HINTS & SOLUTIONS

EXERCISE - 1

Single Choice

1. $(1-2x+5x^2)^n$

For sum of coefficients put x = 1

$\therefore$ $a = (1-2+5)^n = 4^n$

$(1+x)^{2n}$

For sum of coefficients put x = 1;

$b = (1+x)^{2n} = 2^{2n} = (2^2)^n = 4^n$

$\therefore$ a = b

9. Given $(1+x)^n = a + b$ .....(i)

then, $(1-x)^n = a - b$ .....(ii)

Multiplying equation (i) & (ii)

we get, $(1-x^2)^n = a^2 - b^2$

11. Let $b = \sum_{r=0}^{n} \frac{r}{{}^nC_r} = \sum_{r=0}^{n} \frac{n-(n-r)}{{}^nC_r}$

$$= n\sum_{r=0}^{n} \frac{1}{{}^nC_r} - \sum_{r=0}^{n} \frac{n-r}{{}^nC_r}$$

$$= na_n - \sum_{r=0}^{n} \frac{n-r}{{}^nC_{n-r}} \qquad \because {}^nC_r = {}^nC_{n-r}$$

$= na_n - b$

$\Rightarrow 2b = na_n \Rightarrow b = \frac{na_n}{2}$

13. $P = {}^{2n}C_n$ and $Q = {}^{2n-1}C_n$

14. $\binom{47}{4} + \sum_{j=1}^{5}\binom{52-j}{3} = \binom{x}{y}$

L.H.S.

${}^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{47}C_4$

Using property ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$ we get

$= {}^{52}C_4 = \binom{x}{y} \Rightarrow x = 52, y = 4.$

15. Let $x = \frac{1}{2}$, then the sum of the given series

$$= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ... \text{ to } \infty = \log_e(1+x)$$

$$= \log_e\left(1+\frac{1}{2}\right) = \log_e\left(\frac{3}{2}\right)$$

17. $(1+x)^{10} = a_0 + a_1 + a_2x^2 + ... + a_{10}x^{10}$

Put x = i,

$(1+i)^{10} = a_0 - a_2 + a_4 + ...+ a_{10} + i\,(a_1 - a_3 + ...+ a_9)$

$a_0 - a_2 + a_4 + ...+ a_{10}$ = real part of $(1+i)^{10} = 2^5\cos 10\pi/4$ ....(i)

$a_1 - a_3 + ...$ = imaginary part of $(1+i)^{10} = 2^5 \sin 10\pi/4$ ....(ii)

$(i)^2 + (ii)^2 = 2^{10}$

18. Given expression can be rewritten as

$$\frac{2}{2^7\sqrt{4x+1}}\left[{}^7C_1(\sqrt{4x+1}) + {}^7C_3(\sqrt{4x+1})^3 + ... + {}^7C_7(\sqrt{4x+1})^7\right]$$

$$\frac{1}{2^6}\left[{}^7C_1 + {}^7C_3(\sqrt{4x+1})^2 + ... + {}^7C_7(\sqrt{4x+1})^6\right]$$

$\therefore$ Last term becomes $(4x+1)^3$

Hence degree is 3

21. $\left(\sum_{r=0}^{10} {}^{10}C_r\right)\left(\sum_{k=0}^{10} (-1)^k \frac{{}^{10}C_k}{2^k}\right)$

$$= ({}^{10}C_0 + ... + {}^{10}C_{10})\left({}^{10}C_0 - \frac{{}^{10}C_1}{2} + \frac{{}^{10}C_2}{2^2} .... + \frac{{}^{10}C_{10}}{2^{10}}\right)$$

$$= 2^{10} \times \left(1 - \frac{1}{2}\right)^{10} = 1$$

23. Let $R' = (5\sqrt{5}-11)^{31}$

Now $R - R' = (5\sqrt{5}+11)^{31} - (5\sqrt{5}-11)^{31}$

$\Rightarrow$ R – R' = Integer

$\Rightarrow$ I + f – R' = Integer

$\Rightarrow$ f – R' is an Integer but $-1 < f - R' < 1$

so $f - R' = 0 \Rightarrow f = R'$

so $R.f = R.R' = (5\sqrt{5}+11)^{31}(5\sqrt{5}-11)^{31}$

$= 4^{31} = 2^{62}$

25. $(4+x+7x^2)\left(x-\frac{3}{x}\right)^{11}$

$$\Rightarrow 4\left(x-\frac{3}{x}\right)^{11} + x.\left(x-\frac{3}{x}\right)^{11} + 7x^2\left(x-\frac{3}{x}\right)^{11}$$

term independent of x in above will be

$4 \times$ coefficient of x^0 in expansion of $\left(x-\frac{3}{x}\right)^{11}$

$+ 1 \times$ coefficient of x^{-1} in expansion of $\left(x-\frac{3}{x}\right)^{11}$

$+ 7 \times$ coefficient of x^{-2} in expansion of $\left(x-\frac{3}{x}\right)^{11}$

$\because$ x^r in expansion of $\left(x-\frac{3}{x}\right)^{11}$

$= {}^{11}C_r x^{11-r}\left(\frac{-3}{x}\right)^r \Rightarrow {}^{11}C_r \cdot x^{11-2r} \cdot (-3)^r$

x^0 will not exist in expansion of $\left(x-\frac{3}{x}\right)^{11}$ for integral r.

x^{-1} will occur at r = 6

$\therefore$ coefficient of $x^{-1} = {}^{11}C_6(-3)^6 = 3^6 \cdot {}^{11}C_6$

Also x^2 will not exist in expansion of $\left(x-\frac{3}{x}\right)^{11}$ for integral r.

$\therefore$ term independent of x in expansion will be $= 3^6 \cdot {}^{11}C_6$

26. $T_2 = {}^nC_1 (a^{1/13})^{n-1} (a^{3/2}) = 14a^{5/2} \Rightarrow n = 14$

$\therefore \frac{{}^nC_3}{{}^nC_2} = 4$

29. Calculate $m = \frac{n+1}{1+\left|\frac{a}{b}\right|}$ as in $(a+b)^n$

$m = \frac{13+1}{1+\left|\frac{2x}{5y}\right|} = \frac{14}{1+2} = \frac{14}{3}$

m is not integer so greatest term is $T_{[m]+1}$

$T_5 = {}^{13}C_4(2x)^9.(5y)^4$

$= {}^{13}C_4 \cdot 20^9 \cdot 10^4$

[$\because$ x = 10, y = 2]

EXERCISE - 2

Part # I : Multiple Choice

1. $(1+2x^2+x^4)(1+x)^n = A_0 + A_1x + A_2x^2 + ...$

Here $A_0 = 1, A_1 = n, A_2 = 2 + {}^nC_2$

Given A_0, A_1, A_2 are in A.P.

$\therefore$ $n - 1 = 2 + \frac{n(n-1)}{2} - n$

$\Rightarrow n^2 - 5n + 6 = 0 \Rightarrow n = 2, 3$

10. $\left(4^{1/3}+\frac{1}{6^{1/4}}\right)^{20}$

$\Rightarrow T_{r+1} = {}^{20}C_r(4^{1/3})^{20-r}(6^{-1/4})^r$

For rational terms

$20 - r = 3k$ & $r = 4p$, where $k, p \in I$

$\Rightarrow r = 20$ & $r = 8$

$\therefore$ no. of rational terms = 2

$\therefore$ no. of irrational terms = 19

13. $7^9 + 9^7 = (8-1)^9 + (8+1)^7 = {}^9C_0(8)^9 - {}^9C_1(8)^8 + {}^9C_2(8)^7$ $.....+ {}^9C_8(8) - {}^9C_9 + {}^7C_0(8)^7 +$ $......+ {}^7C_6(8) + {}^7C_7$

This is divisible by 64 & 16

14. $\left(x^3+3.2^{-\log_2 x^3}\right)^{11} = \left(x^3+\frac{3}{x^3}\right)^{11}$

$T_{r+1} = {}^{11}C_r(x^3)^{11-r}\left(\frac{3}{x^3}\right)^r$

$= {}^{11}C_r(x)^{33-6r}(3)^r$

Now $33 - 6r = 2 \Rightarrow 6r = 31$ (not possible)

$33 - 6r = -3 \Rightarrow r = 6$

$33 - 6r = 3 \Rightarrow r = 5$

$\therefore \frac{\text{coeff. of } x^3}{\text{coeff. of } x^{-3}} = \frac{{}^{11}C_5 3^5}{{}^{11}C_6 3^6} = \frac{1}{3}$

17. Constant term in $P_1(x)$ is 4

If the constant term in $P_k(x)$ is also 4, then

$P_k(x) = 4 + a_1x + a_2x^2 + ...$

and $P_{k+1}(x) = (P_k(x) - 2)^2 = (a_1x + a_2x^2 + ...+ 2)^2$

18. $(1+x+x^2+x^3)^{100}$

$= a_0 + a_1x + a_2x^2 + ... + a_{300}x^{300}$ ....(i)

put x = 1

$\Rightarrow 4^{100} = a_0 + a_1 ...... a_{300}$

So divisible by 2^{10}

Put x = –1

$\Rightarrow$ $0 = a_0 - a_1 + a_2 - .... + a_{300}$

$\Rightarrow$ $a_0 + a_2 + a_4 .... = a_1 + a_3 + ...$

replace x by $\frac{1}{x}$ we get $(x^3 + x^2 + x + 1)^{100}$

$= a_0x^{300} + a_1x^{399} + ...+a_{300}$ ......(ii)

by (i) & (ii)

$\Rightarrow a_0 = a_{300}, a_1 = a_{299},.....$ so 'B' is true

Diff. (i) we get $100(1+x+x^2+x^3)^{99}(1+2x+3x^2)$

$= a_1 + 2a_2x + ... + 300x^{299}a_{300}$

put $x = 0$

$\Rightarrow$ $100 = a_1$

21. The number of term in the expansion of $(1 + x)^{2n}$ is $2n + 1$ (odd), its middle term is $(n + 1)^{th}$ term

$$\text{coefficient} = {}^{2n}C_n = \frac{2n!}{n!n!} = \frac{1.2.3.......(2n-1).2n}{n!n!}$$

$$= \frac{(1.3.5.......2n-1)(2.4.6.......2n)}{n!n!}$$

$$= \frac{(1.3.5.......2n-1)2^n(1.2.3.......n)}{n!n!}$$

$$= \frac{(1.3.5.......2n-1)2^n}{n!}$$

Part # II : Assertion & Reason

3. Using expansion we get

$$\frac{(14!)}{r_1!\times r_2!\times r_3!\times r_4!}(a^{r_1}.b^{r_2}.c^{r_3}.d^{r_4})$$

where $r_1 + r_2 + r_3 + r_4 = 14$

$\Rightarrow$ $r_1 = 1, r_2 = 8, r_3 = 3, r_4 = 2$

$\therefore$ Coefficient of $ab^8c^3d^4$ is $\frac{14!}{1!\,8!\,3!\,2!}$

$\therefore$ Statement I is true & statement II explain I

4. Statement 1 : $(\sqrt{2}-1)^2 = 2 + 1 - 2\sqrt{2} = 3 - 2\sqrt{2}$

$= \sqrt{9} - \sqrt{8}$

and $(\sqrt{2}-1)^3 = 2\sqrt{2} - 1 - 3\sqrt{2}(\sqrt{2}-1)$

$= -7 + 5\sqrt{2} = \sqrt{50} - \sqrt{49}$

and so on

Statement 1 is correct (can be proved by induction)
Statement 2 is also correct but not correct explanation of statement 1 since any integral power of $(\sqrt{2}-1)$ will have rational and irrational part. The irrational part will have only one surd $\sqrt{2}$. Thus $(\sqrt{2}-1)^n = A + B\sqrt{2}$, where A and B are integers.

8. Statement-1 : Let $I + f = (\sqrt{5}+2)^n = {}^nC_0(\sqrt{5})^n(2)^0 + {}^nC_1(\sqrt{5})^{n-1}(2)^1 + ...$

$f' = (\sqrt{5}-(2))^n = {}^nC_0(\sqrt{5})^n 2^0 - {}^nC_1(\sqrt{5})^{n-1}(2)^1$

$+ .... I + f - f' = 2$

$\left[{}^nC_1(\sqrt{5})^{n-1}2 + {}^nC_3(\sqrt{5})^{n-3}2^3 + ... + {}^nC_n2^n\right]$

$\Rightarrow$ $I + f - f'$

$\left[{}^nC_1(\sqrt{5})^{n-1}2 + {}^nC_3(\sqrt{5})^{n-3}.2^3 + ... + {}^nC_{n-2}\,5.2^{n-2}\right]$

$+ 2^{n+1}$

Here $f - f' = 0$ $\Rightarrow$ $I - 2^{n+1}$ is clearly divisible by 20n hence true.

Statement-2 is obviously true

EXERCISE - 3

Part # I : Matrix Match Type

1. **(A)** $(2n+1)(2n+3)(2n+5)\ldots(4n-1)$

$$=\frac{(2n!)(2n+1)(2n+2)(2n+3)(2n+4)\ldots(4n-1)(4n)}{(2n!)(2n+2)(2n+4)(2n+6)\ldots(4n)}$$

$$=\frac{(4n!)(n!)}{(n!)(2n)!2^n(n+1)(n+2)\ldots(2n)}$$

$$=\frac{(4n!)(n!)}{2^n.(2n)!(2n)!}$$

(B) $$\sum_{r=1}^{n} r\frac{{}^nC_r}{{}^nC_{r-1}}=\frac{r.n!}{r!(n-r)!}\cdot\frac{(r-1)!(n-r+1)!}{n!}$$

$$=\sum_{r=1}^{n}(n-r+1)$$

$$=n(n+1)-\frac{n(n+1)}{2}=\frac{n(n+1)}{2}$$

(C) $(C_0+C_1)(C_1+C_2)(C_2+C_3)\ldots\ldots(C_{n-1}+C_n)$
$=mC_1C_2\ldots C_{n-1}$

$$=\left(\frac{C_0}{C_1}+1\right)\left(\frac{C_1}{C_2}+1\right)\left(\frac{C_2}{C_3}+1\right)\ldots\left(\frac{C_{n-1}}{C_n}+1\right)=m$$

$$=\frac{n+1}{n}\cdot\frac{n+1}{n-1}\cdot\frac{n+1}{n-2}\ldots\frac{n+1}{1}=m$$

$$\frac{(n+1)^n}{n!}=m$$

(D) $$\sum_{i=1}^{n}\left\{\sum_{j=1}^{n}iC_iC_j+\sum_{j=1}^{n}j.C_iC_j\right\}$$

$$=\sum_{i=1}^{n}iC_i(2^n-1)+\sum_{i=1}^{n}nC_i2^{n-1}$$

$=n\,.\,2^{n-1}(2^n-1)+n\,.\,2^{n-1}(2^n-1)$
$=n\,.\,2^n(2^n-1)$

3. **(A)** $I+f=(7+4\sqrt{3})^{2n}$

Here $(7-4\sqrt{3})^{2n}=f'=1-f \quad\Rightarrow\quad (I+f)(1-f)=1$

(B) $T_2={}^nC_1(x)^{n-1}.a=240$ ......**(i)**

$T_3={}^nC_2(x)^{n-2}a^2=720$ ......**(ii)**

$T_4={}^nC_3(x)^{n-3}a^3=1080$ ......**(iii)**

From **(i)** and **(ii)**

Here $$\frac{{}^nC_1(x)^{n-1}a}{{}^nC_2x^{n-2}a^2}=\frac{2x}{(n-1)a}=\frac{240}{720}=\frac{1}{3}$$

$\Rightarrow\ 6x=(n-1)a$

From **(ii)** and **(iii)**

$\Rightarrow\ 9x=2(n-2)a$

On dividing $\dfrac{3}{2}=\dfrac{2(n-2)}{(n-1)}$

$\Rightarrow\ 3n-3=4n-8$

$\Rightarrow\ n=5$

(C) $C_0C_4-C_1C_3+C_2C_2-C_3C_1+C_4C_0=2-2.4.4+6.6=6$

(D) $$\sqrt{\frac{x}{x+y}}\sqrt{\frac{x}{x-y}}=\left(\frac{1}{1+\frac{y}{x}}\right)^{1/2}\left(\frac{1}{1-\frac{y}{x}}\right)^{1/2}$$

$$=\left(1-\frac{y^2}{x^2}\right)^{-1/2}=1+\frac{1}{2}\cdot\frac{y^2}{x^2}$$

$\Rightarrow\ k=2$

Part # II : Comprehension

Comprehension–1

1. $(1+4x+4x^2)^n=a_0+a_1x+a_2x^2+\ldots+a_{2n}.x^{2n}$

$9^n=a_0+a_1+a_2+\ldots..+a_{2n}$ ......**(i)**

$1=a_0-a_1+a_2+\ldots..+a_{2n}$ ......**(ii)**

adding **(i)** & **(ii)** we get

$$9^n+1=2\sum_{r=0}^{n}a_{2r}$$

2. Subtracting **(ii)** from **(i)** we get

$$a^n-1=2\sum_{r=1}^{n}a_{2r-1}$$

3. a_{2n-1} = coefficient of x^{2n-1} in
$(1+4x+4x^2)^n=(1+2x)^{2n}$
$T_{r+1}={}^{2n}C_r(2x)^r$
$a_{2n-1}={}^{2n}C_{2n-1}\,.\,2^{2n-1}=2n\,.\,2^{2n-1}=2n\,.\,2^{2n}$

4. $a_2={}^{2n}C_2\,.\,2^2=2n(2n-1)^2=8n^2-4n$

Comprehension–2

1. The expression $(2+x)^2(3+x)^3(4+x)^4=(x+2)(x+2)$
$(x+3)(x+3)(x+3)(x+4)(x+4)(x+4)(x+4)$
$=x^9+(2+2+3+3+3+4+4+4+4)x^8+\ldots\ldots\ldots\ldots.$
$\Rightarrow$ Co-efficient of $x^8=29$

2. Expression $= x.\, x^2.\, x^3 \ldots\ldots\ldots x^{20}$

$$\left(1-\frac{1}{x}\right)\left(1-\frac{2}{x^2}\right)\left(1-\frac{3}{x^3}\right)\ldots\ldots\left(1-\frac{20}{x^{20}}\right)$$

Let $E=\left(1-\frac{1}{x}\right)\left(1-\frac{2}{x^2}\right)\left(1-\frac{3}{x^3}\right)\ldots\ldots\left(1-\frac{20}{x^{20}}\right)$

Now Co-efficient of x^{203} in original expression

$\Rightarrow$ Co-efficient of x^{-7} in E.

$$\text{But } E = 1-\left(\frac{1}{x}+\frac{2}{x^2}+\frac{3}{x^3}+\ldots\ldots\right)+\left(\frac{1}{x}.\frac{6}{x^6}+\frac{2}{x^2}.\frac{5}{x^5}+\frac{3}{x^3}.\frac{4}{x^4}+\ldots\ldots\right)-\left(\frac{1}{x}.\frac{2}{x^2}.\frac{4}{x^4}+\ldots\ldots\right)$$

$=$ Co-efficient of $x^{-7} = -7+6+10+12-8=13$

3. The Co-efficient of $x^{98} = (1.2+2.3+\ldots\ldots\ldots 99.100)$

= Sum of product of first 100 natural numbers taken two at a time

$$=\frac{1}{2}[(1+2+3+\ldots\ldots+100^2)^2-(1^2+2^2+3^2+\ldots\ldots\ldots+100^2)]$$

Comprehension–3

1. $\sum\sum_{0\le i<j\le n} {}^nC_i\ {}^nC_j$

$$=\frac{\left(\sum_{i=0}^{n}\sum_{j=0}^{n}{}^nC_i\,{}^nC_j\right)-\sum_{i=0}^{n}({}^nC_i)^2}{2}$$

$$=\frac{\left(\sum_{i=0}^{n}{}^nC_i\,2^n\right)-\sum_{i=0}^{n}({}^nC_i)^2}{2}=\frac{2^n2^n-\sum_{i=0}^{n}({}^nC_i)^2}{2}$$

$$=\frac{2^{2n}-{}^{2n}C_n}{2}$$

2. $\sum_{m=0}^{n}\sum_{p=0}^{m}{}^nC_m.{}^mC_p=\sum_{m=0}^{n}{}^nC_m\left(\sum_{p=0}^{m}{}^mC_p\right)$

$$=\sum_{m=0}^{n}{}^nC_m\left(2^m\right)=3^n$$

3. $\sum\sum_{0\le i\le j\le n}\left({}^nC_i+{}^nC_j\right)$

$$=\frac{\left(\sum_{i=0}^{n}\sum_{j=0}^{n}\left({}^nC_i+{}^nC_j\right)\right)-\sum_{i=0}^{n}2\,{}^nC_i}{2}$$

$$=\frac{\left(\sum_{i=0}^{n}\left(\sum_{j=0}^{n}{}^nC_i+\sum_{j=0}^{n}{}^nC_j\right)\right)-2\times2^n}{2}$$

$$=\frac{\left(\sum_{i=0}^{n}\left({}^nC_i\sum_{j=0}^{n}1+2^n\right)\right)-2^{n+1}}{2}$$

$$=\frac{\sum_{i=0}^{n}\left({}^nC_i(n+1)+2^n\right)-2^{n+1}}{2}$$

$$=\frac{(n+1)\sum_{i=1}^{n}{}^nC_i+2^n\sum_{i=0}^{n}1-2^{n+1}}{2}$$

$$=\frac{(n+1)2^n+2^n(n+1)-2^{n+1}}{2}$$

$=(n+1)2^n-2^n=n2^n$

EXERCISE - 4

Subjective Type

1. $r = 5$ or 9

2. 7^{th} term from beginning $T_7 = {}^nC_6 (2)^{\frac{n-6}{3}} \left(\frac{1}{3}\right)^2$

7^{th} term from the end $T_{n-5} = {}^nC_{n-6} (2)^2 \left(\frac{1}{3}\right)^{\frac{n-6}{3}}$

$$\frac{T_7}{T_{n-5}} = \frac{1}{6} = \frac{2^{\left(\frac{n-6}{3}-2\right)}}{\left(\frac{1}{3}\right)^{\left(\frac{n-6}{3}-2\right)}} \Rightarrow \frac{1}{6} = (6)^{\frac{n-12}{3}}$$

$$\Rightarrow \frac{n-12}{3} = -1 \Rightarrow n = 9$$

3. $\because \sum_{r=0}^{2n} a_r (x-2)^r = \sum_{r=0}^{2n} b_r (x-3)^r$

Let $y = x - 3 \Rightarrow y + 1 = x - 2$

so the given expression reduces to :

$$\sum_{r=0}^{2n} a_r (1+y)^r = \sum_{r=0}^{2n} b_r \cdot y^r$$

$\Rightarrow a_0 + a_1(1+y) + a_2(1+y)^2 + \dots a_{2n}(1+y)^{2n}$
$= b_0 + b_1 y + \dots\dots + b_{2n} \cdot y^{2n}$

using $a_{k=1}$ for all $k \ge n$, then we get

$\Rightarrow a_0 + a_1(1+y) + a_2(1+y)^2 + \dots + a_{n-1}(1+y)^{n-1} + (1+y)^n + (1+y)^{n+1} + \dots + (1+y)^{2n}$
$= b_0 + b_1 y + \dots + b_n y^n + \dots + b_{2n} \cdot y^{2n}$

Compare the co-efficients of y^n on both sides we get

${}^nC_n + {}^{n+1}C_n + {}^{n+2}C_n + \dots + {}^{2n}C_n = b_n$

$\Rightarrow {}^{n+1}C_{n+1} + {}^{n+1}C_n + {}^{n+2}C_n + \dots + {}^{2n}C_n = b_n$

(use ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$)

${}^{n+2}C_{n+1} + {}^{n+2}C_n + \dots + {}^{2n}C_n = b_n$

(adding the first two terms).

$\Rightarrow$ If we combine terms on LHS, finally we get

${}^{2n+1}C_{n+1} = b_n$

4. 101^{50}

5. LHS $= {}^rC_r + {}^{r+1}C_r + {}^{r+2}C_r + \dots + {}^{n-2}C_r + {}^{n-1}C_r$

$= {}^{r+1}C_{r+1} + {}^{r+1}C_r + {}^{r+2}C_r + \dots + {}^{n-2}C_r + {}^{n-1}C_r$

$(\because {}^rC_r = {}^{r+1}C_{r+1})$

$S = {}^{r+2}C_{r+1} + {}^{r+3}C_{r+1} + \dots = {}^nC_{r+1}$ $(\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r)$

8. $40x(1-x)^{39} + 2 \times \frac{40}{2} x^2 (1-x)^{35} + 3 \times \frac{40}{3} x^3 (1-x)^{37}$
$+ \dots 40.x^{40}$

$= 40x[(1-x)^{39} + x(1-x)^{38} + x^2(1-x)^{37} + \dots x^{39}]$

$= 40x[(1-x) + x]^{39}] \quad = 40x = ax + b$

$\Rightarrow a = 40$ & $b = 0$

9. $n = 12$

11. $S = \sum_{r=0}^{n} \frac{{}^nC_r \cdot 2^{r+2}}{(r+1)(r+2)} = \frac{1}{n+1} \sum_{r=0}^{n} \frac{{}^{n+1}C_{r+1} \cdot 2^{r+2}}{r+2}$

$$= \frac{1}{(n+1)(n+2)} \sum_{r=0}^{n} {}^{n+2}C_{r+2} \cdot 2^{r+2}$$

$(\because (1+2)^{n+2} = {}^{n+2}C_0 + {}^{n+2}C_1 2^1 + \dots \sum_{r=0}^{n} {}^{n+2}C_{r+2})$

$$S = \frac{1}{(n+1)(n+2)} \{3^{n+2} - 1 - 2n - 4\} = \frac{3^{2n+2} - 2n - 5}{(n+1)(n+2)}$$

13. $S = {}^nC_0.\sin(0x).\cos nx + {}^nC_1.\sin x.\cos(n-1)x + \dots\dots$
$+ {}^nC_n \sin nx.\cos(0x) \dots\dots$ **(i)**

$S = {}^nC_0.\sin nx.\cos(0x) + {}^nC_1.\sin(n-1)x.\cos x + \dots\dots$
$+ {}^nC_n \sin(0x)\cos nx \dots\dots$ **(ii)**

Add (i) & (ii)

$2S = ({}^nC_0 + {}^nC_1 + \dots + {}^nC_n)\sin nx \Rightarrow S = 2^{n-1}\sin nx$

14. $(1+x)^n = C_0 + C_1 x + \dots\dots\dots\dots + C_n x^n$

$\Rightarrow x(1+x)^n = C_0 x + C_1 x^2 + \dots\dots\dots\dots + C_n x^{n+1}$

Differentiating w.r.t. x

$(1+x)^n + n\,x\,(1+x)^{n-1} = C_0 + 2C_1 x + \dots\dots + (n+1)\,C_n x^n$

Putting $x = -1$

$C_0 - 2C_1 + \dots\dots\dots + (-1)^n (n+1)\,C_n = 0$

15. (i) $(1+x)^n = C_0 + C_1 x + \dots\dots\dots + C_n x^n$

$\Rightarrow$ $(x+1)^n = C_0 x^n + C_1 x^{n-1} + \dots\dots\dots + C_n$

$C_0C_3 + C_1C_4 + \dots\dots\dots\dots + C_{n-3}C_n$

$=$ Co-efficient of x^{n-3} in $(1+x)^{2n} = {}^{2n}C_{n-3}$

(ii) $C_0C_r + C_1C_{r+1} + \dots\dots\dots\dots + C_{n-r}C_n$

$=$ co-efficient of x^{n-r} in $(1+x)^{2n} = {}^{2n}C_{n-r}$

(iii) $(1+x)^n = C_0 + C_1 x + \dots\dots\dots + C_n x^n$

$\Rightarrow$ $(x-1)^n = C_0 x^n - C_1 x^{n-1} + \dots\dots\dots + (-1)^n C_n$

$C_0^2 - C_1^2 + \dots\dots\dots\dots + (-1)^n C_n^2$

$=$ co-efficient of x^n in $(x^2-1)^n = 0$ (if n is odd)

$= {}^nC_{n/2}(-1)^{n/2}$ (if n is even)

16. $$\frac{1}{{}^{2n+1}C_r} + \frac{1}{{}^{2n+1}C_{r+1}} = \frac{r!(2n+1-r)!}{(2n+1)!} + \frac{(r+1)!(2n-r)!}{(2n+1)!}$$

$$= \frac{r!(2n-r)!}{(2n+1)!}\{2n+1-r+r+1\}$$

$$= \frac{2n+2}{2n+1} \cdot \frac{r!(2n-r)!}{2n!} = \frac{2n+2}{2n+1} \cdot \frac{1}{{}^{2n}C_r}$$

17. $y = \left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots\text{to } \infty\right) = \log_e(1+x)$

$\Rightarrow$ $e^y = (1+x)$

$\Rightarrow$ $x = (e^y - 1) = \left[\left(1 + y + \frac{y^2}{2!} + \frac{y^3}{3!} \dots\right) - 1\right]$

$\Rightarrow$ $x = \left(y + \frac{y^2}{2!} + \frac{y^3}{3!} \dots \text{to} \infty\right)$

18. $$\sum_{r=0}^{n-2} \binom{n-1}{r}\binom{n}{r+2} = \binom{2n-1}{n-2}$$

L.H.S. $\sum_{r=0}^{n-2} {}^{n-1}C_r \cdot {}^nC_{r+2}$

$\sum_{r=0}^{n-2} {}^{n-1}C_{n-r-1} \cdot {}^nC_{r+2}$

Coefficient of x^{n+1} in the expansion of

$(1+x)^{n-1}(1+x)^n$ i.e. $(1+x)^{2n-1}$

$= {}^{2n-1}C_{n+1} = {}^{2n-1}C_{n-2} = \binom{2n-1}{n-2}$

19. $$\frac{1}{2}\left(\frac{1}{5}\right)^2 + \frac{2}{3}\left(\frac{1}{5}\right)^3 + \frac{3}{4}\left(\frac{1}{5}\right)^4 + \dots$$

$$= \left(1 - \frac{1}{2}\right)\left(\frac{1}{5}\right)^2 + \left(1 - \frac{1}{3}\right)\left(\frac{1}{5}\right)^3 + \left(1 - \frac{1}{4}\right)\left(\frac{1}{5}\right)^4 + \dots\dots$$

$$= \left(\frac{1}{5}\right)^2 + \left(\frac{1}{5}\right)^3 + \left(\frac{1}{5}\right)^4 + \dots$$

$$- \left[\frac{1}{2}\left(\frac{1}{5}\right)^2 + \frac{1}{3}\left(\frac{1}{5}\right)^3 + \frac{1}{4}\left(\frac{1}{5}\right)^4 + \dots\dots\right]$$

$$= \left[\frac{1}{5} + \left(\frac{1}{5}\right)^2 + \left(\frac{1}{5}\right)^3 + \left(\frac{1}{5}\right)^4 + \dots\right]$$

$$- \left[\frac{1}{5} + \frac{1}{2}\left(\frac{1}{5}\right)^2 + \frac{1}{3}\left(\frac{1}{5}\right)^3 + \frac{1}{4}\left(\frac{1}{5}\right)^4 + \dots\dots\right]$$

adding and subtracting $\frac{1}{5}$

$$= \frac{\frac{1}{5}}{1 - \frac{1}{5}} - \left(-\log_e\left(1 - \frac{1}{5}\right)\right) = \frac{1}{4} + \log_e \frac{4}{5}$$

EXERCISE - 5

Part # I : AIEEE/JEE-MAIN

1. $(1+2x+3x^2+.....)^{-3/2} = [(1-x)^{-2}]^{-3/2} = (1-x)^3$

So, coefficient of x^5 in $(1+2x+3x^2+....)^{-3/2}$

= coefficient of x^5 in $(1-x)^3 = 0$.

2. $(r+1)^{th}$ term of $\left(\sqrt{3}+\sqrt[8]{5}\right)^{256}$

i.e., $T_{r+1} = {}^{256}C_r\,(3)^{(256-r)/2}\,(5)^{r/8}$

The terms are integral, if $\frac{256-r}{2}$ and $\frac{r}{8}$ are both positive integer.

$\Rightarrow$ r = 0, 8, 16, 24, 32,....., 256

Hence total terms are 33.

3. $\because$ $(r+1)^{th}$ term in the expansion of $(1+x)^{27/5}$

$$= \frac{\frac{27}{5}\left(\frac{27}{5}-1\right)....\left(\frac{27}{5}-r+1\right)}{r!}x^r$$

Now this term will be negative, if the last factor in numerator is the only negative factor.

$\Rightarrow \frac{27}{5}-r+1<0 \quad \Rightarrow \quad \frac{32}{5}<r \quad \Rightarrow \quad 6.4<r$

$\Rightarrow$ least value of r is 7.

Thus first negative term will be 8th.

4. Coefficient of middle term in $(1+\alpha x)^4 = {}^4C_2\,\alpha^2$

coefficient of middle term in $(1-\alpha x)^6 = {}^6C_3\,(-\alpha)^3$

${}^4C_2\,\alpha^2 = -{}^6C_3\,\alpha^3 \quad \Rightarrow \quad -\frac{6}{20} = \alpha \quad \Rightarrow \quad \alpha = \frac{-3}{10}$

5. $(1-x)(1-x)^n = (1-x)^n + x(1-x)^n$

Coefficient of $x^n = (-1)^n + (-1)^{n-1}.\,n = (-1)^n(1-n)$

6. $s_n = \sum_{r=0}^{n}\frac{1}{{}^nC_r} \quad \Rightarrow \quad t_n = \sum_{r=0}^{n}\frac{r}{{}^nC_r}$

$\Rightarrow t_n = \sum_{r=0}^{n}\frac{n-r}{{}^nC_r}$

$\therefore \quad 2t_n = n\sum_{r=0}^{}\frac{1}{{}^nC_r} = ns_n$

$\Rightarrow \frac{t_n}{s_n} = \frac{n}{2}$

7. $(1+y)^m$

$\Rightarrow T_r = {}^mC_{r-1}\,.\,y^{r-1}$

$\Rightarrow T_{r+1} = {}^mC_r\,.\,y^r$

$\Rightarrow T_{r+2} = {}^mC_{r+1}\,.\,y^{r+1}$

${}^mC_{r-1} + {}^mC_{r+1} = 2\,{}^mC_r$

$\Rightarrow \frac{{}^mC_{r-1}}{{}^mC_r} + \frac{{}^mC_{r+1}}{{}^mC_r} = 2$

$\Rightarrow m^2 - m(4r+1) + 4r^2 - 2 = 0$

8. ${}^{50}C_4 + {}^{55}C_3 + {}^{54}C_3 + ... + {}^{50}C_3 = {}^{56}C_4$

9. $$\frac{(1+x)^{3/2}-\left(1+\frac{1}{2}x\right)^3}{(1-x)^{1/2}}$$

$$\Rightarrow \frac{\left(1+\frac{3}{2}x+\frac{3}{2}.\frac{1}{2}.\frac{x^2}{2!}\right)-\left(1+\frac{3x}{2}+\frac{3.2}{2}.\frac{x^2}{4}\right)}{(1-x)^{1/2}}$$

$$= \left(-\frac{3}{8}x^2\right)(1-x)^{-1/2} = -\frac{3}{8}x^2$$

10. $(1-ax)^{-1}(1-bx)^{-1}$

$= (1+ax+(ax)^2+.......)(1+bx+(bx)^2+............)$

so, $a_n = a^n + a^{n-1}b + a^{n-2}b^2 + ... + b^n$

$$= a^n\frac{1-\left(\frac{b}{a}\right)^{n+1}}{1-\frac{b}{a}} = \frac{a^{n+1}-b^{n+1}}{b-a}$$

11. $(1-y)^m(1+y)^n = 1 + a_1y + a_2y^2 + ...$

$(1 - my + {}^mC_2\,y^2\,........)(1 + ny + {}^nC_2\,y^2\,.........)$

$= 1 + a_1y + a_2y^2 + ...$

$a_1 = n - m = 10$ ..... **(i)**

$\Rightarrow a_2 = {}^mC_2 + {}^nC_2 - mn = 10$ ..... **(ii)**

solving (i) & (ii), we get $(m, n) \equiv (35, 45)$

12. $S = {}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2\; ..... + {}^{20}C_{10}$

We know, ${}^{20}C_0 - {}^{20}C_1 + ... + {}^{20}C_{20} = 0$

$\Rightarrow 2({}^{20}C_0 - {}^{20}C_1 + ... - {}^{20}C_9) + {}^{20}C_{10} = 0$

$\therefore \quad {}^{20}C_0 - {}^{20}C_1 + ... - {}^{20}C_9 = -\frac{1}{2}\,{}^{20}C_{10}$

So, $S = \frac{1}{2}\,{}^{20}C_{10}$

13. Statement -1 : $\sum_{r=0}^{n}(r+1)\,{}^nC_r = \sum_{r=0}^{n} r.{}^nC_r + \sum_{r=0}^{n}{}^nC_r$

$= n\,.\,2^{n-1} + 2^n = (n+2)\,2^{n-1}$

Statement-2 : $\sum_{r=0}^{n}(r+1)^n C_r x^r = \sum_{r=0}^{n} r.{}^nC_r x^r + \sum_{r=0}^{n}{}^nC_r x^r$

$= xn\,(1+x)^{n-1} + (1+x)^n$

14. $S_1 = \sum_{j=1}^{10} j(j-1)\,.\,\frac{10(10-1)}{j(j-1)}\;{}^8C_{j-2}$

$\Rightarrow\; S_1 = 9\times 10\sum_{j=2}^{10}{}^8C_{j-2} \;\Rightarrow\; S_1 = 90\,.\,2^8$

$S_2 = \sum_{j=1}^{10} j\,.\,\frac{10}{j}\;{}^9C_{j-1} = 10.2^9$

$S_3 = \sum_{j=1}^{10}(j(j-1)+j)\;{}^{10}C_j$

$= \sum_{j=1}^{10} j(j-1)\;{}^{10}C_j + \sum_{j=1}^{10} j\;{}^{10}C_j$

$= 90\sum_{j=2}^{10}{}^8C_{j-2} + 10\sum_{j=1}^{10}{}^9C_{j-1}$

$= 90\times 2^8 + 10\times 2^9 = (45+10)\,.\,2^9 = (45+10)\,.\,2^9 = 55.2^9$

so statement-1 is true and statement 2 is false.

Hence correct option is (2)

15. $(1-x-x^2+x^3)^6$

$(1-x)^6\,(1-x^2)^6$

$({}^6C_0 - {}^6C_1x^1 + {}^6C_2x^2 - {}^6C_3x^3 + {}^6C_4x^4 - {}^6C_5x^5 + {}^6C_6x^6)$

$({}^6C_0 - {}^6C_1x^2 + {}^6C_2x^4 - {}^6C_3x^6 + {}^6C_4x^8 + ... + {}^6C_6x^{12})$

Now coefficient of $x^7 = {}^6C_1{}^6C_3 - {}^6C_3{}^6C_2 + {}^6C_5{}^6C_1$

$= 6\times 20 - 20\times 15 + 36 = 120 - 300 + 36 = 156 - 300$

$= -144$

16. $(\sqrt{3}+1)^{2n} - (\sqrt{3}-1)^{2n} = 2[{}^{2n}C_1(\sqrt{3})^{2n-1}$

$+ {}^{2n}C_3(\sqrt{3})^{2n-3} + {}^{2n}C_5(\sqrt{3})^{2n-5} + ...]$

= which is an irrational number

17. $\left((x^{1/3}+1) - \left(\frac{\sqrt{x}+1}{\sqrt{x}}\right)\right)^{10} \;\Rightarrow\; (x^{1/3} - x^{-1/2})^{10}$

$\Rightarrow\; T_{r+1} = {}^{10}c_r\,(x^{1/3})^{10-r}\,(-x^{-1/2})^r$

$\frac{10-r}{3} - \frac{r}{2} = 0$

$\Rightarrow\; 20 - 2r - 3r = 0 \;\Rightarrow\; r = 4$

$T_5 = {}^{10}C_4 = \frac{10\times 9\times 8\times 7}{4\times 3\times 2\times 1} = 210$

20. Number to terms is $2n+1$ which is odd but it is given 28.
If we take $(x+y+z)^n$ then number of terms is $n + {}^2C_2 = 28$
Hence $n = 6$

$\left(1 - \frac{2}{x} + \frac{4}{x^2}\right)^6 = a_0 + a_1x + a_2x^2 + ... + a_6x^6$

Sum of coefficients can be obtained by x = 1
$(1-2+4)^6 = 3^6 = 729$
So according to what the examiner is trying to ask option 3 can be correct.

Part # II : IIT-JEE ADVANCED

1. $(1+x)^m(1-x)^n$

$3 = {}^mC_0\,{}^nC_1(-1) + {}^mC_1\,{}^nC_0$

or $3 = m - n$ .....(i)

$-6 = {}^mC_0.\,{}^nC_2 + {}^mC_2.\,{}^nC_0 - {}^mC_1\,{}^nC_1$

or -6

$= \frac{n(n-1)}{2} + \frac{m(m-1)}{2} - mn$ .....(ii)

from (i) & (ii) $n = 9$ & $m = 12$

2. ${}^nC_r + 2.{}^nC_{r-1} + {}^nC_{r-2} = {}^nC_r + {}^nC_{r-1} + {}^nC_{r-1} + {}^nC_{r-2}$
$= {}^{n+1}C_r + {}^{n+1}C_{r-1} = {}^{n+2}C_r$

3. ${}^nC_m + {}^{n-1}C_m + {}^{n-2}C_m + ... + {}^mC_m$
= Co-efficient of x^m in $(1+x)^n + (1+x)^{n-1} + ... + (1+x)^m$

= Co-efficient of x^m in $(1+x)^m\left[\frac{(1+x)^{n-m+1}-1}{x}\right] = {}^{n+1}C_{m+1}$

$S = {}^nC_m + 2.\,{}^{n-1}C_m + 3.\,{}^{n-2}C_m + ...$

$\Rightarrow$ S = Co-efficient of x^m in $(1+x)^n + 2.(1+x)^{n-1} + 3(1+x)^{n-2} + ...$

Let $S' = (1+x)^n + 2.(1+x)^{n-1} + 3(1+x)^{n-2} + ...+ (n-m+1)(1+x)^m$ ......(i)

$\Rightarrow\; \frac{S'}{(1+x)} = (1+x)^{n-1} + 2.(1+x)^{n-2} + ... + (n-m+1)(1+x)^{m-1}$

......(ii)

from (i) – (ii)

$\Rightarrow \dfrac{xS'}{1+x} = (1+x)^n + (1+x)^{n-1} + ... + (1+x)^m - (n-m+1)(1+x)^{m-1}$

$\Rightarrow \dfrac{xS'}{1+x} = (1+x)^m \left[\dfrac{(1+x)^{n-m+1}-1}{x}\right] - (n-m+1)(1+x)^{m-1}$

$\Rightarrow S' = \dfrac{(1+x)^{n+2}-(1+x)^{m+1}}{x^2} - \dfrac{(n-m+1)(1+x)^m}{x}$

$\Rightarrow$ S = Co-efficient of x^m in $S' = {}^{n+2}C_{m+2}$

4. $(a - b)^n$

$T_5 + T_6 = {}^nC_4(a)^{n-4}(-b)^4 + {}^nC_5(a)^{n-5}(-b)^5 = 0$

$\Rightarrow a + \dfrac{n-4}{5}(-b) = 0 \quad \Rightarrow a/b = \dfrac{n-4}{5}$

5. $S = \sum_{i=0}^{m} \binom{10}{i}\binom{20}{m-i} = {}^{10}C_0 . {}^{20}C_m + {}^{10}C_1 . {}^{20}C_{m-1} + ...$

$\Rightarrow$ S = coefficient of x^m in $(1+x)^{10}(1+x)^{20}$

$= {}^{30}C_m$

S is maximum when m = 15

6. $(1+t^2)^{12}(1+t^{12}+t^{24}+t^{36}) = (1+t^{12}+t^{24})(1+t^2)^{12}$

coefficient of $t^{24} = {}^{12}C_{12} + {}^{12}C_6 + {}^{12}C_0 = {}^{12}C_6 + 2$

7. $S = 2^k\, {}^nC_0 . {}^nC_k - 2^{k-1}\, {}^nC_1\, {}^{n-1}C_{k-1} + 2^{k-2}\, {}^nC_2\, {}^{n-2}C_{k-2} + ...$

$\Rightarrow S = \sum_{r=0}^{k} (-1)^r\, {}^nC_r\, {}^{n-r}C_{k-r} . 2^{k-r}$

$\Rightarrow S = \sum_{r=0}^{k} (-1)^r \dfrac{n!}{r!(n-r)!} \times \dfrac{(n-r)!.2^{k-r}}{(n-k)!(k-r)!}$

$= \sum_{r=0}^{k} (-1)^r . 2^{k-r} \dfrac{n!}{k!(n-k)!} \times \dfrac{k!}{r!(k-r)!}$

$= 2^k\, {}^nC_k \left(1-\dfrac{1}{2}\right)^k = {}^nC_k$

8. ${}^{(n-1)}C_r = (k^2-3)\, {}^nC_{r+1}$ or ${}^{(n-1)}C_{n-(r+1)} = (k^2-3)\, {}^nC_{n-(r+1)}$

$1 \geq k^2 - 3 > 0 \quad \Rightarrow k \in [-2, -\sqrt{3}) \cup (\sqrt{3}, 2]$

9. $S = {}^{30}C_0\, {}^{30}C_{20} - {}^{30}C_1\, {}^{30}C_{19} + {}^{30}C_2\, {}^{30}C_{18} ....$

S = Co-efficient of x^{20} in $(1-x)^{30}(1+x)^{30}$

S = Co-efficient of x^{20} in $(1-x^2)^{30} = {}^{30}C_{10}$

10. $B_{10} \sum_{r=1}^{10} A_r B_r - C_{10} \sum_{r=1}^{10} (A_r)^2 = {}^{20}B_{10}({}^{30}C_{20} - 1) - {}^{30}C_{10}$

$({}^{20}C_{10} - 1) = {}^{30}C_{10} - {}^{20}C_{10} = C_{10} - B_{10}$

13. Coeff. x^2

${}^2C_2 + {}^3C_2 + {}^4C_2 + ... + {}^{49}C_2 + {}^{50}C_2\, m^2 = (3n+1)\, {}^{51}C_3$

${}^3C_3 + {}^3C_2 + {}^4C_2 + ... + {}^{49}C_2 + {}^{50}C_2\, m^2 = (3n+1)\, {}^{51}C_3$

${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$\Rightarrow {}^{50}C_3 + {}^{50}C_2 . m^2 = (3n+1)\, {}^{51}C_3$

$\Rightarrow {}^{50}C_3 + {}^{50}C_2 + (m^2-1)\, {}^{50}C_2 = 3n. \dfrac{51}{3} . {}^{50}C_2 + {}^{51}C_3$

$\Rightarrow {}^{51}C_3 + (m^2-1)\, {}^{50}C_2 = 51n . {}^{50}C_2 + {}^{51}C_3$

$m^2 - 1 = 51n \quad \Rightarrow \quad m^2 = 51n + 1$

min value of m^2 for $51n + 1$ is integer for n = 5

MOCK TEST

1. Using rationalizing

$= \left(\sqrt{2x^2+1}+\sqrt{2x^2-1}\right)^6 + \left(\sqrt{2x^2+1}-\sqrt{2x^2-1}\right)^6$

$= 2\big({}^6C_0(2x^2+1)^3 + {}^6C_2(2x^2+1)^2(2x^2-1) +$

$\quad {}^6C_4(2x^2+1)(2x^2-1)^2 + {}^6C_6(2x^2-1)^3\big)$

clearly '6'

2. (B)

$t_{r+1} = {}^{21}C_r \left(\frac{a}{b}\right)^{\frac{21-r}{3}} \frac{b^{r/3}}{a^{r/6}} = {}^{21}C_r\ a^{\frac{42-3r}{6}}\ b^{\frac{2r-21}{3}}$

$\therefore$ $42 - 3r = 4r - 42$ i.e. $r = 12$

$\therefore$ 13^{th} term contains same powers of a and b

3. Co-efficient of x^{15} in $(1 + x + x^3 + x^4)^n$

$=$ Co-efficient of x^{15} in $(1 + x^3)^n (1 + x)^n$

$= {}^nC_0\,{}^nC_{15} + {}^nC_1\,{}^nC_{12} + {}^nC_2\,{}^nC_9 + {}^nC_3\,{}^nC_6 + {}^nC_4\,{}^nC_3 + {}^nC_5\,{}^nC_0$

4. (B)

$E = (x - \alpha_1)(x - \alpha_2)(x - \alpha_3)\ldots\ldots(x - \alpha_n)$

where $\alpha_1 = 1, \alpha_2 = 2$ etc

$= x^n - \left(\sum \alpha_1\right) x^{n-1} + \left(\sum \alpha_1 \alpha_2\right) x^{n-2} + \ldots\ldots$

Hence co-efficient of x^{n-2} = sum of all the products of the first 'n' natural numbers taken two at a time

$= \frac{(1+2+3+\ldots\ldots+n)^2 - (1^2+2^2+\ldots\ldots\ldots+n^2)}{2}$

$= \frac{n\left(n^2-1\right)\left(3n+2\right)}{24}$

5. Co-efficient of $x^n = {}^{2n+1}C_0 + {}^{2n+1}C_1 + \ldots.. + {}^{2n+1}C_n = 2^{2n}$

6. (C)

Let $x - 1 = t$, then

$\sum_{r=0}^{2n} a_r t^r = \sum_{r=0}^{2n} b_r (t-1)^r$

$\therefore$ a_n = coefficient of t^n in $\sum_{r=0}^{2n} b_r (t-1)^r$

$=$ coefficient of t^n in $(b_0 + b_1(t-1) + \ldots.+ b_n(t-1)^n$

$\quad + b_{n+1}(t-1)^{n+1} + \ldots\ldots.. + b_{2n}(t-1)^{2n})$

$= b_n\,{}^nC_0 + b_{n+1}\,{}^{n+1}C_1(-1)^1 + b_{n+2}\,{}^{n+2}C_2(-1)^2 + \ldots.$

$\quad + b_{2n}\,{}^{2n}C_n(-1)^n$

$= (-1)^{n-n}.\,{}^nC_0 + (-1)^{n+1-n+1}.\,{}^{n+1}C_1 + \ldots\ldots.$

$\quad + (-1)^{2n-n+n}\ {}^{2n}C_n$

$= {}^nC_0 + {}^{n+1}C_1 + {}^{n+2}C_2 + \ldots\ldots\ldots. + {}^{2n}C_n = {}^{2n+1}C_{n+1}$

$= {}^{2n+1}C_n$

7. $\sum_{r=1}^{n}\left(\sum_{p=0}^{r-1} {}^nC_r\,{}^rC_p 2^p\right) = \sum_{r=1}^{n} {}^nC_r \sum_{p=0}^{r-1} {}^rC_p.2^p$

$= \sum_{r=1}^{n} {}^nC_r\,[{}^rC_0 + {}^rC_1 . 2 + \ldots\ldots. + {}^rC_{r-1} 2^{r-1}]$

$= \sum_{r=1}^{n} {}^nC_r\ (3^r - 2^r) = 4^n - 3^n$

8. (B)

$2(1 + x^3)^{100} = \sum_{k=0}^{100}\left(a_k x^k - \cos\frac{\pi}{2}(x+k)\right)$

$= \sum_{k=0}^{100} a_k.x^k - \sum_{k=0}^{100}\cos\frac{\pi}{2}(x+k)$

$= \sum_{k=0}^{100} a_k.x^k -$

$\left[\cos\frac{\pi}{2}x + \cos\left(\frac{\pi}{2}x+\frac{\pi}{2}\right) + \cos\left(\frac{\pi}{2}x+\pi\right) + \cos\left(\frac{\pi}{2}x+\frac{3\pi}{2}\right) + \ldots..\right]$

$2(1 + x^3)^{100} = \sum_{k=0}^{100} a_k.x^k - (0)$

put $x = -1$ $\quad 0 = a_0 - a_1 + a_2 - a_3 + \ldots\ldots..$

put $x = 1$ $\quad 2^{101} = a_0 + a_1 + a_2 + a_3$

add

$2^{101} = 2[a_0 + a_2 + a_4 + \ldots\ldots..]$

$a_0 + a_2 + a_4 + \ldots\ldots + a_{100} = 2^{100}$

9. $(1 + x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \ldots\ldots\ldots. + {}^nC_n x^n$

Multiply it by x

$x(1 + x)^n = {}^nC_0 x + {}^nC_1 x^2 + {}^nC_2 x^3 + \ldots\ldots\ldots..\ {}^nC_n x^{n+1}$

Differentiate w.r. to x and put $x = -3$

$n x (1 + x)^{n-1} + (1 + x)^n$

$= {}^nC_0 + 2\,{}^nC_1 x + 3\,{}^nC_2 x^2 + 4\,{}^nC_3 x^3 + \ldots\ldots.. + (n + 1)\,{}^nC_n x^n$

So answer, $\quad -3n(-2)^{n-1} + (-2)^n = (-2)^n\left(1+\frac{3n}{2}\right)$

$= (-1)^n 2^n\left(\frac{3n}{2}+1\right)$

10. (B)

S_1 : $(1+x)^n = {}^nC_0 + {}^nC_1 x + ...+ {}^nC_n x^n$ ....**(i)**

$(x+1)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1} + ...+ {}^nC_n$ ....**(ii)**

coefficient of x^{n-1} in the product of two series

${}^nC_0 . {}^nC_1 + {}^nC_1 . {}^nC_2 + ... = {}^{2n}C_{n-1}$

S_2 : $(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + ...+ {}^nC_n x^n$ ....**(i)**

$(x+1)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1} + ...+ {}^nC_n$ ....**(ii)**

coefficient of x^n in the product of two series

$C_0^2 + C_1^2 .... = {}^{2n}C_n$

S_3 : $(1+x)^{11} = {}^{11}C_0 + {}^{11}C_1 x + ...+ {}^{11}C_{11} x^{11}$

$(1-x)^{11} + {}^{11}C_0 - {}^{11}C_1 x + ...+ {}^{11}C_{11} (-1)^{11} x^{11}$

Coefficient of x^{11} in product of two series

${}^{11}C_0\, {}^{11}C_{11} - {}^{11}C_1\, {}^{11}C_{10} + ...+ (-1)^{11} . {}^{11}C_{11}$

${}^{11}C_0$ = coefficient of x^{11} in the expansion of $(1-x^2)^{11} = 0$

S_4 : $\sum_{r=0}^{n} \frac{2^{r+1}\, {}^nC_r}{r+1} = \frac{1}{(n+1)} \sum_{r=0}^{n} {}^{n+1}C_{r+1}\, 2^{r+1}$

$= \frac{1}{n+1} ({}^{n+1}C_0 2^0 + {}^{n+1}C_1 2^1 + ... + {}^{n+1}C_{n+1} 2^{n+1} - {}^{n+1}C_0)$

$= \frac{3^{n+1}-1}{n+1}$

11. $n^n\left(\frac{n+1}{2}\right)^{2n} = \left(\frac{\left(\frac{n(n+1)}{2}\right)^2}{n}\right)^n$

$= \left(\frac{1^3+2^3+...+n^3}{n}\right)^n$

$\Rightarrow \frac{1^3+2^3+....+n^3}{n} \ge \sqrt[n]{(n!)^3}$

12. (A, B, C)

We have $a_1 = 2$ and for $n \ge 2$,

$a_n = \left(1+\frac{1}{n}\right)^n = {}^nC_0 + {}^nC_1\left(\frac{1}{n}\right) + {}^nC_2\left(\frac{1}{n}\right)^2 + ...$

$+ {}^nC_r\left(\frac{1}{n}\right)^r + ... + {}^nC_n\left(\frac{1}{n}\right)^n$

$= 1 + 1 + \frac{n(n-1)}{2!}\frac{1}{n^2} + ... + \frac{n(n-1)....(n-r+1)}{r!}$

$\frac{1}{n^r} + ... + \frac{n(n-1)....2.1}{n!}\frac{1}{n^n}$ ....**(i)**

$= 2 + \frac{1}{2!}\left(1-\frac{1}{n}\right) + \frac{1}{3!}\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right) + ...$

$+ \frac{1}{r!}\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right) .... \left(1-\frac{r-1}{n}\right)$

$+ ..... + \frac{1}{n!}\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right) .... \left(1-\frac{n-1}{n}\right)$

Hence from **(i)**, $a_n \ge 2$ for all $n \in N$. Also

$a_n \le 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + ... + \frac{1}{r!} + ... + \frac{1}{n!}$

For $2 \le r \le n$, we have $r! = 1 . 2 . 3 ..... r \ge 2^{r-1}$.

Thus, $a_n \le 1 + 1 + \frac{1}{2} + \frac{1}{2^2} + ... + \frac{1}{2^{r-1}} + ... + \frac{1}{2^{n-1}}$

$= 1 + \frac{1-(1/2^n)}{1-(1/2)} = 1 + 2\left(1-\frac{1}{2^n}\right) = 3 - \frac{1}{2^{n-1}}$

$a_n \le 3 - \frac{1}{2^{n-1}} < 3\ \forall\ n \ge 1 \Rightarrow a_n < 4\ \forall\ n \ge 1$

13. $a_n = \frac{(1000)(1000)............(1000)}{1.2.........n}$

$a_{999} = a_{1000}$

a_n is maximum for $n = 999$ and $n = 1000$

14. $y = (1-x)^{-1}(1+x)^n$

$\Rightarrow y = (1 + x + x^2 + ... \infty)(1+x)^n$

$y = (1+x)^n + x(1+x)^n + x^2(1+x)^n + ...$

Co-efficient of $x^r = {}^nC_r + {}^nC_{r-1} + ... {}^nC_0 = 2^n$

$r \ge n$ (As ${}^nC_{n+1}) = 0$

15. (A,C,D)

$I + f = (4 + \sqrt{15})^n$

Let $g = (4 - \sqrt{15})^n$, then $0 < g < 1$

$I + f = {}^nC_0 4^n + {}^nC_1 4^{n-1}\sqrt{15} + {}^nC_2 4^{n-2} 15 + {}^nC_3 4^{n-3} (\sqrt{15})^3 + ...$

$g = {}^nC_0 4^n - {}^nC_1 4^{n-1}\sqrt{15} + {}^nC_2 . 4^{n-2} . 15 - {}^nC_3 4^{n-3}(\sqrt{15})^3 + ...$

$\therefore I + f + g = 2({}^nC_0 4^n + {}^nC_2 4^{n-2} . 15 + ...)$ = even integer

$\because$ $0 < f + g < 2 \Rightarrow f + g = 1 \Rightarrow 1 - f = g$

thus I is an odd integer

$1 - f = g = (4 - \sqrt{15})^n$

$(I + f)(1 - f) = (I + f) . g = 1$

16. (D) $\because$ $(a + b + c)^n = \sum \frac{n!}{p! \cdot q! \cdot r!} . a^p b^q c^r$,

$p + q + r = n$

in statement-1 $p + q + r$ exceeds n

17. (D) $\sum_{r=0}^{n} r\ {}^nc_r p^r q^{n-r} = np \sum_{r=0}^{n} {}^{n-1}c_{r-1} p^{r-1} q^{n-r}$

$= np(q + p)^{n-1} = np$

18. (A)

If $n \geq 4$, then term containing $x^{\frac{n(n+1)}{2} - 4}$ is

$(-4)\, x^{1+2+3+5+6+7+...+n} + (-1)(-3)\, x^{2+4+5+6+...+n}$

$\therefore$ coefficient of $x^{\frac{n(n+1)}{2} - 4}$ is $-4 + (-1)(-3) = -1$

Statement : 2 is true and it explains statement-1

19. (A)

The number of ways of selecting committee of r persons among 40 women and 60 men = ${}^{100}C_r$

This will assume greatest value at r = 50

20. (A)

$1 + x = {}^nC_n + {}^nC_{n-1} + {}^{n+1}C_{n-1} + ... + {}^{2n}C_{n-1} = {}^{2n+1}C_n$

Since 2n + 1 and n are co-prime for every natural number n.

$\therefore$ ${}^{2n+1}C_n$ is divisible by 2n + 1

$\therefore$ $\frac{x+1}{2n+1}$ is an integer

21. (A) → (p), (B) → (q), (C) → (t), (D) → (s)

(A) $({}^mC_1\, {}^nC_m - {}^mC_2\, {}^{2n}C_m + {}^mC_3\, {}^{3n}C_m - .... (-1)^{m-1}\, {}^mC_m\, {}^{mn}C_m)$

= Coefficient of x^m in the expansion of

$({}^mC_1 (1 + x)^n - {}^mC_2 (1 + x)^{2n} + {}^mC_3(1 + x)^{3n} .... + (-1)^{m-1}.{}^mC_m . (1 + x)^{mn})$

= Coefficient of x^m in the expansion of

$({}^mC_0 - [{}^mC_0 - {}^mC_1(1 + x)^n + {}^mC_2(1 + x)^{2n} - {}^mC_3(1 + x)^{3n} + ... + (-1)^m\, {}^mC_m(1 + x)^{mn}])$

= Coefficient of x^m in the expansion of $(1 - (1 - (1 + x)^n)^m)$

= Coefficient of x^m in the expansion of $-[1 - (1+x)^n]^m$

(B) ${}^nC_m + {}^{n-1}C_m + {}^{n-2}C_m + ... + {}^mC_m$ is the coefficient of x^m in the expansion of

$(1 + x)^n + (1 + x)^{n-1} + (1 + x)^{n-2} + ... + (1 + x)^m$

$= (1 + x)^m[1 + (1 + x) + (1 + x)^2 + ... + (1 + x)^{n-m}]$

$= (1 + x)^m \left(\frac{1 - (1 + x)^{n-m+1}}{1 - (1 + x)}\right) = \frac{(1 + x)^{n+1} - (1 + x)^m}{x}$

Thus the given expression is equal to the coefficient of x^m in the expansion of $\frac{(1 + x)^{n+1}}{x}$

(C) $(1 + x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + {}^nC_3x^3 + ... + {}^nC_nx^n$ .... (A)

$(1 + x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + {}^nC_3x^3 + ... + {}^nC_nx^n$ .... (B)

multiplying eq. (A) and (B) and equating coefficient of x^n on both the sides.

coefficient of x^n in the expansion of $(1 + x)^n (1 + x)^n$

$= {}^nC_0\, {}^nC_n + {}^nC_1\, {}^nC_{n-1} + {}^nC_2\, {}^nC_{n-2} + ... + {}^nC_n\, {}^nC_0$

$\therefore$ Coefficient of x^n in the expansion of $(1 + x)^{2n}$

$= {}^{2n}C_n$

(D) $2^m\, {}^nC_m$ = Coefficient of x^m in the expansion of $(1 + 2x)^n$

$2^{m-1}\, {}^{n-1}C_{m-1}$ = Coefficient of x^{m-1} in the expansion of $(1 + 2x)^{n-1}$

= Coefficient of x^m in the expansion of $x(1 + 2x)^{n-1}$

$\therefore$ given expression = coefficient of x^m in the expansion of

${}^nC_0(1 + 2x)^n - {}^nC_1x(1 + 2x)^{n-1} + {}^nC_2x^2(1 + 2x)^{n-2} - ......$

= coefficient of x^m in the expansion of $(1 + 2x - x)^n = {}^nC_m$

22. (A) → (r), (B) → (p), (C) → (s), (D) → (q)

(A) $((1 + x)^5)^{20} = (1 + x)^{100}$

no. of terms = 101 = λ

$3^{101} = 3.3^{100} = 3.(9^{50}) = 3.(10 - 1)^{50}$

$= 3.(1 - 10)^{50} = 3(1 - {}^{50}C_1.(10) + {}^{50}C_2.(10)^2 - ....)$

= 3(1 – 500 + 122500 + ...) = 3[1 + some number with zero at unit and tenth place]

= 3[......01] = ...03

O + T = 3

(B) $8.\left\{\frac{(8+1)^n}{8}\right\} = 8.\left\{\frac{1}{8}+\frac{C_1.8+C_2.8^2+.......}{8}\right\} = 8.\frac{1}{8} = 1$

(C) $(x+\sqrt{3x^2+1})^7 = {}^7C_0x^7 + {}^7C_1.x^6.$

$(\sqrt{3x^2+1})^1 + {}^7C_2.x^5.(\sqrt{3x^2+1})^2+.......$

$(x-\sqrt{3x^2+1})^7 = {}^7C_0x^7 - {}^7C_1.x^6.(\sqrt{3x^2+1})+.........$

$\Rightarrow 2\left({}^7C_1x^6.\sqrt{3x^2+1}+{}^7C_3x^4.(\sqrt{3x^2+1})^3+ {}^7C_5.x^2(\sqrt{3x^2+1})^5+{}^7C_7.x^0(\sqrt{3x^2+1})^7\right)$

multiply by $\sqrt{3x^2+1}$ =

$\left({}^7C_1x^6.(3x^2+1)+{}^7C_5x^4(3x^2+1)^2+{}^7C_5.x^2(3x^2+1)^3+.....\right)$

Degree = 8

(D) $\frac{(2002+1)}{1.2002} \cdot \frac{(2001+2)}{2.2001} \cdot \frac{(2000+3)}{3.2000} + \ldots\ldots\ldots\ldots$

$\frac{(1002+1001)}{1001.1002}$

$\frac{(2003)^{1001}}{(2002)!}$

23.

1 (C)

$\sum_{r=0}^{6m} {}^{6m}C_r 2^{r/2}$ put $x=\sqrt{2} = (1+\sqrt{2})^{6m}$

$= (3+2\sqrt{2})^{3m}$

2 (B)

$\sum_{r=0}^{3m}(-1)^r\ {}^{6m}C_{2r}$

$=\frac{1}{2}\left[\left(\sqrt{2}\right)^{6m}\left(\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}\right)^{6m}+\left(\sqrt{2}\right)^{6m}\left(\frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}}\right)^{6m}\right]$

$= 2^{3m}.\cos\frac{3m\pi}{2} = \begin{cases} 0 & \text{if } m \text{ is odd} \\ (-1)^{\frac{m}{2}}2^{3m} & \text{if } m \text{ is even}\end{cases}$

3 (A)

$\sum_{r=1}^{3m}(-3)^{r-1}\ {}^{6m}C_{2r-1}$

$=\frac{1}{\sqrt{3}i}$

$\left\{\sqrt{3}i\ {}^{6m}C_1+(\sqrt{3}i)^3\ {}^{6m}C_3+(\sqrt{3}i)^5\ {}^{6m}C_5+...+(\sqrt{3}i)^{6m-1}\ {}^{6m}C_{6m-1}\right\}$

$(1+\sqrt{3}i)^{6m} = {}^{6m}C_0 + \sqrt{3}i\ {}^{6m}C_1 + (\sqrt{3}i)^2\ {}^{6m}C_2 + (\sqrt{3}i)^3\ {}^{6m}C_3 + ...$

$(1-\sqrt{3}i)^{6m} = {}^{6m}C_0 - \sqrt{3}i\ {}^{6m}C_1 + (\sqrt{3}i)^2\ {}^{6m}C_2 - (\sqrt{3}i)^3\ {}^{6m}C_3 + ...$

$\therefore (1+\sqrt{3}i)^{6m} - (1-\sqrt{3}i)^{6m} = 2[\sqrt{3}i\ {}^{6m}C_1 + (\sqrt{3}i)^3\ {}^{6m}C_3 + ...]$

$\therefore$ given expression

$\frac{1}{2\sqrt{3}i}\left[(1+\sqrt{3}i)^{6m}-(1-\sqrt{3}i)^{6m}\right]$

$=\frac{2^{6m}}{2\sqrt{3}i}(\cos 2m\pi + i\sin 2m\pi - \cos 2m\pi + i\sin 2m\pi)$

$= 0$

24.

1 $S_2(n)+S_1(n) = \sum n^2+\sum n = \sum n(n+1) = 1.2+2.3+3.4+.........+n(n+1) = B(2,n)$

2 $S_3(n)+3S_2(n)+2S_1(n)-2S_1(n)$

$=\sum n^3+3\sum n^2+2\sum n-2\sum n = \sum n(n+1)(n+2)-2\sum n$

$= B(3,n)-2B(1,n)$

3 $(1+x)^{k+1} = {}^{k+1}C_{k+1} + {}^{k+1}C_k x + {}^{k+1}C_{k-1}x^2 + {}^{k+1}C_{k-2}x^3 + ... + {}^{k+1}C_0 x^{k+1}$

Put $x = 1, 2, ..., n$

$2^{k+1} = {}^{k+1}C_{k+1} + {}^{k+1}C_k.1 + {}^{k+1}C_{k-1}.1^2 + {}^{k+1}C_{k-2}.1^3 + ... + {}^{k+1}C_0.1^{k+1}$

$3^{k+1} = {}^{k+1}C_{k+1} + {}^{k+1}C_k.2 + {}^{k+1}C_{k-1}.2^2 + ... + {}^{k+1}C_0.2^{k+1}$

$\vdots$

$(1+n)^{k+1} = {}^{k+1}C_{k+1} + {}^{k+1}C_k.n + {}^{k+1}C_{k-1}, n^2 + ... + {}^{k+1}C_0 n^{k+1}$

$2^{k+1}+3^{k+1}+...........+(1+n)^{k+1} = {}^{k+1}C_{k+1}S_0(n) + {}^{k+1}C_kS_1(n) + {}^{k+1}C_{k-1}S_2(n) + ... + {}^{k+1}C_0S_{(k+1)}(n)$

$2^{k+1}+3^{k+1}+............+(n+1)^{k+1} = {}^{k+1}C_{k+1}S_0(n) + {}^{k+1}C_kS_1(n) + ... + {}^{k+1}C_1S_k(n) + 1^{k+1}+2^{k+1}+3^{k+1}+..........+n^{k+1}$

So $(n+1)^{k+1}-1$

25.

$x^3 - 1$

$x = 1, \omega, \omega^2$ or $x = \omega, \omega^2, \omega^3$

$x = 1 \quad : \quad C_0 + C_1 + C_2 + C_3 + C_4 + ... = 2^n$

$x = \omega \quad : \quad C_0 + C_1\omega + C_2\omega^2 + C_3\omega^3 + C_4\omega^4 + ... = (1+\omega)^n$

$x = \omega^2 \quad : \quad C_0 + C_1\omega^2 + C_2\omega^4 + C_3\omega^6 + ... = (1+\omega^2)^n$

$3(C_0 + 0 + 0 + C_3 + 0 + 0 + C_6 + ...)$

$= 2^n + (-\omega^2)^n + (-\omega)^n = 2^n + (-1)^n + (-1)^n$

$\therefore \quad C_0 + C_3 + C_6 + ... = \dfrac{2^n + 2(-1)^n}{3}$

$x^4 - 1 = 0 \quad \Rightarrow \quad x = \pm 1, \pm i$

$\therefore$ Sum of values of $x = 1 + (-1) + i + (-i) = 0$

26. (7)

$$\text{If } (1-x^3)^n = \sum_{r=0}^{n} a_r x^r (1-x)^{3n-2r}$$

$$\Rightarrow [(1-x)(1+x+x^2)]^n = (1-x)^n \sum_{r=0}^{n} a_r x^r [(1-x)^2]^{n-r}$$

$$\Rightarrow [3x + (1-x)^2]^n = \sum_{r=0}^{n} a_r x^r [(1-x)^2]^{n-r}$$

$$\Rightarrow \sum_{r=0}^{n} {}^nC_r (3x)^r [(1-x)^2]^{n-r} = \sum_{r=0}^{n} a_r x^r [(1-x)^2]^{n-r}$$

comparing the coefficients of like power of x on both sides, we get $a_r = {}^nC_r \cdot 3^r$.

$\therefore \quad a_1 = 3 \cdot {}^nC_1,\ a_2 = 9 \cdot {}^nC_2$ and $a_3 = 27 \cdot {}^nC_3$

$\therefore \quad a_1, a_2, a_3$ are in A.G.P. iff ${}^nC_1, {}^nC_2, {}^nC_3$ are in A.P.

$\therefore \quad n = 7$

27. $(1-x)^{-n} = a_0 + a_1x + a_2x^2 + ... a_nx^n = \sum_{r=0}^{n} {}^{n+r-1}C_r x^r$

$a_0 + a_1 + ... + a_n = {}^{n-1}C_0 + {}^nC_1 + {}^{n+1}C_2 + ... + {}^{2n-1}C_n$

$= {}^{n-1}C_{n-1} + {}^nC_{n-1} + {}^{n+1}C_{n-1} + ... + {}^{2n-1}C_{n-1}$

$= {}^nC_n + {}^nC_{n-1} + {}^{n+1}C_{n-1} + ... + {}^{2n-1}C_{n-1} = {}^{2n}C_n$

$\{{}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r\}$

28. (1)

Any term of the expansion is of the form

$$\frac{6!}{a!b!c!}(5^{1/3}x)^a (3^{1/2}y)^b z^c$$

a,b, c non- negative integers and a+ b+ c =6. For rational coefficients 'a' must be multiple of 3 and b must be multiple of 2.

The following are the possibilities

a	b	c
0	0	6
0	2	4
0	4	2
0	6	0
3	0	3
3	2	1
6	0	0

Sum of coefficients

$$= \frac{6!}{6!} + \frac{6!}{2!4!}.3 + \frac{6!}{4!2!}3^2 + \frac{6!}{6!}3^3 + \frac{6!}{3!3!}5 + \frac{6!}{3!2!}.5.3 + \frac{6!}{6!}5^2$$

$= 1233$

29. $32^{32} = (2^5)^{32} = 2^{160}$

$(3-1)^{160} = 3\lambda + 1$

$32^{32^{32}} = (2^5)^{(3\lambda+1)} = 2^{(15\lambda+3)+2} = 4 \cdot (2^3)^{(5\lambda+1)}$

$= 4(7+1)^\beta = 4(7\mu + 1)$

$\therefore$ remainder is 4

30. (2)

$x^3 - 3xy^2 = 2005$

$$\Rightarrow \left[\left(\frac{x}{y}\right)^3 - 3\left(\frac{x}{y}\right) = \frac{2005}{y^3}\right] \times 2004 \qquad .....(i)$$

$y^3 - 3x^2y = 2004$

$$\Rightarrow \left[1 - 3\left(\frac{x}{y}\right)^2 = \frac{2004}{y^3}\right] \times 2005 \qquad .....(ii)$$

subtract (i) & (ii) & put $\dfrac{x}{y} = t$

$2004t^3 + 6015t^2 - 6012t - 2005 = 0 \to t_1, t_2, t_3$

$$\frac{y_1 \cdot y_2 \cdot y_3}{(y_1 - x_1)(y_2 - x_2)(y_3 - y_3)} = \frac{1}{(1-t_1)(1-t_2)(1-t_3)}$$

$$= \frac{1}{1 + (t_1t_2 + t_2t_3 + t_3t_1) - (t_1 + t_2 + t_3) - t_1t_2t_3}$$

put values $= 1002$