HINTS & SOLUTIONS

EXERCISE - 1

Single Choice

9. Sum of interior angles of a n sided polygon

$= (n-2)\times 180$

$= \frac{n}{2}[240 + (n-1)5]$

$\Rightarrow$ $n = 9, 16$

n = 16 is to be rejected.

$(T_{16} = 120° + 15\times 5° = 195° > 180°)$

11. $\frac{S_{kx}}{S_x} = \frac{\frac{kx}{2}[2a+(kx-1)d]}{\frac{x}{2}[2a+(x-1)d]} = k$

$\left[\frac{2a+(kx-1)d}{2a+(x-1)d}\right]$

If $2a - d = 0$, then $\frac{S_{kx}}{S_x}$ is independent of x

So $d = 2a$

12. $a_1 \cdot a_2 \cdot a_3 \cdot ....a_n = c$

$\frac{a_1 + a_2 + a_3 + ...2a_n}{n} \ge (a_1a_2a_3 \,...\, 2a_n)^{1/n} \ge (2c)^{1/n}$

$\Rightarrow$ $a_1 + a_2 + a_3 + ...2a_n \ge n(2c)^{1/n}$

17. Horizontal $1 + \frac{1}{4} + \frac{1}{16} + ...\infty = \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$

Vertical $\frac{1}{2} + \frac{-1}{8} + \frac{1}{32} .....\infty = \frac{\frac{1}{2}}{1+\frac{1}{4}} = \frac{2}{5}$

21. $\alpha + \beta = 3$, $\alpha\beta = a$

$\gamma + \delta = 12$, $\gamma\delta = b$

$\alpha, \beta, \gamma, \delta$ are in G.P

Let r be the common ratio

So $\alpha(1+r) = 3$

$\alpha r^2(1+r) = 12$ $\Rightarrow$ $r^2 = 4$

$r = 2$

So $\alpha = 1$

So $a = 2$, $b = 32$

22. We have $b^2 = ac$ .... (i)

and $2\log\left(\frac{3b}{5c}\right) = \log\left(\frac{5c}{a}\right) + \log\left(\frac{a}{3b}\right)$

$= \log\left(\frac{5c}{a}.\frac{a}{3b}\right) = \log\left(\frac{5c}{3b}\right) = -\log\left(\frac{3b}{5c}\right)$

$\Rightarrow$ $3\log\left(\frac{3b}{5c}\right) = 0$ $\Rightarrow$ $b = \frac{5}{3}c$ .... (ii)

From (i) & (ii), we have $a = \frac{b^2}{c} = \frac{25c}{9}$

Now, we have $b + c = \frac{5c}{3} + c = \frac{8c}{3} < \frac{25c}{9} = a$

Hence a, b, c cannot form the sides of a triangle.

25. $a + b + c + d = 2$

$\Rightarrow$ $a, b, c, d > 0$

$\frac{(a+b)+(c+d)}{2} \ge \sqrt{(a+b)(c+d)}$

$1 \ge \sqrt{(a+b)(c+d)} \ge 0$

$\Rightarrow$ $0 \le (a+b)(c+d) \le 1$

$0 \le M \le 1$

29. $\Rightarrow$ $\frac{a+rd}{a+nd} = \frac{a+nd}{a+md}$

$\Rightarrow$ $\frac{1+r(d/a)}{1+n(d/a)} = \frac{1+n(d/a)}{1+m(d/a)}$ Let $\frac{d}{a} = x$

$\Rightarrow$ $(1+nx)^2 = (1+rx)(1+mx)$

$\Rightarrow$ $(n^2 - mr)x^2 + (2n - r - m)x = 0$ $\Rightarrow$ $x = 0$

or $x = -\left(\frac{2n-r-m}{n^2-mr}\right) = \left(\frac{2n-r-m}{\frac{n(m+r)}{2} - n^2}\right) = \frac{-2}{n}$

(m, n, r are in H.P. $\Rightarrow$ $mr = \frac{n(m+r)}{2}$)

EXERCISE - 2

Part # I : Multiple Choice

1. $\alpha+\beta+\gamma=-3a, \qquad \alpha\beta\gamma=-c$

$\alpha\beta+\beta\gamma+\gamma\alpha=3b \Rightarrow \frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=\frac{3b}{\alpha\beta\gamma}$

$\Rightarrow \frac{2}{\beta}+\frac{1}{\beta}=\frac{3b}{-c}$ $\qquad$ ($\because$ α, β, γ in H.P)

$\Rightarrow \frac{1}{\beta}=-\frac{b}{c} \qquad \Rightarrow \qquad \beta=\frac{-c}{b}$

2. $x_1+x_2=3 \qquad x_3+x_4=12 \qquad x_1x_2=A$

$x_3x_4=B$

let $x_1=a x_2=ar \qquad x_3=ar^2, x_4=ar^3$

$a+ar=3 \qquad ar^2(1+r)=12$

$a^2r=A \qquad r^2(3)=12$

$1, 2, 4, 8 \qquad r^2=4$

$A=2 \qquad x_1+x_3=4$

$r=2 \;\; a=1$

$B=32 \qquad x_2+x_4=10$

3. Using AM $\geq$ GM,

$\frac{1+a_1+a_1^2}{3}\geq(1.a_1.a_1^2)^{1/3} \Rightarrow 1+a_1+a_1^2\geq 3a_1$

$\Rightarrow 1+a_2+a_2^2\geq 3a_2$

$\Rightarrow 1+a_n+a_n^2\geq 3a_n$

Multiplying these,

$(1+a+a_1^2)\ldots(1+a_n+a_n^2)\geq 3^n(a_1a_2a_3\ldots a_n)=3^n.1$

5. $a_1+a_{10}=a_2+a_9=\ldots\ldots=(a+b)$

$g_1g_{10}=g_2\,g_9=\ldots\ldots=ab$

$\Rightarrow \frac{5(a+b)+4(a+b)+3(a+b)+2(a+b)+(a+b)}{ab}$

$=15\frac{(a+b)}{ab}=\frac{30}{h} \qquad \left(\because \; h=\frac{2ab}{a+b}\right)$

6. $a_1+(a_1+d), (a_1+2d), \ldots\ldots\ldots b_1+(b_1+d_1),$

$(b_1+2d_1), \ldots\ldots\ldots$

hence $\quad a_{100}=a_1+99d$

$b_{100}=b_1+99d_1$

add

$a_{100}+b_{100}=100+99(d+d_1)$

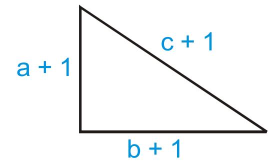
hence $\quad d+d_1=0 \quad \Rightarrow \quad d=-d_1 \Rightarrow$ (A)

(B) and (C) are obviously true.

$\sum_{r=1}^{100}(a_r+b_r)=\frac{100}{2}[(a_1+b_1)+(a_{100}+b_{100})]=\frac{100\times 200}{2}$

$=10^4 \Rightarrow$ (D) (using $S_n=\frac{n}{2}(a+d)$)

9. $\sum_{r=1}^{n}(2r-1)=n^2$



$\Rightarrow a=2n-1$

$\Rightarrow n=\frac{a+1}{2}$

$\Rightarrow (a+1)^2+(b+1)^2=(c+1)^2$

$=8 \quad =6 \quad =10 \qquad$ ($\because$ $a+b+c=21$)

$\Rightarrow a=7 \qquad b=5 \qquad c=9$

Hence $\quad G=9 \qquad L=5$

$G-L=4 \quad \& \quad a-b=2$

12. $\sum_{r=1}^{n} r(r+1)(2r+3)=an^4+bn^3+cn^2+dn+e$

$\sum_{r=1}^{n}(r^2+r)(2r+3)=\sum_{r=1}^{n}(2r^3+5r^2+3r)$

$=2.\frac{n^2(n+1)^2}{4}+5\frac{n(n+1)(2n+1)}{6}+3\frac{n(n+1)}{2}$

$=\frac{n(n+1)}{2}\left[n(n+1)+\frac{5}{3}(2n+1)+3\right]$

$=\frac{n(n+1)}{2}\left[\frac{6(n^2+n)+10(2n+1)+18}{6}\right]$

$=\frac{n(n+1)}{12}[6n^2+26n+28]$

$=\frac{1}{12}[6n^4+26n^3+28n^2+6n^3+26n^2+28n]$

$=\frac{1}{12}[6n^4+32n^3+54n^2+28n]$

$a=\frac{6}{12}, \quad b=\frac{32}{12}, \; c=\frac{54}{12}, \; d=\frac{28}{12}=\frac{7}{3}$

$e=0$

So $a+c=b+d$

$b-\frac{2}{3}=\frac{32}{12}-\frac{2}{3}=\frac{24}{12} \qquad c-1=\frac{42}{12}$

so a, b–2/3, c–1 are in A.P & $\frac{c}{a}=\frac{54}{6}=9$ is an integer

16. since x , |x + 1|, |x – 1| are in A.P.

so $2|x+1| = x + |x-1|$ .... **(i)**

Case-I If $x < -1$, then **(i)** becomes

$-2(x+1) = x - (x-1)$

$\Rightarrow x = -\frac{3}{2}$

then series $-\frac{3}{2}, \frac{1}{2}, \frac{5}{2}, ...$

$\therefore S_{20} = \frac{20}{2}[-3 + (20-1)2] = 350$

21. $a + b + c = xb$

Divide by b, $\frac{a}{b} + 1 + \frac{c}{b} = x$

or $\frac{1}{r} + 1 + r = x$ where r is common ratio of G.P.

$\Rightarrow r^2 + r(1-x) + 1 = 0$

since r is real & distinct $\Rightarrow D > 0$

$\therefore (1-x)^2 - 4 > 0 \Rightarrow x^2 - 2x - 3 > 0$

or $(x+1)(x-3) > 0 \Rightarrow x > 3$ or $x < -1$

Part # II : Assertion & Reason

5. $r = 4R\sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right) \le 4R\left(\frac{1}{8}\right)$

$\left(\because \sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right) \le \frac{1}{8}\right)$

$\Rightarrow R \ge 2r$

if $R = 12$ and $r = 8$

then, $12 \ge 16$ (impossible)

6. **Statement - I**

Let $E = \frac{(\sin x + 1)^3 + (\cos^3 x + 1)}{(\sin x + 1)\cos x}$

$\therefore E = \frac{(\sin x+1)^2}{\cos x} + \frac{1}{\cos x(\sin x + 1)} + \frac{\cos^2 x}{(1+\sin x)} \ge 3$

$\left\{\frac{(1+\sin x)^2}{\cos x}.\frac{1}{\cos x(1+\sin x)}\frac{\cos^2 x}{(1+\sin x)}\right\}^{1/3} \ge 3$

$\Rightarrow E \ge 3$

Hence minimum value of given expression is 3.

Statement - II

$-\sqrt{a^2+b^2} \le a\sin\theta + b\cos\theta \le \sqrt{a^2+b^2}$

7. $S = \ell n2 + 2\ell n2 + 5\ell n2 + ... T_n$ ....**(i)**

$S = \quad \ell n2 + 2\ell n2 + 5\ell n2 + ... T_n$ ....**(ii)**

(i) – (ii) $\Rightarrow T_n = \ell n2 + \ell n2 + 3\ell n2 + ...$ n terms

$T_n = \ell n2 + \ell n2(n-1)^2$

now successice difference in A.P.

$\Rightarrow T_n = an^2 + bn + c$

St.-II is correct.

8. Given $(abc)^{1/3} \ge \frac{a+b+c}{3}$

$\Rightarrow a = b = c$ (GM $\ge$ AM which is possible only if GM = AM)

$\therefore 3a + 4b + 5c = 12 \Rightarrow a = b = c = 1$

$\therefore \frac{1}{a^2} + \frac{1}{b^3} + \frac{1}{c^5} = 3$

10. In any ΔABC, we have $r_1 + r_2 + r_3 = 4R + r \le \frac{9R}{2}$

11. **Statement -I**

$$\frac{\frac{1}{b}+\frac{1}{a}}{\frac{2}{b}-\frac{1}{a}} + \frac{\frac{1}{b}+\frac{1}{c}}{\frac{2}{b}-\frac{1}{c}} = \frac{\frac{1}{b}+\frac{1}{a}}{\frac{1}{c}} + \frac{\frac{1}{b}+\frac{1}{c}}{\frac{1}{a}} \quad \left(\because \frac{2}{b} = \frac{1}{a} + \frac{1}{c}\right)$$

$$= \frac{c}{b} + \frac{c}{a} + \frac{a}{b} + \frac{a}{c} = \frac{a+c}{b} + \left(\frac{c}{a} + \frac{a}{c}\right)$$

$$= \frac{(a+c)^2}{2ac} + \left(\frac{c}{a} + \frac{a}{c}\right) = \frac{1}{2}\left(\frac{a}{c} + \frac{c}{a} + 2\right) + \left(\frac{c}{a} + \frac{a}{c}\right) > 4$$

$[\because x + \frac{1}{x} > 2$ when $x > 0, x \ne \frac{1}{x}]$

Statement - II

Is False $\therefore$ Numbers should be positive

12. Let a, b, c in G.P. then $b^2 = ac$

then $a + b, 2b, b + c$ in HP

$\frac{1}{a+b}, \frac{1}{2b}, \frac{1}{b+c}$ in AP

$\frac{2}{2b} = \frac{1}{a+b} + \frac{1}{b+c}$

$(a+b)(b+c) = (a+c+2b)b$

$ab + b^2 + ac + bc = ab + bc + 2b^2$

$\therefore b^2 = ac$

So statement (1) and (2) is true

15. **Statement - I**

$(AM)(HM) = (GM)^2$

True for any 3 numbers in G.P.

Statement II

False if number are not in G.P.

EXERCISE - 3

Part # I : Matrix Match Type

4. (A) $S_n = 4 + 11 + 22 + 37 \text{.........} T_n$

$S_n = \quad 4 + 11 + 22 + 37 \text{.........} T_n$

$T_n = 4 + 7 + 11 + 15 + \text{.........}$ n terms

$$T_n = 4 + \frac{(n-1)}{2}(14 + (n-2)4) = 1 + 2n^2 + n$$

(B) $|1^2 - 2^2 + 3^2 - 4^2 \text{..........}$ 2n terms$|$

$= |(1-2)(1+2)+(3-4)(3+4)+(5-6)(5+6)|$

$= |-3 - 7 - 11 - 15 \text{........}$n terms$|$

$= 3 + 7 + 11 + \text{.......}$ n terms

$$= \left(\frac{n}{2}(6+(n-1)4)\right) = \frac{n}{2}(4n+2) = (2n^2+n)$$

(C) $3 + 7 + 11 + 15 \text{.........} = 2n^2 + n$

(D) Coefficient of x^n is

$$-2(1+2+3+\text{...term}) = -\frac{2n(n+1)}{2} = -(n^2+n)$$

Part # II : Comprehension

Comprehension–3

$ax^2 + 2bx + b = 5x^2 - 3bx - a$

$\Rightarrow$ $(a-5)x^2 + 5bx + (b+a) = 0$

If $a \neq 5$, then since $x \in R$

$D = 25b^2 - 4(b+a)(a-5) \geq 0 \quad \forall \; b \in R$

$\Rightarrow$ $25b^2 - 4(a-5)b - 4a(a-5) \geq 0 \quad \forall \; b \in R$

$\therefore$ $16(a-5)^2 + 16(25)\,a(a-5) \leq 0$

$\Rightarrow$ $16(a-5)(a-5+25a) \leq 0$

$\Rightarrow$ $(a-5)(26a-5) \leq 0$

$\therefore$ $a \in \left[\frac{5}{26}, 5\right)$

If $a = 5$,

$5bx + (b+5) = 0$

not satisfied for $b = 0$

$\therefore a_m \in \{1, 2, 3, 4\}$

$t_r = (r-1)(r-2)(r-3)(r-4)$

$$S_n = \frac{1}{5}\sum_{r=1}^{n} (r-4)(r-3)(r-2)(r-1)\,(r-(r-5))$$

$$= \frac{1}{5}\sum_{r=1}^{n} ((r-4)(r-3)(r-2)(r-1)\,r - (r-5)(r-4)(r-3)(r-2)(r-1))$$

$$= \frac{1}{5}n(n-1)(n-2)(n-3)(n-4)$$

$S_n = 0 \Rightarrow n = 1, 2, 3, 4 \qquad n = 0$ (rejected)

$$\sum \frac{1}{t_r} = \frac{1}{3}\lim_{n\to\infty}\sum_{r=5}^{n} \frac{(r-1)-(r-4)}{(r-4)(r-3)(r-2)(r-1)}$$

$$= \frac{1}{3}\lim_{n\to\infty}\sum_{r=5}^{n}\left(\frac{1}{(r-4)(r-3)(r-2)} - \frac{1}{(r-3)(r-2)(r-1)}\right)$$

$$= \lim_{n\to\infty}\frac{1}{3}\left[\frac{1}{6} - \frac{1}{(n-3)(n-2)(n-1)}\right] = \frac{1}{18}$$

4. $g(n) - g(n-1) = 1^2 + 2^2 + 3^2 + \text{.........} + (n-1)^2 + n^2 - (1^2 + 2^2 + 3^2 + \text{........} + (n-1)^2)$

$= n^2$

Compherension 4

1. $g(n) - g(n-1) = 1^2 + 2^2 + 3^2 + \text{.........} + (n-1)^2 + n^2 - (1^2 + 2^2 + 3^2 + \text{........} + (n-1)^2)$

$= n^2$

2. $$g(n) - f(n) = \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} = \frac{n(n+1)}{2}$$

$$\left(\frac{2n+1}{3} - 1\right) = \frac{n(n+1)}{2}\;\frac{2n-2}{3}$$

$$= \frac{n(n+1)(n-1)}{3} = \frac{(n-1)\,n\,(n+1)}{3}$$

for $n = 2$ $\dfrac{(n-1)\,n\,(n+1)}{3} = \dfrac{1.2.3}{3}$ which is divisible by 2 but not by 2^2

$\therefore$ greatest even integer which divides $\dfrac{(n-1)\,n\,(n+1)}{3}$, for every $n \in N$, $n \geq 2$, is 2

3. $f(n) + 3g(n) + h(n)$

$$= \frac{n(n+1)}{2} + \frac{n(n+1)(2n+1)}{2} + \left(\frac{n(n+1)}{2}\right)^2$$

$$= \frac{n(n+1)}{2}\left(1 + 2n + 1 + \frac{n(n+1)}{2}\right) = (1 + 2 + 3 + \text{......} + n)$$

$$\left(2n + 2 + \frac{n(n+1)}{2}\right)$$

$\Rightarrow$ for all $n \in N$

Comprehension 5

$a+b+c=5$ (a, b, c > 0) $x^2y^3=243=3^5$

1. $\dfrac{a+\frac{b}{3}+\frac{b}{3}+\frac{b}{3}+c}{5} \geq \left(\dfrac{ab^3c}{27}\right)^{1/5}$ ($\because$ AM ≥ GM)

$\Rightarrow 1^5 \geq \dfrac{ab^3c}{27} \quad \Rightarrow \quad ab^3c \leq 27$

2. Using AM ≥ HM

$$\frac{a+\frac{b}{3}+\frac{b}{3}+\frac{b}{3}+c}{5} \geq \frac{5}{\frac{1}{a}+\frac{3}{b}+\frac{3}{b}+\frac{3}{b}+\frac{1}{c}}$$

$$\Rightarrow \frac{1}{5} \geq \frac{1}{\frac{1}{a}+\frac{9}{b}+\frac{1}{c}}$$

3. Using AM ≥ GM

$$\frac{x^2+y+y+y+1}{5} \geq (x^2.y.y.y.1)^{1/5}$$

$$\Rightarrow x^2+3y+1 \geq 5.(243)^{1/5} \geq 15$$

But x^2, $y \neq 1$, hence $x^2+3y+1>15$

4. $\dfrac{x+x+y+y+y}{5} \geq (x^2y^3)^{1/5} \geq \dfrac{5}{\frac{2}{x}+\frac{3}{y}}$

$\Rightarrow \dfrac{2x+3y}{5} \geq 3 \geq \dfrac{5xy}{3x+2y}$

EXERCISE - 4

Subjective Type

1. $a_1+a_n=a_2+a_{n-1}=a_3+a_{n-2}$...............

$$=\frac{1}{a_1+a_n}\left[\frac{a_1+a_n}{a_1a_n}+\frac{a_2+a_{n-1}}{a_2a_{n-1}}+\frac{a_3+a_{n-2}}{a_3a_{n-2}}+....+\frac{a_1+a_n}{a_na_1}\right]$$

$$=\frac{1}{a_1+a_n}\left[\frac{1}{a_n}+\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_{n-1}}.......+\frac{1}{a_1}+\frac{1}{a_n}\right]$$

$$=\frac{2}{a_1+a_n}\left[\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}.......+\frac{1}{a_n}\right]$$

2. $T_r=(2r+1)\,2^r$

$S_n=3\cdot2^1+5\cdot2^2+7\cdot2^3+$........$(2n+1)2^n$

Sub

$2S_n=3\cdot2^2+5\cdot2^3+$........ $(2n-1)2^n+(2n+1)2^{n+1}$

$-S_n=\underbrace{3\cdot2^1}+2^3+2^4+$...............$2^{n+1}-(2n+1)2^{n+1}$

$\therefore\ -S_n=\underbrace{2+2^2}+2^3+2^4+$...............$2^{n+1}-(2n+1)2^{n+1}$

$\therefore\ -S_n=\dfrac{2[2^{n+1}-1]}{2-1}-(2n+1)2^{n+1}$

$S_n=(2n+1)2^{n+1}-(2^{n+2}-2)$

$n\cdot2^{n+2}+\underbrace{2^{n+1}-2^{n+2}}+2$

$n\cdot2^{n+2}-\underbrace{2^{n+1}}+2$

$4n\cdot2^n-2\cdot2^n+2$

$R=4$; $S=-2$; $T=2$

$\therefore\ R+S+T=4$

3. 160

6. $n.2^{n+2}-2^{n+1}+2.$

7. LHS $=9+8\times10+8\times10^2$.....$+8\times10^{n-1}+4\times10^n+4\times10^{n+1}$ ....$+4\times10^{n+n-1}$

$$=9+8\times10^1\left(\frac{10^{n-1}-1}{9}\right)+4\times10^n\left[\frac{10^n-1}{9}\right]$$

$$=\frac{81+8\times10^n-80+4\times10^{2n}-4\times10^n}{9}$$

$$=\frac{1+4\times10^n+4\times10^{2n}}{9}$$

$$=\frac{3+6\times10^n}{9}=\left(\frac{1+2\times10^n}{3}\right)^2$$

RHS $=7+6\times10^1\dfrac{(10^{n-1}-1)}{9}=\dfrac{63+6\times10^n-60}{9}$

$$=\frac{3+6\times10^n}{9}=\frac{1}{3}(1+2\times10^n)$$

8. $\dfrac{1}{201}$

9. n^2

10. $(a-3d)(a-d)(a+d)(a+3d)+16d^4$

$= (a^2-9d^2)(a^2-d^2)+16d^4$

$= a^4 - a^2d^2 - 9a^2d^2 + 9d^4 + 16d^4$

$= a^4 - 10a^2d^2 + 25d^4 = [a^2 - 5d^2]^2$

$= (a^2 - d^2 - 4d^2)^2 = ((a-d)(a+d) - (2d)^2)^2$

$\because$ $(a-d), (a+d),$ $2d$ are integers.

Hence Proved

11. $a = 1, b = 9$ or $b = 1, a = 9$

12. **(i)** Let $b = ar$

$c = ar^2$ and $d = ar^3$

So $a^2(1-r^2), a^2(r^2)(1-r^2), a^2r^4(1-r^2)$ these are in G.P.

So $(a^2-b^2), (b^2-c^2), (c^2-d^2)$ are in G.P.

(ii) $\dfrac{1}{a^2+b^2}, \dfrac{1}{b^2+c^2}, \dfrac{1}{c^2+d^2}$

$= \dfrac{1}{a^2(1+r^2)}, \dfrac{1}{a^2r^2(1+r^2)}, \dfrac{1}{a^2r^4(1+r^2)}$ are in G.P.

13. 6, 3

14. 3, 7, 11 or 12, 7, 2

15. $\dfrac{T_q}{T_p} = \dfrac{T_r}{T_q}$ = common ratio

$$\frac{a+(q-1)d}{a+(p-1)d} = \frac{a+(r-1)d}{a+(q-1)d}$$

using dividendo

$$\frac{(q-p)}{a+(p-1)d} = \frac{(r-q)}{a+(q-1)d}$$

$$\Rightarrow \frac{T_q}{T_p} = \frac{r-q}{q-p} = \frac{q-r}{p-q}$$

16. $[n(n+1)(n+2)]/6$

EXERCISE - 5

Part # I : AIEEE/JEE-MAIN

2. $\dfrac{a}{1-r} = 20$ ... **(i)** $\dfrac{a^2}{1-r^2} = 100$ ... **(ii)**

from **(i)** and **(ii)**

$\dfrac{a}{1+r} = 5$ $(\therefore a = 20(1-r)$ by (i))

$\Rightarrow \dfrac{20(1-r)}{1+r} = 5 \Rightarrow 5r = 3 \Rightarrow r = 3/5$

6. Given that A.M. = 9 and G.M. = 4

If α, β are roots of quadratic equations then quadratic equation is

$x^2 - x(\alpha+\beta) + \alpha\beta = 0$ ... **(1)**

A.M. $= \dfrac{\alpha+\beta}{2} = 9$

$\Rightarrow \alpha + \beta = 18$ ... **(2)**

G.M. $= \sqrt{\alpha\beta} = 4$

$\Rightarrow \alpha\beta = 16$ ... **(3)**

so the required equation will be

$x^2 - 18x + 16 = 0$

9. $\because \dfrac{(S_m)_{IAP}}{(S_n)_{IIAP}} = \dfrac{m^2}{n^2}$ then $\dfrac{T_m}{T_n} = \dfrac{2m-1}{2n-1}$

so $\dfrac{a_6}{a_{21}} = \dfrac{2\times 6-1}{2\times 21-1} = \dfrac{11}{41}$

12. $a + ar = 12$ ... **(1)**

$ar^2 + ar^3 = 48$ ... **(2)**

$ar^2(a + ar) = 48$ ... **(3)**

so $r^2 = 4$ $a(1+r) = 12$

$r = 2$ $a(3) = 12$

because +ve G.P. $\boxed{a = 4}$

13. $S = 1 + \dfrac{2}{3} + \dfrac{6}{3^2} + \dfrac{10}{3^3} + \dfrac{14}{3^4} + \ldots\ldots\infty$

$\dfrac{1}{3}S = \dfrac{1}{3} + \dfrac{2}{3^2} + \dfrac{6}{3^3} + \dfrac{10}{3^4} + \ldots\ldots\infty$

$\dfrac{2}{3}S = 1 + \dfrac{1}{3} + \dfrac{4}{3^2} + \dfrac{4}{3^3} + \dfrac{4}{3^4} + \dfrac{4}{3^5} + \ldots\ldots\infty$

$= \dfrac{4}{3} + \dfrac{4/9}{1-\frac{1}{3}} = \dfrac{4}{3} + \dfrac{2}{3} = \dfrac{6}{3} = 2 \Rightarrow S = 3$

14. $4500 = 150 \times 10 + \{148 + 146 + \ldots \text{ upto n terms}\}$

$$= 1500 + \frac{n}{2}\{296 + (n-1)(-2)\}$$

$\Rightarrow$ $n^2 - 149n + 3000 = 0$

$\Rightarrow$ $(n-24)(n-125) = 0$

$\Rightarrow$ $n = 24 \quad \because n \neq 125$

So total time taken $= 10 + 24 = 34$ min.

15. Saving after first 3 month = 600

$$600 + \left\{\frac{240 + 280 + \ldots.}{\textit{let n month}}\right\} = 11040$$

$|240 + 280 + \ldots\ldots \text{ n terms}| = 10440$

$n/2\,[480 + (n-1)40] = 10440$

$n\,\{440 + 40n\} = 20880$

$n^2 + 11n - 522 = 0$

$n = 18, -29 \quad (-29 \text{ rejected})$

Total months = n + 3

$18 + 3 = 21$ Months

16. $\sum_{r=1}^{100} a_{2r} = a_2 + a_4 + a_6 + \ldots\ldots + a_{200} = \alpha$

$= (a+d) + (a+3d) + \ldots. + (a+199d) = \alpha$

$\sum_{r=1}^{100} a_{2r-1} = \beta = a_1 + a_3 + \ldots. + a_{199} = \beta$

$= a + (a+2d) + \ldots.. + (a+198d) = \beta$

$\frac{100}{2}[a + d + a + 199d] = \alpha$

$\Rightarrow 50(2a + 200d) = \alpha$ ...... **(i)**

$\frac{100}{2}[a + a + 198d] = \beta$

$\Rightarrow 50(2a + 198d) = \beta$ ...... **(ii)**

(1) – (2)

$\alpha - \beta = 50(2d)$

$= d = \frac{\alpha - \beta}{100}$

17. Statement– I :

$(1^3-0^3) + (2^3-1^3) + (3^3-2^3) + \ldots. + (20^3-19^3)$

$= 20^3 = 8000$

Statement–I is true.

Statement–II :

$\sum_{k=1}^{n} k^3 - (k-1)^3 = (1^3 - 0^3) + (2^3-1^3) + (3^3-2^3)$

$+ \ldots.\ n^3 + (n-1)^3 = n^3.$

Statement–2 is true and Statement–2 is a correct explanation of Statement–1.

18. $100T_{100} = 50T_{50}$

$100\,(a + 99d) = 50(a + 4d)$

$a + 149d = 0$

$T_{150} = a + 149d = 0$

19. $S = \frac{7}{10} + \frac{77}{100} + \frac{777}{1000} + \ldots..$

$$S = \frac{7}{9}\left\{\frac{10-1}{10} + \frac{100-1}{100} + \frac{1000-1}{1000} + \ldots.\right\}$$

$$= \frac{7}{9}\left\{20 - \frac{1}{10}\left(\frac{1-10^{-20}}{9/10}\right)\right\}$$

$$= \frac{7}{9}\left\{20 - \frac{1}{9}\left(1 - 10^{-20}\right)\right\} = \frac{7}{81}(179 + 10^{-20})$$

25. a + d, a + 4d, a + 8d are in G.P.

$(a+4d)^2 = (a+d)(a+8d)$

$\Rightarrow$ $a^2 + 8ad + 16d^2 = a^2 + 9ad + 8d^2$

$\Rightarrow$ $8d^2 = ad \quad \Rightarrow \quad \frac{a}{d} = 8$

$\therefore$ Common ratio $= \frac{a+4d}{a+d}$

$$= \frac{8+4}{4+1} = \frac{4}{3}$$

26. $\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \ldots.. = \frac{16}{5}m$

$\Rightarrow$ $\left(\frac{8}{5}\right)^2 + \left(\frac{12}{5}\right)^2 + \left(\frac{16}{5}\right)^2 + \left(\frac{20}{5}\right)^2 + \ldots..\ 10 \text{ terms} = \frac{16}{6}m$

$\Rightarrow$ $\left(\frac{4}{5}\right)^2 [2^2 + 3^2 + 4^2 + 5^2 \ldots..\ 10 \text{ terms}] = \frac{16}{5}m$

$\Rightarrow$ $\left(\frac{4}{5}\right)^2 [2^2 + 3^2 + 4^2 \ldots.. + 11^2] = \frac{16}{5}m$

$\Rightarrow$ $\left(\frac{4}{5}\right)^2 [1^2 + 2^2 + 3^2 \ldots..+ 11^2 - 1^2] = \frac{16}{5}m$

$\Rightarrow$ $\left(\frac{4}{5}\right)^2 \left[\frac{11.12.23}{6} - 1\right] = \frac{16}{5}m$ (given)

$\Rightarrow$ $\frac{16}{25}[22.23 - 1] = \frac{16}{5}m \quad \Rightarrow \quad \frac{1}{5}(505) = m$

$\Rightarrow$ $m = 101$

Part # II : IIT-JEE ADVANCED

6. $\frac{x}{1-r} = 5 \Rightarrow 5 - 5r = x \Rightarrow r = 1 - \frac{x}{5}$

As $|r| < 1$ i.e. $\left|1 - \frac{x}{5}\right| < 1;\ -1 < 1 - \frac{x}{5} < 1$

$-5 < 5 - x < 5 = -10 < -x < 0 = 10 > x > 0$

i.e. $0 < x < 10$

13. $A_1 = \frac{a+b}{2}$; $G_1 = \sqrt{ab}$; $H_1 = \frac{2ab}{a+b}$

$A_n = \frac{A_{n-1} + H_{n-1}}{2}$; $G_n = \sqrt{A_{n-1}H_{n-1}}$

$H_n = \frac{2A_{n-1}H_{n-1}}{A_{n-1} + H_{n-1}}$

We know that

$G^2 = AH$

So clearly

$G_1 = G_2 = G_3 = \ldots.\ G_n = \sqrt{ab}$

14. A_2 is AM of A_1, H_1 and $A_1 > H_1$

$\Rightarrow A_1 > A_2 > H_1$

A_3 is A.M. of A_2, H_2

$\Rightarrow A_2 > A_3 > H_2$

$\therefore A_1 > A_2 > A_3 \ldots.$

15. as above

$A_1 > H_2 > H_1$

$A_2 > H_3 > H_2$

....

so $\ldots.. H_3 > H_2 > H_1$

$\Rightarrow H_1 < H_2 < H_3 \ldots..$

16. Let $a_1 = a$, $a_2 = ar$, $a_3 = ar^2$, $a_4 = ar^3$

Now $b_1 = a$, $b_2 = a + ar$, $b_3 = a + ar + ar^2$

$b_4 = a + ar + ar^2 + ar^3$

so b_1, b_2, b_3, b_4 are neither in A.P. nor in G.P. & nor in H.P.

so S(I) is true & S(II) is False.

17. $S_n = cn^2$

$S_{n-1} = c(n-1)^2$

$T_n = S_n - S_{n-1} = c\,(2n - 1)$

$T'_n = T_n^{\,2} = c^2\,(4n^2 - 4n + 1)$

$\sum T'_n = nc^2\left(\frac{4n^2 - 1}{3}\right)$

18. $S_k = \frac{\frac{k-1}{k!}}{1 - \frac{1}{k}} = \frac{1}{(k-1)!}$ for $k \geq 2$, $S_1 = 0$

Now $\frac{100^2}{100!} + \sum_{k=2}^{100}\left|(k^2 - 3k + 1)\cdot\frac{1}{(k-1)!}\right| + S_1$

$\frac{100^2}{100!} + \frac{1}{1!} + \sum_{k=3}^{100}\left|\frac{1}{(k-3)!} - \frac{1}{(k-1)!}\right| + 0$

$\left(\because S_2 = \frac{1}{1!}\right)$

$= \frac{100^2}{100!} + 1 + \left|\frac{1}{0!} - \frac{1}{2!}\right| + \left|\frac{1}{1!} - \frac{1}{3!}\right| + \left|\frac{1}{2!} - \frac{1}{4!}\right| + \ldots$

$+ \left|\frac{1}{97!} - \frac{1}{99!}\right|$

$= \frac{100^2}{100!} + 3 - \frac{1}{98!} - \frac{1}{99!}$

$= \frac{100^2}{100!} + 3 - \frac{100}{99!} = 3$

19. $a_1 = 15$

$27 - 2a_2 > 0$

$a_k = 2a_{k-1} - a_{k-2}\ \forall\ k = 3, 4, 5 \ldots.\ 11$

$\frac{a_1^2 + a_2^2 + \ldots\ldots..a_{11}^2}{11} = 90$

$a_{k-1} = \frac{a_k + a_{k-2}}{2}$

all $a_i (i = 1, 2, \ldots\ldots.. 11)$ are in A.P.

Let the numbers are

$(a_6+5d), (a_6+4d), \ldots\ldots., a_6, \ldots.., (a_6 - 4d), (a - 5d)$

$11a_6^2 + 110d^2 = 990$

$a_6 = 15 - 5d$

$a_6^2 + 10d^2 = 90$

$(15 - 5d)^2 + 10d^2 = 90 \Rightarrow 7d^2 - 30d + 27 = 0$

$\Rightarrow d = 3, \frac{9}{7}$

for $d = 3 \Rightarrow a_2 = 12$ (possible)

for $d = 9/7 \Rightarrow a_2 = 13.7$

(not possible since $a_2 < 13.5$)

$a_6 = 0 \Rightarrow \frac{a_1 + a_2 + \ldots + a_{11}}{11} = a_6 = 0$

20. As $a > 0$

and all the given terms are positive

hence considering A.M. $\geq$ G.M. for given numbers :

$$\frac{a^{-5}+a^{-4}+a^{-3}+a^{-3}+a^{-3}+a^{8}+a^{10}}{7}$$

$$\geq \left(a^{-5}.a^{-4}.a^{-3}.a^{-3}.a^{-3}.a^{8}.a^{10}\right)^{\frac{1}{7}}$$

$$\Rightarrow \frac{a^{-5}+a^{-4}+a^{-3}+a^{-3}+a^{-3}+a^{8}+a^{10}}{7} \geq 1$$

$$\Rightarrow \left(a^{-5}+a^{-4}+3a^{-3}+a^{8}+a^{10}\right)_{min} = 7$$

where $a^{-5} = a^{-4} = a^{-3} = a^{8} = a^{10}$ i.e. $a = 1$

$\Rightarrow (a^{-5} + a^{-4} + 3a^{-3} + a^{8} + a^{10} + 1)_{min} = 8$

when $a = 1$

21. Consider $d \neq 0$ the solution is

$a_1, a_2, a_3, ..., a_{100} \to$ AP

$a_1 = 3$; $S_p = \sum_{i=1}^{p} a_i$ $\quad 1 \leq n \leq 20$

$m = 5n$

$$\frac{S_m}{S_n} = \frac{\frac{m}{2}[2a_1 + (m-1)d]}{\frac{n}{2}[2a_1 + (n-1)d]}$$

$$\frac{S_m}{S_n} = \frac{5[(2a_1 - d) + 5nd]}{[(2a_1 - d) + nd]}$$

for $\frac{S_m}{S_n}$ to be independent of n

$\therefore$ $2a_1 - d = 0$ $\Rightarrow$ $d = 2a_1$

$\Rightarrow$ $d = 6$ $\quad\Rightarrow$ $a_2 = 9$

If $d = 0$ $\quad\Rightarrow$ $a_2 = a_1 = 3$

22. a_1, a_2, a_3..........be in H.P

$\Rightarrow \frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}$.... be in A.P.

in A.P. $T_1 = \frac{1}{a_1} = \frac{1}{5}$ and $T_{20} = \frac{1}{a_{20}} = \frac{1}{25}$

$\therefore T_{20} = T_1 + 19d$

$\frac{1}{25} = \frac{1}{5} + 19d$ $\quad\Rightarrow$ $d = -\frac{4}{19 \times 25}$

$T_n = T_1 + (n-1)d < 0$

$\Rightarrow \frac{1}{5} - \frac{(n-1).4}{19 \times 25} < 0$ $\quad\Rightarrow$ $\frac{1}{5} < \frac{4(n-1)}{25 \times 19}$

$\Rightarrow \frac{5 \times 19}{4} + 1 < n$ $\quad\Rightarrow \frac{99}{4} < n$

$\Rightarrow$ least positive integer n is 25.

23. $S_n = -1^2 - 2^2 + 3^2 + 4^2 - 5^2 - 6^2 + 7^2 + 8^2$ ....

$S_n = (3^2 - 1^2) + (4^2 - 2^2) +$ ....

$S_n = 2(1 + 2 + 3 + ... + 4n)$

$$= \frac{2(4n)(4n+1)}{2}$$

$S_n = 4n(4n+1)$

$S_n = 4n(4n+1) = 1056$ is possible when $n = 8$

$4n(4n+1) = 1088$ not possible

$4n(4n+1) = 1120$ not possible

$4n(4n+1) = 1332$ possible when $n = 9$.

24. When 1 and 2 are removed from numbers 1 to n then we get maximum possible sum of remaining numbers and when $n-1, n$ are removed then we get minimum possible sum of remaining numbers.

$$\Rightarrow \frac{n(n+1)}{2} - (2n-1) \leq 1224 \leq \frac{n(n+1)}{2} - 3$$

$$\Rightarrow \begin{cases} n^2 + n - 2454 \geq 0 \\ n^2 - 3n - 2446 \leq 0 \end{cases}$$

$$\Rightarrow \begin{cases} n \geq 50 \\ n \leq 50 \end{cases} \Rightarrow n = 50$$

Now let x and x + 1 be two consecutive numbers

$$\Rightarrow \frac{50(50+1)}{2} - x - x - 1 = 1224$$

$\Rightarrow x = 25$

$\Rightarrow$ 25th and 26th cards are removed from pack

$\Rightarrow k = 25$ $\quad\Rightarrow$ $k - 20 = 5$

27. $\log_e b_1, \log_e b_2, \log_e b_3$, ... $\log_e b_{101}$ are in A.P.

b_1, b_2, b_3, ... b_{101}

Given $\log_e(b_2) - \log_e(b_1) = \log_e(2)$ $\quad\Rightarrow \frac{b_2}{b_1} = 2 = r$

a_1, a_2, a_3, ... a

$a_1 = b_1 = a$

$b_1 + b_2 + b_3 +$ .... $b_{51} = t$,

$S = a_1 + a_2 +$ .... a_{51}

$t = \text{sum of 51 terms of G.P.} = b_1\dfrac{(r^{51}-1)}{r-1} = \dfrac{a(2^{51}-1)}{2-1} = a(2^{51}-1)$

$s = \text{sum of 51 terms of A.P.} = \dfrac{51}{2}[2a_1+(n-1)d] = \dfrac{51}{2}(2a+50d)$

Given $a_{51} = b_{51}$

$a + 50d = a(2)^{50}$

$50d = a(2^{50}-1)$

Hence, $s = a\dfrac{51}{2}[2^{50}+1] \Rightarrow s = a\left(51.2^{49} + \dfrac{51}{2}\right)$

$s = 2\left(4.2^{49} + 47.2^{49} + \dfrac{51}{2}\right)$

$\Rightarrow s = a\left((2^{51}-1) + 47.2^{49} + \dfrac{53}{2}\right)$

$s - t = a\left(47.2^{49} + \dfrac{53}{2}\right)$

Clearly : $s > t$

$a_{101} = a_1 + 100d = a + 2a.2^{50} - 2a = a(2^{51}-1)$

$b_{101} = b_1 r^{100} = a.2^{100}$

Hence, $b^{101} > a^{101}$

MOCK TEST

1. If $1^2 + 2^2 + 3^2 + \ldots\ldots + 2003^2 = (2003)(4007)(334)$

$(1)(2003) + (2)(2002) + (3)(2001) + \ldots\ldots + (2003)(1) = (2003)(334)(x)$

$\Rightarrow \sum_{r=1}^{2003} r(2003 - r + 1) = (2003)(334)(x)$

$\Rightarrow 2004 . \sum_{r=1}^{2003} r - \sum_{r=1}^{2003} r^2 = (2003)(334)(x)$

$\Rightarrow 2004 . \left(\dfrac{2003.2004}{2}\right) - 2003 . (4007) . 334 = (2003)(334)(x)$

$\Rightarrow x = 2005$

2. **(B)**

$S = \dfrac{a(1-r^n)}{1-r}, \; P = a^n . r^{\frac{n(n-1)}{2}}$

$R = \dfrac{1}{a} + \dfrac{1}{ar} + \dfrac{1}{ar^2}$ .... to n terms $= \dfrac{1-r^n}{a(1-r)r^{n-1}}$

$S^n = R^n P^k \quad \Rightarrow \left(\dfrac{S}{R}\right)^n = P^k \quad \Rightarrow (a^2 r^{n-1})^n = P^k$

$\Rightarrow P^2 = P^k \quad \Rightarrow k = 2$

3. $a + 6d = 9$

$T_1 T_2 T_7 = a(a + d)(a + 6d)$

$= 9a(a + d) = 9(9 - 6d)(9 - 5d)$

$\because T_7 = a + 6d = 9$

Let $A = T_1 T_2 T_7$

$\dfrac{dA}{d(d)} = 9[-45 - 54 + 60d] = 0$

$60d = 99$

$d = \dfrac{33}{20}$

4. **(D)**

$a_1 + a_3 + a_5 + \ldots\ldots\ldots\ldots\ldots a_{199} = \beta$

$a_2 + a_4 + a_6 + \ldots\ldots\ldots\ldots + a_{200} = \alpha$

$a_2 - a_1 + a_4 - a_3 + a_6 - a_5 \ldots\ldots\ldots\ldots a_{200} - a_{199} = \alpha - \beta$

$d + d + d \ldots\ldots\ldots\ldots d = \alpha - \beta$

$d = \dfrac{\alpha - \beta}{100}$

5. $S_1 + S_2 + S_3 + ... + S_p$

$\Rightarrow$ $S_1 = \frac{n}{2}\,[2.1 + (n-1)\,1]$

$S_2 = \frac{n}{2}\,[2.2 + (n-1)\,3]$

$S_3 = \frac{n}{2}\,[2.3 + (n-1)\,5]$

$S_p = \frac{n}{2}\,[2.p + (n-1)\,(2p-1)]$

So $S_1 + S_2 + ... + S_p = \frac{n}{2}\,[2(1+2+.....\,p) + (n-1)\,(1+3+5+.....\,(2p-1))]$

$= \frac{n}{2}\left[2.\frac{p(p+1)}{2} + (n-1)p^2\right] = \frac{np}{2}\,(np+1)$

6. (B)

$S = 1 + 4x + 7x^2 + 10x^3 + ...$

$x.S = x + 4x^2 + 7x^3 + ...$

Subtract

$S(1-x) = 1 + 3x + 3x^2 + 3x^3 + ...$

$S(1-x) = 1 + 3x\left(\frac{1}{1-x}\right) \qquad |x| < 1$

$S = \frac{1+2x}{(1-x)^2}$

Given $\frac{1+2x}{(1-x)^2} = \frac{35}{16}$

$\Rightarrow$ $16 + 32x = 35 + 35x^2 - 70x$ $\Rightarrow$ $35x^2 - 102x + 19 = 0$

$\Rightarrow$ $35x^2 - 7x - 95x + 19 = 0$

$\Rightarrow$ $7x(5x-1) - 19(5x-1) = 0$

$\Rightarrow$ $(5x-1)(7x-19) = 0$ $\Rightarrow$ $x = \frac{1}{5}, \frac{19}{7}$

But $|x| < 1$ $\therefore$ $x = \frac{1}{5}$

7. $a = A + (p-1)\,d$ $\Rightarrow$ $d = \frac{a-b}{p-q}$

$b = A + (q-1)\,d$

$S = \frac{p+q}{2}\,[2A + (p+q-1)\,d] = \frac{p+q}{2}\,[A + (p-1)d + A + (q-1)d + d] = \frac{p+q}{2}\,[a + b + d]$

$= \frac{p+q}{2}\left[a + b + \frac{a-b}{p-q}\right]$

8. (A)

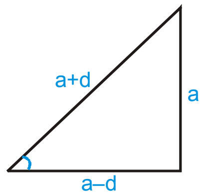
Let the sides be $a - d$, a, $a + d$

where $a > d > 0$

we have

$(a+d)^2 = (a-d)^2 + a^2$

$\Rightarrow$ $d = \frac{a}{4}$ we have $\sin\theta = \frac{a}{a+d}$



$\Rightarrow$ $\cos\theta = \frac{3}{5}$, $\sin\theta = \frac{4}{5}$

9. $t_n = S_n - S_{n-1}$

$= \frac{n}{6}\,(2n^2 + 9n + 13) - \frac{(n-1)}{6}\,\{2(n-1)^2 + 9(n-1) + 13\}$

$= \frac{1}{6}\,[2n^3 + 9n^2 + 13n - 2(n-1)^3 - 9\,(n-1)^2 - 13\,(n-1)]$

$= (n+1)^2$

$\sum_{r=1}^{\infty} \frac{1}{r.(r+1)} = \sum_{r=1}^{\infty}\left(\frac{1}{r} - \frac{1}{r+1}\right) = 1$

10. (C)

S_I **:** To get the maximum value of $P = a_1 a_2 + a_2 a_3 + ...+ a_7 a_8$ maximize one term and minimize other.

$\therefore$ $a_3 = a_4 = ... = a_8 = 0$

$\therefore$ $P = a_1 a_2$

$\because$ $\left(\frac{a_1 + a_2}{2}\right)^2 \geq a_1 a_2$

$\therefore$ $\left(\frac{16}{2}\right)^2 \geq a_1 a_2$

$\Rightarrow$ $a_1 a_2 \leq 64$

$\therefore$ $P_{max} = 64$

S_{II} **:** $\frac{x^2 + r^2}{2} \geq \sqrt{x^2 r^2}$ and $\frac{y^2 + s^2}{2} \geq \sqrt{y^2 s^2}$

$\therefore$ $x^2 + r^2 \geq 2xr$ and $y^2 + s^2 \geq 2ys$

$\therefore$ $2(xr + ys) \leq x^2 + y^2 + r^2 + s^2$

$\Rightarrow$ $2(xr + ys) \leq 2$ $\Rightarrow$ $(xr + ys) \leq 1$

$\therefore$ $(xr + ys)_{maximum} = 1$

S_{III} : Let α and β be two positive real numbers such that

$$A = \frac{\alpha+\beta}{2} \qquad G^2 = \alpha\beta$$

so $\alpha = \frac{\alpha+\beta}{2} + \frac{\alpha-\beta}{2}$ and $\frac{\alpha-\beta}{2}$

$$= \sqrt{\frac{(\alpha+\beta)^2 - 4\alpha\beta}{4}} = \sqrt{\left(\frac{\alpha+\beta}{2}\right)^2 - \alpha\beta} = \sqrt{A^2 - G^2}$$

so $\alpha = A + \sqrt{A^2 - G^2}$

and $\beta = \frac{\alpha+\beta}{2} - \left(\frac{\alpha-\beta}{2}\right) = A - \sqrt{A^2 - G^2}$

S_4 : $q = \frac{p+r}{2} \Rightarrow q^3 = \frac{(p+r)^3}{8}$

$\Rightarrow 8q^3 = p^3 + r^3 + 3pr(p+r)$

$\Rightarrow 8q^3 = p^3 + r^3 + 3pr(2q)$

$\Rightarrow p^3 + r^3 = 8q^3 - 6pqr$

11. $\sum_{r=1}^{n} \frac{1}{\sqrt{a+rx} + \sqrt{a+(r-1)x}}$

$$\sum_{r=1}^{n} \frac{\sqrt{(a+rx)} - \sqrt{a+(r-1)x}}{(a+rx)-(a+(r-1)x)}$$

$$\sum_{r=1}^{n} \frac{\sqrt{a+rx} - \sqrt{a+(r-1)x}}{x}$$

$$= \frac{1}{x}[(\sqrt{a+x} - \sqrt{a+0.x}) + (\sqrt{a+2x} - \sqrt{a+x})$$

$$+ (\sqrt{a+3x} - \sqrt{a+2x}) + ... + (\sqrt{a+nx} - \sqrt{a+(n-1)x})]$$

$$= \frac{1}{x}\left[\sqrt{a+nx} - \sqrt{a}\right] = \frac{n}{\sqrt{a} + \sqrt{a+nx}}$$

12. (A, C)

$$S = 1 + \frac{1}{(1+3)}(1+2)^2 + \frac{1}{(1+3+5)}(1+2+3)^2$$

$$+ \frac{1}{(1+3+5+7)}(1+2+3+4)^2 + ...$$

r^{th} term

$$T_r = \frac{1}{r^2}(1+2+......+r)^2 = \frac{1}{r^2}\left\{\frac{r(r+1)}{2}\right\}^2 = \frac{r^2+2r+1}{4}$$

$\therefore T_7 = 16$ and $S_{10} = \sum_{r=1}^{10} T_r = \frac{1}{4}$

$$\left\{\frac{(10)(10+1)(20+1)}{6} + (10)(10+1) + 10\right\} = \frac{505}{4}$$

13. 1, $\log_y x$, $\log_z y$, $-15 \log_x z$ are in AP.

Let common diff. is d.

$\log_y x = 1 + d \Rightarrow x = (y)^{1+d}$

$\log_z y = 1 + 2d \Rightarrow y = (z)^{1+2d}$

$-15 \log_x z = 1 + 3d \Rightarrow z = x^{\left(\frac{1+3d}{-15}\right)}$

So $x = (y)^{1+d}$

$= ((z)^{1+2d})^{1+d}$

$x = (x)^{\left(\frac{1+3d}{-15}\right)(1+d)(1+2d)}$

So $(1+d)(1+2d)(1+3d) = -15$

So $d = -2$

$x = (y)^{-1}$

$y = (z)^{-3}$

$z = (x)^{1/3}$

$z^3 = x$

14. (A,B,C)

$$\sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r = \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$= \frac{1}{6} n.(n+1)[2n+1+3] = \frac{n(n+1)(n+2)}{3}$$

$a = 0, b = 1, c = 2$

15. (A, B, C, D)

Since a_{912}, a_{951} and a_{480}, is divisible by 3 then a_{91} is not prime

$$a_{91} = \frac{10^{91}-1}{10-1} = \frac{10^{91}-1}{10^7-1} \times \frac{10^7-1}{10-1}$$

$$= (1 + 10^7 + ... + 10^{84})(1 + 10 + ... + 10^6)$$

$\Rightarrow a_{91}$ is not prime

16. (A)

$3(a^2 + b^2 + c^2 + 1) - 2(a + b + c + ab + bc + ca) = 0$

$\Rightarrow (a-1)^2 + (b-1)^2 + (c-1)^2 + (a-b)^2 + (b-c)^2 + (c-a)^2 = 0$

$\Rightarrow a = b = c = 1$

17. (A)

obviously both the statements are true and statement-II explains statement-I.

19. (A)

Statement -I : Consider 'a' quantities equal to $\frac{1}{a}$, 'b' quantities equal to $\frac{1}{b}$ & 'c' quantities equal to $\frac{1}{c}$

$$A.M. = \frac{a.\frac{1}{a}+b.\frac{1}{b}+c.\frac{1}{c}}{a+b+c} = \frac{3}{a+b+c}$$

$$G.M. = \left(\frac{1}{a^a}.\frac{1}{b^b}.\frac{1}{c^c}\right)^{\frac{1}{a+b+c}}$$

A.M. ≥ G.M.

$$\frac{3}{a+b+c} \geq \frac{1}{a^{\frac{a}{a+b+c}}, b^{\frac{b}{a+b+c}}, c^{\frac{c}{a+b+c}}}$$

$$\Rightarrow \quad a^{\frac{a}{a+b+c}} \cdot b^{\frac{b}{a+b+c}} \cdot c^{\frac{c}{a+b+c}} \geq \frac{a+b+c}{3}$$

Statement-II : $A = \frac{a_1+a_n}{2}$, $G = \sqrt{a_1 a_n}$

We may assume that

$a_1a_n - a_2a_{n-1} = a_1\{a_1+(n-1)d\} - (a_1+d)\{a_1+(n-2)d\}$

$= -(n-2)d^2 < 0$

$a_1a_n < a_2a_{n-1}$

Similarly $a_1a_n < a_3a_{n-2}$

If n is even $(a_1a_n)^{n/2} < a_1a_2a_3 \ldots\ldots a_n$

If n is odd

$\Rightarrow$ $n = 2m+1$

$a_1\, a_{2m+1} < a_{m+1}^2$

similarly $a_1\, a_{2m+1} < a_m.a_{m+2}$

$(a_1a_n)^{n/2} < a_1.a_2\ldots\ldots a_n$

$\because$ A.M. ≥ G.M.

$$\left(\frac{a_1+a_2+a_3\ldots\ldots a_n}{n}\right)^n > a_1a_2\ldots\ldots a_n$$

$$\Rightarrow \quad (a_1a_n)^{n/2} < a_1a_2\ldots\ldots\ldots a_n < \left(\frac{a_1+a_n}{2}\right)^n$$

$$\Rightarrow \quad G^n < a_1a_2\ldots\ldots\ldots a_n < A^n$$

20. (B)

Statement-I a, A, b are in A.P.

$\Rightarrow$ $2A = a+b$ ..... **(i)**

a, p, q, b are in G.P.

$\Rightarrow$ $pq = ab$ ..... **(ii)**

and let common ratio of G.P. be r

$\therefore$ $b = ar^3$ $\Rightarrow$ $r = \left(\frac{b}{a}\right)^{\frac{1}{3}}$

$p = ar$ $\Rightarrow$ $p = a.\left(\frac{b}{a}\right)^{\frac{1}{3}}$

$\Rightarrow$ $p^3 = a^2 b$ ..... **(iii)**

$q = ar^2$ $\Rightarrow$ $q = a\left(\frac{b}{a}\right)^{\frac{2}{3}}$

$\Rightarrow$ $q^3 = ab^2$ ..... **(iv)**

from **(i)**, **(ii)**, **(iii)** & **(iv)**

$p^3 + q^3 = 2A\, pq$

statement-II is obviously true

21. (A) → (r), (B) → (p, t), (C) → (s), (D) → (q)

(A) $F(n+1) = \frac{2\,F(n)+1}{2} = F(n) + \frac{1}{2}$

$\therefore$ F(1), F(2), F(3), ...

is an AP with common difference $\frac{1}{2}$

(B) $a_1 + 2d + a_1 + 4d + a_1 + 10d + a_1 + 16d + a_1 + 18d$

$= 5a_1 + 50d$

$= 5(a_1 + 10d) = 10$ i.e. $a_1 + 10d = 2$

Now, $\sum_{i=1}^{21} a_i = \frac{21}{2}[2a_1 + 20d] = 21(a_1+10d) = 42$

(C) $S = 1 + 5 + 13 + 29 + \ldots\ldots + t_{10}$

$S = \quad 1 + 5 + 13 + \ldots\ldots + t_9 + t_{10}$

Subtrating

$t_{10} = 1 + 4 + 8 + 16 + \ldots\ldots.$ up to 10 terms

$= 1 + (4 + 8 + 16 + \ldots\ldots$ up to 9 terms)

$= 2045$

(D) Sum of all two digit numbers

$= \frac{90}{2}(10+99) = (45)(109)$

Sum of all two digit numbers is divisible by 2

$= \frac{45}{2}(10+98) = (45)(54)$

Sum of all two digit numbers is divisible by 3

$$= \frac{30}{2}(12+99) = 15(111)$$

Sum of all two digit numbers divisible by 6

$$= \frac{15}{2}(12+96) = 15(54)$$

The required sum is

$45(109) + 15(54) - (45)(54) - 15(111) = 1620$

22. (A) → (q), (B) → (r), (C) → (p), (D) → (t)

(A) $a + b = 12$

$$ab + \frac{6ab}{a+b} = 48$$

$$ab + \frac{ab}{2} = 48$$

$\therefore$ $ab = 32$

(B) $S = \frac{5}{1^2.4^2} + \frac{11}{4^2.7^2} + \frac{17}{7^2.10^2} + ...$

$$\Rightarrow 3S = \frac{3.5}{1^2.4^2} + \frac{3.11}{4^2.7^2} + \frac{3.17}{7^2.10^2} + ...$$

$$\Rightarrow 3S = \frac{(4-1).(4+1)}{1^2.4^2} + \frac{(7-4)(7+4)}{4^2.7^2} + \frac{(10-7)(10+7)}{7^2.10^2} + ...$$

$$\Rightarrow 3S = \frac{4^2-1^2}{1^2.4^2} + \frac{7^2-4^2}{4^2.7^2} + \frac{10^2-7^2}{7^2.10^2} + ...$$

$$\Rightarrow 3S = 1 - \frac{1}{4^2} + \frac{1}{4^2} - \frac{1}{7^2} + \frac{1}{7^2} - \frac{1}{10^2} + ...$$

$$\Rightarrow 3S = 1 \quad \Rightarrow \quad S = \frac{1}{3}$$

(C) H.M of $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ is $\frac{4}{2+3+4+5} = \frac{2}{7}$

(D) Since G.M. lies between the numbers

$$GM = -\sqrt{(-4)\times(-9)} = -6$$

23.

1. (B)

$$G_1 G_2 .... G_n = \left(\sqrt{1\times 1024}\right)^n = 2^{5n}$$

$\therefore$ $2^{5n} = 2^{45}$

$\therefore$ $n = 9$

2. (B)

$$A_1 + A_2 + A_3 + ... + A_{m-1} + A_m = 1025 \times 171$$

$$\therefore \quad m\left(\frac{-2+1027}{2}\right) = 1025\times 171$$

$\therefore$ $m = 342$

3. (A)

Since $n = 9$, $\therefore$ $r = (1024)^{\frac{1}{9+1}} = 2$

$\therefore$ $G_1 = 2, r = 2$

$$G_1 + G_2 + ... + G_n = \frac{2.(2^9 - 1)}{2-1} = 1024 - 2$$

$= 1022$

24.

1. (C)

2. (B)

3. (C)

Let numbers in set A be a – D, a, a + D and in set B be b – d, b , b + d

$3a = 3b = 15 \Rightarrow a = b = 5$

set A = {5 – D, 5, 5 + D}

set B = {5 – d, 5, 5 + d}

where $D = d + 1$

$$\frac{p}{q} = \frac{5(25-D^2)}{5(25-d^2)} = \frac{7}{8}$$

$$25(8-7) = 8(d+1)^2 - 7d^2$$

$\Rightarrow$ $d = -17, 1$ but $d > 0$

$\Rightarrow$ $d = 1$

So numbers in Set A are 3, 5, 7

number in Set B are 4, 5, 6

Now $p = 3\times 5\times 7 = 105$

$q = 4\times 5\times 6 = 120$

value of $D + d = 3$

25.

1. (C)

2. (B)

3. (C)

Let four integers be a–d, a , a + d and a + 2d where a and d are integers and d > 0.

$\because \quad a + 2d = (a - d)^2 + a^2 + (a + d)^2$

$\Rightarrow \quad 2d^2 - 2d + 3a^2 - a = 0$ ....**(i)**

$\therefore \quad d = \frac{1}{2}\left[1 \pm \sqrt{1 + 2a - 6a^2}\right]$ ....**(ii)**

Since d is positive integer

$\therefore \quad 1 + 2a - 6a^2 > 0$

$6a^2 - 2a - 1 < 0$

$\Rightarrow \quad \frac{1-\sqrt{7}}{6} < a < \frac{1+\sqrt{7}}{6} \qquad \because$ a is an integer

$\therefore \quad a = 0$ Put in **(ii)**

$\therefore \quad d = 1$ or 0 but $\because \quad d > 0$

$\therefore \quad d = 1$

$\therefore$ The four numbers are : –1, 0, 1, 2

26. (12)

Let the series be a, ax, ax^2, ax^3... given that $|x| < 1$ and $x \neq 0$.

Also, $\frac{T_4}{T_2} = \frac{ax^3}{ax} = \frac{1}{16} \quad \Rightarrow \quad x^2 = \frac{1}{16}$

$\Rightarrow \quad x = \pm\frac{1}{4}$

But since it is a decreasing G.P. $\Rightarrow \quad x = \frac{1}{4}$

Also, $\frac{T_3}{T_2^2} = \frac{ax^2}{(ax)^2} = \frac{1}{9} \quad \Rightarrow \quad \frac{1}{a} = \frac{1}{9}$

$\Rightarrow \quad a = 9$

$S_\infty = \frac{a}{1-r} = \frac{9}{1-\frac{1}{4}} = \frac{9\times 4}{3} = 12$

27. Let first installment be = 'a' and the common difference of the A.P. be 'd'

So $a + (a + d) + (a + 2d) + ... + (a + 39d) = 3600$

$\Rightarrow \quad \frac{40}{2}[2a + 39\,d] = 3600$

$\Rightarrow \quad 2a + 39\,d = 180$ ..... **(i)**

and $\frac{30}{2}[2a + 29\,d] = 2400$

$\Rightarrow \quad 2a + 29d = 160$ ..... **(ii)**

By equations **(i)** & **(ii)**, we get $d = 2$ and $a = 51$

28. (7)

$(1^2 + 2^2 + ... + n^2) - (a_1 + a_2 + ... + a_n) = \frac{1}{3}n(n^2 - 1)$ ......**(i)**

Replacing n by (n – 1), then

$(1^2 + 2^2 + ... + (n-1)^2) - (a_1 + a_2 + ... + a_{n-1})$

$= \frac{1}{3}(n-1)((n-1)^2 - 1)$ ......**(ii)**

Subtracting **(ii)** from **(i)**

$n^2 - a_n = n^2 - n \quad \Rightarrow \quad a_n = n$

$\Rightarrow \quad a_7 = 7$

29. $q = \frac{p}{2}[2A + (p-1)d]$

$\Rightarrow \quad \frac{2q}{p} = 2A + (p-1)d$ ......**(i)**

$p = \frac{q}{2}[2A + (q-1)d]$

$\Rightarrow \quad \frac{2p}{q} = 2A + (q-1)\,d$ ......**(ii)**

on subtracting equation **(i)** from **(ii)**, we get

$\frac{2}{pq}(q^2 - p^2) = (p - q)\,d$

$\Rightarrow \quad d = \frac{-2}{pq}(p+q)$

$\therefore$ Sum of (p + q) terms is

$= \frac{p+q}{2}[2A + (p + q - 1)d]$

$= \frac{p+q}{2}[2A + (p-1)\,d + qd]$

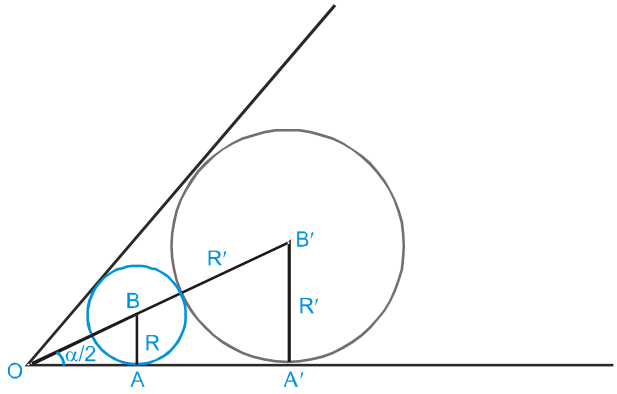
$= \frac{p+q}{2}\left[\frac{2q}{p} + q\left\{\frac{-2}{pq}(p+q)\right\}\right]$

$= \frac{p+q}{2}\left[\frac{2q}{p} - 2 - \frac{2q}{p}\right] = -(p+q)$

30. Radius of first circle = R

Let Radius of second circle be = R′

$$\frac{OB}{BA} = \operatorname{cosec}\frac{\alpha}{2}$$



$\therefore$ $OB = R\operatorname{cosec}\frac{\alpha}{2}$

Now $\frac{OB'}{A'B'} = \operatorname{cosec}\frac{\alpha}{2}$

$\Rightarrow$ $OB' = R'\operatorname{cosec}\frac{\alpha}{2} = OB + R + R'$

$\Rightarrow$ $R + R\operatorname{cosec}\frac{\alpha}{2} + R' = R'\operatorname{cosec}\frac{\alpha}{2}$

$\Rightarrow$ $R(1 + \operatorname{cosec}\alpha/2) = R'(\operatorname{cosec}\alpha/2 - 1)$

$\Rightarrow$ $R' = R\left(\frac{1+\sin\alpha/2}{1-\sin\alpha/2}\right)$

if radius of the third circle be R″, then

Similarly $R'' = R\left(\frac{1+\sin\alpha/2}{1-\sin\alpha/2}\right)^2$

So R + R′ + R″ + ... n terms

$$= R\left[\frac{\left(\frac{1+\sin\alpha/2}{1-\sin\alpha/2}\right)^n - 1}{\left(\frac{1+\sin\alpha/2}{1-\sin\alpha/2}\right) - 1}\right]$$

$$= R\left(\frac{1-\sin\alpha/2}{2\sin\alpha/2}\right)\left[\left(\frac{1+\sin\alpha/2}{1-\sin\alpha/2}\right)^n - 1\right]$$