\left(y\frac{dy}{dx} - x\right)^2$ **(C)** $\left(x\frac{dy}{dx} + y\right)^2$ **(D)** None of these

10. S_1 : If $y = \sin 2x$, then $\frac{d^6y}{dx^6}$ at $x = \frac{\pi}{2}$ is equal to 1

S_2 : If $x = e^{y+e^{y+....\infty}}$, then $\frac{dy}{dx}$ at $x = 1$ is 0

S_3 : If $y = 2t^2, x = 4t$, then $\frac{d^2y}{dx^2}$ at $x = \frac{1}{2}$ is $\frac{1}{2}$

S_4 : If $x = t^2 + 3t - 8, y = 2t^2 - 2t - 5$, then $\frac{dy}{dx}$ at $(2, -1)$ is $\frac{16}{7^3}$

(A) FFFT **(B)** FTFT **(C)** FTTF **(D)** TTTF

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

11. If $y = \sec(\tan^{-1} x)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to :

(A) $\frac{1}{\sqrt{2}}$ **(B)** $\sin^{-1}\left(\sin\frac{1}{\sqrt{2}}\right)$ **(C)** 1 **(D)** none of these

12. If $y = \tan^{-1}\left(\frac{2^x}{1+2^{2x+1}}\right)$, then $\frac{dy}{dx}$ at $x = 0$ is

(A) 1 **(B)** 2 **(C)** ln 2 **(D)** none of these

13. If $y = 10^{10^x}$ and $\frac{1}{y}\frac{dy}{dx} = 10^x . \lambda$, then value of λ is–

(A) $\ell n\ 10$ **(B)** $(\ell n\ 10)^2$ **(C)** $e^{\ell n(\ell n 10)^2}$ **(D)** $(\log_{10} e)^2$

14. **S_1 :** If $f(x) = [x]$, then $f\left(\left|f\left(\frac{1}{2}\right)\right|\right) = 0$ **S_2 :** If $f(x) = \frac{1}{\sin|x|}$, then $f'(n\pi) = 0$

S_3 : If $f(x) = \log|\sin x|$, then $f'(x) < 0\ \forall\ x \in \left(\frac{\pi}{2}, \pi\right)$ **S_4 :** $f(x) = e^{|\sin x|}$ is differentiable everywhere

(A) FFFT **(B)** TFTF **(C)** FTTF **(D)** TTTF

15. Given $f(x) = -\frac{x^3}{3} + x^2 \sin 1.5a - x \sin a . \sin 2a - 5\sin^{-1}(a^2 - 8a + 17)$ then:

(A) $f'(x) = -x^2 + 2x \sin 6 - \sin 4 \sin 8$ **(B)** $f'(\sin 8) > 0$

(C) $f'(x)$ is not defined at $x = \sin 8$ **(D)** $f'(\sin 8) < 0$

SECTION - III : ASSERTION AND REASON TYPE

16. **Statement-I :** Let f(x) is a continuous function defined from R to Q and f(5) = 3 then differential coefficient of f(x) w.r.t. x will be 0.

Statement-II : Differentiation of constant function is always zero.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

17. **Statement-I :** For $x < 0$, $\frac{d}{dx}(\ell n|x|) = \frac{1}{x}$.

Statement-II : For $x < 0, |x| = -x \Rightarrow \frac{d}{dx}|x| = -1$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

18. Consider $f(x) = \frac{x}{x^2 - 1}$ & $g(x) = f''(x)$.

Statement-I : Graph of g(x) is concave up for $x > 1$.

Statement-II : $\frac{d^n}{dx^n}(f(x)) = \frac{(-1)^n n!}{2}\left\{\frac{1}{(x+1)^{n+1}} + \frac{1}{(x-1)^{n+1}}\right\}, n \in N$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

19. **Statement-I :** $\frac{d}{dx}\{\tan^{-1}(\sec x + \tan x)\} = \frac{d}{dx}\{\cot^{-1}(\operatorname{cosec} x + \cot x)\}, x \in \left(0, \frac{\pi}{4}\right)$.

Statement-II : $\sec^2 x - \tan^2 x = 1 = \operatorname{cosec}^2 x - \cot^2 x$.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

20.. **Statement-I :** Let $f : [0, \infty) \to [0, \infty)$ be a function defined by $y = f(x) = x^2$, then $\left(\frac{d^2y}{dx^2}\right)\left(\frac{d^2x}{dy^2}\right) = 1$.

Statement-II : $\frac{d^2y}{dx^2} = -\frac{d^2x}{dy^2} \cdot \left(\frac{dy}{dx}\right)^3$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

SECTION - IV : MATRIX - MATCH TYPE

21.

Column – I		Column – II	
(A)	$\lim\limits_{x\to\frac{1}{\sqrt{2}}} \dfrac{x-\cos(\sin^{-1}x)}{1-\tan(\sin^{-1}x)}$ is equal to	**(p)**	does not exist
(B)	If $f(x) = \log_{x^2}(\log x)$, then $f'\left(\dfrac{1}{2}\right)$ is equal to	**(q)**	$-\dfrac{1}{\sqrt{2}}$
(C)	For the function $f(x) = \ell n \tan\left(\dfrac{\pi}{4}+\dfrac{x}{2}\right)$ if $\dfrac{dy}{dx} = \sec x + p$, then p is equal to	**(r)**	28
(D)	$\lim\limits_{x\to 0} \dfrac{1}{x}\sqrt{\dfrac{1-\cos 2x}{1+\cos 2x}}$ is equal to	**(s)**	1
		(t)	0

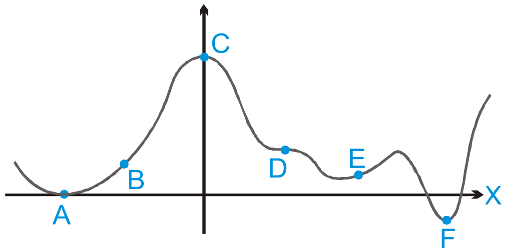
22.

Column – I		Column – II	
(A)	If $y = \cos^{-1}(\cos x)$, then y' at $x = 5$ is equal to	**(p)**	does not exist
(B)	For the function $f(x) = \ell n\,\lvert\tan x\rvert$ $f'\left(-\dfrac{\pi}{4}\right)$ is equal to	**(q)**	2
(C)	The derivative of $\tan^{-1}\left(\dfrac{1+x}{1-x}\right)$ at $x = -1$ is	**(r)**	$\dfrac{1}{2}$
(D)	The derivative of $\dfrac{\log\lvert x\rvert}{x}$ at $x = -1$ is	**(s)**	1
		(t)	–1

SECTION - V : COMPREHENSION TYPE

23. **Read the following comprehension carefully and answer the questions.**

The graph of $y = f(x)$ is given with six labelled points. Out of these points answer the following questions.



1. The point which has the greatest value of $f'(x)$ is

(A) B **(B)** C **(C)** D **(D)** E

2. The point where f′ and f″ are non-zero and of the same sign are

(A) B & D **(B)** D & E **(C)** B & E **(D)** None of these

3. The points where atleast two of f, f′ and f″ are zero

(A) C & D **(B)** A and D **(C)** A & F **(D)** None of these

24. Read the following comprehension carefully and answer the questions.

In certain problem the differentiation of {f(x) . g(x)} appears. One student commits mistake and differentiates as $\frac{df}{dx} \cdot \frac{dg}{dx}$ but he gets correct result if $f(x) = x^3$ and $g(0) = \frac{1}{3}$.

1. The function g(x) is

(A) $\frac{3}{|x-3|^3}$ **(B)** $\frac{4}{|x-3|^3}$ **(C)** $\frac{9}{|x-3|^3}$ **(D)** $\frac{27}{|x-3|^3}$

2. Derivative of {f(x – 3) . g(x)} with respect to x at x = 100 is

(A) 0 **(B)** 1 **(C)** – 1 **(D)** 2

3. $\lim_{x \to 0} \frac{f(x).g(x)}{x(1+g(x))}$ will be

(A) 0 **(B)** – 1 **(C)** 1 **(D)** 2

25. Read the following comprehension carefully and answer the questions.

Let $f(x) = \frac{1}{1+x^2}$. Let m be the slope, a be the x-intercept and b be the y-intercept of a tangent to y = f(x), then

1. Abscissa of the point of contact of the tangent for which m is greatest

(A) $\frac{1}{\sqrt{3}}$ **(B)** 1 **(C)** – 1 **(D)** $-\frac{1}{\sqrt{3}}$

2. The greatest value of b is-

(A) $\frac{9}{8}$ **(B)** $\frac{3}{8}$ **(C)** $\frac{1}{8}$ **(D)** $\frac{5}{8}$

3. The abscissa of the point of contact of tangent for which $\frac{1}{a}$ is greatest, is-

(A) $\frac{1}{\sqrt{3}}$ **(B)** 1 **(C)** – 1 **(D)** $-\frac{1}{\sqrt{3}}$

SECTION - VI : INTEGER TYPE

26. If $y = \tan^{-1}\frac{4x}{1+5x^2} + \tan^{-1}\frac{2+3x}{3-2x}$, where $0 < x < \frac{2}{3}$ and $\frac{dy}{dx} = \frac{\lambda}{1+25x^2}$, then find λ

27. If $y = \left[\ell n\left(\frac{x+\sqrt{x^2-a^2}}{a}\right)\right]^2 + k\,\ell n\,(x+\sqrt{x^2-a^2})$, then find the value of $(x^2 - a^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx}$

28. If $y = \tan^{-1}\left(\frac{\ell n(e/x^2)}{\ell n(ex^2)}\right) + \tan^{-1}\left(\frac{3+2\ell nx}{1-6\ell nx}\right)$, then find $\frac{d^2y}{dx^2}$.

29. If $y = \frac{2}{\sqrt{a^2-b^2}}\tan^{-1}\left(\frac{\sqrt{a-b}}{\sqrt{a+b}}\tan\frac{x}{2}\right)$, then find $\frac{dy}{dx}$, at $a = 1, b = 1, x = 0$.

30. The function $y = f(x)$ defined by the parametric equations $x = e^t \sin t$, $y = e^t \cos t$ satisfies the relation $y''(x + y)^2 = \lambda(xy' - y)$, then find λ

ANSWER KEY

EXERCISE - 1

1. A **2.** B **3.** B **4.** D **5.** B **6.** B **7.** D **8.** C **9.** C **10.** B **11.** B **12.** C **13.** B
14. D **15.** C **16.** B **17.** D **18.** C **19.** B **20.** C **21.** C **22.** D **23.** B **24.** D **25.** C **26.** D
27. C **28.** D **29.** D **30.** D **31.** A **32.** B **33.** B **34.** B **35.** B

EXERCISE - 2 : PART # I

1. AB, **2.** AD **3.** ABC **4.** ABC **5.** BC **6.** ABC **7.** BCD **8.** AC **9.** AC
10. ABCD **11.** ABCD **12.** AC **13.** ACD **14.** CD **15.** AB **16.** ABCD **17.** ACD **18.** BC
19. BD **20.** ABC

PART - II

1. A **2.** D **3.** A **4.** C **5.** B **6.** C **7.** A

EXERCISE - 3 : PART # I

1. $A \to q$ $B \to s$ $C \to p$ $D \to r$ **2.** $A \to p$ $B \to q$ $C \to r$ $D \to s$ **3.** $A \to s$ $B \to q$ $C \to q,s$ $D \to s$
4. $A \to s$ $B \to r$ $C \to s$ $D \to p$

PART - II

Comprehension # 1 : **1.** C **2.** A **3.** A, C
Comprehension # 2 : **1.** B **2.** C **3.** D
Comprehension # 3 : **1.** A **2.** A **3.** B
Comprehension # 4 : **1.** C **2.** B **3.** C
Comprehension # 5 : **1.** B **2.** A **3.** C
Comprehension # 6 : **1.** B **2.** A **3.** A
Comprehension # 7 : **1.** C **2.** D **3.** A

EXERCISE - 5 : PART # I

1. 1 **2.** 1 **3.** 3 **4.** 4 **5.** 2 **6.** 4 **7.** 2 **8.** 3 **9.** 4 **10.** 3 **11.** 2 **12.** 2 **13.** 1
14. 4

PART - II

1. **(A)** A **(B)** $a = 1$ **2.** **(A)** C **(B)** B **(C)** B **(D)** $g'(0) = 0$ **3.** C **4.** D **5.** **(A)** A **(B)** A **6.** 2
7. 1

MOCK TEST

1. A **2.** D **3.** D **4.** C **5.** A **6.** A **7.** D **8.** D **9.** A **10.** B **11.** A, B **12.** D **13.** B, C
14. B **15.** A, D **16.** A **17.** A **18.** A **19.** B **20.** D
21. $A \to q$ $B \to p$ $C \to t$ $D \to p$ **22.** $A \to t$ $B \to q$ $C \to r$ $D \to s$ **23.** **1.** B **2.** C **3.** C **24.** **1.** C
2. A **3.** A **25.** **1.** D **2.** A **3.** A **26.** 5 **27.** 2 **28.** 0 **29.** 1/2 **30.** 2