Electromagnetic Induction

Whenever the magnetic flux linked with an electric circuit changes, an emf is induced in the circuit. This phenomenon is called **electromagnetic induction.**

Magnetic Flux

The total number of magnetic field lines crossing through any surface normally, when it is placed in a magnetic field is known as magnetic flux of that surface.

$$d\phi = \mathbf{B} \cdot d\mathbf{s} = Bds \cos\theta$$

Its SI unit is tesla-metre (or weber)

$$\text{CGS unit of } \phi = \text{maxwell, 1 weber} = 10^8 \text{ maxwell},$$

Dimensional formula of magnetic flux

$$[\phi] = [ML^2 T^{-2} A^{-1}]$$

Faraday's Laws of Electromagnetic Induction

(i) Whenever the magnetic flux linked with a circuit changes, an induced emf is produced in it.

(ii) The induced emf lasts, so long as the change in magnetic flux continues.

(iii) The magnitude of induced emf is directly proportional to the rate of change in magnetic flux, *i.e.*

$$e \propto \frac{d\phi}{dt} \Rightarrow e = -\frac{d\phi}{dt}$$

where, constant of proportionality is one and negative sign indicates that the induced emf in the circuit due to the changing flux always opposes the change in magnetic flux.

Induced current is given as

$$I = \frac{1}{R} \cdot \left(\frac{-d\phi}{dt} \right)$$

If induced current is produced in a coil rotated in uniform magnetic field, then

$$I = \frac{NBA\,\omega \sin \omega t}{R} = I_0 \sin \omega t$$

where, $I_0 = NBA\omega$ = peak value of induced current,
N = number of turns in the coil ,
B = magnetic induction,
ω = angular velocity of rotation and
A = area of cross-section of the coil.

Induced charge is given as $q = \frac{1}{R}(d\phi)$.

Lenz's Law

The direction of induced emf or induced current is always in such a way, that it opposes the cause due to which it is produced.

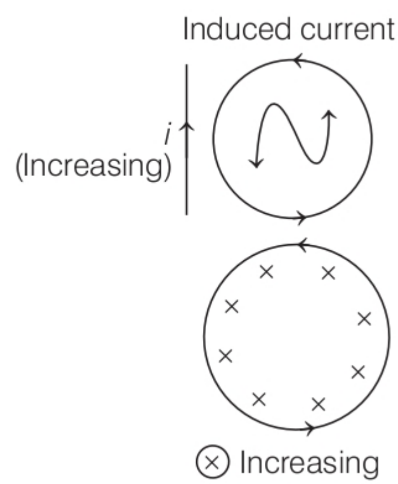
Lenz's law is in accordance with the conservation of energy.

Direction of the induced current can be determined as

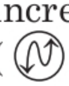
(i) if flux is decreasing, the magnetic field due to induced current will be along the existing magnetic field.

(ii) if flux is increasing, the magnetic field due to induced current will be opposite to existing magnetic field.

Also to apply Lenz's law, you can remember RIN or ⊗ In (when the loop lies on the plane of paper), where



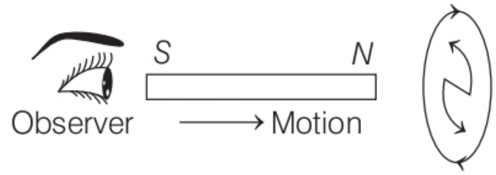
(i) **RIN** In RIN, R stands for right, I stands for increasing and N for north pole (anti-clockwise). It means, if a loop is placed on the right side of a straight current-carrying conductor and the current in the conductor is increasing, then induced current in the loop is anti-clockwise ().

(ii) ⊗ **IN** In ⊗ IN, suppose the magnetic field in the loop is perpendicular to paper inwards ⊗ and this field is increasing, then induced current in the loop is anti-clockwise ().

Direction of Induced Current in Coil or Ring by Moving Bar Magnet

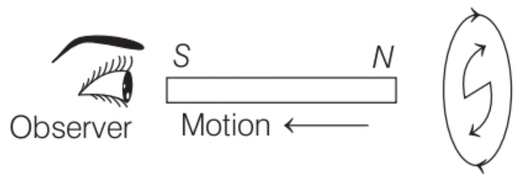
The following are some important points that will explain the direction of induced current according to Lenz's law.

(i) When north pole moves towards ring, then flux will increase, induced current will oppose this, so north pole will be formed in loop as seen by observer.



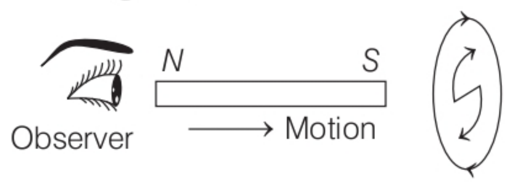
Induced current will be anti-clockwise.

(ii) When north pole moves away from ring, then flux will decrease, induced current will oppose this, so south pole will be formed in loop as seen by observer.

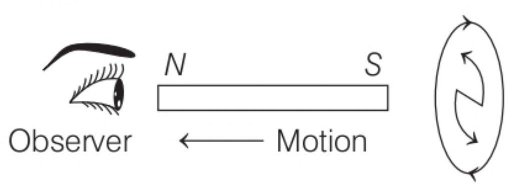


Induced current will be clockwise.

(iii) Similar observations (as in case (a) can be observed when south pole moves towards ring. So, induced current will be clockwise here.



(iv) Similar observations (as in case (b) can be observed when south pole moves away from the ring. So, induced current in this case will be anti-clockwise.



Motional Emf

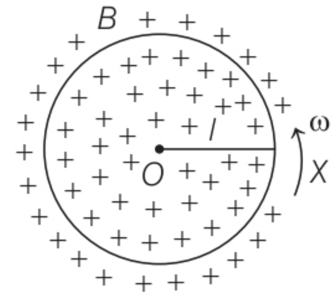
If a rod of length l moves perpendicular to a magnetic field B, with a velocity v, then induced emf produced in it given by $e = Bvl$

If a rectangular coil moves linearly in a field, when coil moves with constant velocity in an uniform magnetic field, flux and induced emf will be zero.

A rod moves at an angle θ with the direction of magnetic field, with velocity v , then

$$e = -Blv \sin\theta$$

If a metallic rod of length l rotates about one of its ends in a plane perpendicular to the magnetic field, then the induced emf produced across, its ends is given by



$$e = \frac{1}{2} B\omega l^2 = BAf$$

where, ω = angular frequency of rotation,

$A = \pi l^2$ = area of circle

and f = frequency of rotation.

If a metallic disc of radius r rotates about its own centre in a plane perpendicular to the magnetic field B, then the induced emf produced between the centre and the edge is given by $e = \frac{1}{2} B\omega r^2 = BAf$

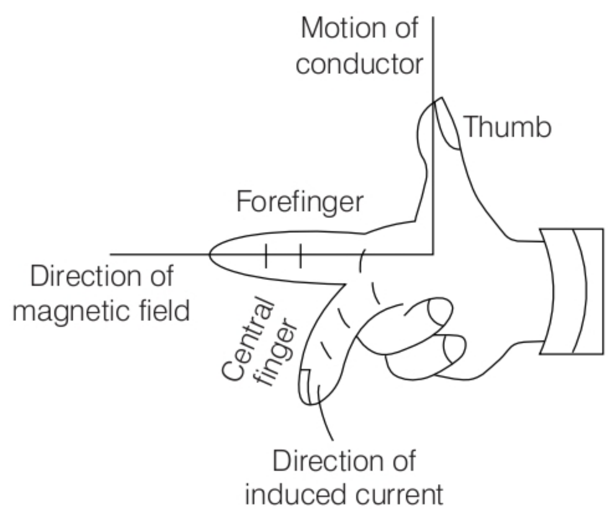
where, ω = angular velocity of rotation,

f = frequency of rotation and $A = \pi r^2$ = area of disc.

The direction of induced current in any conductor can be obtained from Fleming's right hand rule.

Fleming's Right Hand Rule

If we stretch the thumb, the forefinger and the central finger of right hand in such a way that all three are perpendicular to each other, then if thumb represent the direction of motion, the forefinger represent the direction of magnetic field, then central finger will represent the direction of induced current.



Note Integral form of Faraday's law of electromagnetic induction is $\oint \mathbf{F} \cdot d\mathbf{l} = \frac{-d\phi}{dt}$

Eddy Currents

If a piece of metal is placed in a varying magnetic field or rotated with high speed in an uniform magnetic field, then induced currents set up in the piece are like whirlpool of air, called eddy currents.

The magnitude of eddy currents is given by $I = -\frac{e}{R} = \frac{d\phi / dt}{R}$

where, R is the resistance.

Eddy currents are also known as Foucault's currents.

Eddy currents causes unnecessary heating and wastage of power. The heat, thus produced may even damage the insulation of coils of dynamos and generators.

Eddy currents can be reduced by laminations of metal to make a metal core.

Self-Induction

The phenomena of production of induced emf in a circuit due to change in current flowing in its own, is called self-induction.

The magnetic flux linked with a coil, $\phi = LI$

where, $L =$ coefficient of self-induction.

The induced emf in the coil, $e = -L\frac{dI}{dt}$

SI unit of self-induction is henry (H) and its dimensional formula is $[ML^2 T^{-2} A^{-2}]$.

- Self-inductance of a **long solenoid** is given by normal text,

$$L = \frac{\mu_0 N^2 A}{l} = \mu_0 n^2 Al$$

where, $N =$ total number of turns in the solenoid,

$l =$ length of the coil,

$n =$ number of turns in the coil

and $A =$ area of cross-section of the coil.

- If core of the solenoid is of any other magnetic material, then

$$L = \frac{\mu_0 \mu_r N^2 A}{l}$$

- Self-inductance of a **toroid**, $L = \frac{\mu_0 N^2 A}{2\pi r}$

  where, $r =$ radius of the toroid.

- Energy stored in an inductor, $E = \frac{1}{2} LI^2$.

Mutual Induction

The phenomena of production of induced emf in a circuit due to the change in magnetic flux in its neighbouring circuit, is called mutual induction.

If two coils are coupled with each other, then magnetic flux linked with a coil (secondary coil)

$$\phi = MI$$

where, M is coefficient of mutual induction and I is current flowing through primary coil.

The induced emf in the secondary coil, $e = -M\dfrac{dI}{dt}$

where, $\dfrac{dI}{dt}$ is the rate of change of current through primary coil.

The unit of coefficient of mutual induction is henry (H) and its dimensional formula is $[ML^2T^{-2}A^{-2}]$.

The coefficient of mutual induction depends on geometry of two coils, distance between them and orientation of the two coils.

Mutual inductance of **two long co-axial solenoids** is given by,

$$M = \frac{\mu_0 N_1 N_2 A}{l} = \mu_0 n_1 n_2 A l$$

where, N_1 and N_2 are total number of turns in both coils, n_1 and n_2 are number of turns per unit length in coils, A is area of cross-section of coils and l is length of the coils.

Coefficient of Coupling

Coefficient of coupling of two coils gives a measure of the manner in which the two coils are coupled together.

$$K = \sqrt{\frac{M}{L_1 L_2}}$$

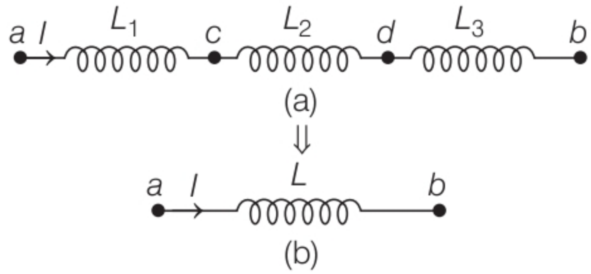
where, L_1 and L_2 are coefficients of self-induction of the two coils and M is coefficient of mutual induction of the two coils.

Co-efficient of coupling is maximum ($K = 1$) in case (a), when coils are co-axial and minimum in case (b), when coils are placed a right angles.

Grouping of Coils

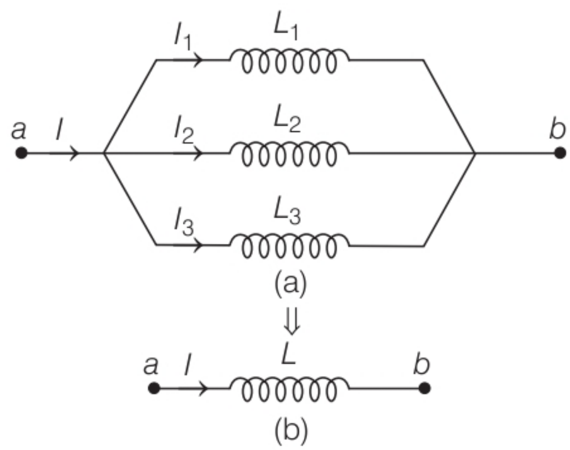
(a) When three coils of inductances L_1, L_2 and L_3 are connected in series and the coefficient of coupling $K = 0$ as in series, then

$$L = L_1 + L_2 + L_3$$



(b) When three coils of inductances L_1, L_2 and L_3 are connected in parallel and the coefficient of coupling $K = 0$ as in parallel, then

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$



If coefficient of coupling $K = 1$, then

(i) **In series**

(a) If current in two coils are in the same direction, then

$$L = L_1 + L_2 + 2M$$

(b) If current in two coils are in opposite directions, then

$$L = L_1 + L_2 - 2M$$

(ii) **In parallel**

(a) If current in two coils are in same direction, then

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

(b) If current in two coils are in opposite directions, then

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$