HINTS & SOLUTIONS

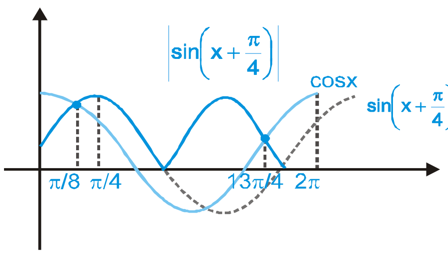
EXERCISE - 1

Single Choice

1. $2\cos x = \sqrt{2 + 2\sin 2x}$

$\Rightarrow \quad \sqrt{2}\cos x = \sqrt{1+\sin 2x} = |\sin x + \cos x|$

$\Rightarrow \quad \cos x = \left|\frac{1}{\sqrt{2}}(\sin x + \cos x)\right|$



$\Rightarrow \quad \cos x = \left|\sin\left(x+\frac{\pi}{4}\right)\right|$

$\Rightarrow$ see from graph or we can put values given in options to verify.

2. $4\sin\theta.\cos\theta - 2\cos\theta - 2\sqrt{3}\sin\theta + \sqrt{3} = 0$

$\Rightarrow 2\cos\theta\,(2\sin\theta - 1) - \sqrt{3}\,(2\sin\theta - 1) = 0$

$\Rightarrow (2\sin\theta - 1)\,(2\cos\theta - \sqrt{3}) = 0$

$\Rightarrow \quad \sin\theta = \frac{1}{2}, \cos\theta = \frac{\sqrt{3}}{2} \quad \Rightarrow \quad \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}$

3. $\tan^2 x - \sec^{10}x + 1 = 0$

$\Rightarrow \quad \sec^2 x - 1 - \sec^{10}x + 1 = 0$

$\Rightarrow \quad \sec^2 x\,(1 - \sec^8 x) = 0 \quad \Rightarrow \quad \sec^8 x = 1$

$\Rightarrow \quad \cos x = \pm 1 \quad \Rightarrow \quad x = \pi, 2\pi, 3\pi$

5. Since $\tan\theta < 0$ and $\cos\theta > 0$, θ lies in the fourth quadrant.
Then $\theta = 7\pi/4$
Hence, the general value of θ is $2n\pi + 7\pi/4$, $n \in Z$

6. For the given relation

$\cos\theta = (2\sin\theta\cos\theta)\sin\theta,\ \sin\theta \neq 0$

or $\quad \sin\theta = \pm\frac{1}{\sqrt{2}}$ or $\cos\theta = 0$

or $\quad \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{2}$

Then the sum of roots is $3\pi/2$

7. $\sin x + 3\sin 2x + \sin 3x = \cos x + 3\cos 2x + \cos 3x$

$\Rightarrow \quad 2\sin 2x\cos x + 3\sin 2x = 2\cos 2x\cos x + 3\cos 2x$

$\Rightarrow \quad \sin 2x\,[2\cos x + 3] = \cos 2x\,[2\cos x + 3]$

$\Rightarrow \quad \tan 2x = 1$

$\Rightarrow \quad 2x = n\pi + \frac{\pi}{4} \Rightarrow x = \frac{n\pi}{2} + \frac{\pi}{8}$

$\therefore \quad x = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{13\pi}{8}, \frac{9\pi}{8}$

8. $\frac{\cos 3\theta}{2\cos 2\theta - 1} = \frac{1}{2}$

$\Rightarrow \quad 2(4\cos^3\theta - 3\cos\theta) = 2(2\cos^2\theta - 1) - 1$

$\Rightarrow \quad 8\cos^3\theta - 4\cos^2\theta - 5\cos\theta\ 3 = 0$

$\Rightarrow \quad (4\cos^2\theta - 3)(2\cos\theta - 1) = 0$

$\Rightarrow \quad \cos\theta = \frac{1}{2}, \pm\frac{\sqrt{3}}{2}$

But when $\cos\theta = \pm\frac{\sqrt{3}}{2}$ then $2\cos 2\theta - 1 = 0$

$\therefore$ rejecting this value,

$\cos\theta = \frac{1}{2}$ is valid only

$\Rightarrow \quad \theta = 2n\pi \pm \frac{\pi}{3}, n \in I$

9. $\cos x + \cos 2x + \cos 3x + \cos 4x + \cos 5x = 0$

$2\cos 3x\cos 2x + 2\cos 3x\cos x + \cos 3x = 0$

$\cos 3x\,[2\cos 2x + 2\cos x - 1] = 0$

$x = (2n - 1)\frac{\pi}{6} \quad \Rightarrow \quad \frac{\pi}{6}, \frac{3\pi}{6}, \frac{5\pi}{6} = 3$

2^{nd} equation gives $\cos x = \frac{1 \pm \sqrt{2}}{4} = 2$

10. We know that

$\tan x \cdot \tan 2x \cdot \tan 3x = \tan 3x - \tan 2x - \tan x$

hence $\quad \tan x + \tan 2x + \tan 3x = \tan 3x - \tan 2x - \tan x$

$\Rightarrow \quad \tan x + \tan 2x = 0$

$\therefore \quad \tan 2x = \tan(-x)$

$2x = n\pi - x$

$x = \frac{n\pi}{3}, n \in I$

11. $\dfrac{1}{\cos x} = \dfrac{1}{1-\cos x}$

$\Rightarrow \quad \cos x = \dfrac{1}{2} = \cos\dfrac{\pi}{3}; \quad x = 2n\pi \pm \dfrac{\pi}{3}$

12. $f(x) = \cos^2 x + \cos^2 2x + \cos^2 3x$

$= 1 + \cos^2 x + \cos^2 2x - \sin^2 3x = 1 + \cos^2 x + \cos 5x \cdot \cos x$

$= 1 + \cos x[\cos x + \cos 5x] = 1 + 2\cos x \cdot \cos 2x \cdot \cos 3x$

$\Rightarrow \quad \cos x \cdot \cos 2x \cdot \cos 3x = 0$

now $f(x) = 1$ if $\cos x = 0$ or $\cos 2x = 0$

or $\cos 3x = 0$

$x = (2n-1)\dfrac{\pi}{2}$ or $x = (2n-1)\dfrac{\pi}{4}$

or $x = (2n-1)\dfrac{\pi}{6}$

$\Rightarrow \quad x \in \left\{\dfrac{\pi}{2}, \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{\pi}{6}, \dfrac{5\pi}{6}\right\}$

$\Rightarrow$ number of values of $x = 5$

13. $f(x) = \dfrac{(\operatorname{cosec}^2 x - 1)^2}{(\operatorname{cosec}^2 x - 1) + 1 - \cot x + \cot x}$,

defined for $R - n\pi, n \in I$

$f(x) = \dfrac{(\cot x)^4}{1 + \cot^2 x}$

$f(x) = 0 \quad \Rightarrow \quad \cot x = 0 \quad \Rightarrow \quad x = (2n-1)\dfrac{\pi}{2}$

$\therefore \quad x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}, \dfrac{5\pi}{2}, \ldots\ldots\ldots, \dfrac{199\pi}{2}$

$\text{sum} = \dfrac{\pi}{2}[1 + 3 + 5 + \ldots\ldots\ldots + 199]$

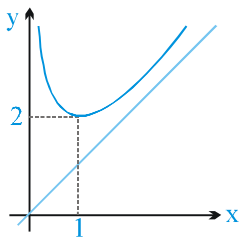
(100 solutions)

$= \dfrac{\pi}{2} \cdot \dfrac{100}{2} \cdot 200 = 5000\pi$

14. $\cos x \cdot \sin\left(x + \dfrac{1}{x}\right) = 0$

$\cos x = 0 \quad \Rightarrow \quad x = \pi/2$

$\sin\left(x + \dfrac{1}{x}\right) = 0 \quad \Rightarrow \quad x + \dfrac{1}{x} = n\pi, \; n \in I$



if $x \in (0, 1)$ then $x + \dfrac{1}{x} \in (2, \infty)$ for $x > 0$

Hence there are infinite solution

15. $2\sin x + 7\cos px = 9$ is possible only if $\sin x = 1$ and $\cos px = 1$

$x = (4n+1)\dfrac{\pi}{2}$ and $px = 2m\pi$

$\Rightarrow \quad x = \dfrac{2m\pi}{p} \quad (m, n \in I)$

$\therefore \quad (4n+1)\dfrac{\pi}{2} = \dfrac{2m\pi}{p}$

$\Rightarrow \quad p = \dfrac{4m}{4n+1}$

$\therefore \quad p \in$ rational

16. $2^{2+4\sin^2 x} = 2^{6\sin x}$

$\Rightarrow \quad 2 + 4\sin^2 x = 6\sin x$

$\Rightarrow \quad 2\sin^2 x - 3\sin x + 1 = 0$

$\Rightarrow \quad (2\sin x - 1)(\sin x - 1) = 0$

$\therefore \quad \sin x = \dfrac{1}{2}$ or $\sin x = 1$

$\therefore \quad x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{\pi}{2}$

17. $\sin^2 x + a\cos x + a^2 > 1 + \cos x$

Putting $x = 0$, we get

or $\quad a + a^2 > 2$

or $\quad a^2 + a - 2 > 0$

or $\quad (a+2)(a-1) > 0$

or $\quad a < -2$ or $a > 1$

Therefore, we have the largest negative integral value of $a = -3$.

EXERCISE - 2

Part # I : Multiple Choice

1. $\sin^6x + \cos^6x = a^2$

$\Rightarrow (\sin^2 x + \cos^2 x)(\sin^4 x + \cos^4 x - \sin^2 x \cos^2 x) = a^2$

$\Rightarrow (\sin^2 x + \cos^2 x)^2 - 3\sin^2 x \cos^2 x = a^2$

$\Rightarrow 1 - 3\sin^2 x \cos^2 x = a^2$

$\Rightarrow 1 - \frac{3}{4}\sin^2 2x = a^2 \quad \Rightarrow \frac{4(1-a^2)}{3} = \sin^2 2x$

$\Rightarrow 0 \le \frac{4}{3}(1-a^2) \le 1$

$1 - a^2 \ge 0$ and $4 - 4a^2 \le 3$

$a^2 \le 1$ and $\frac{1}{4} \le a^2$

$-1 \le a \le 1$ and $a \ge \frac{1}{2}$ or $a \le -\frac{1}{2}$

$a \in \left[-1, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, 1\right]$

2. $\sin^2 x - \cos 2x = 2 - \sin 2x$

$\Rightarrow \sin^2 x - (1 - 2\sin^2 x) = 2 - 2\sin x \cos x$

$\Rightarrow 3\sin^2 x + 2\sin x \cos x = 3$

Case-I : $\cos x \ne 0$

$\therefore 3\tan^2 x + 2\tan x = 3(1 + \tan^2 x) \Rightarrow \tan x = \frac{3}{2}$

Case-II : $\cos x = 0$

$\therefore 3(1) + 2(\pm 1)(0) = 3$ which is true $\therefore x = (2n+1)\frac{\pi}{2}$

4. $\sin^4x - \cos^2x \sin x + 2\sin^2x + \sin x = 0$

or $\sin x\,[\sin^3x - \cos^2x + 2\sin x + 1] = 0$

or $\sin x[\sin^3 x - 1 + \sin^2x + 2\sin x + 1] = 0$

or $\sin x[\sin^3 x + \sin^2x + 2\sin x] = 0$

or $\sin^2x = 0$ or $\sin^2x + \sin x + 2 = 0$ (not possible for real x)

or $\sin x = 0$

Hence, the solutions are $x = 0, \pi, 2\pi, 3\pi$.

6. $\cos^2x + \cos^2 2x + \cos^2 3x = 1$

$\Rightarrow \cos^2x - \sin^2 2x + \cos^2 3x = 0$

$\Rightarrow \cos x \cos 3x + \cos^2 3x = 0$

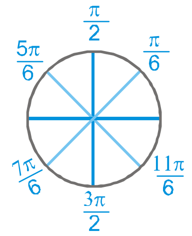
$\Rightarrow \cos 3x(\cos x + \cos 3x) = 0$

$\Rightarrow \cos 3x \cos 2x \cos x = 0$

$\Rightarrow x = (2p+1)\frac{\pi}{6}$

$x = (2q+1)\frac{\pi}{4}$,

$x = (2r+1)\frac{\pi}{2}, n \in I$



$\Rightarrow x = n\pi \pm \frac{\pi}{6}$ also satisfy the equation.

7. $y + \frac{1}{y} \ge 2 \quad \Rightarrow \sqrt{y + \frac{1}{y}} \ge 2$

But $\sin x + \cos x \le \sqrt{2}$

$\Rightarrow y + \frac{1}{y} = 2$ and $\sin x + \cos x = \sqrt{2}$

$y = 1$ and $\sin\left(x + \frac{\pi}{4}\right) = 1$ or $y = 1$ and $x = \frac{\pi}{4}$

8. $3 - 2\cos\theta - 4\sin\theta - \cos 2\theta + \sin 2\theta = 0$

$\sin 2\theta - 2\cos\theta + 3 - 4\sin\theta - (1 - 2\sin^2\theta) = 0$

$2\cos\theta(\sin\theta - 1) + 2\sin^2\theta - 4\sin\theta + 2 = 0$

$\cos\theta(\sin\theta - 1) + (\sin\theta - 1)^2 = 0$

$(\sin\theta - 1)(\cos\theta + \sin\theta - 1) = 0$

$\sin\theta = 1$ or $\cos\left(\theta - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$\theta = 2n\pi + \frac{\pi}{2}$ or $\theta - \frac{\pi}{4} = 2m\pi + \frac{\pi}{4}, 2m\pi - \frac{\pi}{4}$

$\theta = 2m\pi + \frac{\pi}{2}, 2m\pi$

9. $3\theta = n\pi + (-1)^n(3\alpha)$

$\therefore 3\theta = 3\alpha;\ 3\theta = \pi - 3\alpha;\ 3\theta = -\pi - 3\alpha$

or $3\theta = 2\pi + 3\alpha;\ 3\theta = -2\pi + 3\alpha$

Hence $\theta = \alpha;\quad \theta = \frac{\pi}{3} - \alpha;\ \theta = -\left(\frac{\pi}{3} + \alpha\right);$

$\theta = \left(\frac{2\pi}{3} + \alpha\right);\quad \theta = -\frac{2\pi}{3} + 3\alpha$

$\Rightarrow \cos\theta = \cos\left(\frac{\pi}{3} \pm \alpha\right)$ or $\cos\theta = \cos\left(\frac{2\pi}{3} \pm \alpha\right)$

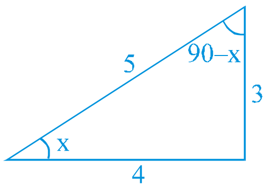
$\Rightarrow$ **(A), (B), (C) and (D)** all are correct

10. Squaring and adding the given equations,
$4 + 9 + 12\sin(x + y) = 25$

$\Rightarrow \sin(x + y) = 1 = \sin\frac{\pi}{2}$

$\therefore x + y = 2n\pi + \frac{\pi}{2} = (4n + 1)\frac{\pi}{2} \quad n \in I \Rightarrow$ **(A)**

if $x + y = \frac{\pi}{2} \Rightarrow y = \frac{\pi}{2} - x$



$5\sin x = 3 \Rightarrow \sin x = \frac{3}{5}$ or $\cos x = \frac{4}{5}$

also $\cos y = \frac{3}{5}$ and $\sin y = \frac{4}{5}$; hence $y > x \Rightarrow$ **(D)**

11. only $x = \frac{\pi}{4}$ and $\frac{11\pi}{12}$ in $(0, \pi)$ satisfies

13. $2\cos^3 3\theta + 3\cos 3\theta + 4 = 3\sin^2 3\theta$

$\Rightarrow \left(\cos 3\theta + \frac{1}{2}\right)\left(2\cos^2 3\theta + 2\cos 3\theta + 2\right) = 0$

$\Rightarrow \cos 3\theta = -\frac{1}{2} \qquad \Rightarrow 3\theta = 2n\pi \pm \frac{2\pi}{3}$

$\Rightarrow \theta = \frac{2n\pi}{3} \pm \frac{2\pi}{9}$

14. $\sin\alpha - \cos\alpha \cdot \tan\beta = \tan(\alpha - \beta)$

$\frac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\cos\beta} = \frac{\sin(\alpha - \beta)}{\cos(\alpha - \beta)}$

$\sin(\alpha - \beta)\cos(\alpha - \beta) = \sin(\alpha - \beta)\cos\beta$

$\sin(\alpha - \beta)[\cos(\alpha - \beta) - \cos\beta] = 0$

$\therefore \sin(\alpha - \beta) = 0$ or $\cos(\alpha - \beta) = \cos\beta$

$\therefore \alpha - \beta = n\pi \qquad \alpha - \beta = 2m\pi \pm \beta$

$\therefore \alpha = n\pi + \beta \qquad (+)\ \alpha = 2m\pi + 2\beta$

$(-)\ \alpha = 2m\pi$

$\therefore$ A, B, C are correct

15. $\cos 3\theta = \cos 3\alpha \qquad$ put $n = 0, 1$

$3\theta = 2n\pi \pm 3\alpha$

$\therefore 3\theta = 3\alpha$ or -3α or $2\pi + 3\alpha$ or $2\pi - 3\alpha$

$\theta = \alpha$ or $-\alpha$ or $\frac{2\pi}{3} + \alpha$ or $\frac{2\pi}{3} - \alpha$

$\Rightarrow$ **(A)**, **(C)**, **(D)** are correct

if $n = -1$

$3\theta = -2\pi \pm 3\alpha$

$\theta = -\frac{2\pi}{3} \pm \alpha$

$\sin\theta = \sin\left(-\frac{2\pi}{3} \pm \alpha\right) = -\sin\left(\frac{2\pi}{3} \pm \alpha\right)$

$= -\sin\left(\pi - \frac{\pi}{3} \pm \alpha\right) = -\sin\left(\pi - \left(\frac{\pi}{3} \pm \alpha\right)\right)$

$= -\sin\left(\frac{\pi}{3} \pm \alpha\right)$

hence **(B)** is not correct.

16. $\cos 4x\cos 8x - \cos 5x\cos 9x = 0$

$\Rightarrow 2\cos 4x\cos 8x = 2\cos 5x\cos 9x$

$\Rightarrow \cos 12x + \cos 4x = \cos 14x + \cos 4x$

$\Rightarrow 14x = 2n\pi \pm (12x)$

$\Rightarrow 2x = 2n\pi$ or $26x = 2n\pi$

$\Rightarrow x = n\pi$ or $\frac{n\pi}{13}$

$\therefore \sin x = 0$ or $\sin 13x = 0$

17. Let $E = \sin x - \cos^2 x - 1$

$\Rightarrow E = \sin x - 1 + \sin^2 x - 1 = \sin^2 x + \sin x - 2$

$= \left(\sin x + \frac{1}{2}\right)^2 - \frac{9}{4}$ assumes least value

when $\sin x = -\frac{1}{2} \Rightarrow x = n\pi + (-1)^n\left(-\frac{\pi}{6}\right)$.

18. $\cos x.\cos 6x = -1$

$\Rightarrow$ Either $\cos x = 1$ and $\cos 6x - 1$

or $\cos x = -1$ and $\cos 6x = 1$

$\Rightarrow x = 2n\pi$ and $\cos 6x = -1$

or $x = (2n + 1)\pi$ and $\cos 6x = 1$

If $x = 2n\pi$ then $\cos 6x$ cannot be -1

However if $x = (2n + 1)\pi$ then $\cos 6x = 1$

$\therefore x = (2n + 1)\pi$

$x = (2n - 1)\pi$ is also as above.

Part # II : Assertion & Reason

1. $(\sin x + \cos x)^{1+\sin 2x} = 2$

$\Rightarrow$ $(\sin x + \cos x)^{(\sin x+\cos x)^2} = 2$

Now we know that the maximum value of $\sin x + \cos x$ is $\sqrt{2}$ which occurs at $x = \pi/4$ for $0 \le x \le \pi/4$.

Also, the given equation has roots only if

$\sin x + \cos x = \sqrt{2}$

Hence, there is only one one solution for $0 \le x \le \pi/4$.

Thus, the correct answer is **(A)**.

2. We know that $\sin^2 x \le 1$ and $\cos^2 y \le 1$, then

$\sin^2 x + \cos^2 y \le 2$

Also, $\sec^2 z \ge 1$, then $2 \sec^2 z \ge 2$.

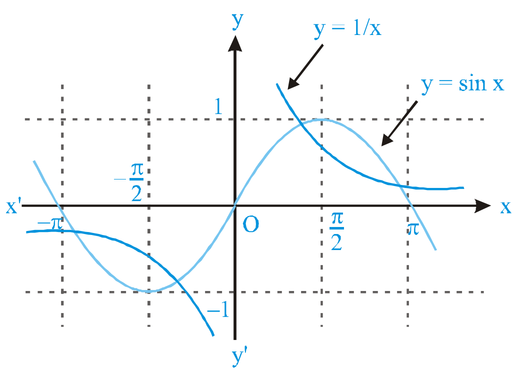
Hence, the given equation is solvable only if $\sin^2 x + \cos^2 y = 2$ and $2\sec^2 z = 2$, for which

$\sin^2 x, \cos^2 y, \sec^2 z = 1$

Then $\sin z, \cos y, \sec z = \pm 1$.

Hence, statement 1 is false and statement 2 is true.

3. Draw the graph of $y = \sin x$ and $y = 1/x$ and verify.



EXERCISE - 3

Part # I : Matrix Match Type

1. **(A)** $|\tan x| = \frac{m}{n} \Rightarrow \tan x = \frac{m}{n}$ & $\tan x = -\frac{m}{n}$

In $[0, 2\pi]$ it has 4 solutions

(B) $\cos x + \cos 2x + \cos 3x + \cos 4x + \cos 5x = 5$

$\Rightarrow \cos x = \cos 2x = \cos 3x = \cos 4x = \cos 5x = 1$

$\Rightarrow x = 2n_1\pi, x = n_2\pi,$

$x = 2n_3\frac{\pi}{3}, x = \frac{n_4\pi}{2}, x = \frac{2n_5\pi}{5}$

$\Rightarrow$ $x = 0, 2\pi$ are common solutions.

(C) $2^{\frac{1}{1-|\cos x|}} = 4$

$\Rightarrow \frac{1}{1-|\cos x|} = 2 \Rightarrow 1 - |\cos x| = \frac{1}{2}$

$\Rightarrow |\cos x| = \frac{1}{2} \Rightarrow \cos x = \pm\frac{1}{2}$

$\therefore$ In $(-\pi, \pi)$ there are 4 solutions

(D) $\tan\theta + \tan 2\theta + \tan 3\theta = \tan\theta \tan 2\theta \tan 3\theta$

$\Rightarrow \theta + 2\theta + 3\theta = n\pi \Rightarrow \theta = n\pi/6$

$\Rightarrow \theta = \frac{\pi}{3}, \frac{2\pi}{3}$ satisfy equation only.

2. **(A)** $\sin^2\theta + 3\cos\theta = 3 \Rightarrow 1 - \cos^2\theta + 3\cos\theta = 3$

$\Rightarrow \cos^2\theta - 3\cos\theta + 2 = 0 \Rightarrow \cos\theta = 1, 2$

$\Rightarrow \cos\theta = 1$ $(\because \cos\theta \ne 2) \Rightarrow \theta = 0$ in $[-\pi, \pi]$

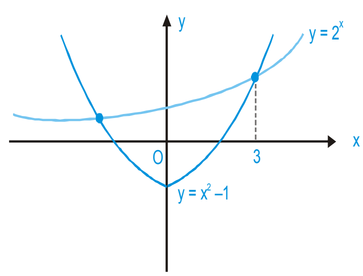
$\therefore$ No. of solution = 1

(B) $\sin x \,.\, \tan 4x = \cos x \Rightarrow \sin x.\frac{\sin 4x}{\cos 4x} = \cos x$

$\Rightarrow \sin 4x \sin x - \cos 4x \cos x = 0 \Rightarrow \cos 5x = 0$

$\Rightarrow 5x = (2n + 1)\,\pi/2 \Rightarrow x = (2n + 1)\,\pi/10$

$\Rightarrow x = \frac{\pi}{10}, \frac{3\pi}{10}, \frac{5\pi}{10}, \frac{7\pi}{10}, \frac{9\pi}{10}$ in $(0, \pi)$



So there are five solutions.

(C) $(1 - \tan^2\theta)\sec^2\theta + 2^{\tan^2\theta} = 0$

$\Rightarrow (1 - \tan^4\theta) + 2^{\tan^2\theta} = 0$

$\Rightarrow (1 - x^2) + 2^x = 0$ where $x = \tan^2\theta$

$\Rightarrow 2^x = x^2 - 1 \quad \Rightarrow x = 3$

$\Rightarrow \tan^2\theta = 3 \quad \Rightarrow \tan\theta = \pm\sqrt{3}$

$\Rightarrow \theta = \pm\frac{\pi}{3}$ in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\therefore$ Number of solutions = 2

(D) $[\sin x] + [\sqrt{2}\cos x] = -3$

$\Rightarrow [\sin x] = -1$ and $[\sqrt{2}\cos x] = -2$

$\Rightarrow \pi < x < 2\pi$ and $-2 \le \sqrt{2}\cos x < -1$

$\Rightarrow -2 \le \cos x < -\frac{1}{\sqrt{2}}$

$\Rightarrow -1 \le \cos x < -\frac{1}{\sqrt{2}}$

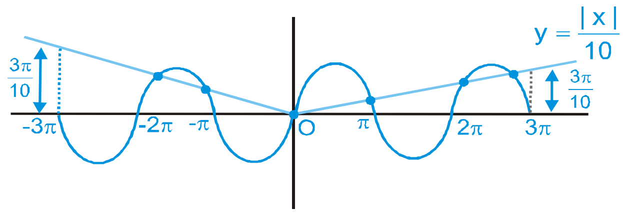
$\Rightarrow \pi \le x < \frac{5\pi}{4}$ for $x \in [0, 2\pi]$

$\pi \le x < \frac{5\pi}{4}, x \in [0, 2\pi]$

$\therefore \pi < x < \frac{5\pi}{4} \quad \Rightarrow 2\pi < 2x < \frac{5\pi}{2}$

$\Rightarrow 0 < \sin 2x < 1 \quad \Rightarrow [\sin 2x] = 0$

3. (A)

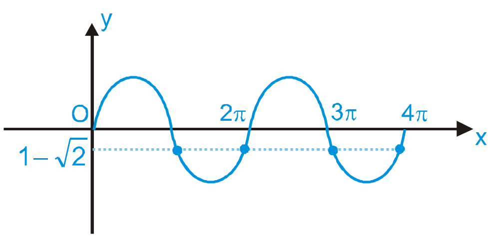


Number of solutions = 6

(B) $\sin x = \frac{2 \pm \sqrt{8}}{2} = 1 \pm \sqrt{2}$

$\Rightarrow \sin x = 1 - \sqrt{2}$

As sin x takes at least four values



in $[0, n\pi]$ $\therefore$ $n \ge 4$

(C) $1 + \sin^4 x = \cos^2 3x$

L.H.S. ≥ 1 and R.H.S. ≤ 1

$\therefore$ L.H.S. = R.H.S. = 1

$\Rightarrow \sin^4 x = 0$ and $\cos^2 3x = 1 \quad \Rightarrow x = n\pi$ and $3x = m\pi$

$\Rightarrow x = n\pi$ and $3x = m\pi \quad \Rightarrow x = n\pi$ and $x = \frac{m\pi}{3}$

$\Rightarrow x = n\pi$

$\Rightarrow x = -2\pi, -\pi, 0, \pi, 2\pi$ in $\left[\frac{-5\pi}{2}, \frac{5\pi}{2}\right]$

$\therefore$ Number of solutions = 5.

(D) A, B, C are in A.P. $\quad \Rightarrow B = 60^\circ$

As $\sin(2A + B) = \frac{1}{2} \quad \Rightarrow 2A + B = 30^\circ$ or 150°

$\Rightarrow 2A = -30^\circ$ or $90^\circ \quad \Rightarrow 2A = 90^\circ$

$\Rightarrow A = 45^\circ$

$\therefore C = 180^\circ - A - B = 75^\circ = \frac{5\pi}{12} \quad \therefore p = 12.$

Part # II : Comprehension

Comprehension 1

1. $x^3 - (1 + \cos\theta + \sin\theta)x^2 + (\cos\theta\sin\theta + \cos\theta + \sin\theta)x - \sin\theta\cos\theta = 0$

Given cubic function is

$f(x) = (x - 1)(x - \cos\theta)(x - \sin\theta)$

Therefore, roots are 1, $\sin\theta$, and $\cos\theta$

Hence, $x_1^2 + x_2^2 + x_3^2 = 1 + \sin^2\theta + \cos^2\theta = 2$

2. Now if $1 = \sin\theta$, we get $\theta = \pi/2$

If $1 = \cos\theta$, then $\theta = 0, 2\pi$

and if $\sin\theta = \cos\theta$, we get $\tan\theta = 1$.

Therefore, $\theta = \frac{\pi}{4}, \frac{5\pi}{4}$

Therefore, the number of values of θ in $[0, 2\pi]$ is 5.

3. Again the maximum possible difference between the two roots is 2.

$1 - \sin\theta = 2$, when $\theta = 3\pi/2$

or $1 - \cos\theta = 2$, when $\theta = \pi$

Comprehension 2

1. $4\sin^3 x + 2\sin^2 x - 2\sin x - 1$

$= (2\sin x + 1)(2\sin^2 x - 1) = 0$

$\therefore \sin x = -\frac{1}{2}, \pm\frac{1}{\sqrt{2}}$

$\therefore$ there are 6 solutions.

2. $3 = \cos 4x + \dfrac{10\tan x}{1+\tan^2 x} = \cos 4x + 5\sin 2x$

i.e $3 = 1 - 2\sin^2 2x + 5\sin 2x$

i.e $\sin 2x = \dfrac{1}{2}$

$\therefore \quad 2x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$

Thus there are two solutions.

3. **(i)** when $\tan x \geq 0$, then the equation becomes

$\tan x = \tan x + \dfrac{1}{\cos x}$ i.e $\dfrac{1}{\cos x} = 0$ (not possible)

(ii) when $\tan x < 0$, then the equation becomes

$-\tan x = \tan x + \dfrac{1}{\cos x}$ i.e $\sin x = -\dfrac{1}{2}$

$\therefore \quad x = \dfrac{11\pi}{6}$ is the only solution.

Comprehension 3

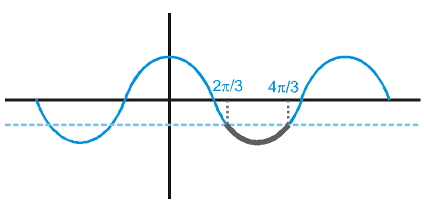
1. $\sin^6 x + \cos^6 x < \dfrac{7}{16} \Rightarrow 1 - 3\sin^2 x\cos^2 x < \dfrac{7}{16}$

$\Rightarrow \sin^2 x\cos^2 x > \dfrac{3}{16} \Rightarrow \sin^2 2x > \dfrac{3}{4}$

$\Rightarrow \dfrac{1-\cos 4x}{2} > \dfrac{3}{4} \Rightarrow 1 - \cos 4x > \dfrac{3}{2}$

$\Rightarrow \cos 4x < -\dfrac{1}{2}$

$\Rightarrow$ Principal is value $4x \in \left(\dfrac{2\pi}{3}, \dfrac{4\pi}{3}\right)$



$\Rightarrow$ General value is $4x \in \left(2n\pi + \dfrac{2\pi}{3}, 2n\pi + \dfrac{4\pi}{3}\right)$

$\Rightarrow \quad x \in \left(\dfrac{n\pi}{2} + \dfrac{\pi}{6}, \dfrac{n\pi}{2} + \dfrac{\pi}{3}\right), n \in I$

2. $\cos 2x + 5\cos x + 3 \geq 0$

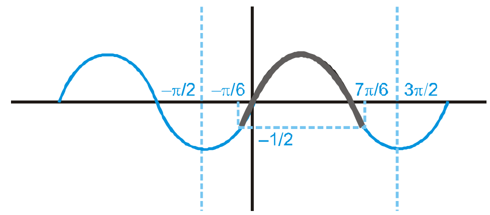
$\Rightarrow \quad 2\cos^2 x + 5\cos x + 2 \geq 0$

$\Rightarrow \quad (\cos x + 2)(2\cos x + 1) \geq 0$

$\Rightarrow \quad 2\cos x + 1 \geq 0 \qquad (\because \cos x + 2 > 0)$

$\Rightarrow \quad \cos x \geq -\dfrac{1}{2} \qquad \Rightarrow \quad x \in \left[-\dfrac{2\pi}{3}, \dfrac{2\pi}{3}\right]$

3. $2\sin^2\left(x + \dfrac{\pi}{4}\right) + \sqrt{3}\cos 2x \geq 0$



$\Rightarrow \quad 1 - \cos\left(2x + \dfrac{\pi}{2}\right) + \sqrt{3}\cos 2x \geq 0$

$\Rightarrow \quad \sqrt{3}\cos 2x + \sin 2x \geq -1$

$\Rightarrow \quad \dfrac{\sqrt{3}}{2}\cos 2x + \dfrac{1}{2}\sin 2x \geq -\dfrac{1}{2}$

$\Rightarrow \quad \sin\left(2x + \dfrac{\pi}{3}\right) \geq -\dfrac{1}{2}$

$\Rightarrow \quad 2x + \dfrac{\pi}{3} \in \left[2n\pi - \dfrac{\pi}{6}, 2n\pi + \dfrac{7\pi}{6}\right]$

$\Rightarrow \quad 2x \in \left[2n\pi - \dfrac{\pi}{2}, 2n\pi + \dfrac{5\pi}{6}\right]$

$\Rightarrow \quad x \in \left[n\pi - \dfrac{\pi}{4}, n\pi + \dfrac{5\pi}{12}\right]$

$\Rightarrow \quad x \in \left[-\pi, \dfrac{-7\pi}{12}\right] \cup \left[-\dfrac{\pi}{4}, \dfrac{5\pi}{12}\right] \cup \left[\dfrac{3\pi}{4}, \pi\right]$

in $[-\pi, \pi]$

EXERCISE - 4

Subjective Type

1. $\sin^4\left(\frac{x}{3}\right) + \cos^4\left(\frac{x}{3}\right) > \frac{1}{2} (n \in Z)$

or $1 - 2\sin^2\left(\frac{x}{3}\right)\cos^2\left(\frac{x}{3}\right) > \frac{1}{2}$

or $1 - \frac{1}{2}\sin^2\left(\frac{2x}{3}\right) > \frac{1}{2}$

or $\sin^2\left(\frac{2x}{3}\right) < 1$

which is always true except when $\sin^2(2x/3) = 1$.
This means $2x/3 = n\pi \pm (\pi/2)$
or $x = (3n\pi/2) \pm (3\pi/4), n \in Z$
Hence, solution set of the inequality is
$R - \{x : x = (3n\pi/2) \pm (3\pi/4), n \in Z\}$.

2. $\sin x + \sin y = \sin (x + y)$

or $2\sin\frac{x+y}{2}\left[\cos\frac{x-y}{2} - \cos\frac{x+y}{2}\right] = 0$

or $4\sin\frac{x+y}{2}\sin\frac{x}{2}\sin\frac{y}{2} = 0$

(a) $\sin\frac{x+y}{2} = 0 \Rightarrow x + y = 2n\pi, n \in Z$

$\Rightarrow x + y = 0$

($\because |x| + |y| = 1 \Rightarrow -1 \le x, y \le 1$)

(b) $\sin\frac{x}{2} = 0 \Rightarrow x = 2m\pi, m \in Z$

$\Rightarrow x = 0$

(c) $\sin\frac{y}{2} = 0 \Rightarrow y = 2p\pi, p \in Z$

From $|x| + |y| = 1$
If $x = 0$, then $|y| = 1 \Rightarrow y = \pm 1$
If $y = 0$, then $|x| = 1 \Rightarrow x = \pm 1$
If $y = -x$, then $|x| + |-x| = 2$
$\Rightarrow$ Hence, solutions are

$(0, 1), (0, -1), (1, 0), (-1, 0), \left(\frac{1}{2}, -\frac{1}{2}\right)$, and $\left(-\frac{1}{2}, \frac{1}{2}\right)$.

3. $\tan\left(\frac{\pi}{2}\cos\theta\right) = \cot\left(\frac{\pi}{2}\sin\theta\right) = \tan\left(\frac{\pi}{2} - \frac{\pi}{2}\sin\theta\right)$

$\Rightarrow \frac{\pi}{2}\cos\theta = n\pi + \frac{\pi}{2} - \frac{\pi}{2}\sin\theta, n \in Z$

or $\frac{\pi}{2}(\sin\theta + \cos\theta) = n\pi + \frac{\pi}{2} = \left(n + \frac{1}{2}\right)\pi$

or $\sin\theta + \cos\theta = (2n + 1)$

$\Rightarrow \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right) = (2n + 1)$

Hence, n = 0, –1 are the only possibilities.

So, $\sin\left(\theta + \frac{\pi}{4}\right) = \pm\frac{1}{\sqrt{2}} = \sin\left(\pm\frac{\pi}{4}\right)$

or $\theta + \frac{\pi}{4} = m\frac{\pi}{2} + \frac{\pi}{4}, m \in Z$

or $\theta = m\frac{\pi}{2}, m \in Z$

However, for the values of $m = 2k, k \in Z$, the equation is not defined.

Hence, $\theta = (2k + 1)\frac{\pi}{2}$, where $k \in Z$

4. $\sin^2 x + \frac{1}{4}\sin^2 3x = \sin x \sin^2 3x$

or $\sin^2 x - \sin x \sin^2 3x + \frac{1}{4}\sin^2 3x = 0$

or $\left(\sin x - \frac{1}{2}\sin^2 3x\right)^2 + \frac{1}{4}\sin^2 3x(1 - \sin^2 3x)$

or $\left(\sin x - \frac{1}{2}\sin^2 3x\right)^2 + \frac{1}{4}\sin^2 3x\cos^2 3x = 0$

or $\left(\sin x - \frac{1}{2}\sin^2 3x\right)^2 + \frac{1}{16}\sin^2 6x = 0$

or $\sin x - \frac{1}{2}\sin^2 3x = 0$ and $\sin 6x = 0$

or $2\sin x = \sin^2 3x$ and $\sin 6x = 0$
From $\sin 6x = 0$, $x = k\pi/6, k \in Z$
From here, we choose those values which satisfy the equation $2\sin x = \sin^2 3x$ .Now,

$$\sin^2 3\left(\frac{k\pi}{3}\right) = + \sin^2\frac{k\pi}{2} = \begin{cases} 1, \text{ if } k \text{ is odd} \\ 0, \text{ if } k \text{ is even} \end{cases}$$

$\Rightarrow \sin x = 0$ or $\frac{1}{2}$

$x = n\pi$ or $x = n\pi + \frac{\pi}{6}(-1)^n, n \in Z$

5. $\sin^4 x + \cos^4 x - 2\sin^2 x + \frac{3}{4}\sin^2 2x = 0$

or $(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x - 2\sin^2 x$

$+\frac{3}{4}.4\sin^2 x \cdot \cos^2 x = 0$

or $1 - 2\sin^2 x + \sin^2 x \cdot \cos^2 x = 0$

or $\sin^4 x + \sin^2 x - 1 = 0$

or $\sin^2 x = \frac{\sqrt{5}-1}{2}$

$\therefore$ $\cos 2x = 2 - \sqrt{5}$

$\Rightarrow$ $x = n\pi \pm \frac{1}{2}\cos^{-1}(2 - \sqrt{5}),\ n \in Z$

6. Given $\sin^3 x \cos 3x + \cos^3 x \sin 3x + \frac{3}{8} = 0$

$\Rightarrow$ $\sin^3 x(4\cos^3 x - 3\cos x)$

$+ \cos^3 x\,(3\sin x - 4\sin^3 x) + \frac{3}{8} = 0$

or $3\sin x \cos x(\cos^2 x - \sin^2 x) + \frac{3}{8} = 0$

or $8(\sin x \cos x)\cos 2x + 1 = 0$

or $2\sin 4x = -1$

or $\sin 4x = -\frac{1}{2}$

$\therefore$ $x = \frac{n\pi}{4} + (-1)^{n+1}\frac{\pi}{6}$; $n \in Z$

7. $\sin^{10} x + \cos^{10} x = \frac{29}{16}\cos^4 2x$

$\Rightarrow$ $\left(\frac{1-\cos 2x}{2}\right)^5 + \left(\frac{1+\cos 2x}{2}\right)^5 = \frac{29}{16}\cos^4 2x$

Let $\cos 2x = t$. Then,

$\left(\frac{1-t}{2}\right)^5 + \left(\frac{1+t}{2}\right)^5 = \frac{29}{16}t^4$

or $24t^4 - 10t^2 - 1 = 0$

or $(2t^2 - 1)(12t^2 + 1) = 0$

or $t^2 = \frac{1}{2}$

or $\cos^2 2x = \frac{1}{2} = \left(\frac{1}{\sqrt{2}}\right)^2 = \left(\cos\frac{\pi}{4}\right)^2$

or $2x = n\pi \pm \frac{\pi}{4}$

or $x = \frac{n\pi}{2} \pm \frac{\pi}{8},\ n \in Z$

8. $\sqrt{13 - 18\tan x} = 6\tan x - 3$ ......**(i)**

$\Rightarrow$ $13 - 18\tan x = 36\tan^2 x + 9 - 36\tan x$

$\Rightarrow$ $\tan x = \frac{2}{3},\ -\frac{1}{6}$

Put in (1) $\Rightarrow$ $\tan x = \frac{2}{3}$ is correct

$\Rightarrow$ $x = n\pi + \tan^{-1}\frac{2}{3}$

$= n\pi + \alpha = \alpha,\ \pi + \alpha,\ -\pi + \alpha,\ -2\pi + \alpha$ in $(-2\pi, 2\pi)$

9. $\left(\sin 2\theta + \sqrt{3}\cos 2\theta\right)^2 - 5 = \cos\left(\frac{\pi}{6} - 2\theta\right)$

$\Rightarrow$ $4\left(\frac{1}{2}\sin 2\theta + \frac{\sqrt{3}}{2}\cos 2\theta\right)^2 - \cos\left(\frac{\pi}{6} - 2\theta\right) - 5 = 0$

$\Rightarrow$ $4\cos^2\left(\frac{\pi}{6} - 2\theta\right) - \cos\left(\frac{\pi}{6} - 2\theta\right) - 5 = 0$

$\Rightarrow$ $\cos\left(\frac{\pi}{6} - 2\theta\right) = \frac{5}{4}, -1$

$\Rightarrow$ $\cos\left(\frac{\pi}{6} - 2\theta\right) = -1 = \cos\pi$

$\Rightarrow$ $\frac{\pi}{6} - 2\theta = 2n\pi \pm \pi$ $\Rightarrow$ $2\theta = \frac{\pi}{6} - 2n\pi \mp \pi$

$\Rightarrow$ $\theta = \frac{2n\pi}{2} + \frac{\pi}{12} \pm \frac{\pi}{2}$ $\Rightarrow$ $\theta = \frac{7\pi}{12}, \frac{19\pi}{12}$.

10. $1 + 2\operatorname{cosec} x = \frac{-\sec^2\left(\frac{x}{2}\right)}{2}$

$\Rightarrow$ $1 + \frac{2}{\sin x} = \frac{-1}{1+\cos x}$

$\Rightarrow$ $(2 + \sin x)(1 + \cos x) = -\sin x$

$\Rightarrow$ $2 + 2\cos x + \sin x + \sin x\cos x = -\sin x$

$\Rightarrow$ $2(\sin x + \cos x) + \sin x\cos x + 2 = 0$

Put $\sin x + \cos x = t$

$\Rightarrow$ $1 + 2\sin x\cos x = t^2$

$\therefore$ $2t + \frac{t^2 - 1}{2} + 2 = 0$

$\Rightarrow$ $t^2 + 4t + 3 = 0$

$\Rightarrow$ $t = -1, -3$

$\Rightarrow$ $\sin x + \cos x = -1$ $\Rightarrow \cos\left(x - \frac{\pi}{4}\right) = \frac{-1}{\sqrt{2}} = \cos\frac{3\pi}{4}$

$\Rightarrow$ $x - \frac{\pi}{4} = 2n\pi \pm \frac{3\pi}{4} \Rightarrow x = 2n\pi + \pi, 2n\pi - \frac{\pi}{2}$

$\Rightarrow$ $x = 2n\pi + \pi$ at which cosec x is not defined

$\therefore$ $x = 2n\pi - \frac{\pi}{2}$.

11. $\sin^2 4x + \cos^2 x = 2 \sin 4x \cdot \cos^4 x$

$\Rightarrow$ $\sin^2 4x - 2\sin 4x\cos^4 x + \cos^2 x = 0$

$\Rightarrow$ $(\sin 4x - \cos^4 x)^2 + \cos^2 x - \cos^8 x = 0$

$\Rightarrow$ $(\sin 4x - \cos^4 x)^2 + \cos^2 x\,(1 - \cos^6 x) = 0$

$\Rightarrow$ $\sin 4x - \cos^4 x = 0$ ......**(i)**

and $\cos^2 x\,(1 - \cos^6 x) = 0$ ......**(ii)**

From (2) $\cos^2 x = 0, 1$

Case-I $\cos^2 x = 0$ $\Rightarrow$ $x = n\pi \pm \frac{\pi}{2}$

$\Rightarrow$ $4x = 4n\pi \pm 2\pi$

$\therefore$ $\sin 4x = 0$

$\Rightarrow$ equation (1) is also true

Case-II $\cos^2 x = 1$ $\Rightarrow$ $\sin^2 x = 0$

$\Rightarrow$ $x = n\pi$ $\therefore$ equation (1) becomes

$0 - 1 = 0$ false $\therefore$ solution is $x = n\pi \pm \frac{\pi}{2}$

EXERCISE - 5

Part # I : AIEEE/JEE-MAIN

1. Clearly, given equation is defined for $x \neq \pi/2, 3\pi/2$.

Now, $\tan x + \sec x = 2\cos x$

$\Rightarrow$ $1 + \sin x = 2\cos^2 x$

$\Rightarrow$ $2\sin^2 x + \sin x - 1 = 0$

$\Rightarrow$ $(2\sin x - 1)(\sin x + 1) = 0$

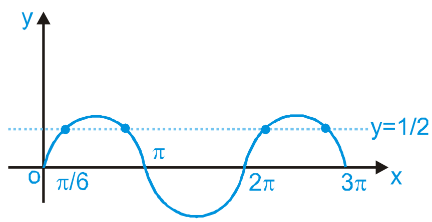
$\Rightarrow$ $\sin x = \frac{1}{2}, \sin x = -1$

$\Rightarrow$ $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$ $\Rightarrow$ $x = \frac{\pi}{6}, \frac{5\pi}{6}$

2. Given equation is $2\sin^2 x + 5\sin x - 3 = 0$

$\Rightarrow$ $(2\sin x - 1)(\sin x + 3) = 0$

$\Rightarrow \sin x = \frac{1}{2}$ ($\because$ $\sin x \neq -3$)



It is clear from figure that the curve intersect the line at four points in the given interval.

Hence, number of solutions are 4.

3. We have,

$\sin\theta = \sin 4\theta + \sin 7\theta = 0$

$\Rightarrow$ $(\sin\theta + \sin 7\theta) + \sin 4\theta = 0$

$\Rightarrow$ $2\sin 4\theta \cos 3\theta + \sin 4\theta = 0$

$\Rightarrow$ $\sin 4\theta = 0$ or $\cos 3\theta = -\frac{1}{2}$

$\Rightarrow$ $4\theta = \pi, 2\pi, 3\pi$ or $3\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}$

$\Rightarrow$ $\theta = \frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{4\pi}{9}, \frac{3\pi}{4}, \frac{8\pi}{9}$

4. $\cos^2\left(x+\frac{\pi}{6}\right) + \cos^2 x - 2\cos\left(x+\frac{\pi}{6}\right)\cos\frac{\pi}{6} = \sin^2\frac{\pi}{6}$

$\Rightarrow \cos^2\left(x+\frac{\pi}{6}\right) + \cos^2 x - \sin^2\frac{\pi}{6} - 2\cos\left(x-\frac{\pi}{6}\right)\cos\frac{\pi}{6} = 0$

$\Rightarrow \cos^2\left(x+\frac{\pi}{6}\right) + \cos\left(x+\frac{\pi}{6}\right)\cos\left(x-\frac{\pi}{6}\right)$

$\Rightarrow -2\cos\left(x+\frac{\pi}{6}\right)\cos\frac{\pi}{6} = 0$

$\Rightarrow \cos\left(x+\frac{\pi}{6}\right)\left\{\cos\left(x+\frac{\pi}{6}\right)+\cos\left(x-\frac{\pi}{6}\right)-2\cos\frac{\pi}{6}\right\} = 0$

$\Rightarrow \cos\left(x+\frac{\pi}{6}\right)\left\{2\cos x\cos\frac{\pi}{6}-2\cos\frac{\pi}{6}\right\} = 0$

$\Rightarrow 2\cos\left(x+\frac{\pi}{6}\right)\cos\frac{\pi}{6}(\cos x - 1) = 0$

$\Rightarrow \cos\left(x+\frac{\pi}{6}\right)(\cos x - 1) = 0$

$\Rightarrow x + \frac{\pi}{6} = \pm\frac{\pi}{2}$ or, $x = 0$

$\Rightarrow x = \frac{\pi}{3}, -\frac{2\pi}{3}, 0$

$\Rightarrow x = 0, \frac{\pi}{3}$ $\left[\because x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$

5. $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$

$2\cos\frac{5x}{2} \cdot \cos\frac{3c}{2} + 2\cos\frac{5x}{2} \cdot \cos\frac{x}{2} = 0$

$2\cos\frac{5x}{2} \times 2\cos x \cdot \cos\frac{x}{2} = 0$

$x = \frac{(2n+1)\pi}{5}, \frac{(2k+1)\pi}{2}, (2r+1)\pi,$

where $n, k \in \mathbb{Z}$ or $0 \le x \le 2\pi$

Hence $x = \frac{\pi}{5}, \frac{3\pi}{5}, \pi, \frac{2\pi}{5}, \frac{9\pi}{5}, \frac{\pi}{2}, \frac{3\pi}{2}$

Part # II : IIT-JEE ADVANCED

1. To simplify the determinant, let sin x = a ; cos x = b. Then the equation becomes

$$\begin{vmatrix} a & b & b \\ b & a & b \\ b & b & a \end{vmatrix} = 0$$

Operating $C_2 \to C_2 - C_1$; $C_3 \to C_3 - C_2$, we get

$$\begin{vmatrix} a & b-a & 0 \\ b & a-b & b-a \\ b & 0 & a-b \end{vmatrix} = 0$$

or $a(a-b)^2 - (b-a)[b(a-b) - b(b-a)] = 0$

or $a(a-b)^2 - 2b(b-a)(a-b) = 0$

or $(a-b)^2(a-2b) = 0$

or $a = b$ or $a = 2b$

or $\frac{a}{b} = 1$ or $\frac{a}{b} = 2$

$\Rightarrow$ tan x = 1 or tan x = 2

But we have $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$

$\Rightarrow \tan\left(\frac{\pi}{4}\right) \le \tan x \le \tan\left(\frac{\pi}{4}\right)$

$\Rightarrow -1 \le \tan x \le 1$

$\therefore$ tanx = 1 $\Rightarrow$ $x = \pi/4$

Therefore, there is only one real root.

2. We know that

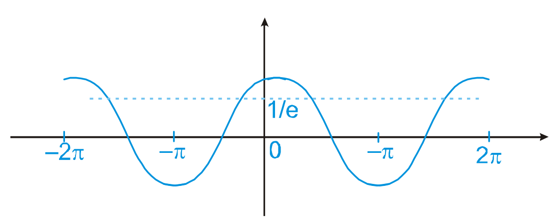
$$-\sqrt{a^2+b^2} \le a\cos\theta + b\sin\theta \le \sqrt{a^2+b^2}$$

$\Rightarrow -\sqrt{74} \le 2k+1 \le \sqrt{74}$

$\Rightarrow -8 \le 2k+1 \le 8$ $\Rightarrow -4.5 \le k \le 3.5$

Considering only integral values, which means k can take eight integral values.

3.



$\alpha - \beta = 0, -2\pi$ or 2π

$\alpha - \beta = 0 \Rightarrow \alpha = \beta \Rightarrow \cos 2\beta = \frac{1}{e}$

This is true for '4' value of 'α', 'β'

If $\alpha - \beta = -2\pi$

$\Rightarrow \alpha = -\pi$ and $\beta = \pi$ and $\cos(\alpha+\beta) = 1$

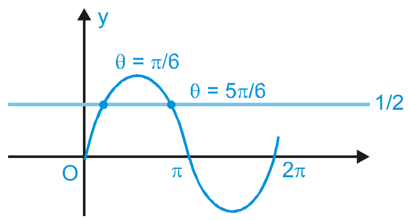
$\Rightarrow$ (No solution)

similarly if $\alpha - \beta = 2\pi$

$\Rightarrow \alpha = \pi$ and $\beta = -\pi$ again no solution results

4. $2\sin^2\theta - 5\sin\theta + 2 > 0$

$\Rightarrow (\sin\theta - 2)(2\sin\theta - 1) > 0$



$\Rightarrow \sin\theta < \frac{1}{2}$ $[\because -1 \le \sin\theta \le 1]$

From graph, we get $\theta \in \left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$

6. We have,

$2\sin^2\theta + \sin^2 2\theta = 2$

$\Rightarrow 1 - \cos2\theta + 1 - \cos^2 2\theta = 2$

$\Rightarrow \cos2\theta(1 + \cos2\theta) = 0 \Rightarrow \cos2\theta = 0$ or $\cos2\theta = -1$

$\Rightarrow 2q = \frac{\pi}{2}$ or $\cos2\pi = \pi \Rightarrow \theta = \frac{\pi}{4}, \frac{\pi}{2}$

7. We have

$(y + z)\cos3\theta = xyz\sin3\theta$ ....(i)

$x\sin3\theta = \frac{2\cos3\theta}{y} + \frac{2\sin3\theta}{z}$ ....(ii)

$xyz\sin3\theta = (y + 2z)\cos3\theta + y\sin3\theta$ ....(iii)

From (i) and (ii), we get

$y(\cos3\theta - 2\sin3\theta) - z\cos3\theta = 0$

From (ii) and (iii), we get

$y(\cos3\theta - \sin3\theta) = 0$

Since the given system of equations have a solution (x_0, y_0, z_0) such that $y_0, z_0 \neq 0$

$\therefore \cos3\theta - \sin3\theta = 0 \Rightarrow \tan3\theta = 1$

$\Rightarrow 3\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4} \Rightarrow \theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}$

Hence, that are three values of θ.

5. $2\sin^2\theta - \cos2\theta = 0$ ....(i)

$\Rightarrow \sin\theta = \pm\frac{1}{2} \Rightarrow 2\cos^2\theta - 3\sin\theta = 0$ ....(ii)

$-2\sin^2\theta - 3\sin\theta + 2 = 0 \Rightarrow \sin\theta = \frac{1}{2}, -2$

So $\sin\theta = \frac{1}{2}$ is the only solution at $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

8. $\tan\theta = \cot5\theta$

$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{\cos5\theta}{\sin5\theta} \Rightarrow \cos6\theta = 0$

$\Rightarrow 6\theta = (2n+1)\frac{\pi}{2} \Rightarrow \theta = (2n+1)\frac{\pi}{12};\ n \in I$

$\Rightarrow \theta = -\frac{5\pi}{12}, -\frac{\pi}{4}, -\frac{\pi}{12}, \frac{\pi}{12}, \frac{\pi}{4}, \frac{5\pi}{12}$ ......(i)

$\sin2\theta = \cos4\theta$

$\Rightarrow \sin2\theta = 1 - 2\sin^2 2\theta \Rightarrow 2\sin^2 2\theta + \sin2\theta - 1 = 0$

$\Rightarrow \sin2\theta = -1, \frac{1}{2} \Rightarrow 2\theta = (4m-1)\frac{\pi}{2}, p\pi + (-1)^p\frac{\pi}{6} \Rightarrow \theta = (4m-1)\frac{\pi}{4}, \frac{p\pi}{2} + (-1)^p\frac{\pi}{12};\ m, p \in I$

$\Rightarrow \theta = -\frac{\pi}{4}, \frac{\pi}{12}, \frac{5\pi}{12}$ ......(ii)

From (i) & (ii)

$\theta \in \left\{-\frac{\pi}{4}, \frac{\pi}{12}, \frac{5\pi}{12}\right\}$ Number of solution is 3.

9. As $\tan(2\pi - \theta) > 0, -1 < \sin\theta < -\frac{\sqrt{3}}{2}, \theta \in [0, 2\pi]$

$\Rightarrow \frac{3\pi}{2} < \theta < \frac{5\pi}{3}$

Now $2\cos\theta(1 - \sin\phi) = \sin^2\theta(\tan\theta/2 + \cot\theta/2)\cos\phi - 1$

$\Rightarrow 2\cos\theta(1 - \sin\phi) = 2\sin\theta\cos\phi - 1$

$\Rightarrow 2\cos\theta + 1 = 2\sin(\theta + \phi)$

As $\theta \in \left(\frac{3\pi}{2}, \frac{5\pi}{3}\right) \Rightarrow 2\cos\theta + 1 \in (1, 2)$

$\Rightarrow 1 < 2\sin(\theta + \phi) < 2 \Rightarrow \frac{1}{2} < \sin(\theta + \phi) < 1$

As $\theta + \phi \in [0, 4\pi]$

$\Rightarrow \theta + \phi \in \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ or $\theta + \phi \in \left(\frac{13\pi}{6}, \frac{17\pi}{6}\right)$

$\Rightarrow \frac{\pi}{6} - \theta < \phi < \frac{5\pi}{6} - \theta$ or $\frac{13\pi}{6} - \theta < \phi < \frac{17\pi}{6} - \theta$

$\Rightarrow \phi \in \left(-\frac{3\pi}{2}, \frac{-2\pi}{3}\right) \cup \left(\frac{2\pi}{3}, \frac{7\pi}{6}\right)$

$\left(\because \theta \in \left(\frac{3\pi}{2}, \frac{5\pi}{3}\right)\right)$ $\therefore$ correct option is (A, C, D)

11. $\sqrt{3}\sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$

$\Rightarrow \sqrt{3}\sin x + \cos x + 2(\sin^2 x - \cos^2 x) = 0$

$\Rightarrow \sqrt{3}\sin x + \cos x - 2\cos2x = 0 \Rightarrow \sin\left(x + \frac{\pi}{3}\right) = \cos2x$

$\cos(\pi/3 - x) = \cos2x \Rightarrow 2x = 2n\pi \pm (\pi/3 - x)$

$x = \frac{2n\pi}{3} + \frac{\pi}{9}$ or $x = 2n\pi - \frac{\pi}{3}$

$-100° - 60° + 20° + 140° = 0$

MOCK TEST

1. $\cot\theta = -\sqrt{3}$ $\Rightarrow$ $\tan\theta = -\dfrac{1}{\sqrt{3}}$

$\text{cosec}\theta = -2$ $\Rightarrow$ $\sin\theta = -\dfrac{1}{2}$

$\therefore$ θ lies in IV quadrant $\therefore$ $\theta = 2n\pi - \dfrac{\pi}{6}$

2. $\dfrac{2.2\cos\theta/2.\cos\theta/2}{2\sin\theta/2\,.\cos\theta/2} = (1+\cot\theta)^2$

or $\dfrac{2(1+\cos\theta)}{\sin\theta} = \text{cosec}^2\theta + 2\cot\theta$

or $2 + 2\cos\theta = \text{cosec}\theta + 2\cos\theta$

or $\sin\theta = 1/2$ $\Rightarrow$ $\theta = n\pi + (-1)^n \dfrac{\pi}{6}$

3. **(C)**

We have $\tan\theta = -1$ and $\cos\theta = 1/\sqrt{2}$

The value of θ lying between 3π/2 and 2π and satisfying these two is 7π/4. Therefore the most general solution is $\theta = 2n\pi + 7\pi/4$ where $n \in Z$.

4. $12\cos^2\theta - 6 + 1 + \cos\theta + 2 - 2\cos^2\theta = 0$

or $10\cos^2\theta + \cos\theta - 3 = 0$

or $(5\cos\theta + 3)(2\cos\theta - 1) = 0$

$\Rightarrow$ $\cos\theta = -\dfrac{3}{5}, \dfrac{1}{2}$

$\Rightarrow$ $\theta = \dfrac{\pi}{3}, \pi - \cos^{-1}\left(\dfrac{3}{5}\right), -\dfrac{\pi}{3}$

5. **(D)**

Since $\sin\theta - \cos\theta \neq 0$

$\tan\theta \neq 1$

and also to define tanθ , $\theta \neq \dfrac{\pi}{2}, \dfrac{3\pi}{2}$

$\therefore$ $\theta \neq \dfrac{\pi}{4}, \dfrac{5\pi}{4}, \dfrac{\pi}{2}, \dfrac{3\pi}{2}$

Now

$\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta - |\sin\theta|\cos\theta - 2\tan\theta\cot\theta = -1$

$\Rightarrow$ $1 + \cos\theta(\sin\theta - |\sin\theta|) - 2 = -1$

$\Rightarrow$ $\cos\theta(\sin\theta - |\sin\theta|) = 0$

$\therefore$ $\theta \in (0, \pi) - \left\{\dfrac{\pi}{4}, \dfrac{\pi}{2}\right\}$

6. $\cos 6x\,(1 + \tan^2 x) = 1 - \tan^2 x$

or $\cos 6x = \dfrac{1-\tan^2 x}{1+\tan^2 x}$

or $\cos 6x = \cos 2x$ or $6x = 2n\pi \pm 2x$

$\Rightarrow$ $x = \dfrac{n\pi}{2}, \dfrac{n\pi}{4}$

$\Rightarrow$ $x = 0, \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \pi, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}, 2\pi$

(At, $x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$, tan x does not exist)

7. **(B)**

note : $\sin\theta \neq \cos\theta$

$\Rightarrow$ $\theta \notin \left(0, \dfrac{\pi}{2}\right) \cup \left(\pi, \dfrac{3\pi}{2}\right)$; $\theta \neq \dfrac{\pi}{2}, \dfrac{3\pi}{2}$, 0, π, 2π

and equality holds if $\theta \in \left(\dfrac{\pi}{2}, \pi\right)$

8. **(B)**

$\therefore$ $-\dfrac{\pi}{2} \le \theta \le \dfrac{\pi}{2}$ $\Rightarrow$ $-1 \le \sin\theta \le 1$

Here $0 < \sin\theta < 1$ $\therefore$ $\log_{\sin\theta}\cos 2\theta = 2$

$\Rightarrow$ $\cos 2\theta = \sin^2\theta$ $\Rightarrow$ $1 - 2\sin^2\theta = \sin^2\theta$

$\Rightarrow$ $3\sin^2\theta = 1$ $\Rightarrow$ $\sin^2\theta = \dfrac{1}{3}$

$\therefore$ $\sin\theta = \dfrac{1}{\sqrt{3}}$ $\{\because 0 < \sin\theta < 1\}$ a unique solution.

9. $\dfrac{\dfrac{1}{1+\sin x}}{\dfrac{1}{1-\sin x}} = \dfrac{1-\cos 2x}{1+\cos 2x}$

$\Rightarrow$ $\dfrac{1-\sin x}{1+\sin x} = \dfrac{1-\cos 2x}{1+\cos 2x}$

$\Rightarrow$ $\dfrac{1-\sin x}{1+\sin x} = \dfrac{\sin^2 x}{(1-\sin^2 x)}$ $\Rightarrow \sin x = \dfrac{1}{2}$

($\because$ $\sin x \neq \pm 1$)

10. **(C)**

Given equation $\sec^2(a+2)x + a^2 - 1 = 0$

$\Rightarrow$ $\tan^2(a+2)x + a^2 = 0$

$\Rightarrow$ $\tan^2(a+2)x = 0$ and $a = 0$

$\Rightarrow$ $\tan^2 2x = 0$ $\Rightarrow$ $x = 0, \dfrac{\pi}{2}, -\dfrac{\pi}{2}$

$\therefore$ (0, 0), (0, π/2), (0, – π/2) are ordered pairs satisfying the equation.

11. $4\sin^4x + (1-\sin^2x)^2 = 1$

$5\sin^4x - 2\sin^2x = 0$

$\sin^2x(5\sin^2x - 2) = 0$

$\Rightarrow \sin^2x = 0;\quad \sin^2x = \frac{2}{5}$

$\Rightarrow x = n\pi;\ n \in I$ or $\cos2x = 1 - 2\sin^2x = 1 - \frac{4}{5}$

$\therefore \cos2x = \frac{1}{5} = \cos\alpha$

$\therefore 2x = 2n\pi \pm \alpha$

$\therefore x = n\pi \pm \frac{1}{2}\cos^{-1}\left(\frac{1}{5}\right);\ n \in I$

12. $\because 2\sin2x\cos x + \sin2x = 2\cos2x\cos x + \cos2x$

$\Rightarrow \sin2x(1+2\cos x) = \cos2x(2\cos x + 1)$

$\Rightarrow (2\cos x + 1)(\sin2x - \cos2x) = 0$

$\Rightarrow \cos x = -\frac{1}{2}$ or $\tan2x = 1$

$\Rightarrow x = 2n\pi \pm \frac{2\pi}{3},\ n \in I$

or $x = \frac{n\pi}{2} + \frac{\pi}{8},\ n \in I$

15. $\sin^2x - \cos2x = 2 - \sin2x$

$\Rightarrow \sin^2x - (1 - 2\sin^2x) = 2 - 2\sin x\cos x$

$\Rightarrow 3\sin^2x + 2\sin x\cos x = 3$

Case-I : $\cos x \neq 0$

$\therefore 3\tan^2x + 2\tan x = 3(1 + \tan^2x) \Rightarrow \tan x = \frac{3}{2}$

Case-II : $\cos x = 0$

$\therefore 3(1) + 2(\pm1)(0) = 3$ which is true $\therefore x = (2n+1)\frac{\pi}{2}$

16. $2^x + 2^{-x} \geq 2$

$\Rightarrow \sin x \geq 2$ (impossible)

$\because |\sin x| \leq 1$

17. $y + \frac{1}{y} \geq 2$ (AM ≥ GM)

$\Rightarrow \sqrt{\left(y + \frac{1}{y}\right)} \geq \sqrt{2}$

or $\sin x + \cos x \geq \sqrt{2}$

and $|\sin x + \cos x| \leq \sqrt{2}$

Hence, $y + \frac{1}{y} = 2$ and $\sin x + \cos x = \sqrt{2}$

which is possible for $y = 1,\ x = \frac{\pi}{4}$.

18. The given equation is equivalent to

$\tan(\theta + 2\theta + 3\theta) = 0$

or $\tan6\theta = 0$

Then, $6\theta = n\pi$

$\therefore \theta = \frac{n\pi}{6},\ n \in I$

In $(0, \pi)$ we have $\theta = \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{5\pi}{6}$

However $\tan\theta$ and $\tan3\theta$ are not defined at $\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

$\therefore \frac{\pi}{3}, \frac{2\pi}{3}$ are the only solutions.

19. $\sin(\cos x) = \cos(\sin x)$

$\Rightarrow \cos(\sin x) = \cos\left(\frac{\pi}{2} - \cos x\right)$

$\Rightarrow \sin x = 2n\pi \pm \left(\frac{\pi}{2} - \cos x\right),\ n \in I$

$\Rightarrow \sin x \pm \cos x = \left(2n \pm \frac{1}{2}\right)\pi$

Squaring

$\Rightarrow 1 \pm \sin2x = \left(2n \pm \frac{1}{2}\right)^2\pi^2$

$\Rightarrow |\sin2x| = \left(2n \pm \frac{1}{2}\right)^2\pi^2 - 1$

But $\left(2n \pm \frac{1}{2}\right)^2\pi^2 > 2$ for all $n \in I$

$\therefore |\sin2x| > 1$ which is inadmissive.

Hence, the given equation does not possess real roots.

and $\sin x > 0$

(x lies in I and II quadrant)

$\therefore 2n\pi < x < (2n+1)\pi,\ n \in I$

20. $\sin^2 A = \sin^2 B$ and $\cos^2 A = \cos^2 B$

$\therefore \quad \cos 2A = \cos 2B$

$\Rightarrow \quad 2A = 2n\pi \pm B, n \in I$

or $\quad A = n\pi \pm B$

or $\quad A = n\pi + B, n \in I$

($\because$ Both sides square given)

Now, $\sin A = \sin B$

$\Rightarrow \quad A = n\pi + (-1)^n B, n \in I$

If n is even, then

$A = n\pi + B$

and $\cos A = \cos B$

$\Rightarrow \quad A = 2n\pi \pm B, n \in I$

Hence, Assertion is true but Reason is false.

22. **(A)** $\sin x = -\frac{1}{2}$

$= -\sin\frac{\pi}{6} \quad \Rightarrow \quad \sin\left(\pi + \frac{\pi}{6}\right), \sin\left(2\pi - \frac{\pi}{6}\right)$

$\therefore \quad x = \frac{7\pi}{6}, \frac{11\pi}{6}$ ....**(i)**

and $\cos x = -\frac{\sqrt{3}}{2} = -\cos\frac{\pi}{6}$

$= \cos\left(\pi - \frac{\pi}{6}\right), \cos\left(\pi + \frac{\pi}{6}\right)$

$\therefore \quad x = \frac{5\pi}{6}, \frac{7\pi}{6}$ ....**(ii)**

From eq. **(i)** and **(ii)**. It is clear that

$\alpha = \frac{7\pi}{6}, \beta = \frac{11\pi}{6}, \gamma = \frac{5\pi}{6}$

$\Rightarrow \quad \alpha + \beta = 3\pi(S), \beta - \gamma = \pi(Q)$

(B) $\cot x = -\sqrt{3}$

$= -\cot\frac{\pi}{6}$

$= \cot\left(\pi - \frac{\pi}{6}\right), \cot\left(2\pi - \frac{\pi}{6}\right)$

$\therefore \quad x = \frac{5\pi}{6}, \frac{11\pi}{6}$ ....**(i)**

and $\operatorname{cosec} x = -2 = -\operatorname{cosec}\frac{\pi}{6}$

$= \operatorname{cosec}\left(\pi + \frac{\pi}{6}\right), \operatorname{cosec}\left(2\pi - \frac{\pi}{6}\right)$

$\therefore \quad x = \frac{7\pi}{6}, \frac{11\pi}{6}$ ....**(ii)**

from eq. **(i)** and **(ii)**, It is clear that

$\alpha = \frac{11\pi}{6}, \beta = \frac{5\pi}{6}, \gamma = \frac{7\pi}{6}$

$\Rightarrow \quad \beta + \gamma = 2\pi(T), \alpha - \beta = \pi(P)$

(C) $\sin x = -\frac{1}{2}$

$= -\sin\frac{\pi}{6}$

$= \sin\left(\pi + \frac{\pi}{6}\right), \sin\left(2\pi - \frac{\pi}{6}\right)$

$\therefore \quad x = \frac{7\pi}{6}, \frac{11\pi}{6}$ ....**(i)**

and $\tan x = \frac{1}{\sqrt{3}}$

$= \tan\frac{\pi}{6}$

$= \tan\frac{\pi}{6}, \tan\left(\pi + \frac{\pi}{6}\right)$

$\therefore \quad x = \frac{\pi}{6}, \frac{7\pi}{6}$ ....**(ii)**

from eq. **(i)** and **(ii)**, It is clear that

$\alpha = \frac{7\pi}{6}, \beta = \frac{11\pi}{6}, \gamma = \frac{\pi}{6}$

$\Rightarrow \quad \alpha + \beta = 3\pi, (S), \beta + \gamma = 2\pi\,(T), \alpha - \gamma = \pi(R)$

24.

1. AM $\geq$ GM

$\therefore \quad 3^x - 3^{-x} \geq 2$

$\Rightarrow \quad 2\cos\left(\frac{x}{2}\right) \geq 2$

or $\quad \cos\left(\frac{x}{2}\right) \geq 1$

$\therefore \quad \cos\left(\frac{x}{2}\right) = 1 \qquad \left(\because \cos\frac{x}{2} \text{ is never} > 1\right)$

$\Rightarrow \quad \frac{x}{2} = 2n\pi, n \in I$

$\therefore \quad x = 4n\pi$

Hence **(A)** corresponding to n = 0,

because other values of n do not satisfy the equation.

2 Let $y = 2\cos^2\left(\frac{x}{2}\right)\sin^2 x = x^2 + x^{-2}$

$\therefore \quad y = x^2 + x^{-2} \geq 2 \qquad (\because \text{ AM} \geq \text{GM})$

$\Rightarrow \quad y \geq 2 \qquad$ ....**(i)**

and $y = 2\cos^2\left(\frac{x}{2}\right)\sin^2 x$

$= (1 + \cos x) . \sin^2 x$

$= (\text{a number} < 2) . (\text{a number} \leq 1) < 2$

i.e., $\quad y < 2 \qquad$ ....**(ii)**

No value of y can be obtained satisfying eq. **(i)** and **(ii)** simultaneously.

$\Rightarrow$ No real solution of the equation exists.

3 AM $\geq$ GM

$\therefore \quad 5^x + 5^{-x} \geq 2 \qquad (\because \sin e^x = 5^x + 5^{-x})$

$\Rightarrow \quad \sin e^x \geq 2$

But the value of $\sin(e^x)$ can never be > 1.

Hence, the given equation has no solution.

25.

1. Let $\cos x - \sin x = t$

$\therefore \quad 1 - 2\sin x\cos x = t^2$

Then, the given equation can be written as

$t^2 = t$

$\Rightarrow \quad t(t-1) = 0$

$\therefore \quad t = 0, t = 1$

$\Rightarrow \quad \cos x - \sin x = 0, \cos x - \sin x = 1$

$\therefore \quad \tan x = 1, \frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad \tan x = 1, \cos\left(x + \frac{\pi}{4}\right) = \cos\frac{\pi}{4}$

$\therefore \quad x = n\pi + \frac{\pi}{4}, x + \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$

Hence, $x = 2n\pi, 2n\pi - \frac{\pi}{2}, n\pi + \frac{\pi}{4}, n \in I$

2. Let $\quad \sin x + \cos x = t$

$\therefore \quad 1 + 2\sin x\cos x = t^2$

Then, the given equation can be written as

$\Rightarrow \quad t = 1 + \frac{t^2 - 1}{2}$

$\Rightarrow \quad 2t = 2 + t^2 - 1$

$\Rightarrow \quad (t-1)^2 = 0$

or $\quad t = 1$

$\Rightarrow \quad \sin x + \cos x = 1$

$\Rightarrow \quad \frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad \cos\left(x - \frac{\pi}{4}\right) = \cos\frac{\pi}{4},$

or $\quad x - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}, n \in I$

$\therefore \quad x = 2n\pi, 2n\pi + \frac{\pi}{2}$

3. $\sin^4 x + \cos^4 x = \sin x\cos x$

$\Rightarrow \quad (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x = \sin x\cos x$

$\Rightarrow \quad 1 - 2\sin^2 x\cos^2 x = \sin x\cos x$

Let $\sin x\cos x = \lambda$, then

$1 - 2\lambda^2 = \lambda$

$\Rightarrow \quad 2\lambda^2 + \lambda - 1 = 0$

$\Rightarrow \quad (\lambda + 1)(2 . \lambda - 1) = 0$

$\therefore \quad \lambda = -1, \frac{1}{2}$

$\Rightarrow \quad \sin x\cos x = -1, \frac{1}{2}$

$\Rightarrow \quad \sin 2x = -2, 1$

$\Rightarrow \quad \sin 2x \neq -2$

$\Rightarrow \quad \sin 2x = 1$

or $\quad 2x = 2n\pi + \frac{1}{2}, n \in I$

$\therefore \quad x = (4n+1)\frac{\pi}{4}, n \in I$

26. (0)

$|\sin x\cos x| + |\tan x + \cot x| = \sqrt{3}$

$\Rightarrow \quad |\sin x\cos x| + \frac{1}{|\sin x\cos x|} = \sqrt{3}$

$|\sin x\cos x| + \frac{1}{|\sin x\cos x|} \geq 2$

Hence, there is no solution.

27. (6)

$$\frac{\sin x}{\cos 3x}+\frac{\sin 3x}{\cos 9x}+\frac{\sin 9x}{\cos 27x}=0$$

or $$\frac{2\sin x\cos x}{2\cos 3x\cos x}+\frac{2\sin 3x\cos 3x}{2\cos 9x\cos 3x}+\frac{2\sin 9x\cos 9x}{2\cos 27x\cos 9x}=0$$

or $$\frac{\sin(3x-x)}{2\cos 3x\cos x}+\frac{\sin(9x-3x)}{2\cos 9x\cos 3x}+\frac{\sin(27x-9x)}{2\cos 27x\cos 9x}=0$$

or $(\tan 3x-\tan x)+(\tan 9x-\tan 3x)+(\tan 27x-\tan 9x)=0$

or $\tan 27x-\tan x=0$

or $\tan x=\tan 27x$

$\Rightarrow$ $27x=n\pi+x, n\in I$

or $x=\frac{n\pi}{26}, n\in I$

or $x=\frac{\pi}{26},\frac{2\pi}{26},\frac{3\pi}{26},\frac{4\pi}{26},\frac{5\pi}{26},\frac{6\pi}{26}$

Hence, there are six solutions.

28. (1)

Adding given equations, we get

$$2=\frac{3a}{2}+\frac{a^2}{2}$$

or $a^2+3a-4=0$

or $(a+4)(a-1)=0$

or $a=1$ (as $a=-4$ is rejected)

29. (6)

$a\sin x+1-2\sin^2x=2a-7$

or $2\sin^2x-a\sin x+(2a-8)=0$

or $$\sin x=\frac{a\pm\sqrt{a^2-8(2a-8)}}{4}=\frac{a\pm(a-8)}{4}$$

$=2$ or $\frac{a-4}{2}$

For a solution $-1\le\frac{a-4}{2}\le 1$, we have $2\le a\le 6$.

30. (4)

$\sin^4x-\cos^2x\sin x+2\sin^2x+\sin x=0$

or $\sin x[\sin^3x-\cos^2x+2\sin x+1]=0$

or $\sin x[\sin^3x+\sin^2x+2\sin x]=0$

or $\sin^2x[\sin^2x+\sin x+2]=0$

or $\sin x=0$, where $x=0,\pi,2\pi,3\pi$

Hence, there are four solutions.