HINTS & SOLUTIONS

EXERCISE - 1

Single Choice

1. $U = \frac{1}{2}LI^2$

$P = I^2R$ or $\frac{2U}{P} = \frac{L}{R} = \tau.$

3. $L_{eff} = 2H$

Energy stored in inductor $= \frac{1}{2}LI^2 = \frac{1}{2} \times (2) \times (1)^2 = 1J.$

Energy developed in resistance $= I^2RT = 1^2 \times 10 \times 10$
$= 100\ J$

Hence the required ratio is $\frac{1}{100}$.

5. In the loop containing wire AB the flow of current will be from B to A because emf generated in that loop is less than the emf generated in the loop containing CD.

7. $I = I_1 + I_2$

$I_1 = E/R$

$L\frac{dI}{dt} = E. \quad I_2 = \frac{Et}{L}$

$I = E/R + \frac{Et}{L}$

$I = 12A.$

9. $P = F.V \quad = Bi\ell V = B\left(\frac{Bv\ell}{R}\right)\ell V, P \propto V^2$

11. Since the magnitude flux in the ring due to motion of charge particle is zero hence the induced emf will be zero.

13. $E = \frac{1}{2}LI^2$

$E = \frac{1}{2}L\frac{V^2}{R^2} = \frac{1}{2} \times 5 \times 10^{-3} \times (1)^2 = 2.5\ mJ.$

15. Since the tube is very long the force on magnet due to induced current will continue to oppose its motion till it acquires a constant speed.

17. Flux through a closed circuit containing an inductor does not change instantaneously.

$\therefore \quad L\left(\frac{E}{R}\right) = \frac{L}{4}(i) \Rightarrow i = \frac{4E}{R}$ **Ans.**

19. (B) Using ; $V_A - V_B = RI + L\frac{dI}{dt}$

$140 = 5R + 10\ L$

$60 = 5R - 10\ L$

$\Rightarrow \quad L = 4H.$ **Ans.**

21. Since the other resistance is attached parallel to the battery hence the time constant of the circuit will be $\frac{L}{R}$. At steady State the inductor offers zero resistance have hence at that time current in inductor will be $\frac{E}{R}$.

23. Magnetic lines of force do noy pass inside a super conducting loop

hence $\quad \varepsilon = 0$

$\frac{d\phi}{dt} = 0$

or $\quad \phi = \text{constant}.$

25. EMF induced $= -L\frac{di}{dt} \neq 0$, rest quantities are zero.

27. $U = \frac{1}{2}LI^2$

$\frac{dU}{dt} = LI\frac{dI}{dt} = RI_0^2(1 - e^{-t/\tau})e^{-t/\tau}$

$\frac{dU}{dt}$ is maximum when $e^{-t/\tau} = \frac{1}{2}$

or $\left(\frac{dU}{dt}\right)_{max} = \frac{E^2}{4R} = 1\ W.$

therefore the current at that instant is $\frac{E}{2R} = 1A$

EXERCISE - 2

Part # I : Multiple Choice

2. Since $\Delta\phi = 0$ hence EMF induced is zero.

3. The direction of current in the loop such that it opposes the the change in magnetic flux in it.

5. Since the magnetic flux in the loop is zero hence the current induced in it is zero.

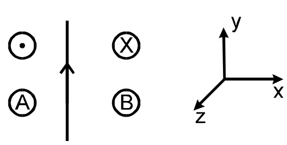
8. By moving away from solenoid the ring will resist the changing flux in it.

6. The repulsion is to resist the increasing magnetic flux in coil B.

11. The decrease in current in to oppose increasing magnetic flux in the circular loops.

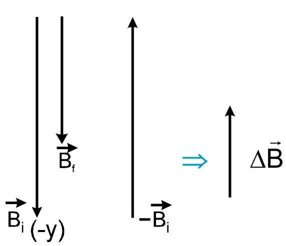
10. When the magnetic goes away from the ring the flux in the ring decreases hence the induced current will be such that it opposes the decreasing flux in it hence ring will behave like a magnet having face A as north pole and face B as south pole.

13. The field at A and B are out of the paper and inside the paper respectively.



As the current in the straight wire decreases the field also decreases.

For B :



The change in the magnetic field which causes induced current $(\Delta\vec{B})$ is along (+)z direction.

Hence, induced emf and hence current should be such as to oppose this change $\Delta\vec{B}$.

Hence, induced emf should be along – z direction which results in a clockwise current in 'B'. Similarly, there will be anticlockwise current in 'A'. Hence (B).

15. If $\vec{\upsilon} \,||\, \vec{\ell}$ or $\vec{\upsilon} \,||\, \vec{B}$ or $\vec{\ell} \,||\, \vec{B}$ then $\frac{d\phi}{dt}$ is zero. Hence potential difference is zero.

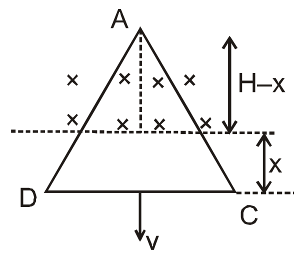
19. $\varepsilon = \vec{B}.(\vec{V} \times \vec{\ell})$

$= (3\hat{i} + 4\hat{j} + 5\hat{k}).[1\hat{i} \times 5\hat{j}]$

$\varepsilon = 25$ volt.

21. $\phi = -\frac{1}{2}(2)\frac{H-I}{\sqrt{3}}(H-X)$

$|-d\phi/dt| = \varepsilon = \frac{2(H-x)}{\sqrt{3}}$



$i = \frac{2}{\sqrt{3}R}(H–x)$

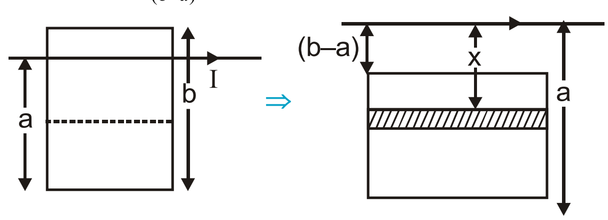
Hence answer is **(B)**

23. Force acting on the rod because of the induced current due to change in magnetic flux will try to oppose the motion of rod. Hence the acceleration of the rod will decrease with time $\frac{dP}{dt} = F\frac{dv}{dt} = F \times a$. Thus, rate of power delivered by external force will be decreasing continuously.

25. It the magnitude of I_A is very large such that force due to magnetic field on PQ exceeds its weight then it will move upwards otherwise it will move downwards.

24. $\int d\varphi = \int \frac{\mu_0 I}{2\pi x}(bdx)$

$\phi = \frac{\mu_0 Ib}{2\pi}\int_{(b-a)}^{a}\frac{dx}{x}$



$\phi = \frac{\mu_0 Ib}{2\pi}\ell n\left(\frac{a}{b-a}\right).$

28. Since there is no magnetic flux change due to rotation of rod hence the potential difference between two ends of the rod is zero.

30. Considering pure rolling of OA about A: the induced emf across OA will be:

$$|\vec{e}| = \frac{B\omega(r)^2}{2}.$$

From Lenz law, O will be the negative end, while A will be the positive end.

Hence $v_0 - v_A = -\frac{B\omega r^2}{2}$

32. $I = \dfrac{\frac{1}{2}B\omega L^2}{R} = \dfrac{\frac{1}{2}\times 0.10\times 40\times(5\times10^{-2})^2}{1} .= 5\text{ mA}$

33. $\text{EMF} = -\dfrac{d\phi}{dt} = -\dfrac{dB\pi r^2}{dt} = -\pi r^2\dfrac{dB}{dt}$

or $E = \left(\dfrac{\text{EMF}}{r}\right) = \left(\dfrac{\pi dB}{dt}\right)r$

or $E \propto r$ for $r \le R$.

$E \propto \dfrac{1}{r}$ for $r > R$.

35. $a = \dfrac{qE}{m} = \dfrac{1}{2}\dfrac{eR}{m}\dfrac{dB}{dt}$.(towards lefts)

34. Since $P_2 = P_2$ or $i_1v_1 = i_2v_2$ & $\dfrac{L_1\frac{di_1}{dt}}{L_2\frac{di_2}{dt}} = \dfrac{v_1}{v_2}$ o r

$\dfrac{v_1}{v_2} = 4$ & $\dfrac{i_1}{i_2} = \dfrac{1}{4}$ $\dfrac{w_2}{w_1} = \dfrac{\frac{1}{2}L_2I_2^2}{\frac{1}{2}L_1I_1^2} = 4$.

36. $L_1\dfrac{di_1}{dt} = L_2\dfrac{di_2}{dt}$

or $L_1di_1 = L_2di_2$ or $L_1i_1 = L_2i_2$

$\therefore \dfrac{i_1}{i_2} = \dfrac{L_2}{L_1}$

39. Initially the inductor offers infinite resistance hence i_1 is 1A. Finally, at steady state inductor offers zero resistance and current i_2 is 1.25 A in the battery.

40. $\text{EMF} = \left|-M\dfrac{dI}{dt}\right|$ $25\times10^{-3} = M\times 15$

or $M = \dfrac{5}{3}\times10^{-3}\text{ H}$

$\phi = MI = \dfrac{5}{3}\times10^{-3}\times3.6 = 6.00\text{ mWb.}$

42. $M_{max} = \sqrt{L_1L_2} = \sqrt{100\times400}\text{ mH} = 200\text{ mH.}$

44. $f = \dfrac{1}{2\pi}\dfrac{1}{\sqrt{L_{eff}\times c_{eff}}} = \dfrac{1}{2\pi\sqrt{3L\times3C}} = \dfrac{1}{6\pi\sqrt{LC}}$.

46. When switch K_1 is opended and K_2 is closed it becomes L–C circuit so applying energy conservation :

$$\frac{Q_0^2}{2C} = \frac{1}{2}Li^2 ;\ Q_0 = C_{eq}V = \frac{C_1C_2}{C_1+C_2}\cdot V = (20\times10^{-6})$$

$$\frac{(20\times10^{-6})^2}{2\times2\times10^{-6}} = \frac{1}{2}(0.2\times10^{-3})\,i^2 \Rightarrow i = 1\text{ A}$$

Part # II : Assertion & Reason

1. A

2. Magnetic field cannot do work, hence statement-1 is false.

3. Obviously statement 2 is correct explanation of statement-1

EXERCISE - 3

Part # I : Matrix Match Type

1. **(A)** q,s **(B)** p,r **(C)** p,r **(D)** q,s

2. When both S_1 and S_2 are either open or closed; current through ad is zero. With S_1 closed, current 2×10^{-7} A flows from a to d. With S_2 closed, current 2×10^{-7} A flows from d to a.

Part # II : Comprehension

Comprehension#1

1. C

2. Even after insertion of the rod the current in circuit will increase with time till steady state is reached.

3. C

Comprehension#2

1. $\frac{dB}{dt} = 2T/s$

$$E = -\frac{AdB}{dt} = -800 \times 10^{-4} m^2 \times 2 = -0.16 \text{ V}$$

$$i = \frac{0.16}{1\Omega} = 0.16 \text{ A, clockwise}$$

2. B

3. At t = 2s, length of the wire

= (2 × 30 cm) + 20 cm = 0.8 m

Resistance of the wire = 0.8 Ω

Current through the rod = $\frac{0.08}{0.8} = \frac{1}{10}$ A

Force on the wire = il B= $\frac{1}{10} \times (0.2) \times 4 = 0.08$ N

Same force is applied on the rod in opposite direction to make net force zero.

EXERCISE - 4

Subjective Type

1. $i = \frac{\varepsilon}{R} = \frac{\Delta\varphi}{\Delta t} \times \frac{1}{R} \; \frac{\Delta q}{\Delta t} = \frac{\Delta\varphi}{\Delta t} \times \frac{1}{R} \Rightarrow \Delta q = \frac{\Delta\varphi}{R}$

2. 1.0 V ,anticlockwise

3. $\varepsilon = -\frac{d\varphi}{dt} = -(0.4t + 0.4)$

(i) $\varepsilon_{t=2} = -1.2$ volt

(ii) $<\varepsilon> = \frac{\Delta\varphi}{\Delta t} = \frac{[0.2(5)^2 + 0.4(5) + 0.6] - [0.6]}{5-0}$

= 1.4 volt

(iii) $\Delta q = \frac{\Delta\varphi}{R} = 17.5$ C.

(iv) $<i> = \frac{\Delta q}{\Delta t} = 3.5$ Anticlockwise

(v) $H = \int \frac{\varepsilon^2}{R}.dt = \int_0^5 \frac{[0.4t + 0.4]^2}{R}.dt = \frac{86}{3}$ J

4. **(a)** –1 mV, –2 mV, 2 mV, 1mV

(b) 10 ms to 20 ms and 20 ms to 30 ms.

5. Since $\Delta\phi = 0$ hence induced current is zero.

6. 2.5 mV

7. $<i> = \frac{<\varepsilon>}{R} = \frac{BA}{Rt} = \frac{(\mu_0 nI)\,AN}{Rt}$

$= \frac{4\pi \times 10^{-7} \times 400 \times 0.40 \times 6 \times 10^{-4} \times 10}{1.5 \times 0.050}$.

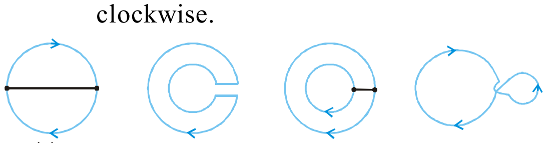
8. 493 μV

9. **(a)** In the round conductor the current flows clockwise, there is no current in the connector;

(b) in the outside conductor clockwise;

(c) in both round conductors, clockwise; no current in the connector,

(d) in the left-hand side of the figure eight, clockwise.



(a) (b) (c) (d)

10. $\frac{\pi}{8} \times 10^{-4}$ A

11. $B = \frac{\mu_o I R^2 N}{2[R^2 + y^2]^{3/2}}$

$\varepsilon = -\frac{d\varphi}{dt} = -A\frac{dB}{dt} = -\pi r^2 \frac{dB}{dt}$ ....(1)

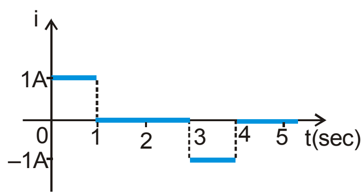
here $\frac{dB}{dt} = -\frac{\mu_o I R^2 N}{2} \times \frac{3}{2}\frac{1}{[R^2+y^2]^{5/2}} \times 2y \times \frac{dy}{dt}$

$= -\frac{3\mu_o I R^2 N}{2}\frac{yv}{[R^2 + y^2]^{5/2}}$

put this value in eq (i) and get Ans.

12. $\Delta q = \frac{\Delta\varphi}{R}\ \frac{2\times 200\times\pi(25\times 10^{-2})^2\times 10^{-2}}{10}$

13.



14. $H = i^2 R1 + 0 + i^2 R1$

$= 1 + 0 + 1 = 2$ J

15. This is in accordance with Lenz's law.

16. $\varepsilon = BVL = 0.2 \times 2 \times 10^{-2}$ volt

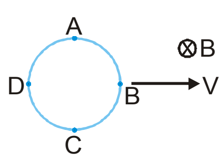
17. **(a)** zero **(b)** vB (bc), positive at b

(c) vB(bc), positive at a **(d)** zero

18. $\varepsilon = BV(L\sin\theta) = 0.1 \times 0.2 \times 1 \sin 60^o = \sqrt{3}\times 10^{-2}$ V

19. 1 mV

20. $\varepsilon_{max} = \varepsilon_{AC} = BV(2r)$



$\varepsilon_{min} = \varepsilon_{BD} = 0$

21. $V^2 = u^2 + 2ay = 0 + 2ay$

$V = \sqrt{2ay}$

$\varepsilon = BV(2x) = B\sqrt{2ay} \times 2\sqrt{\frac{y}{k}}$

$\varepsilon = 2By\sqrt{\frac{2a}{k}} = By\sqrt{\frac{8a}{k}}$.

22. **(a)** $F = ILB = \frac{\varepsilon}{R}(LB) = \frac{B^2L^2V}{R}$, 1 = V/4, V = 4 m/s

$3.2\times 10^{-5} = \frac{(0.02)^2(8\times 10^{-2})^2\times V}{2}$

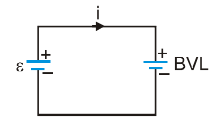
(b) $\varepsilon = BVL = 1\times 4\times 1 = 4$ Volt

(c) $V_{ab} = \varepsilon - IR_{ab} = 4 - (4/4)(1) = 3V$

(d) $V_{cd} = IR_{cd} = (4/4)(1) = 1$ Volt

23. **(a)** 0.1 mA **(b)** 0.2 mA

24. **(a)** $i = \frac{\varepsilon - BV\ell}{r}$ Clockwise



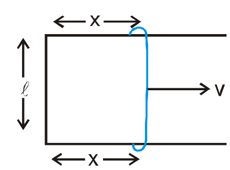
(b) $F = i\ell B = \left(\frac{\varepsilon - BV\ell}{r}\right)\ell B$

(c) F is towards right so v will keep on increasing after some time current will not flow in circuit then it will move with constant velocity. v will be maximum (or constant) when F = 0.

So $\varepsilon = BV\ell,\ V = \frac{\varepsilon}{B\ell}$

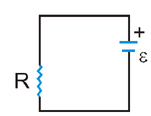
25. zero

26. $\varepsilon = BV\ell$



Net resistance $R = (2\ell + 2x)\,r$

$i = \frac{\varepsilon}{R} = \frac{BV\ell}{(2\ell + 2x)r}$



$= \frac{BV\ell}{2r(\ell + Vt)}$

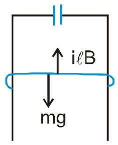
27. **(a)** $\frac{B^2\ell^2 v}{2r(\ell + vt)}$ **(b)** ℓ/v.

28. acceleration $a = \frac{mg - i\ell B}{m}$ ....(i)

here $i = \frac{dq}{dt} = \frac{d}{dt}(c\varepsilon) = C\frac{d\varepsilon}{dt}$

$i = C\frac{d}{dt}(BV\ell) = CB\ell\frac{dV}{dt}$

$i = CB\ell a$ .....(ii)



from eq. (i) and (ii)

$ma = mg - (CB\ell a)\ell B$

$[m + CB^2\ell^2]a = mg$

$$a = \frac{mg}{m + CB^2\ell^2}, \; v = 0 + at = \frac{mgt}{m + CB^2\ell^2}$$

29. **(a)** $\phi = \int B.ds$

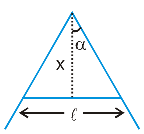
$$= \int_b^{a+b} \frac{\mu_0 i}{2\pi r} \times adr = \frac{\mu_0 ia}{2\pi} \ell n\left(\frac{a+b}{b}\right).$$

(b) $$\varepsilon = -\frac{d\varphi}{dt} = \frac{\mu_0 a i_0}{2\pi} \frac{2\pi}{T} \sin\frac{2\pi t}{T} \ell n\left(\frac{a+b}{b}\right)$$

(c) $$Q = \int_0^{10T} \frac{\varepsilon^2}{r} dt = \left(\frac{5\mu_0^2 i_0^2 a^2}{Tr}\right)\left[\ell n\left(\frac{a+b}{b}\right)\right]^2$$

30. $$\frac{B_0 v_0 L}{2}$$

31.



$\varepsilon = BV\ell$

$$i = \frac{\varepsilon}{R} = \frac{BV\ell}{r\left[\ell + \frac{2x}{\cos\alpha}\right]} \quad \text{where } \cot\alpha = \frac{x}{\ell/2}$$

32. $$\varepsilon = \frac{1}{2}B\omega L^2$$

$$= \frac{1}{2}\, 0.1 \times 60 \times (15 \times 10^{-2})^2.$$

33. $$\frac{B\omega a^2}{R}$$ from C to D

34. $$\varepsilon = \frac{1}{2}B\omega L^2 = 4 \times 10^{-5} \times [\pi \times (30 \times 10^{-2})^2] \times \frac{50}{60}$$

35. $$B\ell\sqrt{g\ell}\,\sin\frac{\theta}{2}$$

36. **(a)** $$\varepsilon_{PQ} = \frac{1}{2}B\omega(2R)^2 = \frac{1}{2}B\left(\frac{v}{R}\right)(2R)^2 = 2BvR$$

(b) $$\varepsilon_{PC} = \frac{1}{2}B\omega R^2 = \frac{1}{2}B\left(\frac{v}{R}\right)R^2 = \frac{BvR}{2}.$$

(c) $$\varepsilon_{QC} = 2BvR - \frac{BvR}{2} = \frac{3}{2}vBR$$

37. **(a)** 2.0×10^{-3} V **(b)** zero **(c)** 50 μC

(d) $\pi \times 10^{-3} \sin(10\pi t)$ **(e)** π mV **(f)** $\frac{\pi^2}{2} \times 10^{-6}$ V

38. **(a)** $$\varepsilon = A\frac{dB}{dt} = \pi(1)^2 6 = 6\pi \text{ V}$$

(b) Find the electric field in the tangential direction, induced due to the changing magnetic field.

39. **(a)** $16\pi^2 \times 10^{-10} = 1.6 \times 10^{-8}$ Weber

(b) $4\pi \times 10^{-8}$ V/m

(c) $18\pi \times 10^{-8} = 5.6 \times 10^{-7}$ V/m

40. $m = \pi r^2 L\delta$ ....(i)

Here $L = 2\pi R$

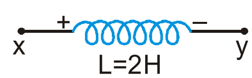
$$\varepsilon = \pi R^2 . \frac{dB}{dt} = \pi\left(\frac{L}{2\pi}\right)^2 \frac{dB}{dt} = \frac{L^2}{4\pi}\frac{dB}{dt}$$

$$i = \frac{\varepsilon}{\text{Resistance}} = \frac{\varepsilon 2\pi R}{\rho\pi r^2}$$

After solving,

$$i = \frac{m}{4\pi\rho\delta} . \frac{dB}{dt}.$$

41.



From KVL

$$V_x - V_y = L\frac{dI}{dt} = 2 \times 5 = 10 \text{ volt.}$$

42. 2.2 A/s, decreasing

43. $P_{cell} = \varepsilon I = 5 \times 1 \times 5$ J/sec. = 5 watt.

$P_{res.} = I^2R = (1)^2 \times 3 = 3$ watt.

$P = 5 - 3 = 2$ watt.

44. 2.55×10^{-14} J

45. $B = \frac{\mu_0 I}{2r} = \frac{\mu_0 fe}{2r}$ ....(i)

$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = \frac{mv^2}{r}, f2\pi r = V$ So

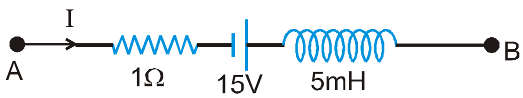
$f = \sqrt{\frac{e^2}{4\pi\varepsilon_0 mr}}\left(\frac{1}{2\pi r}\right)$ .... (ii)

Energy density $= \frac{B^2}{2\mu_0}$ ....(iii)

Solve (i), (ii) & (iii) and get

46. $42 + 20\,t$ volt

47. $\frac{di}{dt} = 10^3$ A / s



$\therefore$ Induced emf across inductance, $|e| = L\frac{di}{dt}$

$|e| = (5 \times 10^{-3})(10^3)\,V = 5\,V$

Since , the current is decreasing , the polarity of this emf would be so as to increase the existing current. The circuit can be redrawn as

Now, $V_A - 5 + 15 + 5 = V_B$

$\therefore$ $V_A - V_B = -15\,V$

or $V_B - V_A = 15\,Vs$

48. (a) $I = I_0(1 - e^{-t/\tau})$

where $I_0 = \frac{4}{20} = \frac{1}{5}$

$\tau = \frac{L}{R} = \frac{2}{20} = \frac{1}{10}$

and $t = 0.2$ sec.

(b) $E = \frac{1}{2}LI^2$

49. $(L/R)\,ln 2 = 1.109$ s, 640 J

50. $U = \frac{1}{2}Li^2$ i.e. $U \propto i^2$

U will reach $\frac{1}{4}$th of its maximum value when current is reached half of its maximum value. In L-R circuit, equation of current growth is written as

$i = i_0(1 - e^{-t/\tau})$

Here i_0 = Maximum value of current

τ = Time constant = L / R

$\tau = \frac{10\text{ henry}}{2\text{ ohm}} = 5s^{-1}$

Therefore, $i = i_0/2 = i_0(1 - e^{-t/5})$

or $\frac{1}{2} = 1 - e^{-t/5}$ or $e^{-t/5} = \frac{1}{2}$

or $-t/5 = \ln\left(\frac{1}{2}\right)$ or $t/5 = \ln(2) = 0.693$

$\therefore$ $t = (5)(0.693)s$ or $t = 3.465s$ **Ans.**

51. 4.0 H

52. $v = v_0[1 - e^{-t/\tau}] = 2[1 - e^{-0.4}] = 0.66$ V.

53. $\frac{2}{e}$ A/s, $\frac{2}{e}$ V

54. $\varepsilon = L\frac{dI}{dt} = \varepsilon_0 e^{-Rt/L}$

$\frac{d\varepsilon}{dt} = -\varepsilon_0 R/L\, e^{-Rt/L}$

55. (a) $\frac{\varepsilon(R_1 + R_2)}{R_1R_2}$ (b) $\frac{L}{R_1 + R_2}$ (c) $\frac{\varepsilon}{R_1 e}$

56. $\varepsilon = \frac{d\varphi_T}{dt} = iR$

as R = 0 for super conducting loop

So ϕ_T = constant

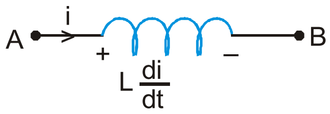
$B\pi R^2 + 0 = -B\pi R^2 - LI$

$I = \frac{2B\pi R^2}{L}$

57. $\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} = \frac{1}{L} + \frac{1}{L} = \frac{2}{L}$

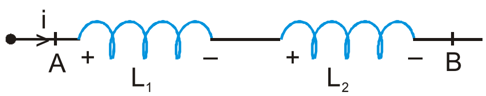
or $L_{eq} = \frac{L}{2}$

58. Equivalent self inductance :



$$L = \frac{V_A - V_B}{di/dt} \qquad ....(1)$$

Series combination



$$V_A - L_1\frac{di}{dt} - L_2\frac{di}{dt} = V_B \qquad ....(2)$$

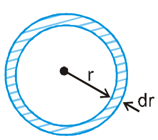
from (1) and (2)

$L = L_1 + L_2$ (separation is large to neglect mutual inductance) **Ans.**

59. $\text{EMF} = \left|M\frac{-\Delta I}{\Delta t}\right|$ or $M = \dfrac{\text{EMF}}{\left|\frac{-\Delta I}{\Delta t}\right|}$

60. 2.5 V

61.



$$d\varepsilon = -\frac{Ndr}{a}\times \pi r^2 . \frac{dB}{dt}$$

here $\frac{dB}{dt} = B_o\,\omega\,\cos\omega t \quad d\,\varepsilon = -\frac{N}{a}\,\pi r^2\,.\,B_o\,\omega$ $\cos\omega t\,dr$

$$\varepsilon = \int d\varepsilon = \frac{B_o\omega\pi N\cos\omega t}{a}\int_o^a r^2.dr$$

$$\varepsilon = -\frac{B_o\,\omega\pi\,N\cos\omega t}{a}\left[\frac{r^3}{3}\right]_o^a$$

$$\varepsilon = -\frac{B_o\omega\pi\,N\cos\omega t.a^2}{3}$$

Amplitude $\varepsilon_o = \frac{1}{3}\,\pi\,a^2\,N\,\omega\,B_o$

Amplitude $\varepsilon_o = \frac{1}{3}\,\pi\,a^2\,N\,\omega\,B_o$

EXERCISE - 5

Part # I : AIEEE/JEE-MAIN

1. Mutual inductance of the pair of coils depends on distance between two coils and geometry of two coils

2. $e = -L\frac{di}{dt} = L\frac{(-2-2)}{0.05} \Rightarrow 8 = L\frac{(4)}{0.05}$

$$L = \frac{8\times 0.05}{4} = 0.1\text{H}$$

3. $\frac{1}{2}Li^2 = \frac{1}{2}\frac{q^2}{C}$

$\Rightarrow q^2 = Li^2C$.....(i)

and $\qquad U_{E(max)} = U_{B(max)}$ (given) .....(ii)

Since, $\qquad U_B = \frac{1}{2}Li^2$

where $\qquad i = \frac{\varepsilon}{R}(1 - e^{Rt/L})$

at $t = \infty\,; = i_\infty = \frac{\varepsilon}{R}$

$U_{B(max)} = \frac{1}{2}\,Li_\infty^{\,2}$ .....(iii)

and $\quad U_{E(max)} = \frac{Q^2}{2C}$ .....(iv)

Where Q is maximum charge on capacitor. The energy is equally divided between electric and magnetic fields. Therefore

$$\therefore\ U_B = \frac{U_{B(max)}}{2} \Rightarrow \frac{1}{2}Li^2 = \frac{1}{2}\left(\frac{1}{2}Li_\infty^2\right)$$

$\Rightarrow i_\infty^2 = 2i^2$ ......(v)

From equation (ii), (iii), (iv) and (v), we get

$$\therefore\ \frac{1}{2}L.\,2i^2 = \frac{1}{2}\frac{Q^2}{C} \Rightarrow LCi^2 = \frac{Q^2}{2}$$

$\therefore\ q^2 = \frac{Q^2}{2}$ [from equation (i)]

$$q = \frac{Q}{\sqrt{2}}$$

Method II

$$\frac{q^2}{2C} = \frac{1}{2}\left(\frac{Q^2}{2C}\right) \Rightarrow q = \frac{Q}{\sqrt{2}} \qquad \textbf{Ans.}$$

4. The rate of change of flux or emf induced in the coil is

$$\varepsilon = \frac{-\Delta\varphi}{\Delta t} \quad \therefore \text{ Induced current}$$

$$i = \frac{\varepsilon}{R_{eq}} = -\frac{1}{R}\frac{\Delta\varphi}{\Delta t} \qquad \text{...(i)}$$

Given:

$R_{eq.} = R + 4R = 5R, \Delta\phi = n(w_2 - w_1)A, \Delta t = t.$

(Here W_1 and W_2 are associated with one turn.)

Putting the given values in eq. (i), we get

$$\therefore \qquad i = -\frac{n}{5R}\frac{(W_2 - W_1)A}{t}$$

5. The flux associated with coil of area A and magnetic induction B is

$$\phi = BA\cos\theta \qquad = \frac{1}{2}B\pi r^2 \cos\omega t$$

$$\left[\because A = \frac{1}{2}\pi r^2\right] \quad \therefore e_{induced} = -\frac{d\varphi}{dt}$$

$$= -\frac{d}{dt}\left(\frac{1}{2}B\pi r^2 \cos\omega t\right) = \frac{1}{2}B\pi r^2 \omega \sin\omega t$$

$$\therefore \text{Power } P = \frac{e^2_{induced}}{R} = \frac{B^2\pi^2 r^4\omega^2 \sin^2\omega t}{4R}$$

Hence, $P_{mean} = \langle P \rangle$

$$= \frac{B^2\pi^2 r^4\omega^2}{4R} \cdot \frac{1}{2}\left[\because \langle \sin\omega t \rangle = \frac{1}{2}\right] = \frac{(B\pi r^2\omega)^2}{8R}$$

6. The emf induced between ends of conductor

$$e = \frac{1}{2}B\omega L^2 = \frac{1}{2} \times 0.2 \times 10^{-4} \times 5 \times (1)^2$$

$$= 0.5 \times 10^{-4}\,V = 5 \times 10^{-5}\,V = 50\,\mu V$$

7. Relative velocity $= \upsilon - (-\upsilon) = 2\upsilon = \frac{dl}{dt}$

Now, $e = \frac{d\varphi}{dt} \Rightarrow e = \frac{BIdl}{dt}\left(\frac{dl}{dt} = 2\upsilon\right)$

Induced emf $e = 2Bl\upsilon$

8. The current at any instant is given by

$I = I_0(1 - e^{-Rt/L})$

$$\frac{I_0}{2} = I_0(1 - e^{-Rt/L}) \Rightarrow \frac{1}{2} = (1 - e^{-Rt/L})$$

$$e^{-Rt/L} = \frac{1}{2} \Rightarrow \frac{Rt}{L} = \ln 2 \quad \therefore \quad t = \frac{L}{R}\ \ln 2$$

$$= \frac{300 \times 10^{-3}}{2} \times 0.693 = 150 \times 0.693 \times 10^{-3}$$

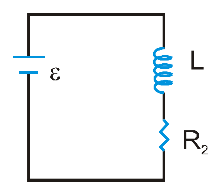
$= 0.10395$ sec $= 0.1$ sec.

9. $I = I_0 e^{-Rt/L} = \frac{1}{e}$ A.

10. $M = \mu_0 n_1 A N_2 = (4\pi \times 10^{-7})\left(\frac{300}{0.20}\right)(10 \times 10^{-4})$

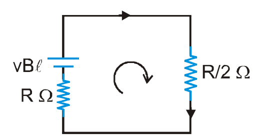
$(400) = 2.4\,\pi \times 10^{-4}\,H$

11.



$$V_L = \varepsilon\, e^{-\frac{R_2 t}{L}} = 12.\, e^{-\frac{2t}{400 \times 10^{-3}}} = 12\, e^{-5t}.$$

12. Current $I = \frac{vB\ell}{R/2 + R} = \frac{2vB\ell}{3R}$



$$I_1 = I_2 = \frac{vB\ell}{3R}$$

13. In LC oscillation energy is transferred C toL or L to C maximum energy in L is $= \frac{1}{2}LI^2_{max}$

Maximum energy in C is $= \frac{q^2_{max}}{2C}$

Equal energy will be when

$$\frac{1}{2}LI^2 = \frac{1}{2}\frac{1}{2}LI^2_{max} \Rightarrow I = \frac{1}{\sqrt{2}}I_{max}$$

$$I = I_{max} \sin\omega t = \frac{1}{\sqrt{2}} I_{max}$$

$$\omega t = \frac{\pi}{4} \quad \text{or} \quad \frac{2\pi}{T} t = \frac{\pi}{4} \quad \text{or} \quad t = \frac{T}{8}$$

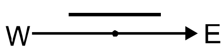
$$t = \frac{1}{8} 2\pi\sqrt{LC} = \frac{\pi}{4}\sqrt{LC}$$ **Ans.**

14. $E_{ind} = B \times v \times \ell \quad = 5.0 \times 10^{-5} \times 1.50 \times 2$

$= 10.0 \times 10^{-5} \times 1.5$

$= 15 \times 10^{-5}$ vot. $= 0.15$ mv

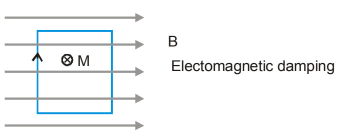
15.



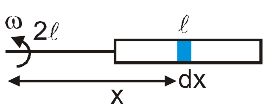
$\varepsilon_{ind} = Bv\ell \quad = 0.3 \times 10^{-4} \times 5 \times 20$

$= 3 \times 10^{-3}$ v $= 3$ mv.

16.



17. $$e = \int_{2\ell}^{3\ell} (\omega x) B dx = B\omega \frac{[(3\ell)^2 - (2\ell)^2]}{2}$$



$$= \frac{5B\ell^2\omega}{2}$$

18. $$\frac{\mu_0 (2)(20 \times 10^{-2})^2}{2\left[(0.2)^2 + (0.15)^2\right]} \times \pi (0.3 \times 10^{-2})^2$$

on solving

$= 9.216 \times 10^{-11}$

$\approx 9.2 \times 10^{-11}$ weber **Ans (1)**

19. $q = CV\,(1 - e^{t/\tau})$

at पर $t = 2\tau$

$q = CV\,(1 - e^{-2})$ **Ans (3)**

20. 1

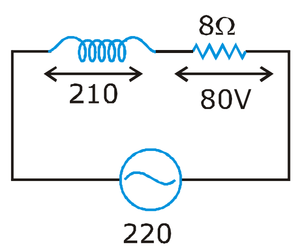
21. According to given conditions:

$$i_0 = \frac{V}{R} = \frac{15}{0.15 \times 10^3} = 0.1\text{A}$$

$$i = i_0\, e^{-\frac{Rt}{L}}$$

$$= 0.1 \times e^{-\frac{0.15 \times 10^3 \times 10^{-3}}{0.03}} = 0.1 \times e^{-5} = \frac{0.1}{150} = 0.67\text{ mA}$$

22.



$V_L^2 + 6400 = 220 \times 220 \quad IR = 80$

$V_L = \sqrt{48400 - 6400}$

$I = \frac{80}{8} = 10 \quad = \sqrt{42000} = 210$

$I x_L = 210 \quad X_L = 2\pi f L = 210$

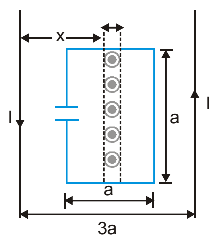
$$L = \frac{210}{10 \times 100\ \pi} = 0.065\text{ H}$$

Part # II : IIT-JEE ADVANCED

1. (i) For an element strip of thickness dx at a distance x from left wire, net magnetic field (due to both wires)

$$B = \frac{\mu_0}{2\pi}\frac{I}{x} + \frac{\mu_0}{2\pi}\frac{I}{3a-x} \quad \text{(out wards)}$$

$$= \frac{\mu_0 I}{2\pi}\left(\frac{I}{x} + \frac{1}{3a-x}\right)$$



Magnetic flux in this strip, $d\phi = BdS$

$$= \frac{\mu_0 I}{2\pi}\left(\frac{I}{x} + \frac{1}{3a-x}\right) a\,dx$$

$\therefore$ total flux $\phi = \int_a^{2a} d\phi = \frac{\mu_0 Ia}{2\pi}$

$$\int_a^{2a}\left(\frac{1}{x} + \frac{1}{3a-x}\right)dx \quad \text{or} \quad \phi = \frac{\mu_0 Ia}{\pi}\ln(2)$$

$$\phi = \frac{\mu_0 a\ \ln(2)}{\pi}(I_0 \sin\omega t) \quad \ldots\ldots\ldots\ldots\text{(i)}$$

Magnitude of induced emf,

$$e = \left|-\frac{d\phi}{dt}\right| = \frac{\mu_0 a I_0 \omega \ln(2)}{\pi}\cos\omega t = e_0 \cos\omega t$$

Where $e_0 = \frac{\mu_0 a I_0 \omega \ln(2)}{\pi}$

Charged stored in the capacitor,

$q = Ce = Ce_0 \cos\omega t$ ....(ii)

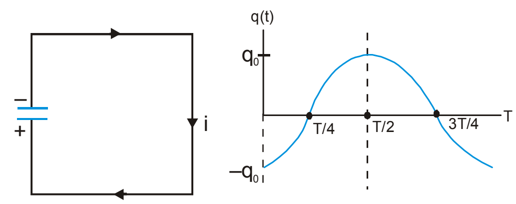
and current in the loop

$$i = \frac{dq}{dt} = C\omega e_0 \sin\omega t \ \ldots\ldots\ldots\ldots\text{(iii)}$$

$$i_{max} = C\omega e_0 = \frac{\mu_0 a I_0 \omega^2 C \ln(2)}{\pi} \quad \textbf{Ans.}$$

(i) Magnetic flux passing through the square loop $\phi \propto \sin\omega t$ [From equation (i)]

i.e.,⦿ magnetic field passing through the loop is increasing at t = 0. Hence, the included current will produce ⊗ magnetic field (from Lenz's law). Or the current in the circuit at t = 0 will be clockwise (or negative as per the given convention). Therefore, charge on upper plate could be written as,



$q = -q_0 \cos\omega t$ [from equation (ii)]

Here $q_0 = Ce_0 = \frac{\mu_0 a C I_0 \omega \ln(2)}{\pi}$

The corresponding q – t graph is shown in figures.

2. zero, as there is no flux change.

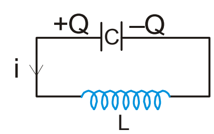
3. $Q = Q_0(1 - e^{-t/\tau})$

$Q = CV(1 - e^{-t/\tau})$ after time interval 2τ.

4. $q = Q_0 \cos\omega t$

$$i = -\frac{dq}{dt} = Q_0\,\omega \sin\omega t \Rightarrow i_{max} = C\omega V = V\sqrt{\frac{C}{L}}$$

5.



$$L\frac{di}{dt} - \frac{Q}{C} = 0,$$

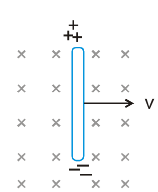
$$-L\frac{d^2Q}{dt^2} - \frac{Q}{C} = 0, \qquad Q = -LC\frac{d^2Q}{dt^2}$$

6. There is a no friction between the train and the track because train is not in contact with track.

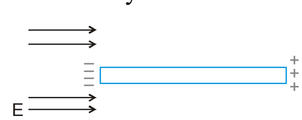
7. By Lenz's law there is train experience a drag.

8. Electro-magnetic force elevates it.

9. EMF = VBℓ is generated, constant charges will also appear at the end of wire. There is no heat loss after steady state which will come within a very small time.



(C)

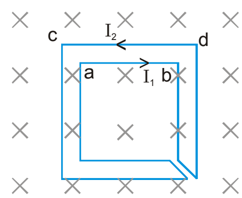


Charge will be induced but net p.d. = 0 because net electric field inside the conductor is zero.

(D) $i = \dfrac{emf}{R}$ Heat $= i^2 Rt$

10. Due to induce current in coil, force between two coil is generated.

11. Current $I_1 = I_2$,
Since magnetic field increases with time
So induced net flux should be outward i.e. current will flow from a to b



12. **Hint** $F = B_H i \ell$

$F\Delta t = MV$

$i = \dfrac{E_{induced}}{R}$

$h = \dfrac{v^2}{2g}$

Solution : The horizotanl component of magnetic field due to solenoid will exert force on ring in vertical direction

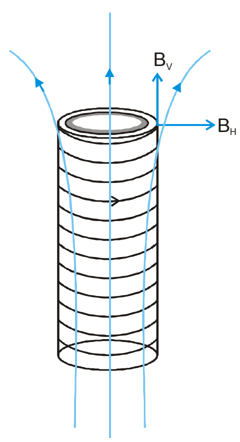
$F = B_H i(2\pi r)$

$F\Delta t = MV$

$i = \dfrac{(\Delta\varphi/\Delta t)}{\left(\rho\dfrac{(2\pi r)}{A}\right)}$

$B_H i(2\pi r)\Delta t = MV$

$V = \dfrac{B_H \Delta\varphi A}{\rho M} = \dfrac{K}{\rho M}$



$h = \dfrac{V^2}{2g} = \dfrac{K^2}{\rho^2 M^2}$

$h_A > h_B$

$\dfrac{K^2}{\rho_A^2 M_A^2} > \dfrac{K^2}{\rho_B^2 M_B^2}$

$\Rightarrow \rho_B M_B > \rho_A M_A \rightarrow$ Using this we get

Answer **(B) and (D)**

13. As the magnetic field is greater, the critical temperature is lower and as B_2 is larger than B_1. Graph 'A' is correct.

14. For $B = 0$ $T_C = 100\,k$
$B = 7.5\,T\ T_C = 75\,k$

15. Flux through circular ring

$\phi = (\mu_0 ni)\pi r^2$

$\phi = \dfrac{\mu_0}{L}\pi r^2\ I_0 \cos 300\,t \Rightarrow i = \dfrac{d\varphi}{Rdt}$

$i = \dfrac{\mu_0 \pi r^2 I_0}{RL}$. sin 300 t × 300

$= \mu_0 I_0 \sin 300\,t \left[\dfrac{\pi r^2 .300}{RL}\right] \Rightarrow M = I\,.\,\pi r^2$

$= \mu_0 I_0 \sin 300\,t \left[\dfrac{\pi^2 r^4 .300}{RL}\right]$ (Take $\pi^2 = 10$)

$= \dfrac{10\times 10^{-4}\times 300}{100\times 10}$ $N = 6$ **Ans.**

16. True for induced electric field and magnetic field.

17. $B = \dfrac{\mu_0 i R^2}{2(R^2 + X^2)^{3/2}}$

$B = \dfrac{\mu_0 i R^2}{2(R^2 + 3R^2)^{3/2}} = \dfrac{\mu_0 i R^2}{2(4R^2)^{3/2}}$

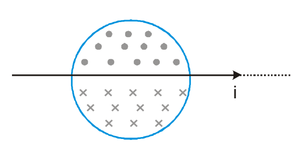
$= \dfrac{\mu_0 i R^2}{2\,.\,2^3\,.\,R} = \dfrac{\mu_0 i}{16R}$

$\phi = NBA\cos 45° = 2\dfrac{\mu_0 i}{16R}a^2\dfrac{1}{\sqrt{2}}$

$\phi = \dfrac{\mu_0 i a^2}{8\sqrt{2}R} \Rightarrow M = \dfrac{\phi}{i}$

$M = \dfrac{\mu_0 a^2}{2^{7/2}R} = \dfrac{\mu_0 a^2}{2^{P/2}R} \Rightarrow P = 7$

18.



$(\phi)_{loop} = 0$ for all cases

so induced emf = 0

19. 8

MOCK TEST

1. $q = \int I dt = \int -\frac{1}{r}\frac{d\varphi}{dt} dt = -\frac{\Delta\varphi}{r} = \frac{\mu_0 I a}{\pi r}\ell n 2$.

2. When the rod rotates, there will be an induced current in the rod. The given situation can be treated as if a rod 'A' of length '3ℓ' rotating in the clockwise direction, while an other say rod 'B' of length '2ℓ' rotating in the anti clockwise direction with same angular speed 'ω'.

As we know that $e = \frac{1}{2}B\omega\ell^2$

For 'A' : For 'B' :

$e_A = \frac{1}{2}B\omega(3\ell)^2$ & $e_B = \frac{1}{2}B(-\omega)(2\ell)^2$

Resultant induced emf will be :

$e = e_A + e_B = \frac{1}{2}B\omega\ell^2(9-4)$ $\qquad e = \frac{5}{2}B\omega\ell^2$

3. Induced emf $\int_a^b Bvdx = \int_a^b \frac{\mu_0 I}{2\pi x}vdx$

$\Rightarrow$ Induced emf $= \frac{\mu_0 I v}{2\pi}\ell n\left(\frac{b}{a}\right)$

$\Rightarrow$ Power dissipated $= \frac{E^2}{R}$

Also, power = F.V $\Rightarrow$ $F = \frac{E^2}{VR}$

$\Rightarrow$ $F = \frac{1}{VR}\left[\frac{\mu_0 I V}{2\pi}\ell n\left(\frac{b}{a}\right)\right]^2$ **Ans.**

4. Rate of increment of energy in inductor $= \frac{dU}{dt}$

$= \frac{d}{dt}\left(\frac{1}{2}Li^2\right) = Li\frac{di}{dt}$

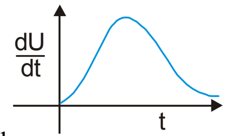
Current in the inductor at time t is:

$i = i_0(1 - e^{-\frac{t}{\tau}})$ and $\frac{di}{dt} = \frac{i_0}{\tau}e^{-\frac{t}{\tau}}$

$\therefore \frac{dU}{dt} = \frac{Li_0^2}{\tau}e^{-\frac{t}{\tau}}(1 - e^{-\frac{t}{\tau}})$

$\frac{dU}{dt} = 0$ at $t = 0$

and $t = \infty$



Hence E is best represented by :

5. $L = \frac{\mu_0 N^2 \pi r^2}{\ell}$

length of wire = N 2πr = Constant (= C, suppose)

$\therefore L = \mu_0\left(\frac{C}{2\pi r}\right)^2\frac{\pi r^2}{\ell}$

$\therefore \qquad L \propto \frac{1}{\ell}$

$\therefore$ Self inductance will become 2L.

6. $\int \vec{E}.d\vec{r} = -\frac{d\varphi}{dt}$

and take the sign of flux according to right hand curl rule.

$\int \vec{E}.d\vec{r} = -(-(-\alpha A) - (-\alpha A) + (-\alpha A)) = -\alpha A$

7. Given :

Voltage in primary $V_p = 200$ volt

Current in primary $i_p = 2$ amp

Voltage in secondary $V_s = 2000$ volt

The relation for the current in the secondary is

$\frac{V_s}{V_p} = \frac{i_p}{i_s}$

$\frac{2000}{200} = \frac{2}{i_s}$ or $i_s = \frac{2 \times 200}{2000} = 0.2$ amp.

8. Flux can't change in a superconducting loop.
$\Delta\phi = 2\pi R^2 . B$

Initially current was zero. So self flux was zero.

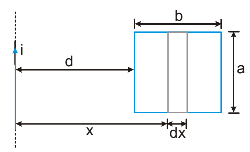
$\therefore$ Finally $Li = 2\pi R^2 \times B$.

$i = \frac{2\pi R^2 \times B}{L}$

9. $BlV = iR + \frac{q}{C}$ or $BlV = \left(\frac{dq}{dt}\right)R + \frac{q}{C}$

Hence the charge on capacitor increases with time.

10. The flux in rectangular loop due to current i in wire is



$$\phi = \int_d^{d+b} \frac{\mu_0 i}{2\pi x} a\,dx = \frac{\mu_0 ia}{2\pi} \ell n \frac{b+d}{d}$$

Mutual inductance is

$$M = \frac{\varphi}{i} = \frac{\mu_0 a}{2\pi} \ell n \frac{b+d}{d}$$

$\therefore$ Mutual inductance is proportional to ‘a’.

11. When the magnetic goes away from the ring the flux in the ring decreases hence the induced current will be such that it opposes the decreasing flux in it hence ring will behave like a magnet having face A as north pole and face B as south pole.

12. Just before opening the switch, the current in the inductor is ε/R. Energy stored in it $= \frac{1}{2}L\left(\frac{\varepsilon}{R}\right)^2$.

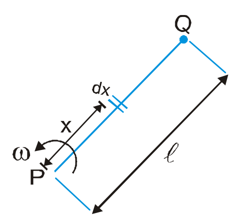
This energy will dissipate in the resistors R_1 and R_2 in the ratio $\frac{1}{R_1}$ & $\frac{1}{R_2}$.

13. Charge on the differential element dx, $dq = \frac{Q}{\ell}.dx$

equivalent current $di = f\,dq$

$\therefore$ magnetic moment of this element $d\mu = (di)\,NA$ $(N = 1)$

$$= \left(\pi x^2\right) f \frac{Q}{\ell} dx$$



$\Rightarrow \mu = \int_0^{\mu} d\mu = \frac{\pi f Q}{\ell} \int_0^{\ell} x^2\,dx$; $\mu = \frac{1}{3} \pi fQ \ell^2$ ....

14. $B_H = B_v \cot\theta = B\cot\theta$

Hence the induced e.m.f. in the rod is $B \ell v \cot\theta$

15. Since all the wires are connected between rim and axle so they will generate induced emf in parallel, hence it is same for any number of spokes.

16. $\frac{dP}{dt} = \frac{dF.v}{dt} = \frac{Fdv}{dt} = Fa$

as 'a' is decreasing continuously hence the rate of power delivered by external force will be decreasing continuously.

17. $e = (\vec{v}\times\vec{B}).\ell$

$e = [\hat{i}\times(3\hat{i}+4\hat{j}+5\hat{k})].5\hat{j} \Rightarrow e = 25$ volt

18. Time constant $= \frac{1}{20} = 50$ msec

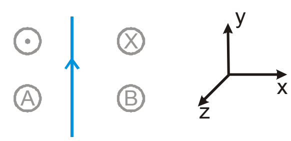
So $i = 0.633\, i_{max} = 0.633\,\frac{E}{R}$

$\Rightarrow E = \frac{3.165\times 20}{0.633} = 100$ V

19. $W = (L)F$

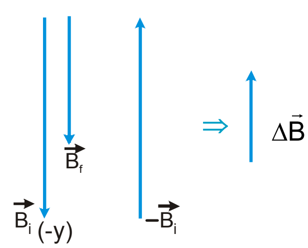
$= L\times ILB = L\times\frac{L^2B^2V}{R} = 1$ J

20. The field at A and B are out of the paper and inside the paper respectively.



As the current in the straight wire decreases the field also decreases.

For B :



The change in the magnetic field which causes induced current $(\Delta\vec{B})$ is along (+)z direction.

Hence, induced emf and hence current should be such as to oppose this change $\Delta\vec{B}$.

Hence, induced emf should be along – z direction which results in a clockwise current in 'B'. Similarly, there will be anticlockwise current in 'A'. Hence (B).

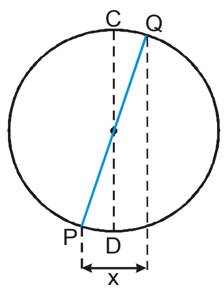
21. When the ring falls vertically, there will be an induced emf across A & B (e = Bv (2r)).

Note that there will be a potential difference across any two points on the ring of line joining them has a projected length in the horizontal plane.

For example, between points 'P' & 'Q' there is a projected length 'x' in the horizontal plane.

$\therefore$ P.d. across P & Q is :

V = Bvx.



But for points C and D : x = 0.

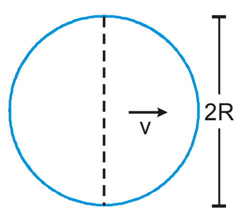
Therefore; P.d. = 0.

Hence **(B)**.

22. Considering a projected length 2R on the ring in vertical plane.

This length will move at a speed v perpendicular to the field. This results in an induced emf :

e = Bv(2R) in the ring.



In Ring "A" : eA = B(-v)(2R)

In Ring "B" : eB = B(v)(2R)

Therefore, potential difference between A & B = B(v)(2R) - B(-v)(2R) = 4 BvR.

Note : there will be no p.d. across a diameter due to rotation.

$$e = \frac{B\omega(2r)^2}{2} = 2Bvr \text{ in A} \quad \text{(since pure rotation).}$$

and e = – 2Bvr in B.

Hence **(C)**

23. The graph of current is given by :

$$i = i_0(1 - e^{-t/\tau}) \Rightarrow \frac{di}{dt} = \frac{i_0}{\tau}e^{-t/\tau}$$

Energy stored in the form of magnetic field energy is :

$$U_B = \frac{1}{2}Li^2$$

$\therefore$ Rate of increase of magnetic field energy is :

$$R = \frac{dU_B}{dt} = Li\frac{di}{dt} = \frac{Li_0^2}{\tau}(1 - e^{-t/\tau})\,e^{-t/\tau}$$

This will be maximum when $\frac{dR}{dt} = 0$

$$\frac{dR}{dt} = 0 \Rightarrow e^{-t/\tau} = 1/2$$

Substituting :

$$R_{max} = \frac{Li_0^2}{\tau}\left(1 - \frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{Li_0^2}{4\tau} = \left[\frac{L(E/R)^2}{4(L/R)}\right] = \frac{E^2}{4R}$$

24. When the switch is at position 1 :

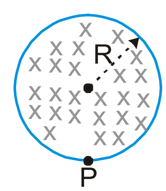
$$U_B = \frac{1}{2}Li_0^2 = \frac{LE^2}{2R^2}$$

Just after the switch is shifted to position 2, current $I = \frac{E}{R}$ is flowing across the resistance. Hence, at that instant P.d. across resistance will be :

$$\Delta V = IR = \frac{E}{R}.R = E$$

Hence **(B)**.

25. If we consider the cylindrical surface to be a ring of radius R, there will be an induced emf due to changing field.



$$\int \vec{E}.\vec{d\ell} = \frac{d\phi}{dt} = -A\frac{dB}{dt}$$

$\Rightarrow$ $E(2\pi R) = -A\dfrac{dB}{dt} = -\pi R^2\dfrac{dB}{dt}$

$\Rightarrow$ $E = \dfrac{R}{2}\dfrac{dB}{dt}$ $\qquad\therefore$ Force on the electron.

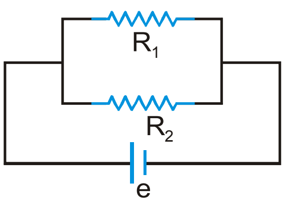
$F = -Ee = -\dfrac{eR}{2}\dfrac{dB}{dt}$

$\Rightarrow$ $\quad \text{acceleration} = \dfrac{1}{2}\dfrac{eR}{m}\dfrac{dB}{dt}$

As the field is increasing being directed inside the paper, hence there will be anticlockwise induced current (in order to oppose the cause) in the ring (assumed). Hence there will be a force towards left on the electron.

26. The equivalent diagram is :

The induced emf across the centre and any point on the circumference is :



$|\vec{e}| = \dfrac{1}{2}B\omega\ell^2 = \dfrac{B\omega r^2}{2}$

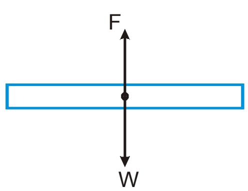
$\therefore$ Current through $R_1 = \dfrac{B\omega r^2}{2R_1}$

27. There is a force $\vec{F}_M = I(d\vec{\ell}\times\vec{B})$ acting on the rod carrying a current I.

By the rule of cross product, this force is vertically upward.

F.B.D. of the rod :

$F - W = ma$



$a = \dfrac{F-W}{m}$

The magnitude of acceleration will be constant, but the direction will depend on the mass of the rod.

Hence **(D)** is correct option.

28. Considering pure rolling of OA about A: the induced emf across OA will be :

$|\vec{e}| = \dfrac{B\omega(r)^2}{2}$.

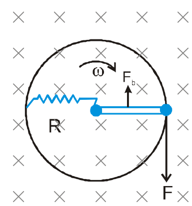
From Lenz law, O will be the negative end, while A will be the positive end.

Hence $\quad v_0 - v_A = -\dfrac{B\omega r^2}{2}$

Hence **(C)** is correct option.

29. $F_b = BIL$

Induced current :



$I = \dfrac{(B\omega r^2/2)}{R}$

$\therefore \qquad F_b = B\left(\dfrac{B\omega r^2}{2R}\right)r = \dfrac{B^2\omega r^3}{2R}$

To maintain constant angular velocity :

$F(r) = F_B(r/2) \quad\Rightarrow\quad F = \dfrac{F_B}{2} = \dfrac{B^2\omega r^3}{4R}$.

30. Flux through a closed circuit containing an inductor does not change instantaneously.

$\therefore\ L\left(\dfrac{E}{R}\right) = \dfrac{L}{4}(i) \ \Rightarrow\ i = \dfrac{4E}{R}$

31. $E = \dfrac{d\phi}{dt} = \dfrac{Bd(b\ell)}{dt}$

$= Bbv = B\times 2\times 10^{-2}\times 20 = 0.40\,B$

$\Delta t = \dfrac{1\times 10^{-2}}{20} = 5\times 10^{-4}\text{ sec} = 500\ \mu\text{ sec}$

$t = \dfrac{6\times 10^{-2}}{20} = 3\times 10^{-3}\text{ sec} = 3000\ \mu\text{ sec}$

32. $i = \dfrac{|E|}{R} = \dfrac{B}{R}\dfrac{dA}{dt}$

$Q = \int i\,dt = \dfrac{B}{R}\int dA = \dfrac{B}{R}A$

using values $Q = 1.2\times 10^{-6}$ C

33. $|\varepsilon| = B\,\ell v$ where l is the edge perpendicular to both B and $\vec{v}$ i.e.c.

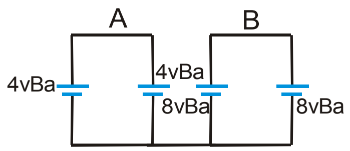
$\therefore |\varepsilon| = B\,v\,c.$

Now by right hand thumb rule magnetic force an a positive charge moving towards right is in down ward direction Hence end P will be positive.

$\therefore$ $V_p - V_Q$ is positive

$\Rightarrow$ $\varepsilon = +Bv\ell.$

34.



$V_{AB} = 4vBa$

35. The magnetic flux in inner loop due to current in outer loop is

$$\phi = BA = \left(\frac{\mu_0}{4\pi}\frac{2\pi i}{b}\right).\pi a^2 = \left(\mu_0 \frac{\pi a^2}{2b}\right) i_0 \cos\omega t$$

$$\therefore \quad e = \frac{d\varphi}{dt} = \mu_0 \frac{\pi a^2 \omega i_0}{2b} \sin\omega t$$

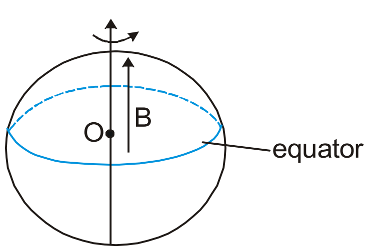
36. $L = \mu_0 n^2 \pi r^2 \ell$

$A = \pi r^2$

$$n = \frac{N}{\ell} \Rightarrow L = \mu_0 \frac{N^2}{\ell} A$$

By putting the given values, it can be seen that it is maximum for solenoid no.4.

37. The equator can be seen as a conducting ring of radius R_e revolving with angular velocity ω in a perpendicular magnetic field B.



$\therefore$ Potential difference. across its center and

$$\text{periphery} = \frac{B\omega R_e^2}{2}$$

Potential at pole = potential of the axis of earth i.e. potential at point O

$$\therefore V_{eqvator} - V_{pole} = \frac{B\omega R_e^2}{2}.$$

38. When switch K_1 is opened and K_2 is closed it becomes L–C circuit so applying energy conservation

$$\frac{Q_0^2}{2C} = \frac{1}{2}Li^2 \,;\, Q_0 = C_{eq}V = \frac{C_1C_2}{C_1 + C_2}\cdot V = (20\times10^{-6})$$

$$\frac{(20\times10^{-6})^2}{2\times2\times10^{-6}} = \frac{1}{2}(0.2\times10^{-3})\,i^2 \Rightarrow i = 1\text{ A}$$

39. $$i = \frac{\varepsilon}{R}(1 - e^{-Rt/L})$$

charge passing through battery

$$q = \int i\,dt = \frac{\varepsilon}{R}\int_0^{L/R}(1 - e^{-Rt/L})dt = \frac{\varepsilon}{R}\left\{\frac{L}{R} + \frac{L}{R}\left(e^{-1} - 1\right)\right\}$$

$$= \frac{\varepsilon L}{eR^2} \quad \therefore W_b = q\,\varepsilon = \frac{\varepsilon^2 L}{eR^2}$$

40. Rate of work done by external agent is :

$\frac{dw}{dt} = \frac{BIL.dx}{dt} = BILv$ & thermal power dissipated in the resistor = eI = (BvL) I

clearly both are equal, hence (A) .

If applied external force is doubled, the rod will experience a net force and hence acceleration. As a result velocity increases, hence (B)

Since ; $I = \frac{e}{R}$

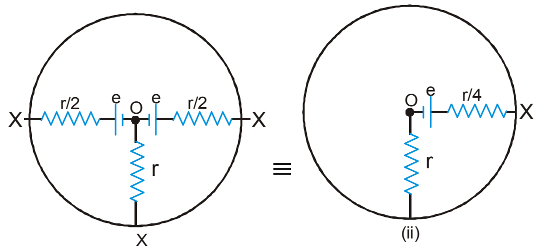
On doubling 'R', current and hence required power becomes half.

Since, P = BILv

Hence (D).

41. Equivalent circuit :

$$\text{Induced emf } e = \frac{B\omega r^2}{2} = \left(\frac{B\omega a^2}{2}\right)$$



($\because$ Radius $= a$)

By nodal equation :

$$4\left(\frac{X-e}{r}\right)+\left(\frac{X-0}{r}\right)=0 \qquad 5X = 4e$$

$$\Rightarrow X = \frac{4e}{5} = \frac{2B\omega a^2}{5} \text{ and } I = \frac{X}{r} = \frac{2B\omega a^2}{5r}$$

also direction of current in 'r' will be towards negative terminal i.e. from rim to origin.

Alternatively; by equivalent of cells (figure (ii)) :

$$I = \frac{e}{r+\frac{r}{4}} = \frac{4e}{5r} = \frac{2B\omega a^2}{5r}$$

$$(\because \; e = \frac{B\omega r^2}{2})$$

42. Since curve 'b' is below curve 'a' hence it is possible that E & R are kept constant and L is increased or E & R are both halved and L is kept constant

43. At steady state no resistance will be offered by inductor hence, time constant is L/R and steady state current in inductor is E/R.

44. $|e| = Ba\,\omega \sin\omega t$

$\phi = Ba \cos\omega t$;

* $|e|$ is maximum when $\omega t = \pi/2$

So ϕ is zero.

* $|e|$ is zero then $\omega t = 0$ So $\phi =$ is maximum.

45. By principal of energy conservation.

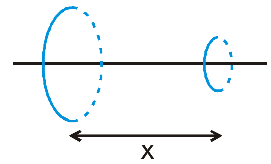
$P_B = P_R + P_L$

Near the starting of the circuit

$$P_R = i^2 R \text{ and } P_L = L\,i\,\frac{di}{dt}.$$

As $\frac{di}{dt}$ has greater value at the starting of the circuit, $P_L > P_R$

46.



It is obvious that flux linkage in one ring due to current in other coaxial ring is maximum when $x = 0$ (as shown) or the rings are also coplanar. Hence under this condition their mutual induction is maximum.

47. **(D)** Magnetic field cannot do work, hence statement-1 is false.

48. **(A)** Obviously statement 2 is correct explanation of statement-1

49. **(A)** $\oint \vec{B}\cdot d\vec{\ell}$ along any closed path within a uniform magnetic field is always zero. Hence the closed path can be chosen of any size, even very small size enclosing a very small area. Hence we can prove that net current through each area of infinitesimally small size within region of uniform magnetic field is zero. Hence we can say no current (rather than no net current) flows through region of uniform magnetic field. Hence statement -2 is correct explanation of statement-1.

50. **(C)**

51. to 53.

The fan is operating at 200 V, consuming 1000 W,

then $I = \frac{1000}{200} = 5A$

But as coil resistance is 1Ω then power dissipated by internal resistance heat is $P_1 = I^2R = 25W$

If V is net emf across coil then

$$\frac{V^2}{R} = 25\text{ W} \qquad V = 5 \text{ volt}$$

Net emf = source emf – back emf

$V = V_s - e \Rightarrow e = 195$ V

The work done $P_2 = 1000 - 25 = 975$ W.

54. $\frac{dB}{dt} = 2T/s$

$$E = -\frac{AdB}{dt} = -800\times10^{-4}\,m^2\times 2 = -0.16\text{ V}$$

$$i = \frac{0.16}{1\Omega} = 0.16 \text{ A, clockwise}$$

55. At t = 2s $\qquad B = 4T; \ \frac{dB}{dt} = 2T/s$

$t = 2\text{ s} \quad B = 4T; \ \frac{dB}{dt} = 2T/s$

$A = 20 \times 30\ cm^2$

$= 600 \times 10^{-4}\ m^2; \ \frac{dA}{dt} = -(5 \times 20)\ cm^2/s$

$= -100 \times 10^{-4}\ m^2/s$

$$E = -\frac{d\varphi}{dt} = -\left[\frac{d(BA)}{dt}\right] = -\left[\frac{BdA}{dt} + \frac{AdB}{dt}\right]$$

$= -[4\times(-100 \times 10^{-4}) + 600 \times 10^{-4} \times 2]$

$= -[-0.04 + 0.120] = -0.08\ v$

Alternative :

$\phi = BA = 2t \times 0.2\ (0.4 - vt)$

$= 0.16t - 0.4\ vt^2$

$E = -\frac{d\varphi}{dt} = 0.8\ vt - 0.16$

at t = 2s

t = 2s

E = – 0.08 V

56. **(C)** At t = 2s, length of the wire

= (2 × 30 cm) + 20 cm = 0.8 m

Resistance of the wire $= 0.8\ \Omega$

Current through the rod $= \frac{0.08}{0.8} = \frac{1}{10}$ A

Force on the wire $= ilB = \frac{1}{10} \times (0.2) \times 4 = 0.08$ N

Same force is applied on the rod in opposite direction to make net force zero.

57. **(B)** Power $= \frac{\mu mg.v}{\text{fractional efficiency}}$

$$= \frac{0.45 \times 10^2 \times 10^3 \times 10 \times 20}{0.9} = 10^6 \text{ watt}$$

58. **(C)** $\varepsilon = B_{\perp} V\ell = B_V.V\ell$

$= 2 \times 10^{-5} \times 20 \times 1 = 40 \times 10^{-5}$ Volts

59. **(D)** Time taken $= \frac{324}{72} = \frac{9}{2}$ hour

extra power engine = Power dissipated in resistor

$$= \frac{\varepsilon^2}{R} = \frac{(40 \times 10^{-5})^2}{10^{-3}} = 16 \times 10^{-5} \text{ watt.}$$

For this the extra power consumed by the train engine will be $\frac{16 \times 10^{-5}}{0.9}$ watt.

$\therefore$ energy consumed $= \frac{16 \times 10^{-5}}{0.9}$ watt $\times \frac{9}{2}$ hour

$= 8 \times 10^{-7}$ KW hour

$= 8 \times 10^{-7}$ KW hour

60. Magnetic field is along x axis because when the cube is moved along x-axis, there is no motional emf as $\vec{v} \times \vec{B} = 0$. When the block is moved along y axis, force on the electrons is in direction

$-(j \times i) = k$

Therefore electric field will be created along z-axis.

Now, $\quad c\,.\,v.\,B = 24mV \quad \Rightarrow \quad c = 20$ cm

similarly $b\,vB = 36$ mV $\quad \Rightarrow \quad b = 30$ cm

$\therefore \quad a = 25$ cm

61. **(A)** Speed of charged particle cannot be changed by magnetic force because magnetic force does no work on charged particle. Only electric field in case (p), (t) and induced electric field in case (r) can change speed of charged particle.

(B) Magnetic field cannot exert a force on charged particle at rest. Only electric field in case (p), (t) and induced electric field in case (r) can exert force on charge initially at rest.

In case (r) after the charge starts moving even the magnetic field can exert force on charge.

(C) A charged particle can move in a circle within uniform speed due to uniform and constant magnetic field in case . Even within a region of non uniform magnetic field, at all points on a circle field may be uniform for example on any circle coaxial with a current carrying ring

(D) A moving charged particle is accelerated by electric field and also accelerated by magnetic field (provided v is not parallel to B).

62. (A) Due to current carrying wire, the magnetic field in loop will be inwards w.r.t. the paper. As current is increased, magnetic flux associated with loop increases. So a current will be induced so as to decrease magnetic flux inside the loop. Hence Induced current in the loop will be anticlockwise. The current in left side of loop shall be downwards and hence repelled by wire. The current in right side of loop is upwards and hence attracted by wire. Since left side of loop is nearer to wire, repulsive force will dominate. Hence wire will repel the loop

(B) Options in (B) will be opposite of that in (A)

(C) When the loop is moved away from wire, mag netic flux decreases in the loop. Hence the op tions for this case shall be same as in (B)

(D) When the loop is moved towards the wire, mag netic flux increases in the loop. Hence the op tions for this case shall be same as in (A)

63.

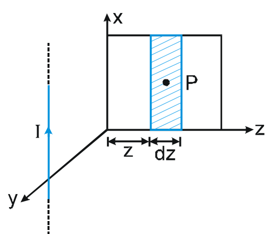


Figure - 1

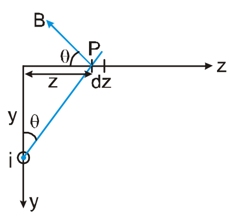


Figure - 2

The magnetic field at point P (figure -2) is

$$B = \frac{\mu_0}{2\pi}\frac{i}{\sqrt{y^2+z^2}}$$

The magnetic flux through the shaded strip in figure -1 is $d\phi = (W\,dz)\,\dfrac{\mu_0}{2\pi}\dfrac{i}{\sqrt{y^2+z^2}}\sin\theta$

where $\sin\theta = \dfrac{z}{\sqrt{y^2+z^2}}$

$\therefore$ total magnetic flux through rectangular loop is

$$\phi = \int_0^L \frac{\mu_0}{2\pi}\frac{i_0 \sin\omega t\; W\; z\,dz}{y^2+z^2}$$

$$= \frac{\mu_0}{4\pi} W \ln\left(\frac{y^2+L^2}{y^2}\right) i_0 \sin\omega t$$

$\therefore$ induced emf in the loop is

$$e = \frac{d\varphi}{dt} = \frac{\mu_0}{4\pi} i_0\, W\, \omega \cos\omega t \ln\left(\frac{L^2+y^2}{y^2}\right)$$

$$\therefore \quad e = \frac{d\varphi}{dt} = \frac{\mu_0}{4\pi} i_0\, W\, \omega \cos\omega t \ln\left(\frac{L^2+y^2}{y^2}\right)$$

$$\frac{\mu_0 i_0 W\omega\cos\omega t}{4\pi}\,\ell n\left(\frac{L^2}{y^2}+1\right)$$

64. $\displaystyle\int \vec{E}\,.\,d\vec{\ell} = -A\,.\,\frac{dB}{dt}$

As $B = 17 + (0.2)\sin(\omega t + \phi)$;

$E\,(2\pi r) = -\pi r^2 (0.2)\omega\,.\,\cos(\omega t + \phi)$

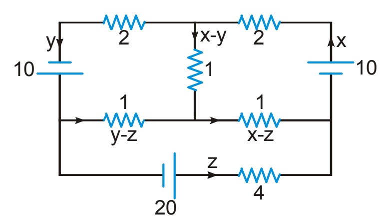
$E = -\dfrac{r}{2}(0.2)\omega\,.\,\cos(\omega t + \phi)$

Magnitude of the amplitude $= \dfrac{r}{2}(0.2)\,.\,\omega = 240$ mN/C

65. The induced emf in loop ABHFG

$= \dfrac{d}{dt}(BA) = A\dfrac{dB}{dt} = 2 \times 10 = 20$ V

The induced emf in loop BCDH & DEFH



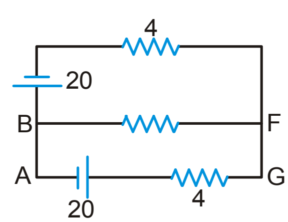
$= 1 \times 10 = 10$ volt . KVL in top left loop is ,

$10 - (y - z) + (x - y) - 2y = 0$

$\Rightarrow \quad x - 4y + z = -10 \qquad$ ...... **(1)**

$\Rightarrow \quad -4x + y + z = -10 \qquad$ ......**(2)**

no current in DH



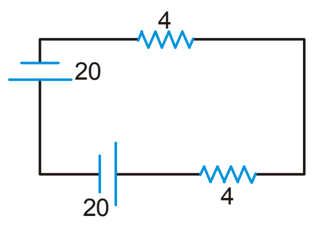
{ This can also be seen by symmetry }

This makes solution very simple now the circuit is , Assume $v_B = 0$, & $v_F = v$,

then $\frac{v+20}{4} + \frac{v-20}{4} + \frac{v-0}{2} = 0$

$\Rightarrow$ **v = 0** $\Rightarrow$ no current in FB

$\therefore$ circuit is :



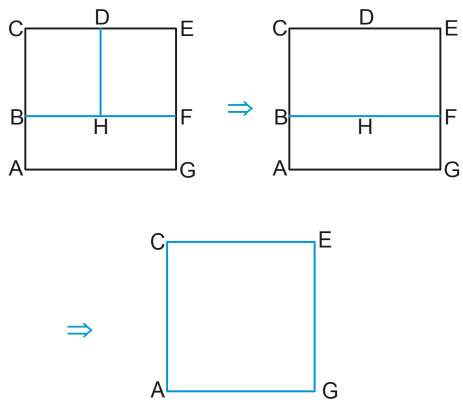
$\therefore$ rate of heat production = $(40)^2/8$ = 200 watt . Ans.

Note : If you are not able to observe the symmetry or decide $x - y = 0$, then write kVL in the lower loop . It will be $x + y - 6z = -20$ —— **(3)**

Solving (1) , (2) and (3) you will get $x = +5$, $y = 5$, $z = 5$ A . Heat rate will be ,

$10x + 10y + 20z = 40 \times 5 = 200$ W

Method II



$$H = I^2R = \frac{\varepsilon^2}{R}$$

where $R = 2 \times 4 \times 1 = 8\ \Omega$ and $\varepsilon = 4 \times 10 = 40$

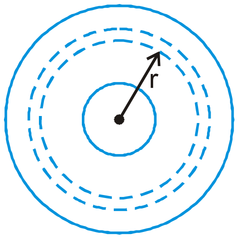
66. The magnetic field inside is only due to current of inner cylinder.

$$B = \frac{\mu_0 i}{2\pi r}$$

Magnetic field energy density is not uniform in the space in between cylinders. At distance r from the centre

$$u_B = \frac{B^2}{2\mu_0} = \frac{\mu_0 i^2}{8\pi^2 r^2}$$

Energy in volume of element (length ℓ)



$$dU_B = u_B \cdot dV = \frac{\mu_0 i^2}{8\pi^2 r^2} \cdot (2\pi r\ell)\, dr = \frac{\mu_0 i^2 \ell}{4\pi} \cdot \frac{dr}{r}$$

$$U_B = \int dU_B = \frac{\mu_0 i^2 \ell}{4\pi} \int_a^b \frac{dr}{r} = \frac{\mu_0 i^2 \ell}{4\pi} \ell n \frac{b}{a}$$

Using values

U = 140 nJ