# Probability

1. **Experiments**

An operation which can produce some well-defined outcomes, is known as experiment. There are two types of experiments, *viz.*

(i) **Deterministic experiments** An experiment which gives the same result every time, when it is repeated under identical conditions, is called deterministic experiments.

(ii) **Random experiments** In our daily life, we perform many experimental activities, where the result may not be same, when they are repeated under identical conditions.

2. **Outcomes and Sample Space**

A possible result of a random experiment is called its outcome. The set of all possible outcomes in a random experiment is called a sample space and it is generally denoted by $S$.

i.e., sample space $(S) = \{\text{All possible outcomes}\}$

Each element of a sample space is called a sample point or an event point.

3. **Events**

An event is a subset of a sample space associated with a random experiment.

4. **Types of Events**

(i) **Impossible and sure events** The empty set $\phi$ and the sample space $S$ describes events. Infact $\phi$ is called the impossible event and $S$ i.e. whole sample space is called sure event.

(ii) **Simple or elementary event** Each outcome of a random experiment is called an elementary event.

(iii) **Compound events** If an event has more than one outcome is called compound event.

(iv) **Equally likely events** Events are said to be equally likely, if none of them is expected to occur in preference to the other.

(v) **Independent events** Two events $A$ and $B$ are said to be independent, if the occurrence of one of them does not depend upon the occurrence of the other.

(vi) **Mutually exclusive events** Two events $A$ and $B$ of a sample space $S$ are mutually exclusive, if the occurrence of any one of them excludes the occurrence of the other event. Hence, the two events $A$ and $B$ cannot occur simultaneously and thus $P(A \cap B) = 0$.

(vii) **Exhaustive events** For a random experiment, a set of events is said to be exhaustive, if one of them necessarily occurs whenever the experiment is performed.

If $E_1, E_2, \ldots, E_n$ are $n$ events of a sample space $S$ and if $E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_n = S$,

then $E_1, E_2, \ldots, E_n$ are called exhaustive events.

(viii) **Mutually exclusive and exhaustive events** If $E_1, E_2, \ldots, E_n$ are $n$ events of a sample space $S$ and if $E_i \cap E_j = \phi$ for every $i \neq j$ i.e. $E_i$ and $E_j$ are pairwise disjoint and $E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_n = S$, then the events are called mutually exclusive and exhaustive events.

5. **Algebra of Events**

(i) $E'$ or not $E$, is called the **complementary event** to $E$. i.e. Not $E = E' = S - E$.

(ii) Let $E_1$ and $E_2$ be two events associated with $S$, then

(a) the event either $E_1$ or $E_2$ or both $= E_1 \cup E_2$.

(b) the event $E_1$ and $E_2 = E_1 \cap E_2$.

(c) the event $E_1$ but not $E_2 = E_1 - E_2$ or $E_1 \cap E_2'$.

## 6. Axiomatic Approach to Probability

Let $S$ be the sample space of a random experiment, then probability $P$ is a real valued function whose domain is the power set of $S$ and range in the interval [0, 1] satisfying the following axioms

(i) For any event $E$, $P(E) \geq 0$ (ii) $P(S) = 1$.

(iii) If $E$ and $F$ are mutually exclusive events, then
$$P(E \cup F) = P(E) + P(F)$$

## 7. Probability of an Event

If there are $n$ elementary events associated with a random experiment and $m$ of them are favourable to an event $A$, then the probability of occurrence of $A$ is defined as

$$P(A) = \frac{m}{n} = \frac{\text{Favourable number of outcomes}}{\text{Total number of outcomes}}$$

**Note** *If an event $E$ occurs in $m$ ways and not occur in $n$ ways, then we say that*

*(i) odds in favour of event $E = \frac{m}{n}$ or $m : n$*

*(ii) odds against the event $E = \frac{n}{m}$ or $n : m$*

*(iii) $P(E) = \frac{m}{m+n}$ and $P(\text{not } E) = \frac{n}{m+n}$*

*(iv) $P(E) + P(\text{not } E) = 1$*

## 8. Addition Rule of Probabilities

If $A$ and $B$ are two events associated with a random experiment, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Similarly, for three events $A$, $B$ and $C$, we have

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

**Note** *If $A$ and $B$ are mutually exclusive events, then $P(A \cup B) = P(A) + P(B)$*

$\hookrightarrow P(A \cap B) = 0$

## 9. Important Results

(i) For any event $A$, $0 \leq P(A) \leq 1$

(ii) For any two events $A$ and $B$, $A \subseteq B \Rightarrow P(A) \leq P(B)$

(iii) For any two events $A$ and $B$,

$P(A - B) = P(A \cap \bar{B}) = P(A) - P(A \cap B)$

$P(B - A) = P(B \cap \bar{A}) = P(B) - P(A \cap B)$

(iv) $P(\bar{E}) = 1 - P(E)$