HINTS & SOLUTIONS

1. $\left(1+\cos\dfrac{\pi}{10}\right)\left(1+\cos\dfrac{3\pi}{10}\right)\left(1-\cos\dfrac{3\pi}{10}\right)\left(1-\cos\dfrac{\pi}{10}\right)$

$= \left(1-\cos^2\dfrac{\pi}{10}\right)\left(1-\cos^2\dfrac{3\pi}{10}\right)$

$\sin^2\dfrac{\pi}{10}\cdot\sin^2\dfrac{3\pi}{10}$

$= \left(\dfrac{\sqrt{5}-1}{4}\cdot\dfrac{\sqrt{5}+1}{4}\right)^2 = \left(\dfrac{4}{16}\right)^2 = \dfrac{1}{16}$

2. Since $f(x) = \sin x$ is an increasing function for $0 < x < \pi/2$ and 1 rad is approximately 57°, we have $1° < 1^R \Rightarrow \sin 1° < \sin 1$

3. On adding and subtracting

$x = \dfrac{3-\cos 4\theta + 4\sin 2\theta}{2}$; $y = \dfrac{3-\cos 4\theta - 4\sin 2\theta}{2}$

$x = \dfrac{4(1+\sin 2\theta)-(1+\cos 4\theta)}{2}$;

$y = \dfrac{4(1-\sin 2\theta)-(1+\cos 4\theta)}{2}$

$x = 2(1+\sin 2\theta) - \cos^2 2\theta$; $y = 2(1-\sin 2\theta)-\cos^2 2\theta$

$x = 1 + 2\sin 2\theta + \sin^2 2\theta$; $y = 1 - 2\sin 2\theta + \sin^2 2\theta$

$x = (1+\sin 2\theta)^2$; $y = (1-\sin 2\theta)^2$

$\Rightarrow \quad \sqrt{x} + \sqrt{y} = 2$

Alternate : Or put $\theta = \dfrac{\pi}{4}$ and verify

4. $\tan(A+B) = \dfrac{\tan A + \tan B}{1-\tan A\tan B} = \dfrac{\tan A + \dfrac{n\sin A\cos A}{1-n\cos^2 A}}{1-\tan A\cdot\dfrac{n\sin A\cos A}{1-n\cos^2 A}}$

$= \dfrac{\sin A(1-n\cos^2 A)+n\sin A\cos^2 A}{\cos A(1-n\cos^2 A)-n\sin^2 A\cos A}$

$= \dfrac{\sin A - 0}{\cos A(1-n\cos^2 A - n\sin^2 A)} = \dfrac{\sin A}{(1-n)\cos A}$

5. $A^2 + B^2 = 3 + 2\left[\cos\dfrac{2\pi}{7}+\cos\dfrac{4\pi}{7}+\cos\dfrac{6\pi}{7}\right]$

$= 3 + 2\left(-\dfrac{1}{2}\right) = 2 \quad \Rightarrow \quad \sqrt{A^2+B^2} = \sqrt{2}$

6. Given expression reduce to

$\dfrac{\sin 22\cos 8 + \cos 22\sin 8}{\sin 23\cos 7 + \cos 23\sin 7} = \dfrac{\sin 30}{\sin 30} = 1$

7. $\sin\theta\sec^7\theta + \cos\theta\,\text{cosec}^7\theta = \tan\theta\sec^6\theta + \cot\theta\,\text{cosec}^6\theta$

$= \sqrt{\dfrac{a}{b}}\left(1+\dfrac{a}{b}\right)^3 + \sqrt{\dfrac{b}{a}}\left(1+\dfrac{b}{a}\right)^3$

$= (a+b)^3\left[\dfrac{\sqrt{a}}{b^{7/2}}+\dfrac{\sqrt{b}}{a^{7/2}}\right] = \dfrac{(a+b)^3(a^4+b^4)}{(ab)^{7/2}}$.

8. $\dfrac{\sin 2\alpha + \sin 4\alpha - \sin 3\alpha}{\cos 2\alpha + \cos 4\alpha - \cos 3\alpha}$

$= \dfrac{2\sin 3\alpha\cos\alpha - \sin 3\alpha}{2\cos 3\alpha\cos\alpha - \cos 3\alpha}$

$= \dfrac{\sin 3\alpha(2\cos\alpha - 1)}{\cos 3\alpha(2\cos\alpha - 1)} = \tan 3\alpha$

9. $\cos 20° + 2\sin^2 55° - \sqrt{2}\sin 65°$

$= \cos 20° + 1 - \cos 110° - \sqrt{2}\sin 65°$

$= 2\sin 65°\sin 45° + 1 - \sqrt{2}\sin 65° = 1.$

10. Use $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$.

11. $\cot 123°\cot 147°\cot 133°\cot 137°$

$= \dfrac{\cos 123°\cos 147°}{\sin 123°\sin 147°}\cdot\dfrac{\cos 133°\cos 137°}{\sin 133°\sin 137°}$

$= \dfrac{\cos 270° + \cos 24°}{\cos 24° - \cos 270°}\cdot\dfrac{\cos 270° + \cos 24°}{\cos 24° - \cos 270°} = 1$

12. $5\tan\theta = 4$ or $\tan\theta = \dfrac{4}{5}$

now, $\dfrac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta} = \dfrac{5\dfrac{\sin\theta}{\cos\theta} - 3}{5\dfrac{\sin\theta}{\cos\theta} + 2}$

$= \dfrac{5\tan\theta - 3}{5\tan\theta + 2}$

$= \dfrac{5 \times \dfrac{4}{5} - 3}{5 \times \dfrac{4}{5} + 2} = \dfrac{1}{6}$

13. $4x^2 - 2\sqrt{5}\,x + 1 = 0$

Let α and β be the roots, we have

$a + b = \dfrac{2\sqrt{5}}{4} = \dfrac{\sqrt{5}}{2},\ \alpha\beta = \dfrac{1}{4}$

Since $\sin 18° = \dfrac{\sqrt{5} - 1}{4}$, $\cos 36° = \dfrac{\sqrt{5} + 1}{4}$

we have

$\sin 18° + \cos 36° = \dfrac{2\sqrt{5}}{4} = \dfrac{\sqrt{5}}{2}\quad \sin 18° \cos 36°$

$= \dfrac{5 - 1}{16} = \dfrac{4}{16} = \dfrac{1}{4}$

Here, the required roots are $\sin 18°$, $\cos 36°$.

14. $\cos(A - B) = \dfrac{3}{5}$

or $\quad 5\cos A \cos B + 5\sin A \sin B = 3$

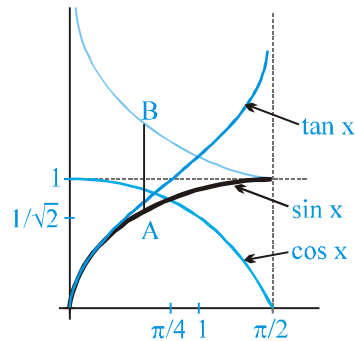From the second relation, we have

$\quad \sin A \sin B = 2\cos A \cos B$

$\Rightarrow \quad \cos A \cos B = \dfrac{1}{5}\quad$ and $\quad \sin A \sin B = \dfrac{2}{5}$

15. $\dfrac{1}{\cos\alpha} = \dfrac{2 - \cos\beta}{2\cos\beta - 1}\quad$ Applying C/D

$\Rightarrow \dfrac{1 - \cos\alpha}{1 + \cos\alpha} = \dfrac{3(1 - \cos\beta)}{1 + \cos\beta}$

$\Rightarrow \tan^2\dfrac{\alpha}{2} = 3\tan^2\dfrac{\beta}{2} \Rightarrow \tan^2\dfrac{\alpha}{2}\ \cot^2\dfrac{\beta}{2} = 3$

16. Given $\quad \tan x = \cos x$

or $\quad \sin x = \cos^2 x = 1 - \sin^2 x \qquad \text{....(1)}$

now, $\quad \cosec x - \sin x = \dfrac{1 - \sin^2 x}{\sin x} = 1\ (\text{from}\,(1)\,)$

17. On rationalizing ; we get

$\dfrac{1 - \sin x + 1 + \sin x + 2\,|\cos x\,|}{1 - \sin x - 1 + \sin x} = \dfrac{2(1 + |\cos x\,|)}{-2(\sin x)}$

$= \dfrac{1 + \cos x}{-(\sin x)}\quad \Rightarrow\ \textbf{(D)}$

18. $\cot x + \dfrac{\cos(60 + x)}{\sin(60 + x)} + \dfrac{\cos(x - 60)}{\sin(x - 60)}$

$= \dfrac{\cos x}{\sin x} + \dfrac{\sin(2x)}{\sin(x + 60)\sin(x - 60)}$

$= \dfrac{\cos x}{\sin x} + \dfrac{8\sin x\ \cos x}{4\sin^2 x - 3}$

$= \dfrac{4\sin^2 x\ \cos x - 3\cos x + 8\sin^2 x\cos x}{4\sin^3 x - 3\sin x}$

$= \dfrac{3[3\cos x - 4\cos^3 x]}{\sin^3 x} = 3\cot 3x$

$\Rightarrow \dfrac{3[1 - 3\tan^2 x]}{3\tan x - \tan^3 x}$

19. $\sin\dfrac{7\pi}{8} = \sin\left(\pi - \dfrac{\pi}{8}\right) = \sin\dfrac{\pi}{8}$

$\sin\dfrac{5\pi}{8} = \sin\left(\pi - \dfrac{3\pi}{8}\right) = \sin\dfrac{3\pi}{8}$

Therefore, the given value $= 2\left[\sin^2\dfrac{\pi}{8} + \sin^2\dfrac{3\pi}{8}\right]$

$= 2\left[\sin^2\dfrac{\pi}{8} + \cos^2\dfrac{\pi}{8}\right]$

$= 2(1) = 2$

$\left[\because \sin\dfrac{3\pi}{8} = \sin\left(\dfrac{\pi}{2} - \dfrac{\pi}{8}\right) = \cos\dfrac{\pi}{8}\right]$

20. We have $\tan 90° - \tan 27° - \tan 63° + \tan 81°$

$= (\tan 9° + \tan 81°) - (\tan 27° + \tan 63)$

$= \dfrac{1}{\sin 9° \cos 9°} - \dfrac{1}{\sin 27° \cos 27°}$

$= \dfrac{2}{\sin 18°} - \dfrac{2}{\sin 54°}$

$= 2\left[\dfrac{\sin 54° - \sin 18°}{\sin 54° \sin 18°}\right]$

$= 2\left[\dfrac{\cos 36° \sin 18°}{\sin 18° \cos 36°}\right] = 4$

1. $\cos\dfrac{\pi}{10} \cdot \cos\dfrac{2\pi}{10} \cdot \cos\dfrac{4\pi}{10} \cdot \cos\dfrac{8\pi}{10} \cdot \cos\dfrac{16\pi}{10}$

$= \dfrac{\sin 2^5 \dfrac{\pi}{10}}{2^5 \sin\dfrac{\pi}{10}} = \dfrac{1}{32}\; \dfrac{\sin\dfrac{32\pi}{10}}{\sin\dfrac{\pi}{10}} = \dfrac{1}{32}\; \dfrac{\sin\left(3\pi + \dfrac{2\pi}{10}\right)}{\sin\left(\dfrac{\pi}{10}\right)}$

$= -\dfrac{1}{32} \cdot \dfrac{2\sin\dfrac{\pi}{10}\cos\dfrac{\pi}{10}}{\sin\dfrac{\pi}{10}}$

$= -\dfrac{1}{16}\; \cos\dfrac{\pi}{10} = -\dfrac{1}{64}\; \sqrt{10 + 2\sqrt{5}}$

2. $\cos^2 x + \cos^2 y + \cos^2 z - 2\cos x \cos y \cos z$

(Given $x + y = z$)

$= 1 + \cos(x + y)\cos(x - y) + \cos^2 z - 2\cos x \cos y \cos z$

$= 1 + \cos z[\cos(x - y) + \cos(x + y)] - 2\cos x \cos y \cos z$

$= 1 + \cos z . 2\cos x \cos y - 2\cos x \cos y \cos z$

$= 1$

$= \cos(x + y - z)$

4. $\tan A + \tan B + \tan C = 6$, $\tan A \tan B = 2$

In any ΔABC,

$\tan A + \tan B + \tan C = \tan A \tan B \tan C$

$\Rightarrow \quad 6 = 2\tan C \qquad \Rightarrow \qquad \tan C = 3$

$\therefore \quad \tan A + \tan B + 3 = 6$

$\Rightarrow \quad \tan A + \tan B = 3 \quad \& \quad \tan A \tan B = 2$

Now $(\tan A - \tan B)^2 = (\tan A + \tan B)^2 - 4\tan A \tan B$

$= 9 - 8$

$= 1$

$\Rightarrow \quad \tan A - \tan B = \pm 1$

$\therefore \quad \tan A - \tan B = 1 \quad$ or $\quad \tan A - \tan B = -1$

$\quad \tan A + \tan B = 3 \qquad\qquad \tan A + \tan B = 3$

on solving $\qquad\qquad\qquad$ on solving

$\quad \tan A = 2 \qquad\qquad\qquad \tan A = 1$

$\quad \tan B = 1 \qquad\qquad\qquad \tan B = 2$

5. $\sqrt{\sin\theta + \left(\sqrt{\sin\theta + \sqrt{\sin\theta + ... + \infty}}\right)} = (\sec^4\alpha - \sin\theta)$

$\sqrt{\sin\theta + (\sec^4\alpha - \sin\theta)} = \sec^4\alpha - \sin\theta$

$\sin\theta = \sec^4\alpha - \sec^2\alpha = \sec^2\alpha \tan^2\alpha$

(B) $\dfrac{2(2\sin^2\alpha)}{4(\cos^2\alpha)^2} = \tan^2\alpha \sec^2\alpha$

6. $\cos 4\theta - \cos 4\phi = 2\cos^2 2\theta - 2\cos^2 2\phi$

$= 2(\cos 2\theta + \cos 2\phi)(\cos 2\theta - \cos 2\phi)$

$= 2(2\cos^2\theta - 2\sin^2\phi)\, 2(\cos^2\theta - \cos^2\phi)$

$= 8(\cos\theta + \sin\phi)(\cos\theta - \sin\phi)(\cos\theta - \cos\phi)$

$\quad (\cos\theta + \cos\phi)$

8. $\sqrt{\dfrac{1-\sin A}{1+\sin A}} + \dfrac{\sin A}{\cos A} = \dfrac{1}{\cos A}$

$\Rightarrow \quad \sqrt{\dfrac{(1-\sin A)^2}{1-\sin^2 A}} + \dfrac{\sin A}{\cos A} = \dfrac{1}{\cos A}$

$\Rightarrow \quad \dfrac{|1-\sin A|}{|\cos A|} + \dfrac{\sin A}{\cos A} = \dfrac{1}{\cos A} \quad (\because\ 1-\sin A \geq 0)$

$\Rightarrow \quad \dfrac{1}{\cos A} = \dfrac{1}{\cos A} \quad \text{when } \cos A > 0$

$\Rightarrow \quad$ A belongs to I^{st} & IVth quadrant.

9. **(A)** $\dfrac{\cos 2\alpha\ \tan\left(\dfrac{\pi}{4}+\alpha\right)}{1+\cos\left(\dfrac{\pi}{2}-2\alpha\right)}$

$= \dfrac{(\cos^2\alpha - \sin^2\alpha)}{1+\sin 2\alpha}.\tan\left(\dfrac{\pi}{4}+x\right)$

$= \dfrac{\cos^2\alpha - \sin^2\alpha}{(\cos\alpha + \sin\alpha)^2}\ \tan\left(\dfrac{\pi}{4}+x\right)$

$= \dfrac{1-\tan\alpha}{1+\tan\alpha}.\dfrac{1+\tan\alpha}{1-\tan\alpha} = 1.$

(B) $\dfrac{\sin\alpha}{\sin\alpha - \cos\alpha\,\tan\dfrac{\alpha}{2}} - \cos\alpha$

$= \dfrac{\sin\alpha\,\cos\dfrac{\alpha}{2}}{\sin\dfrac{\alpha}{2}} - \cos\alpha = \dfrac{\sin\dfrac{\alpha}{2}}{\sin\dfrac{\alpha}{2}} = 1$

10. $\cos A + \cos B = 1$

$2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2} = 1 \qquad \Rightarrow \quad \cos\dfrac{A-B}{2} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \quad \cos(A-B) = 2\cos^2\dfrac{A-B}{2} - 1 = \dfrac{2}{3} - 1 = -\dfrac{1}{3}$

& $|\cos A - \cos B| = \left|2\sin\dfrac{A+B}{2}\sin\dfrac{B-A}{2}\right| = \sqrt{\dfrac{2}{3}}$

(C) $\dfrac{1}{\sin^2 2\alpha} + \dfrac{(\cos^2\alpha - \sin^2\alpha)^2}{\sin^2 2\alpha}$

$= \dfrac{1+\cos^2 2\alpha}{\sin^2 2\alpha} \neq 1$

(D) $\dfrac{(\sin\alpha + \cos\alpha)^2}{(\sin\alpha + \cos\alpha)^2} = 1$

11. $f(x) = \dfrac{\sin x}{|\sec x|} + \dfrac{\cos x}{|\cosec\ x|} = \sin x\,|\cos x| + \cos x\,|\sin x|$

$\Rightarrow f(x)$ is constant when $\sin x$ and $\cos x$ are of opposite sign, i.e. $f(x)$ is in IInd or IVth quadrant.

12. $f_n(\theta) = \dfrac{\sin\dfrac{\theta}{2}(1+\cos\theta)(1+\cos 2\theta)(1+\cos 4\theta).....(1+\cos 2^n\theta)}{\cos\dfrac{\theta}{2}\cos\theta\cos 2\theta\cos 4\theta......\cos 2^n\theta}$

$= \dfrac{\sin\dfrac{\theta}{2}\left(2\cos^2\dfrac{\theta}{2}\right)(2\cos^2\theta)(2\cos^2 2\theta)...(2\cos 2^{n-1}\theta)}{\cos\dfrac{\theta}{2}\cos\theta\cos 2\theta\cos 4\theta...\cos 2^n\theta}$

$= \dfrac{2^{n+1}\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}\cos\theta\cos 2\theta.....\cos 2^{n-1}\theta}{\cos 2^n\theta}$

$= \dfrac{\sin 2^n\theta}{\cos 2^n\theta} = \tan 2^n\theta$

$f_2\left(\dfrac{\pi}{16}\right) = \tan 2^2\left(\dfrac{\pi}{16}\right) = 1$

$f_3\left(\dfrac{\pi}{32}\right) = 1$, $f_4\left(\dfrac{\pi}{64}\right) = 1$, $f_5\left(\dfrac{\pi}{128}\right) = 1$

13. Let $\tan\dfrac{\alpha}{2} = t$

$(a+2)\,2t + (2a-1)(1-t^2) = (2a+1)(t^2+1)$

$\Rightarrow \quad 2at + 4t + 2a - 2at^2 - 1 + t^2 = 2a + 1 + 2at^2 + t^2$

$\Rightarrow \quad 4at^2 - 2t(2+a) + 2 = 0$

$\Rightarrow \quad 2at^2 - 2t - at + 1 = 0$

$\Rightarrow \quad 2t(at-1) - 1(at-1) = 0$

$\Rightarrow \quad t = 1/2,\ t = 1/a$

$\Rightarrow \quad \tan\alpha = \dfrac{2\tan\alpha/2}{1-\tan^2\alpha/2}$

$\Rightarrow \quad \tan\alpha = \dfrac{2 \times 1/2}{1 - 1/4} = 4/3$

or $\quad \tan\alpha = \dfrac{2/a}{1 - 1/a^2} = \dfrac{2a}{a^2 - 1}$

14. $y = a\cos^2 x + 2b\sin x \cos x + c\sin^2 x$

& $\quad \tan x = \dfrac{2b}{a - c}$

$z = a\sin^2 x - 2b\sin x \cos x + c\cos^2 x$

$\Rightarrow \quad y + z = a + c$

and $y - z = (a - c)\left(\cos^2 x - \sin^2 x\right) + 4b\sin x \cos x$

$\qquad = (a - c)\cos 2x + 2b\sin 2x$

$(\because 2b = (a - c)\tan x)$

$\qquad = (a - c)[\cos 2x + \tan x . \sin 2x] = (a - c)$

$\left[2\cos 2x + \dfrac{\sin x}{\cos x}\sin 2x\right] = \dfrac{(a - c)\cos(2x - x)}{\cos x} = (a - c).$

15. $\sin^6 x + \cos^6 x = a^2$

$\Rightarrow \quad (\sin^2 x + \cos^2 x)(\sin^4 x + \cos^4 x - \sin^2 x \cos^2 x) = a^2$

$\Rightarrow \quad (\sin^2 x + \cos^2 x)^2 - 3\sin^2 x \cos^2 x = a^2$

$\Rightarrow \quad 1 - 3\sin^2 x \cos^2 x = a^2$

$\Rightarrow \quad 1 - \dfrac{3}{4}\sin^2 2x = a^2 \qquad \Rightarrow \quad \dfrac{4(1 - a^2)}{3} = \sin^2 2x$

$\Rightarrow \quad 0 \le \dfrac{4}{3}(1 - a^2) \le 1$

$1 - a^2 \ge 0 \qquad \text{and} \quad 4 - 4a^2 \le 3$

$a^2 \le 1 \qquad \text{and} \quad \dfrac{1}{4} \le a^2$

$-1 \le a \le 1 \qquad \text{and} \quad a \ge \dfrac{1}{2} \text{ or } a \le -\dfrac{1}{2}$

$a \in \left[-1, -\dfrac{1}{2}\right] \cup \left[\dfrac{1}{2}, 1\right]$

Part # II : Assertion & Reason

1. Statement-I: $\cos\alpha + \cos\left(\alpha + \dfrac{2\pi}{3}\right) + \cos\left(\alpha + \dfrac{4\pi}{3}\right) = 0$

$\qquad = \cos\alpha + 2\cos(\alpha + \pi)\cos\left(\dfrac{\pi}{3}\right) = 0$

$\qquad = \cos\alpha - \cos\alpha = 0$

i.e. $a + b + c = 0$ (say)

$\Rightarrow \quad a^3 + b^3 + c^3 = 3abc$

hence statement I is true

But statement II is false as vice versa is not true.

2. If A is obtuse than $0° < B + C < 90°$

$\Rightarrow \quad \tan(B + C) = \dfrac{\tan B + \tan C}{1 - \tan B \tan C} > 0$

as numerator is positive

$1 - \tan B \tan C > 0$

$\tan B \tan C < 1$

Statement II is obviously true & it explain I

3. $\cos^2\theta = \dfrac{(x + y)^2}{4xy} = \dfrac{x^2 + y^2}{4xy} + \dfrac{1}{2}$

$\dfrac{x^2 + y^2}{2} \ge xy \qquad (AM \ge GM)$

$(x - y)^2 \ge 0$ only when $x = y$

Statement-II is true.

4. $(\sin\theta - 1)^2 = 0$

$\Rightarrow \quad \sin\theta = 1$

$\Rightarrow \quad \theta = \dfrac{\pi}{2}$

$\Rightarrow \quad$ Statement-I only hold for $n = 1$ hence false.

Statement-II is true.

6. Statement I is wrong as z can be written as $\dfrac{-(x+y)}{1-xy}$.

It implies that for any values of xy(xy ≠ 1), we get a value of z and statement II is correct.

7. In the first quadrant cosθ > sinθ for θ ∈ (π/4, π/2) Hence, cos1 < sin1.

Also in the first quadrant, cosine is decreasing and sine is increasing, but this is not the correct reason for which cos1 < sin1. Thus, the correct answer is (B).

8. Statement II is true, because each trigonometric function has a principle period of π and 2π and hence 2π is one of the periods of every trigonometric function. Thus,

f(2A) = f(2B).

⇒ 2A = 2nπ + 2B, for some n ∈ Z.

or A = nπ + B

1. **(A)** $\dfrac{\cos10°-\sqrt{3}\sin10°}{\sin10°\cos10°}=\dfrac{2\left(\dfrac{1}{2}\cos10°-\dfrac{\sqrt{3}}{2}\sin10°\right)}{\dfrac{2\sin10°\cos10°}{2}}$

$=\dfrac{4\cos70°}{\sin20°}=4$

(B) $\dfrac{4\cos20°\sin20°-\sqrt{3}\cos20°}{\sin20°}$

$=\dfrac{2\sin40°-\sqrt{3}\cos20°}{\sin20°}$

$=\dfrac{2\sin90°\sin40°-2\cos30°\cos20°}{\sin20°}$

$=\dfrac{\cos50°-\cos130°-\cos50°-\cos10°}{\sin20°}$

$=-\dfrac{2\cos70°\cos60°}{\sin20°}=-1.$

(C) $\dfrac{\cos40°+\cos40°-\cos20°}{\sin20°}$

$=\dfrac{\cos40°-2\sin30°\sin10°}{\sin20°}$

$=\dfrac{\cos40°-\sin10°}{\sin20°}=\dfrac{\sin50°-\sin10°}{\sin20°}$

$=\dfrac{2\cos30°\sin20°}{\sin20°}=\sqrt{3}$

(D) $2\sqrt{2}\,\sin10°\left[\dfrac{1}{2\cos5°}+\dfrac{\cos40°}{\sin5°}-2\sin35°\right]$

$=2\sqrt{2}\,\sin10°\left[\dfrac{\sin5°+2\cos5°\cos40°-4\sin35°\sin5°.\cos5°}{2\cos5°\sin5°}\right]$

$=2\sqrt{2}\,[\sin5°+\cos45°+\cos35°-\cos25°+\cos45°]$

$=2\sqrt{2}\,[\sin5°+2\cos45°-2\sin30°\sin5°]$

$=4.$

Part # II : Comprehension

1. a. **2.** b **3.** c

1. **(a)**

$\sin\alpha = A\sin(\alpha + \beta) = A(\sin\alpha\,\cos\beta + \sin\beta\,\cos\alpha)$

$\Rightarrow \sin\alpha(1 - A\cos\beta) = A\sin\beta\,\cos\alpha$ ....**(i)**

$\Rightarrow \tan\alpha = \dfrac{A\sin\beta}{(1 - A\cos\beta)}$ ....**(ii)**

2. **(b)**

$\tan\beta = \dfrac{\sin\beta}{\cos\beta} = \dfrac{(1 - A\cos\beta)\tan\alpha}{A\cos\beta}$

$= \dfrac{(1 - A\cos\beta)\sin\alpha}{A\cos\alpha\,\cos\beta}$ [from eq. **(i)** and **(ii)**]

3. **(c)**

$\tan(\alpha + \beta)$

$= \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\,\tan\beta}$

$= \dfrac{\dfrac{A\sin\beta}{1 - A\cos\beta} + \dfrac{\sin\beta}{\cos\beta}}{1 - \dfrac{A\sin\beta\,\sin\beta}{(1 - A\cos\beta)\cos\beta}}$

$= \dfrac{A\sin\beta\,\cos\beta + \sin\beta - A\sin\beta\,\cos\beta}{\cos\beta - A\cos^2\beta - A\sin^2\beta}$

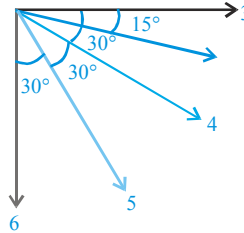$= \dfrac{\sin\beta}{\cos\beta - A}$

Also $\tan(\alpha + \beta)$

$= \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\,\tan\beta}$

$= \dfrac{\dfrac{\sin\alpha}{\cos\alpha} + \dfrac{\sin\alpha(1 - A\cos\beta)}{A\cos\alpha\,\cos\beta}}{1 - \dfrac{\sin^2\alpha(1 - A\cos\beta)}{A\cos^2\alpha\,\cos\beta}}$ [from eq. **(ii)**]

Comprehension # 2

1. **Hint :** $1^c = 57°$

$\Rightarrow \sin1° < \sin57°$ $\Rightarrow \cos1° > \cos57°$

2.

$\Rightarrow$ Answer is $90° - 15° = 75°$

3. Let the number of side of a polygon be 5x and angle α.

& for other polygon number of side be 4x and angle β.

$\alpha = \dfrac{(n-2)\pi}{n} = \dfrac{(5x-2)\pi}{5x}$; $\beta = \dfrac{(4x-2)\pi}{4x}$

$\alpha - \beta = \dfrac{\pi}{20} = \dfrac{\pi}{x}\left(\left(\dfrac{5x-2}{5}\right) - \left(\dfrac{4x-2}{4}\right)\right)$

$\Rightarrow x = 2$ $\Rightarrow$ side : 10, 8

4. $\dfrac{4x}{3} \times \dfrac{90}{100} + 3x + \dfrac{2\pi x}{75} \times \left(\dfrac{180°}{\pi}\right) = 180°$

$\Rightarrow x = 20$

Angles are $24°, 60°, 96°$

Comprehension # 3

1. $\cos\dfrac{2\pi}{7}\cos\dfrac{4\pi}{7}\cos\left(\pi - \dfrac{\pi}{7}\right)$

$= -\cos\dfrac{\pi}{7}\cos\dfrac{2\pi}{7}\cos\dfrac{4\pi}{7}$

$\Rightarrow \alpha = \dfrac{\pi}{7} = \dfrac{\pi}{2^3 - 1}$

Then value is $-\left(\dfrac{-1}{8}\right) = \dfrac{1}{8}$

2. $\cos\dfrac{\pi}{15}\cos\dfrac{2\pi}{15}\cos\dfrac{3\pi}{15}\cos\dfrac{4\pi}{15}\cos\dfrac{5\pi}{15}\cos\dfrac{6\pi}{15}\cos\dfrac{7\pi}{15}$

Put $\cos\dfrac{7\pi}{15}=-\cos\dfrac{8\pi}{15}$ and arrange

$=\cos\dfrac{\pi}{15}\cos\dfrac{2\pi}{15}\cos\dfrac{4\pi}{15}\left(-\cos\dfrac{8\pi}{15}\right)\cos\dfrac{3\pi}{15}$

$\cos\dfrac{5\pi}{15}\cos\dfrac{6\pi}{15}$

$=-\left(\dfrac{\sin 2^4\,\dfrac{\pi}{15}}{2^4\sin\dfrac{\pi}{15}}\right)\cos 36°.\cos 72°.\dfrac{1}{2}$

$=-\dfrac{1}{2^4}\left(\dfrac{\sqrt5+1}{4}\times\dfrac{\sqrt5-1}{4}\right)\dfrac{1}{2}=1/128.$

3. $\sin\dfrac{\pi}{14}\sin\dfrac{3\pi}{14}\sin\dfrac{5\pi}{14}(1)\sin\dfrac{5\pi}{14}\sin\dfrac{3\pi}{14}\sin\dfrac{\pi}{14}$

$=\sin^2\dfrac{\pi}{14}\sin^2\dfrac{3\pi}{14}\sin^2\dfrac{5\pi}{14}$

$=\left[\cos\left(\dfrac{\pi}{2}-\dfrac{\pi}{14}\right)\cos\left(\dfrac{\pi}{2}-\dfrac{3\pi}{14}\right)\cos\left(\dfrac{\pi}{2}-\dfrac{5\pi}{14}\right)\right]^2$

$=\left[\cos\dfrac{3\pi}{7}\cos\dfrac{2\pi}{7}\cos\dfrac{\pi}{7}\right]^2$

$=\left[\cos\dfrac{\pi}{7}\cos\dfrac{2\pi}{7}\cos\dfrac{4\pi}{7}\right]^2=\left[\dfrac{\sin\dfrac{8\pi}{7}}{2^3\sin\dfrac{\pi}{7}}\right]^2=\dfrac{1}{64}$

1. $\dfrac{m}{n}=\dfrac{\tan(\theta+120°)}{\tan(\theta-30°)}$

use C & D

$\dfrac{m+n}{m-n}=\dfrac{\tan(\theta+120°)+\tan(\theta-30)}{\tan(\theta+120)-\tan(\theta-30°)}$

$=\dfrac{\sin(\theta+120°)\cos(\theta-30°)+\sin(\theta-30°)\cos(\theta+120°)}{\sin(\theta+120°)\cos(\theta-30°)-\sin(\theta-30°).\cos(\theta+120°)}$

$=\dfrac{\sin(2\theta+90°)}{\sin 150°}$

$\dfrac{m+n}{m-n}=2\cos 2\theta$

4. If $a=\sin(\theta+\alpha)$, $b=\sin(\theta+\beta)$

$\therefore\quad 2ab=2\sin(\theta+\alpha)\sin(\theta+\beta)$

$2ab=\cos(\alpha-\beta)-\cos(2\theta+\alpha+\beta)$

Multiply both sides by $2\cos(\alpha-\beta)$

$\Rightarrow\quad 4ab\cos(\alpha-\beta)=2\cos^2(\alpha-\beta)$

$\qquad\qquad\qquad\quad -2\cos(2\theta+\alpha+\beta).\cos(\alpha-\beta)$

$\qquad =1+\cos 2(\alpha-\beta)-\cos 2(\theta+\alpha)-\cos 2(\theta+\beta)$

$\Rightarrow\quad \cos 2(\alpha-\beta)-4ab\cos(\alpha-\beta)$

$\qquad =\cos 2(\theta+\alpha)+\cos 2(\theta+\beta)-1$

$\qquad =1-2\sin^2(\theta+\alpha)+1-2\sin^2(\theta+\beta)-1$

$\qquad =1-2a^2-2b^2$

6. $\tan\alpha=\dfrac{p}{q}$

LHS $=\dfrac{1}{2}(p\cosec 2\beta-q\sec 2\beta)\times\dfrac{\sqrt{p^2+q^2}}{\sqrt{p^2+q^2}}$

$=\dfrac{1}{2}\left\{\dfrac{p}{\sqrt{p^2\times q^2}}\cosec 2\beta-\dfrac{q}{\sqrt{p^2\times q^2}}\sec 2\beta\right\}\times\sqrt{p^2+q^2}$
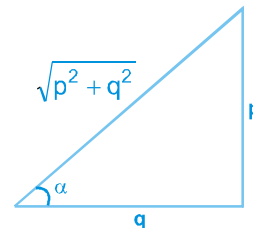
$$\sin\alpha = \frac{p}{\sqrt{p^2+q^2}}, \quad \cos\alpha = \frac{q}{\sqrt{p^2+q^2}}$$

$$= \frac{1}{2}\left(\frac{\sin\alpha\cos2\beta - \cos\alpha\,\sin2\beta}{\sin2\beta\,\cos2\beta}\right) \times \sqrt{p^2+q^2}$$

$$= \frac{\sin(\alpha-2\beta)}{\sin4\beta} \times \sqrt{p^2+q^2} = \frac{\sin4\beta}{\sin4\beta} \times \sqrt{p^2+q^2}$$

$$(\because \quad \alpha = 6\beta)$$

7. (i) $\cot 7\frac{1}{2}^\circ = \tan 82\frac{1}{2}^\circ = \dfrac{\cos 7\frac{1}{2}^\circ}{\sin 7\frac{1}{2}^\circ} = \dfrac{2\cos^2 7\frac{1}{2}^\circ}{\sin 15^\circ}$

$$= \frac{1+\cos(45^\circ - 30^\circ)}{\sin(45^\circ - 30^\circ)} = \frac{1 + \dfrac{\sqrt3+1}{2\sqrt2}}{\dfrac{\sqrt3-1}{2\sqrt2}}$$

$$= \frac{2\sqrt2 + \sqrt3 + 1}{\sqrt3 - 1}$$

$$= \frac{(2\sqrt2 + \sqrt3 + 1)(\sqrt3 + 1)}{3-1} = \sqrt2 + \sqrt3 + 2 + \sqrt6$$

$$= \sqrt2 + \sqrt3 + \sqrt4 + \sqrt6$$

$$= (\sqrt2 + \sqrt3)\,(\sqrt2 + 1)$$

(ii) $\tan 142\frac{1}{2}^\circ = -\cot 52\frac{1}{2}^\circ = \dfrac{-1}{\tan 52\frac{1}{2}^\circ}$

$$= \frac{-1}{\tan\left(45 + 7\frac{1}{2}^\circ\right)}$$

$$= -\frac{1 - \tan 7\frac{1}{2}^\circ}{1 + \tan 7\frac{1}{2}^\circ} = -\frac{\cos 7\frac{1}{2}^\circ - \sin 7\frac{1}{2}^\circ}{\cos 7\frac{1}{2}^\circ + \sin 7\frac{1}{2}^\circ}$$

$$= -\frac{\left(\cos 7\frac{1}{2}^\circ - \sin 7\frac{1}{2}^\circ\right)^2}{\cos 15^\circ} = -\frac{1 - \sin 15^\circ}{\cos 15^\circ}$$

$$= -\left(\frac{1 - \dfrac{\sqrt3-1}{2\sqrt2}}{\dfrac{\sqrt3+1}{2\sqrt2}}\right) = -\frac{(2\sqrt2 - \sqrt3 + 1)(\sqrt3 - 1)}{2}$$

$$= -\frac{[2\sqrt2(\sqrt3 - 1) - (\sqrt3 - 1)^2]}{2}$$

$$= -\frac{[2\sqrt2(\sqrt3 - 1) - (4 - 2\sqrt3)]}{2}$$

$$= -[\sqrt2(\sqrt3 - 1) - (2 - \sqrt3)]$$

$$= -\sqrt6 + \sqrt2 + 2 - \sqrt3 = 2 + \sqrt2 - \sqrt3 - \sqrt6$$

9. (i) $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$

$$= (\tan 9^\circ + \tan 81^\circ) - (\tan 27^\circ + \tan 63^\circ)$$

$$= \frac{\sin 90^\circ}{\cos 9^\circ \cos 81^\circ} - \frac{\sin 90^\circ}{\cos 27^\circ \cos 63^\circ}$$

$$= \frac{2}{2\sin 9^\circ \cos 9^\circ} - \frac{2}{2\sin 27^\circ \cos 27^\circ}$$

$$= \frac{2}{\sin 18^\circ} - \frac{2}{\sin 54^\circ} = \frac{2}{\dfrac{\sqrt5-1}{4}} - \frac{2}{\dfrac{\sqrt5+1}{4}}$$

$$= \frac{8(\sqrt5 + 1 - \sqrt5 + 1)}{4} = 4$$

(ii) $\operatorname{cosec} 10^\circ - \sqrt3\,\sec 10^\circ = 2\left(\dfrac{1}{2}\cos 10^\circ - \dfrac{\sqrt3}{2}\sin 10^\circ\right)$

$$\times \frac{1}{\sin 10^\circ \cos 10^\circ} \times \frac{2}{2} = 4$$

(iii) $2\sqrt2 \, \sin 10^\circ \left(\dfrac{\sec 5^\circ}{2} + \dfrac{\cos 40^\circ}{\sin 5^\circ} - 2\sin 35^\circ\right)$

$$= 2\sqrt2$$

$$\left(\frac{2\sin 5^\circ \cos 5^\circ \sec 5^\circ}{2} + \frac{2\sin 5^\circ \cos 5^\circ \cos 40^\circ}{\sin 5^\circ} - 2\sin 35^\circ \sin 10^\circ\right)$$

$$= 2\sqrt2\,(\sin 5^\circ + 2\cos 45^\circ + \cos 35^\circ - \cos 25^\circ + \cos 45^\circ)$$

$$= 2\sqrt2\,(\sin 5^\circ + 2\cos 45^\circ + 2\sin 30^\circ \sin(-5^\circ))$$

$$= 2\sqrt2\,(\sqrt2) = 4$$

(iv) $\cot 70° + 4 \cos 70°$

$$= \frac{\cos 70°}{\sin 70°} + 4 \cos 70° = \frac{\cos 70° + 4 \cos 70° \sin 70°}{\sin 70°}$$

$$= \frac{\cos 70° + 2 \sin 140°}{\sin 70°}$$

$$= \frac{(\cos 70° + \sin 140°) + \sin 140°}{\sin 70°} = \frac{(\sin 20° + \sin 140°) + \sin 140°}{\sin 70°}$$

$$= \frac{2 \sin 80° \times \cos 60° + \sin 140°}{\sin 70°} = \frac{2 \sin 120° \times \cos 20°}{\sin 70°}$$

$$= 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$$

(v) $\tan 10° - \tan 50° + \tan 70°$
$= \tan 10° - \tan (60° - 10°) + \tan (60° + 10°)$

$$= \tan 10° - \frac{\sqrt{3} + \tan 10°}{1 - \sqrt{3} \tan 10°} + \frac{\sqrt{3} - \tan 10°}{1 + \sqrt{3} \tan 10°}$$

$$= \frac{9 \tan 10° - 3 \tan^3 10°}{1 - 3 \tan^2 10°}$$

$$= 3 \left(\frac{3 \tan 10° - \tan^3 10°}{1 - 3 \tan^2 10°} \right) = 3 \tan 30°$$

$$= \sqrt{3}$$

13. $P_n - P_{n-2} = \cos^n \theta + \sin^n \theta - \cos^{n-2} \theta - \sin^{n-2} \theta$
$= \cos^{n-2} \theta (\cos^2 \theta - 1) + \sin^{n-2} \theta (\sin^2 \theta - 1)$
$= \cos^{n-2} \theta (-\sin^2 \theta) + \sin^{n-2} \theta (-\cos^2 \theta)$
$= (-\sin^2 \theta \cos^2 \theta) \{\cos^{n-4} \theta + \sin^{n-4} \theta \}$
$= (-\sin^2 \theta \cos^2 \theta) P_{n-4}$

put $n = 4$
$\Rightarrow P_4 - P_2 = (-\sin^2 \theta \cos^2\theta) P_0$
$\Rightarrow P_4 = P_2 - 2 \sin^2 \theta \cos^2\theta$
$= 1 - 2 \sin^2 \theta \cos^2\theta$

similarly we can prove the other result also.

19. $\sqrt{13 - 18 \tan x} = 6 \tan x - 3$ ......**(i)**

$\Rightarrow 13 - 18 \tan x = 36 \tan^2 x + 9 - 36 \tan x$

$\Rightarrow \tan x = \dfrac{2}{3}, \ -\dfrac{1}{6}$

Put in **(i)**

$\Rightarrow \tan x = \dfrac{2}{3}$ is correct

$\Rightarrow \ x = n\pi + \tan^{-1} \dfrac{2}{3}$

$= n\pi + \alpha = \alpha, \pi + \alpha, -\pi + \alpha, -2\pi + \alpha$ in $(-2\pi, 2\pi)$

21. $\left(\sin 2\theta + \sqrt{3} \cos 2\theta \right)^2 - 5 = \cos \left(\dfrac{\pi}{6} - 2\theta \right)$

$\Rightarrow 4 \left(\dfrac{1}{2} \sin 2\theta + \dfrac{\sqrt{3}}{2} \cos 2\theta \right)^2 - \cos(\dfrac{\pi}{6} - 2\theta) - 5 = 0$

$\Rightarrow 4 \cos^2 \left(\dfrac{\pi}{6} - 2\theta \right) - \cos \left(\dfrac{\pi}{6} - 2\theta \right) - 5 = 0$

$\Rightarrow \cos \left(\dfrac{\pi}{6} - 2\theta \right) = \dfrac{5}{4}, -1$

$\Rightarrow \cos \left(\dfrac{\pi}{6} - 2\theta \right) = -1 = \cos \pi$

$\Rightarrow \dfrac{\pi}{6} - 2\theta = 2n\pi \pm \pi \qquad \Rightarrow 2\theta = \dfrac{\pi}{6} - 2n\pi \mp \pi$

$\Rightarrow \theta = \dfrac{2n\pi}{2} + \dfrac{\pi}{12} \pm \dfrac{\pi}{2}$

$\Rightarrow \theta = \dfrac{7\pi}{12}, \dfrac{19\pi}{12}$.

23. $a \cos 2\theta + b \sin 2\theta = c$

$\Rightarrow \dfrac{a(1 - t^2)}{1 + t^2} + \dfrac{b(2t)}{1 + t^2} = c \qquad$ where $\quad t = \tan \theta$

$\Rightarrow (c + a)t^2 - 2bt + (c - a) = 0$

$\Rightarrow t_1 + t_2 = \dfrac{2b}{c + a}, t_1 t_2 = \dfrac{c - a}{c + a}$

$\therefore \cos^2 \alpha + \cos^2 \beta = \dfrac{1 + \cos 2\alpha + 1 + \cos 2\beta}{2} = 1 + \dfrac{1}{2}$

$[\cos 2\alpha + \cos 2\beta]$

$= 1 + \dfrac{1}{2} \left[\dfrac{1 - t_1^2}{1 + t_1^2} + \dfrac{1 - t_2^2}{1 + t_2^2} \right]$

simplifying and using values for t_1, t_2 we get

$$\cos^2 \alpha + \cos^2 \beta = 1 + \dfrac{ac}{a^2 + b^2} = \dfrac{a^2 + b^2 + ac}{a^2 + b^2}.$$

25. $1 + 2\,\text{cosec}\,x = \dfrac{-\sec^2\left(\dfrac{x}{2}\right)}{2} \Rightarrow 1 + \dfrac{2}{\sin x} = \dfrac{-1}{1+\cos x}$

$\Rightarrow (2+\sin x)(1+\cos x) = -\sin x$

$\Rightarrow 2 + 2\cos x + \sin x + \sin x \cos x = -\sin x$

$\Rightarrow 2(\sin x + \cos x) + \sin x \cos x + 2 = 0$

Put $\sin x + \cos x = t$

$\Rightarrow 1 + 2\sin x \cos x = t^2$

$\therefore\ 2t + \dfrac{t^2 - 1}{2} + 2 = 0 \Rightarrow t^2 + 4t + 3 = 0$

$\Rightarrow t = -1, -3 \Rightarrow \sin x + \cos x = -1$

$\Rightarrow \cos\left(x - \dfrac{\pi}{4}\right) = \dfrac{-1}{\sqrt{2}} = \cos\dfrac{3\pi}{4}$

$\Rightarrow x - \dfrac{\pi}{4} = 2n\pi \pm \dfrac{3\pi}{4} \Rightarrow x = 2n\pi + \pi,\ 2n\pi - \dfrac{\pi}{2}$

$\Rightarrow x = 2n\pi + \pi$ at which cosec x is not defined

$\therefore\ x = 2n\pi - \dfrac{\pi}{2}$.

26. $\sin^2 4x + \cos^2 x = 2\sin 4x \cdot \cos^4 x$

$\Rightarrow \sin^2 4x - 2\sin 4x \cos^4 x + \cos^2 x = 0$

$\Rightarrow (\sin 4x - \cos^4 x)^2 + \cos^2 x - \cos^8 x = 0$

$\Rightarrow (\sin 4x - \cos^4 x)^2 + \cos^2 x(1 - \cos^6 x) = 0$

$\Rightarrow \sin 4x - \cos^4 x = 0$ ......(i)

and $\cos^2 x(1 - \cos^6 x) = 0$ ......(ii)

From (ii) $\cos^2 x = 0, 1$

Case-I $\cos^2 x = 0 \Rightarrow x = n\pi \pm \dfrac{\pi}{2}$

$\Rightarrow 4x = 4n\pi \pm 2\pi$

$\therefore\ \sin 4x = 0$

$\Rightarrow$ equation (1) is also true

Case-II $\cos^2 x = 1 \Rightarrow \sin^2 x = 0$

$\Rightarrow x = n\pi \qquad \therefore$ equation (1) becomes

$0 - 1 = 0$ false $\qquad \therefore$ solution is $x = n\pi \pm \dfrac{\pi}{2}$

1. since α is a root of

$\qquad 25\cos^2\theta + 5\cos\theta - 12 = 0$

$\therefore\ 25\cos^2\alpha + 5\cos\alpha - 12 = 0$

$\Rightarrow (5\cos\alpha - 3)(5\cos\alpha + 4) = 0$

$\Rightarrow \cos\alpha = -\dfrac{4}{5}$ and $\dfrac{3}{5}$

But $\dfrac{\pi}{2} < \alpha < \pi$ i.e., in second quadrant

$\therefore\ \cos\alpha = -\dfrac{4}{5}$

$\Rightarrow \sin\alpha = \dfrac{3}{5}$

now, $\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\qquad = 2 \times \dfrac{3}{5} \times \left(-\dfrac{4}{5}\right) = -\dfrac{24}{25}$

3. $u = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$

$\Rightarrow u^2 = a^2\cos^2\theta + b^2\sin^2\theta + a^2\sin^2\theta + b^2\cos^2\theta +$

$\qquad 2\sqrt{a^2\cos^2\theta + b^2\sin^2\theta} \times \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$

$\Rightarrow u^2 = (a^2 + b^2) +$

$\qquad 2\sqrt{\{a^2 + (b^2 - a^2)\sin^2\theta\} \times \{a^2 + (b^2 - a^2)\cos^2\theta\}}$

$\Rightarrow u^2 = (a^2 + b^2) +$

$\qquad 2\sqrt{a^4 + a^2(b^2 - a^2) + (b^2 - a^2)^2\sin^2\theta\cos^2\theta}$

$\Rightarrow u^2 = (a^2 + b^2) + 2\sqrt{a^2 b^2 + \left(\dfrac{b^2 - a^2}{2}\right)^2 \sin^2 2\theta}$.

$\therefore\ \min(u^2) = a^2 + b^2 + 2ab = (a+b)^2$ and $\max(u^2)$

$\qquad = a^2 + b^2 + (a^2 + b^2) = 2(a^2 + b^2)$

Now, $\max(u^2) - \min(u^2) = (a-b)^2$

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4. $\sin\alpha + \sin\beta = -\dfrac{21}{65}$ and $\cos\alpha + \cos\beta = -\dfrac{27}{65}$

squaring and adding, we get

$\sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta + \cos^2\alpha + \cos^2\beta$

$\qquad\qquad\qquad\qquad\qquad\qquad + 2\cos\alpha.\cos\beta$

$\qquad = \left(-\dfrac{21}{65}\right)^2 + \left(-\dfrac{27}{65}\right)^2$

$\Rightarrow\ 2 + 2\cos(\alpha - \beta) = \dfrac{1170}{4225}$

$\Rightarrow\ \cos^2\left(\dfrac{\alpha - \beta}{2}\right) = \dfrac{1170}{4\times 4225} = \dfrac{9}{130}$

$\Rightarrow\ \cos\left(\dfrac{\alpha - \beta}{2}\right) = \dfrac{-3}{\sqrt{130}}$ $(\because \pi < \alpha - \beta < 3\pi$

$\Rightarrow\ \dfrac{\pi}{2} < \left(\dfrac{\alpha - \beta}{2}\right) < \dfrac{3\pi}{2}$)

5. $\because$ $\tan\dfrac{P}{2}$ and $\tan\dfrac{Q}{2}$ are the roots of equation

$ax^2 + bx + c = 0$

$\because$ $\tan\dfrac{P}{2} + \tan\dfrac{Q}{2} = -\dfrac{b}{a}$ and $\tan\dfrac{P}{2}\tan\dfrac{Q}{2} = \dfrac{c}{a}$

$\dfrac{P}{2} + \dfrac{Q}{2} + \dfrac{R}{2} = \dfrac{\pi}{2}$ $(\because P + Q + R = \pi)$

$\Rightarrow\ \dfrac{P+Q}{2} = \dfrac{\pi}{2} - \dfrac{R}{2}$ $\Rightarrow\ \dfrac{P+Q}{2} = \dfrac{\pi}{4}$

$(\because \angle R = \dfrac{\pi}{2})$

$\Rightarrow\ \tan\left(\dfrac{P+Q}{2}\right) = 1$ $\Rightarrow\ \dfrac{\tan\dfrac{P}{2} + \tan\dfrac{Q}{2}}{1 - \tan\dfrac{P}{2}.\tan\dfrac{Q}{2}} = 1$

$\Rightarrow\ \dfrac{-b/a}{1 - c/a} = 1$ $\Rightarrow\ c = a + b$

6. $\cos x + \sin x = \dfrac{1}{2}$

$\therefore\ \dfrac{1 - \tan^2 x/2}{1 + \tan^2 x/2} + \dfrac{2\tan x/2}{1 + \tan^2 x/2} = \dfrac{1}{2}$, Let $\tan\dfrac{x}{2} = t$

$\dfrac{1 - t^2}{1 + t^2} + \dfrac{2t}{1 + t^2} = \dfrac{1}{2}$ $\Rightarrow\ 3t^2 - 4t - 1 = 0$

$\therefore\ t = \dfrac{2 \pm \sqrt{7}}{3}$

as $0 < x < \pi \Rightarrow 0 < \dfrac{x}{2} < \dfrac{\pi}{2}$

$\therefore\ \tan\dfrac{x}{2}$ is positive
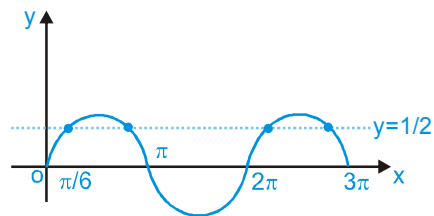
$\therefore\ t = \tan\dfrac{x}{2} = \dfrac{2 + \sqrt{7}}{3}$

Now $\tan x = \dfrac{2\tan x/2}{1 - \tan^2 x/2} = \dfrac{2t}{1 - t^2}$

$\Rightarrow\ \tan x = \dfrac{2\left(\dfrac{2 + \sqrt{7}}{3}\right)}{1 - \left(\dfrac{2 + \sqrt{7}}{3}\right)^2} = -\left(\dfrac{4 + \sqrt{7}}{3}\right)$

7. Given equation is $2\sin^2 x + 5\sin x - 3 = 0$
$\Rightarrow\ (2\sin x - 1)(\sin x + 3) = 0$

$\Rightarrow\ \sin x = \dfrac{1}{2}$ $(\because \sin x \neq -3)$

It is clear from figure that the curve intersect the line at four points in the given interval.

Hence, number of solutions are 4.

8. Given, $\cos x + \sin x = \dfrac{1}{2}$

$\therefore\ \dfrac{1 - \tan^2\dfrac{x}{2}}{1 + \tan^2\dfrac{x}{2}} + \dfrac{2\tan\dfrac{x}{2}}{1 + \tan^2\dfrac{x}{2}} = \dfrac{1}{2}$

Let $\tan \dfrac{x}{2} = t$

$\Rightarrow \dfrac{1-t^2}{1+t^2} + \dfrac{2t}{1+t^2} = \dfrac{1}{2} \Rightarrow 3t^2 - 4t - 1 = 0$

$\Rightarrow t = \dfrac{2 \pm \sqrt{7}}{3}$

As $0 < x < \pi \Rightarrow 0 < \dfrac{x}{2} < \dfrac{\pi}{2}$

$\therefore \tan \dfrac{x}{2}$ is positive.

$\therefore t = \tan \dfrac{x}{2} = \dfrac{2+\sqrt{7}}{3}$

Now, $\tan x = \dfrac{2\tan \dfrac{x}{2}}{1 - \tan^2 \dfrac{x}{2}} = \dfrac{2t}{1-t^2}$

$\Rightarrow \tan x = \dfrac{2\left(\dfrac{2+\sqrt{7}}{3}\right)}{1 - \left(\dfrac{2+\sqrt{7}}{3}\right)^2}$

$\Rightarrow \tan x = -\dfrac{3(2+\sqrt{7})}{1+2\sqrt{7}} \times \dfrac{1-2\sqrt{7}}{1-2\sqrt{7}}$

$\Rightarrow \tan x = -\left(\dfrac{4+\sqrt{7}}{3}\right).$

11. $2\{\cos(\beta-\gamma) + \cos(\gamma-\alpha) + \cos(\alpha-\beta)\} + 3 = 0$
$(\cos\alpha + \cos\beta + \cos\gamma)^2 + (\sin\alpha + \sin\beta + \sin\gamma)^2 = 0$
$\Sigma \cos\alpha = 0 = \Sigma \sin\alpha$

12. $\tan 2\alpha = \tan((\alpha+\beta)+(\alpha-\beta)) = \dfrac{\tan(\alpha+\beta) + \tan(\alpha-\beta)}{1 - \tan(\alpha+\beta)\tan(\alpha-\beta)}$

$= \dfrac{\dfrac{3}{4} + \dfrac{5}{12}}{1 - \dfrac{3}{4}\cdot\dfrac{5}{12}} = \dfrac{(9+5)4}{48-15} = \dfrac{14\times4}{33} = \dfrac{56}{33}$

Hence correct option is (1)

13. $A = \sin^2 x + \cos^4 x = \sin^2 x + (1-\sin^2 x)^2 = \sin^4 x - \sin^2 x + 1$

$= \left(\sin^2 x - \dfrac{1}{2}\right)^2 + \dfrac{3}{4} = \dfrac{3}{4} \le A \le 1$

14. $3\sin P + 4\cos Q = 6$ .....(i)
$\Rightarrow 4\sin Q + 3\cos P = 1$ .....(ii)

Squaring and adding **(i)** & **(ii)** we get $\sin(P+Q) = \dfrac{1}{2}$

$\Rightarrow P+Q = \dfrac{\pi}{6}$ or $\dfrac{5\pi}{6} \Rightarrow R = \dfrac{5\pi}{6}$ or $\dfrac{\pi}{6}$

If $R = \dfrac{5\pi}{6}$ then $0 < P, Q < \dfrac{\pi}{6}$

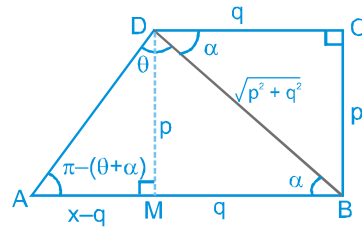$\Rightarrow \cos Q < 1$ and $\sin P < \dfrac{1}{2} \Rightarrow 3\sin P + 4\cos Q < \dfrac{11}{2}$

So $R = \dfrac{\pi}{6}$

15. Let $AB = x$

$\tan(\pi - \theta - \alpha) = \dfrac{p}{x-q} \Rightarrow \tan(\theta+\alpha) = \dfrac{p}{q-x}$

$\Rightarrow q - x = p\cot(\theta+\alpha)$
$\Rightarrow x = q - p\cot(\theta+\alpha)$

$= q - p\left(\dfrac{\cot\theta\cot\alpha - 1}{\cot\alpha + \cot\theta}\right)$

$= q - p\left(\dfrac{\dfrac{q}{p}\cot\theta - 1}{\dfrac{q}{p} + \cot\theta}\right) = q - p\left(\dfrac{q\cot\theta - p}{q + p\cot\theta}\right)$

$= q - p\left(\dfrac{q\cos\theta - p\sin\theta}{q\sin\theta + p\cos\theta}\right)$

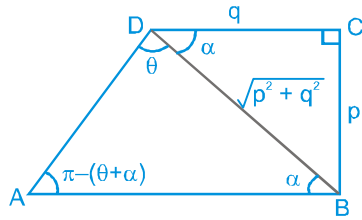$\Rightarrow x = \dfrac{q^2\sin\theta + pq\cos\theta - pq\cos\theta + p^2\sin\theta}{p\cos\theta + q\sin\theta}$

$\Rightarrow AB = \dfrac{(p^2+q^2)\sin\theta}{p\cos\theta + q\sin\theta}.$

Alternative

From Sine Rule

$\dfrac{AB}{\sin\theta} = \dfrac{\sqrt{p^2+q^2}}{\sin(\pi - (\theta+\alpha))}$

$$AB = \frac{\sqrt{p^2+q^2}\,\sin\theta}{\sin\theta\cos\alpha+\cos\theta\sin\alpha}$$

$$= \frac{(p^2+q^2)\sin\theta}{q\sin\theta+p\cos\theta} \qquad \left(\because \cos\alpha = \frac{q}{\sqrt{p^2+q^2}}\right)$$

$$= \frac{(p^2+q^2)\sin\theta}{p\cos\theta+q\sin\theta}.$$

16. Given expression

$$= \frac{\sin A}{\cos A}\times\frac{\sin A}{\sin A-\cos A}+\frac{\cos A}{\sin A}\times\frac{\cos A}{\cos A-\sin A}$$

$$= \frac{1}{\sin A-\cos A}\left\{\frac{\sin^3 A-\cos^3 A}{\cos A\sin A}\right\}$$

$$= \frac{\sin^2 A+\sin A\cos A+\cos^2 A}{\sin A\cos A}=1+\sec A\,\mathrm{cosec}\,A$$

18. $f_k(x)=\dfrac{1}{k}(\sin^k x+\cos^k x)$, where $x\in R$ and $k\ge1$

Now, $\quad f_4(x)-f_6(x)$

$$= \frac{1}{4}(\sin^4 x+\cos^4 x)-\frac{1}{6}(\sin^6 x+\cos^6 x)$$

$$= \frac{1}{4}(1-2\sin^2 x\cdot\cos^2 x)-\frac{1}{6}(1-3\sin^2 x\cdot\cos^2 x)$$

$$= \frac{1}{4}-\frac{1}{6}=\frac{1}{12}$$

19. In $\Delta OA_1 B_1$,

$$\tan 45° = \frac{AB_1}{OB_1} \qquad\Rightarrow\qquad \frac{20}{OB_1}=1$$

$$\Rightarrow\quad OB=20$$

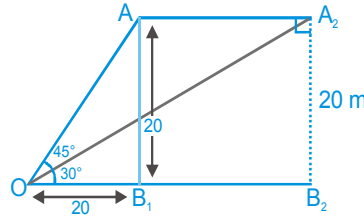In ΔOA_2, $\quad \tan 30° = \dfrac{20}{OB_2}$

$$\Rightarrow\quad OB_2=20\sqrt{3}$$
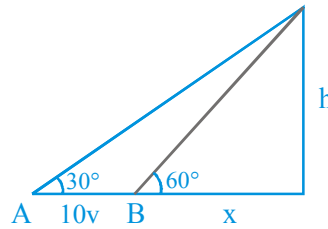
$$\Rightarrow\quad B_1B_2+OB_1=20\sqrt{3}$$

$$\Rightarrow\quad B_1B_2=20\sqrt{3}-20$$

$$\Rightarrow\quad B_1B_2=20(\sqrt{3}-1)\,m$$

$$\therefore\quad \text{Speed}=\frac{\text{Distance}}{\text{Time}}=\frac{20(\sqrt{3}-1)}{1}$$

21.

Let speed $=v$ units/min

$$\frac{h}{10v+x}=\tan 30°$$

$$\frac{h}{x}=\tan 60°$$

$$\Rightarrow\quad \frac{x}{10v+x}=\frac{1}{3} \qquad\Rightarrow\quad x=5v$$

So, time $=5$ minutes.

Part # II : IIT-JEE ADVANCED

1. We are given that

$$(\cot\alpha_1)(\cot\alpha_2)\,....\,(\cot\alpha_n)=1$$

$$\Rightarrow\quad (\cos\alpha_1)(\cos\alpha_2)\,....\,(\cos\alpha_n)$$
$$= (\sin\alpha_1)(\sin\alpha_2)\,....\,(\sin\alpha_n) \qquad ....\textbf{(i)}$$

Let $\quad y=(\cos\alpha_1)(\cos\alpha_2)\,....\,(\cos\alpha_n)$

(to be maximum)

squaring both sides, we get

$$y_2=(\cos^2\alpha_1)(\cos^2\alpha_2)\,....\,(\cos^2\alpha_n)$$
$$= \cos\alpha_1\,\sin\alpha_1\,\cos\alpha_2\,\sin\alpha_2\,....\,\cos\alpha_n\,\sin\alpha_n$$
$$\text{[using eq. (i)]}$$

$$\frac{1}{2^{n/2}}\,[\sin 2\alpha_1\,2\alpha_2\,....\,\sin 2\alpha_n\le1]$$

As $0\le\alpha_1,\alpha_2,\,....\,\alpha_n\le\pi/2$, we have

$$0\le\sin 2\alpha_1,\,\sin 2\alpha_2,\,....\,\sin 2\alpha_n\le1$$

$$\therefore\quad y^2\le\frac{1}{2^n}\times1 \quad\text{or}\quad y\times\frac{1}{2^{n/2}}$$

2. Clearly $\theta = 30°$ and $\phi \in (60°, 90°)$

Hence $\theta + \phi$ lies in $(90°, 120°)$.

3. Let $y = 2 \sin t$

$$y = \frac{1 - 2x + 5x^2}{3x^2 - 2x - 1}$$

$$(3y - 5) x^2 - 2x (y - 1) - (y + 1) = 0$$

$$x \in R - \left\{1, -\frac{1}{3}\right\}$$

$\therefore \quad D \geq 0$

$\Rightarrow \quad y^2 - y - 1 \geq 0$

$$\therefore \quad y \geq \frac{1 + \sqrt{5}}{2} \qquad \text{or} \quad y \leq \frac{1 - \sqrt{5}}{2}$$

$$\Rightarrow \quad \sin t \geq \frac{1 + \sqrt{5}}{4} \qquad \text{or} \quad \sin t \leq \frac{1 - \sqrt{5}}{4}$$

$$\therefore \quad \text{range of t is} \quad \left[-\frac{\pi}{2}, -\frac{\pi}{10}\right] \cup \left[\frac{3\pi}{10}, \frac{\pi}{2}\right]$$

4. $\theta \in \left(0, \dfrac{\pi}{4}\right)$

$\because \quad \tan \theta \uparrow$ in $\theta \in \left(0, \dfrac{\pi}{4}\right)$ and $0 < \tan \theta < 1$

$\cot \theta \downarrow$ in $\theta \in \left(0, \dfrac{\pi}{4}\right)$ and $\cot \theta > 1$

Let $\tan \theta = 1 - \lambda_1$ and $\cot \theta = 1 + \lambda_2$ where λ_1 and λ_2 are very small and positive, then

$t_1 = (1 - \lambda_1)^{1 - \lambda_1}$, $t_2 = (1 - \lambda_1)^{1 + \lambda_2}$,

$t_3 = (1 + \lambda_2)^{1 - \lambda_1}$, $t_4 = (1 + \lambda_2)^{(1 + \lambda_2)}$

$\therefore \quad t_4 > t_3 > t_1 > t_2$

5. $\dfrac{\sin^4 x}{2} + \dfrac{\cos^4 x}{3} = \dfrac{1}{5}$

$\Rightarrow \quad \dfrac{\sin^4 x}{2} + \dfrac{(1 - \sin^2 x)^2}{3} = \dfrac{1}{5}$

$\Rightarrow \quad \dfrac{\sin^4 x}{2} + \dfrac{1 + \sin^4 x - 2\sin^2 x}{3} = \dfrac{1}{5}$

$\Rightarrow \quad 5 \sin^4 x - 4 \sin^2 x + 2 = \dfrac{6}{5}$

$\Rightarrow \quad 25 \sin^4 x - 20 \sin^2 x + 4 = 0 \quad \Rightarrow \quad (5 \sin^2 x - 2)^2 = 0$

$\Rightarrow \quad \sin^2 x = \dfrac{2}{5}, \cos^2 x = \dfrac{3}{5} \qquad \Rightarrow \quad \tan 2x = \dfrac{2}{3}$

and $\dfrac{\sin^8 x}{8} + \dfrac{\cos^8 x}{27} = \dfrac{1}{125}$

6. $f(\theta) = \dfrac{1}{\sin^2 \theta + 3\sin \theta \cos \theta + 5\cos^2 \theta}$

$$= \dfrac{1}{\dfrac{1 - \cos 2\theta}{2} + \dfrac{3}{2}\sin 2\theta + \dfrac{5(1 + \cos 2\theta)}{2}}$$

$$= \dfrac{2}{6 + 3\sin 2\theta + 4\cos 2\theta}$$

$\therefore \quad f(\theta)_{max} = \dfrac{2}{6 - 5} = 2$

7. $\dfrac{1}{\sin \dfrac{\pi}{n}} - \dfrac{1}{\sin \dfrac{3\pi}{n}} = \dfrac{1}{\sin \dfrac{2\pi}{n}}$

$$\dfrac{2\cos \dfrac{2\pi}{n} \sin \dfrac{\pi}{n}}{\sin \dfrac{\pi}{n} \sin \dfrac{3\pi}{n}} = \dfrac{1}{\sin \dfrac{2\pi}{n}}$$

$$\sin \dfrac{4\pi}{n} = \sin \dfrac{3\pi}{n}$$

$$\dfrac{4\pi}{n} = (-1)^k \dfrac{3\pi}{n} + k\pi, \, k \in I$$

If $\quad k = 2m \quad \Rightarrow \quad \dfrac{\pi}{n} = 2m\pi$

$\dfrac{1}{n} = 2m$, not possible

If $\quad k = 2m + 1 \quad \Rightarrow \quad \dfrac{7\pi}{n} = (2m + 1)\pi$

$\Rightarrow \quad n = 7, \, m = 0$

Ans. $\quad n = 7$

8. $P = \{\theta : \sin \theta - \cos \theta = \sqrt{2} \cos \theta\}$

$\sin \theta = (\sqrt{2} + 1) \cos\theta \qquad \Rightarrow \quad \tan \theta = \sqrt{2} + 1$

$\Rightarrow \quad \theta = n\pi + \dfrac{3\pi}{8} \, ; n \in I$

$Q = \{\theta : \sin \theta + \cos \theta = \sqrt{2} \sin \theta\}$

$\therefore \quad \cos \theta = (\sqrt{2} - 1) \sin \theta$

$\Rightarrow \quad \tan \theta = \dfrac{1}{\sqrt{2} - 1} = \sqrt{2} + 1$

$\Rightarrow \quad \theta = n\pi + x\dfrac{3\pi}{8} \, ; n \in I \quad \therefore \quad P = Q$

9. As $\tan(2\pi - \theta) > 0, -1 < \sin\theta < -\dfrac{\sqrt{3}}{2}, \theta \in [0, 2\pi]$

$\Rightarrow \quad \dfrac{3\pi}{2} < \theta < \dfrac{5\pi}{3}$

Now $2\cos\theta(1 - \sin\phi) = \sin^2\theta(\tan\theta/2 + \cot\theta/2)\cos\phi - 1$

$\Rightarrow \quad 2\cos\theta(1 - \sin\phi) = 2\sin\theta\cos\phi - 1$

$\Rightarrow \quad 2\cos\theta + 1 = 2\sin(\theta + \phi)$

As $\theta \in \left(\dfrac{3\pi}{2}, \dfrac{5\pi}{3}\right) \qquad \Rightarrow \quad 2\cos\theta + 1 \in (1, 2)$

$\Rightarrow \quad 1 < 2\sin(\theta + \phi) < 2 \quad \Rightarrow \quad \dfrac{1}{2} < \sin(\theta + \phi) < 1$

As $\theta + \phi \in [0, 4\pi]$

$\Rightarrow \quad \theta + \phi \in \left(\dfrac{\pi}{6}, \dfrac{5\pi}{6}\right) \quad$ or $\quad \theta + \phi \in \left(\dfrac{13\pi}{6}, \dfrac{17\pi}{6}\right)$

$\Rightarrow \dfrac{\pi}{6} - \theta < \phi < \dfrac{5\pi}{6} - \theta$ or $\dfrac{13\pi}{6} - \theta < \phi < \dfrac{17\pi}{6} - \theta$

$\Rightarrow \phi \in \left(-\dfrac{3\pi}{2}, \dfrac{-2\pi}{3}\right) \cup \left(\dfrac{2\pi}{3}, \dfrac{7\pi}{6}\right)$

$\left(\because \theta \in \left(\dfrac{3\pi}{2}, \dfrac{5\pi}{3}\right)\right)$

$\therefore \quad$ correct option is (A, C, D)

10. $\cos x + \cos y + \cos z = 0$

and $\sin x + \sin y + \sin z = 0$

$\cos x + \cos y = -\cos z \qquad$ ....(i)

and $\sin x + \sin y = -\sin z \qquad$ ....(ii)

Squaring and adding we get,

$1 + 1 + 2(\cos x \cos y + \sin x \sin y) = 1$

$\Rightarrow \quad 2 + 2\cos(x - y) = 1$

or $\quad 2\cos(x - y) = 1$ or $\cos(x - y) = \dfrac{1}{2}$

or $\quad 2\cos^2\left(\dfrac{x - y}{2}\right) - 1 = -\dfrac{1}{2} \quad$ or $\cos\left(\dfrac{x - y}{2}\right) = \dfrac{1}{2}$

11. $\cos\left(\dfrac{\pi}{4} - x\right)\cos 2x + \sin x \sin 2x \sec x$

$= \cos x \sin 2x \sec x + \cos\left(\dfrac{\pi}{4} + x\right)\cos 2x$

or $\left[\cos\left(\dfrac{\pi}{4} - x\right) - \cos\left(\dfrac{\pi}{4} + x\right)\right]\cos 2x$

$= (\cos x \sin 2x \sec x - \sin x \sin 2x \sec x)$

or $\quad \dfrac{2}{\sqrt{2}}\sin x \cos 2x = (\cos x - \sin x)\sin 2x \sec x$

or $\quad \sqrt{2}\sin x \cos 2x = (\cos x - \sin x) 2 \sin x$

or $\quad \dfrac{1}{\sqrt{2}} = \dfrac{1}{\cos x + \sin x}$

or $\quad x = \dfrac{\pi}{4} \quad \Rightarrow \quad \sec x = \sec\dfrac{\pi}{4} = \sqrt{2}$

11. $x^2 - 2x\sec\theta + 1 = 0$

$\Rightarrow \quad x = \dfrac{2\sec\theta \pm \sqrt{4\sec^2\theta - 4}}{2}$

$\Rightarrow \quad x = \sec\theta + \tan\theta, \ \sec\theta - \tan\theta$

$\Rightarrow \quad \alpha_1 = \sec\theta - \tan\theta$

Now $\quad x^2 + 2x\tan\theta - 1 = 0$

$\Rightarrow \quad x = \dfrac{-2\tan\theta \pm \sqrt{4\tan^2\theta + 4}}{2}$

$\Rightarrow \quad x = -\tan\theta \pm \sec\theta \quad \Rightarrow \quad \alpha_2 = (\sec\theta - \tan\theta)$

$\Rightarrow \quad \beta_2 = -(\sec\theta + \tan\theta)$

$\therefore \quad \alpha_1 + \beta_2 = -2\tan\theta$

13. $\displaystyle\sum_{k=1}^{13} \dfrac{\sin\left[\left(\dfrac{\pi}{4} + \dfrac{k\pi}{6}\right) - \left(\dfrac{\pi}{4} + (k-1)\dfrac{\pi}{6}\right)\right]}{\sin\dfrac{\pi}{6}\left(\sin\left(\dfrac{\pi}{4} + \dfrac{k\pi}{6}\right)\sin\left(\dfrac{\pi}{4} + (k-1)\dfrac{\pi}{6}\right)\right)}$

$= 2\displaystyle\sum_{k=1}^{13}\left(\cot\left(\dfrac{\pi}{4} + (k-1)\dfrac{\pi}{6}\right) - \cot\left(\dfrac{\pi}{4} + \dfrac{k\pi}{6}\right)\right)$

$= 2\left(\cot\dfrac{\pi}{4} - \cot\left(\dfrac{\pi}{4} + \dfrac{13\pi}{6}\right)\right) = 2\left(1 - \cot\left(\dfrac{29\pi}{12}\right)\right)$

$= 2\left(1 - \cot\left(\dfrac{5\pi}{12}\right)\right) = 2(1 - (2 - \sqrt{3})) = 2(-1 + \sqrt{3})$

$= 2(\sqrt{3} - 1)$

MOCK TEST

1. $2\cos x + \sin x = 1$ ....(i)

$4\cos^2 x = (1 - \sin x)^2$

$4 - 4\sin^2 x = 1 + \sin^2 x - 2\sin x$

$5\sin^2 x - 2\sin x - 3 = 0$

$(\sin x - 1)(5\sin x + 3) = 0$

$\Rightarrow \sin x = 1, \sin x = -\dfrac{3}{5}$

$\because \cos x = \dfrac{1 - \sin x}{2}$ (from equation **(i)**)

$\therefore$ when $\sin x = 1$

$7\cos x + 6\sin x = 7\left(\dfrac{1 - \sin x}{2}\right) + 6\sin x$

$= 7\left(\dfrac{1 - 1}{2}\right) + 6 \times 1 = 6$ Ans.

and when $\sin x = \dfrac{-3}{5}$

then $7\cos x + 6\sin x = 7\left(\dfrac{1 + \dfrac{3}{5}}{2}\right) - \dfrac{6 \times 3}{5} = \dfrac{28 - 18}{5} = 2$

2. **(C)**

$\cos^2 10° - \cos 10°\cos 50° + \cos^2 50°$

$= \dfrac{1}{2}[1 + \cos 20° - (\cos 60° + \cos 40°) + (1 + \cos 100°)]$

$= \dfrac{1}{2}[1 + \cos 20° - \dfrac{1}{2} - \cos 40° + 1 - \cos 80°]$

$= \dfrac{1}{2}\left[\dfrac{3}{2} + \cos 20° - (2\cos 60°\cos 20°)\right] = \dfrac{3}{4}$

3. **(B)**

$\dfrac{3 + \dfrac{\cos 80°\cos 20°}{\sin 80°\sin 20°}}{\dfrac{\cos 80°}{\sin 80°} + \dfrac{\cos 20°}{\sin 20°}}$

$= \dfrac{2\sin 80°\sin 20° + \cos 80°\cos 20° + \sin 80°\sin 20°}{\sin 20°\cos 80° + \cos 20°\sin 80°}$

$= \dfrac{\cos 60° - \cos 100° + \cos 60°}{\sin 100°} = \dfrac{1 - \cos 100°}{\sin 100°}$

$= \tan 50°$

4. $\cos\dfrac{\pi}{19} + \cos\dfrac{3\pi}{19} + \cos\dfrac{5\pi}{19} + ... + \cos\dfrac{17\pi}{19}$

here $A = \dfrac{\pi}{19}, D = \dfrac{2\pi}{19}, n = 9$

$\because \cos A + \cos(A + D) + \cos(A + 2D) + ... + \cos(A + (n-1)D)$

$= \dfrac{\sin\left(\dfrac{nD}{2}\right)}{\sin\dfrac{D}{2}} \cdot \cos\left(\dfrac{2A + (n-1)D}{2}\right)$

$= \dfrac{\sin 9 \times \dfrac{\pi}{19}}{\sin\dfrac{\pi}{19}} \times \cos\left(\dfrac{\dfrac{\pi}{19} + \dfrac{17\pi}{19}}{2}\right)$

$= \dfrac{\sin\dfrac{9\pi}{19}}{\sin\dfrac{\pi}{19}} \times \cos\dfrac{9\pi}{19}$

$= \dfrac{1}{2} \cdot \dfrac{\sin\left(\dfrac{18\pi}{19}\right)}{\sin\dfrac{\pi}{19}} = \dfrac{1}{2} \cdot \dfrac{\sin\dfrac{\pi}{19}}{\sin\dfrac{\pi}{19}} = \dfrac{1}{2}$

5. **(C)**

$\cos x + \sin x = \sqrt{2}\cos x$

$\sin x = (\sqrt{2} - 1)\cos x$

$\cos x = \dfrac{1}{(\sqrt{2} - 1)}\sin x$

$\cos x = (\sqrt{2} + 1)\sin x$

$\cos x - \sin x = \sqrt{2}\sin x$

6. $f(\theta) = \sin^2\theta + \sin^2\left(\theta + \dfrac{2\pi}{3}\right) + \sin^2\left(\theta + \dfrac{4\pi}{3}\right) = \dfrac{1}{2}$

$\left[(1 - \cos 2\theta) + \left(1 - \cos\left(2\theta + \dfrac{4\pi}{3}\right)\right) + \left(1 - \cos\left(2\theta + \dfrac{8\pi}{3}\right)\right)\right]$

$= \dfrac{1}{2}\left[(1 + 1 + 1) - (\cos 2\theta + 2\cos(2\theta + 2\pi))\cos\left(\dfrac{2\pi}{3}\right)\right]$

$= \dfrac{1}{2}[3 - (\cos 2\theta - \cos 2\theta)] = \dfrac{3}{2}$

$f(\theta) = \dfrac{3}{2}$ (constant) $\Rightarrow f\left(\dfrac{\pi}{15}\right) = \dfrac{3}{2}$

7. **(A)**

$\sin 2\beta = \sqrt{\sin\alpha.\cos\alpha}$

$\cos 4\beta = 1 - 2\sin^2 2\beta = 1 - 2\sin\alpha.\sin\alpha = (\sin\alpha - \cos\alpha)^2$

$$= 2\sin^2\left(\alpha - \frac{\pi}{4}\right)$$

$$= 2\sin^2\left(\frac{\pi}{4} - \alpha\right)$$

8. $0° < x < 90°$ & $\cos x = \dfrac{3}{\sqrt{10}}$

$\log_{10}\sin x + \log_{10}\cos x + \log_{10}\tan x$

$= \log_{10}(\sin x \cos x \tan x)$

$= \log_{10}(1 - \cos^2 x) = \log_{10}(1 - 9/10)$

$= \log_{10}\left(\dfrac{1}{10}\right) = -1$

9. $\sin\alpha\sin\beta - \cos\alpha\cos\beta + 1 = 0$

$\Rightarrow \cos(\alpha + \beta) = 1$

$\quad \alpha + \beta = 2n\pi$

$\Rightarrow 1 + \cot\alpha\tan\beta = \dfrac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\sin\alpha\cos\beta}$

$\quad = \dfrac{\sin(\alpha + \beta)}{\sin\alpha\cos\beta} = 0$

10. $\dfrac{\sin A}{\sin B} = \dfrac{\sqrt{3}}{2}$, $\dfrac{\cos A}{\cos B} = \dfrac{\sqrt{5}}{2}$, $0 < A, B < \pi/2$

$\tan A = \dfrac{\sqrt{3}}{\sqrt{5}}\dfrac{\sin B}{\cos B}$

$\tan A = \dfrac{\sqrt{3}}{\sqrt{5}}\tan B$ ....**(i)**

$\dfrac{\sin A \cos A}{\sin B \cos B} = \dfrac{\sqrt{15}}{4}$

$\Rightarrow \dfrac{\tan A.\sec^2 B}{\tan B.\sec^2 A} = \dfrac{\sqrt{15}}{4}$

from **(i)**

$\Rightarrow \dfrac{\sqrt{3}}{\sqrt{5}}\dfrac{(1 + \tan^2 B)}{(1 + \tan^2 A)} = \dfrac{\sqrt{15}}{4}$

$\Rightarrow 4 + 4\tan^2 B = 5 + 5\tan^2 A$

$\Rightarrow -1 + 4\tan^2 B = 5 \times \dfrac{3}{5}\tan^2 B \Rightarrow \tan B = \pm 1$

$\Rightarrow \tan B = +1 \qquad (\because 0 < B < \dfrac{\pi}{2})$

Now $\tan A + \tan B = \dfrac{\sqrt{3}}{\sqrt{5}} + 1 = \dfrac{\sqrt{3} + \sqrt{5}}{\sqrt{5}}$

11. $2(\sec^2\alpha - \csc^2\alpha) + (\csc^2\alpha + \sec^2\alpha)$

$$(\csc^2\alpha - \sec^2\alpha) = \dfrac{15}{4}$$

$\Rightarrow (\csc^2\alpha - \sec^2\alpha)[\csc^2\alpha + \sec^2\alpha - 2] = \dfrac{15}{4}$

$\Rightarrow 4(\cot^2\alpha - \tan^2\alpha)(\cot^2\alpha + \tan^2\alpha) = 15$

$\Rightarrow 4(\cot^4\alpha - \tan^4\alpha) = 15$

$\Rightarrow 4(1 - \tan^8\alpha) = 15\tan^4\alpha$

$\Rightarrow 4\tan^8\alpha + 15\tan^4\alpha - 4 = 0$

$\Rightarrow 4\tan^8\alpha + 16\tan^4\alpha - \tan^4\alpha - 4 = 0$

$\Rightarrow (4\tan^4\alpha - 1)(\tan^4\alpha + 4) = 0$

$\Rightarrow \tan^4\alpha = \dfrac{1}{4}$ or $\tan^4\alpha = -4$ (not possible)

$\Rightarrow \tan^2\alpha = \pm\dfrac{1}{2} \Rightarrow \tan^2\alpha = +\dfrac{1}{2}$

$\left(\because \tan^2\alpha \neq -\dfrac{1}{2}\right)$

$\Rightarrow \tan\alpha = \pm\dfrac{1}{\sqrt{2}}$

12. $3\sin\beta = \sin(2\alpha + \beta)$, (Given)

$\tan(\alpha + \beta) - 2\tan\alpha$

$= \tan(\alpha + \beta) - \tan\alpha - \tan\alpha$

$= \dfrac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} - \dfrac{\sin\alpha}{\cos\alpha} - \tan\alpha$

$= \dfrac{\sin(\alpha + \beta - \alpha)}{\cos\alpha\cos(\alpha + \beta)} - \tan\alpha = \dfrac{\sin\beta}{\cos(\alpha + \beta)\cos\alpha} - \dfrac{\sin\alpha}{\cos\alpha}$

$= \dfrac{\sin\beta - \sin\alpha\cos(\alpha + \beta)}{\cos(\alpha + \beta)\cos\alpha}$

$= \dfrac{2\sin\beta - [\sin(2\alpha + \beta) - \sin\beta]}{2\cos(\alpha + \beta)\cos\alpha}$

$= \dfrac{-[\sin(2\alpha + \beta) - 3\sin\beta]}{2\cos(\alpha + \beta)\cos\alpha} = 0$

13. **(A, B, C, D)**

For option (A)

$\sin^2 x - \cos^2 x = -\cos 2x \le 1$

For option (B)

$\sqrt{\dfrac{6}{5}} \left(\dfrac{1}{\sqrt{2}} \sin x + \dfrac{1}{\sqrt{3}} \cos x \right) = \dfrac{\sqrt{3}}{\sqrt{5}} \sin x + \dfrac{\sqrt{2}}{\sqrt{5}} \cos x$

$= \sin x \cdot \sin\phi + \cos x \cos\phi$ where $\sin\phi = \sqrt{\dfrac{3}{5}}$, $\cos\phi = \sqrt{\dfrac{2}{5}}$

$= \cos(x - \phi) \le 1$

For option (C)

$\cos^6 x + \sin^6 x = (\cos^2 x)^3 + (\sin^2 x)^3$

$= 1 - 3\sin^2 x \cos^2 x = 1 - \dfrac{3}{4}(\sin 2x)^2 \le 1$

For option (D)

$\cos^2 x + \sin^4 x = 1 - \dfrac{(\sin 2x)^2}{4} \le 1$

15. $\cos 4x \cos 8x - \cos 5x \cos 9x = 0$

$\Rightarrow 2\cos 4x \cos 8x = 2\cos 5x \cos 9x$
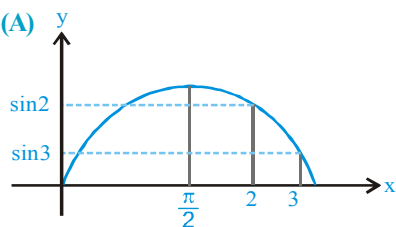
$\Rightarrow \cos 12x + \cos 4x = \cos 14x + \cos 4x$

$\Rightarrow 14x = 2n\pi \pm (12x)$

$\Rightarrow 2x = 2n\pi$ or $26x = 2n\pi$

$\Rightarrow x = n\pi$ or $\dfrac{n\pi}{13}$

$\therefore \sin x = 0$ or $\sin 13x = 0$

16. **(A)**

17. **(C)**

Statement-I $7\cos x + 5\sin x = 2\lambda + 1$

$|2\lambda + 1| \le \sqrt{49 + 25}$

$\Rightarrow |2\lambda + 1| \le \sqrt{74}$ $\Rightarrow -\sqrt{74} \le 2\lambda + 1 \le \sqrt{74}$

$\Rightarrow -8.6 \le 2\lambda + 1 \le 8.6,$ $\Rightarrow -9.6 \le 2\lambda \le 7.6$

$\Rightarrow -4.8 \le \lambda \le 3.8$

$\Rightarrow \lambda$ Integer $= -4, -3, -2, -1, 0, 1, 2, 3$

Statement-II $a\cos\theta + b\sin\theta = c$ has no solution if

$|c| > \sqrt{a^2 + b^2}$

18. From reason (R)

$\displaystyle\prod_{i=1}^{n} \cos\alpha_i = \prod_{i=1}^{n} \sin\alpha_i$

$\Rightarrow \displaystyle\prod_{i=1}^{n} \cos^2\alpha_i = \prod_{i=1}^{n} \left(\dfrac{\sin 2\alpha_i}{2} \right)$ ....(i)

Now, $0 \le \alpha_i \le \dfrac{\pi}{2}$

$0 \le 2\alpha_i \le \pi$

then maximum value of $\sin 2\alpha_i$ is 1 for all i

$\therefore \displaystyle\prod_{i=1}^{n} \cos^2\alpha_i \le \dfrac{1}{2^n}$

$\therefore \displaystyle\prod_{i=1}^{n} \cos\alpha_i \le \dfrac{1}{2^{n/2}}$

19. $\sin\theta + \csc\theta = 2$

$\Rightarrow \dfrac{\sin\theta + \csc\theta}{2} = 1$

and $\sin\theta \csc\theta = 1$

$\Rightarrow \sqrt{\sin\theta \csc\theta} = 1$

i.e., AM = GM

$\Rightarrow \sin\theta = \csc\theta = 1,$

then $\sin^n\theta + \csc^n\theta = 1^n + 1^n = 2$

20. **(D)**

Statement-II $\alpha + \beta = \dfrac{\pi}{2} - \gamma$

$\Rightarrow \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \dfrac{1}{\tan\gamma}$

$\Rightarrow \sum \tan\alpha \tan\beta = 1$

$\therefore$ Statement-2 is true.

Statement-I $\tan\alpha\tan\beta = \dfrac{a!}{6}$,

$\tan\beta\tan\gamma = \dfrac{b!}{2}$ and $\tan\alpha\tan\gamma = \dfrac{c!}{3}$

$\dfrac{a!}{6} + \dfrac{b!}{2} + \dfrac{c!}{3} = 1$

$\Rightarrow a! = 1$ $b! = 1$ $c! = 1$

$\Rightarrow \tan\alpha\tan\beta, \tan\gamma\tan\alpha$ and $\tan\beta\tan\gamma$ are not in A.P.

$\therefore$ **Statement-I** is false

21. $(A) \rightarrow (r), \quad (B) \rightarrow (p), \quad (C) \rightarrow (p), \quad (D) \rightarrow (q)$

(A) $|\cot x| = \cot x + \dfrac{1}{\sin x}$

If $0 < x < \dfrac{\pi}{2} \implies \cot x > 0$

so $\cot x = \cot x + \dfrac{1}{\sin x} \implies \dfrac{1}{\sin x} = 0$ no solution

If $\dfrac{\pi}{2} < \cot x < \pi, \quad -\cot x = \cot x + \dfrac{1}{\sin x}$

$\implies \dfrac{2\cos x}{\sin x} + \dfrac{1}{\sin x} = 0$

$\implies 1 + 2\cos x = 0 \quad \text{and} \quad \sin x \neq 0 \implies x = \dfrac{2\pi}{3}$

(B) since $\sin\phi + \sin\theta = \dfrac{1}{2}$ ....(i)

and $\cos\theta + \cos\phi = 2$ ....(ii)

(ii) is true only if $\theta = \phi = 0$ or 2π but $\theta = \phi = 0$ or 2π do not satisfy (i)

Hence given system of equation has no solution.

(C) $\sin^2\alpha + \sin\left(\dfrac{\pi}{3} - \alpha\right) \cdot \sin\left(\dfrac{\pi}{3} + \alpha\right)$

$= \sin^2\alpha + \sin^2\dfrac{\pi}{3} - \sin^2\alpha = \dfrac{3}{4}$

(D) $\tan\theta = 3\tan\phi$

$\tan(\theta - \phi) = \dfrac{\tan\theta - \tan\varphi}{1 + \tan\theta \tan\varphi} = \dfrac{2\tan\varphi}{1 + 3\tan^2\varphi}$

$= \dfrac{2}{\cot\varphi + 3\tan\varphi}$ Max if $\tan\phi > 0$

$\dfrac{\cot\varphi + 3\tan\varphi}{2} \geq \sqrt{3} \qquad \text{(using AM} \geq \text{GM)}$

$\implies (\cot\phi + 3\tan\phi)^2 \geq 12$

$\implies \left[\dfrac{2}{\tan(\theta - \varphi)}\right]^2 \geq 12$

$\implies \tan^2(\theta - \phi) \leq \dfrac{1}{3}$

22. (A) Let $y = \dfrac{7 + 6\tan\theta - \tan^2\theta}{(1 + \tan^2\theta)}$

$= 7\cos^2\theta + 6\sin\theta\cos\theta - \sin^2\theta$

$= 7\left(\dfrac{1 + \cos 2\theta}{2}\right) + 3\sin 2\theta - \left(\dfrac{1 - \cos 2\theta}{2}\right)$

$= 3\sin 2\theta + 4\cos 2\theta + 3$

$-\sqrt{(3^2 + 4^2)} + 3 \leq 3\sin 2\theta + 4\cos 2\theta + 3 \leq \sqrt{(3^2 + 4^2)} + 3$

$\therefore \quad -2 \leq y \leq 8$

$\implies \lambda = 8, \mu = -2$

$\implies \lambda + \mu = 6, \lambda - \mu = 10 \text{ (R, S)}$

(B) Let $y = 5\cos\theta + 3\cos(\theta + \pi/3) + 3$

$= 5\cos\theta + 3\left(\dfrac{1}{2}\cos\theta - \dfrac{\sqrt{3}}{2}\sin\theta\right) + 3$

$= \dfrac{13}{2}\cos\theta - \dfrac{3\sqrt{3}}{2}\sin\theta + 3$

$\therefore \quad 3 - \sqrt{\left(\dfrac{13}{2}\right)^2 + \left(\dfrac{-3\sqrt{3}}{2}\right)^2} \leq \dfrac{13}{2}\cos\theta - \dfrac{3\sqrt{3}}{2}\sin + 3$

$\leq 3 + \sqrt{\left(\dfrac{13}{2}\right)^2 + \left(\dfrac{-3\sqrt{3}}{2}\right)^2}$

$\implies 3 - 7 \leq y \leq 3 + 7$

$\implies -4 \leq y \leq 10$

$\therefore \quad \lambda = 10, \mu = -4$

$\implies \lambda + \mu = 6, \lambda - \mu = 14 \text{ (R, T)}$

(C) Let $y = 1 + \sin\left(\dfrac{\pi}{4} + \theta\right) + 2\cos\left(\dfrac{\pi}{4} - \theta\right)$

$= 1 + \cos\left(\dfrac{\pi}{2} - \left(\dfrac{\pi}{4} + \theta\right)\right) + 2\cos\left(\dfrac{\pi}{4} - \theta\right)$

$= 1 + \cos\left(\dfrac{\pi}{4} - \theta\right) + 2\cos\left(\dfrac{\pi}{4} - \theta\right)$

$= 1 + 3\cos\left(\dfrac{\pi}{4} - \theta\right)$

$\therefore \quad -1 \le \cos\left(\dfrac{\pi}{4} - \theta\right) \le 1$

$\Rightarrow \quad -3 \le 3\cos\left(\dfrac{\pi}{4} - \theta\right) \le 3$

$\Rightarrow \quad 1 - 3 \le 1 + 3\cos\left(\dfrac{\pi}{4} - \theta\right) \le 1 + 3$

$\therefore \quad -2 \le y \le 4$

$\Rightarrow \quad \lambda = 4, \mu = -2$

$\therefore \quad \lambda + \mu = 2, \lambda - \mu = 6 \ (P, Q)$

Comprehension Type

23. $P_0 = 2, P_2 = 1$

and $P_n - P_{n-2} = (\sin^n\theta + \cos^n\theta) - (\sin^{n-2}\theta + \cos^{n-2}\theta)$

$= -\sin^{n-2}\theta(1 - \sin^2\theta) - \cos^{n-2}\theta(1 - \cos^2\theta)$

$= -\sin^{n-2}\theta \cos^2\theta - \cos^{n-2}\theta \sin^2\theta$

$= -\sin^2\theta \cos^2\theta(\sin^{n-4}\theta + \cos^{n-4}\theta)$

$= -\sin^2\theta \cos^2\theta \, P_{n-4}$

$\therefore \quad P_n - P_{n-2} = -\sin^2\theta \cos^2\theta \, P_{n-4}$ ....**(i)**

for $n = 4$,

$P_4 - P_2 = -\sin^2\theta \cos^2\theta \, P_0$

$\Rightarrow \quad P_4 - 1 = -2\sin^2\theta \cos^2\theta$ $(\because P_2 = 1, P_0 = 2)$

$\therefore \quad P_4 = 1 - 2\sin^2\theta \cos^2\theta$ ....**(ii)**

for $n = 6$,

$\Rightarrow \quad P_6 - P_4 = -\sin^2\theta \cos^2\theta P_2$

$P_6 = P_4 - \sin^2\theta \cos^2\theta P_2$

$= 1 - 2\sin^2\theta \cos^2\theta - \sin^2\theta\cos^2\theta$

$\therefore \quad P_6 = 1 - 3\sin^2\theta \cos^2\theta$ ....**(iii)**

1 $P_1 = m$

$\Rightarrow \quad P_1^2 = m^2$

$\sin^2\theta + \cos^2\theta + 2\sin\theta \cos\theta = m^2$

$\Rightarrow \quad \sin\theta \cos\theta = \dfrac{(m^2 - 1)}{2}$

Now, from eq. **(iii)**,

$P_6 = 1 - 3\sin^2\theta \cos^2\theta$

$\Rightarrow \quad (1 - P_6) = 3(\sin\theta \cos\theta)^2$

$= \dfrac{3(m^2 - 1)^2}{4}$

or $\quad 4(1 - P_6) = 3(m^2 - 1)^2$

2. $2P_6 - 3P_4 + 10$

$= 2(1 - 3\sin^2\theta \cos^2\theta) - 3(1 - 2\sin^2\theta \cos^2\theta) + 10$

(from eq. **(ii)** and **(iii)**)

$= 2 - 3 + 10$

$= 9$

3. Let $\sin^2\theta \cos^2\theta = k$,

then from eq. **(i)**,

$P_n - P_{n-2} = -k \, P_{n-4}$

from eq. **(ii)**,

$P_4 = 1 - 2k$

and from eq. **(iii)**

$P_6 = 1 - 3k$

Put $n = 10$,

then $\quad P_{10} - P_8 = -k P_6 = -k(1 - 3k)$

$\therefore \quad P_{10} - P_8 = 3k^2 - k$ ....**(iv)**

and put $n = 8$,

then $\quad P_8 - P_6 = -k P_4 = -k(1 - 2k)$

$\therefore \quad P_8 = P_6 + 2k^2 - k$

$= 1 - 3k + 2k^2 - k$

$\Rightarrow \quad P_8 = 2k^2 - 4k + 1$

from eq. **(iv)**, $P_{10} = 5k^2 - 5k + 1$

$\therefore \quad 6P_{10} - 15P_8 + 10P_6 + 7$

$= 6(5k^2 - 5k + 1) - 15(2k^2 - 4k + 1)$

$\qquad + 10(1 - 3k) + 7$

$= 8$

24.

1. (D)

Angle subtended by two consecutive marks at centre $= 30°$
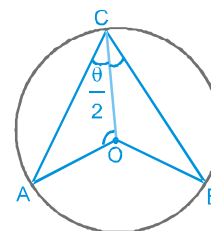
Hence at "half past 4", the angle is $45°$

2. (A)

Distance covered in 1 second $= 5\left(2\pi \cdot \dfrac{1}{2}\right) = 5\pi$ m

Distance covered in 1 hour $= \dfrac{5\pi}{1000} \times 60 \times 60$ km $= 56.52$ km

3. (A)

Area of region $ABC = \dfrac{\pi r^2}{3}$

Area of OAB $= \dfrac{1}{2} r^2 . 2\theta = r^2 \theta$

Area of ΔOAC $= \dfrac{1}{2} r^2 \sin\theta = $ Area of ΔOBC

$\therefore \quad \dfrac{1}{2} r^2 \sin\theta + \dfrac{1}{2} r^2 \sin\theta + r^2\theta = \dfrac{\pi r^2}{3}$

$\Rightarrow \quad 3\sin\theta + 3\theta = \pi$

25.

1. (C)

$\sin \dfrac{9\pi}{14} \, \sin \dfrac{11\pi}{14} \, \sin \dfrac{13\pi}{14} = \sin \dfrac{5\pi}{14} \, \sin \dfrac{3\pi}{14} . \sin \dfrac{\pi}{14}$

$= \cos \dfrac{\pi}{7} \, \cos \dfrac{2\pi}{7} \, \cos \dfrac{3\pi}{7} = -\cos \dfrac{\pi}{7} \, \cos \dfrac{2\pi}{7} \, \cos \dfrac{4\pi}{7}$

$= -\dfrac{\sin \dfrac{8\pi}{7}}{8 \sin \dfrac{\pi}{7}} = \dfrac{1}{8}$

2. (B)

$\cos 2^3 \dfrac{\pi}{10} \, \cos 2^4 \dfrac{\pi}{10} . \cos 2^5 \dfrac{\pi}{10} \, ... \cos 2^{10} \dfrac{\pi}{10}$

$= \dfrac{\sin 2^{11} \dfrac{\pi}{10}}{256 \sin 2^3 \dfrac{\pi}{10}} = \dfrac{1}{256}$

3. (C)

$\cos \dfrac{\pi}{11} \, \cos \dfrac{2\pi}{11} \, \cos \dfrac{3\pi}{11} \, ... \cos \dfrac{11\pi}{11}$

$= \left(\cos\dfrac{\pi}{11}\cos\dfrac{2\pi}{11}\cos\dfrac{3\pi}{11}\cos\dfrac{4\pi}{11}\cos\dfrac{5\pi}{11} \right)^2$

$= \left(\cos\dfrac{\pi}{11}\cos\dfrac{2\pi}{11}\cos\dfrac{4\pi}{11}\cos\dfrac{8\pi}{11}\cos\dfrac{5\pi}{11} \right)^2$

$= \left(\dfrac{\sin 16 \dfrac{\pi}{11}}{16 \sin \dfrac{\pi}{11}} . \cos\dfrac{5\pi}{11} \right)^2 = \left(\dfrac{2\sin \dfrac{5\pi}{11}\cos\dfrac{5\pi}{11}}{32 \sin \dfrac{\pi}{11}} \right)^2 = \dfrac{1}{1024}$

26. (4)

L.H.S. $= 2 \sin 5 (A + B) \cos 5(A - B) + 2 \sin 5C \cos 5C.$

$= 2 \sin (5\pi - 5C) \cos 5 (A - B) + 2 \sin 5C \cos 5C$

$= 2 \sin 5C [\cos (5A - 5B) + \cos 5C]$

$= 2 \sin 5C [\cos (5A - 5B) + \cos 5 (\pi - \overline{A + B})]$

$= 2 \sin 5C [\cos (5A - 5B) - \cos (5A + 5B)]$

$= 4 \sin 5A \sin 5B \sin 5C = $ R.H.S.

27. Let $\theta = \dfrac{\pi}{16}; \; 8\theta = \dfrac{\pi}{2}$

$\therefore \quad y = \tan\theta + \tan 5\theta + \tan 9\theta + \tan 13\theta$

$\therefore \quad y = (\tan\theta - \cot\theta) + (\tan 5\theta - \cot 5\theta)$

$[\; \tan 13\theta = \tan(8\theta + 5\theta) = -\cot 5\theta; \; \tan 9\theta$

$\qquad\qquad\qquad = \tan(8\theta + \theta) = -\cot\theta \;]$

$= (\tan\theta - \cot\theta) + (\cot 3\theta - \tan 3\theta)$

$= \dfrac{\sin^2\theta - \cos^2\theta}{\sin\theta\cos\theta} + \dfrac{\cos^2 3\theta - \sin^2 3\theta}{\sin 3\theta\cos 3\theta}$

$y = 2\left[\dfrac{\cos 6\theta}{\sin 6\theta} - \dfrac{\cos 2\theta}{\sin 2\theta} \right]$

$= 2\left[\dfrac{\sin 2\theta\cos 6\theta - \cos 2\theta\sin 6\theta}{\sin 6\theta\sin 2\theta} \right] = -2 \cdot \dfrac{\sin 4\theta}{\cos 2\theta\sin 2\theta}$

$= -4$

Hence absolute value $= 4$

28. $\dfrac{\tan\alpha + \tan\gamma}{1 + \tan\alpha\tan\gamma} = \dfrac{\sin(\alpha + \gamma)}{\cos(\alpha - \gamma)}$

$\therefore \quad \sin 2\beta = \dfrac{2\tan\beta}{1 + \tan^2\beta} = \dfrac{2\dfrac{\sin(\alpha+\gamma)}{\cos(\alpha-\gamma)}}{1 + \dfrac{\sin^2(\alpha+\gamma)}{\cos^2(\alpha-\gamma)}}$

$= \dfrac{2\sin(\alpha+\gamma)\cos(\alpha-\gamma)}{\cos^2(\alpha-\gamma) + \sin^2(\alpha+\gamma)}$

$= \dfrac{\sin 2\alpha + \sin 2\gamma}{\dfrac{1+\cos 2(\alpha-\gamma)}{2} + \dfrac{1-\cos 2(\alpha+\gamma)}{2}}$

$= \dfrac{\sin 2\alpha + \sin 2\gamma}{1 + \dfrac{1}{2}\big(\cos 2(\alpha-\gamma) - \cos 2(\alpha+\gamma)\big)}$

$= \dfrac{\sin 2\alpha + \sin 2\gamma}{1 + \sin 2\alpha\sin 2\gamma}$

$= $ R.H.S.

29. 6

30. (9)

We have, $f(\theta) = \dfrac{(\sin\theta)^x}{(\cos\theta)^x + (\sin\theta)^x} \Rightarrow f(\theta) + f\left(\dfrac{\pi}{2} - \theta\right) = 1.$

$\therefore \quad S = \sum_{\theta=1°}^{89°} f(\theta) = f(1°) + f(2°) + ... + f(88°) + f(89°)$

$= \left(\underbrace{1+1+1+.....+1}_{44 \text{ times}} \right) + \dfrac{1}{2} = 44 + \dfrac{1}{2} = \dfrac{89}{2}$

$\therefore \quad 2S - 8 = 81$