# SOLVED EXAMPLES

**Ex. 1** The values of x and y satisfying the equation $\frac{(1+i)x-2i}{3+i}+\frac{(2-3i)y+i}{3-i}=i$ are

**Sol.** $\frac{(1+i)x-2i}{3+i}+\frac{(2-3i)y+i}{3-i}=i \quad \Rightarrow \quad (4+2i)\,x+(9-7i)\,y-3i-3=10i$

Equating real and imaginary parts, we get $2x-7y=13$ and $4x+9y=3$.

Hence $x=3$ and $y=-1$.

**Ex. 2** Find the square root of $7+24\,i$.

**Sol.** Let $\sqrt{7+24i}=a+ib$

Squaring $a^2-b^2+2iab=7+24i$

Compare real & imaginary parts $a^2-b^2=7$ & $2ab=24$

By solving these two equations

We get $a=\pm4$, $b=\pm3$

$\sqrt{7+24i}=\pm(4+3i)$

**Ex. 3** Find the value of expression $x^4-4x^3+3x^2-2x+1$ when $x=1+i$ is a factor of expression.

**Sol.** $x=1+i$

$\Rightarrow \quad x-1=i$

$\Rightarrow \quad (x-1)^2=-1$

$\Rightarrow \quad x^2-2x+2=0$

Now $\quad x^4-4x^3+3x^2-2x+1$

$=(x^2-2x+2)\,(x^2-3x-3)-4x+7$

$\therefore \quad$ when $x=1+i$ i.e. $x^2-2x+2=0$

$x^4-4x^3+3x^2-2x+1 \quad =0-4\,(1+i)+7\;=-4+7-4i\;=3-4i$

**Ex. 4** Find modulus and argument for $z=1-\sin\alpha+i\cos\alpha,\ \alpha\in(0,2\pi)$

**Sol.** $|z|=\sqrt{(1-\sin\alpha)^2+(\cos\alpha)^2}=\sqrt{2-2\sin\alpha}=\sqrt{2}\left|\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2}\right|$

**Case I** For $\alpha\in\left(0,\frac{\pi}{2}\right)$, z will lie in I quadrant.

$$\text{amp}(z)=\tan^{-1}\frac{\cos\alpha}{1-\sin\alpha}\Rightarrow \text{amp}(z)=\tan^{-1}\frac{\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2}}{\left(\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2}\right)^2}=\tan^{-1}\frac{\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2}}$$

$\Rightarrow \quad \arg z=\tan^{-1}\tan\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)$

Since $\frac{\pi}{4}+\frac{\alpha}{2}\in\left(\frac{\pi}{4},\frac{\pi}{2}\right)$

$\therefore \quad \text{amp}(z)=\left(\frac{\pi}{4}+\frac{\alpha}{2}\right),\ |z|=\sqrt{2}\left(\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2}\right)$

**Case II** at $\alpha = \frac{\pi}{2}$: $z = 0 + 0i$

$|z| = 0$

amp (z) is not defined.

**Case III** For $\alpha \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$, z will lie in IV quadrant

So $\quad \text{amp}(z) = -\tan^{-1}\tan\left(\frac{\alpha}{2} + \frac{\pi}{4}\right)$

Since $\quad \frac{\alpha}{2} + \frac{\pi}{4} \in \left(\frac{\pi}{2}, \pi\right)$

$\therefore \quad \text{amp}(z) = -\left(\frac{\alpha}{2} + \frac{\pi}{4} - \pi\right) = \frac{3\pi}{4} - \frac{\alpha}{2}$, $|z| = \sqrt{2}\left(\sin\frac{\alpha}{2} - \cos\frac{\alpha}{2}\right)$

**Case IV** at $\alpha = \frac{3\pi}{2}$: $z = 2 + 0i$

$|z| = 2$

amp (z) = 0

**Case V** For $\alpha \in \left(\frac{3\pi}{2}, 2\pi\right)$, z will lie in I quadrant

$\text{arg}(z) = \tan^{-1}\tan\left(\frac{\alpha}{2} + \frac{\pi}{4}\right)$

Since $\quad \frac{\alpha}{2} + \frac{\pi}{4} \in \left(\pi, \frac{5\pi}{4}\right)$

$\therefore \quad \text{arg } z = \frac{\alpha}{2} + \frac{\pi}{4} - \pi = \frac{\alpha}{2} - \frac{3\pi}{4}$, $|z| = \sqrt{2}\left(\sin\frac{\alpha}{2} - \cos\frac{\alpha}{2}\right)$

**Ex. 5** If $x_n = \cos\left(\frac{\pi}{2^n}\right) + i\sin\left(\frac{\pi}{2^n}\right)$ then $x_1 x_2 x_3 \ldots\ldots \infty$ is equal to -

**Sol.** $x_n = \cos\left(\frac{\pi}{2^n}\right) + i\sin\left(\frac{\pi}{2^n}\right) = 1 \times e^{i\frac{\pi}{2^n}}$

$x_1 x_2 x_3 \ldots\ldots \infty$

$$= e^{i\frac{\pi}{2^1}} . e^{i\frac{\pi}{2^2}} --- e^{i\frac{\pi}{2^n}} = e^{i\left(\frac{\pi}{2} + \frac{\pi}{2^2} + --- + \frac{\pi}{2^n}\right)}$$

$$= \cos\left(\frac{\pi}{2} + \frac{\pi}{2^2} + \frac{\pi}{2^3} + \ldots\ldots\right) + i\sin\left(\frac{\pi}{2} + \frac{\pi}{2^2} + \frac{\pi}{2^3} + \ldots\ldots\right) = -1$$

$$\left(\text{as } \frac{\pi}{2} + \frac{\pi}{2^2} + \frac{\pi}{2^3} + \ldots\ldots = \frac{\pi/2}{1 - 1/2} = \pi\right)$$

**Ex. 6** If $\left|\frac{z-i}{z+i}\right| = 1$, then locus of z is -

**Sol.** We have, $\left|\frac{z-i}{z+i}\right| = 1 \Rightarrow \left|\frac{x+i(y-1)}{x+i(y+1)}\right| = 1$

$\Rightarrow \frac{|x+i(y-1)|^2}{|x+i(y+1)|^2} = 1 \Rightarrow x^2+(y-1)^2 = x^2+(y+1)^2 \Rightarrow 4y = 0; y = 0$, which is x-axis

**Ex. 7** Solve for z if $z^2 + |z| = 0$

**Sol.** Let $z = x + iy$

$\Rightarrow \quad (x+iy)^2 + \sqrt{x^2+y^2} = 0$

$\Rightarrow \quad x^2 - y^2 + \sqrt{x^2+y^2} = 0$ and $2xy = 0$

$\Rightarrow \quad x = 0$ or $y = 0$

when $x = 0$ $\quad -y^2 + |y| = 0$

$\Rightarrow \quad y = 0, 1, -1 \quad \Rightarrow \quad z = 0, i, -i$

when $\quad y = 0 \quad x^2 + |x| = 0$

$\Rightarrow \quad x = 0$

$\Rightarrow \quad z = 0$

$z = 0, z = i, z = -i$

**Ex. 8** If $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$ then $\left(\frac{z_1}{z_2}\right)$ is -

**Sol.** Here let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1), |z_1| = r_1$

$z_2 = r_2(\cos\theta_2 + i\sin\theta_2), |z_2| = r_2$

$\therefore \quad |(z_1+z_2)|^2 = |(r_1\cos\theta_1 + r_2\cos\theta_2) + i(r_1\sin\theta_1 + r_2\sin\theta_2)|^2$

$= r_1^2 + r_2^2 + 2r_1r_2\cos(\theta_1-\theta_2) = |z_1|^2 + |z_2|^2$ if $\cos(\theta_1-\theta_2) = 0$

$\therefore \quad \theta_1 - \theta_2 = \pm\frac{\pi}{2}$

$\Rightarrow \quad amp(z_1) - amp(z_2) = \pm\frac{\pi}{2}$

$\Rightarrow \quad amp\left(\frac{z_1}{z_2}\right) = \pm\frac{\pi}{2} \Rightarrow \frac{z_1}{z_2}$ is purely imaginary

**Ex. 9** The locus of the complex number z in argand plane satisfying the inequality

$$\log_{1/2}\left(\frac{|z-1|+4}{3|z-1|-2}\right) > 1 \quad \left(\text{where } |z-1| \neq \frac{2}{3}\right) \text{ is -}$$

**Sol.** We have, $\log_{1/2}\left(\frac{|z-1|+4}{3|z-1|-2}\right) > 1 = \log_{1/2}\left(\frac{1}{2}\right)$

$\Rightarrow \frac{|z-1|+4}{3|z-1|-2} < \frac{1}{2}$ $\quad [\because \log_a x$ is a decreasing function if $a < 1]$

$\Rightarrow 2|z-1|+8 < 3|z-1|-2$ as $|z-1| > 2/3$

$\Rightarrow |z-1| > 10$

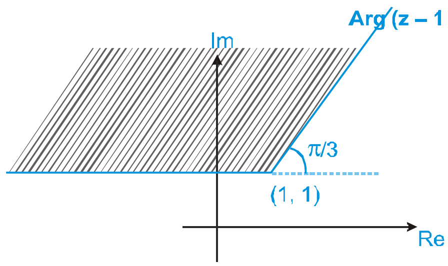
which is exterior of a circle.

**Ex. 10** Sketch the region given by

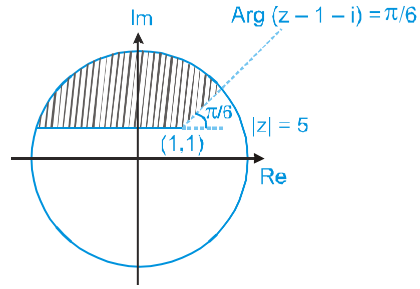
**(i)** $\text{Arg}(z-1-i) \geq \pi/3$ **(ii)** $|z| \leq 5$ & $\text{Arg}(z-i-1) > \pi/6$

**Sol.**

**(i)**




**(ii)**



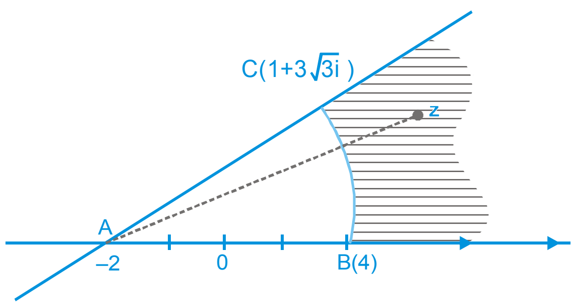

**Ex. 11** Shaded region is given by -

**(A)** $|z+2| \geq 6, 0 \leq \arg(z) \leq \frac{\pi}{6}$

**(B)** $|z+2| \geq 6, 0 \leq \arg(z) \leq \frac{\pi}{3}$

**(C)** $|z+2| \leq 6, 0 \leq \arg(z) \leq \frac{\pi}{2}$

**(D)** None of these



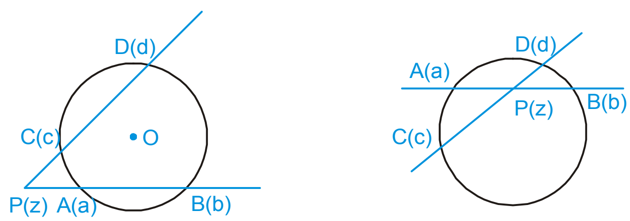

**Sol.** Note that AB = 6 and $1 + 3\sqrt{3}i = -2 + 3 + 3\sqrt{3}i = -2 + 6\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = -2 + 6\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$\therefore \quad \angle BAC = \frac{\pi}{3}$

Thus, shaded region is given by $|z+2| \geq 6$ and $0 \leq \arg(z+2) \leq \frac{\pi}{3}$

**Ex. 12** Two different non parallel lines cut the circle $|z| = r$ in point a, b, c, d respectively. Prove that these lines meet in the point z given by $z = \dfrac{a^{-1} + b^{-1} - c^{-1} - d^{-1}}{a^{-1}b^{-1} - c^{-1}d^{-1}}$

**Sol.** Since point P, A, B are collinear




$$\therefore \quad \begin{vmatrix} z & \bar{z} & 1 \\ a & \bar{a} & 1 \\ b & \bar{b} & 1 \end{vmatrix} = 0 \qquad \Rightarrow \qquad z\left(\bar{a} - \bar{b}\right) - \bar{z}\,(a - b) + \left(a\bar{b} - \bar{a}b\right) = 0 \qquad .....\textbf{(i)}$$

Similarly, points P, C, D are collinear, so

$$z\left(\bar{c} - \bar{d}\right) - \bar{z}\,(c - d) + \left(c\bar{d} - \bar{c}d\right) = 0 \qquad .....\textbf{(ii)}$$

On applying **(i)** × (c – d) – **(ii)** (a – b), we get

$$\therefore \quad z\left(\bar{a} - \bar{b}\right)(c - d) - z\left(\bar{c} - \bar{d}\right)(a - b) = \left(c\bar{d} - \bar{c}d\right)(a - b) - \left(a\bar{b} - \bar{a}b\right)(c - d) \qquad .....\textbf{(iii)}$$

$$\because \quad z\bar{z} = r^2 = k \text{ (say)} \therefore \qquad \bar{a} = \frac{k}{a},\ \bar{b} = \frac{k}{b},\ \bar{c} = \frac{k}{c} \text{ etc.}$$

From equation **(iii)** we get

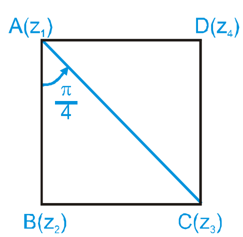
$$z\left(\frac{k}{a} - \frac{k}{b}\right)(c - d) - z\left(\frac{k}{c} - \frac{k}{d}\right)(a - b) = \left(\frac{ck}{d} - \frac{kd}{c}\right)(a - b) - \left(\frac{ak}{b} - \frac{bk}{a}\right)(c - d)$$

$$\therefore \quad z = \frac{a^{-1} + b^{-1} - c^{-1} - d^{-1}}{a^{-1}b^{-1} - c^{-1}d^{-1}}$$

**Ex. 13** If the vertices of a square ABCD are $z_1, z_2, z_3$ & $z_4$ then find $z_3$ & $z_4$ in terms of $z_1$ & $z_2$.

**Sol.** Using vector rotation at angle A

$$\frac{z_3 - z_1}{z_2 - z_1} = \frac{|z_3 - z_1|}{|z_2 - z_1|} e^{i\frac{\pi}{4}}$$




$$\because \quad |z_3 - z_1| = AC \text{ and } |z_2 - z_1| = AB$$

Also $AC = \sqrt{2}\,AB$

$$\therefore \quad |z_3 - z_1| = \sqrt{2}\,|z_2 - z_1|$$

$$\Rightarrow \quad \frac{z_3 - z_1}{z_2 - z_1} = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

$$\Rightarrow \quad z_3 - z_1 = (z_2 - z_1)(1 + i)$$

$$\Rightarrow \quad z_3 = z_1 + (z_2 - z_1)(1 + i)$$

Similarly $z_4 = z_2 + (1 + i)(z_1 - z_2)$

**Ex. 14** If A(2 + 3i) and B(3 + 4i) are two vertices of a square ABCD (take in anticlock wise order) then find C and D.

**Sol.** Let affix of C and D are $z_3$ and $z_4$ respectively.

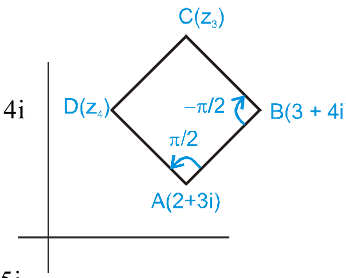
Considering $\angle DAB = 90^\circ$ and AD = AB

we get $\dfrac{z_4-(2+3i)}{(3+4i)-(2+3i)} = \dfrac{AD}{AB}e^{\frac{i\pi}{2}}$

$\Rightarrow$ $z_4 - (2 + 3i) = (1 + i)\, i$ $\Rightarrow$ $z_4 = 2 + 3i + i - 1 = 1 + 4i$

and $\dfrac{z_3-(3+4i)}{(2+3i)-(3+4i)} = \dfrac{CB}{AB}e^{-\frac{i\pi}{2}}$

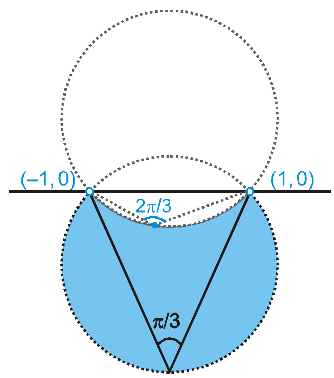
$\Rightarrow$ $z_3 = 3 + 4i - (1 + i)(-i)$ $\Rightarrow$ $z_3 = 3 + 4i + i - 1 = 2 + 5i$




**Ex. 15** Plot the region represented by $\dfrac{\pi}{3} \le \arg\left(\dfrac{z+1}{z-1}\right) \le \dfrac{2\pi}{3}$ in the Argand plane.

**Sol.** Let us take $\arg\left(\dfrac{z+1}{z-1}\right) = \dfrac{2\pi}{3}$, clearly z lies on the minor arc of the circle passing through (1, 0) and (–1, 0). Similarly, $\arg\left(\dfrac{z+1}{z-1}\right) = \dfrac{\pi}{3}$ means that 'z' is lying on the major arc of the circle passing through (1, 0) and (–1, 0). Now if we take any point in the region included between two arcs say $P_1(z_1)$ we get $\dfrac{\pi}{3} \le \arg\left(\dfrac{z+1}{z-1}\right) \le \dfrac{2\pi}{3}$

Thus $\dfrac{\pi}{3} \le \arg\left(\dfrac{z+1}{z-1}\right) \le \dfrac{2\pi}{3}$ represents the shaded region (excluding points (1, 0) and (–1, 0)).




**Ex. 16** If $z_1$, $z_2$ & $z_3$ are the affixes of three points A, B & C respectively and satisfy the condition $|z_1 - z_2| = |z_1| + |z_2|$ and $|(2 - i)\, z_1 + iz_3| = |z_1| + |(1 - i)\, z_1 + iz_3|$ then prove that $\Delta$ ABC in a right angled.

**Sol.** $|z_1 - z_2| = |z_1| + |z_2|$

$\Rightarrow$ $z_1$, $z_2$ and origin will be collinear and $z_1$, $z_2$ will be opposite side of origin

Similarly $|(2 - i)\, z_1 + iz_3| = |z_1| + |(1 - i)\, z_1 + iz_3|$

$\Rightarrow$ $z_1$ and $(1 - i)\, z_1 + iz_3 = z_4$ say, are collinear with origin and lies on same side of origin.

Let $z_4 = \lambda z_1$, $\lambda$ real

then $(1 - i)\, z_1 + iz_3 = \lambda z_1$

$\Rightarrow$ $i(z_3 - z_1) = (\lambda - 1)\, z_1$ $\Rightarrow$ $\dfrac{(z_3-z_1)}{-z_1} = (\lambda - 1)\, i$

$\Rightarrow$ $\dfrac{z_3-z_1}{0-z_1} = me^{i\pi/2}$, $m = \lambda - 1$ $\Rightarrow$ $z_3 - z_1$ is perpendicular to the vector $0 - z_1$.

i.e. also $z_2$ is on line joining origin and $z_1$

so we can say the triangle formed by $z_1$, $z_2$ and $z_3$ is right angled.

**Ex. 17** If $\alpha$, $\beta$, $\gamma$ are roots of $x^3 - 3x^2 + 3x + 7 = 0$ (and $\omega$ is imaginary cube root of unity), then find the value of $\frac{\alpha-1}{\beta-1}+\frac{\beta-1}{\gamma-1}+\frac{\gamma-1}{\alpha-1}$.

**Sol.** We have $x^3 - 3x^2 + 3x + 7 = 0$

$\therefore \quad (x-1)^3 + 8 = 0$

$\therefore \quad (x-1)^3 = (-2)^3$

$\Rightarrow \quad \left(\frac{x-1}{-2}\right)^3 = 1 \Rightarrow \frac{x-1}{-2} = (1)^{1/3} = 1, \omega, \omega^2$ (cube roots of unity)

$\therefore \quad x = -1, 1-2\omega, 1-2\omega^2$

Here $\quad \alpha = -1, \beta = 1-2\omega, \gamma = 1-2\omega^2$

$\therefore \quad \alpha - 1 = -2, \beta - 1 = -2\omega, \gamma - 1 = -2\omega^2$

Then $\quad \frac{\alpha-1}{\beta-1}+\frac{\beta-1}{\gamma-1}+\frac{\gamma-1}{\alpha-1} = \left(\frac{-2}{-2\omega}\right)+\left(\frac{-2\omega}{-2\omega^2}\right)+\left(\frac{-2\omega^2}{-2}\right) = \frac{1}{\omega}+\frac{1}{\omega}+\omega^2 = \omega^2+\omega^2+\omega^2$

Therefore $\frac{\alpha-1}{\beta-1}+\frac{\beta-1}{\gamma-1}+\frac{\gamma-1}{\alpha-1} = 3\omega^2$.

**Ex. 18** If z is a point on the Argand plane such that $|z - 1| = 1$, then $\frac{z-2}{z}$ is equal to -

**Sol.** Since $|z - 1| = 1$,

$\therefore$ let $z - 1 = \cos\theta + i\sin\theta$

Then, $z - 2 = \cos\theta + i\sin\theta - 1$

$$= -2\sin^2\frac{\theta}{2} + 2i\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 2i\sin\frac{\theta}{2}\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right) \quad \text{..... (i)}$$

and $z = 1 + \cos\theta + i\sin\theta$

$$= 2\cos^2\frac{\theta}{2} + 2i\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right) \quad \text{..... (ii)}$$

From **(i)** and **(ii)**, we get $\frac{z-2}{z} = i\tan\frac{\theta}{2} = i\tan(\arg z) \left(\because \arg z = \frac{\theta}{2} \text{ from (ii)}\right)$

**Ex. 19** Let a be a complex number such that $|a| < 1$ and $z_1, z_2, \ldots\ldots, z_n$ be the vertices of a polygon such that $z_k = 1 + a + a^2 + \ldots. a^k$, then show that vertices of the polygon lie within the circle $\left|z - \frac{1}{1-a}\right| = \frac{1}{|1-a|}$.

**Sol.** We have, $z_k = 1 + a + a^2 + \ldots.. + a^k = \frac{1-a^{k+1}}{1-a}$

$\Rightarrow z_k - \frac{1}{1-a} = \frac{-a^{k+1}}{1-a} \Rightarrow \left|z_k - \frac{1}{1-a}\right| = \frac{|a|^{k+1}}{|1-a|} < \frac{1}{|1-a|} \quad (\because |a| < 1)$

$\therefore$ Vertices of the polygon $z_1, z_2, \ldots\ldots, z_n$ lie within the circle $\left|z - \frac{1}{1-a}\right| = \frac{1}{|1-a|}$

**Ex. 20** If $z_1 = a + ib$ and $z_2 = c + id$ are complex number such that $|z_1| = |z_2| = 1$ and Re $(z_1\bar{z}_2) = 0$, then show that the pair of complex numbers $w_1 = a + ic$ and $w_2 = b + id$ satisfies the following

**(i)** $|w_1| = 1$ **(ii)** $|w_2| = 1$ **(iii)** Re $(w_1\bar{w}_2) = 0$

**Sol.** $a = \cos\theta$ , $b = \sin\theta$

$c = \cos\phi$, $d = \sin\phi$

Re $(z_1\bar{z}_2) = 0$ $\Rightarrow$ $\theta - \phi = \frac{n\pi}{2}$ $n = \pm 1$ $\Rightarrow$ $c = \sin\theta$, $d = -\cos\theta$

$\Rightarrow$ $w_1 = \cos\theta + i\sin\theta$

$w_2 = \sin\theta - i\cos\theta$

$\Rightarrow$ $|w_1| = 1, |w_2| = 1$

$w_1\bar{w}_2 = \cos\theta\sin\theta - \sin\theta\cos\theta + i(\sin^2\theta - \cos^2\theta) = -i\cos 2\theta$

$\Rightarrow$ Re $(w_1\bar{w}_2) = 0$

**Ex. 21** If $\theta \in [\pi/6, \pi/3]$, $i = 1, 2, 3, 4, 5$ and $z^4\cos\theta_1 + z^3\cos\theta_2 + z^2\cos\theta_3 + z\cos\theta_4 + \cos\theta_5 = 2\sqrt{3}$ , then show that $|z| > \frac{3}{4}$

**Sol.** Given that $\cos\theta_1.z^4 + \cos\theta_2.z^3 + \cos\theta_3.z^2 + \cos\theta_4.z + \cos\theta_5 = 2\sqrt{3}$

or $\left|\cos\theta_1.z^4 + \cos\theta_2.z^3 + \cos\theta_3.z^2 + \cos\theta_4.z + \cos\theta_5\right| = 2\sqrt{3}$

$2\sqrt{3} \le \left|\cos\theta_1.z^4\right| + \left|\cos\theta_2.z^3\right| + \left|\cos\theta_3.z^2\right| + \left|\cos\theta_4.z\right| + \left|\cos\theta_5\right|$

$\because$ $\theta_i \in [\pi/6, \pi/3]$

$\therefore$ $\frac{1}{2} \le \cos\theta_i \le \frac{\sqrt{3}}{2}$

$$2\sqrt{3} \le \frac{\sqrt{3}}{2}|z|^4 + \frac{\sqrt{3}}{2}|z|^3 + \frac{\sqrt{3}}{2}|z|^2 + \frac{\sqrt{3}}{2}|z| + \frac{\sqrt{3}}{2}$$

$\Rightarrow$ $3 \le |z|^4 + |z|^3 + |z|^2 + |z|$ $\Rightarrow$ $3 < |z| + |z|^2 + |z|^3 + |z|^4 + |z|^5 + \text{..........}\infty$

$\Rightarrow$ $3 < \frac{|z|}{1 - |z|}$ $\Rightarrow$ $3 - 3|z| < |z|$

$\Rightarrow$ $4|z| > 3$ $\therefore$ $|z| > \frac{3}{4}$

**Ex. 22** If $z_1$ and $z_2$ are two complex numbers and $C > 0$, then prove that $|z_1 + z_2|^2 \le (1 + C)|z_1|^2 + (1 + C^{-1})|z_2|^2$

**Sol.** We have to prove that : $|z_1 + z_2|^2 \le (1 + C)|z_1|^2 + (1 + C^{-1})|z_2|^2$

i.e. $|z_1|^2 + |z_2|^2 + z_1\bar{z}_2 + \bar{z}_1 z_2 \le (1 + C)|z_1|^2 + (1 + C^{-1})|z_2|^2$

or $z_1\bar{z}_2 + \bar{z}_1 z_2 \le C|z_1|^2 + C^{-1}|z_2|^2$

or $C|z_1|^2 + \frac{1}{C}|z_2|^2 - z_1\bar{z}_2 - \bar{z}_1 z_2 \ge 0$ (using Re $(z_1\bar{z}_2) \le |z_1\bar{z}_2|$)

or $\left(\sqrt{C}|z_1| - \frac{1}{\sqrt{C}}|z_2|\right)^2 \ge 0$ which is always true.

**Ex. 23** Let $z_1$ and $z_2$ be complex numbers such that $z_1 \neq z_2$ and $|z_1| = |z_2|$. If $z_1$ has positive real part and $z_2$ has negative imaginary part, then show that $\dfrac{z_1+z_2}{z_1-z_2}$ is purely imaginary.

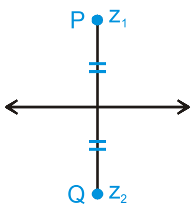
**Sol.** $z_1 = r(\cos\theta + i\sin\theta), \quad -\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$

$z_2 = r(\cos\phi + i\sin\phi), \quad -\pi < \phi < 0$

$$\Rightarrow \quad \frac{z_1+z_2}{z_1-z_2} = -i\cot\left(\frac{\theta-\varphi}{2}\right), \qquad -\frac{\pi}{4} < \frac{\theta-\varphi}{2} < \frac{3\pi}{4}$$

Hence purely imaginary

**Ex. 24** Two given points P & Q are the reflection points w.r.t. a given straight line if the given line is the right bisector of the segment PQ. Prove that the two points denoted by the complex numbers $z_1$ & $z_2$ will be the reflection points for the straight line $\bar{\alpha}z + \alpha\bar{z} + r = 0$ if and only if ; $\bar{\alpha}z_1 + \alpha\bar{z}_2 + r = 0$, where r is real and $\alpha$ is non zero complex constant.




**Sol.** Let $P(z_1)$ is the reflection point of $Q(z_2)$ then the perpendicular bisector of $z_1$ & $z_2$ must be the line

$$\bar{\alpha}z + \alpha\bar{z} + r = 0 \qquad \text{......(i)}$$

Now perpendicular bisector of $z_1$ & $z_2$ is, $|z - z_1| = |z - z_2|$

or $\quad (z-z_1)(\bar{z}-\bar{z}_1) = (z-z_2)(\bar{z}-\bar{z}_2)$

$-z\bar{z}_1 - z_1\bar{z} + z_1\bar{z}_1 = -z\bar{z}_2 - z_2\bar{z} + z_2\bar{z}_2 \qquad$ ( $z\bar{z}$ cancels on either side)

or $\quad (\bar{z}_2 - \bar{z}_1)z + (z_2 - z_1)\bar{z} + z_1\bar{z}_1 - z_2\bar{z}_2 = 0 \qquad$ ......**(ii)**

Comparing **(i)** & **(ii)** $\dfrac{\bar{\alpha}}{\bar{z}_2 - \bar{z}_1} = \dfrac{\alpha}{z_2 - z_1} = \dfrac{r}{z_1\bar{z}_1 - z_2\bar{z}_2} = \lambda$

$\therefore \quad \bar{\alpha} = \lambda(\bar{z}_2 - \bar{z}_1) \qquad$ ......**(iii)**

$\alpha = \lambda(z_2 - z_1) \qquad$ ......**(iv)**

$r = \lambda(z_1\bar{z}_1 - z_2\bar{z}_2) \qquad$ ......**(v)**

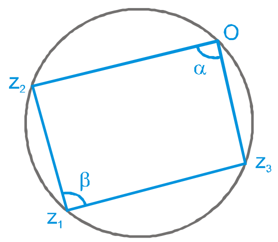
Multiplying **(iii)** by $z_1$; **(iv)** by $\bar{z}_2$ and adding

$$\bar{\alpha}z_1 + \alpha\bar{z}_2 + r = 0$$

Note that we could also multiply **(iii)** by $z_2$ & **(iv)** by $\bar{z}_1$ & add to get the same result.

**Ex. 25** If $z_1, z_2, z_3$ are complex numbers such that $\dfrac{2}{z_1} = \dfrac{1}{z_2} + \dfrac{1}{z_3}$, show that the points represented by $z_1, z_2, z_3$ lie on a circle passing through the origin.

**Sol.** We have, $\frac{2}{z_1} = \frac{1}{z_2} + \frac{1}{z_3}$




$\Rightarrow \quad \frac{1}{z_1} - \frac{1}{z_2} = \frac{1}{z_3} - \frac{1}{z_1} \qquad \Rightarrow \quad \frac{z_2 - z_1}{z_1 z_2} = \frac{z_1 - z_3}{z_1 z_3}$

$\Rightarrow \quad \frac{z_2 - z_1}{z_3 - z_1} = \frac{-z_2}{z_3} \qquad \Rightarrow \quad \arg\left(\frac{z_2 - z_1}{z_3 - z_1}\right) = \arg\left(\frac{-z_2}{z_3}\right)$

$\arg\left(\frac{z_2 - z_1}{z_3 - z_1}\right) = \pi + \arg\left(\frac{z_2}{z_3}\right)$

$\Rightarrow \quad$ or $\beta = \pi - \arg\frac{z_3}{z_2} = \pi - \alpha \;\; = \alpha + \beta = \pi$

Thus the sum of a pair of opposite angle of a quadrilateral is 180°. Hence, the points 0, $z_1$, $z_2$ and $z_3$ are the vertices of a cyclic quadrilateral i.e. lie on a circle.

# Exercise # 1

## [Single Correct Choice Type Questions]

**1.** The argument of the complex number $\sin\frac{6\pi}{5} + i\left(1+\cos\frac{6\pi}{5}\right)$ is

**(A)** $\frac{6\pi}{5}$ **(B)** $\frac{5\pi}{6}$ **(C)** $\frac{9\pi}{10}$ **(D)** $\frac{2\pi}{5}$

**2.** The principal value of the arg(z) and $|z|$ of the complex number $z = 1 + \cos\left(\frac{11\pi}{9}\right) + i\sin\left(\frac{11\pi}{9}\right)$ are respectively

**(A)** $\frac{11\pi}{18}, 2\cos\frac{\pi}{18}$ **(B)** $-\frac{7\pi}{18}, 2\cos\frac{7\pi}{18}$ **(C)** $\frac{2\pi}{9}, 2\cos\frac{7\pi}{18}$ **(D)** $-\frac{\pi}{9}, -2\cos\frac{\pi}{18}$

**3.** The inequality $|z-4| < |z-2|$ represents :

**(A)** $Re(z) > 0$ **(B)** $Re(z) < 0$ **(C)** $Re(z) > 2$ **(D)** $Re(z) > 3$

**4.** The sequence $S = i + 2i^2 + 3i^3 + \ldots\ldots$ upto 100 terms simplifies to where $i = \sqrt{-1}$ -

**(A)** $50(1-i)$ **(B)** $25i$ **(C)** $25(1+i)$ **(D)** $100(1-i)$

**5.** The region of Argand diagram defined by $|z-1| + |z+1| \le 4$ is :

**(A)** interior of an ellipse **(B)** exterior of a circle
**(C)** interior and boundary of an ellipse **(D)** none of these

**6.** The system of equations $\left.\begin{matrix} |z+1-i| & = 2 \\ \text{Re } z & \ge 1 \end{matrix}\right\}$, where z is a complex number has :

**(A)** no solution **(B)** exactly one solution
**(C)** two distinct solutions **(D)** infinite solution

**7.** If $z_1, z_2, z_3$ are 3 distinct complex numbers such that $\frac{3}{|z_2-z_3|} = \frac{4}{|z_3-z_1|} = \frac{5}{|z_1-z_2|}$,

then the value of $\frac{9}{z_2-z_3} + \frac{16}{z_3-z_1} + \frac{25}{z_1-z_2}$ equals

**(A)** 0 **(B)** 3 **(C)** 4 **(D)** 5

**8.** The complex numbers $\sin x + i\cos 2x$ and $\cos x - i\sin 2x$ are conjugate to each other, for

**(A)** $x = n\pi$ **(B)** $x = 0$ **(C)** $x = \frac{n\pi}{2}$ **(D)** no value of x

**9.** Real part of $e^{e^{i\theta}}$ is -

**(A)** $e^{\cos\theta}[\cos(\sin\theta)]$ **(B)** $e^{\cos\theta}[\cos(\cos\theta)]$ **(C)** $e^{\sin\theta}[\sin(\cos\theta)]$ **(D)** $e^{\sin\theta}[\sin(\sin\theta)]$

**10.** If $z\ (\neq -1)$ is a complex number such that $\frac{z-1}{z+1}$ is purely imaginary, then $|z|$ is equal to

**(A)** 1 **(B)** 2 **(C)** 3 **(D)** 5

**11.** Let A, B, C represent the complex numbers $z_1, z_2, z_3$ respectively on the complex plane. If the circumcentre of the triangle ABC lies at the origin, then the orthocentre is represented by the complex number :

**(A)** $z_1 + z_2 - z_3$ **(B)** $z_2 + z_3 - z_1$ **(C)** $z_3 + z_1 - z_2$ **(D)** $z_1 + z_2 + z_3$

**12.** If $(1 + i)(1 + 2i)(1 + 3i) \ldots (1 + ni) = \alpha + i\beta$ then $2 . 5 . 10 \ldots (1 + n^2) =$

**(A)** $\alpha - i\beta$ **(B)** $\alpha^2 - \beta^2$ **(C)** $\alpha^2 + \beta^2$ **(D)** none of these

**13.** $\sin^{-1}\left\{\frac{1}{i}(z-1)\right\}$, where z is nonreal, can be the angle of a triangle if

**(A)** $\text{Re}(z) = 1, \text{Im}(z) = 2$ **(B)** $\text{Re}(z) = 1, 0 < \text{Im}(z) \le 1$

**(C)** $\text{Re}(z) + \text{Im}(z) = 0$ **(D)** none of these

**14.** If $z = \frac{\pi}{4}(1+i)^4\left(\frac{1-\sqrt{\pi}\,i}{\sqrt{\pi}+i} + \frac{\sqrt{\pi}-i}{1+\sqrt{\pi}\,i}\right)$, then $\left(\frac{|z|}{\text{amp}(z)}\right)$ equals

**(A)** 1 **(B)** $\pi$ **(C)** $3\pi$ **(D)** 4

**15.** If $1, \alpha_1, \alpha_2, \ldots, \alpha_{2008}$ are $(2009)^{th}$ roots of unity, then the value of $\sum_{r=1}^{2008} r(\alpha_r + \alpha_{2009-r})$ equals

**(A)** 2009 **(B)** 2008 **(C)** 0 **(D)** –2009

**16.** If $x^2 + x + 1 = 0$, then the numerical value of

$$\left(x+\frac{1}{x}\right)^2 + \left(x^2+\frac{1}{x^2}\right)^2 + \left(x^3+\frac{1}{x^3}\right)^2 + \left(x^4+\frac{1}{x^4}\right)^2 + \ldots\ldots + \left(x^{27}+\frac{1}{x^{27}}\right)^2$$ is equal to

**(A)** 54 **(B)** 36 **(C)** 27 **(D)** 18

**17.** Let $i = \sqrt{-1}$. Define a sequence of complex number by $z_1 = 0$, $z_{n+1} = z_n^2 + i$ for $n \ge 1$. In the complex plane, how far from the origin is $z_{111}$?

**(A)** 1 **(B)** $\sqrt{2}$ **(C)** $\sqrt{3}$ **(D)** $\sqrt{100}$

**18.** Number of values of x (real or complex) simultaneously satisfying the system of equations

$1 + z + z^2 + z^3 + \ldots\ldots + z^{17} = 0$ and $1 + z + z^2 + z^3 + \ldots\ldots + z^{13} = 0$ is -

**(A)** 1 **(B)** 2 **(C)** 3 **(D)** 4

**19.** Let $z_1$ and $z_2$ be two non real complex cube roots of unity and $|z - z_1|^2 + |z - z_2|^2 = \lambda$ be the equation of a circle with $z_1, z_2$ as ends of a diameter then the value of $\lambda$ is

**(A)** 4 **(B)** 3 **(C)** 2 **(D)** $\sqrt{2}$

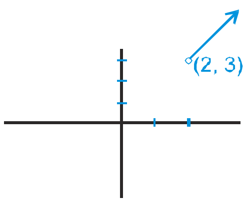
**20.** In G.P. the first term & common ratio are both $\frac{1}{2}\left(\sqrt{3}+i\right)$, then the absolute value of its $n^{th}$ term is :

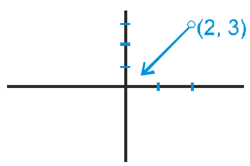
**(A)** 1 **(B)** $2^n$ **(C)** $4^n$ **(D)** none

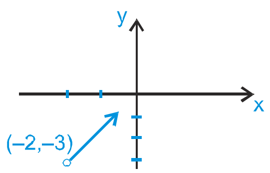
**21.** If P and Q are represented by the complex numbers $z_1$ and $z_2$ such that $\left|\frac{1}{z_1}+\frac{1}{z_2}\right|=\left|\frac{1}{z_1}-\frac{1}{z_2}\right|$, then the circumcentre of $\Delta OPQ$ (where O is the origin) is

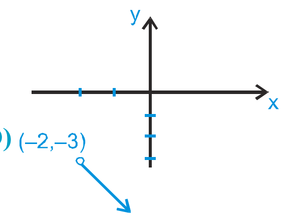
**(A)** $\frac{z_1-z_2}{2}$ **(B)** $\frac{z_1+z_2}{2}$ **(C)** $\frac{z_1+z_2}{3}$ **(D)** $z_1+z_2$

**22.** If Arg $(z-2-3i)=\frac{\pi}{4}$, then the locus of z is

**(A)**



**(B)**



**(C)**



**(D)**



**23.** The points $z_1, z_2, z_3, z_4$ in the complex plane are the vertices of a parallelogram taken in order if and only if :

**(A)** $z_1+z_4=z_2+z_3$ **(B)** $z_1+z_3=z_2+z_4$ **(C)** $z_1+z_2=z_3+z_4$ **(D)** none

**24.** The set of points on the complex plane such that $z^2+z+1$ is real and positive (where $z=x+iy,\ x,\ y\in R$) is-

**(A)** Complete real axis only

**(B)** Complete real axis or all points on the line $2x+1=0$

**(C)** Complete real axis or a line segment joining points $\left(-\frac{1}{2},\frac{\sqrt{3}}{2}\right)$ & $\left(-\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$ excluding both.

**(D)** Complete real axis or set of points lying inside the rectangle formed by the lines.

$2x+1=0$ ; $2x-1=0$ ; $2y-\sqrt{3}=0$ & $2y+\sqrt{3}=0$

**25.** If $z_1, z_2, z_3$ are vertices of an equilateral triangle inscribed in the circle $|z|=2$ and if $z_1=1+i\sqrt{3}$, then

**(A)** $z_2=-2,\ z_3=1+i\sqrt{3}$ **(B)** $z_2=2,\ z_3=1-i\sqrt{3}$

**(C)** $z_2=-2,\ z_3=1-i\sqrt{3}$ **(D)** $z_2=1-i\sqrt{3},\ z_3=-1-i\sqrt{3}$

**26.** The vector $z=-4+5i$ is turned counter clockwise through an angle of 180º & stretched 1.5 times. The complex number corresponding to the newly obtained vector is :

**(A)** $6-\frac{15}{2}i$ **(B)** $-6+\frac{15}{2}i$ **(C)** $6+\frac{15}{2}i$ **(D)** none of these

**27.** If $|z| = 1$ and $|\omega - 1| = 1$ where $z, \omega \in C$, then the largest set of values of $|2z-1|^2 + |2\omega - 1|^2$ equals

**(A)** $[1, 9]$ **(B)** $[2, 6]$ **(C)** $[2, 12]$ **(D)** $[2, 18]$

**28.** If $(\cos\theta + i\sin\theta)(\cos 2\theta + i\sin 2\theta) \ldots (\cos n\theta + i\sin n\theta) = 1$, then the value of $\theta$ is

**(A)** $\frac{3m\pi}{n(n+1)}, m \in Z$ **(B)** $\frac{2m\pi}{n(n+1)}, m \in Z$ **(C)** $\frac{4m\pi}{n(n+1)}, m \in Z$ **(D)** $\frac{m\pi}{n(n+1)}, m \in Z$

**29.** Points $z_1$ & $z_2$ are adjacent vertices of a regular octagon. The vertex $z_3$ adjacent to $z_2$ $(z_3 \neq z_1)$ can be represented by -

**(A)** $z_2 + \frac{1}{\sqrt{2}}(1 \pm i)(z_1 + z_2)$ **(B)** $z_2 + \frac{1}{\sqrt{2}}(-1 \pm i)(z_1 - z_2)$

**(C)** $z_2 + \frac{1}{\sqrt{2}}(-1 \pm i)(z_2 - z_1)$ **(D)** none of these

**30.** If $\log_{1/2}\left(\frac{|z-1| + 4}{3|z-1| - 2}\right) > 1$, then find locus of z

**(A)** Exterior to circle with center 1 + i0 and radius 10

**(B)** Interior to circle with center 1 + i0 and radius 10

**(C)** Circle with center 1 + i0 and radius 10

**(D)** None of these

**31.** If $A_1, A_2, \ldots\ldots, A_n$ be the vertices of an n-sided regular polygon such that $\frac{1}{A_1A_2} = \frac{1}{A_1A_3} + \frac{1}{A_1A_4}$, then find the value of n

**(A)** 5 **(B)** 7 **(C)** 8 **(D)** 9

**32.** If $x = a + b + c$, $y = a\alpha + b\beta + c$ and $z = a\beta + b\alpha + c$, where $\alpha$ and $\beta$ are imaginary cube roots of unity, then $xyz =$

**(A)** $2(a^3 + b^3 + c^3)$ **(B)** $2(a^3 - b^3 - c^3)$ **(C)** $a^3 + b^3 + c^3 - 3abc$ **(D)** $a^3 - b^3 - c^3$

**33.** If z and $\omega$ are two non-zero complex numbers such that $|z\omega| = 1$, and $\text{Arg}(z) - \text{Arg}(\omega) = \pi/2$, then $\bar{z}\,\omega$ is equal to -

**(A)** 1 **(B)** –1 **(C)** i **(D)** –i

**34.** The expression $\left(\frac{1 + i\tan\alpha}{1 - i\tan\alpha}\right)^n - \frac{1 + i\tan n\alpha}{1 - i\tan n\alpha}$ when simplified reduces to :

**(A)** zero **(B)** $2\sin n\alpha$ **(C)** $2\cos n\alpha$ **(D)** none

**35.** If $1, \alpha_1, \alpha_2, \alpha_3, \alpha_4$ be the roots of $x^5 - 1 = 0$, then find the value of $\frac{\omega - \alpha_1}{\omega^2 - \alpha_1} \cdot \frac{\omega - \alpha_2}{\omega^2 - \alpha_2} \cdot \frac{\omega - \alpha_3}{\omega^2 - \alpha_3} \cdot \frac{\omega - \alpha_4}{\omega^2 - \alpha_4}$

(where $\omega$ is imaginary cube root of unity.)

**(A)** $\omega$ **(B)** $\omega^2$ **(C)** 1 **(D)** – 1

# Exercise # 2 Part # I [Multiple Correct Choice Type Questions]

**1.** Which of the following complex numbers lies along the angle bisectors of the line -

$L_1 : z = (1+3\lambda) + i(1+4\lambda)$ $\qquad$ $L_2 : z = (1+3\mu) + i(1-4\mu)$

**(A)** $\frac{11}{5} + i$ **(B)** $11 + 5i$ **(C)** $1 - \frac{3i}{5}$ **(D)** $5 - 3i$

**2.** On the argand plane, let $\alpha = -2 + 3z$ , $\beta = -2 - 3z$ & $|z| = 1$. Then the correct statement is -

**(A)** $\alpha$ moves on the circle, centre at (–2, 0) and radius 3

**(B)** $\alpha$ & $\beta$ describe the same locus

**(C)** $\alpha$ & $\beta$ move on different circles

**(D)** $\alpha - \beta$ moves on a circle concentric with $|z| = 1$

**3.** POQ is a straight line through the origin O . P and Q represent the complex number a + i b and c + i d respectively and OP = OQ. Then

**(A)** $|a + i b| = |c + i d|$ **(B)** $a + c = b + d$

**(C)** arg (a + i b) = arg (c + i d) **(D)** none of these

**4.** The common roots of the equations $z^3 + (1+i)z^2 + (1+i)z + i = 0$, (where $i = \sqrt{-1}$ ) and $z^{1993} + z^{1994} + 1 = 0$ are - (where $\omega$ denotes the complex cube root of unity)

**(A)** 1 **(B)** $\omega$ **(C)** $\omega^2$ **(D)** $\omega^{981}$

**5.** If g(x) and h(x) are two polynomials such that the polynomial $P(x) = g(x^3) + xh(x^3)$ is divisible by $x^2 + x + 1$, then -

**(A)** $g(1) = h(1) = 0$ **(B)** $g(1) = h(1) \neq 0$ **(C)** $g(1) = -h(1)$ **(D)** $g(1) + h(1) = 0$

**6.** The value of $i^n + i^{-n}$, for $i = \sqrt{-1}$ and $n \in I$ is -

**(A)** $\frac{2^n}{(1-i)^{2n}} + \frac{(1+i)^{2n}}{2^n}$ **(B)** $\frac{(1+i)^{2n}}{2^n} + \frac{(1-i)^{2n}}{2^n}$ **(C)** $\frac{(1+i)^{2n}}{2^n} - \frac{2^n}{(1-i)^{2n}}$ **(D)** $\frac{2^n}{(1+i)^{2n}} + \frac{2^n}{(1-i)^{2n}}$

**7.** The equation $|z - i| + |z + i| = k$, $k > 0$, can represent

**(A)** an ellipse if $k > 2$ **(B)** line segment if $k = 2$

**(C)** an ellipse if $k = 5$ **(D)** line segment if $k = 1$

**8.** If the equation $|z|(z+1)^8 = z^8|z+1|$ where $z \in C$ and $z(z+1) \neq 0$ has distinct roots $z_1, z_2, z_3, ...., z_n$ (where $n \in N$) then which of the following is/are true?

**(A)** $z_1, z_2, z_3, ......, z_n$ are concyclic points. **(B)** $z_1, z_2, z_3, ......, z_n$ are collinear points

**(C)** $\sum_{r=1}^{n} Re(z_r) = \frac{-7}{2}$ **(D)** $= 0$

**9.** If $x_r = CiS\left(\frac{\pi}{2^r}\right)$ for $1 \le r \le n$ ; r, $n \in N$ then -

**(A)** $\lim_{n\to\infty} Re\left(\prod_{r=1}^{n} x_r\right) = -1$ **(B)** $\lim_{n\to\infty} Re\left(\prod_{r=1}^{n} x_r\right) = 0$ **(C)** $\lim_{n\to\infty} Im\left(\prod_{r=1}^{n} x_r\right) = 1$ **(D)** $\lim_{n\to\infty} Im\left(\prod_{r=1}^{n} x_r\right) = 0$

**10.** If $|z_1| = |z_2| = |z_3| = 1$ and $z_1, z_2, z_3$ are represented by the vertices of an equilateral triangle then

**(A)** $z_1 + z_2 + z_3 = 0$
**(B)** $z_1z_2z_3 = 1$
**(C)** $z_1z_2 + z_2z_3 + z_3z_1 = 0$
**(D)** none of these

**11.** If S be the set of real values of x satisfying the inequality $1 - \log_2 \dfrac{|x+1+2i|-2}{\sqrt{2}-1} \geq 0$ , then S contains -

**(A)** $[-3, -1)$
**(B)** $(-1, 1]$
**(C)** $[-2, 2]$
**(D)** $[-3, 1]$

**12.** Let $z_1, z_2$ be two complex numbers represented by points on the circle $|z_1| = 1$ and $|z_2|=2$ respectively, then -

**(A)** $\max|2z_1+z_2|=4$
**(B)** $\min|z_1-z_2|=1$
**(C)** $\left|z_2+\dfrac{1}{z_1}\right| \leq 3$
**(D)** none of these

**13.** If z is a complex number then the equation $z^2 + z|z| + |z^2| = 0$ is satisfied by ($\omega$ and $\omega^2$ are imaginary cube roots of unity)

**(A)** $z = k\omega$ where $k \in R$
**(B)** $z = k\omega^2$ where k is non negative real
**(C)** $z = k\omega$ where k is positive real
**(D)** $z = k\omega^2$ where $k \in R$.

**14.** If the complex numbers $z_1, z_2, z_3$ represents vertices of an equilateral triangle such that $|z_1|=|z_2|=|z_3|$, then which of following is correct ?

**(A)** $z_1 + z_2 + z_3 \neq 0$
**(B)** $\text{Re}(z_1 + z_2 + z_3) = 0$
**(C)** $\text{Im}(z_1 + z_2 + z_3) = 0$
**(D)** $z_1 + z_2 + z_3 = 0$

**15.** If $2\cos\theta = x + \dfrac{1}{x}$ and $2\cos\varphi = y + \dfrac{1}{y}$, then

**(A)** $x^n + \dfrac{1}{x^n} = 2\cos(n\theta)$
**(B)** $\dfrac{x}{y} + \dfrac{y}{x} = 2\cos(\theta - \varphi)$
**(C)** $xy + \dfrac{1}{xy} = 2\cos(\theta + \varphi)$
**(D)** none of these

**16.** Value(s) of $(-i)^{1/3}$ is/are -

**(A)** $\dfrac{\sqrt{3}-i}{2}$
**(B)** $\dfrac{\sqrt{3}+i}{2}$
**(C)** $\dfrac{-\sqrt{3}-i}{2}$
**(D)** $\dfrac{-\sqrt{3}+i}{2}$

**17.** If z be a non-real complex number satisfying $|z| = 2$, then which of the following is/are true?

**(A)** $\arg\left(\dfrac{z-2}{z+2}\right) = \pm\dfrac{\pi}{2}$
**(B)** $\arg\left(\dfrac{z+1+i\sqrt{3}}{z-1+i\sqrt{3}}\right) = \dfrac{\pi}{6}$
**(C)** $|z^2 - 1| \geq 3$
**(D)** $|z^2 - 1| \leq 5$

**18.** If $\alpha, \beta$ be any two complex numbers such that $\left|\dfrac{\alpha-\beta}{1-\bar{\alpha}\beta}\right| = 1$, then which of the following may be true -

**(A)** $|\alpha| = 1$
**(B)** $|\beta| = 1$
**(C)** $\alpha = e^{i\theta}, \ \theta \in R$
**(D)** $\beta = e^{i\theta}, \ \theta \in R$

**19.** The equation $||z+i|-|z-i|| = k$ represents

**(A)** a hyperbola if $0 < k < 2$ **(B)** a pair of ray if $k > 2$

**(C)** a straight line if $k = 0$ **(D)** a pair of ray if $k = 2$

**20.** If amp $(z_1z_2) = 0$ and $|z_1| = |z_2| = 1$, then :-

**(A)** $z_1 + z_2 = 0$ **(B)** $z_1z_2 = 1$ **(C)** $z_1 = \bar{z}_2$ **(D)** none of these

**21.** If centre of square ABCD is at z=0. If affix of vertex A is $z_1$, centroid of triangle ABC is/are -

**(A)** $\frac{z_1}{3}(\cos\pi + i\sin\pi)$ **(B)** $4\left[\left(\cos\frac{\pi}{2}\right) - i\left(\sin\frac{\pi}{2}\right)\right]$

**(C)** $\frac{z_1}{3}\left[\left(\cos\frac{\pi}{2}\right) + i\left(\sin\frac{\pi}{2}\right)\right]$ **(D)** $\frac{z_1}{3}\left[\left(\cos\frac{\pi}{2}\right) - i\left(\sin\frac{\pi}{2}\right)\right]$

**22.** Let $z_1, z_2, z_3$ be non-zero complex numbers satisfying the equation $z^4 = iz$. Which of the following statement(s) is/are correct ?

**(A)** The complex number having least positive argument is $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$.

**(B)** $\sum_{k=1}^{3} \text{Amp}(z_k) = \frac{\pi}{2}$

**(C)** Centroid of the triangle formed by $z_1$, $z_2$ and $z_3$ is $\left(\frac{1}{\sqrt{3}}, \frac{-1}{3}\right)$

**(D)** Area of triangle formed by $z_1$, $z_2$ and $z_3$ is $\frac{3\sqrt{3}}{2}$

**23.** If the vertices of an equilateral triangle are situated at z = 0, z = $z_1$, z = $z_2$, then which of the following is/are true -

**(A)** $|z_1| = |z_2|$ **(B)** $|z_1 - z_2| = |z_1|$

**(C)** $|z_1 + z_2| = |z_1| + |z_2|$ **(D)** $|\arg z_1 - \arg z_2| = \pi/3$

**24.** If z satisfies the inequality $|z - 1 - 2i| \le 1$, then

**(A)** min (arg (z)) = $\tan^{-1}\left(\frac{3}{4}\right)$ **(B)** max (arg(z)) = $\frac{\pi}{2}$

**(C)** min (|z|) = $\sqrt{5} - 1$ **(D)** max (|z|) = $\sqrt{5} + 1$

**25.** Let z, ωz and z + ωz represent three vertices of ΔABC, where ω is cube root unity, then -

**(A)** centroid of ΔABC is $\frac{2}{3}(z + \omega z)$ **(B)** orthocenter of ΔABC is $\frac{2}{3}(z + \omega z)$

**(C)** ABC is an obtuse angled triangle **(D)** ABC is an acute angled triangle

## Part # II [Assertion & Reason Type Questions]

These questions contains, Statement I (assertion) and Statement II (reason).

**(A)** Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

**(B)** Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.

**(C)** Statement-I is true, Statement-II is false.

**(D)** Statement-I is false, Statement-II is true.

**1.** **Statement-I :** There are exactly two complex numbers which satisfy the complex equations $|z - 4 - 5i| = 4$ and $\text{Arg}(z - 3 - 4i) = \frac{\pi}{4}$ simultaneously.

**Statement-II :** A line cuts the circle in atmost two points.

**2.** Let $z_1, z_2, z_3$ represent vertices of a triangle.

**Statement - I :** $\frac{1}{z_1 - z_2} + \frac{1}{z_2 - z_3} + \frac{1}{z_3 - z_1} = 0$, when triangle is equilateral.

**Statement - II :** $|z_1|^2 - z_1\bar{z}_0 - \bar{z}_1 z_0 = |z_2|^2 - z_2\bar{z}_0 - \bar{z}_2 z_0 = |z_3|^2 - z_3\bar{z}_0 - \bar{z}_3 z_0$, where $z_0$ is circumcentre of triangle.

**3.** **Statement-I :** If $z = i + 2i^2 + 3i^3 + ............ + 32i^{32}$, then $z, \bar{z}, -z$ & $-\bar{z}$ forms the vertices of square on argand plane.

**Statement-II :** $z, \bar{z}, -z, -\bar{z}$ are situated at the same distance from the origin on argand plane.

**4.** **Statement - 1 :** Roots of the equation $(1 + z)^6 + z^6 = 0$ are collinear.

**Statement - II :** If $z_1, z_2, z_3$ are in A.P. then points represented by $z_1, z_2, z_3$ are collinear

**5.** Let $z_1, z_2, z_3$ satisfy $\left|\frac{z+2}{z-1}\right| = 2$ and $z_0 = 2$. Consider least positive arguments wherever required.

**Statement – I :** $2\arg\left(\frac{z_1 - z_3}{z_2 - z_3}\right) = \arg\left(\frac{z_1 - z_0}{z_2 - z_0}\right)$.

**Statement – II :** $z_1, z_2, z_3$ satisfy $|z - z_0| = 2$.

**6.** Let $1, \alpha_1, \alpha_2, \alpha_3, ........, \alpha_{n-1}$ be the n, $n^{th}$ roots of unity,

**Statement - I :** $\sin\frac{\pi}{n} . \sin\frac{2\pi}{n} . \sin\frac{3\pi}{n} ........ \sin\frac{(n-1)\pi}{n} = \frac{n}{2^{n-1}}$.

**Statement - II :** $(1 - \alpha_1)(1 - \alpha_2)(1 - \alpha_3) ........ (1 - \alpha_{n-1}) = n$.

**7.** **Statement-I :** If $z_1 = 9 + 5i$ and $z_2 = 3 + 5i$ and if $\arg\left(\frac{z - z_1}{z - z_2}\right) = \frac{\pi}{4}$ then $|z - 6 - 8i| = 3\sqrt{2}$

**Statement-II :** If z lies on circle having $z_1$ & $z_2$ as diameter then $\arg\left(\frac{z - z_1}{z - z_2}\right) = \frac{\pi}{4}$.

**8.** **Statement-I :** Let $z_1, z_2, z_3$ be three complex numbers such that $|3z_1 + 1| = |3z_2 + 1| = |3z_3 + 1|$ and $1 + z_1 + z_2 + z_3 = 0$, then $z_1, z_2, z_3$ will represent vertices of an equilateral triangle on the complex plane.

**Statement-II :** $z_1, z_2, z_3$ represent vertices of an equilateral triangle if $z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$.

# Exercise # 3 — Part # I [Matrix Match Type Questions]

Following question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled as A, B, C and D while the statements in Column-II are labelled as p, q, r and s. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II.

**1.**

| Column - I | Column - I |
|---|---|
| **(A)** If z be the complex number such that $\left\lvert z+\frac{1}{z}\right\rvert=2$ then minimum value of $\frac{\lvert z\rvert}{\tan\frac{\pi}{8}}$ is | **(p)** 0 |
| **(B)** $\lvert z\rvert = 1$ & $z^{2n}+1 \neq 0$ then $\frac{z^n}{z^{2n}+1}-\frac{\overline{z}^n}{\overline{z}^{2n}+1}$ is equal to | **(q)** 3 |
| **(C)** If $8iz^3+12z^2-18z+27i=0$ then $2\lvert z\rvert=$ | **(r)** 11 |
| **(D)** If $z_1, z_2, z_3, z_4$ are the roots of equation $z^4+z^3+z^2+z+1=0$, then $\prod_{i=1}^{4}(z_i+2)$ is | **(s)** 1 |

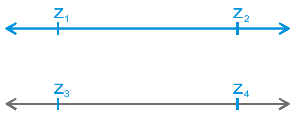
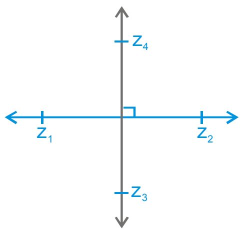
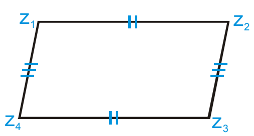
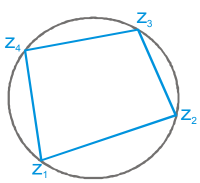
**2.** Let $z_1$ lies on $\lvert z\rvert = 1$ and $z_2$ lies on $\lvert z\rvert = 2$.

| Column – I | Column – II |
|---|---|
| **(A)** Maximum value of $\lvert z_1+z_2\rvert$ | **(p)** 3 |
| **(B)** Minimum value of $\lvert z_1-z_2\rvert$ | **(q)** 1 |
| **(C)** Minimum value of $\lvert 2z_1+3z_2\rvert$ | **(r)** 4 |
| **(D)** Maximum value of $\lvert z_1-2z_2\rvert$ | **(s)** 5 |

**3.**

| Column - I | Column - II |
|---|---|
| **(A)** Let $f(x)=x^4+ax^3+bx^2+cx+d$ has 4 real roots $(a, b, c, d \in R)$. If $\lvert f(-i)\rvert = 1$ (where $i=\sqrt{-1}$), then the value of $a^2+b^2+c^2+d^2$ equals | **(p)** 0 |
| | **(q)** 1 |
| **(B)** If $\arg(z+3)=\frac{\pi}{6}$ and $\arg(z-3)=\frac{2\pi}{3}$, then $\tan^2(\arg z)-2\cos(\arg z)$, is $\sum_{r=1}^{n}\text{Im}(z_r)$ | **(r)** 2 |
| | **(s)** 3 |
| **(C)** If the points $A(z)$, $B(-z)$ and $C(z+1)$ are vertices of an equilateral triangle, then $5+4\,\text{Re}(z)$ equals | |
| **(D)** If $z_1=1+i\sqrt{3}$, $z_2=1-i\sqrt{3}$ and $z_3=2$, then value of $x$ satisfying $z_1^{\,x}+z_2^{\,x}=2^x$ can be | **(t)** 4 |

**4.** Match the figure in column-I with corresponding expression -

| **Column - I** | | **Column - I** | |
|---|---|---|---|
| **(A)** |   two parallel lines | **(p)** | $\frac{z_4 - z_3}{z_2 - z_1} + \frac{\bar{z}_4 - \bar{z}_3}{\bar{z}_2 - \bar{z}_1} = 0$ |
| **(B)** |   two perpendicular lines | **(q)** | $\frac{z_2 - z_1}{z_4 - z_3} = \frac{\bar{z}_2 - \bar{z}_1}{\bar{z}_4 - \bar{z}_3}$ |
| **(C)** |   a parallelogram | **(r)** | $\frac{z_4 - z_1}{z_2 - z_1} \cdot \frac{z_2 - z_3}{z_4 - z_3} = \frac{\bar{z}_4 - \bar{z}_1}{\bar{z}_2 - \bar{z}_1} \cdot \frac{\bar{z}_2 - \bar{z}_3}{\bar{z}_4 - \bar{z}_3}$ |
| **(D)** |   | **(s)** | $z_1 + z_3 = z_2 + z_4$ |

## Part # II [Comprehension Type Questions]

### Comprehension # 1

Let z be any complex number. To factorise the expression of the form $z^n - 1$, we consider the equation $z^n = 1$. This equation is solved using De moiver's theorem. Let 1, $\alpha_1$, $\alpha_2$,........ $\alpha_{n-1}$ be the roots of this equation, then $z^n - 1 = (z - 1)(z - \alpha_1)(z - \alpha_2)........(z - \alpha_{n-1})$ This method can be generalised to factorize any expression of the form $z^n - k^n$.

for example, $z^7 + 1 = \prod_{m=0}^{6}\left(z - CiS\left(\frac{2m\pi}{7} + \frac{\pi}{7}\right)\right)$

This can be further simplified as

$$z^7 + 1 = (z + 1)\left(z^2 - 2z\cos\frac{\pi}{7} + 1\right)\left(z^2 - 2z\cos\frac{3\pi}{7} + 1\right)\left(z^2 - 2z\cos\frac{5\pi}{7} + 1\right) \quad .....(i)$$

These factorisations are useful in proving different trigonometric identities e.g. in eqation **(i)** if we put z = i, then equation **(i)** becomes

$$(1 - i) = (i + 1)\left(-2i\cos\frac{\pi}{7}\right)\left(-2i\cos\frac{3\pi}{7}\right)\left(-2i\cos\frac{5\pi}{7}\right)$$

i.e. $\cos\frac{\pi}{7}\cos\frac{3\pi}{7}\cos\frac{5\pi}{7} = -\frac{1}{8}$

**1.** If the expression $z^5 - 32$ can be factorised into linear and quadratic factors over real coefficients as $(z^5 - 32) = (z-2)(z^2 - pz + 4)(z^2 - qz + 4)$, where p > q, then the value of $p^2 - 2q$ -

**(A)** 8 **(B)** 4 **(C)** –4 **(D)** –8

**2.** By using the factorisation for $z^5 + 1$, the value of $4\sin\frac{\pi}{10}\cos\frac{\pi}{5}$ comes out to be -

**(A)** 4 **(B)** 1/4 **(C)** 1 **(D)** –1

**3.** If $(z^{2n+1} - 1) = (z-1)(z^2 - p_1 z + 1)........ (z^2 - p_n z + 1)$ where $n \in N$ & $p_1, p_2$ ............ $p_n$ are real numbers then $p_1 + p_2 + .......... + p_n =$

**(A)** –1 **(B)** 0 **(C)** $\tan(\pi/2n)$ **(D)** none of these

## Comprehension # 2

Let $z_1, z_2, z_3, z_4$ are three distinct complex numbers such that $|z_1| = |z_2| = |z_3| = |z_4|$, satisfying.
$|(1-d)z_1 + z_2 + z_3 + z_4| = |z_1 + (1-d)z_2 + z_3 + z_4| = |z_1 + z_2 + (1-d)z_3 + z_4|$ where $d \in R - \{0\}$.

**1.** Arg $(z_1 + z_2 + z_3 + z_4)$ is

**(A)** $\frac{\pi}{6}$ **(B)** $\frac{\pi}{2}$ **(C)** $\pi$ **(D)** Not defined.

**2.** $|z_1 + z_2 + z_3 + z_4|$ is

**(A)** 1 **(B)** 2 **(C)** 0 **(D)** $\geq 4$

**3.** The point $dz_1, dz_2, dz_3$ lie on a circle with

**(A)** centre (1, 0), radius $|d|$
**(B)** centre (0, 0), radius $|dz_1|$
**(C)** centre (0, 1), radius $|dz_2|$
**(D)** None of these

## Comprehension # 3

ABCD is a rhombus. Its diagonals AC and BD intersect at the point M and satisfy BD = 2AC. Let the points D and M represent complex numbers 1 + i and 2 – i respecetively.

If θ is arbitary real, then $z = re^{i\theta}$, $R_1 \leq r \leq R_2$ lies in annular region formed by concentric circles $|z| = R_1, |z| = R_2$.

**1.** A possible representation of point A is

**(A)** $3 - \frac{i}{2}$ **(B)** $3 + \frac{i}{2}$ **(C)** $1 + \frac{3}{2}i$ **(D)** $3 - \frac{3}{2}i$

**2.** $e^{iz} =$

**(A)** $e^{-r\cos\theta}(\cos(r\cos\theta) + i\sin(r\sin\theta))$
**(B)** $e^{-r\cos\theta}(\sin(r\cos\theta) + i\cos(r\cos\theta))$
**(C)** $e^{-r\sin\theta}(\cos(r\cos\theta) + i\sin(r\cos\theta))$
**(D)** $e^{-r\sin\theta}(\sin(r\cos\theta) + i\cos(r\sin\theta))$

**3.** If z is any point on segment DM then $w = e^{iz}$ lies in annular region formed by concentric circles.

**(A)** $|w|_{min} = 1, |w|_{max} = 2$
**(B)** $|w|_{min} = \frac{1}{e}, |w|_{max} = e$
**(C)** $|w|_{min} = \frac{1}{e^2}, |w|_{max} = e^2$
**(D)** $|w|_{min} = \frac{1}{2}, |w|_{max} = 1$

## Comprehension # 4

Let A, B, C be three sets of complex numbers as defined below.

$A = \{z : |z+1| \le 2 + \text{Re}(z)\}$, $B = \{z : |z-1| \ge 1\}$ and $C = \left\{z : \left|\frac{z-1}{z+1}\right| \ge 1\right\}$

**1.** The number of point(s) having integral coordinates in the region $A \cap B \cap C$ is

**(A)** 4 **(B)** 5 **(C)** 6 **(D)** 10

**2.** The area of region bounded by $A \cap B \cap C$ is

**(A)** $2\sqrt{3}$ **(B)** $\sqrt{3}$ **(C)** $4\sqrt{3}$ **(D)** 2

**3.** The real part of the complex number in the region $A \cap B \cap C$ and having maximum amplitude is

**(A)** –1 **(B)** $\frac{-3}{2}$ **(C)** $\frac{-1}{2}$ **(D)** – 2

## Comprehension # 5

In the figure |z| = r is circumcircle of ΔABC.D,E & F are the middle points of the sides BC, CA & AB respectively, AD produced to meet the circle at L. If ∠CAD = θ, AD = x, BD = y and altitude of ΔABC from A meet the circle |z|= r at M, $z_a$, $z_b$ & $z_c$ are affixes of vertices A, B & C respectively.

**1.** Area of the ΔABC is equal to -

**(A)** xy cos (θ + C) **(B)** (x + y) sin θ

**(C)** xy sin (θ + C) **(D)** $\frac{1}{2}$ xy sin (θ + C)

**2.** Affix of M is -

**(A)** $2z_be^{i2B}$ **(B)** $z_be^{i(\pi-2B)}$ **(C)** $z_be^{iB}$ **(D)** $2z_be^{iB}$

**3.** Affix of L is -

**(A)** $z_be^{i(2A-2\theta)}$ **(B)** $2z_be^{i(2A-2\theta)}$ **(C)** $z_be^{i(A-\theta)}$ **(D)** $2z_be^{i(A-\theta)}$

# Exercise # 4 [Subjective Type Questions]

**1.** If $x = 1 + i\sqrt{3}$ ; $y = 1 - i\sqrt{3}$ & $z = 2$, then prove that $x^p + y^p = z^p$ for every prime $p > 3$.

**2.** Interpret the following locii in $z \in C$.

**(A)** $1 < |z - 2i| < 3$

**(B)** $\text{Re}\left(\dfrac{z+2i}{iz+2}\right) \le 4 \ (z \ne 2i)$

**(C)** $\text{Arg}(z + i) - \text{Arg}(z - i) = \pi/2$

**(D)** $\text{Arg}(z - a) = \pi/3$ where $a = 3 + 4i$.

**3.** Find the modulus, argument and the principal argument of the complex numbers.

**(A)** $z = 1 + \cos\dfrac{18\pi}{25} + i\sin\dfrac{18\pi}{25}$

**(B)** $z = -2(\cos 30° + i\sin 30°)$

**(C)** $(\tan 1 - i)^2$

**(D)** $\dfrac{i-1}{i\left(1-\cos\dfrac{2\pi}{5}\right)+\sin\dfrac{2\pi}{5}}$

**4.** If $a_1, a_2, a_3 \ldots a_n, A_1, A_2, A_3 \ldots A_n$, k are all real numbers, then prove that

$$\frac{A_1^2}{x-a_1} + \frac{A_2^2}{x-a_2} + \ldots\ldots + \frac{A_n^2}{x-a_n} = k \text{ has no imaginary roots.}$$

**5.** For complex numbers z & ω, prove that, $|z|^2\,\omega - |\omega|^2\,z = z - \omega$ if and only if, $z = \omega$ or $z\bar{\omega} = 1$

**6.** If $|z_1| = |z_2| = \ldots\ldots = |z_n| = 1$ then show that

**(i)** $\bar{z}_1 = \dfrac{1}{z_1}$

**(ii)** $|z_1 + z_2 + \ldots.. + z_n| = \left|\dfrac{1}{z_1} + \dfrac{1}{z_2} + \ldots\ldots + \dfrac{1}{z_n}\right|$.

And hence interpret that the centroid of polygon with 2n vertices $z_1, z_2, \ldots\ldots. z_n, \dfrac{1}{z_1}, \dfrac{1}{z_2}, \ldots\ldots.. \dfrac{1}{z_n}$ (need not be in order) lies on real axis.

**7. (A)** Let $z = x + iy$ be a complex number, where x and y are real numbers. Let A and B be the sets defined by $A = \{z \mid |z| \le 2\}$ and $B = \{z \mid (1 - i)z + (1 + i)\bar{z} \ge 4\}$. Find the area of the region $A \cap B$.

**(B)** For all real numbers x, let the mapping $f(x) = \dfrac{1}{x-i}$, where $i = \sqrt{-1}$. If there exist real numbers a, b, c and d for which f(a), f(b), f(c) and f(d) form a square on the complex plane. Find the area of the square.

**8.** Let circles $C_1$ and $C_2$ on Argand plane be given by $|z + 1| = 3$ and $|z - 2| = 7$ respectively. If a variable circle $|z - z_0| = r$ be inside circle $C_2$ such that it touches $C_1$ externally and $C_2$ internally then locus of '$z_0$' describes a conic E. If eccentricity of E can be written in simplest form as $\dfrac{p}{q}$ where $p, q \in N$, then find the value of $(p + q)$.

**9.** If $z_1, z_2$ are the roots of the equation $az^2 + bz + c = 0$, with a, b, c > 0 ; $2b^2 > 4ac > b^2$ ; $z_1 \in$ third quadrant ; $z_2 \in$ second quadrant in the argand's plane then, show that $\arg\left(\frac{z_1}{z_2}\right) = 2\cos^{-1}\left(\frac{b^2}{4ac}\right)^{1/2}$

**10.** For any two complex numbers $z_1$, $z_2$ and any two real numbers a, b show that
$|az_1 - bz_2|^2 + |bz_1 + az_2|^2 = (a^2 + b^2)(|z_1|^2 + |z_2|^2)$

**11.** If the biquadratic $x^4 + ax^3 + bx^2 + cx + d = 0$ (a, b, c, d $\in$ R) has 4 non real roots, two with sum 3 + 4i and the other two with product 13 + i. Find the value of 'b'.

**12.** If A, B and C are the angle of a triangle $D = \begin{vmatrix} e^{-2iA} & e^{iC} & e^{iB} \\ e^{iC} & e^{-2iB} & e^{iA} \\ e^{iB} & e^{iA} & e^{-2iC} \end{vmatrix}$ where $i = \sqrt{-1}$, then find the value of D.

**13.** If $\alpha$ is imaginary $n^{th}$ $(n \geq 3)$ root of unity then show that $\sum_{r=1}^{n-1}(n-r)\,\alpha^r = \frac{n\alpha}{1-\alpha}$

Hence deduce that $\sum_{r=1}^{n-1}(n-r)\,\sin\frac{2r\pi}{n} = \frac{n}{2}\cot\frac{\pi}{n}$.

**14.** Let A = {a $\in$ R| the equation $(1 + 2i)x^3 - 2(3 + i)x^2 + (5 - 4i)x + 2a^2 = 0$} has at least one real root. Find the value of $\sum_{a \in A} a^2$.

**15.** Consider two concentric circles $S_1 : |z| = 1$ and $S_2 : |z| = 2$ on the Argand plane. A parabola is drawn through the points where '$S_1$' meets the real axis and having arbitrary tangent of '$S_2$' as its directrix. If the locus of the focus of drawn parabola is a conic C then find the area of the quadrilateral formed by the tangents at the ends of the latus-rectum of conic C.

**16.** Let $z_1$ and $z_2$ be two complex numbers such that $\left|\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}\right| = 1$ and $|z_2| \neq 1$, find $|z_1|$.

**17.** If O is origin and affixes of P, Q, R are respectively z, iz, z + iz. Locate the points on complex plane. If $\Delta PQR = 200$ then find **(i)** |z| **(ii)** sides of quadrilateral OPRQ

**18.** If $Z_r$, r = 1, 2, 3,..... 2m, m $\in$ N are the roots of the equation $Z^{2m} + Z^{2m-1} + Z^{2m-2} + \ldots\ldots + Z + 1 = 0$ then prove that $\sum_{r=1}^{2m}\frac{1}{Z_r - 1} = -m$

**19.** ABCD is a rhombus in the Argand plane. If the affixes of the vertices be $z_1, z_2, z_3, z_4$ and taken in anti-clockwise sense and $\angle CBA = \pi/3$, show that

**(A)** $2z_2 = z_1(1 + i\sqrt{3}) + z_3(1 - i\sqrt{3})$ & **(B)** $2z_4 = z_1(1 - i\sqrt{3}) + z_3(1 + i\sqrt{3})$

**20.** Find the locus of mid-point of line segment intercepted between real and imaginary axes, by the line $a\bar{z} + \bar{a}z + b = 0$, where 'b' is real parameter and 'a' is a fixed complex number such that Re(a) $\neq$ 0, Im(a) $\neq$ 0.

**21.** P is a point on the Argand plane. On the circle with OP as diameter two points Q & R are taken such that $\angle POQ = \angle QOR = \theta$. If 'O' is the origin & P, Q & R are represented by the complex numbers $Z_1, Z_2$ & $Z_3$ respectively, show that : $Z_2^2 \cos 2\theta = Z_1 . Z_3 \cos^2\theta$.

**22.** A polynomial $f(z)$ when divided by $(z - w)$ leaves remainder $2 + i\sqrt{3}$ and when divided by $(z - w^2)$ leaves remainder $2 - i\sqrt{3}$. If the remainder obtained when $f(z)$ is divided by $z^2 + z + 1$ is $az + b$ (where w is a non- real cube root of unity and $a, b \in R^+$), then find the value of $(a + b)$.

**23.** The points A, B, C depict the complex numbers $z_1, z_2, z_3$ respectively on a complex plane & the angle B & C of the triangle ABC are each equal to $\frac{1}{2}(\pi - \alpha)$. Show that : $(z_2 - z_3)^2 = 4(z_3 - z_1)(z_1 - z_2)\sin^2\frac{\alpha}{2}$

**24.** Let $z_1, z_2, z_3$ are three pair wise distinct complex numbers and $t_1, t_2, t_3$ are non-negative real numbers such that $t_1 + t_2 + t_3 = 1$. Prove that the complex number $z = t_1z_1 + t_2z_2 + t_3z_3$ lies inside a triangle with vertices $z_1, z_2, z_3$ or on its boundary.

**25.** Let $A \equiv z_1$ ; $B \equiv z_2$ ; $C \equiv z_3$ are three complex numbers denoting the vertices of an acute angled triangle. If the origin 'O' is the orthocentre of the triangle, then prove that $z_1\bar{z}_2 + \bar{z}_1z_2 = z_2\bar{z}_3 + \bar{z}_2z_3 = z_3\bar{z}_1 + \bar{z}_3z_1$.

**26.** If $a = e^{i\alpha}$, $b = e^{i\beta}$, $c = e^{i\gamma}$ and $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$, then prove the following

**(i)** $a + b + c = 0$ **(ii)** $ab + bc + ca = 0$

**(iii)** $a^2 + b^2 + c^2 = 0$ **(iv)** $\Sigma \cos 2\alpha = 0 = \Sigma \sin 2\alpha$

**(v)** $\Sigma \sin^2\alpha = \Sigma \cos^2\alpha = 3/2$

**27.** **(A)** If $\omega$ is an imaginary cube root of unity then prove that :

$(1 - \omega + \omega^2)(1 - \omega^2 + \omega^4)(1 - \omega^4 + \omega^8)$..... to 2n factors $= 2^{2n}$.

**(B)** If $\omega$ is a complex cube root of unity, find the value of ;

$(1 + \omega)(1 + \omega^2)(1 + \omega^4)(1 + \omega^8)$..... to n factors.

**28.** Let $z_i$ $(i = 1, 2, 3, 4)$ represent the vertices of a square all of which lie on the sides of the triangle with vertices (0,0), (2,1) and (3, 0). If $z_1$ and $z_2$ are purely real, then area of triangle formed by $z_3, z_4$ and origin is $\frac{m}{n}$ (where m and n are in their lowest form). Find the value of $(m + n)$.

**29.** The points A, B, C represent the complex numbers $z_1, z_2, z_3$ respectively on a complex plane & the angle B & C of the triangle ABC are each equal to $\frac{1}{2}(\pi - \alpha)$. Show that $(z_2 - z_3)^2 = 4(z_3 - z_1)(z_1 - z_2)\sin^2\frac{\alpha}{2}$.

**30.** Evaluate : $\sum_{p=1}^{32}(3p + 2)\left(\sum_{q=1}^{10}\left(\sin\frac{2q\pi}{11} - i\cos\frac{2q\pi}{11}\right)\right)^p$.

# Exercise # 5

## Part # I [Previous Year Questions] [AIEEE/JEE-MAIN]

**1.** The inequality $|z-4| < |z-2|$ represents the following region **[AIEEE-2002]**

(1) $Re(z) > 0$ (2) $Re(z) < 0$ (3) $Re(z) > 2$ (4) none of these

**2.** Let z and ω are two non-zero complex numbers such that $|z| = |\omega|$ and $\arg z + \arg \omega = \pi$, then z equal to **[AIEEE-2002]**

(1) $\omega$ (2) $-\omega$ (3) $\bar{\omega}$ (4) $-\bar{\omega}$

**3.** Let $z_1$ and $z_2$ be two roots of the equation $z^2 + az + b = 0$, z being complex, Further, assume that the origin $z_3$, $z_1$ and $z_2$ form an equilateral triangle. then- **[AIEEE-2003]**

(1) $a^2 = b$ (2) $a^2 = 2b$ (3) $a^2 = 3b$ (4) $a^2 = 4b$

**4.** If z and ω are two non-zero complex numbers such that $|z\omega| = 1$, and $Arg(z) - Arg(\omega) = \pi/2$, then $\bar{z}\,\omega$ is equal to **[AIEEE-2003]**

(1) 1 (2) –1 (3) i (4) –i

**5.** If $\left(\frac{1+i}{1-i}\right)^x = 1$, then **[AIEEE-2003]**

(1) x = 4n, where n is any positive integer (2) x = 2n, where n is any positive integer

(3) x = 4n + 1, where n is any positive integer (4) x = 2n + 1, where n is any positive integer

**6.** Let z, w be complex numbers such that $\bar{z} + i\,\bar{w} = 0$ and $\arg zw = \pi$. Then arg z equals **[AIEEE-2004]**

(1) $\pi/4$ (2) $\pi/2$ (3) $3\pi/4$ (4) $5\pi/4$

**7.** If $|z^2 - 1| = |z|^2 + 1$, then z lies on **[AIEEE-2004]**

(1) the real axis (2) the imaginary axis (3) a circle (4) an ellipse

**8.** If $z = x - iy$ and $z^{1/3} = p + iq$, then $\frac{\left(\frac{x}{p}+\frac{y}{q}\right)}{(p^2+q^2)}$ is equal to- **[AIEEE-2004]**

(1) 1 (2) –1 (3) 2 (4) –2

**9.** If $z_1$ and $z_2$ are two non zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$ then $\arg z_1 - \arg z_2$ is equal to- **[AIEEE-2005]**

(1) $-\pi$ (2) $\frac{\pi}{2}$ (3) $-\frac{\pi}{2}$ (4) 0

**10.** If $w = \frac{z}{z - \frac{1}{3}i}$ and $|w| = 1$ then z lies on **[AIEEE-2005]**

(1) a circle (2) an ellipse (3) a parabola (4) a straight line

**11.** If $|z + 4| \le 3$, then the maximum value of $|z + 1|$ is- **[AIEEE-2007]**

(1) 4 (2) 10 (3) 6 (4) 0

**12.** The conjugate of a complex number is $\frac{1}{i-1}$, then that complex number is- **[AIEEE-2008]**

**(1)** $\frac{-1}{i-1}$ **(2)** $\frac{1}{i+1}$ **(3)** $\frac{-1}{i+1}$ **(4)** $\frac{1}{i-1}$

**13.** If $\left|Z-\frac{4}{Z}\right|=2$, then the maximum value of $|Z|$ is equal to :- **[AIEEE-2009]**

**(1)** 2 **(2)** $2+\sqrt{2}$ **(3)** $\sqrt{3}+1$ **(4)** $\sqrt{5}+1$

**14.** The number of complex numbers z such that $|z-1| = |z+1| = |z-i|$ equals :- **[AIEEE-2010]**

**(1)** 0 **(2)** 1 **(3)** 2 **(4)** $\infty$

**15.** Let $\alpha$, $\beta$ be real and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line Re z = 1, then it is necessary that :- **[AIEEE-2011]**

**(1)** $|\beta| = 1$ **(2)** $\beta \in (1, \infty)$ **(3)** $\beta \in (0, 1)$ **(4)** $\beta \in (-1, 0)$

**16.** If $\omega(\neq 1)$ is a cube root of unity, and $(1+\omega)^7 = A + B\omega$. Then (A, B) equals :- **[AIEEE-2011]**

**(1)** (1, 0) **(2)** (–1, 1) **(3)** (0, 1) **(4)** (1, 1)

**17.** If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies : **[AIEEE-2012]**

**(1)** on the imaginary axis.
**(2)** either on the real axis or on a circle passing through the origin.
**(3)** on a circle with centre at the origin.
**(4)** either on the real axis or on a circle not passing through the origin.

**18.** If z is a complex number of unit modulus and argument $\theta$, then $\arg\left(\frac{1+z}{1+\bar{z}}\right)$ equals **[JEE (Main)-2013]**

**(1)** $-\theta$ **(2)** $\frac{\pi}{2}-\theta$ **(3)** $\theta$ **(4)** $\pi-\theta$

**19.** If z is a complex number such that $|z| \geq 2$, then the minimum value of $\left|z+\frac{1}{2}\right|$ : **[JEE (Main)-2014]**

**(1)** is equal to $\frac{5}{2}$ **(2)** lies in the interval (1, 2)

**(3)** is strictly greater than $\frac{5}{2}$ **(4)** is strictly greater than $\frac{3}{2}$ but less than $\frac{5}{2}$

**20.** A complex number z is said to be unimodular if $|z| = 1$. Suppose $z_1$ and $z_2$ are complex number such that $\frac{z_1-2z_2}{2-z_1\bar{z}_2}$ is unimodular and $z_2$ is not unimodular. Then the point $z_1$ lies on a : **[JEE (Main)-2015]**

**(1)** circle of radius 2. **(2)** circle of radius $\sqrt{2}$
**(3)** straight line parallel to x-axis **(4)** straight line parallel to y-axis

**21.** A value of $\theta$ for which $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary is : **[JEE (Main)-2016]**

**(1)** $\frac{\pi}{6}$ **(2)** $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ **(3)** $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ **(4)** $\frac{\pi}{3}$

**Part # II** **[Previous Year Questions][IIT-JEE ADVANCED]**

**1.** **(A)** If $z_1, z_2, z_3$ are complex numbers such that $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$ then $|z_1 + z_2 + z_3|$ is -

**(A)** equal to 1 **(B)** less than 1 **(C)** greater than 3 **(D)** equal to 3

**(B)** If arg (z) < 0, then arg (–z) – arg (z) = **[JEE 2000]**

**(A)** $\pi$ **(B)** $-\pi$ **(C)** $-\frac{\pi}{2}$ **(D)** $\frac{\pi}{2}$

**2.** **(A)** The complex numbers $z_1$, $z_2$ and $z_3$ satisfying $\frac{z_1 - z_3}{z_2 - z_3} = \frac{1 - i\sqrt{3}}{2}$ are the vertices of a triangle which is -

**(A)** of area zero **(B)** right-angled isosceles **(C)** equilateral **(D)** obtuse-angled isosceles

**(B)** Let $z_1$ and $z_2$ be nth roots of unity which subtend a right angle at the origin. Then n must be of the form

**(A)** 4k + 1 **(B)** 4k + 2 **(C)** 4k + 3 **(D)** 4k **[JEE 2001]**

**3.** **(A)** Let $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$. Then the value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix}$ is - **[JEE 2002]**

**(A)** $3\omega$ **(B)** $3\omega(\omega - 1)$ **(C)** $3\omega^2$ **(D)** $3\omega(1 - \omega)$

**(B)** For all complex numbers $z_1$, $z_2$ satisfying $|z_1| = 12$ and $|z_2 - 3 - 4i| = 5$, the minimum value of $|z_1 - z_2|$ is **[JEE 2002]**

**(A)** 0 **(B)** 2 **(C)** 7 **(D)** 17

**(C)** Let a complex number $\alpha$, $\alpha \neq 1$, be a root of the equation $z^{p+q} - z^p - z^q + 1 = 0$ where p,q are distinct primes. Show that either $1 + \alpha + \alpha^2 + .... + \alpha^{p-1} = 0$ or $1 + \alpha + \alpha^2 + .. + \alpha^{q-1} = 0$, but not both together. **[JEE 2002]**

**4.** If $|z| = 1$ and $\omega = \frac{z-1}{z+1}$ (where $z \neq -1$), then Re (w) equals – **[JEE 2003]**

**(A)** 0 **(B)** $-\frac{1}{|z+1|^2}$ **(C)** $\left|\frac{z}{z+1}\right| \cdot \frac{1}{|z+1|^2}$ **(D)** $\frac{\sqrt{2}}{|z+1|^2}$

**5.** If $z_1$ and $z_2$ are two complex numbers such that $|z_1| < 1$ and $|z_2| > 1$ then show that $\left|\frac{1 - z_1\bar{z}_2}{z_1 - z_2}\right| < 1$ **[JEE 2003]**

**6.** Show that there exists no complex number z such that $|z| < \frac{1}{3}$ and $\sum_{r=1}^{n} a_r z^r = 1$ where $|a_i| < 2$ for i = 1, 2,.......n. **[JEE 2003]**

**7.** The least positive value of 'n' for which $(1 + \omega^2)^n = (1 + \omega^4)^n$, where $\omega$ is a non real cube root of unity is -

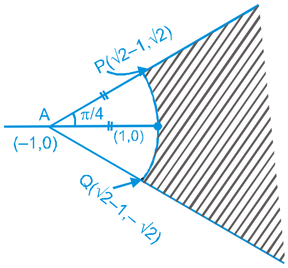
**(A)** 2 **(B)** 3 **(C)** 6 **(D)** 4 **[JEE 2004]**

**8.** Find the centre and radius formed by all the points represented by z = x + i y satisfying the relation $\frac{|z-\alpha|}{|z-\beta|} = K$ $(K \neq 1)$ where $\alpha$ & $\beta$ are constant complex numbers, given by $\alpha = \alpha_1 + i\alpha_2$ & $\beta = \beta_1 + i\beta_2$ **[JEE 2004]**

**9.** If a, b, c are integers not all equal and $\omega$ is cube root of unity $(\omega \neq 1)$ then the minimum value of $|a + b\omega + c\omega^2|$ is - **[JEE 2005]**

**(A)** 0 **(B)** 1 **(C)** $\frac{\sqrt{3}}{2}$ **(D)** $\frac{1}{2}$

**10.** Area of shaded region belongs to - **[JEE 2005]**




**(A)** $z : |z+1| > 2, |\arg(z+1)| < \pi/4$

**(B)** $z : |z-1| > 2, |\arg(z-1)| < \pi/4$

**(C)** $z : |z+1| < 2, |\arg(z+1)| < \pi/2$

**(D)** $z : |z-1| < 2, |\arg(z-1)| < \pi/2$

**11.** If one of the vertices of the square circumscribing the circle $|z - 1| = \sqrt{2}$ is $2 + \sqrt{3}i$. Find the other vertices of square. **[JEE 2005]**

**12.** If $w = \alpha + i\beta$ where $\beta \neq 0$ and $z \neq 1$, satisfies the condition that $\frac{w - \bar{w}z}{1 - z}$ is purely real, then the set of values of z is - **[JEE 2006]**

**(A)** $\{z : |z| = 1\}$ **(B)** $\{z : z = \bar{z}\}$ **(C)** $\{z : z \neq 1\}$ **(D)** $\{z : |z| = 1, z \neq 1\}$

**13.** A man walks a distance of 3 units from the origin towards the north-east (N 45° E) direction. From there, he walks a distance of 4 units towards the north-west (N 45° W) direction to reach a point P. Then the position of P in the Argand plane is : **[JEE 2007]**

**(A)** $3e^{i\pi/4} + 4i$ **(B)** $(3 - 4i)e^{i\pi/4}$ **(C)** $(4 + 3i)e^{i\pi/4}$ **(D)** $(3 + 4i)e^{i\pi/4}$

**14.** If $|z| = 1$ and $z \neq \pm 1$, then all the values of $\frac{z}{1 - z^2}$ lie on : **[JEE 2007]**

**(A)** a line not passing through the origin **(B)** $|z| = \sqrt{2}$

**(C)** the x-axis **(D)** the y-axis

## Comprehension (for 15 to 17)

Let A, B, C be three sets of complex numbers as defined below **[JEE 2008]**

$$A = \{z : \text{Im}\, z \geq 1\}$$
$$B = \{z : |z - 2 - i| = 3\}$$
$$C = \{z : \text{Re}((1 - i)z) = \sqrt{2}\}$$

**15.** The number of elements in the set $A \cap B \cap C$ is -

**(A)** 0 **(B)** 1 **(C)** 2 **(D)** $\infty$

**16.** Let z be any point in $A \cap B \cap C$. Then $|z + 1 - i|^2 + |z - 5 - i|^2$ lies between -

**(A)** 25 and 29 **(B)** 30 and 34 **(C)** 35 and 39 **(D)** 40 and 44

**17.** Let z be any point in $A \cap B \cap C$ and let $\omega$ be any point satisfying $|\omega - 2 - i| < 3$. Then, $|z| - |\omega| + 3$ lies between -

**(A)** –6 and 3 **(B)** –3 and 6 **(C)** –6 and 6 **(D)** –3 and 9

**18.** A particle P starts from the point $z_0 = 1 + 2i$, where $i = \sqrt{-1}$ . It moves first horizontally away from origin by 5 units and then vertically away from origin by 3 units to reach a point $z_1$. From $z_1$ the particle moves $\sqrt{2}$ units in the direction of the vector $\tilde{i} + \tilde{j}$ and then it moves through an angle $\frac{\pi}{2}$ in anticlockwise direction on a circle with centre at origin, to reach a point $z_2$. The point $z_2$ is given by - **[JEE 2008]**

**(A)** $6 + 7i$ **(B)** $-7 + 6i$ **(C)** $7 + 6i$ **(D)** $-6 + 7i$

**19.** Let $z = \cos\theta + i \sin\theta$. Then the value of $\sum_{m=1}^{15} \text{Im}(z^{2m-1})$ at $\theta = 2°$ is - **[JEE 2009]**

**(A)** $\frac{1}{\sin 2°}$ **(B)** $\frac{1}{3\sin 2°}$ **(C)** $\frac{1}{2\sin 2°}$ **(D)** $\frac{1}{4\sin 2°}$

**20.** Let $z = x + iy$ be a complex number where x and y are integers. Then the area of the rectangle whose vertices are the roots of the equation $z\bar{z}^3 + \bar{z}z^3 = 350$ is - **[JEE 2009]**

**(A)** 48 **(B)** 32 **(C)** 40 **(D)** 80

**21.** Match the conics in Column I with the statements/ expressions in Column II. **[JEE 2009]**

| | Column I | | Column II |
|---|---|---|---|
| (A) | Circle | (p) | The locus of the point (h, k) for which the line $hx + ky = 1$ touches the circle $x^2 + y^2 = 4$ |
| (B) | Parabola | (q) | Points z in the complex plane satisfying $\lvert z+2\rvert - \lvert z-2\rvert = \pm 3$ |
| (C) | Ellipse | (r) | Points of the conic have parametric representation $x = \sqrt{3}\left(\frac{1-t^2}{1+t^2}\right), y = \frac{2t}{1+t^2}$ |
| (D) | Hyperbola | (s) | The eccentricity of the conic lies in the interval $1 \le x < \infty$ |
| | | (t) | Points z in the complex plane satisfying $\text{Re}(z+1)^2 = \lvert z\rvert^2 + 1$ |

**22.** Let $z_1$ and $z_2$ be two distinct complex numbers and let $z = (1 - t)z_1 + tz_2$ for some real number t with $0 < t < 1$. If Arg(w) denotes the principal argument of a nonzero complex number w, then **[JEE 2010]**

**(A)** $|z - z_1| + |z - z_2| = |z_1 - z_2|$

**(B)** $\text{Arg}(z - z_1) = \text{Arg}(z - z_2)$

**(C)** $\begin{vmatrix} z - z_1 & \bar{z} - \bar{z}_1 \\ z_2 - z_1 & \bar{z}_2 - \bar{z}_1 \end{vmatrix} = 0$

**(D)** $\text{Arg}(z - z_1) = \text{Arg}(z_2 - z_1)$

**23.** Let ω be the complex number $\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$. Then the number of distinct complex numbers z satisfying

$$\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$$ is equal to **[JEE 2010]**

**24.** Match the statements in Column-I with those in Column-II. **[JEE 2010]**

[**Note :** Here z takes values in the complex plane and Im z and Re z denote, respectively, the imaginary part and the real part of z.]

| Column I | | Column II | |
|---|---|---|---|
| **(A)** | The set of points z satisfying $\left\|z-i\|z\|\right\| = \left\|z+i\|z\|\right\|$ is contained in or equal to | **(p)** | an ellipse with eccentricity $\frac{4}{5}$ |
| **(B)** | The set of points z satisfying $\|z+4\| + \|z-4\| = 10$ is contained in or equal to | **(q)** | the set of points z satisfying Im z = 0 |
| **(C)** | If $\|w\| = 2$, then the set of points $z = w - \frac{1}{w}$ is contained in or equal to | **(t)** | the set of points z satisfying $\|\text{Im } z\| \le 1$ |
| **(D)** | If $\|w\| = 1$, then the set of points $z = w + \frac{1}{w}$ is contained in or equal to | **(s)** | the set of points z satisfying $\|\text{Re } z\| \le 2$ |
| | | **(t)** | the set of points z satisfying $\|z\| \le 3$ |

**25.** **Comprehension (3 questions together)**

Let a,b and c be three real numbers satisfying

$$\begin{bmatrix} a & b & c \end{bmatrix}\begin{bmatrix} 1 & 9 & 7 \\ 8 & 2 & 7 \\ 7 & 3 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \quad \text{...(E)}$$

**(i)** If the point P(a,b,c), with reference to (E), lies on the plane 2x + y + z = 1, then the value of 7a+b+c is

**(A)** 0 **(B)** 12 **(C)** 7 **(D)** 6

**(ii)** Let ω be a solution of $x^3 - 1 = 0$ with Im(ω) > 0. If a = 2 with b and c satisfying (E), then the value of $\frac{3}{\omega^a} + \frac{1}{\omega^b} + \frac{3}{\omega^c}$ is equal to -

**(A)** –2 **(B)** 2 **(C)** 3 **(D)** –3

**(iii)** Let b = 6, with a and c satisfying (E). If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then $\sum_{n=0}^{\infty}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)^n$ is -

**(A)** 6 **(B)** 7 **(C)** $\frac{6}{7}$ **(D)** ∞ **[JEE 2011]**

**26.** If z is any complex number satisfying $|z - 3 - 2i| \le 2$, then the minimum value of $|2z - 6 + 5i|$ is **[JEE 2011]**

**27.** Let $\omega = e^{i\pi/3}$, and a, b, c, x, y, z be non-zero complex numbers such that

$a + b + c = x$

$a + b\omega + c\omega^2 = y$

$a + b\omega^2 + c\omega = z.$

Then the value of $\frac{|x|^2 + |y|^2 + |z|^2}{|a|^2 + |b|^2 + |c|^2}$ is **[JEE 2011]**

**28.** Match the statements given in Column I with the values given in Column II

| Column I | | Column II | |
|---|---|---|---|
| **(A)** | If $\vec{a} = \hat{j} + \sqrt{3}\hat{k}$, $\vec{b} = -\hat{j} + \sqrt{3}\hat{k}$ and $\vec{c} = 2\sqrt{3}\hat{k}$ form a triangle, then the internal angle of the triangle between $\vec{a}$ and $\vec{b}$ is | **(p)** | $\frac{\pi}{6}$ |
| **(B)** | If $\int_a^b (f(x) - 3x)dx = a^2 - b^2$, then the value of $f\left(\frac{\pi}{6}\right)$ is | **(q)** | $\frac{2\pi}{3}$ |
| **(C)** | The value of $\frac{\pi^2}{\ell n 3}\int_{7/6}^{5/6} \sec(\pi x)dx$ is | **(r)** | $\frac{\pi}{3}$ |
| **(D)** | The maximum value of $\left\lvert \text{Arg}\left(\frac{1}{1-z}\right)\right\rvert$ for $\lvert z\rvert = 1, z \neq 1$ is given by | **(s)** | $\pi$ |
| | | **(t)** | $\frac{\pi}{2}$ |

**[JEE 2011]**

**29.** Match the statements given in Column I with the intervals/union of intervals given in Column II

| Column I | | Column II | |
|---|---|---|---|
| **(A)** | The set $\left\{\text{Re}\left(\frac{2iz}{1-z^2}\right) : z \text{ is a complex number, } \lvert z\rvert = 1, z \neq \pm 1\right\}$ is | **(p)** | $(-\infty, -1) \cup (1, \infty)$ |
| **(B)** | The domain of the function $f(x) = \sin^{-1}\left(\frac{8(3)^{x-2}}{1-3^{2(x-1)}}\right)$ is | **(q)** | $(-\infty, 0) \cup (0, \infty)$ |
| **(C)** | If $f(\theta) = \begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$, then the set $\left\{f(\theta) : 0 \le \theta < \frac{\pi}{2}\right\}$ is | **(r)** | $[2, \infty)$ |
| **(D)** | If $f(x) = x^{3/2}(3x - 10), x \ge 0$, then $f(x)$ is increasing in | **(s)** | $(-\infty, -1] \cup [1, \infty)$ |
| | | **(t)** | $(-\infty, 0] \cup [2, \infty)$ |

**[JEE 2011]**

**30.** Let z be a complex number such that the imaginary part of z is nonzero and $a = z^2 + z + 1$ is real. Then a cannot take the value - **[JEE 2012]**

**(A)** $-1$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{3}{4}$

**31.** Let complex numbers $\alpha$ and $\frac{1}{\bar{\alpha}}$ lie on circles $(x - x_0)^2 + (y - y_0)^2 = r^2$ and $(x - x_0)^2 + (y - y_0)^2 = 4r^2$ respectively. If $z_0 = x_0 + iy_0$ satisfies the equation $2|z_0|^2 = r^2 + 2$, then $|\alpha| =$ **[JEE Ad. 2013]**

**(A)** $\frac{1}{\sqrt{2}}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{1}{\sqrt{7}}$ **(D)** $\frac{1}{3}$

**32.** Let $\omega$ be a complex cube root of unity with $\omega \neq 1$ and $P = [p_{ij}]$ be a $n \times n$ matrix with $p_{ij} = \omega^{i+j}$. Then $P^2 \neq 0$, when n = **[JEE Ad.]**

**(A)** 57 **(B)** 55 **(C)** 58 **(D)** 56

**33.** Let $w = \frac{\sqrt{3}+i}{2}$ and $P = \{w^n : n = 1, 2, 3, .....\}$. Further $H_1 = \left\{z \in C : \text{Re}\, z > \frac{1}{2}\right\}$ and $H_2 = \left\{z \in C : \text{Re}\, z < \frac{-1}{2}\right\}$, where C is the set of all complex numbers. If $z_1 \in P \cap H_1$, $z_2 \in P \cap H_2$ and O represents the origin, then $\angle z_1 O z_2 =$ **[JEE-Ad. 2013]**

**(A)** $\frac{\pi}{2}$ **(B)** $\frac{\pi}{6}$ **(C)** $\frac{2\pi}{3}$ **(D)** $\frac{5\pi}{6}$

## Paragraph for Question 34 and 35

Let $S = S_1 \cap S_2 \cap S_3$, where $S_1 = \{z \in C : |z| < 4\}$, $S_2 = \left\{z \in C : \text{Im}\left[\frac{z - 1 + \sqrt{3}i}{1 - \sqrt{3}i}\right] > 0\right\}$ and $S_3 = \{z \in C : \text{Re}\, z > 0\}$.

**34.** $\min_{z \in S} |1 - 3i - z| =$ **[JEE Ad. 2013]**

**(A)** $\frac{2-\sqrt{3}}{2}$ **(B)** $\frac{2+\sqrt{3}}{2}$ **(C)** $\frac{3-\sqrt{3}}{2}$ **(D)** $\frac{3+\sqrt{3}}{2}$

**35.** Area of S = **[JEE Ad. 2013]**

**(A)** $\frac{10\pi}{3}$ **(B)** $\frac{20\pi}{3}$ **(C)** $\frac{16\pi}{3}$ **(D)** $\frac{32\pi}{3}$

**36.** Let $z_k = \cos\left(\frac{2k\pi}{10}\right) + i\sin\left(\frac{2k\pi}{10}\right)$; $k = 1, 2, .....,9$. **[JEE Ad. 2014]**

| | List - I | | List - II |
|---|---|---|---|
| (p) | For each $z_k$ there exists a $z_j$ such $z_k \cdot z_j = 1$ | (1) | True |
| (q) | There exists a $k \in \{1, 2, ....., 9\}$ such that $z_1 \cdot z = z_k$ has no solution z in the set of complex numbers | (2) | False |
| (r) | $\frac{\lvert 1-z_1\rvert \lvert 1-z_2\rvert .......\lvert 1-z_9\rvert}{10}$ equals | (3) | 1 |
| (s) | $1 - \sum_{k=1}^{9} \cos\left(\frac{2k\pi}{10}\right)$ equals | (4) | 2 |

**Codes :**

| | p | q | r | s |
|---|---|---|---|---|
| (A) | 1 | 2 | 4 | 3 |
| (B) | 2 | 1 | 3 | 4 |
| (C) | 1 | 2 | 3 | 4 |
| (D) | 2 | 1 | 4 | 3 |

**37.** For any integer k, let $\alpha_k = \left(\frac{k\pi}{7}\right) + i\sin\left(\frac{k\pi}{7}\right)$, where $i = \sqrt{-1}$. The value of the expresion

$$\frac{\sum_{k=1}^{12}\left|\alpha_{k+1}-\alpha_k\right|}{\sum_{k=1}^{3}\left|\alpha_{4k-1}-\alpha_{4k-2}\right|}$$ is **[JEE Ad. 2015]**

**38.** Let $z = \frac{-1+\sqrt{3i}}{2}$, where $i = \sqrt{-1}$, and r, s $\in \{1, 2, 3\}$. Let $P = \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}$ and I be the identity matrix of order 2.

Then the total number of ordered pairs (r, s) for which $P^2 = -1$ is **[JEE Ad. 2016]**

**39.** Let a, b $\in$ R and $a^2 + b^2 \neq 0$. Suppose $S = \left\{z \in C : z = \frac{1}{a+ibt'} t \in R, t \neq 0\right\}$, where

$i = \sqrt{-1}$. if $z = x + iy$ and $z \in S$, then (x,y) lies on

**(A)** the circle with radius $\frac{1}{2a}$ and centre $\left(\frac{1}{2a}, 0\right)$ for $a < 0, b \neq 0$

**(B)** the circle with radius $-\frac{1}{2a}$ and centre $\left(-\frac{1}{2a}, 0\right)$ for $a < 0, b \neq 0$

**(C)** the x-axis for $a \neq 0, b = 0$

**(D)** the y-axis for $a = 0, b \neq 0$ **[JEE Ad. 2016]**

# MOCK TEST

## SECTION - I : STRAIGHT OBJECTIVE TYPE

**1.** If 'p' and 'q' are distinct prime numbers, than the number of distinct imaginary numbers which are $p^{th}$ as well as $q^{th}$ roots of unity are -

**(A)** $\min^m(p, q)$ **(B)** $\max^m(p, q)$ **(C)** 1 **(D)** zero

**2.** Number of solution of the equation $z^3 + \frac{3(\bar{z})^2}{|z|} = 0$ where z is a complex number is

**(A)** 2 **(B)** 3 **(C)** 6 **(D)** 5

**3.** If 1, $\alpha_1$, $\alpha_2$, $\alpha_3$........ and $\alpha_8$ are nine, ninth roots of unity (taken in counter-clockwise sequence) then $|(2-\alpha_1)(2-\alpha_3)(2-\alpha_5)(2-\alpha_7)|$ is equal to

**(A)** $\sqrt{255}$ **(B)** $\sqrt{511}$ **(C)** $\sqrt{1023}$ **(D)** 15

**4.** The point of intersection the curves $\arg(z-i+2) = \frac{\pi}{6}$ & $\arg(z+4-3i) = -\frac{\pi}{4}$ is given by

**(A)** $(-2+i)$ **(B)** $2-i$ **(C)** $2+i$ **(D)** none of these

**5.** If $|z_2 + iz_1| = |z_1| + |z_2|$ and $|z_1| = 3$ & $|z_2| = 4$ then area of Δ ABC, if affix of A, B & C are $(z_1)$, $(z_2)$ and $\left(\frac{z_2 - iz_1}{1-i}\right)$ respectively, is

**(A)** $\frac{5}{2}$ **(B)** 0 **(C)** $\frac{25}{2}$ **(D)** $\frac{25}{4}$

**6.** The principal argument of the complex number $\frac{(1+i)^5(1+\sqrt{3}\,i)^2}{-2i(-\sqrt{3}+i)}$ is

**(A)** $\frac{19\pi}{12}$ **(B)** $-\frac{7\pi}{12}$ **(C)** $-\frac{5\pi}{12}$ **(D)** $\frac{5\pi}{12}$

**7.** Image of the point, whose affix is $\frac{2-i}{3+i}$, in the line $(1+i)z + (1-i)\bar{z} = 0$ is the point whose affix is

**(A)** $\frac{1+i}{2}$ **(B)** $\frac{1-i}{2}$ **(C)** $\frac{-1+i}{2}$ **(D)** $-\frac{1+i}{2}$

**8.** If a complex number z satisfies $|2z+10+10i| \le 5\sqrt{3} - 5$, then the least principal argument of z is

**(A)** $-\frac{11\pi}{12}$ **(B)** $-\frac{2\pi}{3}$ **(C)** $-\frac{5\pi}{6}$ **(D)** $-\frac{3\pi}{4}$

**9.** If t and c are two complex numbers such that $|t| \neq |c|$, $|t| = 1$ and $z = \frac{at+b}{t-c}$, $z = x + iy$. Locus of z is (where a, b are complex numbers)

**(A)** line segment **(B)** straight line **(C)** circle **(D)** none

**10.** **$S_1$ :** Let $z_k$ (k = 0, 1, 2, 3, 4, 5, 6) be the roots of the equation $(z+1)^7 + (z)^7 = 0$ then $\sum_{k=0}^{6} \text{Re}(z_k)$ is equal to $-\frac{7}{2}$

**$S_2$ :** If α, β, γ and a, b, c are complex numbers such that $\frac{\alpha}{a}+\frac{\beta}{b}+\frac{\gamma}{c} = 1 + i$ and $\frac{a}{\alpha}+\frac{b}{\beta}+\frac{c}{\gamma} = 0$, then the value of $\frac{\alpha^2}{a^2}+\frac{\beta^2}{b^2}+\frac{\gamma^2}{c^2}$ is equal to – 1

**$S_3$ :** If $z_1, z_2, \ldots\ldots z_6$ are six roots of the equation $z^6 - z^5 + z^4 - z^3 + z^2 - z + 1 = 0$ then the value of $\prod_{i=1}^{6}(z_i + 1)$ is equal to 4

**$S_4$ :** Number of solutions of the equation $z^3 = \bar{z}\, i|z|$ are 5

**(A)** TTFT **(B)** TFFT **(C)** FFTF **(D)** TTFF

## SECTION - II : MULTIPLE CORRECT ANSWER TYPE

**11.** If n is the smallest positve integer for which $(a + ib)^n = (a - ib)^n$ where a > 0 & b > 0 then the numerical value of b/a is :

**(A)** $\tan\frac{\pi}{3}$ **(B)** $\sqrt{3}$ **(C)** 3 **(D)** $\frac{1}{\sqrt{3}}$

**12.** If z is a complex number satisfying $|z - i\,\text{Re}(z)| = |z - \text{Im}(z)|$ then z lies on

**(A)** y = x **(B)** y = – x **(C)** y = x + 1 **(D)** y = – x + 1

**13.** If $z_1 = 5 + 12i$ and $|z_2| = 4$ then

**(A)** maximum $(|z_1 + iz_2|) = 17$

**(B)** minimum $(|z_1 + (1+i)z_2|) = 13 - 9\sqrt{2}$

**(C)** minimum $\left|\frac{z_1}{z_2 + \frac{4}{z_2}}\right| = \frac{13}{4}$

**(D)** maximum $\left|\frac{z_1}{z_2 + \frac{4}{z_2}}\right| = \frac{13}{3}$

**14.** If α, β be the roots of the equation $\mu^2 - 2\mu + 2 = 0$ and if $\cot\theta = x + 1$, then $\frac{(x+\alpha)^n - (x+\beta)^n}{\alpha - \beta}$ is equal to

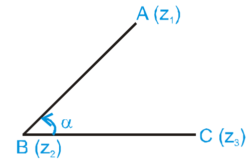
**(A)** $\frac{\sin n\theta}{\sin^n\theta}$ **(B)** $\frac{\cos n\theta}{\cos^n\theta}$ **(C)** $\frac{\sin n\theta}{\cos^n\theta}$ **(D)** $\frac{\text{cosec}^n\theta}{\text{cosec}\, n\theta}$

**15.** If $z_1$ lies on $|z| = 1$ and $z_2$ lies on $|z| = 2$, then

**(A)** $3 \le |z_1 - 2z_2| \le 5$ **(B)** $1 \le |z_1 + z_2| \le 3$ **(C)** $|z_1 - 3z_2| \ge 5$ **(D)** $|z_1 - z_2| \ge 1$

## SECTION - III : ASSERTION AND REASON TYPE

16. **Statement - I :** If A($z_1$), B($z_2$), C($z_3$) are the vertices of an equilateral triangle ABC, then $\arg\left(\dfrac{z_2+z_3-2z_1}{z_3-z_2}\right)=\dfrac{\pi}{4}$




**Statement - II :** If $\angle B=\alpha$, then $\dfrac{z_1-z_2}{z_3-z_2}=\dfrac{AB}{BC}e^{i\alpha}$ or $\arg\left(\dfrac{z_1-z_2}{z_3-z_2}\right)=\alpha$

**(A)** Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.
**(B)** Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
**(C)** Statement-I is True, Statement-II is False
**(D)** Statement-I is False, Statement-II is True

17. **Statement - I :** If $x+\dfrac{1}{x}=1$ and $p=x^{4000}+\dfrac{1}{x^{4000}}$ and q be the digit at unit place in the number $2^{2^n}+1$, $n\in N$ and $n>1$, then the value of $p+q=8$.

**Statement - II :** $\omega,\omega^2$ are the roots of $x+\dfrac{1}{x}=-1$, then $x^2+\dfrac{1}{x^2}=-1$, $x^3+\dfrac{1}{x^3}=2$

**(A)** Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.
**(B)** Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
**(C)** Statement-I is True, Statement-II is False
**(D)** Statement-I is False, Statement-II is True

18. **Statement - I :** If $z_1, z_2, z_3$ are complex number representing the points A, B, C such that $\dfrac{2}{z_1}=\dfrac{1}{z_2}+\dfrac{1}{z_3}$. Then circle through A, B, C passes through origin.

**Statement - II :** If $2z_2=z_1+z_3$ then $z_1, z_2, z_3$ are collinear.

**(A)** Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.
**(B)** Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
**(C)** Statement-I is True, Statement-II is False
**(D)** Statement-I is False, Statement-II is True

19. **Statement - I :** $3+ix^2y$ and $x^2+y+4i$ are complex conjugate numbers, then $x^2+y^2=4$.

**Statement - II :** If sum and product of two complex numbers is real then they are conjugate complex number.

**(A)** Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.
**(B)** Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
**(C)** Statement-I is True, Statement-II is False
**(D)** Statement-I is False, Statement-II is True

20. **Statement - I :** If $|z|<\sqrt{2}-1$, then $|z^2+2z\cos\alpha|<1$

**Statement - II :** $|z_1+z_2|\le|z_1|+|z_2|$ also $|\cos\alpha|\le 1$.

**(A)** Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.
**(B)** Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
**(C)** Statement-I is True, Statement-II is False
**(D)** Statement-I is False, Statement-II is True

## SECTION - IV : MATRIX - MATCH TYPE

**21.** **Column - I** | **Column - II**

**(A)** Locus of the point z satisfying the equation $\text{Re}(z^2) = \text{Re}(z + \bar{z})$ — **(p)** A parabola

**(B)** Locus of the point z satisfying the equation $|z - z_1| + |z - z_2| = \lambda, \lambda \in R^+$ and $\lambda \nless |z_1 - z_2|$ — **(q)** A straight line

**(C)** Locus of the point z satisfying the equation $\left|\frac{2z - i}{z + 1}\right| = m$ where $i = \sqrt{-1}$ and $m \in R^+$ — **(r)** An ellipse

**(D)** If $|\bar{z}| = 25$ then the points representing the complex number $-1 + 75\bar{z}$ will be on a — **(s)** A rectangular hyperbola

**(t)** A circle

**22.** If $z_1, z_2, z_3, z_4$ are the roots of the equation $z^4 + z^3 + z^2 + z + 1 = 0$ then

**Column – I** | **Column – II**

**(A)** $\left|\sum_{i=1}^{4} z_i^4\right|$ is equal to — **(p)** 0

**(B)** $\sum_{i=1}^{4} z_i^5$ is equal to — **(q)** 4

**(C)** $\prod_{i=1}^{4}(z_i + 2)$ is equal to — **(r)** 1

**(D)** least value of $[|z_1 + z_2|]$ is (Where [ ] represents greatest integer function) — **(s)** 11

**(t)** $\left|4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right|$

## SECTION - V : COMPREHENSION TYPE

**23.** **Read the following comprehension carefully and answer the questions.**

The complex slope of a line passing through two points represented by complex numbers $z_1$ and $z_2$ is defined by $\frac{z_2 - z_1}{\bar{z}_2 - \bar{z}_1}$ and we shall denote by $\omega$. If $z_0$ is complex number and c is a real number, then $\bar{z}_0 z + z_0\bar{z} + c = 0$ represents a straight line. Its complex slope is $-\frac{z_0}{\bar{z}_0}$. Now consider two lines

$\alpha\bar{z} + \bar{\alpha}z + i\beta = 0$ ...(i) and $a\bar{z} + \bar{a}z + b = 0$ ...(ii)

where $\alpha$, $\beta$ and a, b are complex constants and let their complex slopes be denoted by $\omega_1$ and $\omega_2$ respectively

**1.** If the lines are inclined at an angle of 120° to each other, then

**(A)** $\omega_2\bar{\omega}_1 = \omega_1\bar{\omega}_1$ **(B)** $\omega_2\bar{\omega}_1^2 = \omega_1\bar{\omega}_2^2$ **(C)** $\omega_1^2 = \omega_2^2$ **(D)** $\omega_1 + 2\omega_2 = 0$

**2.** Which of the following must be true

**(A)** a must be pure imaginary **(B)** $\beta$ must be pure imaginary

**(C)** a must be real **(D)** b must be imaginary

**3.** If line **(i)** makes an angle of 45° with real axis, then $(1+i)\left(-\frac{2\alpha}{\bar{\alpha}}\right)$ is

**(A)** $2\sqrt{2}$ **(B)** $2\sqrt{2}\,i$ **(C)** $2(1-i)$ **(D)** $-2(1+i)$

**24.** **Read the following comprehension carefully and answer the questions.**

Let $(1+x)^n = C_0 + C_1x + C_2x^2 + .......+ C_nx^n$. For sum of series $C_0 + C_1 + C_2 + .........$, put $x = 1$. For sum of series $C_0 + C_2 + C_4 + C_6 + ........$, or $C_1 + C_3 + C_5 + ........$ add or substract equations obtained by putting $x = 1$ and $x = -1$.

For sum of series $C_0 + C_3 + C_6 + ........$ or $C_1 + C_4 + C_7 + .......$ or $C_2 + C_5 + C_8 + .......$ we subsitute $x = 1$, $x = \omega$, $x = \omega^2$ and add or manupulate results.

Similarly, if suffixes differe by 'p' then we substitute $p^{th}$ roots of unity and add.

**1.** $C_0 + C_3 + C_6 + C_9 + ........ =$

**(A)** $\frac{1}{3}\left[2^n - 2\cos\frac{n\pi}{3}\right]$ **(B)** $\frac{1}{3}\left[2^n + 2\cos\frac{n\pi}{3}\right]$ **(C)** $\frac{1}{3}\left[2^n - 2\sin\frac{n\pi}{3}\right]$ **(D)** $\frac{1}{3}\left[2^n + 2\sin\frac{n\pi}{3}\right]$

**2.** $C_1 + C_5 + C_9 + .... =$

**(A)** $\frac{1}{4}\left[2^n - 2^{n/2}2\cos\frac{n\pi}{4}\right]$ **(B)** $\frac{1}{4}\left[2^n + 2^{n/2}2\cos\frac{n\pi}{4}\right]$

**(C)** $\frac{1}{4}\left[2^n - 2^{n/2}2\sin\frac{n\pi}{4}\right]$ **(D)** $\frac{1}{4}\left[2^n + 2^{n/2}2\sin\frac{n\pi}{4}\right]$

**3.** $C_2 + C_6 + C_{10} + ..... =$

**(A)** $\frac{1}{4}\left[2^n - 2^{n/2}\,2.\cos\frac{n\pi}{4}\right]$ **(B)** $\frac{1}{4}\left[2^n + 2^{n/2}\,2.\cos\frac{n\pi}{4}\right]$

**(C)** $\frac{1}{4}\left[2^n - 2^{n/2}\,2.\sin\frac{n\pi}{4}\right]$ **(D)** $\frac{1}{4}\left[2^n + 2^{n/2}\,2.\sin\frac{n\pi}{4}\right]$

**25.** **Read the following comprehension carefully and answer the questions.**

Consider $\Delta ABC$ in Argand plane. Let A(0), B(1) and C(1 + i) be its vertices and M be the mid point of CA. Let z be a variable complex number in the plane. Let u be another variable complex number defined as $u = z^2 + 1$

**1.** Locus of u, when z is on BM, is

**(A)** Circle **(B)** Parabola **(C)** Ellipse **(D)** Hyperbola

**2.** Axis of locus of u, when z is on BM, is

**(A)** real – axis **(B)** Imaginary – axis **(C)** $z + \bar{z} = 2$ **(D)** $z - \bar{z} = 2i$

**3.** Directrix of locus of u, when z is on BM, is

**(A)** real– axis **(B)** imaginary – axis **(C)** $z + \bar{z} = 2$ **(D)** $z - \bar{z} = 2i$

## SECTION - VI : INTEGER TYPE

**26.** If $\left(\frac{1+i}{1-i}\right)^n = \frac{2}{\pi}\left(\sec^{-1}\frac{1}{x} + \sin^{-1}x\right)$ $x \neq 0, -1 \le x \le 1$, then find the number of positive integers less than 20 satisfying above equation.

**27.** Let $f_p(\alpha) = e^{\frac{i\alpha}{p^2}}, e^{\frac{2i\alpha}{p^2}} ......e^{\frac{i\alpha}{p}}$ $p \in N$ (where $i = \sqrt{-1}$, then find the value of $\left|\lim_{n\to\infty} f_n(\pi)\right|$

**28.** If $|z| = \min(|z-1|, |z+1|\}$, then find the value of $|z + \overline{z}|$.

**29.** If z is a complex number and the minimum value of $|z| + |z-1| + |2z-3|$ is $\lambda$ and if $y = 2[x] + 3 = 3[x - \lambda]$, then find the value of $[x + y]$ (where [.] denotes the greatest integer function}

**30.** If $\alpha = e^{\frac{2\pi i}{7}}$ and $f(x) = A_0 + \sum_{k=1}^{20} A_k x^k$, then the value of $\sum_{r=0}^{6} f(\alpha^r x) = n(A_0 + A_n x^n + A_{2n} x^{2n})$ then find the value of n.

# ANSWER KEY

## EXERCISE - 1

**1.** C **2.** B **3.** D **4.** A **5.** C **6.** B **7.** A **8.** D **9.** A **10.** A **11.** D **12.** C **13.** B
**14.** D **15.** D **16.** A **17.** B **18.** A **19.** B **20.** A **21.** B **22.** A **23.** B **24.** B **25.** C **26.** A
**27.** D **28.** C **29.** B **30.** A **31.** B **32.** C **33.** D **34.** A **35.** A

## EXERCISE - 2 : PART # I

**1.** AC **2.** ABD **3.** AB **4.** BC **5.** ACD **6.** BD **7.** ABC **8.** BCD **9.** AD
**10.** AC **11.** AB **12.** ABC **13.** BC **14.** BCD **15.** ABC **16.** AC **17.** ACD **18.** ABCD
**19.** ACD **20.** BC **21.** CD **22.** AB **23.** ABD **24.** ABCD **25.** AC

## PART - II

**1.** D **2.** B **3.** B **4.** B **5.** A **6.** A **7.** C **8.** B

## EXERCISE - 3 : PART # I

**1.** $A\to s$ $B\to p$ $C\to q$ $D\to r$ **2.** $A\to p$ $B\to q$ $C\to$ , $D\to s$ **3.** $A\to p$ $B\to r$ $C\to t$ $D\to q,s$
**4.** $A\to q$ $B\to p$ $C\to q,s$ $D\to r$

## PART - II

**Comprehension # 1 : 1.** A **2.** C **3.** A **Comprehension # 2 : 1.** D **2.** C **3.** B
**Comprehension # 3 : 1.** A **2.** C **3.** B **Comprehension # 4 : 1.** C **2.** A **3.** B
**Comprehension # 5 : 1.** C **2.** B **3.** A

## EXERCISE - 5 : PART # I

**1.** 4 **2.** 4 **3.** 3 **4.** 4 **5.** 1 **6.** 3 **7.** 2 **8.** 4 **9.** 4 **10.** 4 **11.** 3 **12.** 3 **13.** 4
**14.** 2 **15.** B **16.** 4 **17.** 2 **18.** 3 **19.** 2 **20.** 1 **21.** 3

## PART - II

**1.** **(A)** A **(B)** A **2.** **(A)** C, **(B)** D **3.** **(A)** B **(B)** B **4.** A **7.** B

**8.** $\frac{\alpha - k^2\beta}{1-k^2}$ & $\left|\frac{1}{k^2-1}\right|\sqrt{|\alpha - k^2\beta|^2 - (k^2|\beta|^2 - |\alpha|^2)(k^2-1)}$ **9.** B **10.** A

**11.** $(-\sqrt{3}\,i)$, $(1-\sqrt{3})+i$ and $(1+\sqrt{3})-i$ **12.** D **13.** D **14.** D **15.** B **16.** C **17.** D **18.** D **19.** D

**20.** A **21.** $A\to p$ $B\to s,t$ $C\to r$ $D\to q,s$ **22.** A, C, D **23.** 1

**24.** **(A)** $\to q,r$ **(B)** $\to p$ **(C)** $\to p,s,t$ **(D)** $\to q,r,s,t$ **25.** **(i)** D, **(ii)** A, **(iii)** B **26.** 5 **27.** 3

**28.** **(A)** $\to q$ **(B)** $\to p$ **(C)** $\to s$ **(D)** $\to t$ **29.** **(A)** $\to s$ **(B)** $\to t$ **(C)** $\to r$ **(D)** $\to r$ **30.** D

**31.** C **32.** BCD **33.** CD **34.** C **35.** B **36.** C **37.** 4 **38.** 1 **39.** ACD

## MOCK TEST

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **1.** D | **2.** D | **3.** B | **4.** D | **5.** D | **6.** C | **7.** C | **8.** C | **9.** C |
| **10.** B | **11.** AB | **12.** AB | **13.** AD | **14.** AD | **15.** ABCD | **16.** D | **17.** D | **18.** B |
| **19.** D | **20.** A | **21.** $A \to s$ $B \to q,r$ $C \to a,t$ $D \to t$ | | | **22.** $A \to r$ $B \to q,t$ $C \to s$ $D \to p$ | | | |
| **23. 1.** B | **2.** B | **3.** C | **24. 1.** B | **2.** D | **3.** A | **25. 1.** B | **2.** C | **3.** D |
| **26.** 4 | **27.** 1 | **28.** 1 | **29.** 30 | **30.** 7 | | | | |