SOLVED EXAMPLES

Ex. 1 Solve $\cos 3x + \sin 2x - \sin 4x = 0$

Sol. $\cos 3x + \sin 2x - \sin 4x = 0$ $\Rightarrow$ $\cos 3x + 2\cos 3x.\sin(-x) = 0$

$\Rightarrow$ $\cos 3x - 2\cos 3x.\sin x = 0$ $\Rightarrow$ $\cos 3x\,(1 - 2\sin x) = 0$

$\Rightarrow$ $\cos 3x = 0$ or $1 - 2\sin x = 0$

$\Rightarrow$ $3x = (2n+1)\dfrac{\pi}{2}, n \in I$ or $\sin x = \dfrac{1}{2}$

$\Rightarrow$ $x = (2n+1)\dfrac{\pi}{6}, n \in I$ or $x = n\pi + (-1)^n\dfrac{\pi}{6}, n \in I$

$\therefore$ solution of given equation is

$(2n+1)\dfrac{\pi}{6}, n \in I$ or $n\pi + (-1)^n\dfrac{\pi}{6}, n \in I$

Ex. 2 If $x \neq \dfrac{n\pi}{2}, n \in I$ and $(\cos x)^{\sin^2 x - 3\sin x + 2} = 1$, then find the general solutions of x.

Sol. As $x \neq \dfrac{n\pi}{2}$ $\Rightarrow$ $\cos x \neq 0, 1, -1$

So, $(\cos x)^{\sin^2 x - 3\sin x + 2} = 1$ $\Rightarrow$ $\sin^2 x - 3\sin x + 2 = 0$

$\therefore$ $(\sin x - 2)(\sin x - 1) = 0$ $\Rightarrow$ $\sin x = 1, 2$

where $\sin x = 2$ is not possible and $\sin x = 1$ which is also not possible as $x \neq \dfrac{n\pi}{2}$

$\therefore$ no general solution is possible.

Ex. 3 Solve $3\cos x + 4\sin x = 5$

Sol. $\because$ $3\cos x + 4\sin x = 5$ ......(i)

$\because$ $\cos x = \dfrac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$ & $\sin x = \dfrac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$

$\therefore$ equation (i) becomes

$\Rightarrow$ $3\left(\dfrac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right) + 4\left(\dfrac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right) = 5$ ......(ii)

Let $\tan\dfrac{x}{2} = t$

$\therefore$ equation (ii) becomes $3\left(\dfrac{1 - t^2}{1 + t^2}\right) + 4\left(\dfrac{2t}{1 + t^2}\right) = 5$

$\Rightarrow$ $4t^2 - 4t + 1 = 0$ $\Rightarrow$ $(2t - 1)^2 = 0$

$\Rightarrow$ $t = \dfrac{1}{2}$ $\because$ $t = \tan\dfrac{x}{2}$

$\Rightarrow$ $\tan\dfrac{x}{2} = \dfrac{1}{2}$ $\Rightarrow$ $\tan\dfrac{x}{2} = \tan\alpha$, where $\tan\alpha = \dfrac{1}{2}$

$\Rightarrow$ $\dfrac{x}{2} = n\pi + \alpha$ $\Rightarrow$ $x = 2n\pi + 2\alpha$ where $\alpha = \tan^{-1}\left(\dfrac{1}{2}\right), n \in I$

Ex. 4 Solve $2\cos^2x + 4\cos x = 3\sin^2x$

Sol. $2\cos^2x + 4\cos x - 3\sin^2x = 0$

$\Rightarrow$ $2\cos^2x + 4\cos x - 3(1-\cos^2x) = 0$

$\Rightarrow$ $5\cos^2x + 4\cos x - 3 = 0$

$\Rightarrow$ $\left\{\cos x - \left(\frac{-2+\sqrt{19}}{5}\right)\right\}\left\{\cos x - \left(\frac{-2-\sqrt{19}}{5}\right)\right\} = 0$ ....**(ii)**

$\because$ $\cos x \in [-1, 1] \quad \forall x \in R$

$\therefore$ $\cos x \neq \frac{-2-\sqrt{19}}{5}$

$\therefore$ equation **(ii)** will be true if $\cos x = \frac{-2+\sqrt{19}}{5}$

$\Rightarrow$ $\cos x = \cos\alpha$, where $\cos\alpha = \frac{-2+\sqrt{19}}{5}$

$\Rightarrow$ $x = 2n\pi \pm \alpha$ where $\alpha = \cos^{-1}\left(\frac{-2+\sqrt{19}}{5}\right)$, $n \in I$

Ex. 5 Solve $\sin^2\theta - \cos\theta = \frac{1}{4}$ for θ and write the values of θ in the interval $0 \le \theta \le 2\pi$.

Sol. The given equation can be written as

$1 - \cos^2\theta - \cos\theta = \frac{1}{4}$ $\Rightarrow$ $\cos^2\theta + \cos\theta - 3/4 = 0$

$\Rightarrow$ $4\cos^2\theta + 4\cos\theta - 3 = 0$ $\Rightarrow$ $(2\cos\theta - 1)(2\cos\theta + 3) = 0$

$\Rightarrow$ $\cos\theta = \frac{1}{2}, -\frac{3}{2}$

Since, $\cos\theta = -3/2$ is not possible as $-1 \le \cos\theta \le 1$

$\therefore$ $\cos\theta = \frac{1}{2}$ $\Rightarrow$ $\cos\theta = \cos\frac{\pi}{3}$ $\Rightarrow$ $\theta = 2n\pi \pm \frac{\pi}{3}, n \in I$

For the given interval, $n = 0$ and $n = 1$.

$\Rightarrow$ $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

Ex. 6 Solve the equation $5\sin^2x - 7\sin x \cos x + 16\cos^2 x = 4$

Sol. To solve this equation we use the fundamental formula of trigonometric identities,

$\sin^2x + \cos^2x = 1$

writing the equation in the form,

$5\sin^2x - 7\sin x \cdot \cos x + 16\cos^2x = 4(\sin^2x + \cos^2x)$

$\Rightarrow$ $\sin^2x - 7\sin x \cos x + 12\cos^2 x = 0$ dividing by $\cos^2x$ on both side we get,

$\tan^2x - 7\tan x + 12 = 0$

Now it can be factorized as :

$(\tan x - 3)(\tan x - 4) = 0$

$\Rightarrow$ $\tan x = 3, 4$

i.e., $\tan x = \tan(\tan^{-1}3)$ or $\tan x = \tan(\tan^{-1}4)$

$\Rightarrow$ $x = n\pi + \tan^{-1}3$ or $x = n\pi + \tan^{-1}4$, $n \in I$.

Ex. 7 Solve $\sin x + \cos x = \sqrt{2}$

Sol. $\because$ $\sin x + \cos x = \sqrt{2}$ ......**(i)**

Here $a = 1, b = 1.$

$\therefore$ divide both sides of equation **(i)** by $\sqrt{2}$, we get

$\sin x \cdot \frac{1}{\sqrt{2}} + \cos x \cdot \frac{1}{\sqrt{2}} = 1 \Rightarrow \sin x \cdot \sin\frac{\pi}{4} + \cos x \cdot \cos\frac{\pi}{4} = 1 \Rightarrow \cos\left(x - \frac{\pi}{4}\right) = 1$

$\Rightarrow x - \frac{\pi}{4} = 2n\pi, n \in I \Rightarrow x = 2n\pi + \frac{\pi}{4}, n \in I$

$\therefore$ Solution of given equation is $2n\pi + \frac{\pi}{4}, n \in I$

Ex. 8 Solve the equation $\sin^4 x + \cos^4 x = \frac{7}{2}\sin x \cdot \cos x.$

Sol. $\sin^4 x + \cos^4 x = \frac{7}{2}\sin x \cdot \cos x \Rightarrow (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x = \frac{7}{2}\sin x \cdot \cos x$

$\Rightarrow 1 - \frac{1}{2}(\sin 2x)^2 = \frac{7}{4}(\sin 2x) \Rightarrow 2\sin^2 2x + 7\sin 2x - 4 = 0$

$\Rightarrow (2\sin 2x - 1)(\sin 2x + 4) = 0 \Rightarrow \sin 2x = \frac{1}{2}$ or $\sin 2x = -4$ (which is not possible)

$\Rightarrow 2x = n\pi + (-1)^n \frac{\pi}{6}, n \in I$

i.e., $x = \frac{n\pi}{2} + (-1)^n \frac{\pi}{12}, n \in I$

Ex.9 Find the number of distinct solutions of $\sec x + \tan x = \sqrt{3}$, where $0 \le x \le 3\pi$.

Sol. Here, $\sec x + \tan x = \sqrt{3} \Rightarrow 1 + \sin x = \sqrt{3}\cos x$

or $\sqrt{3}\cos x - \sin x = 1$

dividing both sides by $\sqrt{a^2 + b^2}$ i.e. $\sqrt{4} = 2$, we get

$\Rightarrow \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x = \frac{1}{2}$

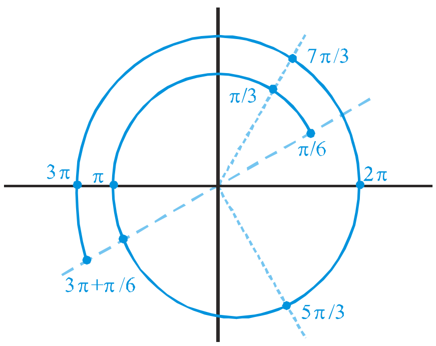
$\Rightarrow \cos\frac{\pi}{6}\cos x - \sin\frac{\pi}{6}\sin x = \frac{1}{2} \Rightarrow \cos\left(x + \frac{\pi}{6}\right) = \frac{1}{2}$

As $0 \le x \le 3\pi$

$\frac{\pi}{6} \le x + \frac{\pi}{6} \le 3\pi + \frac{\pi}{6}$

$\Rightarrow x + \frac{\pi}{6} = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}$

$\Rightarrow x = \frac{\pi}{6}, \frac{3\pi}{2}, \frac{13\pi}{6}$



But at $x = \frac{3\pi}{2}$, tanx and secx is not defined.

$\therefore$ Total number of solutions are 2.

Ex. 10 Solve : $\cos\theta + \cos3\theta + \cos5\theta + \cos7\theta = 0$

Sol. We have $\cos\theta + \cos7\theta + \cos3\theta + \cos5\theta = 0$

$\Rightarrow$ $2\cos4\theta\cos3\theta + 2\cos4\theta\cos\theta = 0$ $\Rightarrow$ $\cos4\theta(\cos3\theta + \cos\theta) = 0$

$\Rightarrow$ $\cos4\theta(2\cos2\theta\cos\theta) = 0$

$\Rightarrow$ Either $\cos\theta = 0$ $\Rightarrow$ $\theta = (2n_1 + 1)\pi/2,\ n_1 \in I$

or $\cos2\theta = 0 \Rightarrow \theta = (2n_2 + 1)\dfrac{\pi}{4},\ n_2 \in I$

or $\cos4\theta = 0 \Rightarrow \theta = (2n_3 + 1)\dfrac{\pi}{8},\ n_3 \in I$

Ex. 11 Find the general solution of equation $\sin^4x + \cos^4x = \sin x \cos x$.

Sol. Using half-angle formulae, we can represent given equation in the form :

$$\left(\frac{1-\cos 2x}{2}\right)^2 + \left(\frac{1+\cos 2x}{2}\right)^2 = \sin x \cos x$$

$\Rightarrow$ $(1 - \cos2x)^2 + (1 + \cos2x)^2 = 4\sin x \cos x$

$\Rightarrow$ $2(1 + \cos^2 2x) = 2\sin2x$ $\Rightarrow$ $1 + 1 - \sin^2 2x = \sin2x$

$\Rightarrow$ $\sin^2 2x + \sin2x = 2$

$\Rightarrow$ $\sin2x = 1$ or $\sin2x = -2$ (which is not possible)

$\Rightarrow$ $2x = 2n\pi + \dfrac{\pi}{2},\ n \in I$ $\Rightarrow$ $x = n\pi + \dfrac{\pi}{4},\ n \in I$

Ex. 12 Solve $\cos4\theta + \sin5\theta = 2$.

Sol. The equation $\cos4\theta + \sin5\theta = 2$.

$4\theta = 2n\pi$ and $5\theta = 2n\pi + \pi/2,\ n, m \in Z$

$\Rightarrow$ $\theta = \dfrac{2n\pi}{4}$ and $\theta = \dfrac{2m\pi}{5} + \dfrac{\pi}{10},\ n, m \in Z$

Putting n; m = 0, ±1, ±2, ..., the common value in $[0, 2\pi]$ is $\theta = \pi/2$

Therefore, the solution is $\theta = 2k\pi + \pi/2,\ k \in Z$.

Ex. 13 Solve $4\cot^2\theta = \cot^2\theta - \tan^2\theta$.

Sol. $\dfrac{4}{\tan 2\theta} = \dfrac{1}{\tan^2\theta} - \tan^2\theta$

or $\dfrac{4(1-\tan^2\theta)}{2\tan\theta} = \dfrac{1-\tan^4\theta}{\tan^2\theta}$ $\left[\text{put } \tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}\right]$

or $(1 - \tan^2\theta)\,[2\tan\theta - (1 + \tan^2\theta)] = 0$

or $(1 - \tan^2\theta)(\tan^2\theta - 2\tan\theta + 1) = 0$

or $(1 - \tan^2\theta)(\tan\theta - 1)^2 = 0$

or $\tan\theta = \pm 1$

$\Rightarrow$ $\theta = n\pi \pm \dfrac{\pi}{4},\ n \in Z$

Ex. 14 Find the solution set of inequality sin x > 1/2.

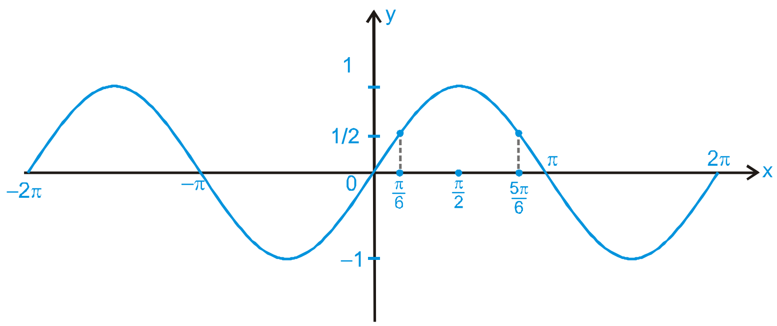
Sol. When $\sin x = \frac{1}{2}$, the two values of x between 0 and 2π are $\pi/6$ and $5\pi/6$.

From the graph of y = sin x, it is obvious that between 0 and 2π,

$\sin x > \frac{1}{2}$ for $\pi/6 < x < 5\pi/6$

Hence, $\sin x > 1/2$

$\Rightarrow \quad 2n\pi + \pi/6 < x < 2n\pi + 5\pi/6, n \in I$



Thus, the required solution set is $\bigcup_{n\in I}\left(2n\pi + \frac{\pi}{6}, 2n\pi + \frac{5\pi}{6}\right)$

Ex. 15 Find the value of x in the interval $\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$ for which $\sqrt{2}\sin 2x + 1 \le 2\sin x + \sqrt{2}\cos x$

Sol. We have, $\sqrt{2}\sin 2x + 1 \le 2\sin x + \sqrt{2}\cos x$

$\Rightarrow \quad 2\sqrt{2}\sin x\cos x - 2\sin x - \sqrt{2}\cos x + 1 \le 0$

$\Rightarrow \quad 2\sin x(\sqrt{2}\cos x - 1) - 1(\sqrt{2}\cos x - 1) \le 0$

$\Rightarrow \quad (2\sin x - 1)(\sqrt{2}\cos x - 1) \le 0$

$\Rightarrow \quad \left(\sin x - \frac{1}{2}\right)\left(\cos x - \frac{1}{\sqrt{2}}\right) \le 0$

Above inequality holds when :

Case-I : $\sin x - \frac{1}{2} \le 0$ and $\cos x - \frac{1}{\sqrt{2}} \ge 0 \quad \Rightarrow \quad \sin x \le \frac{1}{2}$ and $\cos x \ge \frac{1}{\sqrt{2}}$

Now considering the given interval of x :

for $\sin x \le \frac{1}{2}$: $x \in \left[-\frac{\pi}{2}, \frac{\pi}{6}\right] \cup \left[\frac{5\pi}{6}, \frac{3\pi}{2}\right]$ and for $\cos x \ge \frac{1}{\sqrt{2}}$: $x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

For both to simultaneously hold true : $x \in \left[-\frac{\pi}{4}, \frac{\pi}{6}\right]$

Case-II : $\sin x - \frac{1}{2} \geq 0$ and $\cos x \leq \frac{1}{\sqrt{2}}$

Again, for the given interval of x :

for $\sin x \geq \frac{1}{2}$: $x \in \left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$ and for $\cos x \leq \frac{1}{\sqrt{2}}$: $x \in \left[-\frac{\pi}{2}, -\frac{\pi}{4}\right] \cup \left[\frac{\pi}{4}, \frac{3\pi}{2}\right]$

For both to simultaneously hold true : $x \in \left[\frac{\pi}{4}, \frac{5\pi}{6}\right]$

$\therefore$ Given inequality holds for $x \in \left[-\frac{\pi}{4}, \frac{\pi}{6}\right] \cup \left[\frac{\pi}{4}, \frac{5\pi}{6}\right]$

Ex. 16 Solve $\sin x + \cos x = 1 + \sin x.\cos x$

Sol. $\sin x + \cos x = 1 + \sin x.\cos x$ ....**(i)**

Let $\sin x + \cos x = t$

$\Rightarrow$ $\sin^2 x + \cos^2 x + 2 \sin x.\cos x = t^2$

$\Rightarrow$ $\sin x.\cos x = \frac{t^2 - 1}{2}$

Now put $\sin x + \cos x = t$ and $\sin x.\cos x = \frac{t^2 - 1}{2}$ in **(i)**, we get $t = 1 + \frac{t^2 - 1}{2}$

$\Rightarrow$ $t^2 - 2t + 1 = 0$

$\Rightarrow$ $t = 1$ $\quad \because$ $t = \sin x + \cos x$

$\Rightarrow$ $\sin x + \cos x = 1$ ....**(ii)**

divide both sides of equation **(ii)** by $\sqrt{2}$, we get

$\Rightarrow$ $\sin x.\frac{1}{\sqrt{2}} + \cos x.\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\Rightarrow$ $\cos\left(x - \frac{\pi}{4}\right) = \cos\frac{\pi}{4}$ $\quad \Rightarrow$ $x - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$

(i) if we take positive sign, we get $x = 2n\pi + \frac{\pi}{2}$, $n \in I$

(ii) if we take negative sign, we get

$x = 2n\pi$, $n \in I$

Ex. 17 If the set of all values of x in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ satisfying $|4 \sin x + \sqrt{2}| < \sqrt{6}$ is $\left(\frac{a\pi}{24}, \frac{b\pi}{24}\right)$ then find the value of $\left|\frac{a-b}{3}\right|$.

Sol. $|4 \sin x + \sqrt{2}| < \sqrt{6}$

$\Rightarrow$ $-\sqrt{6} < 4 \sin x + \sqrt{2} < \sqrt{6}$ $\quad \Rightarrow$ $-\sqrt{6} - \sqrt{2} < 4 \sin x < \sqrt{6} - \sqrt{2}$

$\Rightarrow$ $\frac{-(\sqrt{6} + \sqrt{2})}{4} < \sin x < \frac{\sqrt{6} - \sqrt{2}}{4}$ $\quad \Rightarrow$ $-\frac{5\pi}{12} < x < \frac{\pi}{12}$ for $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Comparing with $\frac{a\pi}{24} < x < \frac{b\pi}{24}$, we get, $a = -10, b = 2$

$\therefore \left|\frac{a-b}{3}\right| = \left|\frac{-10-2}{3}\right| = 4$

Ex. 18 Find the values of x in the interval $[0,2\pi]$ which satisfy the inequality : $3|2\sin x-1|\geq 3+4\cos^2 x$.

Sol. The given inequality can be written as :

$3|2\sin x-1|\geq 3+4(1-\sin^2 x) \Rightarrow 3|2\sin x-1|\geq 7-4\sin^2 x$

Let $\sin x = t \Rightarrow 3|2t-1|\geq 7-4t^2$

Case I : For $2t-1\geq 0$ i.e. $t\geq 1/2$

we have , $|2t-1|=(2t-1)$

$\Rightarrow 3(2t-1)\geq 7-4t^2 \Rightarrow 6t-3\geq 7-4t^2$

$\Rightarrow 4t^2+6t-10\geq 0 \Rightarrow 2t^2+3t-5\geq 0$

$\Rightarrow (t-1)(2t+5)\geq 0 \Rightarrow t\leq -\frac{5}{2}$ and $t\geq 1$

Now for $t\geq \frac{1}{2}$, we get $t\geq 1$ from above conditions i.e. $\sin x\geq 1$

The inequality holds true only for x satisfying the equation $\sin x = 1$ $\therefore$ $x=\frac{\pi}{2}$ (for $x\in[0,2\pi]$)

Case II : For $2t-1<0 \Rightarrow t<\frac{1}{2}$

we have, $|2t-1|=-(2t-1)$

$\Rightarrow -3(2t-1)\geq 7-4t^2 \Rightarrow -6t+3\geq 7-4t^2$

$\Rightarrow 4t^2-6t-4\geq 0 \Rightarrow 2t^2-3t-2\geq 0$

$\Rightarrow (t-2)(2t+1)\geq 0 \Rightarrow t\leq -\frac{1}{2}$ and $t\geq 2$

Again, for $t<\frac{1}{2}$ we get $t\leq -\frac{1}{2}$ from above conditions

i.e. $\sin x\leq -\frac{1}{2} \Rightarrow \frac{7\pi}{6}\leq x\leq \frac{11}{6}\pi$ (for $x\in[0,2\pi]$)

Thus, $x\in\left[\frac{7\pi}{6},\frac{11\pi}{6}\right]\cup\left\{\frac{\pi}{2}\right\}$

Exercise # 1

[Single Correct Choice Type Questions]

1. The principal solution set of the equation $2\cos x = \sqrt{2+2\sin 2x}$ is

(A) $\left\{\frac{\pi}{8}, \frac{13\pi}{8}\right\}$ **(B)** $\left\{\frac{\pi}{4}, \frac{13\pi}{8}\right\}$ **(C)** $\left\{\frac{\pi}{4}, \frac{13\pi}{10}\right\}$ **(D)** $\left\{\frac{\pi}{8}, \frac{13\pi}{10}\right\}$

2. The solution set of the equation $4\sin\theta.\cos\theta - 2\cos\theta - 2\sqrt{3}\sin\theta + \sqrt{3} = 0$ in the interval $(0, 2\pi)$ is

(A) $\left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$ **(B)** $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$ **(C)** $\left\{\frac{3\pi}{4}, \pi, \frac{\pi}{3}, \frac{5\pi}{3}\right\}$ **(D)** $\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}\right\}$

3. The number of solutions of the equation $\tan^2 x - \sec^{10} x + 1 = 0$ in $(0, 10)$ is -

(A) 3 **(B)** 6 **(C)** 10 **(D)** 11

4. The number of solution of $\sin^4 x - \cos^2 x \sin x + 2\sin^2 x + \sin x = 0$ in $0 \le x \le 3$ is

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6

5. The most general value for which $\tan\theta = -1$, $\cos\theta = \frac{1}{\sqrt{2}}$ is $(n \in Z)$

(A) $n\pi + \frac{7\pi}{4}$ **(B)** $n\pi + (-1)^n \frac{7\pi}{4}$ **(C)** $2n\pi + \frac{7\pi}{4}$ **(D)** none of these

6. The sum of all the solution of $\cot\theta = \sin 2\theta$ ($\theta \ne n\pi$, n integer), $0 \le \theta \le \pi$, is

(A) $3\pi/2$ **(B)** π **(C)** infinite **(D)** 2π

7. The solutions of the equation $\sin x + 3\sin 2x + \sin 3x = \cos x + 3\cos 2x + \cos 3x$ in the interval $0 \le x \le 2\pi$, are ;

(A) $\frac{\pi}{8}, \frac{5\pi}{8}, \frac{2\pi}{3}$ **(B)** $\frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$ **(C)** $\frac{4\pi}{3}, \frac{9\pi}{3}, \frac{2\pi}{3}, \frac{13\pi}{8}$ **(D)** $\frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{3}, \frac{4\pi}{3}$

8. $\frac{\cos 3\theta}{2\cos 2\theta - 1} = \frac{1}{2}$ if

(A) $\theta = n\pi + \frac{\pi}{3}, n \in I$ **(B)** $\theta = 2n\pi \pm \frac{\pi}{3}, n \in I$

(C) $\theta = 2n\pi \pm \frac{\pi}{6}, n \in I$ **(D)** $\theta = n\pi + \frac{\pi}{6}, n \in I$

9. The number of solution of the equation, $\sum_{r=1}^{5} \cos(r\,x) = 0$ lying in $(0, \pi)$ is :

(A) 2 **(B)** 3 **(C)** 5 **(D)** more than 5

10. The general solution of the trigonometric equation $\tan x + \tan 2x + \tan 3x = \tan x \cdot \tan 2x \cdot \tan 3x$ is

(A) $x = n\pi$ **(B)** $n\pi \pm \frac{\pi}{3}$ **(C)** $x = 2n\pi$ **(D)** $x = \frac{n\pi}{3}$

where $n \in I$

11. General solution of the equation $\sec x = 1 + \cos x + \cos^2 x + \cos^3 x + \ldots\ldots\ldots\, \infty$, is

(A) $n\pi + \frac{\pi}{3}$ **(B)** $2n\pi \pm \frac{\pi}{3}$ **(C)** $n\pi \pm \frac{\pi}{6}$ **(D)** $2n\pi + \frac{\pi}{6}$

where *n* is an integer.

12. Let $f(x) = \cos^2 x + \cos^2 2x + \cos^2 3x$. Number of values of $x \in [0, \pi]$ for which $f(x)$ equals the smallest positive integer is

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6

13. Let $f(x) = \dfrac{\operatorname{cosec}^4 x - 2\operatorname{cosec}^2 x + 1}{\operatorname{cosec} x(\operatorname{cosec} x - \sin x) + \dfrac{\sin x - \cos x}{\sin x} + \cot x}$. The sum of all the solutions of $f(x) = 0$ in $[0, 100\pi]$ is

(A) 2550π **(B)** 2500π **(C)** 5000π **(D)** 5050π

14. Let X be the set of all solutions to the equation $\cos x \cdot \sin\left(x + \dfrac{1}{x}\right) = 0$. Number of real numbers contained by X in the interval $(0 < x < \pi)$, is

(A) 0 **(B)** 1 **(C)** 2 **(D)** more than 2

15. If $2 \sin x + 7 \cos px = 9$ has atleast one solution then p must be

(A) an odd integer **(B)** an even integer

(C) a rational number **(D)** an irrational number

16. Number of principal solution(s) of the equation,

$$4 \cdot 16^{\sin^2 x} = 2^{6 \sin x}, \quad \text{is}$$

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4

17. If the inequality $\sin^2 x + a \cos x + a^2 > 1 + \cos x$ holds for any $x \in R$, then the largest negative integral value of a is

(A) –4 **(B)** –3 **(C)** –2 **(D)** –1

Exercise # 2

Part # I [Multiple Correct Choice Type Questions]

1. The equation $\sin^6 x + \cos^6 x = a^2$ has real solution if

(A) $a \in (-1, 1)$ **(B)** $a \in \left(-1, -\frac{1}{2}\right)$ **(C)** $a \in \left(-\frac{1}{2}, \frac{1}{2}\right)$ **(D)** $a \in \left(\frac{1}{2}, 1\right)$

2. $\sin^2 x - \cos 2x = 2 - \sin 2x$ if

(A) $x = n\pi/2, n \in I$ **(B)** $\tan x = 3/2$

(C) $x = (2n + 1)\, \pi/2, n \in I$ **(D)** $x = n\pi + (-1)^n \sin^{-1}(2/3), n \in I$

3. The solution(s) of the equation cos2x sin6x = cos3x sin5x in the interval $[0, \pi]$ is/are -

(A) $\frac{\pi}{6}$ **(B)** $\frac{\pi}{2}$ **(C)** $\frac{2\pi}{3}$ **(D)** $\frac{5\pi}{6}$

4. The equation $4\sin^2 x - 2(\sqrt{3}+1)\sin x + \sqrt{3} = 0$ has -

(A) 2 solutions in $(0, \pi)$ **(B)** 4 solutions in $(0, 2\pi)$

(C) 2 solutions in $(-\pi, \pi)$ **(D)** 4 solutions in $(-\pi, \pi)$

5. If $\cos^2 2x + 2\cos^2 x = 1$, $x \in (-\pi, \pi)$, then x can take the values -

(A) $\pm\frac{\pi}{2}$ **(B)** $\pm\frac{\pi}{4}$ **(C)** $\pm\frac{3\pi}{4}$ **(D)** none of these

6. If $\cos^2 x + \cos^2 2x + \cos^2 3x = 1$ then -

(A) $x = (2n+1)\frac{\pi}{4}, n \in I$ **(B)** $x = (2n+1)\frac{\pi}{2}, n \in I$ **(C)** $x = n\pi \pm \frac{\pi}{6}, n \in I$ **(D)** none of these

7. If $\sin x + \cos x = \sqrt{y + \frac{1}{y}}$ for $x \in [0, \pi]$, then

(A) $x = \pi/4$ **(B)** $y = 0$ **(C)** $y = 1$ **(D)** $x = 3\pi/4$

8. The value(s) of θ, which satisfy $3 - 2\cos\theta - 4\sin\theta - \cos 2\theta + \sin 2\theta = 0$ is/are -

(A) $\theta = 2n\pi$; $n \in I$ **(B)** $2n\pi + \frac{\pi}{2}$; $n \in I$ **(C)** $2n\pi - \frac{\pi}{2}$; $n \in I$ **(D)** $n\pi$; $n \in I$

9. Given that $\sin 3\theta = \sin 3\alpha$, then which of the following angles will be equal to $\cos\theta$?

(A) $\cos\left(\frac{\pi}{3} + \alpha\right)$ **(B)** $\cos\left(\frac{\pi}{3} - \alpha\right)$ **(C)** $\cos\left(\frac{2\pi}{3} + \alpha\right)$ **(D)** $\cos\left(\frac{2\pi}{3} - \alpha\right)$

10. Let $2 \sin x + 3 \cos y = 3$ and $3 \sin y + 2 \cos x = 4$ then

(A) $x + y = (4n + 1)\pi/2, n \in I$ **(B)** $x + y = (2n + 1)\pi/2, n \in I$

(C) x and y can be the two non right angles of a 3-4-5 triangle with x > y.

(D) x and y can be the two non right angles of a 3-4-5 triangle with y > x.

11. The equation $\text{cosec } x + \sec x = 2\sqrt{2}$ has

(A) no solution in $\left(0, \frac{\pi}{4}\right)$ **(B)** a solution in $\left[\frac{\pi}{4}, \frac{\pi}{2}\right)$

(C) no solution in $\left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$ **(D)** a solution in $\left[\frac{3\pi}{4}, \pi\right)$

12. If $\left(\frac{1-a\sin x}{1+a\sin x}\right)\sqrt{\frac{1+2a\sin x}{1-2a\sin x}} = 1$, where $a \in R$ then -

(A) $x \in \phi$ **(B)** $x \in R\ \forall\ a$

(C) $a = 0, x \in R$ **(D)** $a \in R,\ x \in n\pi$, where $n \in I$

13. The value(s) of θ, which satisfy the equation : $2\cos^3 3\theta + 3\cos 3\theta + 4 = 3\sin^2 3\theta$ is/are -

(A) $\frac{2n\pi}{3} + \frac{2\pi}{9}, n \in I$ **(B)** $\frac{2n\pi}{3} - \frac{2\pi}{9}, n \in I$ **(C)** $\frac{2n\pi}{5} + \frac{2\pi}{5}, n \in I$ **(D)** $\frac{2n\pi}{5} - \frac{2\pi}{5}, n \in I$

14. The equation $\sin\alpha = \tan(\alpha - \beta) + \cos\alpha \cdot \tan\beta$ holds true if

(A) $\alpha = n\pi + \beta$ **(B)** $\alpha = 2n\pi + 2\beta$ **(C)** $\alpha = 2n\pi$ and $\beta \in R$ **(D)** $\beta = 2n\pi$ and $\alpha \in R$

(n is an integer)

15. If $\cos 3\theta = \cos 3\alpha$ then the value of $\sin\theta$ can be given by

(A) $\pm\sin\alpha$ **(B)** $\sin\left(\frac{\pi}{3} \pm \alpha\right)$ **(C)** $\sin\left(\frac{2\pi}{3} + \alpha\right)$ **(D)** $\sin\left(\frac{2\pi}{3} - \alpha\right)$

16. $\cos 4x \cos 8x - \cos 5x \cos 9x = 0$ if

(A) $\cos 12x = \cos 14x$ **(B)** $\sin 13x = 0$ **(C)** $\sin x = 0$ **(D)** $\cos x = 0$

17. $\sin x - \cos^2 x - 1$ assumes the least value for the set of values of x given by:

(A) $x = n\pi + (-1)^{n+1}(\pi/6)$, $n \in I$ **(B)** $x = n\pi + (-1)^n(\pi/6)$, $n \in I$

(C) $x = n\pi + (-1)^n(\pi/3)$, $n \in I$ **(D)** $x = n\pi - (-1)^n(\pi/6)$, $n \in I$

18. The general solution of the equation $\cos x \cdot \cos 6x = -1$, is :

(A) $x = (2n + 1)\pi, n \in I$ **(B)** $x = 2n\pi, n \in I$

(C) $x = (2n - 1)\pi, n \in I$ **(D)** none of these

Part # II [Assertion & Reason Type Questions]

Each question has four choices (A), (B), (C) and (D) out of which only one is correct. These questions contains, Statement-I (assertion) and Statement-II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.
(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for Statement-I.
(C) Statement-I is true, Statement-II is false.
(D) Statement-I is false, Statement-II is true.

1. **Statement - I** The value of x for which $(\sin x + \cos x)^{1+\sin 2x} = 2$, when $0 \le x \le \pi$, is $\pi/4$ only.

Statement - II The maximum value of $\sin x + \cos x$ occurs when $x = \pi/4$

2. **Statement - I** For any real value of $\theta \ne (2n+1)\pi$ or $(2n+1)\pi/2$, $n \in I$, the value of the expression $y = \frac{\cos^2\theta - 1}{\cos^2\theta + \cos\theta}$ is $y \le 0$ or $y \ge 2$ (either less than or equal to zero or greater than or equal to two)

Statement - II $\sec\theta \in (-\infty, -1] \cup [1, \infty)$ for all real values of θ.

3. **Statement-I** The equation $\sqrt{3}\cos x - \sin x = 2$ has exactly one solution in $[0, 2\pi]$.

Statement-II For equations of type $a\cos\theta + b\sin\theta = c$ to have real solutions in $[0, 2\pi]$, $|c| \le \sqrt{a^2 + b^2}$ should hold true.

Exercise # 3 Part # I [Matrix Match Type Questions]

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with **one or more** statement(s) in **Column-II**.

1. On the left, equation with interval is given and on the right number of solutions are given, match the column.

Column-I		Column-II	
(A)	$n\|\sin x\| = m\|\cos x\|$ in $[0, 2\pi]$ where $n > m$ and are positive integers	(p)	2
(B)	$\sum_{r=1}^{5} \cos rx = 5$ in $[0, 2\pi]$	(q)	4
(C)	$2^{1+\|\cos x\|+\|\cos x\|^2 ......\infty} = 4$ in $(-\pi, \pi)$	(r)	3
(D)	$\tan\theta + \tan 2\theta + \tan 3\theta = \tan\theta \tan 2\theta \tan 3\theta$ in $(0, \pi)$	(s)	1

2.

Column – I		Column – II	
(A)	Number of solutions of $\sin^2\theta + 3\cos\theta = 3$ in $[-\pi, \pi]$	(p)	0
(B)	Number of solutions of $\sin x \,.\, \tan 4x = \cos x$ in $(0, \pi)$	(q)	1
(C)	Number of solutions of equation $(1 - \tan\theta)(1 + \tan\theta)\sec^2\theta + 2^{\tan^2\theta} = 0$ where $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	(r)	2
(D)	If $[\sin x] + [\sqrt{2}\cos x] = -3$, where $x \in [0, 2\pi]$ then $[\sin 2x]$ equals (Here [.] denotes G.I.F.)	(s)	5

3.

Column – I		Column – II	
(A)	Number of solutions of $\sin x = \frac{\|x\|}{10}$	(p)	4
(B)	If $\sin^2 x - 2\sin x - 1 = 0$ has minimum four different solutions in $[0, n\pi]$ then n can take values	(q)	5
(C)	If $1 + \sin^4 x = \cos^2 3x$, $x \in \left[\frac{-5\pi}{2}, \frac{5\pi}{2}\right]$, then number of values of x are	(r)	6
(D)	In ΔABC, $\sin(2A + B) = \frac{1}{2}$. A, B, C are in A.P. and $C = \frac{5\pi}{p}$ then p equals	(s)	12

Part # II [Comprehension Type Questions]

Comprehension # 1

Consider the cubic equation
$x^3 - (1 + \cos\theta + \sin\theta)x^2 + (\cos\theta \sin\theta + \cos\theta + \sin\theta)x - \sin\theta \cos\theta = 0$
whose roots are x_1, x_2, and x_3

1. The value of $x_1^2 + x_2^2 + x_3^2$ equals

(A) 1 **(B)** 2 **(C)** $2 \cos\theta$ **(D)** $\sin\theta(\sin\theta + \cos\theta)$

2. The number of values of θ in $[0, 2\pi]$ for which at least two roots are equal

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6

3. The greatest possible difference between two of the roots if $\theta \in [0, 2\pi]$ is

(A) 2 **(B)** 1 **(C)** $\sqrt{2}$ **(D)** $2\sqrt{2}$

Comprehension # 2

Let a, b, c, d $\in$ R. Then the cubic equation of the type $ax^3 + bx^2 + cx + d = 0$ has either one root real or all three roots are real. But in case of trigonometric equations of the type $a \sin^3 x + b \sin^2 x + c \sin x + d = 0$ can possess several solutions depending upon the domain of x.

To solve an equation of the type $a \cos\theta + b \sin\theta = c$. The equation can be written as $\cos(\theta - \alpha) = c/\sqrt{(a^2 + b^2)}$.

The solution is $\theta = 2n\pi + \alpha \pm \beta$, where $\tan \alpha = b/a$, $\cos \beta = c/\sqrt{(a^2 + b^2)}$.

1. On the domain $[-\pi, \pi]$ the equation $4\sin^3 x + 2 \sin^2 x - 2\sin x - 1 = 0$ possess

(A) only one real root **(B)** three real roots

(C) four real roots **(D)** six real roots

2. In the interval $[-\pi/4, \pi/2]$, the equation, $\cos 4x + \dfrac{10 \tan x}{1 + \tan^2 x} = 3$ has

(A) no solution **(B)** one solution **(C)** two solutions **(D)** three solutions

3. $|\tan x| = \tan x + \dfrac{1}{\cos x}$ $(0 \le x \le 2\pi)$ has

(A) no solution **(B)** one solution **(C)** two solutions **(D)** three solutions

Comprehension # 3

To solve a trigonometric inequation of the type sin x ≥ a where |a| ≤ 1, we take a hill of length 2π in the sine curve and write the solution within that hill. For the general solution, we add 2nπ. For instance, to solve $\sin x \geq -\frac{1}{2}$, we take the hill $\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$ over which solution is $-\frac{\pi}{6} < x < \frac{7\pi}{6}$. The general solution is $2n\pi - \frac{\pi}{6} < x < 2n\pi + \frac{7\pi}{6}$, n is any integer. Again to solve an inequation of the type sin x ≤ a, where |a| ≤ 1, we take a hollow of length 2π in the sine curve. (since on a hill, sinx ≤ a is satisfied over two intervals). Similarly cos x ≥ a or cosx ≤ a, |a| ≤ 1 are solved.

1. Solution to the inequation $\sin^6 x + \cos^6 x < \frac{7}{16}$ must be

(A) $n\pi + \frac{\pi}{3} < x < n\pi + \frac{\pi}{2}$
(B) $2n\pi + \frac{\pi}{3} < x < 2n\pi + \frac{\pi}{2}$
(C) $\frac{n\pi}{2} + \frac{\pi}{6} < x < \frac{n\pi}{2} + \frac{\pi}{3}$
(D) none of these

2. Solution to inequality cos 2x + 5 cos x + 3 ≥ 0 over [– π, π] is

(A) [– π, π]
(B) $\left[\frac{-5\pi}{6}, \frac{5\pi}{6}\right]$
(C) [0, π]
(D) $\left[\frac{-2\pi}{3}, \frac{2\pi}{3}\right]$

3. Over [– π, π], the solution of $2\sin^2\left(x + \frac{\pi}{4}\right) + \sqrt{3}\cos 2x \geq 0$ is

(A) [– π, π]
(B) $\left[\frac{-5\pi}{6}, \frac{5\pi}{6}\right]$
(C) [0, π]
(D) $\left[-\pi, \frac{-7\pi}{12}\right] \cup \left[-\frac{\pi}{4}, \frac{5\pi}{12}\right] \cup \left[\frac{3\pi}{4}, \pi\right]$

Exercise # 4 [Subjective Type Questions]

1. Solve $\sin^4(x/3) + \cos^4(x/3) > 1/2$.

2. Solve $\sin x + \sin y = \sin(x + y)$ and $|x| + |y| = 1$.

3. Solve $\tan\left(\frac{\pi}{2}\cos\theta\right) = \cot\left(\frac{\pi}{2}\sin\theta\right)$

4. Solve $\sin^2 x + \frac{1}{4}\sin^2 3x = \sin x \sin^2 3x$.

5. Solve the equation $\sin^4 x + \cos^4 x - 2\sin^2 x + \sin^2 2x = 0$

6. Solve the equation $\sin^3 x \cos 3x + \cos^3 x + \frac{3}{8} = 0$

7. Solve the equation for x, $\sin^{10} x + \cos^{10} x = \frac{29}{16}\cos^4 2x$.

8. Solve for x, the equation $\sqrt{13 - 18\tan x} = 6\tan x - 3$, where $-2\pi < x < 2\pi$.

9. Solve the equation for $0 \le \theta \le 2\pi$; $\left(\sin 2\theta + \sqrt{3}\cos 2\theta\right)^2 - 5 = \cos\left(\frac{\pi}{6} - 2\theta\right)$.

10. Solve the equation $1 + 2\,\text{cosec}\, x = -\dfrac{\sec^2 \frac{x}{2}}{2}$

11. Solve the equation $\sin^2 4x + \cos^2 x = 2\sin 4x \cdot \cos^4 x$

Exercise # 5

Part # I [Previous Year Questions] [AIEEE/JEE-MAIN]

1. The number of solutions of $\tan x + \sec x = 2\cos x$ in $[0, 2\pi]$, is

(1) 2 **(2)** 3 **(3)** 0 **(4)** 1 **[AIEEE 2002]**

2. The number of values of x in the interval $[0, 3\pi]$ satisfying the equation $2\sin^2 x + 5\sin x - 3 = 0$, is

(1) 6 **(2)** 1 **(3)** 2 **(4)** 4 **[AIEEE 2006]**

3. The possible values of $\theta \in (0, \pi)$ such that $\sin\theta + \sin 4\theta + \sin 7\theta = 0$ are :

(1) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{35\pi}{6}$ **(2)** $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

(3) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{4\pi}{9}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{8\pi}{9}$ **(4)** $\frac{\pi}{4}, \frac{5\pi}{2}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$ **[AIEEE 2011]**

4. The number of solutions of the equation $\cos^2\left(x + \frac{\pi}{6}\right) + \cos^2 x - 2\cos\left(x + \frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}\right) = \sin^2\frac{\pi}{6}$ in the interval $(-\pi/2, \pi/2)$ is

(1) 0 **(2)** 1 **(3)** 2 **(4)** 3 **[AIEEE 2012]**

5. If $0 \le x < 2\pi$, then the number of real values of x, which satisfy the equation $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$, is : **[JEE Main 2016]**

(1) 5 **(2)** 7 **(3)** 9 **(4)** 3

Part # II [Previous Year Questions][IIT-JEE ADVANCED]

1. The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ is

(A) 0 **(B)** 2 **(C)** 1 **(D)** 3 **[IIT-JEE-2001]**

2. The number of integral values of k for which the equation $7\cos x + 5\sin x = 2k + 1$ has a solution is

(A) 4 **(B)** 8 **(C)** 10 **(D)** 12 **[IIT-JEE-2002]**

3. $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = \frac{1}{e}$, where $\alpha, \beta \in [-\pi, \pi]$. Pairs α, β which satisfy both the equations is/are **[IIT-JEE-2005]**

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4

4. If $0 < \theta < 2\pi$, then the intervals of values of θ for which $2\sin^2\theta - 5\sin\theta + 2 > 0$, is **[IIT-JEE-2006]**

(A) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$ **(B)** $\left(\frac{\pi}{8}, \frac{5\pi}{6}\right)$ **(C)** $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ **(D)** $\left(\frac{41\pi}{48}, \pi\right)$

5. The number of solutions of the pair of equations $2\sin^2\theta - \cos 2\theta = 0$, $2\cos^2\theta - 3\sin\theta = 0$ in the interval $[0, 2\pi]$ is **[IIT-JEE-2007]**

(A) zero **(B)** one **(C)** two **(D)** four

6. Roots of the equation $2\sin^2\theta + \sin^2\theta = 2$ are

(A) $\frac{\pi}{6}$ **(B)** $\frac{\pi}{4}$ **(C)** $\frac{\pi}{3}$ **(D)** $\frac{\pi}{2}$ **[IIT 2009]**

7. The number of all possible values of θ, where $0 < \theta < \pi$, which the system of equations **[IIT-JEE-2010]**

$$(y + z)\cos 3\theta = xyz \sin 3\theta$$

$$x \sin 3\theta = \frac{2\cos 3\theta}{y} + \frac{2\sin 3\theta}{z}$$

$$xyz \sin 3\theta = (y + 2z)\cos 3\theta + y \sin 3\theta$$

has a solution (x_0, y_0, z_0) with $y_0, z_0 \neq 0$, is

(A) 0 **(B)** 2 **(C)** 3 **(D)** 4

8. The number of values of θ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ such that $\theta \neq \frac{n\pi}{5}$ for $n = 0, \pm 1, \pm 2$ and $\tan\theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$ is **[IIT-JEE-2010]**

9. Let $\theta, \phi \in [0, 2\pi]$ be such that $2\cos\theta(1 - \sin\phi) = \sin^2\theta\left(\tan\frac{\theta}{2} + \cot\frac{\theta}{2}\right)\cos\phi - 1$, $\tan(2\pi - \theta) > 0$ and $-1 < \sin\theta < -\frac{\sqrt{3}}{2}$. Then ϕ cannot satisfy **[IIT-JEE 2012]**

(A) $0 < \phi < \frac{\pi}{2}$ **(B)** $\frac{\pi}{2} < \phi < \frac{4\pi}{3}$ **(C)** $\frac{4\pi}{3} < \phi < \frac{3\pi}{2}$ **(D)** $\frac{3\pi}{2} < \phi < 2\pi$

10. For $x \in (0, \pi)$, the equation $\sin x + 2\sin 2x - \sin 3x = 3$ has **[JEE Ad. 2014]**

(A) Infinitely many solutions **(B)** Three solutions

(C) One solutions **(D)** No solutions

11. Let $S = \left\{x \in (-\pi, \pi) : x \neq 0, \pm\frac{\pi}{2}\right\}$. The sun of all distinct solution of the equation $\sqrt{3}\sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$ in the set S is equal to **[JEE Ad. 2016]**

(A) $-\frac{7\pi}{9}$ **(B)** $-\frac{2\pi}{9}$ **(C)** 0 **(D)** $\frac{5\pi}{9}$

MOCK TEST

SECTION - I : STRAIGHT OBJECTIVE TYPE

1. The most general solution of $\cot\theta = -\sqrt{3}$ and $\operatorname{cosec}\theta = -2$ is :

(A) $n\pi - \frac{\pi}{6}$ (B) $n\pi - (-1)^n \frac{\pi}{6}$ (C) $2n\pi - \frac{\pi}{6}$ (D) none of these , $n \in I$

2. The general solution of the equation , $2 \cot \frac{\theta}{2} = (1 + \cot \theta)^2$ is :

(A) $n\pi + (-1)^n \frac{\pi}{4}, n \in I$ (B) $n\pi + (-1)^n \frac{\pi}{3}, n \in I$ (C) $n\pi + (-1)^n \frac{\pi}{6}, n \in I$ (D) none

3. The most general solution of the equations $\tan \theta = -1, \cos \theta = 1/\sqrt{2}$ is

(A) $n\pi + 7\pi/4$ (B) $n\pi + (-1)^n \frac{7\pi}{4}$ (C) $2n\pi + \frac{7\pi}{4}$ (D) none of these

4. If $6 \cos 2\theta + 2 \cos^2\theta/2 + 2 \sin^2 \theta = 0, -\pi < \theta < \pi$, then $\theta =$

(A) $\pi/3$ (B) $\pi/3, \cos^{-1}(3/5)$ (C) $\cos^{-1}(3/5)$ (D) $\pi/3, \pi - \cos^{-1}(3/5)$

5. If $\frac{\sin^3\theta - \cos^3\theta}{\sin\theta - \cos\theta} - \frac{\cos\theta}{\sqrt{1+\cot^2\theta}} - 2\tan\theta\cot\theta = -1$, then

(A) $\theta \in \left(0, \frac{\pi}{2}\right)$ (B) $\theta \in \left(\frac{\pi}{2}, \pi\right)$ (C) $\theta \in \left(\pi, \frac{3\pi}{2}\right)$ (D) $\theta \in \left(\frac{3\pi}{2}, 2\pi\right)$

6. Number of solutions of the equation $\cos 6x + \tan^2 x + \cos 6x . \tan^2 x = 1$ in the interval $[0, 2\pi]$ is :

(A) 4 (B) 5 (C) 6 (D) 7

7. If $\frac{\sin^3\theta - \cos^3\theta}{\sin\theta - \cos\theta} - \frac{\cos\theta}{\sqrt{(1+\cot^2\theta)}} - 2\tan\theta\cot\theta = -1, \theta \in [0, 2\pi]$, then

(A) $\theta \in \left(0, \frac{\pi}{2}\right) - \left\{\frac{\pi}{4}\right\}$ (B) $\theta \in \left(\frac{\pi}{2}, \pi\right) - \left\{\frac{3\pi}{4}\right\}$ (C) $\theta \in \left(\pi, \frac{3\pi}{2}\right) - \left\{\frac{5\pi}{4}\right\}$ (D) $\theta \in (0, \pi) - \left\{\frac{\pi}{4}, \frac{\pi}{2}\right\}$

8. In the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ the equation $\log_{\sin\theta}(\cos 2\theta) = 2$ has

(A) no solution (B) a unique solution (C) two solutions (D) infinitely many solutions

9. The general solution of the equation, $\frac{1 - \sin x + ... + (-1)^n \sin^n x + ...\infty}{1 + \sin x + ... + \sin^n x + ...\infty} = \frac{1-\cos 2x}{1+\cos 2x}$ is

(A) $(-1)^n \pi/3 + n\pi$ (B) $(-1)^n \pi/6 + n\pi$ (C) $(-1)^{n+1} \pi/6 + n\pi$ (D) $(-1)^{n-1} \pi/3 + n\pi$

10. Number of ordered pairs (a, x) satisfying the equation $\sec^2 (a + 2)x + a^2 - 1 = 0 ; -\pi < x < \pi$ is

(A) 1 (B) 2 (C) 3 (D) 4

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

11. $4\sin^4x + \cos^4x = 1$ if

(A) $x = n\pi$ **(B)** $x = n\pi \pm \frac{1}{2}\cos^{-1}\left(\frac{1}{5}\right)$

(C) $x = \frac{n\pi}{2}$ **(D)** none of these, $(n \in I)$

12. $\sin x + \sin 2x + \sin 3x = \cos x + \cos 2x + \cos 3x$ if

(A) $\cos x = -\frac{1}{2}$ **(B)** $\sin 2x = \cos 2x$ **(C)** $x = \frac{n\pi}{2} + \frac{\pi}{8}$ **(D)** $x = 2n\pi \pm \frac{2\pi}{3}$, $(n \in I)$

13. $\cos 15x = \sin 5x$ if

(A) $x = -\frac{\pi}{20} + \frac{n\pi}{5}, n\in I$ **(B)** $x = \frac{\pi}{40} + \frac{n\pi}{10}, n\in I$ **(C)** $x = \frac{3\pi}{20} + \frac{n\pi}{5}, n\in I$ **(D)** $x = -\frac{3\pi}{40} + \frac{n\pi}{10}, n \in I$

14. $\sin^2x + 2\sin x\cos x - 3\cos^2x = 0$ if

(A) $\tan x = 3$ **(B)** $\tan x = -1$ **(C)** $x = n\pi + \pi/4, n \in I$ **(D)** $x = n\pi + \tan^{-1}(-3), n \in I$

15. $\sin^2x - \cos 2x = 2 - \sin 2x$ if

(A) $x = n\pi/2, n \in I$ **(B)** $\tan x = 3/2$ **(C)** $x = (2n + 1)\pi/2, n \in I$ **(D)** $x = n\pi + (-1)^n \sin^{-1}(2/3), n\in I$

SECTION - III : ASSERTION AND REASON TYPE

16. **Statement-I :** The number of real solutions of the equation $\sin x = 2^x + 2^{-x}$ is zero

Statement-II : Since $|\sin x| \le 1$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

17. **Statement-I :** If $\sin x + \cos x = \sqrt{\left(y + \frac{1}{y}\right)}$, $x \in [0, \pi]$, then $x = \frac{\pi}{4}, y = 1$

Statement-II : AM $\ge$ GM

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

18. **Statement-I :** In $(0, \pi)$, the number of solutions of the equation $\tan\theta + \tan 2\theta + \tan 3\theta = \tan\theta \tan 2\theta \tan 3\theta$ is two

Statement-II : $\tan 6\theta$ is not defined at $\theta = (2n + 1)\frac{\pi}{12}, n \in I$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

19. **Statement-I :** The equation sin(cos x) = cos(sin x) does not possess real roots.

Statement-II : If sin x > 0, then $2n\pi < x < (2n + 1)$, $n \in I$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

20. **Statement-I :** If $\sin^2 A = \sin^2 B$ and $\cos^2 A = \cos^2 B$, then $A = n\pi + B$, $n \in I$

Statement-II : If sinA = sinB and cosA = cosB, then $A = n\pi + B$, $n \in I$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

SECTION - IV : MATRIX - MATCH TYPE

21.

	Column I		Column II
(A)	If number of ordered pairs (x, y) satisfying $\|y\| = \cos x$ and $y = \sin^{-1}(\sin x)$, when $\|x\| \le 2\pi$ is m and when $\|x\| \le 3\pi$ is n, then	**(p)**	$m + n = 0$
(B)	If number of solutions of $\ln\|\sin x\| = -x^2 + 2x$, when $x \in [0, \pi]$ is m and when $x \in \left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$ is n, then	**(q)**	$m + n = 2$
		(r)	$m + n = 10$
(C)	If number of solutions of [sin x] = cos x, (where [.] denotes the greatest integer function) when $x \in [0, \pi]$ is m and when $x \in [0, 3\pi]$ is n, then	**(s)**	$n - m = 0$
		(t)	$n - m = 2$

22.

	Column I		Column II
(A)	If α, β are the solutions of $\sin x = -\frac{1}{2}$ in $[0, 2\pi]$ and α, γ are the solutions of $\cos x = -\frac{\sqrt{3}}{2}$ in $[0, 2\pi]$, then	**(p)**	$\alpha - \beta = \pi$
		(q)	$\beta - \gamma = \pi$
(B)	If α, β are the solutions of $\cot x = -\sqrt{3}$ in $[0, 2\pi]$ and α, γ are the solutions of cosec x = –2 in $[0, 2\pi]$, then	**(r)**	$\alpha - \gamma = \pi$
(C)	If α, β are the solutions of $\sin x = -\frac{1}{2}$ in $[0, 2\pi]$ α, γ are the solution of $\tan x = \frac{1}{\sqrt{3}}$ in $[0, 2\pi]$, then	**(s)**	$\alpha + \beta = 3\pi$
		(t)	$\beta + \gamma = 2\pi$

SECTION - V : COMPREHENSION TYPE

23. Read the following comprehension carefully and answer the questions.

Suppose equation is f(x) – g(x) = 0 of f(x) = g(x) = y say, then draw the graphs of y = f(x) and y = g(x). If graph of y = f(x) and y = g(x) cuts at one, two, three, ..., no points, then number of solutions are one, two, three, ..., zero respectively.

1. The number of solutions of $\sin x = \frac{|x|}{10}$ is

(A) 4 **(B)** 6 **(C)** 8 **(D)** none of these

2. Total number of solutions of the equation $3x + 2\tan x = \frac{5\pi}{2}$ in $x \in [0, 2\pi]$ is equal to

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4

3. Total number of solutions of $\sin\{x\} = \cos\{x\}$, where {.} denotes the fractional part, in $[0, 2\pi]$ is equal to

(A) 3 **(B)** 5 **(C)** 7 **(D)** none of these

24. Read the following comprehension carefully and answer the questions.

When ever the terms on the two sides of the equation are of different nature, then equations are known as Non standard form, some of them are in the form of an ordinary equation but can not be solved by standard procedures. Non standard problems require high degree of logic, they also require the use of graphs, inverse properties of functions, in equalities.

1. The number of solutions of the equation $2\cos\left(\frac{x}{2}\right) = 3^x + 3^{-x}$ is

(A) 1 **(B)** 2 **(C)** 3 **(D)** none of these

2. The equation $2\cos^2\left(\frac{x}{2}\right)\sin^2 x = x^2 + x^{-2}, 0 < x \le \frac{\pi}{2}$ has

(A) one real solutions **(B)** more than one real solutions

(C) no real solution **(D)** none of the above

3. The number of real solutions of the equation $\sin(e^x) = 5^x + 5^{-x}$ is

(A) 0 **(B)** 1 **(C)** 2 **(D)** infinitely many

25. Read the following comprehensions carefully and answer the questions.

An equation of the form f(sinx ± cosx, ± sinx cosx) = 0 can be solved by changing variable.

Let sinx ± cosx = t

$\Rightarrow \quad \sin^2 x + \cos^2 x \pm 2\sin x\cos x = t^2$

$\Rightarrow \quad \pm\sin x\cos x = \left(\frac{t^2 - 1}{2}\right).$

Hence, reduce the given equation into

$$f\left(t, \frac{t^2 - 1}{2}\right) = 0$$

1. If $1 - \sin 2x = \cos x - \sin x$, then x is

(A) $2n\pi, 2n\pi - \frac{\pi}{2}, n \in I$ **(B)** $2n\pi, n\pi + \frac{\pi}{4}, n \in I$

(C) $2n\pi - \frac{\pi}{2}, n\pi + \frac{\pi}{4}$ $n \in I$ **(D)** none of these

2. If $\sin x + \cos x = 1 + \sin x \cos x$, then x is

(A) $2n\pi, 2n\pi + \frac{\pi}{2}, n \in I$ **(B)** $2n\pi, n\pi + \frac{\pi}{4}, n \in I$

(C) $2n\pi - \frac{\pi}{2}, n\pi + \frac{\pi}{4}$ $n \in I$ **(D)** none of these

3. If $\sin^4 x + \cos^4 x = \sin x \cos x$, then x is

(A) $n\pi, n \in I$ **(B)** $(6n + 1)\frac{\pi}{6}, n \in I$

(C) $(4n + 1)\frac{\pi}{4}, n \in I$ **(D)** none of these

SECTION - VI : INTEGER TYPE

26. Number of roots of the equation $|\sin x \cos x| + \sqrt{2 + \tan^2 x + \cot^2 x} = \sqrt{3}$, $x \in [0, 4\pi]$.

27. Number of solutions of the equation $\frac{\sin x}{\cos 3x} + \frac{\sin 3x}{\cos 9x} + \frac{\sin 9x}{\cos 27x} = 0$ in the interval $\left(0, \frac{\pi}{4}\right)$.

28. The value of a for which system of equations $\sin^2 x + \cos^2 y = \frac{3a}{2}$ and $\cos^2 x + \sin^2 y = \frac{a^2}{2}$ has a solutions.

29. The maximum integral value of a for which the equation $a \sin x + \sin 2x = 2a - 7$ has a solution.

30. Number of solution of the equation $\sin^4 x - \cos^2 x \sin x + 2 \sin^2 x + \sin x = 0$ in $0 \le x \le 3\pi$.

ANSWER KEY

EXERCISE - 1

1. A **2.** B **3.** A **4.** B **5.** C **6.** A **7.** B **8.** B **9.** C **10.** D **11.** B **12.** C **13.** C
14. B **15.** C **16.** C **17.** B

EXERCISE - 2 : PART # I

1. BD **2.** BC **3.** ABD **4.** BD **5.** ABC **6.** ABC **7.** AC **8.** AB
9. ABCD **10.** AD **11.** ABCD **12.** CD **13.** AB **14.** AB **15.** ABCD **16.** ABC
17. AD **18.** AC

PART - II

1. A **2.** D **3.** B

EXERCISE - 3 : PART # I

1. $A \rightarrow q$ $B \rightarrow p$ $C \rightarrow q$ $D \rightarrow p$ **2.** $A \rightarrow q$ $B \rightarrow s$ $C \rightarrow r$ $D \rightarrow p$ **3.** $A \rightarrow r$ $B \rightarrow p,q,r,s$ $C \rightarrow q$ $D \rightarrow s$

PART - II

Comprehension # 1 : 1. B **2.** C **3.** A **Comprehension # 2 : 1.** D **2.** C **3.** B
Comprehension # 3 : 1. C **2.** D **3.** D

EXERCISE - 5 : PART # I

1. 1 **2.** 4 **3.** 3 **4.** 3 **5.** 2

PART - II

1. C **2.** B **3.** D **4.** A **5.** C **6.** BD **7.** C **8.** 3 **9.** ACD **10.** C **11.** C

MOCK TEST

1. C **2.** C **3.** C **4.** D **5.** D **6.** D **7.** B **8.** B **9.** B **10.** C **11.** AB **12.** C
13. B **14.** BC **15.** BC **16.** D **17.** A **18.** D **19.** A **20.** C
21. $A \rightarrow r,t$ $B \rightarrow s$ $C \rightarrow t$ **22.** $A \rightarrow s,q$ $B \rightarrow p,t$ $C \rightarrow r,s,t$
23. 1. B **2.** A **3.** C **24. 1.** A **2.** C **3.** A **25. 1.** D **2.** A **3.** C
26. 0 **27.** 6 **28.** 1 **29.** 6 **30.** 4