HINTS & SOLUTIONS

EXERCISE - 1

Single Choice

1. Radius of $Os^{189} = r_0 \; A_{Os}^{1/3}$

$O_s^{189} = r_0 \; A_{Os}^{1/3}$

Radius of that nucleus $= \frac{1}{3} \times r_0 \left(A_{O_s}\right)^{\frac{1}{3}}$

$= r_0 \left(\frac{189}{27}\right)^{\frac{1}{3}} = r_0 \, 7^{1/3}$

$\therefore$ A for that nucleus = 7 $\Rightarrow$ A = 7

2. (D)

3. The binding energy per nucleon in a nucleus varies in a way that depends on the actual value of A.

4. The energy of the reaction $Li^7 + p \longrightarrow 2\,He^4$ is (the binding energy per nucleon in Li^7 and He^4
$Q = (2BE_{He} - BE_{Li})$
$= (2 \times 7.06 \times 4 - 5.60 \times 7)$ Mev **= 17.28 Mev.**

5. (D) **6.** (B)

7. ${}_2^4He + {}_7^{14}N \rightarrow {}_8^{17}O + {}_1^1H$

8. (A)

9. Energy of γ photon

= Difference in energies of α particles = 0.4 MeV

10. No. of nucleus of P, $N_P = \frac{m}{10} \times N_A$

No. of nucleus of Q, $N_Q = \frac{m}{20} \times N_A$

No. of Isotope P after 20 days, $N_P{}' = \frac{N_P}{4}$

Let no. of Isotope Q after 20 days be $N_Q{}'$

$N_Q{}' = 2 \times N_P{}' = \frac{N_P}{2} = NQ$

Thus no change in number of Q. Hence its half life is infinity.

11. $T_{avg.} = \frac{1}{\lambda} \Rightarrow T_{1/2} = \frac{\ln 2}{\lambda} < T_{avg.}$

12. (B)

13. $\frac{dN_2}{dt} = \lambda N_1 - 2\lambda N_2$

for N_2 to be maximum, N_2

$\frac{dN_2}{dt} = 0 \Rightarrow \lambda N_1 = 2\lambda N_2$ or $\frac{N_1}{N_2} = 2$

14. (A)

15. ${}_{92}U^{235} + n \longrightarrow {}_{54}Xe^{139} + {}_{38}Y^{94} + 3n$

16. (D) **17.** (D) **18.** (C)

19. No. of nuclear spliting per second is

$N = \frac{100MW}{200MeV} = \frac{100}{200 \times 1.6 \times 10^{-19}} \; S^{-1}$

No. of neutrons Liberated $= \frac{100}{200} \times \frac{1}{1.6 \times 10^{-19}} \times 2.5 \; S^{-1}$

$= \frac{125}{16} \times 1018 \; S^{-1}$

20. (C)

21. Fusion reaction is possible at high temperature because kinetic energy is high enough to overcome repulsion between nuclei.

22. (A)

23. Energy released $= [(80 \times 7) + (120 \times 8)] - [200 \times 6.5]$
$= 220$ MeV **Ans.**

24. (B)

25. $R = R_0 \; A^{1/3}$

$\ln \frac{R}{R_0} = \frac{1}{3} \ln A$

It is similar to y = mx.

26. $R = \frac{mv}{2B} \Rightarrow R_p = \frac{m_P v}{eb} \Rightarrow R_{235_U} = \frac{m_{235_U} . v}{eB}$

$\Delta x = 2 \times \frac{3M_P v}{2B} = 6 \times 10$ mm **= 60 mm.**

27. $A = A_{0]} \, e^{-\lambda t}$
$\ell n A = \ell n A_0 - \lambda t$
ℓn A versus t is a linearly decreasing graph with slope depending to λ. As λ does not change, slope remains same.

28. (A)

29. Initial activity $= \left|\frac{dN}{dt}\right| = \lambda N_0 = \lambda.\frac{m}{M}.NA$

30. (A) **31.** (D) **32.** (C)

33. As a proton is lighter than a neutron, proton can not be converted into neutron without providing energy from outside. Reverse is possible. The weak interaction force is responsible in both the processes (i) conversion of p to n and (ii) conversion of n to p.

34. (B)

35. $A_P = A_Q e^{-\lambda t} = A_Q e^{-\frac{1}{T}t} \quad \therefore \quad t = T\,\ell n \frac{A_Q}{A_P}$

36. (B)

37. $n = \lambda N = \lambda = \frac{n}{N} \qquad \therefore t_{1/2} = \frac{0.69}{\lambda} = \frac{0.69\ N}{n}$

38. (D)

EXERCISE - 2

Part # I : Multiple Choice

1. (C,D)

2. The total number of nucleons will be A − 4 and the number of neutrons will be A − Z − 3.

3. $\left|\frac{dN}{dt}\right| = \lambda N = \frac{\ln 2}{T_{1/2}} \times \frac{1\times 6.02\times 10^{23}}{238} \Rightarrow T_{\frac{1}{2}}$

$= \frac{\ln 2 \times 6.023\times 10^{23}}{238\times 1.24\times 10^4} =$ **4.5 × 109 yrs.**

The activity = number of disintegration per second = 1.24×10^4 dps

4. Given $\lambda = 0.173 \Rightarrow T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{0.173} \cong 4$

Also $N_0 - N = N_0 e^{-\lambda t}$

For $t = \frac{1}{0.173}$ year : $\Rightarrow N_0 - N = \frac{N_0}{e} = 0.37 N_0$

5. (A,C,D)

6. (C,D)

Part # II : Assertion & Reason

1. (D)

2. Statement-2 is true by definition and correctly explains the staement-1, namely, ${}_{Z}X^{A}$ undergoes 2 α decays, 2β decays (negative β) and 2 γ decays. As a result the daughter product is ${}_{Z-2}Y^{A-8}$.

3. In spontaneous reaction, binding energy increases.

EXERCISE - 3

Part # I : Matrix Match Type

1. A → Q,R,S ; B → Q,R,S ; C → Q,R,S ; D → P,Q,R,S

2. **(A)** Energy is released in all the four processes. Hence mass will decrease

(B) Since energy is released, Binding energy per nucleon will increase.

(C) Mass number conserves in all the processes.

(D) Total charge also conserves in all the processes

3. (A)

Part # II : Comprehension

Comprehension # 1

1. (B) **2.** (D) **3.** (D)

Comprehension # 2

1. In equilibrium, rate of decay = rate of production

2. As Rate of decay = Rate of production

$\Rightarrow P = \lambda N \Rightarrow N = \frac{P}{\lambda} = \frac{P t_{1/2}}{\ell n\, 2} = 1.8 \times 10^{15}$

3. As $N = \frac{P t_{1/2}}{\ell n\, 2}$

it is dependent upon P and $t_{1/2}$. Initial no. of ^{56}Mn nuclei will make no difference as in equilibrium rate of production equals rate of decay. Large initial no. will only make equilibrium come sooner.

EXERCISE - 4

Subjective Type

1. B.E. $= \left[3M_{{}_1H^1} + 4m_{{}_0n^1} - M_{{}_3Li^7}\right] 931$ MeV = 39.22 MeV, $\frac{B.E.}{A} = \frac{39.22}{7} = 5.6$ MeV

2 **(i)** $r_1 = \left[\frac{4\times 10^{30}}{3\times 10^{17}} \times \frac{3}{4\pi}\right]^{1/3} = 14.71$ km

(ii) $r_2 = \left[\frac{6\times 10^{24}}{3\times 10^{17}} \times \frac{3}{4\pi}\right]^{1/3} = 168.4$ m

3. $E = 20 \times (8.03) - 2\times 4\,(7.07) - 12(7.68) = 11.9$ MeV

4. **(a)** (0.680 – 0.180) Me V = 500 ke V

(b) $\frac{500\times 10^3 e}{C} = 2.67 \times 10^{-22}$ kg–m/s

5. When α - particle will escape, the daughter nucleus will recoil back with same momentum. Applying momentum conservation $p_\alpha = p_d$

Total energy released

TE = KE of α + KE of daughter nucleus

$$= \frac{p_\alpha^2}{2m_\alpha} + \frac{p_d^2}{2m_d} = \frac{p_\alpha^2}{2}\left(\frac{1}{m_\alpha}+\frac{1}{m_d}\right)$$

$$= m_\alpha.E_\alpha\left(\frac{1}{m_\alpha}+\frac{1}{m_d}\right) = E_\alpha\left(1+\frac{m_\alpha}{m_d}\right)$$

$$= 4.78\left(1+\frac{4}{222}\right) = 4.87 \text{ Mev.} = 2.67\times10^{-22} \text{ Kg-m/s.}$$

6. Specific activity = No. of particles emitted per second by 1g of the substance

$$\text{specific activity} = \frac{N_A}{24}\times\frac{0.693}{T_{\frac{1}{2}}} = \mathbf{3.2\times10^{17}\ dps/g}$$

For U^{235}

$$\text{specific activity} = \frac{N_A}{235}\times\frac{0.693}{T_{\frac{1}{2}}} = 0.8\times10^5 \text{ dps/g}$$

7. $$\frac{6\times10^{23}\times10^{-6}}{24}\left[1-e^{-0.693/15}\right] = 1.125\times10^{15}$$

8. **(a)** $\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{14\times60} S^{-1} = 8.25\times10^{-4}\ S^{-1}$

(b) $n = p + e^- + \bar{\upsilon} + Q$

$Q = (m_n - m_p - m_{e^-}).C^2$

$= (1.008665 - 1.007276 - 0.0005486)\times931$ Mev.= **782 Kev.**

9. $U^{235} + n \longrightarrow M_0^{95} + La^{139} + 2n + Q$

$Q = (m_u + m_n - m_{M_0} - m_{La} - 2m_n).C^2$

$= (235.0439 + 1.0087 - 94.9058 - 138.9061$
$- 2\times1.0087)\times931$ Mev. = **207.9 MeV.**

10. Let λ = decay constant

N_0 = number of active nuclei = 3.0×10^{16}

number of α -particles emitted per second= λN_0

number of α-particle falling on the window

$$= \frac{\lambda N_0 a}{4\pi R^2} = \frac{4\pi R^2 A}{N_0 a}$$

$$= \frac{4\pi\times5\times10^4}{3.0\times10^{16}\times10^{-4}} = 2.1\times10^{-7}\ s^{-1}.$$

11. $$\frac{2}{Q}\times\frac{100}{30}\times\frac{50}{1.6\times10^{-19}}\times\frac{2}{N_A}\times10^{-3}\text{Kg} = 2.9\times10^{-7}\text{ kg}$$

where $Q = (2M_{{}_1H^2} - M_{{}_2He^4})\times931 = 23.834531$ MeV

12. Each deuterium nucleus produces 17.6 MeV.

1 kg of deuterium = $\frac{1\times10^3}{2} N_A$ no. of deuterium

$\equiv 17.6\times\frac{10^3}{2} N_A$ MeV energy produced.

$\therefore$ To produce 1 MW, amount of deuterium in kg required per second.

$$= \frac{1\times10^6}{17.6\times\frac{10^3}{2}\times N_A\times e\times10^6} \text{ kg/s} = 1.179\times10^{-9} \text{ kg/s}$$

similarly for tritium. $\frac{1\times10^6}{17.6\times\frac{10^3}{2}\times N_A\times e\times10^6}$ kg/s

$= 1.769\times10^{-9}$ kg/s

13. Let N = no. of isotope at any instant

$$\frac{dN}{dt} = R - \lambda N \Rightarrow \frac{dN}{R-\lambda N} = dt$$

On integrating with initial condition t = 0, N = 0, we get

$$\int_O^N \frac{dN}{R-\lambda N} = \int_o^t dt \Rightarrow \frac{1}{-\lambda}\ln\frac{R-\lambda N}{R} = t$$

$$\Rightarrow N = \frac{R}{\lambda}(I - e^{-\lambda t}) \Rightarrow \text{when } t >> t_{1/2}$$

$$N \approx \frac{R}{\lambda} = \frac{Rt_{1/2}}{\ell n2}$$

14. $$\frac{CTM\times100}{m\,N_A\,\ell n2}$$

$$= \frac{480\times75.5\times1.3\times10^9\times365\times86400\times100}{0.693\times N_A} = 0.36\%$$

15. Activity of the sample after time t_1

$$A = N\lambda = N_0\lambda e^{-\frac{t}{\tau}}$$

Enrgy stored in the compaciter

$$E = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{(Q_0 e^{-\frac{t}{RC}})^2}{C} = \frac{1}{2}\frac{Q_0^2}{C}e^{-2\frac{t}{RC}}$$

$$\frac{A}{E} = \frac{N_0\lambda}{Q_0^2/2C} \cdot e^{\left(-\frac{t}{\tau}+\frac{2t}{RC}\right)}$$

For the ratio to remain constant

$$-\frac{1}{\tau} + \frac{2}{RC} = 0 \Rightarrow R = \frac{2\tau}{C}$$

16. $A_\beta = N_0 \lambda_1 \exp.(-\lambda_1 t) = 0.72 \times 10^{11}$ part/s,

$$A_\alpha = \frac{N_0 \lambda_1 \lambda_2}{\lambda_2 - \lambda_1}\left[e^{-\lambda_1 t} - e^{-\lambda_2 t}\right] \approx \lambda_2 N_0\, 2^{-\frac{1}{4.6}}$$

$= 1.4 \times 10^{11}\ s^{-1}$

17. $P = 1 - e^{-\lambda t}$

18. $\frac{1850}{235} eN_A$ sec. $= 8.781$ days

19. $^4He + {}^4He = {}^8Be + Q$

(1) $\Delta m = 2 \times 4.0026 - 8.0053$

$= 8.0052 - 8.0053 = -0.0001$ amu **Ans**

(2) $Q = (2m_{He} - m_{Be}).C^2$

$= (2 \times 4.0026 - 8.0053) \times 931$ MeV

$= -93.1$ KeV

Since Q is negative the fusion is not energetically favorable.

20. 79 gm, $\frac{N_A}{4}$

21. **(i)** $E = [M_U + m_n - M_{Ba} - M_{Kr} - 3m_n]\, 931$

$= 200.57$ MeV

(ii) $\frac{N_A}{235} \times E = 22.84$ MWh

22. $Q = -\frac{11}{12} T_{th} = -3.7$ MeV

23. Applying conservation of energy

$m_A c^2 + K_A + m_B c^2 + K_B = m_c c^2 + K_C +$ excitation energy Z

$(m_A + m_B - m_C) c^2 + K_A + K_B = K_C +$ excitation energy

$4.65 + 5 + 3 = K_c + 10$

or $K_c = 2.65$ MeV **Ans. 2.65 MeV**

24. 6000

25. $N^{14} + \alpha \rightarrow O^{17} +$ proton

Q value $Q = (14.00307 + 4.00260 - 1.00783 - 16.99913)$

$931.5 = -1.20$ MeV

Let m and M be mass of α particle and nitrogen nucleus respectively and let minimum KE of α particle be $\frac{1}{2}mu^2$.

From energy equation

$$\frac{1}{2}mu^2 = |Q| + \text{minimum KE of system}$$

$$= |Q| + \frac{1}{2}(m+M)\left[\frac{mu}{(m+M)}\right]^2$$

$$\therefore \frac{1}{2}mu^2\left(\frac{M}{m+M}\right) = |Q| = 1.2 \times \frac{4}{14} = 0.34 \text{ MeV}$$ **Ans.**

26. $3.32 \times 10^{-5}\ Js^{-1}$

27. $2 \times 10^{11}\ kg/cm^3$, 1×10^{38} nucl./cm^3

28. BE = Original Binding energy + Binding energy due to additional mass defect

$= 70 + \frac{1}{100}(930) = 79.3$ MeV ...

EXERCISE - 5

Part # I : AIEEE/JEE-MAIN

1. N_0 is the initial amount of substance and N is the amount left after decay.

Thus, $N = N_0\left(\frac{1}{2}\right)^n$

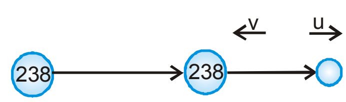
$n = \text{no. of half lives} = \frac{1}{t_1/2} = \frac{15}{5} = 3$

Therefore, $N = N_0\left(\frac{1}{2}\right)^3 = \frac{N_0}{8}$

2. (1)

3. By conservation of linear momentum

$0 = 234\vec{v} + 4\vec{u}$



$\vec{v} = \frac{-4\vec{u}}{234} \Rightarrow$ speed $v = \frac{4u}{234}$

4. Given : $N_0\lambda = 5000, N\lambda = 1250$

$N = N_0 e^{-\lambda t} = N_0 e^{-5\lambda}$

$1250 = N_0 \lambda e^{-5\lambda}$

$$\therefore \frac{N_0\lambda}{N_0\lambda e^{-5\lambda}} = \frac{5000}{1250} = 4$$

$e^{5\lambda} = 4 \Rightarrow 5\lambda = 2\log_e 2 \Rightarrow \lambda = 0.4 \ln 2$

5. Since, 8 α-particles and 2β- particles are emitted so, new atomic number

$Z' = Z - 8 \times 2 + 2 \times 1$

$= 92 - 16 + 2 = 78$

6. Protons cannot be emitted by radioactive substances during decay, because proton remains inside nucleous.

7. $\frac{3}{2}kT = 7.7 \times 10^{-14} J$

$$T = \frac{2 \times 7.7 \times 10^{-14}}{3 \times 1.38 \times 10^{-23}} = 3.7 \times 10^9 K$$

8. Law of conservation of momentum gives

$$m_1 v_1 = m_2 v_2 \Rightarrow \frac{m_1}{m_2} = \frac{v_2}{v_1}$$

But $m = \frac{4}{3}\pi r^3 \rho$ or $m \propto r^3$

$$\therefore \frac{m_1}{m_2} = \frac{r_1^3}{r_2^3} = \frac{v_2}{v_1} \Rightarrow \frac{r_1}{r_2} = \left(\frac{1}{2}\right)^{1/3} \quad \therefore r^1 : r^2 = 1 : 2^{1/3}$$

9. As given

${}_1H^2 + {}_1H^2 \longrightarrow {}_2He^4 + \text{energy}$

The binding energy per nucleon of deuteron (${}_1H^2$)

$= 1.1$ MeV

$\therefore$ Total binding energy

$= 2 \times 1.1 = 2.2$ MeV

The binding energy per nucleon of helium

(${}_2He^4$) = 7 MeV

$\therefore$ Total binding energy

$= 4 \times 7 = 28$ MeV

Hence, energy released in above process

$= 28 - 2 \times 2.2 = 28 - 4.4 = 23.6$ MeV

10. According to law of conservation of energy, kinetic energy of α – particle

= the potential energy of α – particle at distance of closest approach.

i.e $\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$

$$\therefore 5 \text{ Me V} = \frac{9 \times 10^9 \times (2e) \times (92e)}{r}$$

$$\left(\because \frac{1}{2}mv^2 = 5 \text{MeV}\right)$$

$$\Rightarrow r = \frac{9 \times 10^9 \times 2 \times 92 \times (1.6 \times 10^{-19})^2}{5 \times 10^6 \times 1.6 \times 10^{-19}}$$

$\therefore r = 5.3 \times 10^{-14} m \approx 10^{-12} cm$

11. $N = N_0(1 - e^{-\lambda t}) \Rightarrow \frac{N_0 - N}{N_0} = e^{-\lambda t}$

$\therefore \frac{1}{8} = e^{-\lambda t} \Rightarrow 8 = e^{\lambda t} \Rightarrow 3 \ln 2 = \lambda t$

$\Rightarrow \lambda = \frac{3 \times 0.693}{15}$

$t_{1/2} = \frac{0.693}{3 \times 0.693} \times 15 \quad t_{1/2} = 5$ min

12. R R$_0$ $(A)^{1/3} \Rightarrow \frac{R_{Al}}{R_{Te}} = \frac{R_0 (A_{Al})^{1/3}}{R_0 (A_{Te})^{1/3}} = \frac{3}{5}$

$\therefore R_{Te} = \frac{5}{3} \times 3.6 \Rightarrow R_{Te} = 6$ Fermi

13. ${}_Z^AX + {}_0^1n \rightarrow {}_3^7Li + {}_2^4He$

It implies that

$A + 1 = 7 + 4 \Rightarrow A = 10$

and $Z + 0 = 3 + 2 \Rightarrow Z = 5$

Thus, it is Boron ${}_5^{10}B$

14. (4)

15. Gamma-photon.

16. (4)

17. EP = $(8 \times 7.06 - 7 \times 5.60)$ MeV = 17.28 MeV

18. Nuclear binding energy = [mass of nucleus – mass of nucleons] $C^2 = (M_o - 8M_p - 9M_N)C^2$

19. Gamma ray is electromagnetic radiation which does not involve any change in proton number or neutron number

20. $\frac{\ell n 2}{\lambda_x} = \frac{1}{\lambda_Y} = \lambda_Y = 1.4\lambda_X$, $\lambda_Y > \lambda_X$, Y will decay faster than X.

21. (3)

22. If binding energy of product nuclei is greater then energy is released.

23. Energy is released

$\therefore (B.E.)_{product} > (B.E.)_{Reactant}$

24. $Q = \Delta m c^2 = \frac{1}{2} \times \left(\frac{M}{2}\right) v^2 + \frac{1}{2} \times \left(\frac{M}{2}\right) v^2$

$\Delta mc^2 = \frac{1}{2} \times Mv^2 \Rightarrow v = c\sqrt{\frac{2\Delta m}{M}}$

25. ${}_Z X^A \longrightarrow 3\,{}_2He^4 + {}_{Z-8}Y^{A-12} + 2\,{}_{+1}e^0 + 2\upsilon$

number of proton = Z – 8

number of neutron = (A – 12) – (Z – 8) = A – Z – 4

ratio is $\frac{A-Z-4}{Z-8}$

26. $\frac{2}{3}N_0 = N_0e^{-\lambda t_1} \Rightarrow \frac{1}{3}N_0 = N_0e^{-\lambda t_2}$

$2 = e^{\lambda(t_2-t_1)} \Rightarrow \lambda(t_2-t_1) = \ell n\,2$

$\Rightarrow (t_2-t_1) = \frac{\ell n 2}{\lambda} = 20$ min. **Ans.**

27. **Statement–1:** Energy of β^- particle from 0 to maximum so $E_1 - E_2$ is the continuous energy spectrum.
Statement–2 : For energy conservation and momentum at least three particles daughter nucleus $+\beta^{-1}$ and antineutron.

28. ${}_0n^1 \rightarrow {}_1H^1 + {}_{-1}e^0 + \overline{\nu} + Q$

$\Delta m = m_n - m_\alpha - m_e$

$= (1.6725 \times 10^{-27} - 1.6725 \times 10^{-27} - 9 \times 10^{-31})$ kg

$= -9 \times 10^{-31}$ kg

Energy = $9\times 10^{-31} \times (3 \times 10^8)^2 = 0.511$ MeV

Which is nearly equal to 0.73 Mev

but as energy will be required.

since mass is increasing

so answer = –0.511 Mev

either (1) or bonus.

29. $\Delta E = h\nu$

$$\nu = \frac{\Delta E}{h} = k\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right] = \frac{k2n}{n^2(n-1)^2} \approx \frac{2k}{n^3} \propto \frac{1}{n^3}$$

Part # II : IIT-JEE ADVANCED

1. **(i)** During γ-decay atomic number (Z) and mass number (A) does not change. So the correct option is (C) because in all other options either Z, A or both is/are changing.

(ii) $R = R_0\left(\frac{1}{2}\right)^n$ ....**(1)**

Here R = activity of radioactive substance after n half lives

$= \frac{R_0}{16}$ (given)

Substituting in equation (1), we get n = 4

$\therefore$ $t = (n)t_{1/2} = (4)\,(100\ \mu s) = 400\ \mu s$

$R = R_0\left(\frac{1}{2}\right)^n$ ....**(1)**

2. The magnitude of momentum of the daughter nucleus and α-particles will be equal

Q = KE of daughter nucleus + KE of α-particle

$= \frac{p^2}{2m_d} + \frac{p^2}{2m_\alpha}$

KE of α-particle $= \frac{p^2}{2m_\alpha} = \frac{1}{m_\alpha} \times \frac{m_\alpha . m_d}{m_\alpha + m_d} . \alpha$

$= \frac{216}{220} \times 5.5$ Mev. = **5.4 Mev.**

3. Nuclear density is constant hence, mass $\propto$ volume or $m \propto V$

4. Let n_0 be the number of radioactive nuclei at time t = 0. Number of nuclei decayed in time t are given $n_0(1 - e^{-2\lambda})$, which is also equal to the number of beta particles emitted the same interval of time. For the given condition,

$n = n_0\,(1 - e^{-2\lambda})$ ......**(i)**

$(n + 0.75\,n) = n_0\,(1 - e^{-4\lambda})$ ......**(ii)**

Dividing (ii) by (i) we get

$1.75 = \frac{1-e^{-4\lambda}}{1-e^{-2\lambda}}$ or $1.75 - 1.75\,e^{-2\lambda} = 1 - e^{-4\lambda}$

$\therefore$ $1.75\,e^{-2\lambda} - e^{-4\lambda} = \frac{3}{4}$ .....**(iii)**

Let us take $e^{-2\lambda} = x$

Then the above equation is,

$x^2 - 1.75x + 0.75 = 0$

or $x = \frac{1/75\sqrt{(1.75)^2 - (4)(0.75)}}{2}$

or $x = 1$ and $\frac{3}{4}$

$\therefore$ From equation **(iii)** either $e^{-2\lambda} = 1$ or $e^{-2\lambda} = \frac{3}{4}$

but $e^{-2\lambda} = 1$ is not accepted because which means $\lambda = 0$.

Hence

$e^{-2\lambda} = \frac{3}{4}$ or $-2\lambda\ \ell n\,(e) = \ell n(3)$-$\ell n\,(4) = \ell n(3) - 2\,\ell n(2)$

$\therefore$ $\lambda = \ell n(2) - \frac{1}{2}\ell n(3)$

Substituting the given values,

$\lambda = 0.6931 - \frac{1}{2} \times (1.0986) = 0.14395\ s^{-1}$

$\therefore$ Mean life $t_{means} = \frac{1}{\lambda} = 6.947$ sec

5. Activity reduces from 6000 dps to 3000 dps in 140 days. It implies that half-life of the radioactive sample is 140 days. In 280 days (or two half-lives) activity will remain 1/4th of the initial activity. Hence the initial activity of the sample is - 4×6000 dps $=$ 24000 dps
Therefore, the correct option is (D)

6. Let N_0 be the initial number of nuclei of ^{238}U.

After time t, $N_U = N_0 \left(\frac{1}{2}\right)^n$

Here n = number of half-lives $= \frac{t}{t_{1/2}} = \frac{1.5\times10^9}{4.5\times10^9} = \frac{1}{3}$

$N_U = N_0 \left(\frac{1}{2}\right)^{\frac{1}{3}}$ and $N_{Pb} = N_0 - N_U = N_0\left[1-\left(\frac{1}{2}\right)^{\frac{1}{3}}\right]$

$\therefore \frac{N_U}{N_{Pb}} = \frac{\left(\frac{1}{2}\right)^{\frac{1}{3}}}{1-\left(\frac{1}{2}\right)^{3}} = 3.861$

7. $\Delta m = 4m_{He} - m_0$
$\Delta m = .0.011$ amu $\Delta E = \Delta M C^2 = 0$
binding energy per/Nucleon = .176/16 amu = 10.24 meV

8. Number of nuclei left after 2 half lives $= \frac{N_0}{4}$
probability of a nucleus decaying = no. of nuclei decayed total no.
$= \frac{3\, N_0/4}{N_0} = \frac{3}{4}$

9. Fission is the process which involves conversion of some matter into energy and shown by atoms of high atomic number such as $_{92}U^{235}$.
Fusion also converts some matter into energy but shown by atoms of low atomic number such as $_1H^1$, $_1H^2$.
β-decay essentially carried out by weak nuclear forces and also converts matter into energy for example
$_0n^1 \rightarrow {}_1H^1 + {}_{-1}e^0\,(\beta^-) + \bar{\nu}$ + energy
$_1H^1 \rightarrow {}_0n^1 + {}_{+1}e^0\,(\beta^+) + \nu$ + energy
Exothermic nuclear reactions also converts matter into energy and all spontaneous nuclear reaction are exothermic.

10. Since energy is released in a fission process, the rest mass energy must decrease.

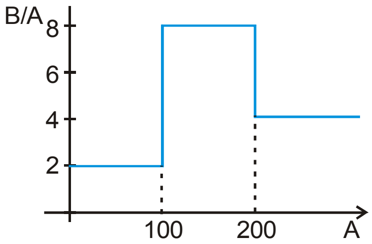
11. Different related laws / processes.

12. (A) For $1 < A < 50$, on fusion mass number for compound nucleus is less than 100. B/A remains same. Hence no energy is released

(B) For $51 < A < 100$, on fusion mass no. of compound nucleus is between 100 and 200. B/A increases. Hence energy is released.

(C) On fission for $100 < A < 200$, the mass no. for fission nuclei will be between 50 to 100. B/A decreases. Hence no energy is released.

(D) On fission for $200 < A < 260$, the mass no. for fission nuclei will be between 100 to 130, B/A will increase. Hence energy is released.



13. Given that $\lambda_1 N_1 = 5\mu Ci$
$\lambda_2 N_2 = 10\mu Ci$
$\lambda_2 N_2 = 2\lambda_1 N_1$
Also $N_1 = 2N_2$
Then $\lambda_2 N_2 = 2\lambda_1(2N_2)$
$\lambda_2 = 4\lambda_1$

15. From energy conservation

$$2\left(\frac{3}{2}kT\right) = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

$$\Rightarrow T = \frac{(1.44\times10^{-9})}{(8.6\times10^{-5})} \times \frac{1}{3\times4\times10^{-15}} = 1.39 \times 10^9\ K$$

16. $nt_0 > 5 \times 10^{14}$ s/cm^3
for deuteron density $= 8.0 \times 10^{14}$ cm^{-3}, confinement time $= 9.0 \times 10^{-1}$ s
$nt_0 = 7.2 \times 10^{14}$ s/cm^3

17. (A)
(P) : Capacitor is charged, hence its energy is increased
(Q) : The temperature is increased, hence its energy is increased or as the external positive work is done, hence energy increases
(R) : The temperature decreases, its energy is decreased
(S) : All natural process, energy of the system decreases
(T) : The current is produced. Hence energy of the system increases

(B)
(P), (R), (S) no mechanical energy is provided to the system
(Q) the mechanical energy is provided which increases the temperature and hence random motion of molecules

(T) Mechanical work is done to change the magnetic field, which increases the mechanical energy of electron and these electrons strike with stationary positive charge and energy is converted in random motion.

(C)

(S) Internal binding energy is converted into mechanical energy

(D)

(S) Mass changes only in nuclear process.

18. $-\dfrac{dN}{dt} = \lambda N \Rightarrow -\dfrac{dN}{dt} = \lambda N_0 e^{-\lambda t}$

$\ell n\left|\dfrac{dN}{dt}\right| = -\lambda t + \ell n(\lambda N_0) \quad \Rightarrow y = mx + c$

$m = -\lambda \quad \Rightarrow \lambda = \dfrac{1}{2}$ [slope by graph $= \dfrac{1}{2}$]

$T = \dfrac{\ell n 2}{\lambda} = 2 \times 0.693 = \dfrac{4.16}{n}$

$n = 3 =$ no. of half life. $\Rightarrow \quad p = z^3 = 8.$

19. $N = N_0 e^{-\lambda t}$

$\dfrac{dN}{dt} = 10^{10} = N_0 (\lambda)\, e^{-10^{-9} t}$

at $(t = 0)$

$10^{10} = N_0 10^{-9}$

$N_0 = 10^{19}$

mass of sample $= N_0 10^{-25}$

$= N_0$ (mass of the atom)

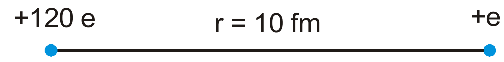
$= 10^{-6}$ kgm

$= 10^{-6} \times 10^3$ gm

$= 10^{-3}$ gm

$= 1$ mg

20.



$\dfrac{(9\times10^9)(120e)(e)}{10\times10^{-15}} = \dfrac{p^2}{2m} \quad \lambda = \dfrac{h}{p} \quad \therefore p^2 = \dfrac{h^2}{\lambda^2}$

$2\left(\dfrac{5}{3}\times10^{-27}\right)10^{15}(9\times10^9)(12)e^2 = \dfrac{h^2}{2m\lambda^2}$

$(120)(3)10^{-27+15+9} \quad \lambda^2 = (4.2)^2 \times 10^{-30}$

$\lambda^2 = \dfrac{4.2\times4.2\times10^{-30}}{360\times10^{-3}} = \dfrac{42\times42}{360}\times10^{-29}$

$= 7^2 \times 10^{-30} \quad \lambda = 7\times10^{-15}\text{ m} = 7\text{ fm}$

21. KE_{max} of $\beta^- \Rightarrow \quad Q = 0.8 \times 10^6$ eV

$KE_P + KE_{\beta^-} + KE_{\bar{\nu}} = Q \Rightarrow KE_P$ is almost zero

When $KE_{\beta^-} = 0 \Rightarrow$ then $KE_{\bar{\nu}} = Q - KE_P \cong Q$

22. $0 \le KE_{\beta^-} \le Q - KE_P - KE_{\bar{\nu}}$

$0 \le KE_{\beta^-} < Q$