SOLVED EXAMPLES

Ex. 1 How many numbers between 10 and 10,000 can be formed by using the digits 1, 2, 3, 4, 5 if

(i) No digit is repeated in any number. **(ii)** Digits can be repeated.

Sol.

(i) Number of two digit numbers = $5 \times 4 = 20$

Number of three digit numbers = $5 \times 4 \times 3 = 60$

Number of four digit numbers = $5 \times 4 \times 3 \times 2 = 120$

Total = 200

(ii) Number of two digit numbers = $5 \times 5 = 25$

Number of three digit numbers = $5 \times 5 \times 5 = 125$

Number of four digit numbers = $5 \times 5 \times 5 \times 5 = 625$

Total = 775

Ex. 2 If 7all the letters of the word 'RAPID' are arranged in all possible manner as they are in a dictionary, then find the rank of the word 'RAPID'.

Sol. First of all, arrange all letters of given word alphabetically : 'ADIPR'

Total number of words starting with A _ _ _ _ = 4! = 24

Total number of words starting with D _ _ _ _ = 4! = 24

Total number of words starting with I_ _ _ _ = 4! = 24

Total number of words starting with P _ _ _ _ = 4! = 24

Total number of words starting with RAD _ _ = 2! = 2

Total number of words starting with RAI _ _ = 2! = 2

Total number of words starting with RAPD _ = 1

Total number of words starting with RAPI _ = 1

$\therefore$ Rank of the word RAPID = 24 + 24 + 24 + 24 + 2 + 2 + 1 + 1 = 102

Ex.3 A delegation of four students is to be selected from a total of 12 students. In how many ways can the delegation be selected, if-

(A) all the students are equally willing ?

(B) two particular students have to be included in the delegation ?

(C) two particular students do not wish to be together in the delegation ?

(D) two particular students wish to be included together only ?

(E) two particular students refuse to be together and two other particular students wish to be together only in the delegation ?

Sol. **(A)** Formation of delegation means selection of 4 out of 12.

Hence the number of ways = ${}^{12}C_4 = 495$.

(B) If two particular students are already selected. Here we need to select only 2 out of the remaining 10. Hence the number of ways = ${}^{10}C_2 = 45$.

(C) The number of ways in which both are selected = 45. Hence the number of ways in which the two are not included together = 495 – 45 = 450

(D) There are two possible cases

(i) Either both are selected. In this case, the number of ways in which the selection can be made = 45.

(ii) Or both are not selected. In this case all the four students are selected from the remaining ten students. This can be done in ${}^{10}C_4 = 210$ ways.

Hence the total number of ways of selection = 45 + 210 = 255

(e) We assume that students A and B wish to be selected together and students C and D do not wish to be together. Now there are following 6 cases.

(i) (A, B, C) selected, (D) not selected

(ii) (A, B, D) selected, (C) not selected

(iii) (A, B) selected, (C, D) not selected

(iv) (C) selected, (A, B, D) not selected

(v) (D) selected, (A, B, C) not selected

(vi) A, B, C, D not selected

For **(i)** the number of ways of selection = $^8C_1 = 8$

For **(ii)** the number of ways of selection = $^8C_1 = 8$

For **(iii)** the number of ways of selection = $^8C_2 = 28$

For **(iv)** the number of ways of selection = $^8C_3 = 56$

For **(v)** the number of ways of selection = $^8C_3 = 56$

For **(vi)** the number of ways of selection = $^8C_4 = 70$

Hence total number of ways = 8 + 8 + 28 + 56 + 56 + 70 = 226.

Ex. 4 A box contains 5 different red and 6 different white balls. In how many ways can 6 balls be drawn so that there are atleast two balls of each color ?

Sol. The selections of 6 balls, consisting of atleast two balls of each color from 5 red and 6 white balls, can be made in the following ways

Red balls (5)	White balls (6)	Number of ways
2	3	$^5C_2 \times {}^6C_4 = 150$
3	3	$^5C_3 \times {}^6C_3 = 200$
4	2	$^5C_4 \times {}^6C_2 = 75$

Therefore total number of ways = 425

Ex. 5 How many functions can be defined from a set A containing 5 elements to a set B having 3 elements ? How many of these are surjective functions ?

Sol. Image of each element of A can be taken in 3 ways.

$\therefore$ Number of functions from A to B = $3^5 = 243$.

Number of into functions from A to B = $2^5 + 2^5 + 2^5 - 3 = 93$.

$\therefore$ Number of onto functions = 150.

Ex. 6 In how many ways the letters of the word "ARRANGE" can be arranged without altering the relative position of vowels & consonants.

Sol. The consonants in their positions can be arranged in $\frac{4!}{2!} = 12$ ways.

The vowels in their positions can be arranged in $\frac{3!}{2!} = 3$ ways

$\therefore$ Total number of arrangements = $12 \times 3 = 36$

Ex. 7 (A) How many permutations can be made by using all the letters of the word HINDUSTAN ?

(B) How many of these permutations begin and end with a vowel ?

(C) In how many of these permutations, all the vowels come together ?

(D) In how many of these permutations, none of the vowels come together ?

(E) In how many of these permutations, do the vowels and the consonants occupy the same relative positions as in HINDUSTAN ?

Sol.

(A) The total number of permutations = Arrangements of nine letters taken all at a time = $\frac{9!}{2!} = 181440$.

(B) We have 3 vowels and 6 consonants, in which 2 consonants are alike. The first place can be filled in 3 ways and the last in 2 ways. The rest of the places can be filled in $\frac{7!}{2!}$ ways.

Hence the total number of permutations = $3 \times 2 \times \frac{7!}{2!} = 15120$.

(C) Assume the vowels (I, U, A) as a single letter. The letters (IUA), H, D, S, T, N, N can be arranged in $\frac{7!}{2!}$ ways. Also IUA can be arranged among themselves in 3! = 6 ways.

Hence the total number of permutations = $\frac{7!}{2!} \times 6 = 15120$.

(D) Let us divide the task into two parts. In the first, we arrange the 6 consonants as shown below in $\frac{6!}{2!}$ ways.

$\times$ C $\times$ C $\times$ C $\times$ C $\times$ C $\times$ C $\times$ (Here C stands for a consonant and $\times$ stands for a gap between two consonants)
Now 3 vowels can be placed in 7 places (gaps between the consonants) in ${}^7C_3.3! = 210$ ways.

Hence the total number of permutations = $\frac{6!}{2!} \times 210 = 75600$.

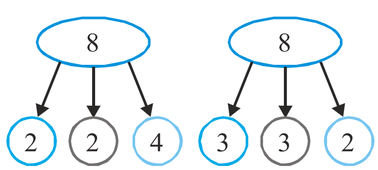
(E) In this case, the vowels can be arranged among themselves in 3! = 6 ways.

Also, the consonants can be arranged among themselves in $\frac{6!}{2!}$ ways.

Hence the total number of permutations = $\frac{6!}{2!} \times 6 = 2160$.

Ex. 8 In how many ways can 8 different books be distributed among 3 students if each receives at least 2 books ?

Sol. If each receives at least two books, then the division trees would be as shown below :



(i) (ii)

The number of ways of division for tree in figure (i) is $\left[\frac{8!}{(2!)^2 4!2!}\right]$.

The number of ways of division for tree in figure (ii) is $\left[\frac{8!}{(3!)^2 2!2!}\right]$.

The total number of ways of distribution of these groups among 3 students is $\left[\frac{8!}{(2!)^2 4!2!}+\frac{8!}{(3!)^2 2!2!}\right]\times 3!$

Ex. 9 In how many ways 10 persons can be divided into 5 pairs ?

Sol. We have each group having 2 persons and the qualitative characteristic are same (Since there is no purpose mentioned or names for each pair).

Thus the number of ways = $\frac{10!}{(2!)^5\, 5!}$ = 945.

Ex. 10 Find the number of all 6 digit numbers such that all the digits of each number are selected from the set {1,2,3,4,5} and any digit that appears in the number appears at least twice.

Sol.

Cases	No. of ways of selection	No. of ways of arrangements	Total
All alike	${}^5C_1$	${}^5C_1 \times 1$	5
4 alike + 2 other alike	${}^5C_2 \times 2!$	———	300
3 alike + 3 other alike	${}^5C_2$	———	200
2 alike + 2 other alike + 2 other alike	${}^5C_3$	———	900
		Total	1405

Ex. 11 A student is allowed to select at most n books from a collection of (2n + 1) books. If the total number of ways in which he can select books is 63, find the value of n.

Sol. Given student selects at most n books from a collection of (2n + 1) books. It means that he selects one book or two books or three books or .... or n books. Hence, by the given condition-

$${}^{2n+1}C_1 + {}^{2n+1}C_2 + {}^{2n+1}C_3 + \ldots\ldots\ldots + {}^{2n+1}C_n = 63 \qquad \ldots\ldots(i)$$

But we know that

$${}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + {}^{2n+1}C_3 + \ldots\ldots + {}^{2n+1}C_{2n+1} = 2^{2n+1} \qquad \ldots\ldots(ii)$$

Since ${}^{2n+1}C_0 = {}^{2n+1}C_{2n+1} = 1$, equation (ii) can also be written as

$$2 + ({}^{2n+1}C_1 + {}^{2n+1}C_2 + {}^{2n+1}C_3 + \ldots\ldots + {}^{2n+1}C_n) +$$
$$({}^{2n+1}C_{n+1} + {}^{2n+1}C_{n+2} + {}^{2n+1}C_{n+3} + \ldots\ldots + {}^{2n+1}C_{2n-1} + {}^{2n+1}C_{2n}) = 2^{2n+1}$$

$$\Rightarrow \quad 2 + ({}^{2n+1}C_1 + {}^{2n+1}C_2 + {}^{2n+1}C_3 + \ldots\ldots\ldots + {}^{2n+1}C_n) + ({}^{2n+1}C_n + {}^{2n+1}C_{n-1} + \ldots\ldots\ldots + {}^{2n+1}C_2 + {}^{2n+1}C_1) = 2^{2n+1}$$

$$(\because\ {}^{2n+1}C_r = {}^{2n+1}C_{2n+1-r})$$

$$\Rightarrow \quad 2 + 2\,({}^{2n+1}C_1 + {}^{2n+1}C_2 + {}^{2n+1}C_3 + \ldots\ldots + {}^{2n+1}C_n) = 2^{2n+1} \qquad \text{[from (i)]}$$

$\Rightarrow \quad 2 + 2.63 = 2^{2n+1} \qquad \Rightarrow \quad 1 + 63 = 2^{2n}$

$\Rightarrow \quad 64 = 2^{2n} \qquad \Rightarrow \quad 2^6 = 2^{2n} \qquad \therefore \quad 2n = 6$

Hence, n = 3.

Ex. 12 Find the number of ways in which 20 different pearls of two colours can be set alternately on a necklace, there being 10 pearls of each color.

Sol. Ten pearls of one color can be arranged in $\frac{1}{2}.(10-1)!$ ways. The number of arrangements of 10 pearls of the other color in 10 places between the pearls of the first color = 10!

$\therefore$ The required number of ways $=\frac{1}{2}\times 9!\times 10! = 5\,(9!)^2$

Ex. 13 Find the number of positive integral solutions of the inequation $x+y+z \geq 150$, where $0 < x \leq 60$, $0 < y \leq 60$, $0 < z \leq 60$.

Sol. Let $x = 60 - t_1$, $y = 60 - t_2$, $z = 60 - t_3$ (where $0 \leq t_1 \leq 59$, $0 \leq t_2 \leq 59$, $0 \leq t_3 \leq 59$)

Given $x + y + z \geq 150$

or $x + y + z - w = 150$ (where $0 \leq w \leq 147$) .....**(i)**

Putting values of x, y, z in equation **(i)**

$60 - t_1 + 60 - t_2 + 60 - t_3 - w = 150$

$30 = t_1 + t_2 + t_3 + w$

Total solutions = ${}^{33}C_3$

Ex. 14 Find sum of all numbers formed using the digits 2,4,6,8 taken all at a time and no digit being repeated.

Sol. All possible numbers = 4! = 24

If 2 occupies the unit's place then total numbers = 6

Hence, 2 comes at unit's place 6 times.

Sum of all the digits occuring at unit's place

$= 6 \times (2+4+6+8)$

Same summation will occur for ten's, hundred's & thousand's place. Hence required sum

$= 6 \times (2+4+6+8) \times (1+10+100+1000) = 133320$

Ex.15 Four slip of papers with the numbers 1, 2, 3, 4 written on them are put in a box. They are drawn one by one (without replacement) at random. In how many ways it can happen that the ordinal number of atleast one slip coincide with its own number ?

Sol. Total number of ways = 4 ! = 24.

The number of ways in which ordinal number of any slip does not coincide with its own number is the number of dearrangements of 4 objects $= 4!\left(\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right) = 9$

Thus the required number of ways. = 24 – 9 = 15

Ex. 16 Find the total number of proper factors of the number 35700. Also find

(i) sum of all these factors, **(ii)** sum of the odd proper divisors,

(iii) the number of proper divisors divisible by 10 and the sum of these divisors.

Sol. $35700 = 5^2 \times 2^2 \times 3^1 \times 7^1 \times 17^1$

The total number of factors is equal to the total number of selections from (5,5), (2,2), (3), (7) and (17), which is given by $3 \times 3 \times 2 \times 2 \times 2 = 72$.

These include 1 and 35700. Therefore, the number of proper divisors (excluding 1 and 35700) is $72 - 2 = 70$

(i) Sum of all these factors (proper) is :

$(5^0 + 5^1 + 5^2)(2^0 + 2^1 + 2^2)(3^0 + 3^1)(7^0 + 7^1)(17^0 + 17^1) - 1 - 35700$

$= 31 \times 7 \times 4 \times 8 \times 18 - 1 - 35700 = 89291$

(ii) The sum of odd proper divisors is :

$$(5^0 + 5^1 + 5^2)(3^0 + 3^1)(7^0 + 7^1)(17^0 + 17^1) - 1$$
$$= 31 \times 4 \times 8 \times 18 - 1 = 17856 - 1 = 17855$$

(iii) The number of proper divisors divisible by 10 is equal to number of selections from (5,5), (2,2), (3), (7), (17) consisting of at least one 5 and at least one 2 and 35700 is to be excluded and is given by $2 \times 2 \times 2 \times 2 \times 2 - 1 = 31$.

Sum of these divisors is :

$$(5^1 + 5^2)(2^1 + 2^2)(3^0 + 3^1)(7^0 + 7^1)(17^0 + 17^1) - 35700$$
$$= 30 \times 6 \times 4 \times 8 \times 18 - 35700 = 67980$$

Ex. 17 Find the number of solutions of the equation xyz = 360 when **(i)** $x,y,z \in N$ **(ii)** $x,y,z \in I$

Sol. (i) $xyz = 360 = 2^3 \times 3^2 \times 5$ $(x,y,z \in N)$

$x = 2^{a_1}3^{a_2}5^{a_3}$ (where $0 \le a_1 \le 3, 0 \le a_2 \le 2, 0 \le a_3 \le 1$)

$y = 2^{b_1}3^{b_2}5^{b_3}$ (where $0 \le b_1 \le 3, 0 \le b_2 \le 2, 0 \le b_3 \le 1$)

$z = 2^{c_1}3^{c_2}5^{c_3}$ (where $0 \le c_1 \le 3, 0 \le c_2 \le 2, 0 \le c_3 \le 1$)

$\Rightarrow$ $2^{a_1}3^{a_2}5^{a_3}.2^{b_1}3^{b_2}5^{b_3}.2^{c_1}3^{c_2}5^{c_3} = 2^3 \times 3^2 \times 5^1$

$\Rightarrow$ $2^{a_1+b_1+c_1}.3^{a_2+b_2+c_2}.5^{a_3+b_3+c_3} = 2^3 \times 3^3 \times 5^1$

$\Rightarrow$ $a_1 + b_1 + c_1 = 3 \rightarrow {}^5C_2 = 10$

$a_2 + b_2 + c_2 = 2 \rightarrow {}^4C_2 = 6$

$a_3 + b_3 + c_3 = 1 \rightarrow {}^3C_2 = 3$

Total solutions $= 10 \times 6 \times 3 = 180$.

(ii) If $x,y,z \in I$ then, **(A)** all positive **(B)** 1 positive and 2 negative.

Total number of ways $= 180 + {}^3C_2 \times 180 = 720$

Ex. 18 Find the number of positive integral solutions of $x + y + z = 20$, if $x \ne y \ne z$.

Sol. $x \ge 1$

$y = x + t_1 t_1 \ge 1$, $z = y + t_2 t_2 \ge 1$, $x + x + t_1 + x + t_1 + t_2 = 20$

$3x + 2t_1 + t_2 = 20$

(i) $x = 1$ $2t_1 + t_2 = 17$

$t_1 = 1,2 \,......... \, 8$ $\Rightarrow$ 8 ways

(ii) $x = 2$ $2t_1 + t_2 = 14$

$t_1 = 1,2 \,......... \, 6$ $\Rightarrow$ 6 ways

(iii) $x = 3$ $2t_1 + t_2 = 11$

$t_1 = 1,2 \,......... \, 5$ $\Rightarrow$ 5 ways

(vi) $x = 4$ $2t_1 + t_2 = 8$

$t_1 = 1,2,3$ $\Rightarrow$ 3 ways

(v) $x = 5$ $2t_1 + t_2 = 5$

$t_1 = 1, 2$ $\Rightarrow$ 2 ways

Total $= 8 + 6 + 5 + 3 + 2 = 24$

But each solution can be arranged by 3! ways.

So total solutions $= 24 \times 3! = 144$.

Exercise # 1

[Single Correct Choice Type Questions]

1. A 5 digit number divisible by 3 is to be formed using the numerals 0, 1, 2,3,4 & 5 without repetition. The total number of ways this can be done is -

(A) 3125 **(B)** 600 **(C)** 240 **(D)** 216

2. The total number of words which can be formed using all the letters of the word "AKSHI" if each word begins with vowel or terminates with vowel -

(A) 84 **(B)** 12 **(C)** 48 **(D)** 60

3. The number of signals that can be made with 3 flags each of different colour by hoisting 1 or 2 or 3 above the other, is:

(A) 3 **(B)** 7 **(C)** 15 **(D)** 16

4. Number of words that can be made with the letters of the word "GENIUS" if each word neither begins with G nor ends in S, is:

(A) 24 **(B)** 240 **(C)** 480 **(D)** 504

5. Number of ways in which 9 different prizes can be given to 5 students, if one particular student receives 4 prizes and the rest of the students can get any numbers of prizes is -

(A) ${}^9C_4 \cdot 2^{10}$ **(B)** ${}^9C_5 \cdot 5^4$ **(C)** $4 \cdot 4^5$ **(D)** none of these

6. Boxes numbered 1, 2, 3, 4 and 5 are kept in a row and they are necessarily to be filled with either a red or a blue ball such that no two adjacent boxes can be filled with blue balls. How many different arrangements are possible, given that the balls of a given colour are exactly identical in all respects ?

(A) 8 **(B)** 10 **(C)** 13 **(D)** 22

7. In a conference 10 speakers are present. If S_1 wants to speak before S_2 & S_2 wants to speak after S_3, then the number of ways all the 10 speakers can give their speeches with the above restriction if the remaining seven speakers have no objection to speak at any number is :

(A) ${}^{10}C_3$ **(B)** ${}^{10}P_8$ **(C)** ${}^{10}P_3$ **(D)** $\frac{10!}{3}$

8. If all the letters of the word "QUEUE" are arranged in all possible manner as they are in a dictionary, then the rank of the word QUEUE is -

(A) 15^{th} **(B)** 16^{th} **(C)** 17^{th} **(D)** 18^{th}

9. Number of ways in which 9 different toys can be distributed among 4 children belonging to different age groups in such a way that distribution among the 3 elder children is even and the youngest one is to receive one toy more is -

(A) $\frac{(5!)^2}{8}$ **(B)** $\frac{9!}{2}$ **(C)** $\frac{9!}{3!(2!)^3}$ **(D)** none of these

10. The number of ways of arranging the letters AAAAA, BBB, CCC, D, EE & F in a row if no two 'C's are together :

(A) ${}^{13}C_3 \cdot \frac{12!}{5!3!2!}$ **(B)** $\frac{13!}{5!3!3!2!}$ **(C)** $\frac{14!}{5!3!2!}$ **(D)** $11 \cdot \frac{13!}{6!}$

11. Passengers are to travel by a double decked bus which can accommodate 13 in the upper deck and 7 in the lower deck. The number of ways that they can be divided if 5 refuse to sit in the upper deck and 8 refuse to sit in the lower deck, is

(A) 25 **(B)** 21 **(C)** 18 **(D)** 15

12. How many nine digit numbers can be formed using the digits 2, 2, 3, 3, 5, 5, 8, 8, 8 so that the odd digits occupy even positions?

(A) 7560 **(B)** 180 **(C)** 16 **(D)** 60

13. A box contains 2 white balls, 3 black balls & 4 red balls. In how many ways can three balls be drawn from the box if atleast one black ball is to be included in draw (the balls of the same color are different).

(A) 60 **(B)** 64 **(C)** 56 **(D)** none

14. Number of ways in which 25 identical pens can be distributed among Keshav, Madhav, Mukund and Radhika such that at least 1, 2, 3 and 4 pens are given to Keshav, Madhav, Mukund and Radhika respectively, is -

(A) ${}^{18}C_4$ **(B)** ${}^{28}C_3$ **(C)** ${}^{24}C_3$ **(D)** ${}^{18}C_3$

15. The number of proper divisors of $a^p b^q c^r d^s$ where a, b, c, d are primes & p, q, r, s $\in$ N is -

(A) pqrs **(B)** $(p+1)(q+1)(r+1)(s+1)-4$

(C) pqrs – 2 **(D)** $(p+1)(q+1)(r+1)(s+1)-2$

16. The number of way in which 10 identical apples can be distributed among 6 children so that each child receives atleast one apple is -

(A) 126 **(B)** 252 **(C)** 378 **(D)** none of these

17. How many divisors of 21600 are divisible by 10 but not by 15?

(A) 10 **(B)** 30 **(C)** 40 **(D)** none

18. If chocolates of a particular brand are all identical then the number of ways in which we can choose 6 chocolates out of 8 different brands available in the market, is:.

(A) ${}^{13}C_6$ **(B)** ${}^{13}C_8$ **(C)** 8^6 **(D)** none

19. The sum of all numbers greater than 1000 formed by using the digits 1, 3, 5, 7 such that no digit is being repeated in any number is -

(A) 72215 **(B)** 83911 **(C)** 106656 **(D)** 114712

20. A set contains (2n + 1) elements. The number of subset of the set which contain at most n elements is : -

(A) 2^n **(B)** 2^{n+1} **(C)** 2^{n-1} **(D)** 2^{2n}

21. The number of ways in which we can arrange n ladies & n gentlemen at a round table so that 2 ladies or 2 gentlemen may not sit next to one another is -

(A) $(n-1)!\,(n-2)!$ **(B)** $(n)!\,(n-!)!$ **(C)** $(n+1)!\,(n)!$ **(D)** none of these

22. If $\alpha = {}^mC_2$, then ${}^{\alpha}C_2$ is equal to

(A) ${}^{m+1}C_4$ **(B)** ${}^{m-1}C_4$ **(C)** $3\ {}^{m+2}C_4$ **(D)** $3\ {}^{m+1}C_4$

23. The number of ways in which the number 27720 can be split into two factors which are co-primes, is:

(A) 15 **(B)** 16 **(C)** 25 **(D)** 49

24. Number of numbers greater than a million and divisible by 5 which can be formed by using only the digits 1, 2, 1, 2, 0, 5 & 2 is -

(A) 120 **(B)** 110 **(C)** 90 **(D)** none of these

25. The number of ways in which 6 red roses and 3 white roses (all roses different) can form a garland so that all the white roses come together, is

(A) 2170 **(B)** 2165 **(C)** 2160 **(D)** 2155

26. Ten different letters of alphabet are given. Words with four letters are formed from these letters, then the number of words which have at least one letter repeated is -

(A) 10^4 **(B)** ${}^{10}P_4$ **(C)** ${}^{10}C_4$ **(D)** 4960

27. Number of positive integral solutions of $x_1 . x_2 . x_3 = 30$, is

(A) 25 **(B)** 26 **(C)** 27 **(D)** 28

28. The number of ways in which 5 different books can be distributed among 10 people if each person can get at most one book is -

(A) 252 **(B)** 10^5 **(C)** 5^{10} **(D)** ${}^{10}C_5 . 5!$

29. A shopkeeper has 10 copies of each of nine different books, then number of ways in which atleast one book can be selected is

(A) $9^{11} - 1$ **(B)** $10^{10} - 1$ **(C)** $11^9 - 1$ **(D)** 10^9

30. Out of seven consonants and four vowels, the number of words of six letters, formed by taking four consonants and two vowels is (Assume that each ordered group of letter is a word) -

(A) 210 **(B)** 462 **(C)** 151200 **(D)** 332640

Exercise # 2 Part # I [Multiple Correct Choice Type Questions]

1. $N = 2^2 . 3^3 . 5^4 . 7$, then -

(A) Number of proper divisors of N(excluding 1 & N) is 118

(B) Number of proper divisors of N(excluding 1 & N) is 120

(C) Number of positive integral solutions of xy = N is 60

(D) Number of positive integral solutions of xy = N is 120

2. There are (p + q) different books on different topics in Mathematics. ($p \neq q$)

If L = the number of ways in which these books are distributed between two students X and Y such that X get p books and Y gets q books.

M = The number of ways in which these books are distributed between two students X and Y such that one of them gets p books and another gets q books.

N = The number of ways in which these books are divided into two groups of p books and q books then -

(A) L = N **(B)** L = 2M = 2N **(C)** 2L = M **(D)** L = M

3. A student has to answer 10 out of 13 questions in an examination. The number of ways in which he can answer if he must answer atleast 3 of the first five questions is:

(A) 276 **(B)** 267 **(C)** ${}^{13}C_{10} - {}^{5}C_{3}$ **(D)** ${}^{5}C_{3} . {}^{8}C_{7} + {}^{5}C_{4} . {}^{8}C_{6} + {}^{8}C_{5}$

4. The number of ways in which 10 students can be divided into three teams, one containing 4 and others 3 each, is

(A) $\frac{10!}{4!3!3!}$ **(B)** 2100 **(C)** ${}^{10}C_{4} . {}^{5}C_{3}$ **(D)** $\frac{10!}{6!3!3!} . \frac{1}{2}$

5. Number of dissimilar terms in the expansion of $(x_1 + x_2 + ... + x_n)^3$ is -

(A) $\frac{n^2(n+1)^2}{4}$ **(B)** $\frac{n(n+1)(n+2)}{6}$ **(C)** ${}^{n+1}C_2 + {}^{n+1}C_3$ **(D)** $\frac{n^3+3n^2}{4}$

6. The number of ways in which 200 different things can be divided into groups of 100 pairs, is:

(A) $\frac{200!}{2^{100}}$ **(B)** $\binom{101}{2}\binom{102}{2}\binom{103}{2}....\binom{200}{2}$

(C) $\frac{200!}{2^{100}(100)!}$ **(D)** (1. 3. 5...... 199)

7. ${}^{50}C_{36}$ is divisible by

(A) 19 **(B)** 5^2 **(C)** 19^2 **(D)** 5^3

8. The number of five digit numbers that can be formed using all the digits 0, 1, 3, 6, 8 which are -

(A) divisible by 4 is 30

(B) greater than 30,000 and divisible by 11 is 12

(C) smaller than 60,000 when digit 8 always appears at ten's place is 6

(D) between 30,000 and 60,000 and divisible by 6 is 18.

9. The number of non-negative integral solutions of $x_1 + x_2 + x_3 + x_4 \leq n$ (where n is a positive integer) is

(A) ${}^{n+3}C_3$ **(B)** ${}^{n+4}C_4$ **(C)** ${}^{n+5}C_5$ **(D)** ${}^{n+4}C_n$

10. Which of the following statement(s) is/are true :-

(A) ${}^{100}C_{50}$ is not divisible by 10

(B) $n(n-1)(n-2)$ ....$(n-r+1)$ is always divisible by $r!$ ($n \in N$ and $0 \le r \le n$)

(C) Morse telegraph has 5 arms and each arm moves on 6 different positions including the position of rest. Number of different signals that can be transmitted is $5^6 - 1$.

(D) There are 5 different books each having 5 copies. Number of different selections is $6^5 - 1$.

11. The number of ways of arranging the letters AAAAA, BBB, CCC, D, EE & F in a row if the letter C are separated from one another is:

(A) ${}^{13}C_3 \cdot \dfrac{12!}{5!\,3!\,2!}$ **(B)** $\dfrac{13\,!}{5\,!\;3\,!\;3\,!\;2\,!}$ **(C)** $\dfrac{14\,!}{3\,!\;3\,!\;2\,!}$ **(D)** $11.\dfrac{13\,!}{6\,!}$

12. Number of ways in which 3 numbers in A.P. can be selected from 1, 2, 3,...... n is:

(A) $\dfrac{(n-2)(n-4)}{4}$ if n is even **(B)** $\dfrac{n^2-4n+5}{2}$ if n is odd

(C) $\dfrac{(n-1)^2}{4}$ if n is odd **(D)** $\dfrac{n(n-2)}{4}$ if n is even

13. All the 7 digit numbers containing each of the digits 1, 2, 3, 4, 5, 6, 7 exactly once and not divisible by 5 are arranged in the increasing order. Then -

(A) 1800^{th} number in the list is 3124567 **(B)** 1897^{th} number in the list is 4213567

(C) 1994^{th} number in the list is 4312567 **(D)** 2001^{th} number in the list is 4315726

14. The kindergarten teacher has 25 kids in her class. She takes 5 of them at a time, to zoological garden as often as she can, without taking the same 5 kids more than once. Then the number of visits, the teacher makes to the garden exceeds that of a kid by:

(A) ${}^{25}C_5 - {}^{24}C_4$ **(B)** ${}^{24}C_5$ **(C)** ${}^{25}C_5 - {}^{24}C_5$ **(D)** ${}^{24}C_4$

15. A persons wants to invite one or more of his friend for a dinner party. In how many ways can he do so if he has eight friends : -

(A) 2^8 **(B)** $2^8 - 1$ **(C)** 8^2 **(D)** ${}^8C_1 + {}^8C_2 + ...+ {}^8C_8$

16. You are given 8 balls of different color (black, white,...). The number of ways in which these balls can be arranged in a row so that the two balls of particular colour (say red & white) may never come together is:

(A) $8! - 2.7!$ **(B)** $6.\,7!$ **(C)** $2.\,6!.\,{}^7C_2$ **(D)** none

17. In an examination, a candidate is required to pass in all the four subjects he is studying. The number of ways in which he can fail is

(A) ${}^4P_1 + {}^4P_2 + {}^4P_3 + {}^4P_4$ **(B)** $4^4 - 1$ **(C)** $2^4 - 1$ **(D)** ${}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4$

18. If P(n, n) denotes the number of permutations of n different things taken all at a time then P(n, n) is also identical to

(A) $n.P(n-1, n-1)$ **(B)** $P(n, n-1)$ **(C)** $r!\,.\,P(n, n-r)$ **(D)** $(n-r)\,.\,P(n, r)$

(where $0 \le r \le n$)

19. There are 12 points in a plane of which 5 are collinear. The number of distinct quadrilaterals which can be formed with vertices at these points is:

(A) $2 \cdot {}^7P_3$ **(B)** ${}^7P_3$ **(C)** $10 \cdot {}^7C_3$ **(D)** 420

20. There are 10 seats in the first row of a theatre of which 4 are to be occupied. The number of ways of arranging 4 persons so that no two persons sit side by side is:

(A) ${}^7C_4$ **(B)** $4.\,{}^7P_3$ **(C)** ${}^7C_3.\,4!$ **(D)** 840

Part # II [Assertion & Reason Type Questions]

These questions contain, Statement I (assertion) and Statement II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.

(C) Statement-I is true, Statement-II is false.

(D) Statement-I is false, Statement-II is true.

1. **Statement-I :** If a, b, c are positive integers such that $a + b + c \le 8$, then the number of possible values of the ordered triplets (a, b, c) is 56.

Statement-II : The number of ways in which n distinct things can be distributed among r girls such that each get at least one is ${}^{n-1}C_{r-1}$.

2. **Statement-I :** Number of terms in the expansion of $(x_1 + x_2 + x_3 + ... + x_{11})^6 = {}^{16}C_6$.

Statement-II : Number of ways of distributing n identical things among r persons when each person get zero or more things = ${}^{n+r-1}C_n$

3. **Statement -I :** The maximum number of points of intersection of 8 unequal circles is 56.

Statement -II : The maximum number of points into which 4 unequal circles and 4 non coincident straight lines intersect, is 50.

4. **Statement-I :** Number of ways in which 400 different things can be distributed between Ramu & Shamu so that each receives 200 things > Number of ways in which 400 different things can be distributed between Sita & Geeta. So that Sita receives 238 things & Geeta receives 162 things.

Statement-II : Number of ways in which (m + n) different things can be distributed between two receivers such that one receives m and other receives n is equal to ${}^{m+n}C_m$, for any two non-negative integers m and n.

5. **Statement-I :** The number of positive integral solutions of the equation xyzw = 770 is 2^8.

Statement-II : The number of ways of selection of atleast one thing from n things of which 'p' are alike of one kind, q are alike of 2nd kind and rest of the things are different is $(p + 1)(q + 1)\, 2^{n-(p+q)} - 1$.

6. **Statement-I :** If a polygon has 45 diagonals, then its number of sides is 10.

Statement-II : Number of ways of selecting 2 points from n non collinear points is ${}^nC_2$.

7. **Statement -I :** If there are six letters $L_1, L_2, L_3, L_4, L_5, L_6$ and their corresponding six envelopes $E_1, E_2, E_3, E_4, E_5, E_6$. Letters having odd value can be put into odd value envelopes and even value letters can be put into even value envelopes, so that no letter go into the right envelopes, the number of arrangement will be equal to 4.

Statement -II : If P_n number of ways in which n letter can be put in 'n' corresponding envelopes such that no letter goes to correct envelope, then $P_n = n!\left(1 - \frac{1}{1!} + \frac{1}{2!} - .... + \frac{(-1)^n}{n!}\right)$

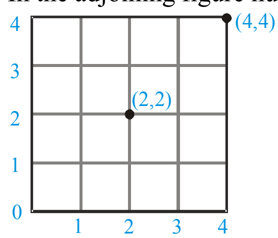
Exercise # 3 Part # I [Matrix Match Type Questions]

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with **one or more** statement(s) in **Column-II**.

1. **Column-I** | **Column-II**

(A) ${}^{24}C_2 + {}^{23}C_2 + {}^{22}C_2 + {}^{21}C_2 + {}^{20}C_2 + {}^{20}C_3$ is equal to | **(p)** 102

(B) In the adjoining figure number of progressive



ways to reach from (0,0) to (4, 4) passing through point (2, 2) are (particle can move on horizontal or vertical line) | **(q)** 2300

(C) The number of 4 digit numbers that can be made with the digits 1, 2, 3, 4, 3, 2 | **(r)** 82

(D) If $\left\{\dfrac{500!}{14^k}\right\} = 0$, then the maximum natural value of k is equal to | **(s)** 36

(where {.} is fractional part function)

2. **Column – I** | **Column – II**

(A) The total number of selections of fruits which can be made from, 3 bananas, 4 apples and 2 oranges is, it is given that fruits of one kind are identical | **(p)** Greater than 50

(B) If 7 points out of 12 are in the same straight line, then the number of triangles formed is | **(q)** Greater than 100

(C) The number of ways of selecting 10 balls from unlimited number of red, black, white and green balls is, it is given that balls of same colours are identical | **(r)** Greater than 150

(D) The total number of proper divisors of 38808 is | **(s)** Greater than 200

3. 5 balls are to be placed in 3 boxes. Each box can hold all the 5 balls. Number of ways in which the balls can be placed so that no box remains empty, if :

	Column-I		**Column-II**
(A)	balls are identical but boxes are different	**(p)**	2
(B)	balls are different but boxes are identical	**(q)**	25
(C)	balls as well as boxes are identical	**(r)**	50
(D)	balls as well as boxes are identical but boxes are kept in a row	**(s)**	6

4. Consider the word "HONOLULU".

	Column – I		Column – II
(A)	Number of words that can be formed using the letters of the given word in which consonants & vowels are alternate is	**(p)**	26
(B)	Number of words that can be formed without changing the order of vowels is	**(q)**	144
(C)	Number of ways in which 4 letters can be selected from the letters of the given word is	**(r)**	840
(D)	Number of words in which two O's are together but U's are separated is	**(s)**	900

5. Consider all the different words that can be formed using the letters of the word HAVANA, taken 4 at a time.

	Column-I		Column-II
(A)	Number of such words in which all the 4 letters are different	**(p)**	36
(B)	Number of such words in which there are 2 alike letters & 2 different letters.	**(q)**	42
(C)	Number of such words in which A's never appear together	**(r)**	37
(D)	If all such 4 letters words are written, by the rule of dictionary then the rank of the word HANA	**(s)**	24

Part # II [Comprehension Type Questions]

Comprehension # 1

Let p be a prime number and n be a positive integer, then exponent of p is n! is denoted by $E_p(n!)$ and is given by

$$E_p(n!) = \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \ldots\ldots + \left[\frac{n}{p^k}\right]$$

where $p^k < n < p^{k+1}$

and [x] denotes the integral part of x.

If we isolate the power of each prime contained in any number N, then N can be written as

$$N = 2^{\alpha_1} \cdot 3^{\alpha_2} \cdot 5^{\alpha_3} \cdot 7^{\alpha_4} \;\ldots.$$

where α_i are whole numbers.

1. The exponent of 7 in ${}^{100}C_{50}$ is -

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3

2. The number of zeros at the end of 108! is -

(A) 10 **(B)** 13 **(C)** 25 **(D)** 26

3. The exponent of 12 in 100! is -

(A) 32 **(B)** 48 **(C)** 97 **(D)** none of these

Comprehension # 2

There are 8 official and 4 non-official members, out of these 12 members a committee of 5 members is to be formed, then answer the following questions.

1. Number of committees consisting of 3 official and 2 non-official members, are
(A) 363 **(B)** 336 **(C)** 236 **(D)** 326

2. Number of committees consisting of at least two non-official members, are
(A) 456 **(B)** 546 **(C)** 654 **(D)** 466

3. Number of committees in which a particular official member is never included, are
(A) 264 **(B)** 642 **(C)** 266 **(D)** 462

Comprehension # 3

S = {0, 2, 4, 6, 8}. A natural number is said to be divisible by 2 if the digit at the unit place is an even number. The number is divisible by 5, if the number at the unit place is 0 or 5. If four numbers are selected from S and a four digit number ABCD is formed.

1. The number of such numbers which are even (all digits are different) is
(A) 60 **(B)** 96 **(C)** 120 **(D)** 204

2. The number of such numbers which are even (all digits are not different) is
(A) 404 **(B)** 500 **(C)** 380 **(D)** none of these

3. The number of such numbers which are divisible by two and five (all digits are not different) is
(A) 125 **(B)** 76 **(C)** 65 **(D)** 100

Comprehension # 4

Consider the letters of the word MATHEMATICS.

1. Possible number of words taking all letters at a time such that at least one repeating letter is at odd position in each word is
(A) $\frac{11!}{2!2!2!}-\frac{9!}{2!2!}$ **(B)** $\frac{9!}{2!2!2!}$ **(C)** $\frac{9!}{2!2!}$ **(D)** $\frac{11!}{2!2!2!}$

2. Possible number of words taking all letters at a time such that in each word both M's are together and both T's are together but both A's are not together is
(A) $\frac{11!}{2!2!2!}-\frac{10!}{2!2!}$ **(B)** $7!\,{}^8C_2$ **(C)** $\frac{6!4!}{2!2!}$ **(D)** $\frac{9!}{2!2!2!}$

3. Possible number of words in which no two vowels are together is
(A) $7!\,{}^8C_4\frac{4!}{2!}$ **(B)** $\frac{7!}{2!}\,{}^8C_4\frac{4!}{2!}$ **(C)** $\frac{7!}{2!2!}\,{}^8C_4\frac{4!}{2!}$ **(D)** $\frac{7!}{2!2!2!}\,{}^8C_4\frac{4!}{2!}$

Comprehension # 5

We have to choose 11 players for cricket team from 8 batsmen. 6 bowlers, 4 allrounders and 2 wicketkeeper, in the following conditions.

1. The number of selections when at most 1 allrounder and 1 wicketkeeper will play -

(A) ${}^4C_1 . {}^{14}C_{10} + {}^2C_1 . {}^{14}C_{10} + {}^4C_1 . {}^2C_1 . {}^{14}C_9 + {}^{14}C_{11}$ **(B)** ${}^4C_1 . {}^{15}C_{11} + {}^{15}C_{11}$

(C) ${}^4C_1 . {}^{15}C_{10} + {}^{15}C_{11}$ **(D)** none of these

2. Number of selections when 2 particular batsmen don't want to play, if a particular bowler will play -

(A) ${}^{17}C_{10} + {}^{19}C_{11}$ **(B)** ${}^{17}C_{10} + {}^{19}C_{11} + {}^{17}C_{11}$

(C) ${}^{17}C_{10} + {}^{20}C_{11}$ **(D)** ${}^{19}C_{10} + {}^{19}C_{11}$

3. Number of selections when a particular batsman and a particular wicketkeeper don't want to play together -

(A) $2\ {}^{18}C_{10}$ **(B)** ${}^{19}C_{11} + {}^{18}C_{10}$ **(C)** ${}^{19}C_{10} + {}^{19}C_{11}$ **(D)** none of these

Exercise # 4 [Subjective Type Questions]

1. In how many ways can a team of 6 horses be selected out of a stud of 16, so that there shall always be 3 out of ABC A'B'C', but never AA', BB' or CC' together ?

2. How many different permutations are possible using all the letters of the word MISSISSIPPI, if no two I's are together ?

3. There are n straight lines in a plane, no 2 of which are parallel & no 3 pass through the same point. Their points of intersection are joined. Show that the number of fresh lines introduced is $\frac{n(n-1)(n-2)(n-3)}{8}$.

4. A family consists of a grandfather, m sons and daughters and 2n grand children. They are to be seated in a row for dinner. The grand children wish to occupy the n seats at each end and the grandfather refuses to have a grand children on either side of him. In how many ways can the family be made to sit.

5. There are 2 women participating in a chess tournament. Every participant played 2 games with the other participants. The number of games that the men played between themselves exceeded by 66 as compared to the number of games that the men played with the women. Find the number of participants & the total number of games played in the tournament.

6. Prove that : $\frac{200!}{(10!)^{20}\,19!}$ is an integer

7. A party of 10 consists of 2 Americans, 2 Britishmen, 2 Chinese & 4 men of other nationalities (all different). Find the number of ways in which they can stand in a row so that no two men of the same nationality are next to one another. Find also the number of ways in which they can sit at a round table.

8. Find the number of words those can be formed by using all letters of the word 'DAUGHTER'. If

(i) Vowels occurs in first and last place.

(ii) Start with letter G and end with letters H.

(iii) Letters G,H,T always occurs together.

(iv) No two letters of G,H,T are consecutive

(v) No vowel occurs together

(vi) Vowels always occupy even place.

(vii) Order of vowels remains same.

(viii) Relative order of vowels and consonants remains same.

(ix) Number of words are possible by selecting 2 vowels and 3 consonants.

9. How many different ways can 15 candy bars be distributed between Ram, Shyam, Ghanshyam and Balram, if Ram can not have more than 5 candy bars and Shyam must have at least two ? Assume all candy bars to be alike.

10. In how many other ways can the letters of the word MULTIPLE be arranged ;

(i) without changing the order of the vowels ?

(ii) keeping the position of each vowel fixed ?

(iii) without changing the relative order/position of vowels & consonants ?

11. Find the number of ways to invite one of the three friends for dinner on 6 successive nights such that no friend is invited more than 3 times.

12. Let N = 24500, then find

(i) The number of ways by which N can be resolved into two factors.

(ii) The number of ways by which 5N can be resolved into two factors.

(iii) The number of ways by which N can be resolved into two coprime factors.

13. A man has 7 relatives, 4 of them are ladies & 3 gentlemen; his wife has also 7 relatives, 3 of them are ladies & 4 gentlemen. In how many ways can they invite a dinner party of 3 ladies & 3 gentlemen so that there are 3 of the man's relatives & 3 of the wife's relatives ?

14. Words are formed by arranging the letters of the word "STRANGE" in all possible manner. Let m be the number of words in which vowels do not come together and 'n' be the number of words in which vowels come together. Then find the ratio of m: n.

15. A firm of Chartered Accountants in Bombay has to send 10 clerks to 5 different companies, two clerks in each. Two of the companies are in Bombay and the others are outside. Two of the clerks prefer to work in Bombay while three others prefer to work outside. In how many ways can the assignment be made if the preferences are to be satisfied ?

16. One hundred management students who read at least one of the three business magazines are surveyed to study the readership pattern. It is found that 80 read Business India, 50 read Business world, and 30 read Business Today. Five students read all the three magazines. Find how many read exactly two magazines?

17. **(A)** Prove that : ${}^{n}P_r = {}^{n-1}P_r + r.\ {}^{n-1}P_{r-1}$

(B) If ${}^{20}C_{r+2} = {}^{20}C_{2r-3}$ find ${}^{12}C_r$

(C) Find r if ${}^{15}C_{3r} = {}^{15}C_{r+3}$

(D) Find the ratio ${}^{20}C_r$ to ${}^{25}C_r$ when each of them has the greatest value possible.

18. Find number of ways in which five vowels of English alphabets and ten decimal digits can be placed in a row such that between any two vowels odd number of digits are placed and both end places are occupied by vowels

19. Prove by combinatorial argument that :

(A) ${}^{n+1}C_r = {}^{n}C_r + {}^{n}C_{r-1}$

(B) ${}^{n+m}C_r = {}^{n}C_0 \,.\, {}^{m}C_r + {}^{n}C_1 \,.\, {}^{m}C_{r-1} + {}^{n}C_2 \,.\, {}^{m}C_{r-2} + \ldots.. + {}^{n}C_r \,.\, {}^{m}C_0$

20. A number lock has 4 dials, each dial has the digits 0, 1, 2, ..., 9. What is the maximum unsuccessful attempts to open the lock ?

21. 5 boys & 4 girls sit in a straight line. Find the number of ways in which they can be seated if 2 girls are together & the other 2 are also together but separated from the first 2.

22. $X = \{1, 2, 3, 4, \ldots\ldots n\}$ and $A \subset X$; $B \subset X$; $A \cup B \subset X$ here $P \subset Q$ denotes that P is subset of $Q (P \neq Q)$.Find number of ways of selecting unordered pair of sets A and B such that $A \cup B \subset X$.

23. In how many ways 11 players can be selected from 15 players, if only 6 of these players can bowl and the 11 players must include atleast 4 bowlers ?

24. There are 20 books on Algebra & Calculus in our library. Prove that the greatest number of selections each of which consists of 5 books on each topic is possible only when there are 10 books on each topic in the library.

25. Find number of divisiors of 1980.

(i) How many of them are multiple of 11 ? find their sum

(ii) How many of them are divisible by 4 but not by 15.

Exercise # 5

Part # I [Previous Year Questions] [AIEEE/JEE-MAIN]

1. Numbers greater than 1000 but not greater than 4000 which can be formed with the digits 0, 1, 2, 3, 4 (repetition of digits is allowed), are **[AIEEE 2002]**

(A) 350 **(B)** 375 **(C)** 450 **(D)** 576

2. A five digit number divisible by 3 has to formed using the numerals 0, 1, 2, 3, 4 and 5 without repetition. The total number of ways in which this can be done is **[AIEEE 2002]**

(A) 216 **(B)** 240 **(C)** 600 **(D)** 3125

3. Total number of four digit odd numbers that can be formed using 0, 1, 2, 3, 5, 7 are **[AIEEE 2002]**

(A) 192 **(B)** 375 **(C)** 400 **(D)** 720

4. The number of ways in which 6 men and 5 women can dine at a round table if no two women are to sit together is given by **[AIEEE 2003]**

(A) $6! \times 5!$ **(B)** 30 **(C)** $5! \times 4!$ **(D)** $7! \times 5!$

5. A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five question. The number of choices available to him is **[AIEEE 2003]**

(A) 140 **(B)** 196 **(C)** 280 **(D)** 346

6. If ${}^{n}C_{r}$ denotes the number of combinations of n things taken r at a time, then the expression

${}^{n}C_{r+1} + {}^{n}C_{r-1} + 2 \times {}^{n}C_{r}$ equals **[AIEEE 2003]**

(A) ${}^{n+2}C_{r}$ **(B)** ${}^{n+2}C_{r+1}$ **(C)** ${}^{n+1}C_{r}$ **(D)** ${}^{n+1}C_{r+1}$

7. How many ways are there to arrange the letters in the word 'GARDEN' with the vowels in alphabetical order ? **[AIEEE 2004]**

(A) 120 **(B)** 240 **(C)** 360 **(D)** 480

8. The number of ways of distributing 8 identical balls in 3 distinct boxes so that none of the boxes is empty is **[AIEEE 2004]**

(A) 5 **(B)** 21 **(C)** 3^8 **(C)** ${}^{8}C_{3}$

9. If the letters of the word 'SACHIN' are arranged in all possible ways and these words are written out as in dictionary, then the word 'SACHIN' appears at serial number **[AIEEE 2005]**

(A) 602 **(B)** 603 **(C)** 600 **(D)** 601

10. The value of ${}^{50}C_{4} + \sum_{r=1}^{6} {}^{56-r}C_{3}$ is **[AIEEE 2005]**

(A) ${}^{56}C_{4}$ **(B)** ${}^{56}C_{3}$ **(C)** ${}^{55}C_{3}$ **(D)** ${}^{55}C_{4}$

11. At an election, a voter may vote for any number of candidates, not greater than the number to be elected. There are 10 candidated and 4 are to be elected. If a voter votes for at least one candidates, then the number of ways in which he can vote is **[AIEEE-2006]**

(A) 385 **(B)** 1110 **(C)** 5040 **(D)** 6210

12. The set S = {1, 2, 3,.....,12} is to partitioned into three sets A, B, C of equal size. Thus, $A \cup B \cup C = S$, $A \cap B = B \cap C = C \cap A = \phi$, then number of ways to partition S are- **[AIEEE-2007]**

(A) $\frac{12!}{3!(3!)^4}$ **(B)** $\frac{12!}{(4!)^3}$ **(C)** $\frac{12!}{(3!)^4}$ **(D)** $\frac{12!}{3!(4!)^3}$

13. In a shop there are five types of ice–creams available. A child buys six ice–creams. **[AIEEE 2008]**

Statement –I : The number of different ways the child can buy the six ice–creams is ${}^{10}C_5$.

Statement –2 : The number of different ways the child can buy the six ice–creams is equal to the number of different ways of arranging 6 A's and 4 B's in a row.

(A) Statement –1 is false, Statement –2 is true

(B) Statement–1 is true, Statement–2 is true; Statement–2 is a correct explanation for Statement–1

(C) Statement–1 is true, Statement–2 is true; Statement–2 is not a correct explanation for Statement–1

(D) Statement–1 is true, Statement–2 is false

14. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. Then the number of such arrangements is :- **[AIEEE 2009]**

(A) At least 750 but less than 1000 **(B)** At least 1000
(C) Less than 500 **(D)** At least 500 but less than 750

15. There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is :- **[AIEEE-2010]**

(A) 3 **(B)** 36 **(C)** 66 **(D)** 108

16. **Statement - I :** The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^9C_3$. **[AIEEE-2011]**

Statement - II : The number of ways of choosing any 3 places from 9 different places is ${}^9C_3$.

(A) Statement-1 is true, Statement-2 is false.
(B) Statement-1 is false, Statement-2 is true
(C) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(D) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

17. There are 10 points in a plane, out of these 6 are collinear. If N is the number of triangles formed by joining these points, then : **[AIEEE-2011]**

(A) $N > 190$ **(B)** $N \le 100$ **(C)** $100 < N \le 140$ **(D)** $140 < N \le 190$

18. Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is :- **[AIEEE-2012]**

(A) 879 **(B)** 880 **(C)** 629 **(D)** 630

19. Let A and B be two sets containing 2 elements and 4 elements respectively. The number of subsets of $A \times B$ having 3 or more elements is **[JEE (Main)-2013]**

(A) 256 **(B)** 220 **(C)** 219 **(D)** 211

20. Let T_n be the number of all possible triangles formed by joining vertices of an n-sided regular polygon. If $T_{n+1} - T_n = 10$, then the value of n is : **[JEE (Main)-2013]**

(A) 7 **(B)** 5 **(C)** 10 **(D)** 8

21. The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices (0, 0), (0, 41) and (41, 0), is **[JEE (Main)-2015]**

(A) 820 **(B)** 780 **(C)** 901 **(D)** 861

22. The number of integers greater than 6,000 that can be formed, using the digits 3, 5, 6, 7 and 8, without repetition, is :

(A) 120 **(B)** 72 **(C)** 216 **(D)** 192 **[JEE (Main)-2015]**

23. Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set $A \times B$, each having at least three elements is : **[JEE (Main)-2015]**

(A) 275 **(B)** 510 **(C)** 219 **(D)** 256

24. Let two fair six-faced dice A and B be thrown simultaneously. If E_1 is the event that die A shows up four, E_2 is the event that die B shows up two and E_3 is the event that the sum of numbers on both dice is odd, then which of the following statements is NOT true ? **[JEE (Main)-2016]**

(A) E_2 and E_3 are independent **(B)** E_1 and E_3 are independent
(C) E_1, E_2 and E_3 are independent **(D)** E_1 and E_2 are independent

Part # II [Previous Year Questions][IIT-JEE ADVANCED]

1. How many different nine digit numbers can be formed from the number 223355888 by rearranging its digits so that the odd digits occupy even positions ? **[JEE 2000]**

(A) 16 **(B)** 36 **(C)** 60 **(D)** 180

2. (A) Let T_n denote the number of triangles which can be formed using the vertices of a regular polygon of 'n' sides. If $T_{n+1} - T_n = 21$, then 'n' equals - **[JEE 2001]**

(A) 5 **(B)** 7 **(C)** 6 **(D)** 4

(B) Let E = {1, 2, 3, 4} and F = {1, 2}. Then the number of onto functions from E to F is -

(A) 14 **(B)** 16 **(C)** 12 **(D)** 8

3. The number of arrangements of the letters of the word BANANA in which two 'N's do not appear adjacently is - **[JEE 2002]**

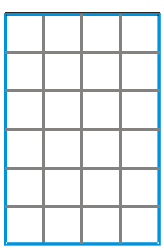
(A) 40 **(B)** 60 **(C)** 80 **(D)** 100

4. Number of points with integral co-ordinates that lie inside a triangle whose co-ordinates are (0, 0), (0, 21) and (21, 0) **[JEE 2003]**

(A) 210 **(B)** 190 **(C)** 220 **(D)** none of these

5. Using permutation or otherwise prove that $\dfrac{(n^2)!}{(n!)^n}$ is an integer, where n is a positive integer. **[JEE 2004]**

6. A rectangle with sides 2m – 1 and 2n – 1 is divided into squares of unit length by drawing



parallel lines as shown then number of rectangles possible with odd side lengths is - **[JEE 2005]**

(A) $(m+n+1)^2$ **(B)** 4^{m+n-1} **(C)** $m^2 n^2$ **(D)** $mn(m+1)(n+1)$

7. If r, s, t are the prime numbers and p, q are the positive integers such that the LCM of p & q is $r^2t^4s^2$, then the number of ordered pair (p, q) is : **[JEE 2006]**

(A) 252 **(B)** 254 **(C)** 225 **(D)** 224

8. The letters of the word COCHIN are permuted and all the permutations are arranged in an alphabetical order as in an English dictionary. The number of words that appear before the word COCHIN is - **[JEE 2007]**

(A) 360 **(B)** 192 **(C)** 96 **(D)** 48

9. Consider all possible permutations of the letters of the word ENDEANOEL. **[JEE 2008]**

Match the Statements / Expressions in Column I with the Statements / Expressions in Column II.

	Column I		Column II
(A)	The number of permutations containing the word ENDEA is	**(p)**	5!
(B)	The number of permutations in which the letter E occurs in the first and the last positions is	**(q)**	$2 \times 5!$
(C)	The number of permutations in which none of the letters D, L, N occurs in the last five positions is	**(r)**	$7 \times 5!$
(D)	The number of permutations in which the letters A, E, O occur only in odd positions is	**(s)**	$21 \times 5!$

10. The number of seven digit integers, with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only is - **[JEE 2009]**

(A) 55 **(B)** 66 **(C)** 77 **(D)** 88

11. Let S = {1,2,3,4}. The total number of unordered pairs of disjoint subsets of S is equal to - **[JEE 2010]**

(A) 25 **(B)** 34 **(C)** 42 **(D)** 41

12. The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is - **[JEE 2012]**

(A) 75 **(B)** 150 **(C)** 210 **(D)** 243

Paragraph for Question 13 and 14

Let a_n denotes the number of all n-digit positive integers formed by the digits 0, 1 or both such that no consecutive digits in them are 0. Let b_n = the number of such n-digit integers ending with digit 1 and c_n = the number of such n-digit integers ending with digit 0.

13. The value of b_6 is **[JEE 2012]**

(A) 7 **(B)** 8 **(C)** 9 **(D)** 11

14. Which of the following is correct ? **[JEE 2012]**

(A) $a_{17} = a_{16} + a_{15}$ **(B)** $c_{17} \neq c_{16} + c_{15}$ **(C)** $b_{17} \neq b_{16} + c_{16}$ **(D)** $a_{17} = c_{17} + b_{16}$

15. Six cards and six envelopes are numbered 1, 2, 3, 4, 5, 6 and cards are to be placed in envelopes so that each envelope contains exactly one card and no card is placed in the envelope bearing the same number and moreover the card numbered 1 is always placed in envelope numbered 2. Then the number of ways it can be done is **[JEE Ad. 2014]**

(A) 264 **(B)** 265 **(C)** 53 **(D)** 67

16. Let $n \geq 2$ be an integer. Take n distinct points on a circle and join each pair of points by a line segment. Colour the line segment joining every pair of adjacent points by blue and the rest by red. If the number of red and blue line segment are equal, then the value of n is **[JEE Ad. 2014]**

17. Let $n_1 < n_2 < n_3 < n_4 < n_5$ be positive integers such that $n_1 + n_2 + n_3 + n_4 + n_5 = 20$. Then the number of such distinct arrangements $(n_1, n_2, n_3, n_4, n_5)$ is **[JEE Ad. 2014]**

18. A debate club consists of 6 girls and 4 boys. A team of 4 members is to be selected from this club including the selection of a captain (from among these 4 members) for the team. If the team has to include at most one boy, then the number of ways of selecting the team is **[JEE Ad. 2016]**

(A) 380 **(B)** 320 **(C)** 260 **(D)** 95

MOCK TEST

SECTION - I : STRAIGHT OBJECTIVE TYPE

1. A shopkeeper has 10 copies of each of nine different books, then number of ways in which atleast one book can be selected is

(A) $9^{11} - 1$ **(B)** $10^{10} - 1$ **(C)** $11^{9} - 1$ **(D)** 10^{9}

2. No. of different squares of any size (side of square be natural no.) which can be made from a rectangle of size 15×8, is

(A) 456 **(B)** 120 **(C)** 228 **(D)** None of these

3. The number of different ways in which five 'alike dashes' and eight 'alike dots' can be arranged, using only seven of these 'dashes' & 'dots' is

(A) 1287 **(B)** 119 **(C)** 120 **(D)** 1235520

4. In a hockey series between team X and Y, they decide to play till a team win 'm' match. Then the no. of ways in which team X wins -

(A) 2^m **(B)** ${}^{2m}P_m$ **(C)** ${}^{2m}C_m$ **(D)** None of these

5. There are three coplanar parallel lines. If any p points are taken on each the lines, the maximum number of triangles with vertices at these points is

(A) $3p^2(p - 1) + 1$ **(B)** $3p^2(p - 1)$ **(C)** $p^2(4p - 3)$ **(D)** none of these

6. A gentleman invites a party of $m + n$ $(m \neq n)$ friends to a dinner & places m at one table and n at another, the table being round. If the clockwise & anticlockwise arrangements are not to be distinguished and assuming sufficient space on both tables, then the number of ways in which he can arrange the guest is

(A) $\dfrac{(m+n)!}{4\ mn}$ **(B)** $\dfrac{1}{2}\dfrac{(m+n)!}{4\ mn}$ **(C)** $2\dfrac{(m+n)!}{4\ mn}$ **(D)** none

7. Number of ways in which 6 different toys can be distributed among two brothers in ratio 1 : 2, is

(A) 30 **(B)** 60 **(C)** 20 **(D)** 40

8. There are m apples and n oranges to be placed in a line such that the two extreme fruits being both oranges. Let P denotes the number of arrangements if the fruits of the same species are different and Q the corresponding figure when the fruits of the same species are alike, then the ratio P/Q has the value equal to :

(A) ${}^nP_2 . {}^mP_m . (n - 2)!$ **(B)** ${}^mP_2 . {}^nP_n . (n - 2)!$ **(C)** ${}^nP_2 . {}^nP_n . (m - 2)!$ **(D)** none

9. **S_1 :** For some natural N, the number of positive integral 'x' satisfying the equation, $1! + 2! + 3! + ... + (x!) = (N)^2$ is 3

S_2 : A women has 11 close friends then the number of ways in which she can invite 5 of them to dinner, if two particular of them are not on speaking terms & will not attend together is 378

S_3 : An old man while dialing a 7 digit telephone number remembers that the first four digits consists of one 1's, one 2's and two 3's. He also remembers that the fifth digit is either a 4 or 5 while has no memorizing of the sixth digit, he remembers that the seventh digit is 9 minus the sixth digit. Maximum number of distinct trials he has to try to make sure that he dials the correct telephone number, is 240

S_4 : The number of times the digit 3 will be written when listing the integers from 1 to 1000 is 300

(A) TTTF **(B)** FTTF **(C)** FTTT **(D)** FTFT

10. A five letter word is to be formed such that the letters appearing in the odd numbered positions are taken from the letters which appear without repetition in the word "MATHEMATICS". Further the letters appearing in the even numbered positions are taken from the letters which appear with repetition in the same word "MATHEMATICS". The number of ways in which the five letter word can be formed is:

(A) 720 **(B)** 540 **(C)** 360 **(D)** none

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

11. Identify the correct statement(s).

(A) Number of zeroes standing at the end of 125 ! is 30.

(B) A telegraph has 10 arms and each arm is capable of 9 distinct positions excluding the position of rest. The number of signals that can be transmitted is $10^{10} - 1$.

(C) Number of numbers greater than 4 lacs which can be formed by using only the digits 0, 2, 2, 4, 4 and 5 is 90.

(D) In a table tennis tournament every player plays with every other player. If the number of games played is 5050 then the number of players in the tournament is 100.

12. Six cards are drawn one by one from a set of unlimited number of cards, each card is marked with numbers – 1, 0 or 1. Number of different ways in which they can be drawn if the sum of the numbers shown by them vanishes, is:

(A) 111 **(B)** 121 **(C)** 141 **(D)** none

13. The number of different seven digit numbers that can be written using only three digits 1, 2 & 3 under the condition that the digit 2 occurs exactly twice in each number is -

(A) 672 **(B)** 640 **(C)** 512 **(D)** none

14. Number of different words that can be formed using all the letters of the word "DEEPMALA", if two vowels are together and the other two are also together but separated from the first two is

(A) 960 **(B)** 1200 **(C)** 2160 **(D)** 1440

15. The number of ways of arranging the letters AAAAA, BBB, CCC, D, EE & F in a row if the letters C are separated from one another is:

(A) ${}^{13}C_3 \cdot \frac{12\,!}{5\,!\,3\,!\,2\,!}$

(B) $\frac{13\,!}{5\,!\,3\,!\,3\,!\,2\,!}$

(C) $\frac{14\,!}{3\,!\,3\,!\,2\,!}$

(D) $\frac{15!}{5!(3!)^2 2!} - \frac{13!}{5!3!2!} - \frac{12!}{5!3!}\,{}^{13}C_2$

SECTION - III : ASSERTION AND REASON TYPE

16. **Statement - I :** Let $E = \left[\frac{1}{3}+\frac{1}{50}\right]+\left[\frac{1}{3}+\frac{2}{50}\right]+\ldots$ upto 50 terms, then E is divisible by exactly two primes.

Statement - II : $[x+n]=[x]+n,\ n \in I$ and $[x+y]=[x]+[y]$ if $x, y \in I$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

17. **Statement I :** The sum of the digits in the tens place of all numbers with the help of 2, 3, 4, 5 taken all at a time is 84.

Statement II: The sum of the digits in the units place of all numbers formed with the help of $(a_1, a_2, ..., a_n)$ taken all at a time is $(n-1)!\,(a_1 + a_2 + ... + a_n)$ (repetition of digits not allowed)

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

18. **Statement - I :** Let $A = \{x \mid x$ is a prime number and $x < 30\}$. Then the number of different rational nubmers, whose numerator and denominator belong to A is 93.

Statement - II : $\frac{p}{q}$ is a rational number $\forall\ q \neq 0$, and $p, q \in I$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

19. **Statement -I :** A five digit number divisible by 3 is to be formed using the digits 0, 1, 2, 3, 4 and 5 with repetition. The total number of numbers formed is 216.

Statement -II : If sum of digits of any number is divisible by 3 then the number must be divisible by 3.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

20. **Statement - I :** The number of ordered pairs (m, n) ; $m,n \in \{1, 2, 3, ...20\}$ such that $3^m + 7^n$ is a multiple of 10, is equal to 100.

Statement - II : $3^m + 7^n$ has last digit zero, when m is of $4k + 2$ type and n is of 4ℓ type where $k, \ell \in W$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

SECTION - IV : MATRIX - MATCH TYPE

21. **Match the following :**

	Column - I		Column - II
(A)	The number of five - digit numbers having the product of digits 20 is	**(p)**	77
(B)	A man took 5 space plays out of an engine to clean them. The number of ways in which he can place atleast two plays in the engine from where they came out is	**(q)**	30
(C)	The number of integers between 1 & 1000 inclusive in which atleast two consecutive digits are equal is	**(r)**	50
(D)	The value of $\frac{1}{15} \sum\limits_{1 \le i \le j \le 9}\sum i \, . \, j$	**(s)**	181
		(t)	31

22. **Column I** **Column II**

Column I		Column II	
(A)	Number of increasing permutations of m numbers from the n set number $\{a_1, a_2, ..., a_n\}$ where the order among the numbers is given by $a_1 < a_2 < a_3 < ....a_{n-1} < a_n$ is	**(p)**	n^m
(B)	There are m men and n monkeys. Number of ways in which every monkey has a master, if a man can have any number of monkeys	**(q)**	${}^mC_n$
(C)	Number of ways in which n red balls and (m –1) green balls can be arranged in a line, so that no two red balls are together, is (balls of the same colour are alike)	**(r)**	${}^nC_m$
(D)	Number of ways in which 'm' different toys can be distributed in 'n' children if every child may receive any number of toys is	**(s)**	m^n

SECTION - V : COMPREHENSION TYPE

23. **Read the following comprehensions carefully and answer the questions.**

Consider the letters of the word MATHEMATICS. There are eleven letters some of them are identical. Letters are classified as repeating and non-repeating letters. Set of repeating letters = {M, A, T}. Set of non-repeating letters = {H, E, I, C, S}

1. Possible number of words taking all letters at a time such that atleast one repeating letter is at odd position in each word, is

(A) $\dfrac{9!}{2!2!2!}$ **(B)** $\dfrac{11!}{2!2!2!}$ **(C)** $\dfrac{11!}{2!2!2!} - \dfrac{9!}{2!2!}$ **(D)** $\dfrac{9!}{2!2!}$

2. Possible number of words taking all letters at a time such that in each word both M's are together and both T's are together but both A's are not together, is

(A) $7!\,.\,{}^8C_2$ **(B)** $\dfrac{11!}{2!2!2!} - \dfrac{10!}{2!2!}$ **(C)** $\dfrac{6!4!}{2!2!}$ **(D)** $\dfrac{9!}{2!2!2!}$

3. Possible number of words in which no two vowels are together, is

(A) $\dfrac{7!}{2!2!}\cdot{}^8C_4\cdot\dfrac{4!}{2!}$ **(B)** $\dfrac{7!}{2!}\cdot{}^8C_4\cdot\dfrac{4!}{2!}$ **(C)** $7!\,.\,{}^8C_4\cdot\dfrac{4!}{2!}$ **(D)** $\dfrac{7!}{2!2!2!}\cdot{}^8C_4\cdot\dfrac{4!}{2!}$

24. **Read the following comprehensions carefully and answer the questions.**

5 ball are to be placed in 3 boxes. Each box can hold all the 5 balls. Number of ways in which the balls can be placed so that no box remains empty, if :

1. balls are identical but boxes are different

(A) 2 **(B)** 25 **(C)** 50 **(D)** 6

2. balls are different but boxes are identical

(A) 2 **(B)** 25 **(C)** 50 **(D)** 6

3. balls as well as boxes are identical

(A) 2 **(B)** 25 **(C)** 50 **(D)** 6

25. **Read the following comprehensions carefully and answer the questions.**

Counting by critical paranthesis method

Suppose we have to arrange n-pairs of paranthesis in such a way that every arrangement is matched i.e. number of left paranthesis are always greater than or equal to number of right paranthesis in any length of the chain from start.

S is the number of ways of arranging n - right and n - left paranthesis in a row = $\frac{2n!}{n!\ n!}$.

Let T be the arrangement of (n + 1) right and (n – 1) left paranthesis = $\frac{(2n)!}{(n+1)!(n-1)!}$.

It can be shown that set of mismatched arrangements of paranthesis in 'S' has bijective relation with the set of arrangements of T.

Since the set of mismatched arrangements in S has bijective relation with the set of arrangements in T.

$\therefore$ number of the mismatched arrangements in S = $\frac{2n!}{(n+1)!(n-1)!}$

$\therefore$ Number of matched arrangements in S = $\frac{2n!}{n!\ n!} - \frac{2n!}{(n+1)!(n-1)!} = \frac{2n!}{n!\ (n+1)!}$

1. The number of ways in which '4' pairs of paranthesis be arranged so that every arrangement is matched is:

(A) 3 **(B)** $\frac{{}^8C_4}{5}$ **(C)** ${}^8C_4$ **(D)** ${}^8C_5$

2. If a stamp vendor sells tickets of 1 rupee each and there are 3 persons having 1 rupee coin and 3 having 2 rupee coin standing in a row. Then the probability that stamp vendor do not run out of change if he does not have any money to start with is:

(A) $\frac{1}{4}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{3}{4}$ **(D)** None of these

3. Number of ways of arranging the 5 pairs of paranthesis, if first pair is matched but the next four pairs are not matched is:

(A) ${}^{10}C_6$ **(B)** ${}^8C_5$ **(C)** ${}^4C_2 \times {}^4C_2$ **(D)** None of these

SECTION - VI : INTEGER TYPE

26. In a row, there are n rooms, whose door no. are 1,2,.......,n, initially all the door are closed. A person takes n round of the row, numbers as 1st round, 2nd round .... nth round. In each round, he interchange the position of those door no., whose no is multiple of the round no. Find out after nth round,How many doors will be open.

27. 10 IIT & 2 PET students sit in a row. The total number of ways in which exactly 3 IIT students sit between 2 PET students is $\lambda 10!$, then find λ.

28. The integers from 1 to 1000 are written in order around a circle. Starting at 1, every fifteenth number is marked (that is 1, 16, 31, ... etc.). This process in continued untill a number is reached which has already been marked, then find number of unmarked numbers.

29. 17 persons can depart from railway station in 2 cars and 3 autos, given that 2 particular person depart by same car are $\frac{15!}{\lambda!(3!)^3}$. (4 persons can sit in a car and 3 persons can sit in an auto), then find the value of λ.

30. Find the number of positive unequal integral solution of the equation x + y + z = 20.

ANSWER KEY

EXERCISE - 1

1. D	2. A	3. C	4. D	5. A	6. C	7. D	8. C	9. C	10. A	11. B	12. D	13. B
14. D	15. D	16. A	17. A	18. A	19. C	20. D	21. B	22. D	23. B	24. B	25. C	26. D
27. C	28. D	29. C	30. C									

EXERCISE - 2 : PART # I

1. AD	2. AC	3. ACD	4. BC	5. BC	6. BCD	7. AB	8. ABD	9. BD
10. ABD	11. AD	12. CD	13. BD	14. AB	15. BD	16. ABC	17. CD	18. ABC
19. AD	20. BCD							

PART - II

1. C 2. A 3. B 4. D 5. B 6. D 7. A

EXERCISE - 3 : PART # I

1. $A \to q$ $B \to s$ $C \to P$ $D \to r$ 2. $A \to p$ $B \to p,q,r$ $C \to p,q,r,s$ $D \to p$ 3. $A \to s$ $B \to q$ $C \to p$ $D \to s$

4. $A \to q$ $B \to r$ $C \to p$ $D \to s$ 5. $A \to s$ $B \to p$ $C \to q$ $D \to r$

PART - II

Comprehension # 1 : 1. A 2. C 3. B Comprehension # 2 : 1. B 2. A 3. D

Comprehension # 3 : 1. B 2. A 3. B Comprehension # 4 : 1. D 2. B 3. C

Comprehension # 5 : 1. A 2. A 3. B

EXERCISE - 5 : PART # I

1. B	2. A	3. A	4. A	5. B	6. B	7. C	8. B	9. D	10. A	11. A	12. B	13. A
14. B	15. A	16. C	17. B	18. A	19. C	20. B	21. B	22. D	23. C	24. C		

PART - II

1. C 2. $A \to p$ $B \to q$ 3. A 4. B 6. C 7. C 8. C 9. $A \to p$ $B \to s$ $C \to q$ $D \to q$

10. C 11. D 12. B 13. B 14. A 15. C 16. 5 17. 7 18. A

MOCK TEST

1. C 2. A 3. C 4. D 5. C 6. C 7. A 8. A 9. C 10. B 11. B, C 12. C

13. A 14. D 15. A, D 16. D 17. A 18. D 19. A 20. B

21. $A \to r$ $B \to t$ $C \to s$ $D \to p$ 22. $A \to r$ $B \to s$ $C \to p$ $D \to q$

23. 1. B 2. A 3. A 24. 1. D 2. B 3. A 25. 1. B 2. A 3. B

26. $\left[\sqrt{n}\right]$ 27. 16 28. 800 29. 4 30. 144

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