HINTS & SOLUTIONS

EXERCISE - 1
Single Choice

2. Total possible words – words do not begin or terminate with vowel

Total words = 5! = 120

Words which do not begin and terminate with vowel

$= 3 \times 3 \times 2 \times 1 \times 2 = 36$

Desired words : 180 – 36 = 84

II-Method $\rightarrow$ words which begin with vowel

(A/I) = 4! × 2 = 48 ways $\rightarrow$ say = n(A)

Similarly words terminating with vowel

= 4! × 2 = 48 ways $\rightarrow$ say = n(B)

Now exclude words which begin as well as terminates with vowel

$2 \times 3 \times 2 \times 1 \times 1 = 12$ ways $\rightarrow n(A \cap B)$

Desired number of words :-

48 + 48 – 12 = 84 ways

$(\because\ n(A \cup B) = n(A) + n(B) - n(A \cap B))$

6. Make cases when all 5 boxes are filled by:

Case I : identical 5 red balls

$^5C_5 \rightarrow 1$ way

Case II : 4 identical red balls and 1 blue ball

$^5C_1 = 5$ ways

Case III : 3 blue and 2 red balls i.e. xRxRx

$\Rightarrow$ 4 gaps, for 2 blue balls

$\therefore$ $^4C_2 = 6$ ways

Case IV : 2 red and 3 blue balls i.e. xRxRx $\Rightarrow$ 3 gaps, 3 blue balls

$\Rightarrow$ $^3C_3 = 1$ way

$\therefore$ Total number of ways are 1+5+6+1 = 13 ways

7. First we select 3 speaker out of 10 speaker and put in any way and rest are no restriction i.e. total number of ways = $^{10}C_3 \,.\, 7.2\,! = \dfrac{10!}{3}$

8. EEQUU

Words starting with E $\rightarrow \dfrac{4!}{2!}$

Words starting with QE $\rightarrow \dfrac{3!}{2!}$

next word will be QUEEU $\rightarrow$ 1

and finally QUEUE $\rightarrow$ 1

Rank is 12 + 3 + 1 + 1 = 17^{th}

17. Here $21600 = 2^5 . 3^3 . 5^2$

$\Rightarrow (2 \times 5) \times 2^4 \times 3^3 \times 5^1$

Now numbers which are divisible by 10

$= (4 + 1)(3 + 1)(1 + 1) = 40$

$(2 \times 3 \times 5) \times (2^4 \times 3^2 \times 5^1)$ now numbers which are divisible by both 10 and 15

$= (4+1)(2+1)(1+1) = 30$

So the numbers which are divisible by only

40 – 30 = 10

18. Using multinomial theorem

Total no. of ways of choosing 6 chocolates out of 8 different brand is = $^{8+6-1}C_6 = {}^{13}C_6$

24. For a number to be divisible by 5,5 or 0 should be at units place.

$\therefore$ Unit place can be filled by 2 ways

Remaining digits can be filled in $\dfrac{6!}{3!\times 2!}$ ways.

$\therefore$ Total ways = $\dfrac{2 \times 6!}{3!2!}$

But these arrangements also include cases where 0 is at millions place and 5 at units place, which are undesirable cases

$\Rightarrow \dfrac{5!}{3!\times 2!}$ ways (undesirable)

subtract it from total ways.

$\therefore$ Desired ways = $2 \times \dfrac{6!}{3!\times 2!} - \dfrac{5!}{3!\times 2!} = 110$

27. Total number of positive integral solution of $x_1 . x_2 . x_3 = 80 = 2 \times 3 \times 5$ is $3 \times 3 \times 3 = 27$

30. Number of selections of 4 consonants out of 7 is $^7C_4$

Number of selections of 2 vowels from 4 is $^4C_2$

Arrangement of words in 6! ways

Desired words : $^7C_4 \times {}^4C_2 \times 6! = 151200$

EXERCISE - 2

Part # I : Multiple Choice

3. Total number of required possibilities

$^5C_3 \cdot {^8C_7} + {^5C_4} \cdot {^8C_6} + {^5C_5} \cdot {^8C_5} \cdot {^5C_5}$

$= {^5C_3} \cdot {^8C_7} + {^5C_4} \cdot {^8C_6} + {^8C_6} = {^{13}C_{10}} - {^5C_3} = 276$

6. $\dfrac{^{200}C_2 \cdot {^{198}C_2} \cdot {^{196}C_2} \ldots\ldots {^2C_2}}{100!} = \dfrac{200!}{2^{100} \cdot 100!}$

$= \dfrac{101.102.103.....200}{2^{100}}$

$= \left(\dfrac{100}{2}\right) \cdot \left(\dfrac{102}{2}\right) \cdot \left(\dfrac{103}{2}\right) \ldots \left(\dfrac{100}{2}\right)$

and $\dfrac{1.2.3.4.5.6.7.8.....200}{2^{100}.100!}$

$= \dfrac{(1.3.5.7.....199)(2.4.6.8......200)}{2^{100}.100!}$

$= \dfrac{(1.3.5......199).2^{100}.100!}{2^{100}.100!} = 1.3.5\ .199$

9. $x_1 + x_2 + x_3 + x_4 \le n$

$\Rightarrow \quad x_1 + x_2 + x_3 + x_4 + y = n$

(where y is known as pseudo variable)

Total no. of required solution is $= {^{n+5-1}C_n}$

$= {^{n+4}C_n}$ or ${^{n+4}C_4}$

11. We have arrange all the letter except 'ccc' is $\dfrac{12!}{5!.3!.2!}$

new there all 13 place where 'i' can be placed $^{13}C_3$

Hence required number of ways is

$= \dfrac{12!}{5!3!2!} \, {^{13}C_3} = 11 \cdot \dfrac{13!}{6!}$

12. Here given no. be 1,2,3,.........n

Let common difference = r

Total way of selection = (1, 1 + r, 1+2r),

(2, 2 + r, 2 + 2r), ..(n – 2r, n – r, n)

Total numbers are = (n – 2r)

Here $r_{min.} = 1$ and $r_{max.} = (n-1)/2$

Case- I When n is odd

$\therefore \quad r_{max} = \dfrac{(n-1)}{2}$ & total no. of selection is

$= \sum\limits_{r=1}^{(n-1)/2} (n-2r) = \dfrac{n(n-1)}{2} - \dfrac{2\left(\dfrac{n-1}{2}\right)\left(\dfrac{n+1}{2}\right)}{2}$

$= \left(\dfrac{n-1}{2}\right)^2$

Case - II when n is even = $r_{max} = \dfrac{n-2}{2}$

so total no. selection is

$= \sum\limits_{r=1}^{(n-2)/2} (n-2r) = \dfrac{n(n-2)}{2} - \dfrac{2\left(\dfrac{n-2}{2}\right)\dfrac{n}{2}}{2}$

$= \left(\dfrac{n-2}{2}\right)\left(n - \dfrac{n}{2}\right) = \dfrac{n(n-2)}{4}$

14. Total no. of visits that a teacher goes is = $^{25}C_5$

(selection of 5 different kids each time & teacher goes every time)

Number of visits of a boy = select one particular boy & 4 from rest 24 = $^{24}C_4$

So extra visits of a teacher from a boy is

$= {^{25}C_5} - {^{24}C_4} = {^{24}C_5}$

17. Number of ways he can fail is either one or two, three or four subject then total of ways.

$^4C_1 + {^4C_2} + {^4C_3} + {^4C_4} = 2^4 - 1$

19. Total number of required quadrilateral

$^7C_4 + {^7C_3} \times {^5C_1} + {^7C_2} \times {^5C_2}$

$= \dfrac{7\times6\times5\times4}{1\times2\times3\times4} + \dfrac{7\times6\times5}{1\times2\times3}.5 + \dfrac{7\times6}{1\times2} \times \dfrac{5\times4}{1\times2}$

$= 35 + 175 + 210 = 420 = 2\ .{^7p_3}$

Part # II : Assertion & Reason

3. **Statement -1:** Two circles intersect in 2 points.

$\therefore$ Maximum number of points of intersection

= 2 × number of selections of two circles from 8 circles.

$= 2 \times {^8C_2} = 2 \times 28 = 56$

Statement -2: 4 lines intersect each other in $^4C_2 = 6$ points.

4 circles intersect each other in $2 \times {^4C_2} = 12$ points.

Further, one lines and one circle intersect in two points.

So, 4 lines will intersect four circles in 32 points.

$\therefore$ Maximum number of points = 6 + 12 + 32 = 50.

5. $x_1 x_2 x_3 x_4 = 2 \times 5 \times 7 \times 11 \quad \Rightarrow \quad N = 4^4$

EXERCISE - 3

Part # II : Comprehension

Comprehension # 01

1. Exponent of 7 in 100! –

$$\left[\frac{100}{7}\right]+\left[\frac{14}{7}\right]=14+2=16$$

exponent of 7 in 50!

$$\left[\frac{50}{7}\right]+\left[\frac{7}{7}\right]=8$$

Exponent of 7 in $^{100}C_{50}=\frac{100!}{50!50!}=\frac{7^{16}}{7^8 7^8}=7^0$

$\therefore$ exponent of 7 will be 0.

2. Product of 5's & 2's constitute 0's at the end of a number

$\Rightarrow$ No. of 0's in 108!

= exponent of 5 in 108!

(Note that exponent of 2 will be more than exponent of 5 in 108 !)

$$\Rightarrow\left[\frac{108}{5}\right]+\left[\frac{21}{5}\right]=21+4=25$$

3. As $12=2^2.3$, here we have to calculate exponent of 2 and exponent of 3 in 100!

exponent of 2

$$=\left[\frac{100}{2}\right]+\left[\frac{50}{2}\right]+\left[\frac{25}{2}\right]+\left[\frac{12}{2}\right]+\left[\frac{6}{2}\right]+\left[\frac{3}{2}\right]=97$$

exponent of 3 $=\left[\frac{100}{3}\right]+\left[\frac{33}{3}\right]+\left[\frac{11}{3}\right]+\left[\frac{3}{3}\right]=48$

Now, $12=2\times2\times3$

we require two 2's & one 3

$\therefore$ exponent of 3 will give us the exponent of 12 in 100!

i.e. 48

Comprehension # 04

1. Since there are 5 even places and 3 pairs of repeated letters, therefore at least one of these must be at an odd place. Therefore, the number of ways is 11!/(2!2!2!).

2. Make a group of both M's and another group of T's. Then except A's we have 5 letters remaining. So M's, T's, and the letters except A's can be arranged in 7! ways. Therefore, total number of arrangements is $7!\times{}^8C_2$.

3. Consonants can be placed in 7!/(2!2!) ways. Then there are 8 places and 4 vowels. Therefore, number of ways is

$$\frac{7!}{2!2!}\,{}^8C_4\,\frac{4!}{2!}.$$

EXERCISE - 4

Subjective Type

1. Selecting 3 horses out of ABC A'B'C' is ${}^6C_3$ ways
When AA' is always selected among (ABC A'B'C')
Remaining (BB'CC') can be selected in ${}^4C_1$ ways similarly, when BB' and CC' is selected
$\therefore$ Undesirable ways will be $({}^4C_1) \times 3$
using, total ways–undesirable ways
= desired ways we get
$({}^6C_3 - ({}^4C_1)3) \rightarrow$ This is selection of 3 horses among (ABC A'B'C') under given condition.
Remaining 3 can be selected in ${}^{10}C_3$ ways.
Hence, desired ways will be $[{}^6C_3 - {}^4C_1 \times 3]{}^{10}C_3 = 792$

Method II : Select one horse each from AA', BB' and CC' hence ${}^2C_1 \times {}^2C_1 \times {}^2C_1$ ways. Now select 3 horses from remaining 10 horses in ${}^{10}C_3$ ways.
Total ways = ${}^{10}C_3 \times {}^2C_1 \times {}^2C_1 \times {}^2C_1$

2. Total no. of M are = 1 Total no. of I are = 4
Total no. of P are = 2 Total no. of S are = 4
First we arrange all the words other than I's are

$$\frac{7!}{2!\,4!} = \frac{7\times6\times5}{1\times2} = 105$$

Now, there are 8 places which can be fulfilled by I's i.e. the number of ways is ${}^8C_4$

Total required no. $= 105 \times {}^8C_4 = \frac{105\times8\times7\times6\times5}{1\times2\times3\times4}$
$= 105 \times 70 = 7350$

3. Step 1[st] : Select 2 lines out of n lines in ${}^nC_2$ ways to get a point (say p).

Step-2[nd] : Now select another 2 lines in ${}^{n-2}C_2$ ways, to get another point (say Q)

Step-3[rd] : When P and Q are joined we get a fresh line.

But when we select P first then Q and Q first then P we get same line.

$$\therefore \frac{{}^nC_2 \times {}^{n-2}C_2}{2} \text{ Fresh lines}$$

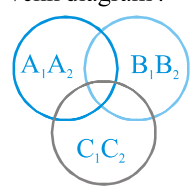
4. First we select n grand children from 2n grand children is ${}^{2n}C_n$
Now arrangement of both group is n! × n!
Now Rest all (m + 1) place where we occupy the grandfather and m sons but grandfather refuse the sit to either side of grand children so the out of m – 1 seat one seat can be selected
Now required number of sitting in
${}^{2n}C_n \times n! \times n! \times {}^{(m-1)}C_1 . m!$

$$= \frac{12n}{n!\times n!} \times n! \times n! \times {}^{(m-1)}C_1 . m! = 2n! . m! . (m-1)$$

6. Number of ways of distributing 200 objects into 20 groups each containing 10 objects

$$= \frac{200!}{(10!)^{20}.20!} \times 20 = \frac{200!}{(10!)^{20}.19!}$$ which must be an integer.

7. **(i)** Total ways = 10!
undesirable cases : when 2 Americans are together (A_1A_2) or two British are together (B_1B_2) or two Chinese are together (C_1C_2)
we plot them on Venn diagram :



we use,
$n(A_1A_2 \cup B_1B_2 \cup C_1C_2)$
$= n(A_1A_2) + n(B_1B_2) + n(C_1C_2) - n[(A_1A_2) \cup (B_1B_2)]$
$- n[(B_1B_2) \cup (C_1C_2)] - n[(C_1C_2) \cup (A_1A_2)]$
$+ n[(A_1A_2) \cap (B_1B_2) \cap (C_1C_2)]$
where $n(A_1A_2)$ denotes $\rightarrow$ when 2 Americans are together = 9! 2! correspondingly for B_1B_2&C_1C_2
$n[(A_1A_2) \cup (B_1B_2)]$ denotes when 2 Americans and 2 Britishmen are together
$= 8! \times 2! \times 2!$
correspondingly same for others.
$n[(A_1A_2) \cap (B_1B_2) \cap (C_1C_2)]$ denotes when 2 Americans, 2 Britishmen and 2 Chinese are together
$= 7! \times 2! \times 2! \times 2! = 86$

Put values we get

$n(A_1A_2 \cup B_1B_2 \cup C_1C_2)$

$= 9! \times 2! \times 3 - 8! \times 2 \times 2 \times 3 + 8!$

$= 8!(43)$

These are undesired ways

Desired ways $= 10! - 8!\,(43) = 8!(47)$

(ii) Now they are on a round table

Total ways $= (n-1)! = (10-1)! = 9!$

Undesired ways :

$n(A_1A_2 \cup B_1B_2 \cup C_1C_2)$

$= 8! \times 2! \times 3 - 7! \times 2! \times 2! \times 3 + 6! \times 2! \times 2! \times 2!$

$= 6! \times 4\,[7 \times 2 \times 2 \times 3 - 7 \times 3 + 2]$

$= 6! \times 260$

Desired ways $= 9! - 6! \times 260$

$= (244)\,6!$ ways

8. **(i)** Vowels → 3, Consenents → 5

${}^3C_2 . 2!\,6! = 6 \times 6! = 4320$

(ii)

G						H

Start with G end with H

$6! = 720$

(iii)

GHT					

$3!\,6! = 6 \times 6! = 4320$

(iv) $\uparrow - \uparrow - \uparrow - \uparrow - \uparrow - \uparrow$ $\quad 5! . {}^6C_3 . 3!$

6 gap

(v) Same as above

(vi) _ _ _ _ _ _ _ _ $\quad {}^4C_3 . 3! . 5!$

(vii) _ _ _ _ _ _ _ _ $\quad {}^8C_3 . 1 . 5! = {}^8C_3 . 5! = 6720$

(viii) $3! . 5! = 720$

(ix) ${}^3C_2 . {}^5C_3 . 5! = 3.10 . 5! = 120 \times 30 = 3600$

9. Distribute 15 candies among.

Ram (R) + Shyam(S) + Ghanshyam(G) + Balram(B)

with condition given : $R+S+G+B=15$ & $R \le 5$ & $S \ge 2$

After giving 2 to Shyam, remaining candies $15-2=13$

Now distribute 13 candies in

R, S, G, B in $\dfrac{\lfloor 13+4-1}{\lfloor 13 . \lfloor 3} = {}^{16}C_3$ ways

In ${}^{16}C_3$ ways, we have to remove undesirable ways, when $R > 5$

Undesirable ways : $R > 5 \;\Rightarrow\; R \ge 6$

give at least 6 to R and 2 to S and distribute remaining between R, S, G, B

$15 - (2 + 6) = 7$ remaining can be distributed between R,

S, G, B in $= \dfrac{\lfloor 7+4-1}{\lfloor 7 . \lfloor 4-1} = {}^{10}C_3$ ways

${}^{10}C_3$ are the undesirable cases

Desired ways $= {}^{16}C_3 - {}^{10}C_3 = 440$

10. **(i)** Without changing the order of the vowels of MULTIPLE

So we choose the first three place in ${}^8C_3$ ways

and the rest are arranged is $\dfrac{8!}{3!5!} \times \dfrac{5!}{2!} = \dfrac{8!}{3!2!} = 3360$

Hence required no. is $3360 - 1 = 3359$

(ii) Keeping the position of each vowel fixed M_LT_PL_

Number of ways $= \dfrac{5!}{2} = 60$

other ways $= 60-1 = 59$

(iii) without changing the relative order/position of vowels & consonants

so number of ways is $= \dfrac{5!}{2!} \times 3! = 60 \times 6 = 360$

Hence required number is $= 360-1 = 359$

13. Husband – H, Wife – W

Given :

Relatives of husband (H) (a) Ladies $(L_H) = 4$
(b) Gentlemen $(G_H) = 3$

Relatives of Wife (W) (a) Ladies $(L_W) = 3$
(b) Gentlemen $(G_W) = 4$

Case 1 : Selecting $(3L_H)$ and $3(G_W)$

ways : ${}^4C_3 \times {}^4C_3 = 16$

Case 2 : Selecting $(3G_H)$ and $3(L_W)$

ways : ${}^3C_3 \times {}^3C_3 = 1$

Case 3 : Selecting $(2L_H \,\&\, 1G_H)$ & $(1L_w \,\&\, 2G_w)$

ways : ${}^4C_2 \times {}^3C_1 \times {}^3C_1 \times {}^4C_2 = 324$

Case 4 : Selecting $(1L_H \,\&\, 2G_H)$ & $(2L_W \,\&\, 1G_w)$

ways : ${}^4C_1 \times {}^3C_2 \times {}^3C_2 \times {}^4C_1 = 144$

Add all cases we get : 485 ways

15. 2 clerks who prefer Bombay are to be sent to 2 different companies in Bombay, and Out of remaining 5 clerks (excluding 3 clerks who prefer for outside) 2 clerks are chosen in $^5C_2$ ways.

Now these 4 can be sent to 2 different companies into 2 groups of 2 each in $^4C_2$ ways

$\Rightarrow$ $^5C_2 \times {}^4C_2$

Now for outside companies we have 6 clerks remaining we select them as (2 for each company)

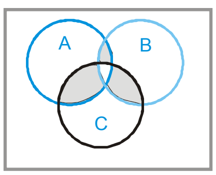
$^6C_2 \times {}^4C_2 \times {}^2C_2$

Desired ways = $(^5C_2 \times {}^4C_2)(^6C_2 \times {}^4C_2 \times {}^2C_2) = 5400$ ways.

16. Total students $n(A \cup B \cup C) = 100$

Students reading Business India = n(A) = 80
Students reading Business World = n(B) = 50
Students reading Business Today = n(C) = 30

Students reading all the three magazines = $n(A \cap B \cap C) = 5$



Hence students reading exactly two magazines

$= n(A) + n(B) + n(C) - n(A \cup B \cup C) - 2 \times n(A \cap B \cap C)$

$= 80 + 50 + 30 - 100 - 2 \times 5 = 50$

19. **(a)** Selection of r things out of n +1 different things = Selection of r things out of n +1 different things, when a particular thing is excluded + a particular thing is included.

(b) Selection of r things out of not m + n different Things can be made by selecting x thing from m and y thing from such that x +y = r

& $(x, y) = (0, r), (1, r-1), (2, r-2), ...(r, 0)$

21. Step 1st : Arrange 5 boys in 5! ways

Step 2nd : Select 2 gaps from 6 gaps for 4 girls (2 girls for each gap) in $^6C_2$ ways.

Step 3rd : Select 2 girls to sit in one of the gaps and other 2 in remaining selected gaps = $^4C_2$ ways

Step 4th : Arrange 1st, 2 girls in 2! and other 2 in 2! ways

Hence, total ways $\rightarrow 5! \times {}^6C_2 \times {}^4C_2 \times 2 \times 2 = 43200$

22. Ordered pair = total – $(A \cup B = X) = 4^n - 3^n$

Subsets of X = 2^n will not repeat in both but here the whole set X has not been taken

So subsets of x which are not repeated $(2^n - 1)$

Hence unordered pair = $\dfrac{(4^n - 3^n) - (2^n - 1)}{2} + (2^n - 1)$

23. Total No. of bowlers = 6

Now,

(i) If 4 bowlers are including the no. of ways selecting 11 players out of 15 players

$= {}^6C_4 \times {}^9C_7 = 15 \times 36 = 540$

(ii) If 5 bowlers are selected $= {}^6C_5 \times {}^9C_6 = 6 \times 84 = 504$

(iii) If all 6 bowlers are selected $= {}^6C_6 \times {}^9C_5 = 1 \times 126 = 126$

Hence total no. of ways = 540 + 504 + 126 = 1170

25. $1980 = 2^2 . 3^2 . 5 . 11,$

number of divisiors of 1980 = 36

(i) 3.3.2 = 18

sum = $11.(1 + 2 + 2^2)$

$. (1 + 3 + 3^2)$

$. (1 + 5)$

(ii) 3.2 + 1.1.2 = 8

EXERCISE - 5

Part # I : AIEEE/JEE-MAIN

1. Numbers greater than 1000 and less than or equal to 4000 will be of 4 digits and will have either 1 (except 1000) or 2 or 3 in the first place with 0 in each of the remaining places.

After fixing 1st place, the second place can be filled by any of the 5 numbers. Similarly third place can be filled up in 5 ways and 4th place can be filled up in 5 ways. Thus there will be $5 \times 5 \times 5 = 125$ ways in which 1 will be in first place but this include 1000 also hence there will be 124 numbers having 1 in the first place. Similarly 125 for each 2 or 3. One number will be in which 4 in the first place and i.e. 4000. Hence the required numbers are $124 + 125 + 125 + 1 = 375$ ways.

2. We know that a five digit number is divisible by 3, if and only if sum of its digits (= 15) is divisible by 3. Therefore we should not use 0 or 3 while forming the five digit numbers. Now,

(i) In case we do not use 0 the five digit number can be formed (from the digit 1, 2, 3, 4, 5) in ${}^5P_5$ ways.

(ii) In case we do not use 3, the five digit number can be formed (from the digit 0, 1, 2, 4, 5) in ${}^5P_5 - {}^4P_4 = 5! - 4! = 120 - 24 = 96$ ways.

$\therefore$ The total number of such 5 digit number $= {}^5P_5 + ({}^5P_5 - {}^4P_4) = 120 + 96 = 216$

4. No. of ways in which 6 men can be arranged at a round table $= (6 - 1)!$

Now women can be arranged in 6! ways.

Total Number of ways $= 6! \times 5!$

5. As for given question two cases are possible

(i) Selecting 4 out of first 5 question and 6 out of remaining 8 questions $= {}^5C_4 \times {}^8C_6 = 140$ choices

(ii) Selecting 5 out of first 5 questions and 5 out of remaining 8 questions $= {}^5C_5 \times {}^8C_5 = 56$ choices.

$\therefore$ Total no. of choices $= 140 + 56 = 196$

7. Number of ways to arrange in which vowels are in alphabetical order $= \dfrac{6!}{2!} = 360$

8. Number of ways $= {}^{n-1}C_{r-1} = {}^{8-1}C_{3-1} = {}^7C_2 = 21$

11. ${}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4 = 385$

12. Number of ways $= \dfrac{12!}{(4!)^3 . 3!} \times 3! = \dfrac{12!}{(4!)^3}$

14. The no. of ways to select 4 novels & 1 dictionary from 6 different novels & 3 different dictionary are ${}^6C_4 \times {}^3C_1$ and to arrange these things in shelf so that dictionary is always in middle _ _ D _ _ are 4!

Required No. of ways ${}^6C_4 \times {}^3C_1 \times 4! = 1080$

15. Urn A $\rightarrow$ 3 Red balls

Urn B $\rightarrow$ 9 Blue balls

So the number of ways = selection of 2 balls from urn A & B each.

$= {}^3C_2 . {}^9C_2 = 108$

16. $B_1 + B_2 + B_3 + B_4 = 10$

Statement - I

$B_1 \geq 1, B_2 \geq 1, B_3 \geq 1, B_4 \geq 1$

so no. of negative integers solution of equation

$x_1 + x_2 + x_3 + x_4 = 10 - 4 = 6$

${}^{6+4-1}C_{4-1} = {}^9C_3$

Statement - II

Selction of 3 places from out of 9 places $= {}^9C_3$

Both statements are true and correct explaination

17. $N = {}^{10}C_3 - {}^6C_3 = \dfrac{10 \times 9 \times 8}{3 \times 2 \times 1} - \dfrac{6 \times 5 \times 4}{3 \times 2 \times 1}$

$= 120 - 20 = 100$

$N \leq 100$

18. W^{10}, G^9, B^7

selection of one or more balls

$= (10 + 1)(9 + 1)(7 + 1) - 1$

$= 11 \times 10 \times 8 - 1 = 879$

19. (A, B)

$\uparrow \ \uparrow$

$2 \times 4 = 8$

${}^8C_3 + {}^8C_4 + ... + {}^8C_8 = 2^8 - {}^8C_0 - {}^8C_1 - {}^8C_2$

$= 256 - 37 = 219$

20. $T_n = {}^nC_3$

$\Rightarrow {}^{n+1}C_3 - {}^nC_3 = 10$

$(n + 1)\, n\, (n - 1) - n(n - 1)(n - 2) = 60$

$n(n - 1) = 20$

$n = 5$

23. $P(E_1) = \dfrac{1}{6}$ $\qquad P(E_2) = \dfrac{1}{6}$

$P(E_1 \cap E_2) = P(\text{A shows 4 and B shows 2})$

$= \dfrac{1}{36} = P(E_1) . P(E_2)$

So E_1, E_2 are independent

Also as $E_1 \cap E_2 \cap E_3 = \phi$

So $P(E_1 \cap E_2 \cap E_3) \neq P(E_1). P(E_2). P(E_3)$

So E_1, E_2, E_3 are not independent.

Part # II : IIT-JEE ADVANCED

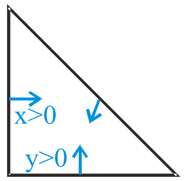
4. $x + y < 21$

$x + y \le 20$

$x + y \le 18$ ($\because$ $x > 0$ & $y > 0$)

Introducing new variable t

$x + y + t = 18$



Now dividing 18 identical things among 3 persons.

$$= {}^{18+3-1}C_{3-1} = \frac{\lfloor 18+3-1}{\lfloor 18\ \lfloor 3-1} = 190$$

5. Total number of ways of distributing n^2 objects into n groups, each containing n objects

$$= \frac{(n^2)!}{(n!)^n\, n!}.n! = \frac{(n^2)!}{(n!)^n} = \text{integer}$$

(Since number of ways are always integer)

7. Since, r, s, t are prime numbers.

$\therefore$ Selection of p and q are as under

p	q	number of ways
r^0	r^2	1 way
r^1	r^2	1 way
r^2	r^0, r^1, r^2	3 ways

$\therefore$ Total number of ways to select r = 5

s^0	s^4	1 way
s^1	s^4	1 way
s^2	s^4	1 way
s^3	s^4	1 way
s^4	s^0, s^1, s^2, s^3, s^4	5 ways

$\therefore$ Total number of ways to select s = 9.

Similarly total number of ways to select t

= 5 number of ways = $5 \times 9 \times 5 = 225$.

11. (D)

Case- I : The number of elements in the pairs can be 1,1; 1,2; 1,3,; 2,2

$$= {}^4C_2 + {}^4C_1 \times {}^3C_2 + {}^4C_1 \times {}^3C_3 + \frac{{}^4C_2 . {}^2C_2}{2} = 25$$

Case- II : Number of pairs with ϕ as one of subsets $= 2^4 = 16$

$\therefore$ Total pairs = 25 + 16 = 41

12 Balls can be distributed as 1, 1, 3 or 1, 2, 2 to each person.

When 1, 1, 3 balls are distributed to each person, then total number of ways :

$$= \frac{5!}{1!\,1!\,3!}.\frac{1}{2!}.3! = 60$$

When 1, 2, 2 balls are distributed to each person, then total number of ways :

$$= \frac{5!}{1!\,2!\,2!}.\frac{1}{2!}.3! = 90$$

$\therefore$ total = 60 + 90 = 150

Paragraph for Question 13 and 14 : For a_n

The first digit should be 1

For b_n

$$\underbrace{1\_\,\_\,\_\,\_\,\_\ldots\_\,\_1}_{(n-2\text{ Places})}$$

Last digit is 1. so b_n is equal to number of ways of a_{n-1} (i.e. remaining (n – 1) places)

$b_n = a_{n-1}$

For c_n

Last digit is 0 so second last digit must be 1

So $c_n = a_{n-2}$

$b_n + c_n = a_n$

So $a_n = a_{n-1} + a_{n-2}$

Similarly $b_n = b_{n-1} + b_{n-2}$

13. (B)

$a_1 = 1,\ a_2 = 2$

So $a_3 = 3,\quad a_4 = 5\ \ a_5 = 8$

$\Rightarrow$ $b_6 = a_5 = 8$

14. (A)

$a_n = a_{n-1} + a_{n-2}$

put n = 17

$a_{17} = a_{16} + a_{15}$ (A) is correct

$c_n = c_{n-1} + c_{n-2}$

So put n = 17

$c_{17} = c_{16} + c_{15}$ (B) is incorrect

$b_n = b_{n-1} + b_{n-2}$

put n = 17

$b_{17} = b_{16} + b_{15}$ (C) is incorrect

$a_{17} = a_{16} + a_{15}$

while (D) says $a_{17} = a_{15} + a_{15}$ (D) is incorrect

MOCK TEST

1. **(C)**

$\underbrace{B_1}_{10}$ $\underbrace{B_2}_{10}$ $\underbrace{B_3}_{10}$ .... $\underbrace{B_9}_{10}$

Selection of atleast one book

$\underbrace{(10+1)(10+1)......(10+1)}_{9\text{ times}} - 1 = 11^9 - 1$

2. maximum size of square can be 8 x 8

so required no. of square be $\sum_{r=1}^{8}(16-r)(9-r)$

$$= \sum_{r=1}^{8} r^2 - 25r + 144 = 456$$

3. **(C)**

Dashes	dots	arrangem
5	2	$^7C_2$
4	3	$^7C_3$
3	4	$^7C_4$
2	5	$^7C_5$
1	6	$^7C_6$
0	7	$^7C_7$

$= {}^7C_2 + {}^7C_3 + {}^7C_{4+} {}^7C_5 + {}^7C_6 + {}^7C_7 = 2^7 - 8 = 120$

4. Team X will win if it wins $(m + r)$th match and wins $m - 1$ match from the first $m + r - 1$ matches ,

so total no. of ways $= \sum_{r=0}^{m} {}^{m+r-1}C_{m-1} = \dfrac{{}^{2m}C_m}{2}$

5. **(C)**

Maximum no. of triangle

$$= \frac{3p(3p-1)(3p-2) - 3p(p-1)(p-2)}{6}$$

$$= \frac{p}{2}\,[9p^2 - 9p + 2 - p^2 + 3p - 2] = p\,[4p^2 - 3p]$$

$$= p^2\,[4p - 3]$$

6. First we select m friends for one table is ${}^{m+n}C_m$ and select a table by ${}^2C_1$ ways.
Now total number of arrangements is

$$2\,({}^{m+n}C_m)\,.\,\frac{(m-1)!}{2}\frac{(n-1)!}{2}$$

$$\Rightarrow \quad 2.\frac{(m+n)!}{m!\,n!}\frac{(m-1)!}{2}\frac{(n-1)!}{2} = 2.\frac{(m+n)!}{4mn}$$

7. **(A)**

Number of ways $= \dfrac{6!}{2!\times 4!} \times 2 = 30$

8. For P $\rightarrow$ If same species are different
Total number of arrangements is
${}^nP_2\,.\,(m+n-2)!$
For Q $\rightarrow$ If same species are alike then number of arrangement is $\dfrac{(m+n-2)!}{m!.(n-2)!}$

Hence $\dfrac{P}{Q} = {}^nP_2\,.\,m!\,.\,(n-2)! = {}^nP_2\,.\,{}^mP_m\,.\,(n-2)!$

9. **(C)**

S_1 : $1! = 1$
$1! + 2! = 3$
$1! + 2! + 3! = 9$
$1! + 2! + 3! + 4! = 33$
$1! + 2! + 3! + 4! + 5! = 153$
further in every number unit digit is 3 so it can't be a perfect square.
Hence only possible values are $x = 1$ or $x = 3$

S_2 : Total – both together
${}^{11}C_5 - {}^9C_3 = 378$

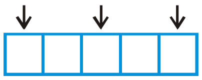
S_3 : $\underbrace{1\ \ 2\ \ 3\ \ 3}_{\frac{4!}{2!}\text{ways}}$ $\underbrace{4\text{ or }5}_{2\text{ ways}}$ $\underbrace{x\ \ 9-x}_{10\text{ ways}}$

Hence $12 \times 2 \times 10 = 240$

S_4 : **(i)** One digit number
number of 3's = 1.

(ii) Two digit number
one 3's _3 + 3_ = 8 + 9 = 17
two 3's 3 3 = 2
$\therefore$ number of 3's in two digit numbers is 17 + 2 = 19

(iii) Three digit numbers one 3's
3 _ _ + _ 3 _ + _ _ 3
= 9.9 + 8.9+8.9 = 225
two 3's 33_ + 3 _ 3 + _3 3
= 2(9+9+8) = 26.2 = 52
three 3's 333 = 3.1 = 3
$\therefore$ number of 3's in three digit numbers is 225 + 52 + 3 = 280
Hence answer is 1 + 19 + 280 = 300

10. 

There are 2M, 2T, 2A and 1 H, E, I, C, S
First find the number of ways if odd's no. position place be filled is ${}^5p_3 = 60$
Now Case I If even place words is same i.e no. of ways = 3
Case II If even place words is different i.e no. of ways = ${}^3c_2 \times 2! = 6$
Hence total no. of arragment is $60 \times (3 + 6) = 540$

11. First find no. of '2' at the end of (125)! is

(A) $\left[\frac{125}{2}\right]+\left[\frac{125}{2^2}\right]+\left[\frac{125}{2^3}\right]+\left[\frac{125}{2^4}\right]+\left[\frac{125}{2^5}\right]+\left[\frac{125}{2^6}\right]+\left[\frac{125}{2^7}\right]$

$= 62 + 31 + 15 + 7 + 3 + 1 + 0 = 119$

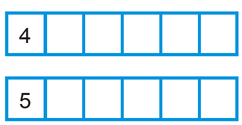
Find the number of '5' at the end of (125)!

is $\left[\frac{125}{5}\right]+\left[\frac{125}{5^2}\right]+\left[\frac{125}{5^3}\right]+\ldots\ldots$

$= 25 + 5 + 1 = 31$

Hence no. of zero is 31

(B) Total no. of signals can made by each arm = 10 so total number of different signals can be formed $= 10^{10} - 1$
(here – 1 is because if all arms are at the position of rest, then no signal will pass away)

(C) 

$\frac{5!}{2!} = 60$ $\qquad$ $\frac{5!}{2!.2!} = 30$

Total number of arrangement = 90

(D) Let number of player is n
then total number of games is ${}^nc_2 = 5050$
$\Rightarrow$ n = 101

12. Here the sum of the numbers are vanishes of six cards i.e
Case I : If selected 3 cards each of number –1 or 1

i.e The number of arrangement $= \frac{6!}{3!\,3!} = 20$

Case II : If selected 2 cards each of no. –1, 0 or 1 i.e

number of arrangement $= \frac{6!}{2!\,2!\,2!} = 90$

Case III : If selected one card each of number –1 and 1 and 4 cards of no. 0.

so no. of arrangement is $\frac{6!}{1!\,1!\,4!} = 30$

Case IV : If all cards selected fram the no. 0

So no. of arrangement is $\frac{6!}{6!} = 1$

Hence total no. of arrangement is
$20 + 90 + 30 + 1 = 141$

13. **(A)**

two 2's	five 1's	
two 2's	four 1's	one 3
two 2's	three 1's	two 3's
two 2's	two 1's	three 3's
two 2's	one1	four 3's
two 2's	five 3's	

$= 2\left(\frac{7!}{2!\;5!}+\frac{7!}{2!\;4!}+\frac{7!}{2!\;3!\;2!}\right) = 672$

14. **(D)**
D P M L can be arranged in 4 ! ways & the two gaps out of 5 gaps can be selected in ${}^5C_2$ ways. {A A and E E} or {A E and A E} can be placed in 6 ways.
Total = $4!.\,{}^5C_2.\,6 = 1440$

15. **(A,D)**
All AAAAA BBB D EEF can be arranged in

$\frac{12!}{5!\,3!\,2!}$ ways

Between the gaps C can be arranged in ${}^{13}C_3$ ways

Total ways $= {}^{13}C_3 \times \frac{12!}{5! \times 3! \times 2!}$

Number of ways
= without considering separation of C – in which all C's are together – in which exactly two C's are

together $= \frac{15!}{5!(3!)^2\,2!} - \frac{13!}{5!\,3!\,2!} - \frac{12!}{5!\,3!}\,{}^{13}C_2$

16. **(D)**

If $\left[\frac{1}{3}+\frac{\lambda}{50}\right] = 0$

$\Rightarrow \quad 0 \le \frac{1}{3}+\frac{\lambda}{50} < 1$

$\Rightarrow \quad -\frac{1}{3} \le \frac{\lambda}{50} < \frac{2}{3}$ or $-\frac{50}{3} \le \lambda < \frac{2}{3}$

$\therefore \quad 1 \le \lambda \le 33$

If $\left[\frac{1}{3} + \frac{\lambda}{50}\right] = 1,\ 1 \le \frac{1}{3} + \frac{\lambda}{50} < 2$

$\Rightarrow \quad \frac{2}{3} \le \frac{\lambda}{50} < \frac{5}{3}$

$\Rightarrow \quad \frac{100}{3} \le \lambda < \frac{250}{3}$

$\therefore \quad 34 \le \lambda \le 83$

$\therefore \quad E = (0 + 0 + 0 \ldots. + 33 \text{ times }) +$

$(1 + 1 + 1 + \ldots\ldots + 17 \text{ times })$

$= 0 + 17 = 0 + 17$

$= 17$ (which a prime number)

17. **(A)**

Sum of the digits in the tens places

= sum of the digits in the unit's place

$= (4 - 1)!\ (2 + 3 + 4 + 5)$

$= 6.14 = 84$

18. **(D)**

$A = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$

Two different numbers for numerator and denominator from these can be obtained in ${}^{10}P_2$

$= 10.9 = 90$ ways and if $\frac{p}{p}$ or $\frac{q}{q} = 1$

(If numerator and denominator same)

$\therefore$ Number of ways $= 90 + 1 = 91$

19. **(A)**

Number of numbers formed by using 1, 2, 3, 4, 5 is

$= 5! = 120$

Number of numbers formed by using 0, 1, 2, 4, 5 is

$= 4.4.3.2.1 = 96$

Total number of numbers formed, which are divisible by 3 (taking numbers without repitition) is = 216

STATEMENT 1 $\rightarrow$ Incorrect

STATEMENT 2 $\rightarrow$ Correct

20. **(B)**

3^m gives 1 at unit place if $m = 4k$

$\Rightarrow$ these are 5 in number

7^n gives 9 at unit place if $n = 4\ell + 2$

$\Rightarrow$ these are 5 in number

$\therefore$ number of ways = 25

3^m gives 3 at unit place if $m = 4k + 1$

$\Rightarrow$ these are 5 in number

7^n gives 7 at unit place if $n = 4\ell + 1$

$\Rightarrow$ these are 5 in number

$\therefore$ number of ways = 25

3^m gives 7 at unit place if $m = 4k + 3$

$\Rightarrow$ these are 5 in number

7^n gives 3 at unit place if $n = 4\ell + 3$

$\Rightarrow$ these are 5 in number

$\therefore$ number of ways = 25

3^m gives 9 at unit place if $m = 4k + 2$

$\Rightarrow$ these are 5 in number

7^n gives 1 at unit place if $n = 4\ell$

$\Rightarrow$ these are 5 in number

$\therefore$ number of ways = 25

$\therefore$ total number of ways = 100

$\therefore$ statement-1 is true

statement-2

if $m = 4k + 2$, then 3^m gives 9 at units place

if $n = 4\ell$, then 7^n gives 1 at units place

$\therefore$ statement-2 is true but does not explain statement-1

21. **(A) $\rightarrow$ (r), (B) $\rightarrow$ (t), (C) $\rightarrow$ (s), (D) $\rightarrow$ (p)**

(a) Two cases

(i) 5, 4, 1, 1, 1 $\quad \frac{5!}{3!} = 20$

(ii) 5, 2, 2, 1, 1 $\quad \frac{5!}{2!\,2!} = 30$

Total 20 + 30 = 50

(b) $5! - D_5 - 5 . D_4$.

(D_5 stands for dearrangements of 5 things)

$= 120 - 44 - 5 \times 9 = 31$

(c) $1000 - 9^3 - 9^2 - 9 = 181$

(d) $\frac{1}{15} \sum_{1 \le i \le j \le 9} \sum i . j$

$$= \frac{1}{15}\left\{\sum_{1 \le i < j \le 9} \sum i . j + (1^2 + 2^2 + \ldots\ldots + 9^2)\right\}$$

$$= \frac{1}{15}\left\{\frac{(1+2+........+9)^2-(1^2+2^2+...+9^2)}{2}+(1^2+2^2+...+9^2)\right\}$$

$$= \frac{1}{30}\left\{\left(\frac{9\times10}{2}\right)^2+\frac{9\times10\times19}{6}\right\}=77$$

22. **(A) → (r) ; (B) → (s) ; (C) → (q) ; (D) → (p)**

(A) Select m out of n = ${}^nC_m$
number of ways of arranging in increasing order = 1
Hence ${}^nC_m$

(B) A monkey has m choice

$\therefore$ $\underbrace{m\times m\times..m}_{n\ \text{times}}=m^n$

(C) Arrange (m –1) green balls then out of m gaps select n positions for red balls and arrange red balls = 1. ${}^mC_n$. 1
= ${}^mC_n$

(D) $\underbrace{n\times n\times..n}_{m\ \text{times}}=n^m$

23.
1 **(B)**
Since there are 5 even places and 3 pairs of repeated letters therefore at least one of these must be at an odd place.

$\therefore$ the number of ways = $\frac{11!}{2!\ 2!\ 2!}$

2 **(A)**
Make a bundle of both M's and another bundle of T's. Then except A's we have 5 letters remaining so M's, T's and the letters except A's can be arranged in 7 ! ways
$\therefore$ total number of arrangements = 7 ! × ${}^8C_2$

3 **(A)**

Consonants can be placed in $\frac{7!}{2!2!}$ ways

Then there are 8 places and 4 vowels

$\therefore$ Number of ways = $\frac{7!}{2!2!}\cdot{}^8C_4\ \frac{4!}{2!}$

24.
1. **(D)**

3	1	1	⇒	${}^3C_1$
2	2	1	⇒	${}^3C_1$

$\therefore$ ${}^3C_1+{}^3C_1=6$

2. **(B)**

□ □ □

3 1 1 ⇒ $\frac{5!}{3!1!1!2!}=10$

2 2 1 ⇒ $\frac{5!}{1!2!2!2!}=15$

$\therefore$ 10 + 15 = 25

3 **(A)**

3	1	1	⇒	1
2	2	1	⇒	1

$\therefore$ 1 + 1 = 2

25.
1. **(B)**
Since n = 4
$\therefore$ Number of matched arrangements

$$=\frac{8!}{4!\ 5!}=\frac{{}^8C_4}{5}$$

2. **(A)**
n = 3
$\therefore$ Number of matched arrangements

$$=\frac{6!}{3!4!}$$

Total number of arrangements = $\frac{6!}{3!3!}$

$\therefore$ probability = $\frac{1}{4}$

3. **(B)**
Since first pair is matched and it can be done in 1 way
$\therefore$ for mismatched pairs n = 4
$\therefore$ Number of mismatched pairs = ${}^8C_5$

26. Here, we note the following.
1. A door will open if it face odd no of changes.
2. No. of changes faced by any door will be equal to no. of factors of the door no.
3. So only those door will open, whose number is perfect square so ans is $[\sqrt{n}]$,
[where [ ] denotes the G.I.F.]

27. **(16)**

10 IIT students $T_1, T_2, \ldots, T_{10}$ can be arranged in 10 ! ways. Now the number of ways in which two PET student can be placed will be equal to the number of ways in which 3 consecutive IIT students can be taken i.e. in 8 ways and can be arranged in two ways $\Rightarrow$ (10!) (8) (2!).

Alternatively 3 IIT students can be selected in ${}^{10}C_3$ ways. Now each selection of 3 IIT and 2 PET students in $P_1 T_1 T_2 T_3 P_2$ can be arranged in (2 !) (3!) ways. Call this box X. Now this X and the remaining IIT students can be arranged in 8! ways

$\Rightarrow$ Total ways ${}^{10}C_3$ (2 !) (3 !) (8 !)

28. In one round, marked numbers are 1, 16, 31, ..., 991

$\rightarrow$ 67 numbers

In second round marked numbers are 6, 21, 36, ..., 996

$\rightarrow$ 67 numbers

In third round marked numbers are 11, 26, 41, ..., 986

$\rightarrow$ 66 numbers

the next number will be 1 which has already been marked

$\therefore$ total marked numbers = 67 + 67 + 66 = 200

$\therefore$ unmarked numbers = 1000 – 200 = 800

29. **(4)**

Make 1 group of 2 persons, 1 group of 4 persons and 3 group of 3 persons among 15 persons (except 2 particular persons)

hence by grouping method = $\dfrac{15!}{2!\,4!\,(3!)^3\,3!}$

Now we add that 2 person in the group of 2 persons and thus number of arrangement of these groups into cars and autos is

$$\frac{15!}{2!\,4!\,(3!)^3\,3!} \times 2! \times 3! = \frac{15!}{4!\,(3!)^3}$$

30. The given equation is $x + y + z = 20$ ....**(1)**

We have to find the number of different values of x, y, z

Such that $x \neq y \neq z$ and $x, y, z \geq 1$

Let us assume that $x < y < z$

and $x = x_1,\ y - x = x_2$ and $z - y = x_3$

then $x = x_1;\ y = x_1 + x_2$ and $z = x_1 + x_2 + x_3$

also $x_1, x_2, x_3 \geq 1$

Substitution these values in (1) we get

$3x_1 + 2x_2 + x_3 = 20$ ....**(2)**

Where $x_1, x_2, x_3 \geq 1$

Now No. of solution of equation (2) is

$= \text{Co-efficient of } x^{20} \text{ in } (x^3 + x^6 + x^9 + \ldots.) \times (x^2 + x^4 + x^6 + \ldots.) \times (x + x^2 + x^3 + \ldots)$

$= \text{Co-efficient of } x^{14} \text{ in } (1 + x^3 + x^6 + x^9 + \ldots.)(1 + x^2 + x^4 + \ldots.) \times (1 + x + x^2 + \ldots.)$

$= (1 + x^2 + x^3 + x^4 + x^5 + 2x^6 + x^7 + 2x^8 + 2x^9 + 2x^{10} + 2x^{11} + 3x^{12} + 2x^{13} + 3x^{14} + 3x^{15} + \ldots.)(1 + x + x^2 + x^3 + \ldots\ldots.)$

$= \text{Co-efficient of } x^{14} \text{ is } 1 + 1 + 1 + 1 + 1 + 2 + 1 + 2 + 2 + 2 + 2 + 3 + 2 + 3$

$= 24$

But x , y and z are arranged in 3! ways

So Required no of solution = 24 × 6 = 144