SOLVED EXAMPLES

Ex. 1 In triangle ABC, if b = 3, c = 4 and $\angle B = \pi/3$, then find number of such triangles.

Sol. Using sine formulae $\frac{\sin B}{b} = \frac{\sin C}{c}$

$\Rightarrow \frac{\sin \pi/3}{3} = \frac{\sin C}{4} \Rightarrow \frac{\sqrt{3}}{6} = \frac{\sin C}{4} \Rightarrow \sin C = \frac{2}{\sqrt{3}} > 1$ which is not possible.

Hence there exist no triangle with given elements.

Ex. 2 In any ΔABC, prove that $(b^2 - c^2) \cot A + (c^2 - a^2) \cot B + (a^2 - b^2) \cot C = 0$

Sol. Since $a = k \sin A$, $b = k \sin B$ and $c = k \sin C$

$\therefore$ $(b^2 - c^2) \cot A = k^2 (\sin^2 B - \sin^2 C) \cot A = k^2 \sin (B + C) \sin (B - C) \cot A$

$\therefore$ $= k^2 \sin A \sin (B - C) \frac{\cos A}{\sin A}$

$= - k^2 \sin (B - C) \cos (B + C)$ $(\because \cos A = - \cos (B + C))$

$= - \frac{k^2}{2} [2\sin (B - C) \cos (B + C)]$

$= - \frac{k^2}{2} [\sin 2B - \sin 2C]$ ......**(i)**

Similarly $(c^2 - a^2) \cot B = - \frac{k^2}{2} [\sin 2C - \sin 2A]$ ......**(ii)**

and $(a^2 - b^2) \cot C = - \frac{k^2}{2} [\sin 2A - \sin 2B]$ ......**(iii)**

adding equations **(i)**, **(ii)** and **(iii)**, we get

$(b^2 - c^2) \cot A + (c^2 - a^2) \cot B + (a^2 - b^2) \cot C = 0$

Ex. 3 Angles of a triangle are in 4 : 1 : 1 ratio. Then find the ratio between its greatest side and perimeter.

Sol. Angles are in ratio 4 : 1 : 1.

$\Rightarrow$ angles are 120°, 30°, 30°.

If sides opposite to these angles are a, b, c respectively, then a will be the greatest side. Now from sine formula

$$\frac{a}{\sin 120°} = \frac{b}{\sin 30°} = \frac{c}{\sin 30°}$$

$$\Rightarrow \frac{a}{\sqrt{3}/2} = \frac{b}{1/2} = \frac{c}{1/2}$$

$$\Rightarrow \frac{a}{\sqrt{3}} = \frac{b}{1} = \frac{c}{1} = k \text{ (say)}$$

then $a = \sqrt{3}k$, perimeter $= (2 + \sqrt{3})k$

$\therefore$ required ratio $= \frac{\sqrt{3}k}{(2 + \sqrt{3})k} = \frac{\sqrt{3}}{2 + \sqrt{3}}$

Ex. 4 In a triangle ABC, if B = 30° and c = $\sqrt{3}$ b, then find angle A.

Sol. We have $\cos B = \dfrac{c^2 + a^2 - b^2}{2ca}$ $\Rightarrow$ $\dfrac{\sqrt{3}}{2} = \dfrac{3b^2 + a^2 - b^2}{2 \times \sqrt{3}b \times a}$

$\Rightarrow$ $a^2 - 3ab + 2b^2 = 0$ $\Rightarrow$ $(a - 2b)(a - b) = 0$

$\Rightarrow$ Either a = b $\Rightarrow$ A = 30°

or a = 2b $\Rightarrow$ $a^2 = 4b^2 = b^2 + c^2$ $\Rightarrow$ A = 90°.

Ex. 5 In a triangle ABC if a = 13, b = 8 and c = 7, then find sin A.

Sol. $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc} = \dfrac{64 + 49 - 169}{2.8.7}$ $\Rightarrow$ $\cos A = -\dfrac{1}{2}$ $\Rightarrow$ $A = \dfrac{2\pi}{3}$

$\therefore$ $\sin A = \sin\dfrac{2\pi}{3} = \dfrac{\sqrt{3}}{2}$

Ex. 6 In any ΔABC, prove that $\dfrac{a+b}{c} = \dfrac{\cos\left(\dfrac{A-B}{2}\right)}{\sin\dfrac{C}{2}}$.

Sol. Since $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = k$ (let)

$\Rightarrow$ a = k sinA, b = k sinB and c = k sinC

$\therefore$ L.H.S. $= \dfrac{a+b}{c} = \dfrac{k(\sin A + \sin B)}{k \sin C}$

$$= \frac{\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{\sin\frac{C}{2}\cos\frac{C}{2}} = \frac{\cos\frac{C}{2}\cos\left(\frac{A-B}{2}\right)}{\sin\frac{C}{2}\cos\frac{C}{2}} = \frac{\cos\left(\frac{A-B}{2}\right)}{\sin\frac{C}{2}} = \text{R.H.S.}$$

Hence L.H.S. = R.H.S.

Ex. 7 A cyclic quadrilateral ABCD of area $\dfrac{3\sqrt{3}}{4}$ is inscribed in unit circle. If one of its side AB = 1, and the diagonal $BD = \sqrt{3}$, find lengths of the other sides.

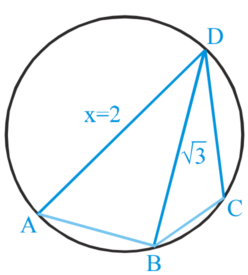
Sol. AB = 1, BD = $\sqrt{3}$, OA = OB = OD = 1

The given circle of radius 1 is also circumcircle of ΔABD

$\Rightarrow$ R = 1 for ΔABD

$\Rightarrow$ $\dfrac{a}{\sin A} = 2R$ $\Rightarrow$ A = 60°

and hence C = 120°



Also by cosine rule on $\Delta ABD, \left(\sqrt{3}\right)^2 = 1^2 + x^2 - 2x\cos 60°$

$\Rightarrow \quad x = 2$

Now, area ABCD = $\Delta ABD + \Delta BCD$

$$\Rightarrow \quad \frac{3\sqrt{3}}{4} = \frac{1}{2}(1.2.\sin 60°) + \frac{1}{2}(c.d.\sin 120°)$$

$\Rightarrow \quad cd = 1$, or $c^2d^2 = 1$

Also by cosine rule on triangle BCD we have

$$\left(\sqrt{3}\right)^2 = c^2 + d^2 - 2cd\cos 120° = c^2 + d^2 + cd$$

$\Rightarrow \quad c^2 + d^2 = 2$ or $cd = 1$

$\Rightarrow \quad c^2$ and d^2 are the roots of $t^2 - 2t + 1 = 0$

$\therefore \quad c^2 = d^2 = 1 \; \therefore$ BC = 1 = CD and AD = x = 2.

Ex. 8 In a ΔABC, prove that $a(b \cos C - c \cos B) = b^2 - c^2$

Sol. Since $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$ & $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

$$\therefore \quad \text{L.H.S.} = a\left\{b\left(\frac{a^2+b^2-c^2}{2ab}\right) - c\left(\frac{a^2+c^2-b^2}{2ac}\right)\right\}$$

$$= \frac{a^2+b^2-c^2}{2} - \frac{(a^2+c^2-b^2)}{2} = (b^2 - c^2)$$

= R.H.S.

Hence L.H.S. = R.H.S.

Ex. 9 If in a triangle ABC, CD is the angle bisector of the angle ACB, then find CD.

Sol. $\Delta CAB = \Delta CAD + \Delta CDB$

$$\Rightarrow \quad \frac{1}{2}ab\sin C = \frac{1}{2}b.CD.\sin\left(\frac{C}{2}\right) + \frac{1}{2}a.CD\sin\left(\frac{C}{2}\right)$$

$$\Rightarrow \quad CD(a+b)\sin\left(\frac{C}{2}\right) = ab\left(2\sin\left(\frac{C}{2}\right)\cos\left(\frac{C}{2}\right)\right)$$

$$\text{So} \quad CD = \frac{2ab\cos(C/2)}{(a+b)}$$

and in ΔCAD, $\dfrac{CD}{\sin\angle DAC} = \dfrac{b}{\sin\angle CDA}$ (by sine rule)

$$\Rightarrow \quad CD = \frac{b\sin\angle DAC}{\sin(B + C/2)}$$

Ex. 10 In a ΔABC, the tangent of half the difference of two angles is one-third the tangent of half the sum of the angles. Determine the ratio of the sides opposite to the angles.

Sol. Here, $\tan\left(\frac{A-B}{2}\right) = \frac{1}{3}\tan\left(\frac{A+B}{2}\right)$ ......**(i)**

using Napier's analogy, $\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b}.\cot\left(\frac{C}{2}\right)$ ......**(ii)**

from **(i)** & **(ii)** ;

$$\frac{1}{3}\tan\left(\frac{A+B}{2}\right) = \frac{a-b}{a+b}.\cot\left(\frac{C}{2}\right) \quad \Rightarrow \quad \frac{1}{3}\cot\left(\frac{C}{2}\right) = \frac{a-b}{a+b}.\cot\left(\frac{C}{2}\right)$$

$\{$as $A + B + C = \pi \quad \therefore \quad \tan\left(\frac{B+C}{2}\right) = \tan\left(\frac{\pi}{2} - \frac{C}{2}\right) = \cot\frac{C}{2}\}$

$$\Rightarrow \quad \frac{a-b}{a+b} = \frac{1}{3} \quad \text{or} \quad 3a - 3b = a + b$$

$$2a = 4b \quad \text{or} \quad \frac{a}{b} = \frac{2}{1} \Rightarrow \frac{b}{a} = \frac{1}{2}$$

Thus the ratio of the sides opposite to the angles is b : a = 1 : 2.

Ex. 11 In a triangle ABC, if a : b : c = 4 : 5 : 6, then find ratio between its circumradius and inradius.

Sol. $\frac{R}{r} = \frac{abc}{4\Delta}\Big/\frac{\Delta}{s} = \frac{(abc)s}{4\Delta^2} \quad \Rightarrow \quad \frac{R}{r} = \frac{abc}{4(s-a)(s-b)(s-c)}$ ......**(i)**

$\because \; a : b : c = 4 : 5 : 6 \quad \Rightarrow \quad \frac{a}{4} = \frac{b}{5} = \frac{c}{6} = k$ (say)

$\Rightarrow \; a = 4k, \quad b = 5k, \quad c = 6k$

$\therefore \; s = \frac{a+b+c}{2} = \frac{15k}{2}, \quad s-a = \frac{7k}{2}, \quad s-b = \frac{5k}{2}, \quad s-c = \frac{3k}{2}$

using **(i)** in these values $\frac{R}{r} = \frac{(4k)(5k)(6k)}{4\left(\frac{7k}{2}\right)\left(\frac{5k}{2}\right)\left(\frac{3k}{2}\right)} = \frac{16}{7}$

Ex. 12 In a ΔABC, prove that (b + c) cos A + (c + a) cos B + (a + b) cos C = a + b + c.

Sol. L.H.S. = (b + c) cos A + (c + a) cos B + (a + b) cos C
= b cos A + c cos A + c cos B + a cos B + a cos C + b cos C
= (b cos A + a cos B) + (c cos A + a cos C) + (c cos B + b cos C)
= a + b + c
= R.H.S.

Hence L.H.S. = R.H.S.

Ex. 13 Value of the expression $\frac{b-c}{r_1}+\frac{c-a}{r_2}+\frac{a-b}{r_3}$.

Sol. $\frac{(b-c)}{r_1}+\frac{(c-a)}{r_2}+\frac{(a-b)}{r_3}$

$\Rightarrow$ $(b-c)\left(\frac{s-a}{\Delta}\right)+(c-a)\left(\frac{s-b}{\Delta}\right)+(a-b).\left(\frac{s-c}{\Delta}\right)$

$\Rightarrow$ $\frac{(s-a)(b-c)+(s-b)(c-a)+(s-c)(a-b)}{\Delta}$

$=\frac{s(b-c+c-a+a-b)-[ab-ac+bc-ba+ac-bc]}{\Delta}=\frac{0}{\Delta}=0$

Thus, $\frac{b-c}{r_1}+\frac{c-a}{r_2}+\frac{a-b}{r_3}=0$

Ex. 14 In a ΔABC, prove that $\sin A+\sin B+\sin C=\frac{s}{R}$

Sol. In a ΔABC, we know that $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=2R$

$\therefore$ $\sin A=\frac{a}{2R}$, $\sin B=\frac{b}{2R}$ and $\sin C=\frac{c}{2R}$.

$\therefore$ $\sin A+\sin B+\sin C=\frac{a+b+c}{2R}=\frac{2s}{2R}$ $\because$ $a+b+c=2s$

$\Rightarrow$ $\sin A+\sin B+\sin C=\frac{s}{R}$.

Ex. 15 In a ΔABC if $b\sin C(b\cos C+c\cos B)=42$, then find the area of the ΔABC.

Sol. $b\sin C\,(b\cos C+c\cos B)=42$ ......**(i)**

$\because$ From **projection rule**, we know that

$a=b\cos C+c\cos B$ put in **(i)**, we get

$ab\sin C=42$ ......**(ii)**

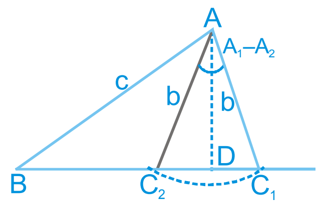
$\because$ $\Delta=\frac{1}{2}ab\sin C$ $\therefore$ from equation **(ii)**, we get

$\therefore$ $\Delta=21$ sq. unit

Ex. 16 If b,c,B are given and $b<c$, prove that $\cos\left(\frac{A_1-A_2}{2}\right)=\frac{c\sin B}{b}$.

Sol. $\angle C_2AC_1$ is bisected by AD.

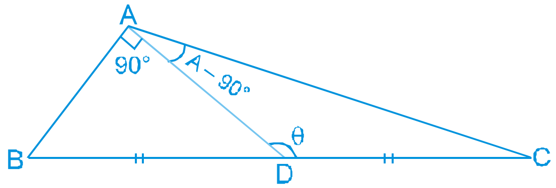
$\Rightarrow$ In ΔAC_2D, $\cos\left(\frac{A_1-A_2}{2}\right)=\frac{AD}{AC_2}=\frac{c\sin B}{b}$



Hence proved.

Ex. 17 If the median AD of a triangle ABC is perpendicular to AB, prove that tan A + 2tan B = 0.

Sol. From the figure, we see that $\theta = 90° + B$ (as θ is external angle of ΔABD)



Now if we apply **m-n rule** in ΔABC, we get $(1 + 1) \cot (90^o + B) = 1. \cot 90° – 1.\cot (A – 90°)$

$\Rightarrow \quad -2 \tan B = \cot (90° – A) \quad \Rightarrow \quad -2 \tan B = \tan A \quad \Rightarrow \quad \tan A + 2 \tan B = 0$

Ex. 18 In a triangle ABC, if cos A + 2 cosB + cosC = 2. Prove that the sides of the triangle are in A.P.

Sol. $\cos A + 2 \cos B + \cos C = 2$ or $\cos A + \cos C = 2(1 – \cos B)$

$$\Rightarrow \quad 2\cos\left(\frac{A+C}{2}\right).\cos\left(\frac{A-C}{2}\right) = 4\sin^2 B/2$$

$$\Rightarrow \quad \cos\left(\frac{A-C}{2}\right) = 2\sin\frac{B}{2} \qquad \left\{\text{as } \cos\left(\frac{A+C}{2}\right) = \cos\left(\frac{\pi}{2} - \frac{B}{2}\right) = \sin\frac{B}{2}\right\}$$

$$\Rightarrow \quad \cos\left(\frac{A-C}{2}\right) = 2\cos\left(\frac{A+C}{2}\right)$$

$$\Rightarrow \quad \cos\frac{A}{2}.\cos\frac{C}{2} + \sin\frac{A}{2}.\sin\frac{C}{2} = 2\cos\frac{A}{2}.\cos\frac{C}{2} - 2\sin\frac{A}{2}.\sin\frac{C}{2}$$

$$\Rightarrow \quad \cot\frac{A}{2}.\cot\frac{C}{2} = 3 \qquad \Rightarrow \quad \sqrt{\frac{s(s-a)}{(s-b)(s-c)}}.\sqrt{\frac{s(s-c)}{(s-a)(s-b)}} = 3$$

$$\Rightarrow \quad \frac{s}{(s-b)} = 3 \Rightarrow s = 3s - 3b \qquad \Rightarrow \quad 2s = 3b$$

$\Rightarrow \quad a + c = 2b, \qquad \therefore$ a, b, c are in A.P.

Ex. 19 In a ΔABC if a, b, c are in A.P., then find the value of $\tan\frac{A}{2}.\tan\frac{C}{2}$

Sol. Since $\tan\frac{A}{2} = \frac{\Delta}{s(s-a)}$ and $\tan\frac{C}{2} = \frac{\Delta}{s(s-c)}$

$\therefore \quad \tan\frac{A}{2}.\tan\frac{C}{2} = \frac{\Delta^2}{s^2(s-a)(s-c)} \qquad \because \quad \Delta^2 = s(s-a)(s-b)(s-c)$

$\therefore \quad \tan\frac{A}{2}.\tan\frac{C}{2} = \frac{s-b}{s} = 1 - \frac{b}{s}$ ....**(i)**

$\because$ it is given that a, b, c are in A.P. $\Rightarrow \quad 2b = a + c$

$\because \quad s = \frac{a+b+c}{2} = \frac{3b}{2}$

$\therefore \quad \frac{b}{s} = \frac{2}{3}$ put in equation **(i)**, we get

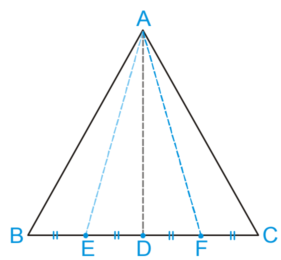
$\therefore \quad \tan\frac{A}{2}.\tan\frac{C}{2} = 1 - \frac{2}{3} \qquad \Rightarrow \quad \tan\frac{A}{2}.\tan\frac{C}{2} = \frac{1}{3}$

Ex. 20 AD is a median of the ΔABC. If AE and AF are medians of the triangles ABD and ADC respectively, and $AD = m_1$, $AE = m_2$, $AF = m_3$, then prove that $m_2^2 + m_3^2 - 2m_1^2 = \frac{a^2}{8}$.

Sol. In ΔABC

$$AD^2 = \frac{1}{4}(2b^2 + 2c^2 - a^2) = m_1^2 \quad \text{......(i)}$$

$\because$ In ΔABD, $AE^2 = m_2^2 = \frac{1}{4}\left(2c^2 + 2AD^2 - \frac{a^2}{4}\right)$ ......**(ii)**



Similarly in ΔADC, $AF^2 = m_3^2 = \frac{1}{4}\left(2AD^2 + 2b^2 - \frac{a^2}{4}\right)$ ......**(iii)**

by adding equations **(ii)** and **(iii)**, we get

$$\because \quad m_2^2 + m_3^2 = \frac{1}{4}\left(4AD^2 + 2b^2 + 2c^2 - \frac{a^2}{2}\right)$$

$$= AD^2 + \frac{1}{4}\left(2b^2 + 2c^2 - \frac{a^2}{2}\right) = AD^2 + \frac{1}{4}\left(2b^2 + 2c^2 - a^2 + \frac{a^2}{2}\right)$$

$$= AD^2 + \frac{1}{4}(2b^2 + 2c^2 - a^2) + \frac{a^2}{8} = AD^2 + AD^2 + \frac{a^2}{8}$$

$$= 2AD^2 + \frac{a^2}{8} = 2m_1^2 + \frac{a^2}{8} \qquad \because \quad AD^2 = m_1^2$$

$$\therefore \quad m_2^2 + m_3^2 - 2m_1^2 = \frac{a^2}{8}$$

Ex. 21 In triangle ABC, prove that the maximum value of $\tan\frac{A}{2}\tan\frac{B}{2}\tan\frac{C}{2}$ is $\frac{R}{2s}$.

Sol. For triangle ABC, we have

$$\tan\frac{A}{2}\tan\frac{B}{2}\tan\frac{C}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}\sqrt{\frac{(s-a)(s-c)}{s(s-b)}}\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

$$= \sqrt{\frac{s(s-a)(s-b)(s-c)}{s^2}}$$

$$= \frac{\Delta}{s^2} = \frac{r}{s} \le \frac{R}{2s}$$

Exercise # 1

[Single Correct Choice Type Questions]

1. In a triangle ABC, a: b: c = 4: 5: 6. Then 3A + B equals to :

(A) 4C **(B)** 2π **(C)** $\pi - C$ **(D)** π

2. A triangle is inscribed in a circle. The vertices of the triangle divide the circle into three arcs of length 3, 4 and 5 units. Then area of the triangle is equal to:

(A) $\frac{9\sqrt{3}\,(1+\sqrt{3})}{\pi^2}$ **(B)** $\frac{9\sqrt{3}\,(\sqrt{3}-1)}{\pi^2}$ **(C)** $\frac{9\sqrt{3}\,(1+\sqrt{3})}{2\pi^2}$ **(D)** $\frac{9\sqrt{3}\,(\sqrt{3}-1)}{2\pi^2}$

3. In a triangle ABC a : b : c = $\sqrt{3}$: 1 : 1, then the triangle is -

(A) right angled triangle **(B)** obtuse angled triangle

(C) acute angled triangle, which is not isosceles **(D)** Equilateral triangle

4. In a $\Delta ABC\left(\frac{a^2}{\sin A}+\frac{b^2}{\sin B}+\frac{c^2}{\sin C}\right)\cdot \sin\frac{A}{2}\ \sin\frac{B}{2}\sin\frac{C}{2}$ simplifies to -

(A) 2Δ **(B)** Δ **(C)** $\frac{\Delta}{2}$ **(D)** $\frac{\Delta}{4}$

(where Δ is the area of triangle)

5. The distance between the middle point of BC and the foot of the perpendicular from A is :

(A) $\frac{-a^2+b^2+c^2}{2a}$ **(B)** $\frac{b^2-c^2}{2a}$ **(C)** $\frac{b^2+c^2}{\sqrt{bc}}$ **(D)** none of these

6. If in a triangle ABC angle B = 90° then $\tan^2 A/2$ is -

(A) $\frac{b-c}{a}$ **(B)** $\frac{b-c}{b+c}$ **(C)** $\frac{b+c}{b-c}$ **(D)** $\frac{b+c}{a}$

7. In a ΔABC if b + c = 3a then $\cot\frac{B}{2}.\cot\frac{C}{2}$ has the value equal to -

(A) 4 **(B)** 3 **(C)** 2 **(D)** 1

8. In any ΔABC, $\frac{(r_1+r_2)(r_2+r_3)(r_3+r_1)}{Rs^2}$ is always equal to

(A) 8 **(B)** 27 **(C)** 16 **(D)** None of these

9. In a ΔABC, a = 1 and the perimeter is six times the arithmetic mean of the sines of the angles. Then measure of $\angle$ A is

(A) $\frac{\pi}{3}$ **(B)** $\frac{\pi}{2}$ **(C)** $\frac{\pi}{6}$ **(D)** $\frac{\pi}{4}$

10. In a ΔABC, a semicircle is inscribed, whose diameter lies on the side c. Then the radius of the semicircle is (Where Δ is the area of the triangle ABC)

(A) $\frac{2\Delta}{a+b}$ **(B)** $\frac{2\Delta}{a+b-c}$ **(C)** $\frac{2\Delta}{s}$ **(D)** $\frac{c}{2}$

11. In a ΔABC, the value of $\frac{a\cos A + b\cos B + c\cos C}{a+b+c}$ is equal to -

(A) $\frac{r}{R}$ **(B)** $\frac{R}{2r}$ **(C)** $\frac{R}{r}$ **(D)** $\frac{2r}{R}$

12. With usual notation in a ΔABC $\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\left(\frac{1}{r_2}+\frac{1}{r_3}\right)\left(\frac{1}{r_3}+\frac{1}{r_1}\right) = \frac{K\ R^3}{a^2\ b^2\ c^2}$ then K has value equal to -

(A) 1 **(B)** 16 **(C)** 64 **(D)** 128

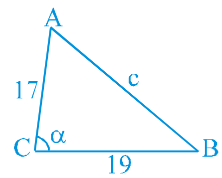
13. With usual notations in a triangle ABC, if $r_1 = 2r_2 = 2r_3$ then -

(A) 4a = 3b **(B)** 3a = 2b **(C)** 4b = 3a **(D)** 2a = 3b

14. If r_1, r_2, and r_3 be the radii of excircles of the triangle ABC, then $\frac{\sum r_1}{\sqrt{\sum r_1 r_2}}$ is equal to -

(A) $\sum \cot\frac{A}{2}$ **(B)** $\sum \cot\frac{A}{2}\cot\frac{B}{2}$ **(C)** $\sum \tan\frac{A}{2}$ **(D)** $\prod \tan\frac{A}{2}$

15. Consider the triangle pictured as shown. If $0 < \alpha < \pi/2$ then the number of integral values of c is



(A) 35 **(B)** 23
(C) 24 **(D)** 25

16. In an acute angled triangle ABC, point D, E and F are the feet of the perpendiculars from A, B and C onto BC, AC and AB respectively. H is the intersection of AD and BE. If sin A = 3/5 and BC = 39, the length of AH is

(A) 45 **(B)** 48 **(C)** 52 **(D)** 54

17. A triangle has sides 6, 7, 8. The line through its incentre parallel to the shortest side is drawn to meet the other two sides at P and Q. The length of the segment PQ is

(A) $\frac{12}{5}$ **(B)** $\frac{15}{4}$ **(C)** $\frac{30}{7}$ **(D)** $\frac{33}{9}$

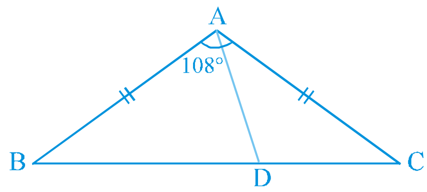
18. Triangle ABC has BC = 1 and AC = 2. The maximum possible value of the angle A is

(A) $\frac{\pi}{6}$ **(B)** $\frac{\pi}{4}$ **(C)** $\frac{\pi}{3}$ **(D)** $\frac{\pi}{2}$

19. Triangle ABC is right angled at A. The points P and Q are on the hypotenuse BC such that BP = PQ = QC. If AP = 3 and AQ = 4 then the length BC is equal to

(A) $\sqrt{27}$ **(B)** $\sqrt{36}$ **(C)** $\sqrt{45}$ **(D)** $\sqrt{54}$

20. In an isosceles triangle ABC, AB = AC, $\angle BAC = 108°$ and AD trisects $\angle BAC$ and BD > DC. The ratio $\frac{BD}{DC}$ is



(A) $\frac{3}{2}$ **(B)** $\frac{\sqrt{5}+1}{2}$
(C) $\sqrt{5}-1$ **(D)** 2

21. In ΔABC if a = 8, b = 9, c = 10, then the value of $\frac{\tan C}{\sin B}$ is

(A) $\frac{32}{9}$ **(B)** $\frac{24}{7}$ **(C)** $\frac{21}{4}$ **(D)** $\frac{18}{5}$

22. In a triangle ABC, CD is the bisector of the angle C. If $\cos\frac{C}{2}$ has the value $\frac{1}{3}$ and $l(CD) = 6$, then $\left(\frac{1}{a}+\frac{1}{b}\right)$ has the value equal to

(A) $\frac{1}{9}$ **(B)** $\frac{1}{12}$ **(C)** $\frac{1}{6}$ **(D)** none

23. Let a, b, c be the three sides of a triangle then the quadratic equation $b^2x^2 + (b^2 + c^2 - a^2)x + c^2 = 0$ has

(A) both imaginary roots **(B)** both positive roots
(C) both negative roots **(D)** one positive and one negative roots.

24. With usual notations, in a triangle ABC, a cos(B – C) + b cos(C – A) + c cos(A – B) is equal to

(A) $\frac{abc}{R^2}$ **(B)** $\frac{abc}{4R^2}$ **(C)** $\frac{4abc}{R^2}$ **(D)** $\frac{abc}{2R^2}$

25. With usual notations in a triangle ABC, $(II_1)\cdot(II_2)\cdot(II_3)$ has the value equal to

(A) R^2r **(B)** $2R^2r$ **(C)** $4R^2r$ **(D)** $16R^2r$

26. A sector OABO of central angle θ is constructed in a circle with centre O and of radius 6. The radius of the circle that is circumscribed about the triangle OAB, is

(A) $6\cos\frac{\theta}{2}$ **(B)** $6\sec\frac{\theta}{2}$ **(C)** $3(\cos\frac{\theta}{2}+2)$ **(D)** $3\sec\frac{\theta}{2}$

27. Let $a \le b \le c$ be the lengths of the sides of a triangle T. If $a^2 + b^2 < c^2$ then which one of the following must be true?

(A) All 3 angles of T are acute. **(B)** Some angle of T is obtuse.
(C) One angle of T is a right angle. **(D)** No such triangle can exist.

28. Let triangle ABC be an isosceles triangle with AB = AC. Suppose that the angle bisector of its angle B meets the side AC at a point D and that BC = BD + AD. Measure of the angle A in degrees, is

(A) 80 **(B)** 100 **(C)** 110 **(D)** 130

29. In a ΔABC, the value of $\frac{a\cos A + b\cos B + c\cos C}{a+b+c}$ is equal to :

(A) $\frac{r}{R}$ **(B)** $\frac{R}{2r}$ **(C)** $\frac{R}{r}$ **(D)** $\frac{2r}{R}$

30. With usual notation in a ΔABC, if $R = k\frac{(r_1+r_2)(r_2+r_3)(r_3+r_1)}{r_1r_2+r_2r_3+r_3r_1}$ where k has the value equal to

(A) 1 **(B)** 2 **(C)** 1/4 **(D)** 4

31. If the incircle of the ΔABC touches its sides respectively at L, M and N and if x, y, z be the circumradii of the triangles MIN, NIL and LIM where I is the incentre then the product xyz is equal to :

(A) Rr^2 **(B)** rR^2 **(C)** $\frac{1}{2}Rr^2$ **(D)** $\frac{1}{2}rR^2$

32. ABC is an acute angled triangle with circumcentre 'O' orthocentre H. If AO = AH then the measure of the angle A is

(A) $\frac{\pi}{6}$ **(B)** $\frac{\pi}{4}$ **(C)** $\frac{\pi}{3}$ **(D)** $\frac{5\pi}{12}$

33. Let L and M be the respective intersections of the internal and external angle bisectors of the triangle ABC at C and the side AB produced. If CL = CM, then the value of $(a^2 + b^2)$ is (where a and b have their usual meanings)

(A) $2R^2$ **(B)** $2\sqrt{2}R^2$ **(C)** $4R^2$ **(D)** $4\sqrt{2}R^2$

34. In a Δ ABC if $b + c = 3a$ then $\cot\frac{B}{2}\cdot\cot\frac{C}{2}$ has the value equal to :

(A) 4 **(B)** 3 **(C)** 2 **(D)** 1

35. Let f, g, h be the lengths of the perpendiculars from the circumcentre of the Δ ABC on the sides a, b and c respectively If $\frac{a}{f}+\frac{b}{g}+\frac{c}{h}=\lambda\frac{a\,b\,c}{f\,g\,h}$ then the value of λ is :

(A) 1/4 **(B)** 1/2 **(C)** 1 **(D)** 2

36. In a ΔABC if $b = a\left(\sqrt{3}-1\right)$ and $\angle C = 30^0$ then the measure of the angle A is

(A) 15^0 **(B)** 45^0 **(C)** 75^0 **(D)** 105^0

37. If x, y and z are the distances of incentre from the vertices of the triangle ABC respectively then $\frac{abc}{xyz}$ is equal to

(A) $\prod\tan\frac{A}{2}$ **(B)** $\sum\cot\frac{A}{2}$ **(C)** $\sum\tan\frac{A}{2}$ **(D)** $\sum\sin\frac{A}{2}$

38. If in a Δ ABC, cosA·cosB + sinA sinB sin2C = 1 then, the statement which is incorrect, is

(A) Δ ABC is isosceles but not right angled **(B)** Δ ABC is acute angled

(C) Δ ABC is right angled **(D)** least angle of the triangle is $\frac{\pi}{4}$

39. In a triangle ABC, $\angle ABC = 120°$, AB = 3 and BC = 4. If perpendicular constructed on the side AB at A and to the side BC at C meets at D then CD is equal to

(A) 3 **(B)** $\frac{8\sqrt{3}}{3}$ **(C)** 5 **(D)** $\frac{10\sqrt{3}}{3}$

40. Let ABC be a triangle with $\angle BAC = \frac{2\pi}{3}$ and AB = x such that (AB)(AC) = 1. If x varies then the longest possible length of the angle bisector AD equals

(A) 1/3 **(B)** 1/2 **(C)** 2/3 **(D)** 3/2

Exercise # 2

Part # I [Multiple Correct Choice Type Questions]

1. If in a ΔABC, a = 5, b = 4 and $\cos(A - B) = \frac{31}{32}$, then

(A) c = 6

(B) $\sin A = \left(\frac{5\sqrt{7}}{16}\right)$

(C) area of $\Delta ABC = \frac{15\sqrt{7}}{4}$

(D) None of these

2. In a ΔABC, $\frac{s}{R}$ is equal to -

(A) $\sin A + \sin B + \sin C$

(B) $4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

(C) $4\sin A\sin B\sin C$

(D) $\frac{\Delta s}{abc}$

3. In triangle ABC, $\cos A + 2\cos B + \cos C = 2$, then -

(A) $\tan\frac{A}{2}\tan\frac{C}{2} = 3$

(B) $\cot\frac{A}{2}\cot\frac{C}{2} = 3$

(C) $\cot\frac{A}{2} + \cot\frac{C}{2} = 2\cot\frac{B}{2}$

(D) $\tan\frac{A}{2}\tan\frac{C}{2} = 0$

4. In a triangle ABC, with usual notations the length of the bisector of internal angle A is :

(A) $\frac{2bc\cos\frac{A}{2}}{b+c}$

(B) $\frac{2bc\sin\frac{A}{2}}{b+c}$

(C) $\frac{abc\,\mathrm{cosec}\frac{A}{2}}{2R(b+c)}$

(D) $\frac{2\Delta}{b+c}\cdot\mathrm{cosec}\frac{A}{2}$

5. In a triangle ABC, right angled at B, then

(A) $r = \frac{AB + BC - AC}{2}$

(B) $r = \frac{AB + AC - BC}{2}$

(C) $r = \frac{AB + BC + AC}{2}$

(D) $R = \frac{s - r}{2}$

6. In a triangle ABC, $(r_1 - r)(r_2 - r)(r_3 - r)$ is equal to -

(A) $4Rr^2$

(B) $\frac{4abc.\Delta}{(a+b+c)^2}$

(C) $16R^3(\cos A + \cos B + \cos C - 1)$

(D) $r^3\mathrm{cosec}\frac{A}{2}\mathrm{cosec}\frac{B}{2}\mathrm{cosec}\frac{C}{2}$

7. If in a triangle ABC, cos A cos B + sin A sin B sin C = 1, then the triangle is

(A) isosceles **(B)** right angled **(C)** equilateral **(D)** None of these

8. If 'O' is the circum centre of the ΔABC and R_1, R_2 and R_3 are the radii of the circumcircles of triangles OBC, OCA and OAB respectively then $\frac{a}{R_1} + \frac{b}{R_2} + \frac{c}{R_3}$ has the value equal to -

(A) $\frac{abc}{2R^3}$ **(B)** $\frac{R^3}{abc}$ **(C)** $\frac{4\Delta}{R^2}$ **(D)** $\frac{abc}{R^3}$

9. The product of the distances of the incentre from the angular points of a Δ ABC is:

(A) $4R^2r$ **(B)** $4Rr^2$ **(C)** $\frac{(abc)R}{s}$ **(D)** $\frac{(abc)r}{s}$

10. In a ΔABC, following relations hold good. In which case(s) the triangle is a right angled triangle ?

(A) $r_2 + r_3 = r_1 - r$ **(B)** $a^2 + b^2 + c^2 = 8R^2$

(C) $r_1 = s$ **(D)** $2R = r_1 - r$

11. With usual notations, in a Δ ABC the value of $\Pi (r_1 - r)$ can be simplified as:

(A) $abc\,\Pi \tan\frac{A}{2}$ **(B)** $4rR^2$ **(C)** $\frac{(abc)^2}{R(a+b+c)^2}$ **(D)** $4Rr^2$

12. Three equal circles of radius unity touches one another. Radius of the circle touching all the three circles is :

(A) $\frac{2-\sqrt{3}}{\sqrt{3}}$ **(B)** $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{2}}$ **(C)** $\frac{2+\sqrt{3}}{\sqrt{3}}$ **(D)** $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{2}}$

13. In a triangle ABC, points D and E are taken on side BC such that BD = DE = EC. If angle ADE = angle AED = θ, then:

(A) $\tan\theta = 3\tan B$ **(B)** $3\tan\theta = \tan C$

(C) $\frac{6\tan\theta}{\tan^2\theta - 9} = \tan A$ **(D)** angle B = angle C

14. In a ΔABC, a semicircle is inscribed, whose diameter lies on the side c. If x is the length of the angle bisector through angle C then the radius of the semicircle is

(A) $\frac{abc}{4R^2(\sin A + \sin B)}$ **(B)** $\frac{\Delta}{x}$

(C) $x\sin\frac{C}{2}$ **(D)** $\frac{2\sqrt{s(s-a)(s-b)(s-c)}}{s}$

Where Δ is the area of the triangle ABC and 's' is semiperimeter.

15. If $r_1 = 2r_2 = 3r_3$, then

(A) $\frac{a}{b} = \frac{4}{5}$ **(B)** $\frac{a}{b} = \frac{5}{4}$ **(C)** $\frac{a}{c} = \frac{3}{5}$ **(D)** $\frac{a}{c} = \frac{5}{3}$

Part # II [Assertion & Reason Type Questions]

These questions contains, Statement I (assertion) and Statement II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.
(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.
(C) Statement-I is true, Statement-II is false.
(D) Statement-I is false, Statement-II is true.

1. **Statement-I :** If R be the circumradius of a ΔABC, then circumradius of its excentral $\Delta I_1I_2I_3$ is 2R.

Statement-II : If circumradius of a triangle be R, then circumradius of its pedal triangle is $\frac{R}{2}$.

2. **Statement-I :** If two sides of a triangle are 4 and 5, then its area lies in (0, 10]

Statement-II : Area of a triangle $=\frac{1}{2}$ ab sinC and sinC $\in$ (0, 1]

3. **Statement-I :** Perimeter of a regular pentagon inscribed in a circle with centre O and radius a cm equals $10a\sin 36°$ cm

Statement-II : Perimeter of a regular polygon inscribed in a circle with centre O and radius a cm equals

$$(3n - 5)\ \sin\left(\frac{360°}{2n}\right) \text{ cm, then it is n sided, where } n \geq 3$$

4. Let ABC be an acute angle triangle and D, E, F are the feet of the perpendicular from A, B, C to the sides BC, CA and AB respectively.

Statement-I : Orthocentre of triangle ABC is the Incentre of triangle DEF.

Statement-II : Triangle DEF is the excentral triangle of triangle ABC.

5. **Statement-I :** The statement that circumradius and inradius of a triangle are 12 and 8 respectively can not be correct.

Statement-II : Circumradius ≥ 2 (inradius)

Exercise # 3 Part # I [Matrix Match Type Questions]

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with **one or more** statement**(s)** in **Column-II**.

1.

Column–I		Column–II	
(A)	In a ΔABC, $2B = A + C$ and $b^2 = ac$. Then the value of $\frac{a^2(a+b+c)}{3abc}$ is equal to	**(p)**	8
(B)	In any right angled triangle ABC, the value of $\frac{a^2+b^2+c^2}{R^2}$ is always equal to (where R is the circumradius of ΔABC)	**(q)**	1
(C)	In a ΔABC if $a = 2$, $bc = 9$, then the value of $2R\Delta$ is equal to	**(r)**	5
(D)	In a ΔABC, $a = 5$, $b = 3$ and $c = 7$, then the value of $3\cos C + 7\cos B$ is equal to	**(s)**	9

2.

Column – I		Column – II	
(A)	In a ΔABC, $a = 4$, $b = 3$ and the medians AA_1 and BB_1 are mutually perpendicular, then square of area of the ΔABC is equal to	**(p)**	27
(B)	In any ΔABC, minimum value of $\frac{r_1 r_2 r_3}{r^3}$ is equal to	**(q)**	7
(C)	In a ΔABC, $a = 5$, $b = 4$ and $\tan\frac{C}{2} = \sqrt{\frac{7}{9}}$, then side 'c' is equal to	**(r)**	6
(D)	In a ΔABC, $2a^2 + 4b^2 + c^2 = 4ab + 2ac$, then value of $(8\cos B)$ is equal to	**(s)**	11

3. If p_1, p_2, p_3 are altitudes of a triangle ABC from the vertices A, B, C respectively and Δ is the area of the triangle and s is semi perimeter of the triangle, then match the columns

Column-I		Column-I	
(A)	If $\frac{1}{p_1}+\frac{1}{p_2}+\frac{1}{p_3}=\frac{1}{2}$ then the least value of $p_1p_2p_3$ is	**(p)**	$\frac{1}{R}$
(B)	The value of $\frac{\cos A}{p_1}+\frac{\cos B}{p_2}+\frac{\cos C}{p_3}$ is	**(q)**	216
(C)	The minimum value of $\frac{b^2p_1}{c}+\frac{c^2p_2}{a}+\frac{a^2p_3}{b}$ is	**(r)**	6Δ
(D)	The value of $p_1^{-2} + p_2^{-2} + p_3^{-2}$ is	**(s)**	$\frac{\Sigma a^2}{4\Delta^2}$

Part # II [Comprehension Type Questions]

Comprehension # 1

An altitude BD and a bisector BE are drawn in the triangle ABC from the vertex B. It is known that the length of side AC = 1, and the magnitudes of the angles BEC, ABD, ABE, BAC form an arithmetic progression.

1. The area of circle circumscribing ΔABC is

(A) $\frac{\pi}{8}$ **(B)** $\frac{\pi}{4}$ **(C)** $\frac{\pi}{2}$ **(D)** π

2. Let 'O' be the circumcentre of ΔABC, the radius of circle inscribed in ΔBOC is

(A) $\frac{1}{8\sqrt{3}}$ **(B)** $\frac{1}{4\sqrt{3}}$ **(C)** $\frac{1}{2\sqrt{3}}$ **(D)** $\frac{1}{2}$

3. Let B' be the image of point B with respect to side AC of ΔABC, then the length BB' is equal to

(A) $\frac{\sqrt{3}}{4}$ **(B)** $\frac{\sqrt{2}}{4}$ **(C)** $\frac{1}{\sqrt{2}}$ **(D)** $\frac{\sqrt{3}}{2}$

Comprehension # 2

The triangle formed by joining the three excentres I_1, I_2 and I_3 of Δ ABC is called the excentral or excentric triangle and in this case internal angle bisector of triangle ABC are the altitudes of triangles $I_1I_2I_3$

1. Incentre I of Δ ABC is the .... of the excentral $\Delta I_1I_2I_3$.

(A) Circumcentre **(B)** Orthocentre **(C)** Centroid **(D)** None of these

2. Angles of the $\Delta I_1I_2I_3$ are

(A) $\frac{\pi}{2}-\frac{A}{2}, \frac{\pi}{2}-\frac{B}{2}$ and $\frac{\pi}{2}-\frac{C}{2}$ **(B)** $\frac{\pi}{2}+\frac{A}{2}, \frac{\pi}{2}+\frac{B}{2}$ and $\frac{\pi}{2}+\frac{C}{2}$

(C) $\frac{\pi}{2}-A, \frac{\pi}{2}-B$ and $\frac{\pi}{2}-C$ **(D)** None of these

3. Sides of the $\Delta I_1I_2I_3$ are

(A) $R\cos\frac{A}{2}, R\cos\frac{B}{2}$ and $R\cos\frac{C}{2}$ **(B)** $4R\cos\frac{A}{2}, 4R\cos\frac{B}{2}$ and $4R\cos\frac{C}{2}$

(C) $2R\cos\frac{A}{2}, 2R\cos\frac{B}{2}$ and $2R\cos\frac{C}{2}$ **(D)** None of these

4. Value of $II_1^2 + I_2I_3^2 = II_2^2 + I_3I_1^2 = II_3^2 + I_1I_2^2 =$

(A) $4R^2$ **(B)** $16R^2$ **(C)** $32R^2$ **(D)** $64R^2$

Comprehension # 3

Let A_n be the area that is outside a n-sided regular polygon and inside it's circumscribing circle. Also B_n is the area inside the polygon and outside the circle inscribed in the polygon. Let R be the radius of the circle circumscribing n-sided polygon.

On the Basis of Above Information, Answer the Following Questions :

1. If n = 6 then A_n is equal to-

(A) $R^2\left(\frac{\pi-\sqrt{3}}{2}\right)$ **(B)** $R^2\left(\frac{2\pi-6\sqrt{3}}{2}\right)$ **(C)** $R^2\left(\pi-\sqrt{3}\right)$ **(D)** $R^2\left(\frac{2\pi-3\sqrt{3}}{2}\right)$

2. If n = 4 then B_n is equal to -

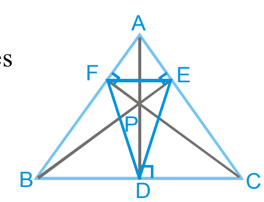
(A) $R^2\frac{(4-\pi)}{2}$ **(B)** $R^2\frac{(4-\pi\sqrt{2})}{2}$ **(C)** $R^2\frac{(4\sqrt{2}-\pi)}{2}$ **(D)** none of these

3. $\frac{A_n}{B_n}$ is equal to $\left(\theta=\frac{\pi}{n}\right)$ -

(A) $\frac{2\theta-\sin 2\theta}{\sin 2\theta-\theta\cos^2\theta}$ **(B)** $\frac{2\theta-\sin\theta}{\sin 2\theta-\theta\cos^2\theta}$ **(C)** $\frac{\theta-\cos\theta\sin\theta}{\cos\theta(\sin\theta-\theta\cos\theta)}$ **(D)** none of these

Comprehension # 4

The triangle DEF which is formed by joining the feet of the altitudes of triangle ABC is called the Pedal Triangle.



1. Angle of triangle DEF are

(A) $\pi-2A$, $\pi-2B$ and $\pi-2C$
(B) $\pi+2A$, $\pi+2B$ and $\pi+2C$
(C) $\pi-A$, $\pi-B$ and $\pi-C$
(D) None of these

2. Sides of triangle DEF are

(A) b cosA, a cosB, c cosC
(B) a cosA, b cosB, c cosC
(C) R sin 2A, R sin 2B, R sin 2C
(D) None of these

3. Circumraii of the triangle PBC, PCA and PAB are respectively

(A) R, R, R
(B) 2R, 2R, 2R
(C) R/2, R/2, R/2
(D) None of these

4. Which of the following is/are correct

(A) $\frac{\text{Perimeter of } \Delta DEF}{\text{Perimeter of } \Delta ABC}=\frac{r}{R}$
(B) Area of $\Delta DEF = 2\,\Delta \cos A\cos B\cos C$
(C) Area of $\Delta AEF = \Delta\cos^2 A$
(D) Circum-radius of $\Delta DEF = \frac{R}{2}$

Comprehension # 5

Consider a triangle ABC with b = 3. Altitude from the vertex B meets the opposite side in D, which divides AC internally in the ratio 1 : 2. A circle of radius 2 passes through the point A and D and touches the circumcircle of the triangle BCD at D.

1. If E is the centre of the circle with radius 2 then angle EDA equals

(A) $\sin^{-1}\left(\frac{\sqrt{15}}{4}\right)$ **(B)** $\sin^{-1}\left(\frac{3}{4}\right)$ **(C)** $\sin^{-1}\left(\frac{1}{4}\right)$ **(D)** $\sin^{-1}\left(\frac{15}{16}\right)$

2. If F is the circumcentre of the triangle BDC then which one of the following does **not** hold good ?

(A) $\angle FCD = \sin^{-1}\left(\frac{\sqrt{15}}{4}\right)$ **(B)** $\angle FDC = \cos^{-1}\left(\frac{1}{4}\right)$

(C) triangle DFC is an isosceles triangle **(D)** Area of $\Delta ADE = (1/4)^{th}$ of the area of Δ DBC

3. If R is the circumradius of the ΔABC, then R equal

(A) 4 **(B)** 6 **(C)** $2\left(\sqrt{\frac{61}{15}}\right)$ **(D)** $4\left(\sqrt{\frac{61}{15}}\right)$

Exercise # 4

[Subjective Type Questions]

1. In ΔABC, show that $a^2(s-a)(+ b^2(s-b) + c^2(s-c) = 4RD\left(1+4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right)$.

2. In any ΔABC, prove that $(b^2 - c^2)\cot A + (c^2 - a^2)\cot B + (a^2 - b^2)\cot C = 0$

3. ABCD is a trapezium such that AB, DC are parallel and BC is perpendicular to them. If angle ADB = θ, BC = p and CD = q, show that $AB = \frac{(p^2 + q^2)\sin\theta}{p\cos\theta + q\sin\theta}$.

4. If in a triangle ABC, $\frac{\cos A + 2\cos C}{\cos A + 2\cos B} = \frac{\sin B}{\sin C}$ prove that the triangle ABC is either isosceles or right angled.

5. In a Δ ABC, $\angle C = 60°$ and $\angle A = 75°$. If D is a point on AC such that the area of the Δ BAD is $\sqrt{3}$ times the area of the Δ BCD, find the $\angle$ ABD.

6. Three circles, whose radii are a, b and c, touch one another externally and the tangents at their points of contact meet in a point, prove that the distance of this point from either of their points of contact is $\left(\frac{abc}{a+b+c}\right)^{\frac{1}{2}}$.

7. In any ΔABC, prove that

(i) $(r_3 + r_1)(r_3 + r_2)\sin C = 2r_3\sqrt{r_2 r_3 + r_3 r_1 + r_1 r_2}$

(ii) $\frac{\tan\frac{A}{2}}{(a-b)(a-c)} + \frac{\tan\frac{B}{2}}{(b-a)(b-c)} + \frac{\tan\frac{C}{2}}{(c-a)(c-b)} = \frac{1}{\Delta}$

(iii) $(r+r_1)\tan\frac{B-C}{2} + (r+r_2)\tan\frac{C-A}{2} + (r+r_3)\tan\frac{A-B}{2} = 0$

(iv) $r^2 + r_1^2 + r_2^2 + r_3^2 = 16R^2 - a^2 - b^2 - c^2$.

8. DEF is the triangle formed by joining the points of contact of the incircle with the sides of the triangle ABC; prove that

(i) its sides are $2r\cos\frac{A}{2}$, $2r\cos\frac{B}{2}$ and $2r\cos\frac{C}{2}$,

(ii) its angles are $\frac{\pi}{2} - \frac{A}{2}$, $\frac{\pi}{2} - \frac{B}{2}$ and $\frac{\pi}{2} - \frac{C}{2}$ and

(iii) its area is $\frac{2\Delta^3}{(abc)s}$, i.e. $\frac{1}{2}\frac{r}{R}\Delta$.

Exercise # 5 Part # I [Previous Year Questions] [AIEEE/JEE-MAIN]

1. The sum of the radii of inscribed and circumscribed circles for an n sided regular polygon of side 'a', is : **[AIEEE - 2003]**

(1) $a \cot\left(\frac{\pi}{n}\right)$ (2) $\frac{a}{2}\cot\left(\frac{\pi}{2n}\right)$ (3) $a \cot\left(\frac{\pi}{2n}\right)$ (4) $\frac{a}{4}\cot\left(\frac{\pi}{2n}\right)$

2. If in a triangle ABC, $a \cos^2\left(\frac{C}{2}\right) + c \cos^2\left(\frac{A}{2}\right) = \frac{3b}{2}$, then the sides a, b and c : **[AIEEE - 2003]**

(1) are in A.P. (2) are in G.P. (3) are in H.P. (4) satisfy a + b = c.

3. In a triangle ABC, medians AD and BE are drawn. If AD = 4, $\angle DAB = \frac{\pi}{6}$ and $\angle ABE = \frac{\pi}{3}$, then the area of the ΔABC is : **[AIEEE - 2003]**

(1) $\frac{8}{3}$ (2) $\frac{16}{3}$ (3) $\frac{32}{3\sqrt{3}}$ (4) $\frac{64}{3}$.

4. The sides of a triangle are $\sin\alpha$, $\cos\alpha$ and $\sqrt{1+\sin\alpha\cos\alpha}$ for some $0 < \alpha < \frac{\pi}{2}$. Then the greatest angle of the triangle is : **[AIEEE - 2004]**

(1) 60º (2) 90º (3) 120º (4) 150º

5. In a triangle ABC, let $\angle C = \pi/2$, if r is the inradius and R is the circumradius of the triangle ABC, then 2(r + R) equals : **[AIEEE - 2005]**

(1) c + a (2) a + b + c (3) a + b (4) b + c

6. If in a ΔABC, the altitudes from the vertices A,B,C on opposite sides are in H.P., then sinA, sinB, sinC are in: **[AIEEE - 2005]**

(1) HP (2) Arithemetico-Geometric Progression

(3) AP (4) GP

7. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A **false** statement among the following is **[AIEEE - 2010]**

(1) There is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$. (2) There is a regular polygon with $\frac{r}{R} = \frac{2}{3}$.

(3) There is a regular polygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$. (4) There is a regular polygon with $\frac{r}{R} = \frac{1}{2}$.

8. ABCD is a trapezium such that AB and CD are parallel and $BC \perp CD$. If $\angle ADB = \theta$, BC = p and CD = q, then AB is equal to : **[AIEEE - 2013]**

(1) $\frac{(p^2+q^2)\sin\theta}{p\cos\theta + q\sin\theta}$ (2) $\frac{p^2+q^2\cos\theta}{p\cos\theta + q\sin\theta}$ (3) $\frac{p^2+q^2}{p^2\cos\theta + q^2\sin\theta}$ (4) $\frac{(p^2+q^2)\sin\theta}{(p\cos\theta + q\sin\theta)^2}$

Part # II [Previous Year Questions][IIT-JEE ADVANCED]

1. Let PQ and RS be tangents at the extremities of the diameter PR of a circle of radius r. If PS and RQ intersect at a point X on the circumference of the circle, then 2r equals **[IIT-JEE-2001]**

(A) $\sqrt{PQ.RS}$ **(B)** $\frac{PQ+RS}{2}$ **(C)** $\frac{2PQ.RS}{PQ+RS}$ **(D)** $\sqrt{\frac{PQ^2+RS^2}{2}}$

2. If I_n is the area of n sided regular polygon inscribed in a circle of unit radius and O_n be the area of the polygon circumscribing the given circle, prove that $I_n = \frac{O_n}{2}\left(1+\sqrt{1-\left(\frac{2I_n}{n}\right)^2}\right)$. **[IIT-JEE-2003]**

3. If the angles of a triangle are in the ratio 4 : 1 : 1, then the ratio of the longest side to the perimeter is– **[IIT-JEE-2003]**

(A) $\sqrt{3} : (2+\sqrt{3})$ **(B)** $1 : \sqrt{3}$ **(C)** $1 : 2+\sqrt{3}$ **(D)** 2 : 3

4. If a,b,c are the sides of a triangle such that a : b : c = 1 : $\sqrt{3}$: 2, then ratio A : B : C is equal to – **[IIT-JEE-2004]**

(A) 3 : 2 : 1 **(B)** 3 : 1 : 2 **(C)** 1 : 2 : 3 **(D)** 1 : 3 : 2

5. If a,b,c denote the lengths of the sides of a triangle opposite to angles A,B,C respectively of a ΔABC, then the correct relation among a,b,c, A,B and C is given by – **[IIT-JEE-2005]**

(A) $(b+c)\sin\left(\frac{B+C}{2}\right) = a\cos\frac{A}{2}$ **(B)** $(b-c)\cos\frac{A}{2} = a\sin\left(\frac{B-C}{2}\right)$

(C) $(b-c)\cos\frac{A}{2} = 2a\sin\left(\frac{B-C}{2}\right)$ **(D)** $(b-c)\sin\left(\frac{B-C}{2}\right) = a\cos\frac{A}{2}$

6. Circles with radii 3, 4 and 5 touch each other externally. If P is the point of intersection of tangents to these circles at their points of contact, find the distance of P from the points of contact. **[IIT-JEE-2005]**

7. Given an isosceles triangle, whose one angle is 120° and radius of its incircle is $\sqrt{3}$ unit. Then the area of the triangle in sq. units is **[IIT-JEE-2006]**

(A) $7+12\sqrt{3}$ **(B)** $12-7\sqrt{3}$ **(C)** $12+7\sqrt{3}$ **(D)** 4π

8. Internal bisector of $\angle A$ of triangle ABC meets side BC at D. A line drawn through D perpendicular to AD intersects the side AC at E and the side AB at F. If a, b, c represent sides of ΔABC, then **[IIT-JEE-2006]**

(A) AE is HM of b and c **(B)** $AD = \frac{2bc}{b+c}\cos\frac{A}{2}$

(C) $EF = \frac{4bc}{b+c}\sin\frac{A}{2}$ **(D)** the triangle AEF is isosceles

9. Let ABC and ABC′ be two non-congruent triangles with sides $AB = 4, AC = AC' = 2\sqrt{2}$ and angle B= 30º. Find the absolute value of the difference between the areas of these triangles. **[IIT-JEE 2009]**

10. In a triangle ABC with fixed base BC, the vertex A moves such that $\cos B + \cos C = 4\sin^2\frac{A}{2}$. If a, b and c denote the lengths of the sides of the triangle opposite to the angles A, B and C respectively, then **[IIT-JEE 2009]**

(A) $b + c = 4a$ **(B)** $b + c = 2a$
(C) locus of points A is an ellipse **(D)** locus of point A is a pair of straight lines

11. If the angle A, B and C of a triangle are in arithmetic progression and if a, b and c denote the lengths of the sides opposite to A, B and C respectively, then the value of the expression $\frac{a}{c}\sin 2C + \frac{c}{a}\sin 2A$ is **[IIT-JEE 2010]**

(A) $\frac{1}{2}$ **(B)** $\frac{\sqrt{3}}{2}$ **(C)** 1 **(D)** $\sqrt{3}$

12. Let ABC be a triangle such that $\angle ACB = \frac{\pi}{6}$ and let a, b and c denote the lengths of the sides opposite to A, B and C respectively. The value(s) of x for which $a = x^2 + x + 1$, $b = x^2 - 1$ and $c = 2x + 1$ is (are) **[IIT-JEE 2010]**

(A) $-\left(2+\sqrt{3}\right)$ **(B)** $1 + \sqrt{3}$ **(C)** $2 + \sqrt{3}$ **(D)** $4\sqrt{3}$

13. Consider a triangle ABC and let a, b and c denote the lengths of the sides opposite to vertices A, B and C respectively. Suppose $a = 6$, $b = 10$ and the area of the triangle is $15\sqrt{3}$. If $\angle ACB$ is obtuse and if r denotes the radius of the incircle of the triangle, then r^2 is equal to **[IIT-JEE 2010]**

14. Let PQR be a triangle of area Δ with $a = 2$, $b = \frac{7}{2}$ and $c = \frac{5}{2}$, where a, b and c are the lengths of the sides of the triangle opposite to the angles at P, Q and R respectively. Then $\frac{2\sin P - \sin 2P}{2\sin P + \sin 2P}$ equals **[IIT-JEE 2012]**

(A) $\frac{3}{4\Delta}$ **(B)** $\frac{45}{4\Delta}$ **(C)** $\left(\frac{3}{4\Delta}\right)^2$ **(D)** $\left(\frac{45}{4\Delta}\right)^2$

15. In a triangle PQR, P is the largest angle and $\cos P = \frac{1}{3}$. Further the incircle of the triangle touches the sides PQ, QR and RP at N, L and M respectively, such that the lengths of PN, QL and RM are consecutive even integers. Then possible length(s) of the side(s) of the triangle is (are) **[JEE (Ad.) 2013]**

(A) 16 **(B)** 18 **(C)** 24 **(D)** 22

16. In a triangle the sum of two sides is x and the product of the same two sides is y. If $x^2 - c^2 = y$, where c is the third side of the triangle, then the ratio of the in-radius to the circum-radius of the triangle is **[JEE Ad. 2014]**

(A) $\frac{3y}{2x(x+c)}$ **(B)** $\frac{3y}{2c(x+c)}$ **(C)** $\frac{3y}{4x(x+c)}$ **(D)** $\frac{3y}{4c(x+c)}$

MOCK TEST

SECTION - I : STRAIGHT OBJECTIVE TYPE

1. In ΔABC let L, M, N be the feet of the altitudes. Then $\sin \angle MLN + \sin \angle LMN + \sin \angle MNL$ equals to

(A) $4 \sin A \sin B \sin C$
(B) $4 \cos A \cos B \cos C$
(C) $\tan A + \tan B + \tan C$
(D) None of these

2. In a triangle ABC, if a : b : c = 7 : 8 : 9, then cos A : cos B equals to

(A) $\frac{11}{63}$ (B) $\frac{22}{63}$ (C) $\frac{2}{9}$ (D) $\frac{14}{11}$

3. In ΔABC, let AD be the median and O, G, P be respectively the circumcentre, centroid and orthocentre. Then ΔOGD is directly similar to

(A) ΔABC (B) ΔPAG (C) ΔPGA (D) None of these

4. In a ΔABC $\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2}$ is equal to

(A) $\frac{\Delta}{r^2}$ (B) $\frac{(a+b+c)^2}{abc} . 2R$ (C) $\frac{\Delta}{r}$ (D) $\frac{\Delta}{Rr}$

5. In ΔABC if $\tan\frac{C}{2}(a \tan A + b \tan B) = a + b$, then the triangle is

(A) Right angled (B) Isosceles (C) Equilateral (D) Obtuse angled

6. In a triangle ABC, if $\angle A = 30^\circ$ and $BC = 2 + \sqrt{5}$, then the distance of the vertex A from the orthocentre of the triangle is

(A) 1 (B) $\left(2+\sqrt{5}\right)\sqrt{3}$ (C) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ (D) $\frac{1}{2}$

7. Consider a given acute angled triangle ABC having O as its circumcentre. Let D be a variable interior point on the side BC. The limiting value of the circumradius of the ΔOCD as point D approaches towards vertex C is equal to

(A) $\frac{R}{2\cos A}$ (B) $\frac{R}{\cos A}$ (C) $\frac{R}{\sin A}$ (D) $\frac{R}{2\sin A}$

8. Four points A, B, C, D are in a plane so that B is in line joining A and C. Also B is due north of D and D is due west of C. BD = 2, $\angle BDA = 45^\circ$ and $\angle BCD = 30^\circ$. Then AD equals to :

(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) $3\sqrt{3}$ (D) None of these

9. ABCD is a quadrilateral circumscribed about a circle of unit radius, then

(A) $AB \sin \frac{C}{2} . \sin \frac{A}{2} = CD \sin \frac{B}{2} \sin \frac{D}{2}$
(B) $AB \sin \frac{A}{2} . \sin \frac{B}{2} = CD \sin \frac{C}{2} \sin \frac{D}{2}$
(C) $AB \sin \frac{A}{2} . \sin \frac{A}{2} = CD \sin \frac{C}{2} \sin \frac{B}{2}$
(D) $AB \sin \frac{A}{2} . \cos \frac{B}{2} = CD \sin \frac{C}{2} \cos \frac{D}{2}$

10. In a ΔABC, following relations hold good . In which case(s) the triangle is a right angled triangle ? (Assume all symbols have their usual meaning)

S_1 : $r_2 + r_3 = r_1 - r$

S_2 : $a^2 + b^2 + c^2 = 8R^2$

S_3 : If the diameter of an excircle be equal to the perimeter of the triangle.

S_4 : $2R = r_1 - r$

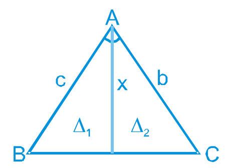
(A) TFTT **(B)** FFTT **(C)** TFTF **(D)** TTTT

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

11. In a triangle ABC, with usual notations the length of the bisector of internal angle A is

(A) $\dfrac{2bc\cos\frac{A}{2}}{b+c}$ **(B)** $\dfrac{2bc\sin\frac{A}{2}}{b+c}$

(C) $\dfrac{abc\,\text{cosec}\frac{A}{2}}{2R(b+c)}$ **(D)** none,



where Δ is the area of triangle ABC.

12. If in a triangle ABC, p, q and r are the altitudes drawn from the vertices A, B, C respectively to the opposite sides, then which of the following hold(s) good.

(A) $(\Sigma p)\left(\Sigma \dfrac{1}{p}\right) = (\Sigma a)\left(\Sigma \dfrac{1}{a}\right)$

(B) $(\Sigma p)(\Sigma a) = \left(\Sigma \dfrac{1}{p}\right)\left(\Sigma \dfrac{1}{a}\right)$

(C) $(\Sigma p)(\Sigma pq)(\Pi a) = (\Sigma a)(\Sigma ab)(\Pi p)$

(D) $\left(\Sigma \dfrac{1}{p}\right)\Pi\left(\dfrac{1}{p}+\dfrac{1}{q}-\dfrac{1}{r}\right)\Pi a^2 = 16R^2$, where R is the circum-radius of ΔABC.

13. The sides of a ΔABC satisfy the equation $2a^2 + 4b^2 + c^2 = 4ab + 2ac$. Then

(A) the triangle is isosceles. **(B)** the triangle is obtuse.

(C) $B = \cos^{-1}\dfrac{7}{8}$ **(D)** $A = \cos^{-1}\dfrac{1}{4}$

14. Let ABC be an isosceles triangle with base BC . If 'r' is the radius of the circle inscribed in the ΔABC and ρ be the radius of the circle described opposite to the angle A, then the product ρr can be equal to :

(A) $R^2\sin^2 A$ **(B)** $R^2\sin^2 2B$ **(C)** $\dfrac{1}{2}a^2$ **(D)** $\dfrac{a^2}{4}$

where R is the radius of the circumcircle of the ΔABC

15. In ΔABC, if $r_1 : r_2 : r_3 = 6 : 3 : 2$, then

(A) $\dfrac{a}{b}=\dfrac{5}{4}$ **(B)** $\dfrac{b}{c}=\dfrac{2}{3}$ **(C)** $\dfrac{c}{a}=\dfrac{3}{5}$ **(D)** $\dfrac{a}{5}=\dfrac{b}{4}=\dfrac{c}{2}$

SECTION - III : ASSERTION AND REASON TYPE

16. All the notations used in statement-1 and statement-2 are usual.

Statement-I : In triangle ABC, if $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$, then value of $\frac{r_1 + r_2 + r_3}{r}$ is equal to 9.

Statement-II : In Δ ABC : $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$, where R is circumradius.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I.

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

17. **Statement-I :** If I is incentre of Δ ABC and I_1 is excentre opposite to A and P is the intersection of II_1 and BC, then IP. I_1P = BP. PC

Statement-II : In a Δ ABC, I is incentre and I_1 is excentre opposite to A. then IBI_1C must be a square.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I.

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

18. **Statement-I :** In a ΔABC, $\sum \frac{\cos^2 \frac{A}{2}}{a}$ has the value equal to $\frac{s^2}{abc}$

Statement-II : In a Δ ABC, $\cos\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$, $\cos\frac{B}{2} = \sqrt{\frac{(s-a)(s-c)}{ac}}$, $\cos\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I.

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

19. **Statement-I :** If the sides of a triangle are 13, 14, 15, then the radius of incircle is equal to 4 unit.

Statement-II : In a Δ ABC, $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{a+b+c}{2}$ and $r = \frac{\Delta}{s}$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I.

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

20. **Statement -I :** In a ΔABC, if $a < b < c$ and r is inradius and r_1, r_2, r_3 are the exradii opposite to angle A, B, C respectively, then $r < r_1 < r_2 < r_3$

Statement-II : For a ΔABC $r_1 r_2 + r_2 r_3 + r_3 r_1 = \frac{r_1 r_2 r_3}{r}$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I.

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

SECTION - IV : MATRIX - MATCH TYPE

21.

Column – I		Column – II	
(A)	In a ΔABC, $(a+b+c)(b+c-a) = \lambda bc$, where $\lambda \in I$, then greatest value of λ is	**(p)**	3
(B)	In a ΔABC, $\tan A + \tan B + \tan C = 9$. If $\tan^2 A + \tan^2 B + \tan^2 C = k$, then least value of k satisfying is	**(q)**	$9(3)^{1/3}$
(C)	In a triangle ABC, then line joining the circumcenter to the incentre is parallel to BC, then value of $\cos B + \cos C$ is	**(r)**	1
(D)	If in a ΔABC, $a = 5$, $b = 4$ and $\cos(A-B) = \frac{31}{32}$, then the third side 'c' is equal to	**(s)**	6
		(t)	2

22.

Column – I		Column – II	
(A)	If $\cos A = \frac{\sin B}{2\sin C}$, then ΔABC is	**(p)**	isosceles
(B)	If $\frac{\cos A + 2\cos C}{\cos A + 2\cos B} = \frac{\sin B}{\sin C}$, then ΔABC may be	**(q)**	obtuse angle
(C)	If $\frac{2\cos A}{a} + \frac{\cos B}{b} + \frac{2\cos C}{c} = \frac{a}{bc} + \frac{b}{ca}$, then ΔABC is	**(r)**	right angle
(D)	If $\frac{a^2 - b^2}{a^2 + b^2} = \frac{\sin(A-B)}{\sin(A+B)}$, then ΔABC may be	**(s)**	acute angle
		(t)	equilateral

SECTION - V : COMPREHENSION TYPE

23. **Read the following comprehension carefully and answer the questions.**

G is the centroid of triangle ABC. Perpendiculars from vertices A, B, C meet the sides BC, CA, AB at D, E, F respectively. P, Q, R are feet of the perpendiculars from G on sides BC, CA, AB respectively. L, M, N are the mid points of sides BC, CA, AB respectively, then

1. Length of the side PG is

(A) $\frac{1}{2}$ b sin C **(B)** $\frac{1}{2}$ c sin C **(C)** $\frac{2}{3}$ b sin C **(D)** $\frac{1}{3}$ c sin B

2. (Area of ΔGPL) to (Area of ΔALD) is equal to

(A) $\frac{1}{3}$ **(B)** $\frac{1}{9}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{4}{9}$

3. Area of ΔPQR is

(A) $\frac{1}{9}(a^2 + b^2 + c^2) \sin A \sin B \sin C$ **(B)** $\frac{1}{18}(a^2 + b^2 + c^2) \sin A \sin B \sin C$

(C) $\frac{2}{9}(a^2 + b^2 + c^2) \sin A \sin B \sin C$ **(D)** $\frac{1}{3}(a^2 + b^2 + c^2) \sin A \sin B \sin C$

24. **Read the following comprehension carefully and answer the questions.**

Consider a triangle ABC, where x, y, z are the length of perpendicular drawn from the vertices of the triangle to the opposite sides a, b, c respectively and let the letters R, r, S, Δ denote the circumradius, inradius semi-perimeter and area of the triangle respectively.

1. If $\frac{bx}{c} + \frac{cy}{a} + \frac{az}{b} = \frac{a^2 + b^2 + c^2}{k}$, then the value of k is :–

(A) R **(B)** S **(C)** 2R **(D)** $\frac{3}{2}$ R

2. If $\cot A + \cot B + \cot C = k\left(\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2}\right)$, then the value of k is

(A) R^2 **(B)** rR **(C)** Δ **(D)** $a^2 + b^2 + c^2$

3. The value of $\frac{c\sin B + b\sin C}{x} + \frac{a\sin C + c\sin A}{y} + \frac{b\sin A + a\sin B}{z}$ is equal to

(A) $\frac{R}{r}$ **(B)** $\frac{S}{R}$ **(C)** 2 **(D)** 6

25. **Read the following comprehension carefully and answer the questions.**

Let a, b, c are the sides opposite to angle A, B, C respectively in a ΔABC and

$\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2}$ and $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$. If a = 6, b = 3 and $\cos(A - B) = \frac{4}{5}$

1. Angle C is equal to

(A) $\frac{\pi}{4}$ **(B)** $\frac{\pi}{2}$ **(C)** $\frac{3\pi}{4}$ **(D)** $\frac{2\pi}{3}$

2. Area of the triangle is equal to

(A) 8 **(B)** 9 **(C)** 10 **(D)** 11

3. Value of sin A is equal to

(A) $\frac{1}{\sqrt{5}}$ **(B)** $\frac{2}{\sqrt{5}}$ **(C)** $\frac{1}{2\sqrt{5}}$ **(D)** $\frac{1}{\sqrt{3}}$

SECTION - VI : INTEGER TYPE

26. In ΔABC, if r = 1, r = 3, and s = 5, then the value of $\frac{a^2+b^2+c^2}{3}$.

27. The sides of triangle ABC satisfy the relations $a + b - c = 2$ and $2ab - c^2 = 4$, then find the square of the area of triangle.

28. If p_1, p_2 and p_3 are the altitudes of a triangle from vertices A , B and C respectively, and Δ is the area of the triangle and $\frac{1}{p_1}+\frac{1}{p_2}-\frac{1}{p_3}=\frac{\lambda ab}{(a+b+c)\Delta}\cos^2\frac{C}{2}$, then find the value of λ.

29. In a ΔABC ,if the angles A, B, C are in A.P. and $\lambda\cos\frac{A-C}{2}=\frac{a+c}{\sqrt{a^2-ac+c^2}}$, then find the value of λ.

30. Let ABC be a triangle with altitudes h_1, h_2, h_3 and inradius r and $\frac{h_1+r}{h_1-r}+\frac{h_2+r}{h_2-r}+\frac{h_3+r}{h_3-r}\geq\lambda$, then find the value of λ.

ANSWER KEY

EXERCISE - 1

1. C **2.** A **3.** B **4.** B **5.** B **6.** B **7.** C **8.** D **9.** C **10.** A **11.** A **12.** C **13.** C
14. C **15.** B **16.** C **17.** C **18.** A **19.** C **20.** B **21.** A **22.** A **23.** A **24.** A **25.** D **26.** D
27. B **28.** B **29.** A **30.** C **31.** C **32.** C **33.** C **34.** C **35.** A **36.** D **37.** B **38.** C **39.** D
40. B

EXERCISE - 2 : PART # I

1. ABC **2.** AB **3.** BC **4.** ACD **5.** AD **6.** ABD **7.** AB **8.** CD **9.** ACD
10. ABCD **11.** ACD **12.** AC **13.** ACD **14.** AC **15.** BD

PART - II

1. A **2.** A **3.** C **4.** C **5.** A

EXERCISE - 3 : PART # I

1. $A \to q$ $B \to p$ $C \to s$ $D \to r$ **2.** $A \to s$ $B \to p$ $C \to r$ $D \to q$ **3.** $A \to q$ $B \to p$ $C \to r$ $D \to s$

PART - II

Comprehension # 1 : 1. B **2.** B **3.** D **Comprehension # 2 : 1.** B **2.** A **3.** B **4.** A
Comprehension # 3 : 1. D **2.** A **3.** C **Comprehension # 4 : 1.** A **2.** BC **3.** A **4.** ABCD
Comprehension # 5 : 1. A **2.** D **3.** C

EXERCISE - 5 : PART # I

1. 2 **2.** 1 **3.** 3 **4.** 3 **5.** 3 **6.** 3 **7.** 2 **8.** 1

PART - II

1. A **3.** A **4.** C **5.** B **6.** $\sqrt{5}$ **7.** C **8.** ABCD **9.** 4 **10.** BC **11.** D **12.** B **13.** 3
14. C **15.** BD **16.** B

MOCK TEST

1. A **2.** D **3.** C **4.** A **5.** B **6.** B **7.** A **8.** D **9.** B **10.** D **11.** AC **12.** ACD
13. ACD **14.** A **15.** AC **16.** A **17.** C **18.** C **19.** A **20.** B
21. $A \to p$ $B \to q$ $C \to r$ $D \to s$ **22.** $A \to p$ $B \to p,r$ $C \to r$ $D \to p,r$
23. 1. D **2.** B **3.** B **24. 1.** C **2.** C **3.** D **25. 1.** B **2.** B **3.** B
26. 8 **27.** 3 **28.** 2 **29.** 2 **30.** 6