SOLVED EXAMPLES

Ex. 1 Find

(a) The coefficient of x^7 in the expansion of $\left(ax^2+\frac{1}{bx}\right)^{11}$

(b) The coefficient of x^{-7} in the expansion of $\left(ax-\frac{1}{bx^2}\right)^{11}$

Also, find the relation between a and b, so that these coefficients are equal.

Sol. (a) In the expansion of $\left(ax^2+\frac{1}{bx}\right)^{11}$, the general term is :

$$T_{r+1} = {}^{11}C_r(ax^2)^{11-r}\left(\frac{1}{bx}\right)^r = {}^{11}C_r.\frac{a^{11-r}}{b^r}.x^{22-3r}$$

putting $22 - 3r = 7$

$\therefore$ $3r = 15$ $\Rightarrow$ $r = 5$

$\therefore$ $T_6 = {}^{11}C_5\frac{a^6}{b^5}.x^7$

Hence the coefficient of x^7 in $\left(ax^2+\frac{1}{bx}\right)^{11}$ is ${}^{11}C_5a^6b^{-5}$.

Note that binomial coefficient of sixth term is ${}^{11}C_5$.

(b) In the expansion of $\left(ax-\frac{1}{bx^2}\right)^{11}$, general term is :

$$T_{r+1} = {}^{11}C_r(ax)^{11-r}\left(\frac{-1}{bx^2}\right)^r = (-1)^r\,{}^{11}C_r\frac{a^{11-r}}{b^r}.x^{11-3r}$$

putting $11 - 3r = -7$

$\therefore$ $3r = 18$ $\Rightarrow$ $r = 6$

$\therefore$ $T_7 = (-1)^6.\ {}^{11}C_6\frac{a^5}{b^6}.x^{-7}$

Hence the coefficient of x^{-7} in $\left(ax-\frac{1}{bx^2}\right)^{11}$ is ${}^{11}C_6a^5b^{-6}$.

Also given :

Coefficient of x^7 in $\left(ax^2+\frac{1}{bx}\right)^{11}$ = coefficient of x^{-7} in $\left(ax-\frac{1}{bx^2}\right)^{11}$

$\Rightarrow$ ${}^{11}C_5a^6b^{-5} = {}^{11}C_6a^5b^{-6}$

$\Rightarrow$ $ab = 1$ $\quad(\because\ {}^{11}C_5 = {}^{11}C_6)$

which is the required relation between a and b.

Ex. 2 Find the numerically greatest term in the expansion of $(3 - 5x)^{15}$ when $x = \frac{1}{5}$.

Sol. Let r^{th} and $(r + 1)^{th}$ be two consecutive terms in the expansion of $(3 - 5x)^{15}$

$$T_{r+1} \geq T_r$$

$$^{15}C_r\, 3^{15-r}\, (|-5x|)^r \geq {}^{15}C_{r-1}\, 3^{15-(r-1)}\, (|-5x|)^{r-1}$$

$$\frac{(15)!}{(15-r)!\,r!}\,|-5x| \geq \frac{3.(15)!}{(16-r)!\,(r-1)!}$$

$$5\,.\,\frac{1}{5}\,(16-r) \geq 3r$$

$$16 - r \geq 3r$$
$$4r \leq 16$$
$$r \leq 4$$

Ex. 3 Given T_3 in the expansion of $(1 - 3x)^6$ has maximum numerical value. Find the range of 'x'.

Sol. Using $\dfrac{n+1}{1+\left|\frac{a}{b}\right|} - 1 \leq r \leq \dfrac{n+1}{1+\left|\frac{a}{b}\right|}$

$$\frac{6+1}{1+\left|\frac{1}{-3x}\right|} - 1 \leq 2 \leq \frac{7}{1+\left|\frac{1}{-3x}\right|}$$

Let $|x| = t$

$$\frac{21t}{3t+1} - 1 \leq 2 \leq \frac{21t}{3t+1}$$

$$\begin{cases} \dfrac{21t}{3t+1} \leq 3 \\ \dfrac{21t}{3t+1} \geq 2 \end{cases} \Rightarrow \begin{cases} \dfrac{4t-1}{3t+1} \leq 0 \Rightarrow t \in \left[-\dfrac{1}{3}, \dfrac{1}{4}\right] \\ \dfrac{15t-2}{3t+1} \geq 0 \Rightarrow t \in \left(-\infty, -\dfrac{1}{3}\right] \cup \left[\dfrac{2}{15}, \infty\right) \end{cases}$$

Common solution $\quad t \in \left[\dfrac{2}{15}, \dfrac{1}{4}\right] \quad \Rightarrow \quad x \in \left[-\dfrac{1}{4}, -\dfrac{2}{15}\right] \cup \left[\dfrac{2}{15}, \dfrac{1}{4}\right]$

Ex. 4 Find the last two digits of the number $(17)^{10}$.

Sol. $(17)^{10} = (289)^5 = (290 - 1)^5$

$= {}^5C_0\,(290)^5 - {}^5C_1\,(290)^4 + \ldots\ldots.. + {}^5C_4\,(290)^1 - {}^5C_5\,(290)^0$

$= {}^5C_0\,(290)^5 - {}^5C_1\,.\,(290)^4 + \ldots\ldots\ldots{}^5C_3\,(290)^2 + 5 \times 290 - 1$

= A multiple of 1000 + 1449

Hence, last two digits are 49

Ex. 5 Find the number of rational terms in the expansion of $(9^{1/4} + 8^{1/6})^{1000}$.

Sol. The general term in the expansion of $(9^{1/4} + 8^{1/6})^{1000}$ is

$$T_{r+1} = {}^{1000}C_r \left(9^{\frac{1}{4}}\right)^{1000-r} \left(8^{\frac{1}{6}}\right)^{r} = {}^{1000}C_r\ 3^{\frac{1000-r}{2}}\ 2^{\frac{r}{2}}$$

The above term will be rational if exponents of 3 and 2 are integers

It means $\dfrac{1000-r}{2}$ and $\dfrac{r}{2}$ must be integers

The possible set of values of r is {0, 2, 4, ..., 1000}

Hence, number of rational terms is 501

Ex. 6 Show that the integer just above $(\sqrt{3} + 1)^{2n}$ is divisible by 2^{n+1} for all $n \in N$.

Sol. Let $(\sqrt{3} + 1)^{2n} = (4 + 2\sqrt{3})^n = 2^n (2 + \sqrt{3})^n = I + f$ ....(i)
where I and f are its integral & fractional parts respectively
$0 < f < 1$.

Now $0 < \sqrt{3} - 1 < 1$

$0 < (\sqrt{3} - 1)^{2n} < 1$

Let $(\sqrt{3} - 1)^{2n} = (4 - 2\sqrt{3})^n = 2^n (2 - \sqrt{3})^n = f'$. ....(ii)
$0 < f' < 1$
adding (i) and (ii)

$I + f + f' = (\sqrt{3} + 1)^{2n} + (\sqrt{3} - 1)^{2n}$

$= 2^n [(2 + \sqrt{3})^n + (2 - \sqrt{3})^n] = 2.2^n [{}^nC_0 2^n + {}^nC_2 2^{n-2} (\sqrt{3})^2 + ...]$

$I + f + f' = 2^{n+1} k$ (where k is a positive integer)
$0 < f + f' < 2 \Rightarrow f + f' = 1$
$I + 1 = 2^{n+1} k$.

$I + 1$ is the integer just above $(\sqrt{3} + 1)^{2n}$ and which is divisible by 2^{n+1}.

Ex. 7 If $(1 + x)^n = \sum_{r=0}^{n} {}^nC_r x^r$, then prove that $C_1^2 + 2.C_2^2 + 3.C_3^2 + ... + n.C_n^2 = \dfrac{(2n-1)!}{((n-1)!)^2}$

Sol. $(1 + x)^n = C_0 + C_1x + C_2x^2 + C_3x^3 + ... + C_nx^n$ .....(i)
Differentiating both the sides, w.r.t. x, we get

$n(1 + x)^{n-1} = C_1 + 2C_2x + 3C_3x^2 + ... + n.C_nx^{n-1}$ .....(ii)

also, we have

$(x + 1)^n = C_0x^n + C_1x^{n-1} + C_2x^{n-2} + ... + C_n$ .....(iii)

Multiplying (ii) & (iii), we get

$(C_1 + 2C_2x + 3C_3x^2 + ... + C_nx^{n-1})(C_0x^n + C_1x^{n-1} + C_2x^{n-2} + ... + C_n) = n(1 + x)^{2n-1}$

Equating the coefficients of x^{n-1}, we get

$$C_1^2 + 2C_2^2 + 3C_3^2 + ... + n.C_n^2 = n.\,{}^{2n-1}C_{n-1} = \frac{(2n-1)!}{((n-1)!)^2}$$

Ex. 8: If $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots\ldots\ldots\ldots + c_nx^n$, then show that

(i) $C_0 + 3C_1 + 3^2C_2 + \ldots\ldots\ldots + 3^n C_n = 4^n$.

(ii) $C_0 + 2C_1 + 3.\,C_2 + \ldots\ldots\ldots + (n+1)\,C_n = 2^{n-1}(n+2)$.

(iii) $C_0 - \dfrac{C_1}{2} + \dfrac{C_2}{3} - \dfrac{C_3}{4} + \ldots\ldots\ldots + (-1)^n \dfrac{C_n}{n+1} = \dfrac{1}{n+1}$.

Sol. (i) $(1+x)^n = C_0 + C_1\,x + C_2x^2 + \ldots\ldots\ldots + C_nx^n$

put $x = 3$

$$C_0 + 3\,.\,C_1 + 3^2\,.\,C_2 + \ldots\ldots\ldots + 3^n\,.\,C_n = 4n$$

(ii) **I Method : By Summation**

L.H.S. $= {}^nC_0 + 2.\,{}^nC_1 + 3\,.\,{}^nC_2 + \ldots\ldots\ldots + (n+1).\,{}^nC_n$.

$$= \sum_{r=0}^{n}(r+1)\,.\,{}^nC_r = \sum_{r=0}^{n} r.\,{}^nC_r + \sum_{r=0}^{n}{}^nC_r$$

$$= n\sum_{r=0}^{n}{}^{n-1}C_{r-1} + \sum_{r=0}^{n}{}^nC_r = n\,.\,2^{n-1} + 2^n = 2^{n-1}(n+2). \qquad \text{RHS}$$

II Method : By Differentiation

$(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots\ldots\ldots + C_nx^n$

Multiplying both sides by x,

$x(1+x)^n = C_0x + C_1x^2 + C_2x^3 + \ldots\ldots\ldots + C_n\,x^{n+1}$.

Differentiating both sides

$(1+x)^n + x\,n\,(1+x)^{n-1} = C_0 + 2.\,C_1x + 3\,.\,C_2x^2 + \ldots\ldots + (n+1)C_nx^n$.

putting $x = 1$, we get

$C_0 + 2.C_1 + 3\,.\,C_2 + \ldots\ldots + (n+1)\,C_n = 2^n + n\,.\,2^{n-1}$

$C_0 + 2.C_1 + 3\,.\,C_2 + \ldots\ldots + (n+1)\,C_n = 2^{n-1}(n+2)$ **Proved**

(iii) **I Method : By Summation**

$$\text{L.H.S.} = C_0 - \frac{C_1}{2} + \frac{C_2}{3} - \frac{C_3}{4} + \ldots\ldots\ldots + (-1)^n.\,\frac{C_n}{n+1}$$

$$= \sum_{r=0}^{n}(-1)^r\,.\,\frac{{}^nC_r}{r+1} = \frac{1}{n+1}\sum_{r=0}^{n}(-1)^r\,.\,{}^{n+1}C_{r+1} \qquad \left\{\frac{n+1}{r+1}.\,{}^nC_r = {}^{n+1}C_{r+1}\right\}$$

$$= \frac{1}{n+1}\left[{}^{n+1}C_1 - {}^{n+1}C_2 + {}^{n+1}C_3 - \ldots\ldots\ldots\ldots + (-1)^n\,.\,{}^{n+1}C_{n+1}\right]$$

$$= \frac{1}{n+1}\left[-{}^{n+1}C_0 + {}^{n+1}C_1 - {}^{n+1}C_2 + \ldots\ldots\ldots + (-1)^n\,.\,{}^{n+1}C_{n+1} + {}^{n+1}C_0\right]$$

$$= \frac{1}{n+1} = \text{R.H.S.}, \quad \text{since } \left\{-{}^{n+1}C_0 + {}^{n+1}C_1 - {}^{n+1}C_2 + \ldots + (-1)^n\,{}^{n+1}C_{n+1} = 0\right\}$$

II Method : By Integration

$(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n.$

Integrating both sides, within the limits – 1 to 0.

$$\left[\frac{(1+x)^{n+1}}{n+1}\right]_{-1}^{0} = \left[C_0x + C_1\frac{x^2}{2} + C_2\frac{x^3}{3} + ... + C_n\frac{x^{n+1}}{n+1}\right]_{-1}^{0}$$

$$\frac{1}{n+1} - 0 = 0 - \left[-C_0 + \frac{C_1}{2} - \frac{C_2}{3} + ... + (-1)^{n+1}\frac{C_n}{n+1}\right]$$

$$C_0 - \frac{C_1}{2} + \frac{C_2}{3} - .... + (-1)^n\frac{C_n}{n+1} = \frac{1}{n+1}$$ **Proved**

Ex. 9 Prove that $({}^{2n}C_0)^2 - ({}^{2n}C_1)^2 + ({}^{2n}C_2)^2 - .... + (-1)^n({}^{2n}C_{2n})^2 = (-1)^n.\ {}^{2n}C_n$

Sol. $(1-x)^{2n} = {}^{2n}C_0 - {}^{2n}C_1x + {}^{2n}C_2x^2 - ....+(-1)^n\,{}^{2n}C_{2n}x^{2n}$ ...**(i)**

and $(x+1)^{2n} = {}^{2n}C_0x^{2n} + {}^{2n}C_1x^{2n-1} + {}^{2n}C_2x^{2n-2} + ...+{}^{2n}C_{2n}$ ....**(ii)**

Multiplying **(i)** and **(ii)**, we get

$(x^2-1)^{2n} = ({}^{2n}C_0 - {}^{2n}C_1x + ... + (-1)^n\,{}^{2n}C_{2n}x^{2n}) \times ({}^{2n}C_0x^{2n} + {}^{2n}C_1x^{2n-1} + \ .... + {}^{2n}C_{2n})$ ....**(iii)**

Now, coefficient of x^{2n} in R.H.S.

$= ({}^{2n}C_0)^2 - ({}^{2n}C_1)^2 + ({}^{2n}C_2)^2 - ...... + (-1)^n({}^{2n}C_{2n})^2$

$\because$ General term in L.H.S., $T_{r+1} = {}^{2n}C_r(x^2)^{2n-r}(-1)^r$

Putting $2(2n-r) = 2n$

$\therefore$ $r = n$

$\therefore$ $T_{n+1} = {}^{2n}C_nx^{2n}(-1)^n$

Hence coeffiecient of x^{2n} in L.H.S. $= (-1)^n.{}^{2n}C_n$

But **(iii)** is an identity, therefore coefficient of x^{2n} in R.H.S. = coefficient of x^{2n} in L.H.S.

$\Rightarrow$ $({}^{2n}C_0)^2 - ({}^{2n}C_1)^2 + ({}^{2n}C_2)^2 - .... + (-1)^n({}^{2n}C_{2n})^2 = (-1)^n.\ {}^{2n}C_n$

Ex. 10 In the expansion of $\left(1+x+\frac{7}{x}\right)^{11}$, find the term independent of x.

Sol. $$\left(1+x+\frac{7}{x}\right)^{11} = \sum_{r_1+r_2+r_3=11}\frac{(11)!}{r_1!\,r_2!\,r_3!}\ (1)^{r_1}\,(x)^{r_2}\left(\frac{7}{x}\right)^{r_3}$$

The exponent 11 is to be divided among the base variables 1, x and $\frac{7}{x}$ in such a way so that we get x^0.

Therefore, possible set of values of (r_1, r_2, r_3) are (11, 0, 0), (9, 1, 1), (7, 2, 2), (5, 3, 3), (3, 4, 4), (1, 5, 5)

Hence the required term is

$$\frac{(11)!}{(11)!}(7^0) + \frac{(11)!}{9!1!1!}7^1 + \frac{(11)!}{7!2!2!}7^2 + \frac{(11)!}{5!3!3!}7^3 + \frac{(11)!}{3!4!4!}7^4 + \frac{(11)!}{1!5!5!}7^5$$

$$= 1 + \frac{(11)!}{9!2!}\cdot\frac{2!}{1!1!}7^1 + \frac{(11)!}{7!4!}\cdot\frac{4!}{2!2!}7^2 + \frac{(11)!}{5!6!}\cdot\frac{6!}{3!3!}7^3 + \frac{(11)!}{3!8!}\cdot\frac{8!}{4!4!}7^4 + \frac{(11)!}{1!10!}\cdot\frac{(10)!}{5!5!}7^5$$

$$= 1 + {}^{11}C_2\,.\,{}^2C_1\,.\,7^1 + {}^{11}C_4\,.\,{}^4C_2\,.\,7^2 + {}^{11}C_6\,.\,{}^6C_3\,.\,7^3 + {}^{11}C_8\,.\,{}^8C_4\,.\,7^4 + {}^{11}C_{10}\,.\,{}^{10}C_5\,.\,7^5$$

$$= 1 + \sum_{r=1}^{5}{}^{11}C_{2r}\ .\ {}^{2r}C_r\,.\,7^r$$

Ex. 11 If $(1 + x)^n = C_0 + C_1 x + C_2x^2 + ... + C_nx^n$ then show that the sum of the products of the C_i's taken two at a time represented by : $\sum\limits_{0\le i<j\le n}\sum C_iC_j$ is equal to $2^{2n-1} - \frac{2n!}{2.n!n!}$

Sol. Since $(C_0 + C_1 + C_2 + ... + C_{n-1} + C_n)^2$

$= C_0^2 + C_1^2 + C_2^2 + ... + C_{n-1}^2 + C_n^2 + 2(C_0C_1 + C_0C_2 + C_0C_3 + ... + C_0C_n + C_1C_2 + C_1C_3 + ...$
$+ C_1C_n + C_2C_3 + C_2C_4 + ... + C_2C_n + ... + C_{n-1}C_n)$

$(2^n)^2 = {}^{2n}C_n + 2 \sum\limits_{0\le i<j\le n}\sum C_iC_j$

Hence $\sum\limits_{0\le i<j\le n}\sum C_iC_j = 2^{2n-1} - \frac{2n!}{2.n!n!}$

Ex. 12 If $(1+x+x^2)^n = \sum\limits_{r=0}^{2n} a_r x^r$, then prove that **(a)** $a_r = a_{2n-r}$ **(b)** $\sum\limits_{r=0}^{n-1} a_r = \frac{1}{2}(3^n - a_n)$

Sol. (a) We have

$$\left(1+x+x^2\right)^n = \sum_{r=0}^{2n} a_r x^r \quad ....\textbf{(A)}$$

Replace x by $\frac{1}{x}$

$$\therefore \quad \left(1+\frac{1}{x}+\frac{1}{x^2}\right)^n = \sum_{r=0}^{2n} a_r \left(\frac{1}{x}\right)^r$$

$$\Rightarrow \quad \left(x^2+x+1\right)^n = \sum_{r=0}^{2n} a_r x^{2n-r}$$

$$\Rightarrow \quad \sum_{r=0}^{2n} a_r x^r = \sum_{r=0}^{2n} a_r x^{2n-r} \quad \{\text{Using } \textbf{(A)}\}$$

Equating the coefficient of x^{2n-r} on both sides, we get

$a_{2n-r} = a_r$ for $0 \le r \le 2n$.

Hence $a_r = a_{2n-r}$.

(b) Putting x=1 in given series, then

$a_0 + a_1 + a_2 + ... + a_{2n} = (1+1+1)^n$

$a_0 + a_1 + a_2 + ... + a_{2n} = 3^n$ ....**(1)**

But $a_r = a_{2n-r}$ for $0 \le r \le 2n$

$\therefore$ series (1) reduces to

$2(a_0 + a_1 + a_2 + ... + a_{n-1}) + a_n = 3^n$.

$\therefore \quad a_0 + a_1 + a_2 + ... + a_{n-1} = \frac{1}{2}(3^n - a_n)$

Ex. 13 Find the value of $e^{-1/5}$ correct to four places of decimal.

Sol. $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + ...\text{to } \infty$ ......(i)

putting $x = \left(-\frac{1}{5}\right)$ in (1), we get

$$e^{-1/5} = 1 - \frac{1}{5} + \frac{1}{2!}\left(-\frac{1}{5}\right)^2 + \frac{1}{3!}\left(-\frac{1}{5}\right)^3 + \frac{1}{4!}\left(-\frac{1}{5}\right)^4 + ...\text{to } \infty$$

$\Rightarrow$ $e^{-1/5} = 1 - \frac{2}{10} + \frac{2^2}{2!}.\frac{1}{10^2} - \frac{2^3}{3!}.\frac{1}{10^3} + \frac{2^4}{4!}.\frac{1}{10^4} + ...$

$\Rightarrow$ $e^{-1/5} = 1 \ -0.200000 + 0.0200000 - 0.001333 + 0.000066$

$\Rightarrow$ $e^{-1/5} = 0.8187$ (correct to 4 decimal places)

Ex. 14 If $(6\sqrt{6}+14)^{2n+1}$ = [N] + F and F = N – [N]; where [.] denotes greatest integer function, then find value of NF.

Sol. Since $(6\sqrt{6}+14)^{2n+1}$ = [N] + F

Let us assume that f = $(6\sqrt{6}-14)^{2n+1}$; where $0 \le f < 1$.

Now, [N] + F – f = $(6\sqrt{6}+14)^{2n+1} - (6\sqrt{6}-14)^{2n+1}$

$$= 2\left[{}^{2n+1}C_1(6\sqrt{6})^{2n}(14) + {}^{2n+1}C_3(6\sqrt{6})^{2n-2}(14)^3 + ...\right]$$

$\Rightarrow$ [N] + F – f = even integer.

Now $0 < F < 1$ and $0 < f < 1$

so $-1 < F - f < 1$ and F – f is an integer so it can only be zero

Thus NF = $(6\sqrt{6}+14)^{2n+1}(6\sqrt{6}-14)^{2n+1} = 20^{2n+1}$.

Ex. 15 If y > 0, then prove that

$\log_e y = 2\left[\left(\frac{y-1}{y+1}\right) + \frac{1}{3}\left(\frac{y-1}{y+1}\right)^3 + \frac{1}{5}\left(\frac{y-1}{y+1}\right)^5 + ...\right]$ and calculate $\log_e 2$ to three places of decimal.

Sol. We know $\log_e\left(\frac{1+x}{1-x}\right) = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + ... \text{ to } \infty\right)$

Putting $x = \frac{y-1}{y+1}$, we get $\log_e\left(\frac{1+\frac{y-1}{y+1}}{1-\frac{y-1}{y+1}}\right) = 2\left[\frac{y-1}{y+1} + \frac{1}{3}\left(\frac{y-1}{y+1}\right)^3 + \frac{1}{5}\left(\frac{y-1}{y+1}\right)^5 + ...\right]$

$$\log_e y = 2\left[\frac{y-1}{y+1} + \frac{1}{3}\left(\frac{y-1}{y+1}\right)^3 + \frac{1}{5}\left(\frac{y-1}{y+1}\right)^5 + ...\right]$$

putting y = 2, we get $\log_e 2 = 2\left[\frac{1}{3} + \frac{1}{3}\left(\frac{1}{3}\right)^3 + \frac{1}{5}\left(\frac{1}{3}\right)^5 + ...\right] = \frac{2}{3}\left[1 + \left(\frac{1}{3}\right)^3 + \frac{1}{5}\left(\frac{1}{3}\right)^4 + ...\right] = 0.693$

Exercise # 1

[Single Correct Choice Type Questions]

1. The sum of the co–efficients in the expansion of $(1 - 2x + 5x^2)^n$ is 'a' and the sum of the co–efficients in the expansion of $(1+x)^{2n}$ is b . Then -

(A) $a = b$ **(B)** $a = b^2$ **(C)** $a^2 = b$ **(D)** $ab = 1$

2. If the coefficients of x^7 & x^8 in the expansion of $\left[2 + \frac{x}{3}\right]^n$ are equal, then the value of n is -

(A) 15 **(B)** 45 **(C)** 55 **(D)** 56

3. The value of the expression ${}^{47}C_4 + \sum_{j=1}^{5} {}^{52-j}C_3$ is equal to:

(A) ${}^{47}C_5$ **(B)** ${}^{52}C_5$ **(C)** ${}^{52}C_4$ **(D)** ${}^{49}C_4$

4. The last two digits of the number 3^{400} are -

(A) 81 **(B)** 43 **(C)** 29 **(D)** 01

5. The sum of the binomial coefficients of $\left[2x + \frac{1}{x}\right]^n$ is equal to 256. The constant term in the expansion is -

(A) 1120 **(B)** 2110 **(C)** 1210 **(D)** none

6. If $|x| < 1$, then the co-efficient of x^n in the expansion of $(1 + x + x^2 + x^3 + ...)^2$ is

(A) n **(B)** $n - 1$ **(C)** $n + 2$ **(D)** $n + 1$

7. Number of rational terms in the expansion of $\left(\sqrt{2} + \sqrt[4]{3}\right)^{100}$ is -

(A) 25 **(B)** 26 **(C)** 27 **(D)** 28

8. Sum of the infinite series $\frac{1}{2!} + \frac{1+2}{3!} + \frac{1+2+3}{4!} + ...$ to ∞

(A) $\frac{e}{3}$ **(B)** e **(C)** $\frac{e}{2}$ **(D)** none of these

9. If 'a' be the sum of the odd terms & 'b' be the sum of the even terms in the expansion of $(1+x)^n$, then $(1-x^2)^n$ is equal to -

(A) $a^2 - b^2$ **(B)** $a^2 + b^2$ **(C)** $b^2 - a^2$ **(D)** none

10. The greatest integer less than or equal to $(\sqrt{2} + 1)^6$ is

(A) 196 **(B)** 197 **(C)** 198 **(D)** 199

11. If $a_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$, then $\sum_{r=0}^{n} \frac{r}{{}^nC_r}$ equals -

(A) $(n-1)\, a_n$ **(B)** $n\, a_n$ **(C)** $n\, a_n/2$ **(D)** none of these

12. $\sum_{r=0}^{n-1} \frac{{}^nC_r}{{}^nC_r + {}^nC_{r+1}} =$

(A) $\frac{n}{2}$ **(B)** $\frac{n+1}{2}$ **(C)** $(n+1)\frac{n}{2}$ **(D)** $\frac{n(n-1)}{2(n+1)}$

13. Let the co-efficients of x^n in $(1 + x)^{2n}$ & $(1 + x)^{2n-1}$ be P & Q respectively, then $\left(\frac{P+Q}{Q}\right)^5 =$

(A) 9 **(B)** 27 **(C)** 81 **(D)** none of these

14. $\binom{47}{4} + \sum_{j=1}^{5} \binom{52-j}{3} = \binom{x}{y}$, then $\frac{x}{y} =$

(A) 11 **(B)** 12 **(C)** 13 **(D)** 14

15. Find the sum of the series $\frac{1}{2} - \frac{1}{2\times 2^2} + \frac{1}{3\times 2^3} - \frac{1}{4\times 2^4} + \ldots\ldots$

(A) $\log_e\left(\frac{3}{2}\right)$ **(B)** $\log_e\left(\frac{2}{3}\right)$ **(C)** $\log_e\left(\frac{4}{7}\right)$ **(D)** none of these

16. $\frac{C_0}{1} + \frac{C_1}{2} + \frac{C_2}{3} + \ldots\ldots + \frac{C_{10}}{11}$ is equal to (here $C_r = {}^{10}C_r$)

(A) $\frac{2^{11}}{11}$ **(B)** $\frac{2^{11}-1}{11}$ **(C)** $\frac{3^{11}}{11}$ **(D)** $\frac{3^{11}-1}{11}$

17. If $(1 + x)^{10} = a_0 + a_1x + a_2x^2 + \ldots\ldots + a_{10}x^{10}$, then value of $(a_0 - a_2 + a_4 - a_6 + a_8 - a_{10})^2 + (a_1 - a_3 + a_5 - a_7 + a_9)^2$ is

(A) 2^{10} **(B)** 2 **(C)** 2^{20} **(D)** None of these

18. The expression $\frac{1}{\sqrt{4x+1}}\left[\left[\frac{1+\sqrt{4x+1}}{2}\right]^7 - \left[\frac{1-\sqrt{4x+1}}{2}\right]^7\right]$ is a polynomial in x of degree -

(A) 7 **(B)** 5 **(C)** 4 **(D)** 3

19. Find numerically greatest term in the expansion of $(2 + 3x)^9$, when $x = 3/2$.

(A) ${}^9C_6 . 2^9 . (3/2)^{12}$ **(B)** ${}^9C_3 . 2^9 . (3/2)^6$ **(C)** ${}^9C_5 . 2^9 . (3/2)^{10}$ **(D)** ${}^9C_4 . 2^9 . (3/2)^8$

20. The value of $\sum_{r=0}^{10} \binom{10}{r}\binom{15}{14-r}$ is equal to -

(A) ${}^{25}C_{12}$ **(B)** ${}^{25}C_{15}$ **(C)** ${}^{25}C_{10}$ **(D)** ${}^{25}C_{11}$

21. The value of the expression $\left(\sum_{r=0}^{10} {}^{10}C_r\right)\left(\sum_{K=0}^{10} (-1)^K \frac{{}^{10}C_K}{2^K}\right)$ is :

(A) 2^{10} **(B)** 2^{20} **(C)** 1 **(D)** 2^5

22. If $k \in R$ and the middle term of $\left(\frac{k}{2} + 2\right)^8$ is 1120, then value of k is:

(A) 3 **(B)** 2 **(C)** -3 **(D)** -4

23. Let $R = (5\sqrt{5} + 11)^{31} = I + f$, where I is an integer and f is the fractional part of R, then $R \cdot f$ is equal to -

(A) 2^{31} **(B)** 3^{31} **(C)** 2^{62} **(D)** 1

24. Sum of the infinite series $\frac{1}{2!}+\frac{1+2}{3!}+\frac{1+2+3}{4!}+.....$ to ∞

(A) $\frac{e}{3}$ **(B)** e **(C)** $\frac{e}{2}$ **(D)** none of these

25. The term independent of x in the product $(4+x+7x^2)\left(x-\frac{3}{x}\right)^{11}$ is -

(A) $7.{}^{11}C_6$ **(B)** $3^6.\ {}^{11}C_6$ **(C)** $3^5.\ {}^{11}C_5$ **(D)** -12.2^{11}

26. If the second term of the expansion $\left[a^{1/13}+\frac{a}{\sqrt{a^{-1}}}\right]^n$ is $14a^{5/2}$, then the value of $\frac{{}^nC_3}{{}^nC_2}$ is:

(A) 4 **(B)** 3 **(C)** 12 **(D)** 6

27. If $n \in N$ & n is even, then $\frac{1}{1.(n-1)!}+\frac{1}{3!(n-3)!}+\frac{1}{5!(n-5)!}+......+\frac{1}{(n-1)!1!}=$

(A) 2^n **(B)** $\frac{2^{n-1}}{n!}$ **(C)** $2^n n!$ **(D)** none of these

28. If the 6^{th} term in the expansion of $\left[\frac{1}{x^{8/3}}+x^2\log_{10}x\right]^8$ is 5600, then x =

(A) 10 **(B)** 8 **(C)** 11 **(D)** 9

29. The greatest terms of the expansion $(2x+5y)^{13}$ when $x=10, y=2$ is -

(A) ${}^{13}C_5 . 20^8 . 10^5$ **(B)** ${}^{13}C_6 . 20^7 . 10^4$ **(C)** ${}^{13}C_4 . 20^9 . 10^4$ **(D)** *none of these*

30. The value of, $\frac{18^3+7^3+3.18.7.25}{3^6+6.243.2+15.81.4+20.27.8+15.9.16+6.3.32+64}$ is :

(A) 1 **(B)** 2 **(C)** 3 **(D)** none

Exercise # 2 Part # I [Multiple Correct Choice Type Questions]

1. Let $(1 + x^2)^2 (1 + x)^n = A_0 + A_1 x + A_2 x^2 + ...$ If A_0, A_1, A_2 are in A.P. then the value of n is -

(A) 2 **(B)** 3 **(C)** 5 **(D)** 7

2. If the coefficients of three consecutive terms in the expansion of $(1 + x)^n$ are in the ratio of 1 : 7 : 42, then n is divisible by -

(A) 9 **(B)** 5 **(C)** 3 **(D)** 11

3 The sum of the co–efficients in the expansion of $(1 - 2x + 5x^2)^n$ is a and the sum of the co–efficients in the expansion of $(1 + x)^{2n}$ is b. then:

(A) $a = b$ **(B)** $(x - a)^2 + (x - b)^2 = 0$, has real roots

(C) $\sin^2 a + \cos^2 b = 1$ **(D)** $ab = 1$

4. If $(1 + 2x + 3x^2)^{10} = a_0 + a_1x + a_2x^2 + ... + a_{20}x^{20}$, then :

(A) $a_1 = 20$ **(B)** $a_2 = 210$ **(C)** $a_4 = 8085$ **(D)** $a_{20} = 2^2 . 3^7 . 7$

5. Let $n \in N$. If $(1 + x)^n = a_0 + a_1x + a_2x^2 + ...+ a_nx^n$ and $a_{n-3}, a_{n-2}, a_{n-1}$ are in AP, then -

(A) a_1, a_2, a_3 are in AP **(B)** a_1, a_2, a_3 are in HP **(C)** $n = 7$ **(D)** $n = 14$

6. In the expansion of $(x + y + z)^{25}$

(A) every term is of the form ${}^{25}C_r . {}^rC_k . x^{25-r} . y^{r-k} . z^k$

(B) the coefficient of $x^8 y^9 z^9$ is 0

(C) the number of terms is 325

(D) none of these

7. In the expansion of $\left(x^{2/3} - \frac{1}{\sqrt{x}}\right)^{30}$, a term containing the power x^{13} -

(A) does not exist **(B)** exists and the co-efficient is divisible by 29

(C) exists and the co-efficient is divisible by 63 **(D)** exists and the co-efficient is divisible by 65

8. The coefficient of x^4 in $\left(\frac{1+x}{1-x}\right)^2$, $|x| < 1$, is

(A) 4 **(B)** –4 **(C)** $10 + {}^4C_2$ **(D)** 16

9. The value r for which $\binom{30}{r}\binom{15}{r} + \binom{30}{r-1}\binom{15}{1} + ... + \binom{30}{0}\binom{15}{r}$ is maximum is/are -

(A) 21 **(B)** 22 **(C)** 23 **(D)** 24

10. In the expansion of $\left(\sqrt[3]{4} + \frac{1}{\sqrt[4]{6}}\right)^{20}$

(A) the number of irrational terms is 19 **(B)** middle term is irrational

(C) the number of rational terms is 2 **(D)** 9th term is rational

11. The number $101^{100} - 1$ is divisible by -

(A) 100 **(B)** 1000 **(C)** 10000 **(D)** 100000

12. If the expansion of $(3x + 2)^{-1/2}$ is valid in ascending powers of x, then x lies in the interval.

(A) (0, 2/3) **(B)** (–3/2, 3/2) **(C)** (–2/3, 2/3) **(D)** $(-\infty, -3/2) \cup (3/2, \infty)$

13. $7^9 + 9^7$ is divisible by :

(A) 16 **(B)** 24 **(C)** 64 **(D)** 72

14. In the expansion of $\left(x^3 + 3.2^{-\log_{\sqrt{2}}\sqrt{x^3}}\right)^{11}$ -

(A) there appears a term with the power x^2

(B) there does not appear a term with the power x^2

(C) there appears a term with the power x^{-3}

(D) the ratio of the co-efficient of x^3 to that of x^{-3} is $\frac{1}{3}$

15. If $\left(9 + \sqrt{80}\right)^n = I + f$, where I, n are integers and $0 < f < 1$, then :

(A) I is an odd integer

(B) I is an even integer

(C) $(I + f)(1 - f) = 1$

(D) $1 - f = \left(9 - \sqrt{80}\right)^n$

16. If the co-efficients of rth, (r + 1)th, and (r + 2)th terms in the expansion of $(1 + x)^{14}$ are in A.P. then r is/are

(A) 5 **(B)** 12 **(C)** 10 **(D)** 9

17. If recursion polynomials $P_k(x)$ are defined as $P_1(x) = (x - 2)^2$, $P_2(x) = ((x - 2)^2 - 2)^2$
$P_3(x) = ((x - 2)^2 - 2)^2 - 2)^2$ .... (In general $P_k(x) = (P_{k-1}(x) - 2)^2$, then the constant term in $P_k(x)$ is

(A) 4 **(B)** 2 **(C)** 16 **(D)** a perfect square

18. If $(1 + x + x^2 + x^3)^{100} = a_0 + a_1x + a_2x^2 + ... + a_{300}x^{300}$, then -

(A) $a_0 + a_1 + a_2 + a_3 + ... + a_{300}$ is divisible by 1024

(B) $a_0 + a_2 + a_4 + ... + a_{300} = a_1 + a_3 + ... + a_{299}$

(C) coefficients equidistant from beginning and end are equal

(D) $a_1 = 100$

19. If the 6^{th} term in the expansion of $\left(\frac{3}{2} + \frac{x}{3}\right)^n$ when $x = 3$ is numerically greatest then the possible integral value(s) of n can be -

(A) 11 **(B)** 12 **(C)** 13 **(D)** 14

20. The sum of the coefficient in the expansion of $(1 + ax - 2x^2)^n$ is

(A) positive, when $a < 1$ and $n = 2k, k \in N$

(B) negative, when $a < 1$ and $n = 2k + 1, k \in N$

(C) positive, when $a > 1$ and $n \in N$

(D) zero, when $a = 1$

21. The co-efficient of the middle term in the expansion of $(1 + x)^{2n}$ is -

(A) $\frac{1.3.5.7......(2n-1)}{n!} 2^n$

(B) ${}^{2n}C_n$

(C) $\frac{(n+1)(n+2)(n+3)....(2n-1)(2n)}{1.2.3..........(n-1)\,n}$

(D) $\frac{2.6.10.14......(4n-6)(4n-2)}{1.2.3.4.....(n-1).n}$

Part # II [Assertion & Reason Type Questions]

These questions contains, Statement I (assertion) and Statement II (reason).

(a) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(b) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.

(c) Statement-I is true, Statement-II is false.

(d) Statement-I is false, Statement-II is true.

1. **Statement-I :** The greatest value of ${}^{40}C_0 \cdot {}^{60}C_r + {}^{40}C_1 \cdot {}^{60}C_{r-1} .... {}^{40}C_{40} \cdot {}^{60}C_{r-40}$ is ${}^{100}C_{50}$

Statement-II : The greatest value of ${}^{2n}C_r$, (where r is constant) occurs at r = n.

2. **Statement - I :** If n is even, then ${}^{2n}C_1 + {}^{2n}C_3 + {}^{2n}C_5 + ... + {}^{2n}C_{n-1} = 2^{2n-1}$.

Statement - II : ${}^{2n}C_1 + {}^{2n}C_3 + {}^{2n}C_5 + ... + {}^{2n}C_{2n-1} = 2^{2n-1}$.

3. **Statement-I :** Coefficient of $ab^8c^3d^2$ in the expansion of $(a + b + c + d)^{14}$ is 180180

Statement-II : General term in the expansion of $(a_1 + a_2 + a_3 + ... + a_m)^n = \sum \frac{n!}{n_1!n_2!n_3!....n_m!} a_1^{n_1} a_2^{n_2} ... a_m^{n_m}$,

where $n_1 + n_2 + n_3 + ... + n_m = n$.

4. **Statement - I :** Any positive integral power of $\left(\sqrt{2}-1\right)$ can be expressed as $\sqrt{N} - \sqrt{N-1}$ for some natural number N > 1.

Statement - II : Any positive integral power of $\sqrt{2} - 1$ can be expressed as $A + B\sqrt{2}$ where A and B are integers.

5. **Statement-I :** If $x = {}^{n}C_{n-1} + {}^{n+1}C_{n-1} + {}^{n+2}C_{n-1} + ... + {}^{2n}C_{n-1}$, then $\frac{x+1}{2n+1}$ is integer.

Because

Statement-II : ${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$ and ${}^{n}C_r$ is divisible by n if n and r are co-prime.

6. **Statement - I :** The term independent of x in the expansion of $\left(x + \frac{1}{x} + 2\right)^m$ is $\frac{(2m)!}{(m!)^2}$.

Statement - II : The coefficient of x^b in the expansion of $(1 + x)^n$ is ${}^{n}C_b$.

7. **Statement-I :** If $q = \frac{1}{3}$ and p + q = 1, then $\sum_{r=0}^{15} r \; {}^{15}C_r p^r q^{15-r} = 15 \times \frac{1}{3} = 5$

Statement-II : If p + q = 1 , 0 < p < 1, then $\sum_{r=0}^{n} r \; {}^{n}C_r p^r q^{n-r} = np$

8. **Statement - I :** If n is an odd prime then $\left[\left(\sqrt{5}+2\right)^n\right] - 2^{n+1}$ is divisible by 20 n, where [.] denotes greatest integer function.

Statement - II : If n is prime then ${}^{n}C_1, {}^{n}C_2, ... {}^{n}C_{n-1}$ must be divisible by n.

Exercise # 3 Part # I [Matrix Match Type Questions]

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with **one or more** statement(s) in **Column-II**.

1. **column-I** | **column-II**

(A) $(2n+1)(2n+3)(2n+5)\ldots\ldots(4n-1)$ is equal to — **(p)** $\dfrac{(n+1)^n}{n!}$

(B) $\dfrac{C_1}{C_0}+\dfrac{2.C_2}{C_1}+\dfrac{3.C_3}{C_2}+\ldots\ldots+\dfrac{n.C_n}{C_{n-1}}$ is equal to — **(q)** $n.2^n.(2^n-1)$

here C_r stand for ${}^nC_r$.

(C) If $(C_0+C_1)(C_1+C_2)(C_2+C_3)\ldots\ldots(C_{n-1}+C_n)$ — **(r)** $\dfrac{(4n)!\ n!}{2^n.(2n)!(2n)!}$

$= m.C_1C_2C_3\ldots.C_{n-1}$, then m is equal to

(D) If C_r are the binomial co–efficients in the expansion of — **(s)** $\dfrac{n(n+1)}{2}$

$(1+x)^n$, the value of $\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n}(i+j)\,C_iC_j$ is

2. **Column – I** | **Column – II**

(A) If $(r+1)^{th}$ term is the first negative term in the expansion of $(1+x)^{7/2}$, then the value of r (where $|x|<1$) is — **(p)** divisible by 2

(B) The coefficient of y in the expansion of $(y^2+1/y)^5$ is — **(q)** divisible by 5

(C) ${}^nC_r$ is divisible by n, $(1<r<n)$ if n is — **(r)** divisible by 10

(D) The coefficient of x^4 in the expression $(1+2x+3x^2+4x^3+\ldots\ldots\text{up to }\infty)^{1/2}$ is c, $(c \in N)$, then $c+1$ (where $|x|<1$) is — **(s)** a prime number

3. **Column - I** | **Column - II**

(A) If $x=\left(7+4\sqrt{3}\right)^{2n}=[x]+f$, then $x(1-f)=$ — **(p)** 6

(B) If second, third and fourth terms in the expansion of $(x+a)^n$ are 240, 720 and 1080 respectively, then n is equal to — **(q)** 1

(C) value of ${}^4C_0{}^4C_4-{}^4C_1{}^4C_3+{}^4C_2{}^4C_2-{}^4C_3{}^4C_1+{}^4C_4{}^4C_0$ is — **(r)** 2

(D) If x is very large as compare to y, then the value of k in $\sqrt{\dfrac{x}{x+y}}\ \sqrt{\dfrac{x}{x-y}}=1+\dfrac{y^2}{kx^2}$ — **(s)** 5

Part # II [Comprehension Type Questions]

Comprehension # 1

If n is positive integer and if $(1 + 4x + 4x^2)^n = \sum_{r=0}^{2n} a_r x^r$, where a_i's are (i = 0, 1, 2, 3, ... , 2n) real numbers.

On the basis of above information, answer the following questions :

1. The value of $2\sum_{r=0}^{n} a_{2r}$ is -

(A) $9^n - 1$ **(B)** $9^n + 1$ **(C)** $9^n - 2$ **(D)** $9^n + 2$

2. The value of $2\sum_{r=1}^{n} a_{2r-1}$ is -

(A) $9^n - 1$ **(B)** $9^n + 1$ **(C)** $9^n - 2$ **(D)** $9^n + 2$

3. The value of a_{2n-1} is -

(A) 2^{2n} **(B)** $(n - 1).2^{2n}$ **(C)** $n.2^{2n}$ **(D)** $(n + 1).2^{2n}$

4. The value of a_2 is -

(A) 8n **(B)** $8n^2 - 4$ **(C)** $8n^2 - 4n$ **(D)** $8n - 4$

Comprehension # 2

Let P be a product given by $P = (x + a_1)(x + a_2)\,.........\,(x + a_n)$

and Let $S_1 = a_1 + a_2 + ... + a_n = \sum_{i=1}^{n} a_i$, $S_2 = \sum\sum_{i<j} a_i.a_j$, $S_3 = \sum\sum\sum_{i<j<k} a_i.a_j.a_k$ and so on,

then it can be shown that

$$P = x^n + S_1 x^{n-1} + S_2 x^{n-2} + ... + S_n.$$

1. The coefficient of x^8 in the expression $(2 + x)^2 (3 + x)^3 (4 + x)^4$ must be

(A) 26 **(B)** 27 **(C)** 28 **(D)** 29

2. The coefficient of x^{203} in the expression $(x - 1)(x^2 - 2)(x^3 - 3)\,..........\,(x^{20} - 20)$ must be

(A) 11 **(B)** 12 **(C)** 13 **(D)** 15

3. The coefficient of x^{98} in the expression of $(x - 1)(x - 2)\,.........\,(x - 100)$ must be

(A) $1^2 + 2^2 + 3^2 + ... + 100^2$

(B) $(1 + 2 + 3 + ... + 100)^2 - (1^2 + 2^2 + 3^2 + ... + 100^2)$

(C) $\frac{1}{2}[(1 + 2 + 3 + ... + 100)^2 - (1^2 + 2^2 + 3^2 + ... + 100^2)]$

(D) None of these

Comprehension # 3

Consider, sum of the series $\sum\sum_{0\le i<j\le n} f(i)\,f(j)$

In the given summation, i and j are not independent.

In the sum of series $\sum_{i=1}^{n}\sum_{j=1}^{n} f(i)\,f(j) = \sum_{i=1}^{n}\left(f(i)\left(\sum_{j=1}^{n} f(j)\right)\right)$ i and j are independent. In this summation, three types of terms occur, those when $i<j$, $i>j$ and $i=j$.

Also, sum of terms when $i<j$ is equal to the sum of the terms when $i>j$ if f(i) and f(j) are symmetrical.

So, in that case

$$\sum_{i=1}^{n}\sum_{j=1}^{n} f(i)f(j) = \sum\sum_{0\le i<j\le n} f(i)f(j) + \sum\sum_{0\le i<j\le n} f(i)f(j) + \sum\sum_{i=j} f(i)f(j) = 2\sum\sum_{0\le i<j\le n} f(i)f(j) + \sum\sum_{i=j} f(i)f(j)$$

$$\Rightarrow \qquad \sum\sum_{0\le i<j\le n} f(i)f(j) = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} f(i)f(j) - \sum\sum_{i=j} f(i)f(j)}{2}$$

When f(i) and f(j) are not symmetrical, we find the sum by listing all the terms.

1. $\sum\sum_{0\le i<j\le n} {}^{n}C_i\ {}^{n}C_j$ is equal to

(A) $\frac{2^{2n} - {}^{2n}C_n}{2}$ **(B)** $\frac{2^{2n} + {}^{2n}C_n}{2}$ **(C)** $\frac{2^{2n} - {}^{n}C_n}{2}$ **(D)** $\frac{2^{2n} + {}^{n}C_n}{2}$

2. $\sum_{m=0}^{n}\sum_{p=0}^{m} {}^{n}C_m \cdot {}^{m}C_p$ is equal to

(A) 2^n-1 **(B)** 3^n **(C)** 3^n-1 **(D)** 2^n

3. $\sum\sum_{0\le i\le j\le n}\left({}^{n}C_i + {}^{n}C_j\right)$

(A) $n2^n$ **(B)** $(n+1)2^n$ **(C)** $(n-1)2^n$ **(D)** $(n+1)2^n-1$

Exercise # 4 [Subjective Type Questions]

1. If the coefficients of the r^{th}, $(r+1)^{th}$ & $(r+2)^{th}$ terms in the expansion of $(1+x)^{14}$ are in AP, find r.

2. In the binomial expansion of $\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{3}}\right)^n$, the ratio of the 7th term from the beginning to the 7th term from the end is 1 : 6 ; find n.

3. If $\sum_{r=0}^{2n} a_r(x-2)^r = \sum_{r=0}^{2n} b_r(x-3)^r$ & $a_k = 1$ for all $k \geq n$, then show that $b_n = {}^{2n+1}C_{n+1}$

4. Which is larger : $(99^{50} + 100^{50})$ or $(101)^{50}$.

5. Prove that : ${}^{n-1}C_r + {}^{n-2}C_r + {}^{n-3}C_r + ... + {}^{r}C_r = {}^{n}C_{r+1}$.

6. Prove that the co-efficient of the middle term in the expansion of $(1+x)^{2n}$ is equal to the sum of the co-efficients of middle terms in the expansion of $(1+x)^{2n-1}$.

7. Prove that : $({}^{2n}C_1)^2 + 2.({}^{2n}C_2)^2 + 3.({}^{2n}C_3)^2 + ... + 2n.({}^{2n}C_{2n})^2 = \frac{(4n-1)!}{[(2n-1)!]^2}$

8. If ${}^{40}C_1 . x(1-x)^{39} + 2 . {}^{40}C_2\, x^2(1-x)^{38} + 3\, {}^{40}C_3\, x^3(1-x)^{37} + ... + 40.\, {}^{40}C_{40}\, x^{40} = ax + b$, then find a & b.

9. Find the index 'n' of the binomial $\left(\frac{x}{5}+\frac{2}{5}\right)^n$ if the 9^{th} term of the expansion has numerically the greatest coefficient $(n \in N)$.

10. Prove that : $\frac{C_0}{2}+\frac{C_1}{3}+\frac{C_2}{4}+\frac{C_3}{5}+.......+\frac{C_n}{n+2}=\frac{1+n.\;2^{n+1}}{(n+1)(n+2)}$

11. If $(1+x)^n = \sum_{r=0}^{n} C_r.x^r$ then prove that ; $\frac{2^2.C_0}{1.2}+\frac{2^3.C_1}{2.3}+\frac{2^4.C_2}{3.4}+.......+\frac{2^{n+2}.C_n}{(n+1)(n+2)}=\frac{3^{n+2}-2n-5}{(n+1)(n+2)}$

12. $(C_0+C_1)(C_1+C_2)(C_2+C_3)(C_3+C_4)........(C_{n-1}+C_n) = \frac{C_0\, C_1\, C_2\, ...\, C_{n-1}\,(n+1)^n}{n\,!}$.

13. Prove that $\sum_{k=0}^{n} {}^nC_k \sin Kx.\cos(n-K)x = 2^{n-1}\sin nx$.

14. $C_o - 2C_1 + 3C_2 - 4C_3 + ... + (-1)^n(n+1)\, C_n = 0$

15. If $(1 + x)^n = C_0 + C_1 x + C_2 x^2 + \ldots\ldots\ldots + C_n x^n$, prove that

(i) $C_0 C_3 + C_1 C_4 + \ldots\ldots\ldots + C_{n-3} C_n = \dfrac{(2n)!}{(n+3)!(n-3)!}$

(ii) $C_0 C_r + C_1 C_{r+1} + \ldots\ldots\ldots\ldots + C_{n-r} C_n = \dfrac{(2n)!}{(n+r)!(n-r)!}$

(iii) $C_0^2 - C_1^2 + C_2^2 - C_3^2 + \ldots\ldots\ldots + (-1)^n C_n^2 = 0$ or $(-1)^{n/2} C_{n/2}$ according as n is odd or even.

16. Prove the identity $\dfrac{1}{{}^{2n+1}C_r} + \dfrac{1}{{}^{2n+1}C_{r+1}} = \dfrac{2n+2}{2n+1}\dfrac{1}{{}^{2n}C_r}$.

17. If $y = \left(x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \dfrac{x^4}{4} + \ldots\text{to } \infty\right)$ and $|x| < 1$, prove that $x = \left(y + \dfrac{y^2}{2!} + \dfrac{y^3}{3!}..\text{to } \infty\right)$.

18. Prove that : $\displaystyle\sum_{r=0}^{n-2} \binom{n-1}{r}\binom{n}{r+2} = \binom{2n-1}{n-2}$

19. Find the sum of the following infinite series : $\dfrac{1}{2}\left(\dfrac{1}{5}\right)^2 + \dfrac{2}{3}\left(\dfrac{1}{5}\right)^3 + \dfrac{3}{4}\left(\dfrac{1}{5}\right)^4 + \ldots.$

20. Prove that : $\displaystyle\sum_{r=1}^{n} \binom{n-1}{n-r}\binom{n}{r} = \binom{2n-1}{n-1}$

Exercise # 5 Part # I [Previous Year Questions] [AIEEE/JEE-MAIN]

1. The coefficient of x^5 in $(1 + 2x + 3x^2 + ...)^{-3/2}$ is : **[AIEEE 2002]**

(A) 21 **(B)** 25 **(C)** 26 **(D)** none of these

2. The number of integral terms in the expansion of $\left(\sqrt{3} + \sqrt[8]{5}\right)^{256}$ is : **[AIEEE 2003]**

(A) 32 **(B)** 33 **(C)** 34 **(D)** 35

3. If x is positive, the first negative term in the expansion of $(1 + x)^{\frac{27}{5}}$ is : **[AIEEE 2003]**

(A) 7th term **(B)** 5th term **(C)** 8th term **(D)** 6th term.

4. The coefficient of the middle term in the binomial expansion in powers of x of $(1 + \alpha x)^4$ and of $(1 - \alpha x)^6$ is the same, if α equals : **[AIEEE 2004]**

(A) $-\frac{5}{3}$ **(A)** $\frac{10}{3}$ **(C)** $-\frac{3}{10}$ **(D)** $\frac{3}{5}$

5. The coefficient of x^n in the expansion of $(1 + x)(1 - x)^n$ is- **[AIEEE 2004]**

(A) $(n - 1)$ **(B)** $(-1)^n (1 - n)$ **(C)** $(-1)^{n-1}(n - 1)^2$ **(D)** $(-1)^{n-1} n$

6. If $s_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$ and $t_n = \sum_{r=0}^{n} \frac{r}{{}^nC_r}$, then $\frac{t_n}{s_n}$ is equal to- **[AIEEE 2004]**

(A) $\frac{n}{2}$ **(B)** $\frac{n}{2} - 1$ **(C)** $n - 1$ **(D)** $\frac{2n-1}{2}$

7. If the coefficients of r^{th}, $(r + 1)^{th}$ and $(r + 2)^{th}$ terms in the binomial expansion of $(1 + y)^m$ are in AP, then m and r satisfy the equation : **[AIEEE 2005]**

(A) $m^2 - m(4r - 1) + 4r^2 + 2 = 0.$ **(B)** $m^2 - m(4r + 1) + 4r^2 - 2 = 0.$

(C) $m^2 - m(4r+1) + 4r^2 + 2 = 0.$ **(D)** $m^2 - m(4r - 1) + 4r^2 - 2 = 0.$

8. The value of ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$ is : **[AIEEE 2005]**

(A) ${}^{56}C_4$ **(B)** ${}^{56}C_3$ **(C)** ${}^{55}C_3$ **(D)** ${}^{55}C_4$

9. If x is so small that x^3 and higher powers of x may be neglected, then $\dfrac{(1+x)^{3/2} - \left(1 + \frac{1}{2}x\right)^3}{(1-x)^{1/2}}$ may be approximated as **[AIEEE 2005]**

(A) $\frac{x}{2} - \frac{3}{8}x^2$ **(B)** $-\frac{3}{8}x^2$ **(C)** $3x + \frac{3}{8}x^2$ **(D)** $1 - \frac{3}{8}x^2$

10. If the expansion in powers of x of the function $\dfrac{1}{(1-ax)(1-bx)}$ is $a_0 + a_1x + a_2x^2 + a_3x^3 + ...$, then a_n is : **[AIEEE 2006]**

(A) $\frac{a^n - b^n}{b - a}$ **(B)** $\frac{a^{n+1} - b^{n+1}}{b - a}$ **(C)** $\frac{b^{n+1} - a^{n+1}}{b - a}$ **(D)** $\frac{b^n - a^n}{b - a}$

11. For natural numbers m, n if $(1 - y)^m (1 + y)^n = 1 + a_1y + a_2y^2 + ...$ and $a_1 = a_2 = 10$, then (m, n) is : **[AIEEE 2006]**

(A) (35, 20) **(B)** (45, 35) **(C)** (35, 45) **(D)** (20, 45)

12. The sum of the series $^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... + {}^{20}C_{10}$ is **[AIEEE 2007]**

(A) $-{}^{20}C_{10}$ (B) $\frac{1}{2}\,{}^{20}C_{10}$ (C) 0 (D) $^{20}C_{10}$

13. **Statement-I :** $\sum_{r=0}^{n}(r+1)\,{}^nC_r = (n+2)\,2^{n-1}$ **[AIEEE 2008]**

Statement-II : $\sum_{r=0}^{n}(r+1)\,{}^nC_r x^r = (1+x)^n + nx\,(1+x)^{n-1}$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

14. Let $S_1 = \sum_{j=1}^{10} j(j-1)\,{}^{10}C_j$, $S_2 = \sum_{j=1}^{10} j\,{}^{10}C_j$ and $S_3 = \sum_{j=1}^{10} j^2\,{}^{10}C_j$. **[AIEEE 2009]**

Statement -I : $S_3 = 55 \times 2^9$.

Statement -II : $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$.

(A) Statement -1 is true, Statement-2 is true ; Statement -2 is not a correct explanation for Statement -1.

(B) Statement-1 is true, Statement-2 is false.

(C) Statement -1 is false, Statement -2 is true.

(D) Statement -1 is true, Statement -2 is true; Statement-2 is a correct explanation for Statement-1.

15. The coefficient of x^7 in the expansion of $(1 - x - x^2 + x^3)^6$ is : **[AIEEE-2011]**

(A) 144 (B) – 132 (C) – 144 (D) 132

16. If n is a positive integer, then $\left(\sqrt{3}+1\right)^{2n} - \left(\sqrt{3}-1\right)^{2n}$ is : **[AIEEE- 2012]**

(A) an irrational number (B) an odd positive integer

(C) an even positive integer (D) a rational number other than positive integers

17. The term independent of x in expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1} - \frac{x-1}{x-x^{1/2}}\right)^{10}$ is : **[AIEEE - 2013]**

(A) 4 (B) 120 (C) 210 (D) 310

18. If the coefficients of x^3 and x^4 in the expansion of $(1 + ax + bx^2)(1 - 2x)^{18}$ in powers of x are both zero, then (a, b) is equal to : **[JEE Main 2014]**

(A) $\left(16, \frac{251}{3}\right)$ (B) $\left(14, \frac{251}{3}\right)$ (C) $\left(14, \frac{272}{3}\right)$ (D) $\left(16, \frac{272}{3}\right)$

19. The sum of coefficients of integral powers of x in the binomial expansion of $\left(1 - 2\sqrt{x}\right)^{50}$ is **[JEE Main 2015]**

(A) $\frac{1}{2}\left(3^{50} - 1\right)$ (B) $\frac{1}{2}\left(2^{50} + 1\right)$ (C) $\frac{1}{2}\left(3^{50} + 1\right)$ (D) $\frac{1}{2}\left(3^{50}\right)$

20. If the number of terms in the expansion of $\left(1 - \frac{2}{x} + \frac{4}{x^2}\right)^n$, $x \neq 0$, is 28, then the sum of the coefficients of all the terms in this expansion, is : **[JEE Main 2016]**

(A) 2187 (B) 243 (C) 729 (D) 64

Part # II [Previous Year Questions][IIT-JEE ADVANCED]

1. If in the expansion of $(1 + x)^m (1 - x)^n$, the coefficients of x and x^2 are 3 and – 6 respectively, then m is **[IIT-JEE - 1999]**

(A) 6 **(B)** 9 **(C)** 12 **(D)** 24.

2. For $2 \le r \le n$, $\binom{n}{r} + 2\binom{n}{r-1} + \binom{n}{r-2} =$ **[IIT-JEE-2000]**

(A) $\binom{n+1}{r-1}$ **(B)** $2\binom{n+1}{r+1}$ **(C)** $2\binom{n+2}{r}$ **(D)** $\binom{n+2}{r}$

3. For any positive integer m, n (with $n \ge m$), let $\binom{n}{m} = {}^nC_m$. Prove that

$\binom{n}{m} + \binom{n-1}{m} + \binom{n-2}{m} + \ldots + \binom{m}{m} = \binom{n+1}{m+1}$. Hence or otherwise, prove that

$\binom{n}{m} + 2\binom{n-1}{m} + 3\binom{n-2}{m} + \ldots + (n-m+1)\binom{m}{m} = \binom{n+2}{m+2}$. **[IIT-JEE 2000]**

4. In the binomial expansion of $(a - b)^n$, $n \ge 5$, the sum of the 5^{th} and 6^{th} terms is zero. Then a/b equals: **[IIT-JEE-2001]**

(A) $\frac{n-5}{6}$ **(B)** $\frac{n-4}{5}$ **(C)** $\frac{5}{n-4}$ **(D)** $\frac{6}{n-5}$

5. The sum $\sum_{i=0}^{m} \binom{10}{i}\binom{20}{m-i}$, (where $\binom{p}{q} = 0$, if $p < q$) is maximum when 'm' is : **[IIT-JEE-2002]**

(A) 5 **(B)** 10 **(C)** 15 **(D)** 20

6. Coefficient of t^{24} in $(1 + t^2)^{12} (1 + t^{12}) (1 + t^{24})$ is : **[IIT-JEE-2003]**

(A) ${}^{12}C_6 + 3$ **(B)** ${}^{12}C_6 + 1$ **(C)** ${}^{12}C_6$ **(D)** ${}^{12}C_6 + 2$

7. Prove that $2^k\binom{n}{0}\binom{n}{k} - 2^{k-1}\binom{n}{1}\binom{n-1}{k-1} + 2^{k-2}\binom{n}{2}\binom{n-2}{k-2} - \ldots\ldots + (-1)^k\binom{n}{k}\binom{n-k}{0} = \binom{n}{k}$. **[IIT-JEE-2003]**

8. If ${}^{(n-1)}C_r = (k^2 - 3)\,{}^nC_{r+1}$, then an interval in which k lies is **[IIT-JEE-2004]**

(A) $(2, \infty)$ **(B)** $(-\infty, -2)$ **(C)** $\left[-\sqrt{3}, \sqrt{3}\right]$ **(D)** $\left(\sqrt{3}, 2\right]$

9. The value of **[IIT-JEE-2005]**

$$\binom{30}{0}\binom{30}{10}-\binom{30}{1}\binom{30}{11}+\binom{30}{2}\binom{30}{12}-\ldots\ldots\ldots+\binom{30}{20}\binom{30}{30}$$ is :

(A) $\binom{60}{20}$ **(B)** $\binom{30}{10}$ **(C)** $\binom{30}{15}$ **(D)** None of these

10. For r = 0, 1, ..., 10, let A_r, B_r and C_r denote, respectively, the coefficient of x^r in the expansions of $(1+x)^{10}$, $(1+x)^{20}$ and $(1+x)^{30}$. Then $\sum_{r=1}^{10} A_r(B_{10}B_r - C_{10}A_r)$ is equal to **[IIT-JEE 2010]**

(A) $B_{10}-C_{10}$ **(B)** $A_{10}(B^2{}_{10}-C_{10}A_{10})$ **(C)** 0 **(D)** $C_{10}-B_{10}$

11. Coefficient of x^{11} in the expansion of $(1+x^2)^4(1+x^3)^7(1+x^4)^{12}$ is **[JEE Ad. 2014]**

(A) 1051 **(B)** 1106 **(C)** 1113 **(D)** 1120

12. The coefficient of x^9 in the expansion of **[JEE Ad. 2015]**

$(1+x)(1+x^2)(1+x^3)...(1+x^{100})$ is.

13. Let m be the smallest positive integer such that the coefficient of x^2 in the expansion **[JEE Ad. 2016]**
$(1+x)^2+(1+x)^3+....+(1+x)^{40}+(1+mx)^{50}$ is $(3n+1)\,{}^{51}C_3$ for some positive integer n. Then the value of n is

MOCK TEST

SECTION - I : STRAIGHT OBJECTIVE TYPE

1. The expression, $\left(\sqrt{2x^2+1}+\sqrt{2x^2-1}\right)^6+\left(\dfrac{2}{\sqrt{2x^2+1}+\sqrt{2x^2-1}}\right)^6$ is a polynomial of degree

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8

2. In the expansion of $\left(\sqrt[3]{\dfrac{a}{b}}+\sqrt[3]{\dfrac{b}{\sqrt{a}}}\right)^{21}$ the term containing same powers a and b is

(A) 11^{th} **(B)** 13^{th} **(C)** 12^{th} **(D)** 6^{th}

3. Co-efficient of x^{15} in $(1+x+x^3+x^4)^n$ is :

(A) $\sum_{r=0}^{5} {}^nC_{15-3r}\,{}^nC_r$ **(B)** $\sum_{r=0}^{5} {}^nC_{5r}$ **(C)** $\sum_{r=0}^{5} {}^nC_{3r}$ **(D)** $\sum_{r=0}^{3} {}^nC_{3-r}\,{}^nC_{5r}$

4. The co-efficient of x^{n-2} in the polynomial $(x-1)(x-2)(x-3)........(x-n)$ is

(A) $\dfrac{n(n^2+2)(3n+1)}{24}$ **(B)** $\dfrac{n(n^2-1)(3n+2)}{24}$

(C) $\dfrac{n(n^2+1)(3n+4)}{24}$ **(D)** none of these

5. The coefficient of x^n in polynomial $(x+{}^{2n+1}C_0)(x+{}^{2n+1}C_1)........(x+{}^{2n+1}C_n)$ is -

(A) $2n+1$ **(B)** $2^{2n+1}-1$ **(C)** 2^{2n} **(D)** none of these

6. If $\sum_{r=0}^{2n} a_r(x-1)^r = \sum_{r=0}^{2n} b_r(x-2)^r$ and $b_r=(-1)^{r-n}$ for all $r \geq n$, then $a_n =$

(A**(C)** $^{+1}C_{n-1}$ **(B)** ${}^{3n}C_n$ **(C)** ${}^{2n+1}C_n$ **(D)** 0

7. $\sum_{r=1}^{n}\left(\sum_{p=0}^{r-1} {}^nC_r\,{}^rC_p\,2^p\right)$ is equal to -

(A) 4^n-3^n+1 **(B)** 4^n-3^n-1 **(C)** 4^n-3^n+2 **(D)** 4^n-3^n

8. $2(1+x^3)^{100} = \sum_{k=0}^{100}\left(a_k x^k - \cos\dfrac{\pi}{2}(x+k)\right)$ then the value of $a_0+a_2+a_4+....+a_{100}$

(A) 2^{99} **(B)** 2^{100} **(C)** 2^{101} **(D)** None of these

9. ${}^nC_0-2.3\,{}^nC_1+3.3^2\,{}^nC_2-4.3^3\,{}^nC_3+..........+(-1)^n(n+1)\,{}^nC_n\,3^n$ is equal to

(A) $(-1)^n 2^n\left(\dfrac{3n}{2}+1\right)$ **(B)** $2^n\left(n+\dfrac{3}{2}\right)$ **(C)** $2^n+5n\,2^n$ **(D)** $(-2)^n$.

10. Consider the following statements :

S_1 : $\quad {}^nC_0 \cdot {}^nC_1 + {}^nC_1 \cdot {}^nC_2 + ... + {}^nC_{n-1} \cdot {}^nC_n = \dfrac{(2n)!}{(n-1)!(n+1)!}$

S_2 : $\quad C_0^2 + C_1^2 + C_2^2 + ... + C_1^2 = \dfrac{(2n)!}{(n!)^2}$

S_3 : $\quad {}^{11}C_0\, {}^{11}C_{11} - {}^{11}C_1\, {}^{11}C_{10} + ... + (-1)^{11}\, {}^{11}C_{11} \cdot {}^{11}C_0 = {}^{22}C_{11}$

S_4 : $\quad 2\, {}^nC_0 + \dfrac{2^2\, {}^nC_1}{2} + \dfrac{2^3\, {}^nC_2}{3} + ... + \dfrac{2^{n+1}\, {}^nC_n}{n+1} = \dfrac{3^{n+1}-1}{n+1}$

State, in order, whether S_1, S_2, S_3, S_4 are true or false

(A) FTFF **(B)** FTTT **(C)** FFFT **(D)** TTFT

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

11. $n^n\left(\dfrac{n+1}{2}\right)^{2n}$ is

(A) Less than $\left(\dfrac{n+1}{2}\right)^3$ **(B)** Greater than or equal to $\left(\dfrac{n+1}{2}\right)^3$

(C) Less than $(n!)^3$ **(D)** Greater than or equal to $(n!)^3$.

12. Let $a_n = \left(1+\dfrac{1}{n}\right)^n$. Then for each $n \in N$

(A) $a_n \geq 2$ **(B)** $a_n < 3$ **(C)** $a_n < 4$ **(D)** $a_n < 2$

13. Let $a_n = \dfrac{1000^n}{n!}$ for $n \in N$, then a_n is greatest, when

(A) $n = 997$ **(B)** $n = 998$ **(C)** $n = 999$ **(D)** $n = 1000$

14. If n is even natural and coefficient of x^r in the expansion of $\dfrac{(1+x)^n}{1-x}$ is 2^n, $(|x| < 1)$, then –

(A) $r \leq n/2$ **(B)** $r \geq (n-2)/2$ **(C)** $r \leq (n+2)/2$ **(D)** $r \geq n$

15. If $(4 + \sqrt{15})^n = I + f$, where n is an odd natural number, I is an integer and $f < 1$, then

(A) I is natural(d)ber **(B)** I is an even integer

(C) $(I + f)(1 - f) = 1$ **(D)** $1 - f = (4 - \sqrt{15})^n$

SECTION - III : ASSERTION AND REASON TYPE

16. **Statement-I :** Coefficient of $a^2b^3c^4$ in the expansion of $(a+b+c)^8$ is $\frac{8!}{2!3!4!}$

Statement-II : Coefficient of $a^{\alpha} b^{\beta} c^{\gamma}$, where $\alpha+\beta+\gamma=n$, in the expansion of $(a+b+c)^n$ is $\frac{n!}{\alpha!\beta!\gamma!}$.

(A) Statement I is true, statement II is true, statement II is a correct explanation for statement I
(B) Statement I is true, statement II is true, statement II is NOT a correct explanation for statement I
(C) Statement I is true, statement II is false
(D) Statement I is false, statement II is true

17. **Statement-I :** If $q=\frac{1}{3}$ and $p+q=1$, then $\sum_{r=0}^{15} r\,{}^{15}C_r\, p^r q^{15-r}=15\times\frac{1}{3}=5$

Statement-II : If $p+q=1,\ 0<p<1$, then $\sum_{r=0}^{n} r\,{}^{n}C_r\, p^r q^{n-r}=np$

(A) Statement I is true, statement II is true, statement II is a correct explanation for statement I
(B) Statement I is true, statement II is true, statement II is NOT a correct explanation for statement I
(C) Statement I is true, statement II is false
(D) Statement I is false, statement II is true

18. **Statement-I :** Coefficient of x^{51} in the expansion of $(x-1)(x^2-2)(x^3-3)\,.........\,(x^{10}-10)$ is -1

Statement-II : Coefficient of $x^{\frac{n(n+1)}{2}-4}$, $n\geq 4, n\in N$, in the expansion of $(x-1)(x^2-2)(x^3-3)\,.......\,(x^n-n)$ is $-4+(-1)(-3)=-1$

(A) Statement I is true, statement II is true, statement II is a correct explanation for statement I
(B) Statement I is true, statement II is true, statement II is NOT a correct explanation for statement I
(C) Statement I is true, statement II is false
(D) Statement I is false, statement II is true

19. **Statement-I :** If $r\geq 40$, then greatest possible value of ${}^{40}C_0\cdot{}^{60}C_r+{}^{40}C_1\,{}^{60}C_{r-1}\,..........\,{}^{40}C_{40}\cdot{}^{60}C_{r-40}$ is ${}^{100}C_{50}$

Statement-II : The greatest value of ${}^{2n}C_r$ occurs at $r=n$.

(A) Statement I is true, statement II is true, statement II is a correct explanation for statement I
(B) Statement I is true, statement II is true, statement II is NOT a correct explanation for statement I
(C) Statement I is true, statement II is false
(D) Statement I is false, statement II is true

20. **Statement-I :** If $x={}^{n}C_{n-1}+{}^{n+1}C_{n-1}+{}^{n+2}C_{n-1}+\,........\,+{}^{2n}C_{n-1}$, then $\frac{x+1}{2n+1}$ is integer.

Statement-II : ${}^{n}C_r+{}^{n}C_{r-1}={}^{n+1}C_r$ and ${}^{n}C_r$ is divisible by n if n and r are co-prime.

(A) Statement I is true, statement II is true, statement II is a correct explanation for statement I
(B) Statement I is true, statement II is true, statement II is NOT a correct explanation for statement I
(C) Statement I is true, statement II is false
(D) Statement I is false, statement II is true

SECTION - IV : MATRIX - MATCH TYPE

21. Match the column

Column – I		Column – II	
(A)	${}^mC_1\,{}^nC_m - {}^mC_2\,{}^{2n}C_m + {}^mC_3\,{}^{3n}C_m - \ldots$ is	**(p)**	the coefficient of x^m in the expansion of $-(1-(1+x)^n)^m$
(B)	${}^nC_m + {}^{n-1}C_m + {}^{n-2}C_m + \ldots\ldots + {}^mC_m$ is	**(q)**	the coefficient of x^m in $\dfrac{(1+x)^{n+1}}{x}$
(C)	$C_0\,C_n + C_1\,C_{n-1} + \ldots\ldots\ldots + C_n\,C_0$ is	**(r)**	the coefficient of x^{n+1} in $(1+x)^{2n}$
(D)	$2^m\,{}^nC_0 \cdot {}^nC_m - 2^{m-1}\,{}^nC_1\,{}^{n-1}C_{m-1} + \ldots\ldots\ldots + (-1)^m\,{}^nC_m\,{}^{n-m}C_0$ is	**(s)**	the coefficient of x^m in the expansion of $(1+x)^n$
		(t)	the coefficient of x^n in $(1+x)^{2n}$

22. Match the column

Column – I		Column – II	
(A)	If λ denotes the number of terms in the expansion of $(1+5x+10x^2+10x^3+5x^4+x^5)^{20}$ & If unit's place & ten's place digits in 3^λ are O and T, then O + T is	**(p)**	1
(B)	The value of $8.\left\{\dfrac{3^{2n}}{8}\right\}$ is (Here $\{.\}$ denotes fraction part function)	**(q)**	$\dfrac{(2003)^{1001}}{(2002)!}$
(C)	If n be the degree of the polynomial $\sqrt{(3x^2+1)}\,\{(x+\sqrt{(3x^2+1)}\,)^7-(x-\sqrt{(3x^2+1)}\,)^7\}$ then n is divisible by	**(r)**	3
(D)	The value of $\left(\dfrac{1}{1}+\dfrac{1}{2002}\right)\left(\dfrac{1}{2}+\dfrac{1}{2001}\right)\left(\dfrac{1}{3}+\dfrac{1}{2000}\right)\ldots\ldots\left(\dfrac{1}{1001}+\dfrac{1}{1002}\right)$ is	**(s)**	8
		(t)	$(2002)^{2001}$

SECTION - V : COMPREHENSION TYPE

23. **Read the following comprehension carefully and answer the questions.**

Consider the identity $(1+x)^{6m} = \sum_{r=0}^{6m} {}^{6m}C_r\,.\,x^r$. By putting different values of x on both sides, we can get summation of several series involving binomial coefficients. For example, by putting $x = \dfrac{1}{2}$ we get

$$\sum_{r=0}^{6m} {}^{6m}C_r \frac{1}{2^r} = \left(\frac{3}{2}\right)^{6m}.$$

23.

1 The value of $\sum_{r=0}^{6m} {}^{6m}C_r 2^{r/2}$ is equal to

(A) $\frac{3^{6m}}{2}$ **(B)** $(1+\sqrt{2})^{3m}$ **(C)** $(3+2\sqrt{2})^{3m}$ **(D)** None of these

2 The value of $\sum_{r=0}^{3m} (-1)^r \, {}^{6m}C_{2r}$ is

(A) 2^{3m} **(B)** 0 if m is odd **(C)** -2^{3m} if m is even **(D)** None of these

3 The value of $\sum_{r=1}^{3m} (-3)^{r-1} \, {}^{6m}C_{2r-1}$ is

(A) 0 **(B)** 1 **(C)** depends on m **(D)** None of these

24. **Read the following comprehension carefully and answer the questions.**

For k, n $\in$ N, we define

$B(k, n) = 1.2.3 .... k + 2.3.4 .... (k+1) + ... + n(n+1) .... (n+k-1)$, $S_0(n) = n$ and $S_k(n) = 1^k + 2^k + ... + n^k$.

To obtain value B(k, n), we rewrite B(k, n) as follows

$$B(k,n) = k!\left[{}^kC_k + {}^{k+1}C_k + {}^{k+2}C_k + ... + {}^{n+k-1}C_k\right] = k!\left({}^{n+k}C_{k+1}\right)$$

$$= \frac{n(n+1)........(n+k)}{k+1} \quad \text{where} \quad {}^nC_k = \frac{n!}{k!\,(n-k)!}$$

1 $S_2(n) + S_1(n)$ equals

(A) $B(2,n)$ **(B)** $\frac{1}{2}B(2,n)$ **(C)** $\frac{1}{6}B(2,n)$ **(D)** none of these

2 $S_3(n) + 3S_2(n)$ equals

(A) $B(3, n)$ **(B)** $B(3,n) - 2B(2, n)$ **(C)** $B(3, n) - 2B(1, n)$ **(D)** $B(3, n) + 2B(1, n)$

3 If $(1+x)^p = 1 + {}^pC_1 x + {}^pC_2 x^2 + ...+ {}^pC_p x^p$, $p \in N$, then ${}^{k+1}C_1 S_k(n) + {}^{k+1}C_2 S_{k-1}(n) + ... + {}^{k+1}C_k S_1(n) + {}^{k+1}C_{k+1} S_0(n)$ equals

(A) $(n+1)^{k+1}$ **(B)** $(n+1)^{k+1} - 1$ **(C)** $n^{k+1} - (n-1)^{k+1}$ **(D)** $(n+1)^{k+1} - (n-1)^{k+1}$.

25. **Read the following comprehension carefully and answer the questions.**

If $(1+x)^n = C_0 + C_1x + C_2x^2 + ...+ C_nx^n$ ....**(i)**

then sum of the series $C_0 + C(b)C_{2k} + ...$ can be obtained by putting all the roots of the equation $x^k - 1 = 0$ in **(i)** and then adding vertically.

For Example : Sum of the series $C_0 + C_2 + C_4 + ...$ can be obtained by putting roots of the equation $x^2 - 1 = 0$

$\Rightarrow$ $x = \pm 1$ in **(i)**

$x = 1$	$C_0 + C_1 + C_2 + C_3 + ... = 2^n$
$x = -1$	$C_0 - C_1 + C_2 - C_3 + ... = 0$
	$2(C_0 + C_2 + C_4 + ...) = 2^n$
$\therefore$	$C_0 + C_2 + C_4 + ... = 2^{n-1}$

1 Values of x, we should substitute in **(i)** to get the sum of the series $C_0 + C_3 + C_6 + C_9$ .... , are

(A) $1, -1, \omega$ **(B)** $\omega, \omega^2, \omega^3$ **(C)** $\omega, \omega^2, -1$ **(D)** None of these

2 If n is a multiple of 3, then $C_0 + C_3 + C_6 + ...$ is equal to

(A) $\frac{2^n + 2}{3}$ **(B)** $\frac{2^n - 2}{3}$ **(C)** $\frac{2^n + 2(-1)^n}{3}$ **(D)** $\frac{2^n - 2(-1)^n}{3}$

3 Sum of values of x, which we should substitute in **(i)** to give the sum of the series $C_0 + C_4 + C_8 + C_{12} + ...$, is

(A) 2 **(B)** $2(1 + i)$ **(C)** $2(1 - i)$ **(D)** 0

SECTION - VI : INTEGER TYPE

26. Let $(1 - x^3)^n = \sum_{r=0}^{n} a_r x^r (1-x)^{3n-2r}$, $n > 2$, then find the value of n so that a_1, a_2, a_3 are in A.G.P.

27. If $(1 - x)^{-n} = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + ...$, find the value of, $a_0 + a_1 + a_2 + ... + a_n$.

28. Find the exponent of three in sum of rational coefficients in expansion of $(\sqrt[3]{5}x + \sqrt{3}y + z)^6$

29. Find the remainder when $32^{32^{32}}$ is divided by 7.

30. The value of $\frac{y_1 . y_2 . y_3}{501(y_1 - x_1)(y_2 - x_2)(y_3 - x_3)}$ when

(x_i, y_i), i = 1, 2, 3 satisfy both $x^3 - 3xy^2 = 2005$ & $y^3 - 3x^2y = 2004$ is

ANSWER KEY

EXERCISE - 1

1. A	**2.** C	**3.** C	**4.** D	**5.** A	**6.** D	**7.** B	**8.** C	**9.** A	**10.** B	**11.** C	**12.** A	**13.** D
14. C	**15.** A	**16.** B	**17.** A	**18.** D	**19.** A	**20.** D	**21.** C	**22.** B	**23.** C	**24.** C	**25.** B	**26.** A
27. B	**28.** A	**29.** C	**30.** A									

EXERCISE - 2 : PART # I

1. AB	**2.** BD	**3.** ABC	**4.** ABC	**5.** AC	**6.** AB	**7.** BCD	**8.** CD	**9.** BC
10. ABCD	**11.** ABC	**12.** AC	**13.** AC	**14.** BCD	**15.** ACD	**16.** AB	**17.** AD	**18.** ABCD
19. BCD	**20.** ABCD	**21.** ABCD						

PART - II

1. C **2.** D **3.** C **4.** B **5.** A **6.** A **7.** D **8.** A

EXERCISE - 3 : PART # I

1. $A \rightarrow r$ $B \rightarrow s$ $C \rightarrow p$ $D \rightarrow q$ **2.** $A \rightarrow q,s$ $B \rightarrow p,q,r$ $C \rightarrow s$ $D \rightarrow ps$ **3.** $A \rightarrow q$ $B \rightarrow s$ $C \rightarrow p$ $D \rightarrow r$

PART - II

Comprehension # 1 : **1.** B **2.** A **3.** C **4.** C **Comprehension # 2 :** **1.** D **2.** C **3.** C

Comprehension # 3 : **1.** A **2.** B **3.** A

EXERCISE - 5 : PART # I

1. D	**2.** B	**3.** C	**4.** C	**5.** B	**6.** A	**7.** B	**8.** A	**9.** B	**10.** C	**11.** C	**12.** B	**13.** A
14. B	**15.** C	**16.** A	**17.** C	**18.** D	**19.** C	**20.** C						

PART - II

1. C **2.** D **4.** B **5.** C **6.** D **8.** D **9.** B **10.** D **11.** B **12.** 8 **13.** 5

MOCK TEST

1. B	**2.** B	**3.** A	**4.** B	**5.** C	**6.** C	**7.** D	**8.** B	**9.** A
10. B	**11.** BD	**12.** ABC	**13.** CD	**14.** D	**15.** ACD	**16.** D	**17.** D	**18.** A

19. A **20.** A **21.** $A \to p$ $B \to q$ $C \to t$ $D \to s$ **22.** $A \to r$ $B \to p$ $C \to s$ $D \to q$

23. **1.** C **2.** B **3.** A **24.** **1.** A **2.** C **3.** B **25.** **1.** B **2.** C **3.** D

26. 7 **27.** $\frac{(2n)!}{(n!)^2}$ **28.** 1 **29.** 4 **30.** 2