SOLVED EXAMPLES

Ex. 1 If $49^n + 16n + \lambda$ is divisible by 64 for all $n \in N$, then find the least negative integral value of λ.

Sol. For $n = 1$, we have

$49^n + 16n + \lambda = 49 + 16 + \lambda = 65 + \lambda$

$= 64 + (\lambda + 1)$, which is divisible by 64 if $\lambda = -1$

For $n = 2$, we have

$49^n + 16n + \lambda = 49^2 + 16 \times 2 + \lambda = 2433 + \lambda$

$= (64 \times 38) + (\lambda + 1)$, which is divisible by 64 if $\lambda = -1$

Hence, $\lambda = -1$

Ex. 2 Prove that $n^2 + n$ is even for all natural numbers n.

Sol. Let P(n) be $n^2 + n$ is even

P(1) is true as $1^2 + 1 = 2$ is an even number.

Let P(k) be true.

To Prove: P(k + 1) is true.

P(k + 1) states that $(k + 1)^2 + (k + 1)$ is even.

Now $(k + 1)^2 + (k + 1)$

$= k^2 + 2k + 1 + k + 1$

$= k^2 + k + 2k + 2$ (rearranging terms)

$= 2\lambda + 2k + 2$ (Since P(k) is true, $k^2 + k$ is an even number, or can be written as 2λ, where λ is some natural number)

$= 2(\lambda + k + 1)$

= a multiple of 2.

thus, $(k + 1)^2 + (k + 1)$ is an even number, or P(k + 1) is true when P(k) is true.

Hence, by PMI, P(n) is true for all n, where n is a natural number.

Ex. 3 Prove that $x^{2n-1} + y^{2n-1}$ is divisible by x + y for all n ε N.

Sol. Let P(n) be the given statement.

P(1) is clearly true, as x + y is divisible by x + y.

Let P(k) be true. Thus, $x^{2k-1} + y^{2k-1} = (x + y)\lambda$

Consider $x^{2(k+1)-1} + y^{2(k+1)-1}$

$= x^{2k-1}.x^2 + y^{2k-1}.y^2$

$= ((x + y)\lambda - y^{2k-1})x^2 + y^{2k-1}.y^2$ (Since P(k) is true)

$= (x + y)\lambda x^2 + y^{2k-1}(y^2 - x^2)$ $\Rightarrow (x + y)\lambda x^2 + y^{2k-1}(y - x)(y + x)$

$= (x + y)(\lambda x^2 + y^{2k-1}(y - x))$ $\Rightarrow$ divisible (x + y)

Hence, P(k + 1) is true when P(k) is true. Thus, P(n) is true for all n e N by PMI.

Ex. 4 Prove that : $1+2+3+...+n<\frac{(2n+1)^2}{8}$ for all $n \in N$.

Sol. Let P(n) be the statement given by

$$P(n): 1+2+3+...+n<\frac{(2n+1)^2}{8}$$

Step-I We have

$$P(1): 1<\frac{(2\times1+1)^2}{8} \qquad \because 1<\frac{(2\times1+1)^2}{8}=\frac{9}{8} \qquad \therefore P(1) \text{ is true}$$

Step-II Let P(m) be true, then

$$1+2+3+...+m<\frac{(2m+1)^2}{8} \qquad ...(i)$$

We shall now show that P(m + 1) is true i.e.

$$1+2+3+...+m+(m+1)<\frac{[2(m+1)+1]^2}{8}$$

Now P(m) is true

$$\Rightarrow 1+2+3+...+m<\frac{(2m+1)^2}{8}$$

$$\Rightarrow 1+2+3+...+m+(m+1)<\frac{(2m+1)^2}{8}+(m+1)$$

$$\Rightarrow 1+2+3+...+m+(m+1)<\frac{(2m+1)^2+8(m+1)}{8}$$

$$\Rightarrow 1+2+3+...+m+(m+1)<\frac{(4m^2+12m+9)}{8}$$

$$\Rightarrow 1+2+3+...+m+(m+1)<\frac{(2m+3)^2}{8}=\frac{[2(m+1)+1]^2}{8}$$

$\therefore$ P(m + 1) is true

thus P(m) is true $\Rightarrow$ P(m + 1) is true

Hence by the principle of mathematical induction P(n) is true for all $n \in N$

Ex.5 Prove that $\cos\alpha \cos2\alpha \cos4\alpha .... \cos(2^{n-1}\alpha)=\frac{\sin 2^n\alpha}{2^n \sin\alpha}$

Sol. Let P(n) be the given statement.

Clearly, P(1) is true. (Expand $\sin 2\alpha$ on RHS to verify this)

Let P(k) be true.

Consider $\cos\alpha \cos2\alpha \cos4\alpha .... \cos(2^{k-1}\alpha)\cos(2^{k+1-1}\alpha)$

$$=\frac{\sin 2^k\alpha}{2^k\sin\alpha}\cos2^k\alpha \Rightarrow \frac{2\sin 2^k\alpha\cos 2^k\alpha}{2.2^k\sin\alpha}$$

$$=\frac{\sin 2^{k+1}\alpha}{2^{k+1}\sin\alpha} \qquad (\text{Using } \sin2\theta=2\sin\theta\cos\theta)$$

Hence, P(k + 1) is true when P(k) is true. By PMI, for all natural numbers n, P(n) is true.

Ex. 6 $\frac{3}{4}+\frac{15}{16}+\frac{63}{64}+$ .... to n terms =

(1) $n-\frac{4^n}{3}-\frac{1}{3}$ (2) $n+\frac{4^{-n}}{3}-\frac{1}{3}$ (3) $n+\frac{4^n}{3}-\frac{1}{3}$ (4) $n-\frac{4^{-n}}{3}+\frac{1}{3}$

Sol. For n = 1, we have

$n-\frac{4^n}{3}-\frac{1}{3}=1-\frac{4}{3}-\frac{1}{3}=-\frac{2}{3}$ $n+\frac{4^{-n}}{3}-\frac{1}{3}=1+\frac{4^{-1}}{3}-\frac{1}{3}=\frac{3}{4}$

$n+\frac{4^n}{3}-\frac{1}{3}=1+\frac{4}{3}-\frac{1}{3}=2$ $n-\frac{4^{-n}}{3}+\frac{1}{3}=1-\frac{4^{-1}}{3}+\frac{1}{3}=\frac{5}{4}$

Also, for n = 2, we have

$$n+\frac{4^{-n}}{3}-\frac{1}{3}=2+\frac{1}{48}-\frac{1}{3}=\frac{27}{16} \text{ and } \frac{3}{4}+\frac{15}{16}=\frac{27}{16}$$

Hence, option (2) is correct

Ex. 7 Prove that

$7 + 77 + 777 + 7777 + 777\ldots7(\text{n digits}) = 7(10^{n+1} - 9n - 10)/81$

Sol. Let P(n) be the given statement.

Clearly, P(1) is true.

Let P(k) be true.

To Prove : P(k + 1) is true.

Consider the LHS of P(k + 1)

$7 + 77 + 777 + 7777 + \ldots + 777\ldots7(\text{k digits}) + 777\ldots7$ (k + 1 digits)

$= 7(10^{k+1} - 9k - 10)/81 + 777\ldots7(k + 1 \text{ digits})$

$= 7(10^{k+1} - 9k - 10 + 81\text{x}111\ldots1(k + 1 \text{ digits}))/81$

Now, 111…1 = 1 + 10 + 100 + 1000 +…(upto k + 1 terms). This is a Geometric Progression with a = 1,

r = 10. Hence 111 ... 1(k + 1 digits) = $\frac{1(10^{k+1}-1)}{10-1}=\frac{(10^{k+1}-1)}{9}$

Thus, LHS becomes

$= 7(10^{k+1} - 9k - 10 + 9(10^{k+1} - 1))/81$

$= 7(10^{k+1}(1 + 9) - 9k - 9 - 10)/81$

$= 7(10^{k+2} - 9(k + 1) - 10)/81$

Hence, P(k+1) is true when P(k) is true. Thus, by PMI, P(n) is true for all n, where n is a Natural Number.

Ex. 8 If P(n) is the statement "$2^{3n} - 1$ is an integral multiple of 7", and if P(r) is true, prove that P(r + 1) is true.

Sol. Let P(r) be true. Then $2^{3r} - 1$ is an integral multiple of 7.

We wish to prove that P(r + 1) is true i.e. $2^{3(r+1)} - 1$ is an integral multiple of 7.

Now P(r) is true

$\Rightarrow$ $2^{3r} - 1$ is an integral multiple of 7 $\Rightarrow$ $2^{3r} - 1 = 7\lambda$ for some $\lambda \in N$

$\Rightarrow$ $2^{3r} = 7\lambda + 1$ ...(i)

Now $2^{3(r+1)} - 1 = 2^{3r} \cdot 2^3 - 1 = (7\lambda + 1) \times 8 - 1$

$\Rightarrow$ $2^{3(r+1)} - 1 = 56\lambda + 8 - 1 = 56\lambda + 7 = 7(8\lambda + 1)$ $\Rightarrow$ $2^{3(r+1)} - 1 = = 7\mu$, where $\mu = 8\lambda + 1 \in N$

$\Rightarrow$ $2^{3(r+1)} - 1$ is an integral multiple of 7 $\Rightarrow$ P(r + 1) is true

Ex. 9 Consider the sequence of real numbers defined by the relations

$$x_1 = 1 \text{ and } x_{n+1} = \sqrt{1 + 2x_n} \text{ for } n \geq 1.$$

Use the Principle of Mathematical Induction to show that $x_n < 4$ for all $n \geq 1$.

Sol. For any $n \geq 1$, let P_n be the statement that $x_n < 4$.

Base Case The statement P_1 says that $x_1 = 1 < 4$, which is true.

Inductive Step Fix $k \geq 1$, and suppose that P_k holds, that is, $x_k < 4$.

It remains to show that P_{k+1} holds, that is, that $x_{k+1} < 4$.

$$x_{k+1} = \sqrt{1 + 2x_k}$$
$$< \sqrt{1 + 2(4)}$$
$$= \sqrt{9}$$
$$= 3$$
$$< 4.$$

Therefore, P_{k+1} holds.

Thus by the principle of mathematical induction, for all $n \geq 1$, P_n holds.

Ex. 10 For $n \in N$, $x^{n+1} + (x + 1)^{2n-1}$ is divisible by -

(1) x **(2)** x + 1 **(3)** $x^2 + x + 1$ **(4)** $x^2 - x + 1$

Sol. For n = 1, we have

$x^{n+1} + (x + 1)^{2n-1} = x^2 + (x + 1) = x^2 + x + 1$, which is divisible by $x^2 + x + 1$.

For n = 2, we have

$x^{n+1} + (x + 1)^{2n-1} = x^3 + (x + 1)^3 = (2x + 1)(x^2 + x + 1)$, which is divisible by $x^2 + x + 1$

Hence, option (3) is true.

Ex. 11 Let $p_0 = 1$, $p_1 = \cos\theta$ (for θ some fixed constant) and $p_{n+1} = 2p_1p_n - p_{n-1}$ for $n \geq 1$. Use an extended Principle of Mathematical Induction to prove that $p_n = \cos(n\theta)$ for $n \geq 0$.

Sol. For any $n \geq 0$, let P_n be the statement that $p_n = \cos(n\theta)$.

Base Cases The statement P_0 says that $p_0 = 1 = \cos(0\theta) = 1$, which is true. The statement P_1 says that $p_1 = \cos\theta = \cos(1\theta)$, which is true.

Inductive Step Fix $k \geq 0$, and suppose that both P_k and P_{k+1} hold, that is, $p_k = \cos(k\theta)$, and $p_{k+1} = \cos((k+1)\theta)$.

It remains to show that P_{k+2} holds, that is, that $p_{k+2} = \cos((k+2)\theta)$.

We have the following identities:

$$\cos(a + b) = \cos a \cos b - \sin a \sin b$$

$$\cos(a - b) = \cos a \cos b + \sin a \sin b$$

Therefore, using the first identity when $a = \theta$ and $b = (k + 1)\theta$, we have

$$\cos(\theta + (k+1)\theta) = \cos\theta \cos(k+1)\theta - \sin\theta \sin(k+1)\theta,$$

and using the second identity when $a = (k + 1)\theta$ and $b = \theta$, we have

$$\cos((k+1)\theta - \theta) = \cos(k+1)\theta \cos\theta + \sin(k+1)\theta \sin\theta.$$

Therefore,

$$
\begin{aligned}
p_{k+2} &= 2p_1p_{k+1} - p_k \\
&= 2(\cos\theta)(\cos((k+1)\theta)) - \cos(k\theta) \\
&= (\cos\theta)(\cos((k+1)\theta)) + (\cos\theta)(\cos((k+1)\theta)) - \cos(k\theta) \\
&= \cos(\theta + (k+1)\theta) + \sin\theta \sin(k+1)\theta + (\cos\theta)(\cos((k+1)\theta)) - \cos(k\theta) \\
&= \cos((k+2)\theta) + \sin\theta \sin(k+1)\theta + (\cos\theta)(\cos((k+1)\theta)) - \cos(k\theta) \\
&= \cos((k+2)\theta) + \sin\theta \sin(k+1)\theta + \cos((k+1)\theta - \theta) - \sin(k+1)\theta \sin\theta - \cos(k\theta) \\
&= \cos((k+2)\theta) + \cos(k\theta) - \cos(k\theta) \\
&= \cos((k+2)\theta).
\end{aligned}
$$

Therefore P_{k+2} holds.

Thus by the principle of mathematical induction, for all $n \geq 1$, P_n holds.

Ex. 12 Prove that for any positive integer number n, $n^3 + 2n$ is divisible by 3

Sol. Statement P (n) is defined by

$n^3 + 2n$ is divisible by 3

STEP 1 : We first show that p (1) is true. Let $n = 1$ and calculate $n^3 + 2n$

$$1^3 + 2(1) = 3$$

3 is divisible by 3

Hence p (1) is true.

STEP 2 : We now assume that p(k) is true

$k^3 + 2k$ is divisible by 3

is equivalent to

$k^3 + 2k = 3M$, where M is a positive integer.

We now consider the algebraic expression $(k + 1)^3 + 2(k + 1)$; expand it and group like terms

$$(k+1)^3 + 2(k+1) = k^3 + 3k^2 + 5k + 3$$

$$= [k^3 + 2k] + [3k^2 + 3k + 3] \quad \Rightarrow \quad 3M + 3[k^2 + k + 1] = 3[M + k^2 + k + 1]$$

Hence $(k + 1)^3 + 2(k + 1)$ is also divisible by 3 and therefore statement $P(k + 1)$ is true.

Ex. 13 Prove by the principle of mathematical induction that for all $n \in N$:

$$1 + 4 + 7 + \ldots + (3n - 2) = \frac{1}{2} n(3n - 1)$$

Sol. Let P(n) be the statement given by

$$P(n) : 1 + 4 + 7 + \ldots + (3n - 2) = \frac{1}{2} n(3n - 1)$$

Step-I We have $P(1) : 1 = \frac{1}{2} \times (1) \times (3 \times 1 - 1)$

$\because 1 = \frac{1}{2} \times (1) \times (3 \times 1 - 1)$

So, P(1) is true

Step-II Let P(m) be true, then

$$1 + 4 + 7 + \ldots + (3m - 2) = \frac{1}{2} m(3m - 1) \qquad \text{...(i)}$$

We wish to show that P(m + 1) is true. For this we have to show that

$$1 + 4 + 7 + \ldots + (3m - 2) + [3(m + 1) - 2] = \frac{1}{2}(m + 1)(3(m + 1) - 1)$$

Now $1 + 4 + 7 + \ldots + (3m - 2) + [3(m + 1) - 2]$

$= \frac{1}{2} m(3m - 1) + [3(m + 1) - 2]$ [Using **(i)**]

$= \frac{1}{2} m(3m - 1) + (3m + 1) = \frac{1}{2}[3m^2 - m + 6m + 2]$

$= \frac{1}{2}[3m^2 + 5m + 2] = \frac{1}{2}(m + 1)(3m + 2) = \frac{1}{2}(m + 1)[3(m + 1) - 1]$

$\therefore$ P(m + 1) is true

Thus P(m) is true $\Rightarrow$ P(m + 1) is true

Hence by the principle of mathematical induction the given result is true for all $n \in N$.

Ex. 14 Prove that $n! > 2^n$ for n a positive integer greater than or equal to 4. (Note: n! is n factorial and is given by 1 * 2 * ...* (n – 1)*n.)

Sol. Statement P (n) is defined by $n! > 2^n$

STEP 1 : We first show that p(4) is true. Let n = 4 and calculate 4 ! and 2^n and compare them

$4! = 24$

$2^4 = 16$

24 is greater than 16 and hence p (4) is true.

STEP 2 : We now assume that p (k) is true

$k! > 2^k$

Multiply both sides of the above inequality by k + 1

$k!\,(k + 1) > 2^k\,(k + 1)$

The left side is equal to (k + 1)!. For k >, 4, we can write

$k + 1 > 2$

Multiply both sides of the above inequality by 2^k to obtain

$2^k (k + 1) > 2 * 2^k$

The above inequality may be written

$2^k (k + 1) > 2^{k+1}$

We have proved that $(k + 1)! > 2^k (k + 1)$ and $2^k (k + 1) > 2$

$^{k+1}$we can now write

$(k + 1)! > 2^{k+1}$

We have assumed that statement P(k) is true and proved that statement P(k + 1) is also true.

Ex. 15 Prove by the principle of mathematical induction that for all n ∈ N,

$$\sin\theta + \sin 2\theta + \sin 3\theta + \ldots + \sin n\theta = \frac{\sin\left(\frac{n+1}{2}\right)\theta\,\sin\frac{n\theta}{2}}{\sin\frac{\theta}{2}}$$

Sol. Let P(n) be the statement given by

$$P(n) : \sin\theta + \sin 2\theta + \sin 3\theta + \ldots + \sin n\theta = \frac{\sin\left(\frac{n+1}{2}\right)\theta\sin\frac{n\theta}{2}}{\sin\frac{\theta}{2}}$$

Step-I We have $P(1) : \sin\theta = \dfrac{\sin\left(\frac{1+1}{2}\right)\theta\sin\left(\frac{1\times\theta}{2}\right)}{\sin\frac{\theta}{2}}$

$\because$ $$\sin\theta = \frac{\sin\left(\frac{1+1}{2}\right)\theta\cdot\sin\left(\frac{1\times\theta}{2}\right)}{\sin\frac{\theta}{2}}$$

$\therefore$ P(1) is true

Step-II Let P(m) be true, then

$$\sin\theta + \sin 2\theta + \ldots + \sin m\theta = \frac{\sin\left(\frac{m+1}{2}\right)\theta\sin\frac{m\theta}{2}}{\sin\frac{\theta}{2}} \qquad \ldots(i)$$

We shall now show that P(m + 1) is true

i.e. $$\sin\theta + \sin 2\theta + \ldots + \sin m\theta + \sin(m+1)\theta = \frac{\sin\left(\frac{(m+1)+1}{2}\right)\theta\sin\left(\frac{m+1}{2}\right)\theta}{\sin\frac{\theta}{2}}$$

We have $\sin\theta + \sin 2\theta + \ldots + \sin m\theta + \sin(m+1)\theta$

$$= \frac{\sin\left(\frac{m+1}{2}\right)\theta \sin\frac{m\theta}{2}}{\sin\frac{\theta}{2}} + \sin(m+1)\theta \qquad \text{[Using (i)]}$$

$$= \frac{\sin\left(\frac{m+1}{2}\right)\theta \sin\frac{m\theta}{2}}{\sin\frac{\theta}{2}} + 2\sin\left(\frac{m+1}{2}\right)\theta \cos\left(\frac{m+1}{2}\right)\theta$$

$$= \sin\left(\frac{m+1}{2}\right)\theta \left\{\frac{\sin\left(\frac{m\theta}{2}\right)}{\sin\frac{\theta}{2}} + 2\cos\left(\frac{m+1}{2}\right)\theta\right\}$$

$$= \sin\left(\frac{m+1}{2}\right)\theta \left\{\frac{\sin\left(\frac{m\theta}{2}\right) + 2\sin\frac{\theta}{2}\cos\left(\frac{m+1}{2}\right)\theta}{\sin\frac{\theta}{2}}\right\}$$

$$= \sin\left(\frac{m+1}{2}\right)\theta \left\{\frac{\sin\left(\frac{m\theta}{2}\right) + \sin\left(\frac{m+2}{2}\right)\theta - \sin\frac{m\theta}{2}}{\sin\frac{\theta}{2}}\right\}$$

$$= \frac{\sin\left(\frac{m+1}{2}\right)\theta \sin\left(\frac{m+2}{2}\right)\theta}{\sin\frac{\theta}{2}} = \frac{\sin\left\{\frac{(m+1)+1}{2}\right\}\theta \sin\left(\frac{m+1}{2}\right)\theta}{\sin\frac{\theta}{2}}$$

$\therefore$ P(m + 1) is true

Thus, P(m) is true $\Rightarrow$ P(m + 1) is true

Hence by principle mathematical induction P(n) is true for all $n \in N$

Ex. 16 If $n \in N$ and $n > 1$, then

(1) $n! > \left(\frac{n+1}{2}\right)^n$ **(2)** $n! \geq \left(\frac{n+1}{2}\right)^n$ **(3)** $n! < \left(\frac{n+1}{2}\right)^n$ **(4)** None of these

Sol. When n = 2 then

$$n = 2, \left(\frac{n+1}{2}\right)^n = \frac{9}{4} \quad \Rightarrow \quad n! < \left(\frac{n+1}{2}\right)^n$$

When $n = 3$, then $n! = 6$, $\left(\frac{n+1}{2}\right)^n = 8$

$\Rightarrow \quad n! < \left(\frac{n+1}{2}\right)^n$

When $\quad n = 4$, then $n! = 24$, $\left(\frac{n+1}{2}\right)^n = \frac{625}{16}$

$\Rightarrow \quad n! < \left(\frac{n+1}{2}\right)^n$

$\therefore$ it is seen that $\Rightarrow \quad n! < \left(\frac{n+1}{2}\right)^n$

Ex. 17 Use mathematical induction to prove De Moivre's theorem

$$[R(\cos t + i \sin t)]^n = R^n(\cos nt + i \sin nt)$$

for n a positive integer.

Sol. **STEP 1 :** For n = 1

$$[R(\cos t + i \sin t)]^1 = R^1(\cos 1*t + i \sin 1*t)$$

It can easily be seen that the two sides are equal.

STEP 2 : We now assume that the theorem is true for n = k,

Hence

$$[R(\cos t + i \sin t)]^k = R^k(\cos kt + i \sin kt)$$

Multiply both sides of the above equation by R (cos t + i sin t)

$$[R(\cos t + i \sin t)]^k R(\cos t + i \sin t) = R^k(\cos kt + i \sin kt)$$

$$R(\cos t + i \sin t)$$

Rewrite the above as follows

$$[R(\cos t + i \sin t)]^{k+1} = R^{k+1}[(\cos kt \cos t - \sin kt \sin t) + i(\sin kt \cos t + \cos kt \sin t)]$$

Trigonometric identities can be used to write the trigonometric expressions (cos kt cos t – sin kt sin t) and (sin kt cos t + cos kt sin t) as follows

$$(\cos kt \cos t - \sin kt \sin t) = \cos(kt + t) = \cos(k + 1)t$$

$$(\sin kt \cos t + \cos kt \sin t) = \sin(kt + t) = \sin(k + 1)t$$

Substitute the above into the last equation to obtain

$$[R(\cos t + i \sin t)]^{k+1} = R^{k+1}[\cos(k + 1)t + \sin(k + 1)t]$$

It has been established that the theorem is true for n=1 and that if it assumed true for n=k it is true for n=k+1.

Ex. 18 Prove by the principle of mathematical induction that for all $n \in N$, 3^{2n} when divided by 8 the remainder is always 1.

Sol. Let P(n) be the statement given by

P(n) : 3^{2n} when divided by 8 the remainder is 1

or $\quad$ P(n) : $3^{2n} = 8\lambda + 1$ for some $\lambda \in N$

Step-I $P(1) : 3^2 = 8\lambda + 1$ for some $\lambda \in N$

$\because\ 3^2 = 8 \times 1 + 1 = 8\lambda + 1$ where $\lambda = 1$

$\therefore$ P(1) is true

Step-II Let P(m) be true then

$3^{2m} = 8\lambda + 1$ for some $\lambda \in N$

We shall now show that P(m + 1) is true for which we have to show that $3^{2(m+1)}$ when divided by 8 the remainder is 1 i.e. $3^{2(m+1)} = 8\mu + 1$ for some $\mu \in N$

Now $3^{2(m+1)} = 3^{2m} \cdot 3^2 = (8\lambda + 1) \times 9$ [Using **(i)**]

$\Rightarrow$ P(m + 1) is true

thus P(m) is true $\Rightarrow$ P(m + 1) is true

Hence by the principle of mathematical induction P(n) is true for all $n \in N$ i.e. 3^{2n} when divided by 8 the remainder is always 1.

Ex. 19 Using the principle of mathematical induction, prove that

$1^2 + 2^2 + 3^2 + ... + n^2 = (1/6)\{n(n+1)(2n+1\}$ for all $n \in N$.

Sol. Let the given statement be P(n). Then,

P(n): $1^2 + 2^2 + 3^2 + ... + n^2 = (1/6)\{n(n+1)(2n+1)\}$.

Putting n =1 in the given statement, we get

LHS = $1^2 = 1$ and RHS = $(1/6) \times 1 \times 2 \times (2 \times 1 + 1) = 1$.

Therefore LHS = RHS.

Thus, P(1) is true.

Let P(k) be true. Then,

P(k): $1^2 + 2^2 + 3^2 + ... + k^2 = (1/6)\{k(k+1)(2k+1)\}$.

Now, $1^2 + 2^2 + 3^2 + ... + k^2 + (k+1)^2$

$= (1/6)\ \{k(k+1)(2k+1) + (k+1)^2$

$= (1/6)\{(k+1)\ .\ (k(2k+1) + 6(k+1))\}$

$= (1/6)\{(k+1)(2k^2 + 7k + 6\})$

$= (1/6)\{(k+1)(k+2)(2k+3)\}$

$= 1/6\{(k+1)(k+1+1)[2(k+1)+1]\}$

$\Rightarrow$ P(k + 1): $1^2 + 2^2 + 3^2 + ... + k^2 + (k+1)^2$

$= (1/6)\{(k+1)(k+1+1)[2(k+1)+1]\}$

$\Rightarrow$ P(k + 1) is true, whenever P(k) is true.

Thus, P(1) is true and P(k + 1) is true, whenever P(k) is true.

Hence, by the principle of mathematical induction, P(n) is true for all $n \in N$.

Ex. 20 Using the principle of mathematical induction, prove that

$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + ... + n(n+1) = (1/3)\{n(n+1)(n+2)\}$.

Sol. Let the given statement be P(n). Then,

P(n): $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + ... + n(n+1) = (1/3)\{n(n+1)(n+2)\}$.

Thus, the given statement is true for n = 1, i.e., P(1) is true.

Let P(k) be true. Then,

P(k): $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + ... + k(k+1) = (1/3)\{k(k+1)(k+2)\}$.

Now, $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + ... + k(k+1) + (k+1)(k+2)$

$= (1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + ... + k(k+1)) + (k+1)(k+2)$

$= (1/3)\, k(k+1)(k+2) + (k+1)(k+2)$ [using **(i)**]

$= (1/3)\, [k(k+1)(k+2) + 3(k+1)(k+2)$

$= (1/3)\{(k+1)(k+2)(k+3)\}$

$\Rightarrow$ P(k + 1): $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + ... + (k+1)(k+2)$

$= (1/3)\{k+1)(k+2)(k+3)\}$

$\Rightarrow$ P(k + 1) is true, whenever P(k) is true.

Thus, P(1) is true and P(k + 1)is true, whenever P(k) is true.

Hence, by the principle of mathematical induction, P(n) is true for all values of $\in$ N.

Ex. 21 Using the principle of mathematical induction, prove that

$1 \cdot 3 + 3 \cdot 5 + 5 \cdot 7 + ... + (2n-1)(2n+1) = (1/3)\{n(4n^2 + 6n - 1)$.

Sol. Let the given statement be P(n). Then,

P(n): $1 \cdot 3 + 3 \cdot 5 + 5 \cdot 7 + ... + (2n-1)(2n+1) = (1/3)n(4n^2 + 6n - 1)$.

When n = 1, LHS = $1 \cdot 3 = 3$ and RHS = $(1/3) \times 1 \times (4 \times 1^2 + 6 \times 1 - 1)$

$= \{(1/3) \times 1 \times 9\} = 3$.

LHS = RHS.

Thus, P(1) is true.

Let P(k) be true. Then,

P(k) : $1 \cdot 3 + 3 \cdot 5 + 5 \cdot 7 + ... + (2k-1)(2k+1) = (1/3)\{k(4k^2 + 6k - 1)$ ......**(i)**

Now,

$1 \cdot 3 + 3 \cdot 5 + 5 \cdot 7 + ... + (2k-1)(2k+1) + \{2k(k+1) - 1\}\{2(k+1) + 1\}$

$= \{1 \cdot 3 + 3 \cdot 5 + 5 \cdot 7 + ... + (2k-1)(2k+1)\} + (2k+1)(2k+3)$

$= (1/3)\, k(4k^2 + 6k - 1) + (2k+1)(2k+3)$ [using **(i)**]

$= (1/3)\, [(4k^3 + 6k^2 - k) + 3(4k^2 + 8k + 3)]$

$= (1/3)(4k^3 + 18k^2 + 23k + 9)$

$= (1/3)\{(k+1)(4k^2 + 14k + 9)\}$

$= (1/3)[k+1)\{4k(k+1)^2 + 6(k+1) - 1\}]$

⇒ $P(k+1) : 1 \cdot 3 + 3 \cdot 5 + 5 \cdot 7 + ... + (2k+1)(2k+3)$

$= (1/3)[(k+1)\{4(k+1)^2 + 6(k+1) - 1)\}]$

⇒ P(k + 1) is true, whenever P(k) is true.

Thus, P(1) is true and P(k + 1) is true, whenever P(k) is true.

Hence, by the principle of mathematical induction, P(n) is true for all $n \in N$.

Ex. 22 Using the principle of mathematical induction, prove that

$1/(1 \cdot 2) + 1/(2 \cdot 3) + 1/(3 \cdot 4) + ... + 1/\{n(n+1)\} = n/(n+1)$

Sol. Let the given statement be P(n). Then,

$P(n) : 1/(1 \cdot 2) + 1/(2 \cdot 3) + 1/(3 \cdot 4) + ... + 1/\{n(n+1)\} = n/(n+1)$.

Putting n = 1 in the given statement, we get

LHS = $1/(1 \cdot 2)$ = and RHS = $1/(1+1) = 1/2$.

LHS = RHS.

Thus, P(1) is true.

Let P(k) be true. Then,

$P(k) : 1/(1 \cdot 2) + 1/(2 \cdot 3) + 1/(3 \cdot 4) + ... + 1/\{k(k+1)\} = k/(k+1)$ .....**(i)**

Now $1/(1 \cdot 2) + 1/(2 \cdot 3) + 1/(3 \cdot 4) + ... + 1/\{k(k+1)\} + 1/\{(k+1)(k+2)\}$

$[1/(1 \cdot 2) + 1/(2 \cdot 3) + 1/(3 \cdot 4) + ... + 1/\{k(k+1)\}] + 1/\{(k+1)(k+2)\}$

$= k/(k+1) + 1/\{(k+1)(k+2)\}$.

$\{k(k+2)+1\}/\{(k+1)2/[(k+1)k+2)]$ [using **(i)**]

$= \{k(k+2)+1\}/\{(k+1)(k+2\}$

$= \{(k+1)^2\}/\{(k+1)(k+2)\}$

$= (k+1)/(k+2) = (k+1)/(k+1+1)$

⇒ $P(k+1): 1/(1 \cdot 2) + 1/(2 \cdot 3) + 1/(3 \cdot 4) + ... + 1/\{k(k+1)\} + 1/\{(k+1)(k+2)\}$

$= (k+1)/(k+1+1)$

⇒ P(k + 1) is true, whenever P(k) is true.

Thus, P(1) is true and P(k + 1)is true, whenever P(k) is true.

Hence, by the principle of mathematical induction, P(n) is true for all $n \in N$.

Ex. 23 Using the principle of mathematical induction, prove that

$\{1/(3 \cdot 5)\} + \{1/(5 \cdot 7)\} + \{1/(7 \cdot 9)\} + ... + 1/\{(2n+1)(2n+3)\} = n/\{3(2n+3)\}$.

Sol. Let the given statement be P(n). Then,

$P(n) : \{1/(3 \cdot 5) + 1/(5 \cdot 7) + 1/(7 \cdot 9) + ... + 1/\{(2n+1)(2n+3)\} = n/\{3(2n+3)$.

Putting n = 1 in the given statement, we get and LHS = $1/(3 \cdot 5) = 1/15$ and RHS = $1/\{3(2 \times 1 + 3)\} = 1/15$.

LHS = RHS

Thus , P(1) is true.

Let P(k) be true. Then,

P(k): $\{1/(3 \cdot 5) + 1/(5 \cdot 7) + 1/(7 \cdot 9) + \ldots\ldots + 1/\{(2k+1)(2k+3)\} = k/\{3(2k+3)\}$ ..... **(i)**

Now, $1/(3 \cdot 5) + 1/(5 \cdot 7) + \ldots\ldots + 1/[(2k+1)(2k+3)] + 1/[\{2(k+1)+1\}2(k+1)+3$

$= \{1/(3 \cdot 5) + 1/(5 \cdot 7) + \ldots\ldots + [1/(2k+1)(2k+3)]\} + 1/\{(2k+3)(2k+5)\}$

$= k/[3(2k+3)] + 1/[2k+3)(2k+5)]$ [using **(i)**]

$= \{k(2k+5)+3\}/\{3(2k+3)(2k+5)\}$

$= (2k^2 + 5k + 3)/[3(2k+3)(2k+5)]$

$= \{(k+1)(2k+3)\}/\{3(2k+3)(2k+5)\}$

$= (k+1)/\{3(2k+5)\}$

$= (k+1)/[3\{2(k+1)+3\}]$

$= P(k+1): 1/(3 \cdot 5) + 1/(5 \cdot 7) + \ldots\ldots + 1/[2k+1)(2k+3)] + 1/[\{2(k+1)+1\}\{2(k+1)+3\}]$

$= (k+1)/\{3\{2(k+1)+3\}]$

$\Rightarrow$ P(k + 1) is true, whenever P(k) is true.

Thus, P(1) is true and P(k + 1) is true, whenever P(k) is true.

Hence, by the principle of mathematical induction, P(n) is true for $n \in N$.

Ex. 24 Using the principle of mathematical induction, prove that

$1/(1 \cdot 2 \cdot 3) + 1/(2 \cdot 3 \cdot 4) + \ldots\ldots + 1/\{n(n+1)(n+2)\} = \{n(n+3)\}/\{4(n+1)(n+2)\}$ for all $n \in N$.

Sol. Let P (n) : $1/(1 \cdot 2 \cdot 3) + 1/(2 \cdot 3 \cdot 4) + \ldots\ldots + 1/\{n(n+1)(n+2)\} = \{n(n+3)\}/\{4(n+1)(n+2)\}$.

Putting n = 1 in the given statement, we get

LHS = $1/(1 \cdot 2 \cdot 3) = 1/6$ and RHS = $\{1 \times (1+3)\}/[4 \times (1+1)(1+2)] = (1 \times 4)/(4 \times 2 \times 3) = 1/6$.

Therefore LHS = RHS.

Thus, the given statement is true for n = 1, i.e., P(1) is true.

Let P(k) be true. Then,

P(k): $1/(1 \cdot 2 \cdot 3) + 1/(2 \cdot 3 \cdot 4) + \ldots\ldots + 1/\{k(k+1)(k+2)\} = \{k(k+3)\}/\{4(k+1)(k+2)\}$ ....**(i)**

Now, $1/(1 \cdot 2 \cdot 3) + 1/(2 \cdot 3 \cdot 4) + \ldots\ldots + 1/\{k(k+1)(k+2)\} + 1/\{(k+1)(k+2)(k+3)\}$

$= [1/(1 \cdot 2 \cdot 3) + 1/(2 \cdot 3 \cdot 4) + \ldots\ldots + 1/\{k(k+1)(k+2\}] + 1/\{(k+1)(k+2)(k+3)\}$

$= [\{k(k+3)\}/\{4(k+1)(k+2)\} + 1/\{(k+1)(k+2)(k+3)\}]$ [using**(i)**]

$= \{k(k+3)^2 + 4\}/\{4(k+1)(k+2)(k+3)\}$

$= (k^3 + 6k^2 + 9k + 4)/\{4(k+1)(k+2)(k+3)\}$

$= \{(k+1)(k+1)(k+4)\}/\{4(k+1)(k+2)(k+3)\}$

$= \{(k+1)(k+4)\}/\{4(k+2)(k+3)$

$\Rightarrow$ P(k + 1) : $1/(1 \cdot 2 \cdot 3) + 1/(2 \cdot 3 \cdot 4) + \ldots\ldots + 1/\{(k+1)(k+2)(k+3)\}$

$= \{(k+1)(k+2)\}/\{4(k+2)(k+3)\}$

$\Rightarrow$ P(k + 1) is true, whenever P(k) is true.

Thus, P(1) is true and P(k + 1) is true, whenever P(k) is true.

Hence, by the principle of mathematical induction, P(n) is true for all $n \in N$.

Ex. 25 Using the Principle of mathematical induction, prove that

$\{1-(1/2)\}\{1-(1/3)\}\{1-(1/4)\} \ldots\ldots \{1-1/(n+1)\} = 1/(n+1)$ for all $n \in N$.

Sol. Let the given statement be P(n). Then,

$P(n) : \{1-(1/2)\}\{1-(1/3)\}\{1-(1/4)\} \ldots\ldots \{1-1/(n+1)\} = 1/(n+1)$.

When $n = 1$, LHS $= \{1-(1/2)\} = ½$ and RHS $= 1/(1+1) = ½$.

Therefore LHS = RHS.

Thus, P(1) is true.

Let P(k) be true. Then,

$P(k) : \{1-(1/2)\}\{1-(1/3)\}\{1-(1/4)\} \ldots\ldots [1-\{1/(k+1)\}] = 1/(k+1)$

Now, $[\{1-(1/2)\}\{1-(1/3)\}\{1-(1/4)\} \ldots\ldots [1-\{1/(k+1)\}] \cdot [1-\{1/(k+2)\}]$

$= [1/(k+1)] \cdot [\{(k+2)-1\}/(k+2)\}]$

$= [1/(k+1)] \cdot [(k+1)/(k+2)]$

$= 1/(k+2)$

Therefore $p(k+1) : [\{1-(1/2)\}\{1-(1/3)\}\{1-(1/4)\} \ldots\ldots [1-\{1/(k+1)\}] = 1/(k+2)$

$\Rightarrow$ P(k + 1) is true, whenever P(k) is true.

Thus, P(1) is true and P(k + 1) is true, whenever P(k) is true.

Hence, by the principle of mathematical induction, P(n) is true for all $n \in N$.

Exercise # 1 [Single Correct Choice Type Questions]

1. The greatest positive integer. which divides (n + 16) (n + 17) (n + 18) (n + 19), for all $n \in N$, is-

(A) 2 **(B)** 4 **(C)** 24 **(D)** 120

2. The sum of the cubes of three consecutive natural numbers is divisible by-

(A) 2 **(B)** 5 **(C)** 7 **(D)** 9

3. For every positive integer

n, $\frac{n^7}{7} + \frac{n^5}{5} + \frac{2n^3}{3} - \frac{n}{105}$ is-

(A) an integer **(B)** a rational number

(C) a negative real number **(D)** an odd integer

4. If $10^n + 3.4^{n+2} + \lambda$ is exactly divisible by 9 for all $n \in N$, then the least positive integral value of λ is-

(A) 5 **(B)** 3 **(C)** 7 **(D)** 1

5. The sum of n terms of $1^2 + (1^2+2^2) + (1^2+2^2+3^2) + ...$ is-

(A) $\frac{n(n+1)(2n+1)}{6}$ **(B)** $\frac{n(n+1)(2n-1)}{6}$

(C) $\frac{1}{12}n(n+1)^2(n+2)$ **(D)** $\frac{1}{12}n^2(n+1)^2$

6. For positive integer n, $3^n < n!$ when-

(A) $n \geq 6$ **(B)** $n > 7$ **(C)** $n \geq 7$ **(D)** $n \leq 7$

7. For all positive integral values of n, $3^{2n} - 2n + 1$ is divisible by-

(A) 2 **(B)** 4 **(C)** 8 **(D)** 12

8. $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + ...+ \frac{1}{n(n+1)} = \frac{n}{n+1}$, $n \in N$, is true for

(A) $n \geq 3$ **(B)** $n \geq 2$ **(C)** $n \geq 4$ **(D)** all n

9. Let P(n) : $n^2 + n$ is an odd integer. It is seen that truth of P(n) $\Rightarrow$ the truth of P(n + 1). Therefore, P(n) is true for all-

(A) $n > 1$ **(B)** n **(C)** $n > 2$ **(D)** None of these

10. If $n \in N$, then $3^{4n+2} + 5^{2n+1}$ is a multiple of-

(A) 14 **(B)** 16 **(C)** 18 **(D)** 20

11. If $A = \begin{pmatrix} a & 1 \\ 0 & a \end{pmatrix}$, then for any $n \in N$, A^n equals-

(A) $\begin{pmatrix} na & n \\ 0 & na \end{pmatrix}$ **(B)** $\begin{pmatrix} a^n & na^{n-1} \\ 0 & a^n \end{pmatrix}$ **(C)** $\begin{pmatrix} na & 1 \\ 0 & na \end{pmatrix}$ **(D)** $\begin{pmatrix} a^n & n \\ 0 & a^n \end{pmatrix}$

12. For every natural number n, n(n + 3) is always-

(A) multiple of 4 **(B)** multiple of 5 **(C)** even **(D)** odd

13. $\frac{1^2}{1} + \frac{1^2+2^2}{1+2} + \frac{1^2+2^2+3^2}{1+2+3} + \ldots$ upto n terms is-

(A) $\frac{1}{3}(2n+1)$ **(B)** $\frac{1}{3}n^2$ **(C)** $\frac{1}{3}(n+2)$ **(D)** $\frac{1}{3}n(n+2)$

14. The sum of n terms of the series

$1 + (1+a) + (1+a+a^2) + (1+a+a^2+a^3) + \ldots$, is-

(A) $\frac{n}{1-a} - \frac{a(1-a^n)}{(1-a)^2}$ **(B)** $\frac{n}{1-a} + \frac{a(1-a^n)}{(1-a)^2}$ **(C)** $\frac{n}{1-a} + \frac{a(1+a^n)}{(1-a)^2}$ **(D)** $-\frac{n}{1-a} + \frac{a(1-a^n)}{(1-a)^2}$

15. If $p(n) : n^2 > 100$ then

(A) p(1) is true **(B)** p(4) is true

(C) p(k) is true $\forall\ k \geq 5, k \in N$ **(D)** p(k + 1) is true whenever p(k) is true where $k \in N$

16. If $n \in N$, then $x^{2n-1} + y^{2n-1}$ is divisible by-

(A) $x+y$ **(B)** $x-y$ **(C)** x^2+y^2 **(D)** x^2+xy

17. For each $n \in N$, $10^{2n+1} + 1$ is divisible by-

(A) 11 **(B)** 13 **(C)** 27 **(D)** None of these

18. The sum of n terms of the series $\frac{\frac{1}{2}.\frac{2}{2}}{1^3} + \frac{\frac{2}{2}.\frac{3}{2}}{1^3+2^3} + \frac{\frac{3}{2}.\frac{4}{2}}{1^3+2^3+3^3} + \ldots$ is-

(A) $\frac{1}{n(n+1)}$ **(B)** $\frac{n}{n+1}$ **(C)** $\frac{n+1}{n}$ **(D)** $\frac{n+1}{n+2}$

19. For all $n \in N$, n^4 is less than-

(A) 10^n **(B)** 4^n **(C)** 10^{10} **(D)** None of these

20. For positive integer n, $10^{n-2} > 81n$ when-

(A) $n < 5$ **(B)** $n > 5$ **(C)** $n \geq 5$ **(D)** $n > 6$

21. $1 + 3 + 6 + 10 + \ldots$ upto n terms is equal to-

(A) $\frac{1}{3}n(n+1)(n+2)$ **(B)** $\frac{1}{6}n(n+1)(n+2)$

(C) $\frac{1}{12}n(n+2)(n+3)$ **(D)** $\frac{1}{12}n(n+1)(n+2)$

22. Sum of n terms of the series $\frac{1}{1} + \frac{1}{1+2} + \frac{1}{1+2+3} + \ldots$ is-

(A) $\frac{n}{n+1}$ **(B)** $\frac{2}{n(n+1)}$ **(C)** $\frac{2n}{n+1}$ **(D)** $\frac{2(n+1)}{n+2}$

23. The inequality $n! > 2^{n-1}$ is true-

(A) for all $n > 1$ **(B)** for all $n > 2$ **(C)** for all $n \in N$ **(D)** None of these

24. $1 + 2 + 3 + ...+ n < \frac{(n+2)^2}{8}$, $n \in N$, is true for

(A) $n \geq 1$ **(B)** $n \geq 2$ **(C)** all n **(D)** none of these

25. For all $n \in N$, $7^{2n} - 48n - 1$ is divisible by-

(A) 25 **(B)** 26 **(C)** 1234 **(D)** 2304

26. A student was asked to prove a statement by induction. He proved

(i) P(5) is true and

(ii) Truth of $P(n) \Rightarrow$ truth of $p(n + 1)$, $n \in N$

On the basis of this, he could conclude that P(n) is true for

(A) no $n \in N$ **(B)** all $n \in N$ **(C)** all $n \geq 5$ **(D)** None of these

27. $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + ...$ upto n terms is-

(A) $\frac{1}{2n+1}$ **(B)** $\frac{n}{2n+1}$ **(C)** $\frac{1}{2n-1}$ **(D)** $\frac{2n}{3(n+1)}$

28. The difference between an +ve integer and its cube is divisible by-

(A) 4 **(B)** 6 **(C)** 9 **(D)** None of these

29. For all $n \in N$, Σn

(A) $< \frac{(2n+1)^2}{8}$ **(B)** $> \frac{(2n+1)^2}{8}$ **(C)** $= \frac{(2n+1)^2}{8}$ **(D)** None of these

30. If P is a prime number then $n^p - n$ is divisible by p when n is a

(A) natural number greater than 1 **(B)** odd number

(C) even number **(D)** None of these

31. For natural number n, $2^n (n - 1)! < n^n$, if-

(A) $n < 2$ **(B)** $n > 2$ **(C)** $n \geq 2$ **(D)** never

32. For all $n \in N$, $\cos\theta \cos 2\theta \cos 4\theta ...... \cos 2^{n-1}\theta$ equals to-

(A) $\frac{\sin 2^n \theta}{2^n \sin\theta}$ **(B)** $\frac{\sin 2^n \theta}{\sin\theta}$ **(C)** $\frac{\cos 2^n \theta}{2^n \cos 2\theta}$ **(D)** $\frac{\cos 2^n \theta}{2^n \sin\theta}$

33. If $x \neq y$, then for every natural number n, $x^n - y^n$ is divisible by

(A) $x - y$ **(B)** $x + y$ **(C)** $x^2 - y^2$ **(D)** all of these

34. $1.2^2 + 2.3^2 + 3.4^2 + ...$ upto n terms, is equal to-

(A) $\frac{1}{12} n(n+1)(n+2)(n+3)$ **(B)** $\frac{1}{12} n(n+1)(n+2)(n+5)$

(C) $\frac{1}{12} n(n+1)(n+2)(3n+5)$ **(D)** None of these

35. If n is a natural number then $\left(\frac{n+1}{2}\right)^n \geq n!$ is true when-

(A) $n > 1$ **(B)** $n \geq 1$ **(C)** $n > 2$ **(D)** Never

36. The n^{th} term of the series

$4 + 14 + 30 + 52 + 80 + 114 + ...$ is-

(A) $5n - 1$ **(B)** $2n^2 + 2n$ **(C)** $3n^2 + n$ **(D)** $2n^2 + 2$

37. The sum of the series

$\frac{3}{1^2} + \frac{5}{1^2+2^2} + \frac{7}{1^2+2^2+3^2} + ...$ upto n terms

(A) $\frac{2n}{n+1}$ **(B)** $\frac{3n}{n+1}$ **(C)** $\frac{3n}{2(n+1)}$ **(D)** $\frac{6n}{n+1}$

35. $n^3 + (n+1)^3 + (n+2)^3$ is divisible for all $n \in N$ by

(A) 3 **(B)** 9 **(C)** 27 **(D)** 81

39. If $n \in N$, then $11^{n+2} + 12^{2n+1}$ is divisible by-

(A) 113 **(B)** 123 **(C)** 133 **(D)** None of these

40. $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + ...$ upto n terms equal to-

(A) $n + \frac{1}{2^n}$ **(B)** $2n + \frac{1}{2^n}$ **(C)** $n - 1 + \frac{1}{2^n}$ **(D)** $n + 1 + \frac{1}{2^n}$

Exercise # 2

[Subjective Type Questions]

1. By using PMI, prove that $2 + 4 + 6 + ...+ 2n = n(n + 1), n \in N$

2. Prove that $1 + 2 + 3 + ...+ n < \frac{1}{8}(2n + 1)^2, n \in N.$

3. Let P(n) be the statement "$n^3 + n$ is divisible by 3". Write P(1), P(4)

4. Prove that $2^n > n, n \in N.$

5. Use the principle of mathematical induction to prove that $n(n + 1)(n + 2)$ is a multiple of 6 for all natural numbers n.

6. Prove that $\sin\theta + \sin 2\theta + ...+ \sin\left(\frac{n+1}{2}\right) n\theta = \sin\frac{n\theta}{2}\,\theta \sin\frac{\theta}{2} \text{cosec}$ for all $n \in N$.

7. Prove that $\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + ...+ \frac{1}{2^n} = 1 - \frac{1}{2^n}, n \in N.$

8. By using PMI, prove that $1.3 + 2.3^2 + 3.3^3 + ...+ n.3^n = \frac{(2n-1)3^{n+1} + 3}{4}, n \in N$

9. If 3^{2n}, where n is a natural number, is divided by 8, prove that the remainder is always 1.

10. Prove that $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + ...+ \frac{1}{n^2} < 2 - \frac{1}{n}$ where $n (> 1) \in N$, by using P.M.I

11. Prove that $2n + 7 < (n + 3)^2, n \in N$. Using this, prove that

$(n + 3)^2 \le 2^{n+3}, n \in N.$

Exercise # 3 [Previous Year Questions] [AIEEE/JEE-MAIN]

1. Let $S(k) = 1 + 3 + 5 + ... + (2k - 1) = 3 + k^2$, then which of the following is true ? **[AIEEE-2004]**

(1) S(1) is true **(2)** $S(k) \Rightarrow S(k+1)$

(3) $S(k) \not\Rightarrow S(k+1)$ **(4)** Principle of mathematical Induction can be used to prove that formula

2. The sum of first n terms of the given series

$1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ...$ is $\frac{n(n+1)^2}{2}$, when n is even. When n is odd, then sum will be- **[AIEEE-2004]**

(1) $\frac{n(n+1)^2}{2}$ **(2)** $\frac{1}{2}n^2(n+1)$

(3) $n(n+1)^2$ **(4)** None of these

3. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which one of the following holds for all $n \geq 1$, (by the principal of mathematical induction) **[AIEEE-2005]**

(1) $A^n = nA + (n-1)I$ **(2)** $A^n = 2^{n-1}A + (n+1)I$

(3) $A^n = nA - (n-1)I$ **(4)** $A^n = 2^{n-1}A - (n-1)I$

4. **Statement –1 :** For every natural number $n \geq 2$

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{n}} > \sqrt{n}$$

Statement –2 : For every natural number $n \geq 2$, $\sqrt{n(n+1)} < n + 1$. **[AIEEE-2008]**

(1) Statement –1 is false, Statement –2 is true

(2) Statement–1 is true, Statement–2 is false

(3) Statement–1 is true, Statement–2 is true; Statement–2 is a correct explanation for Statement–1

(4) Statement–1 is true, Statement–2 is true;Statement–2 is not a correct explanation for Statement–1

5. **Statement - 1:** For each natural number n, $(n + 1)^7 - n^7 - 1$ is divisible by 7.
Statement - 2: For each natural number n, $n^7 - n$ is divisible by 7. **[AIEEE-2011]**

(1) Statement –1 is false, Statement –2 is true

(2) Statement–1 is true, Statement–2 is false

(3) Statement–1 is true, Statement–2 is true; Statement–2 is a correct explanation for Statement–1

(4) Statement–1 is true, Statement–2 is true;Statement–2 is not a correct explanation for Statement–1

ANSWER KEY

EXERCISE - 1

1. C	**2.** D	**3.** A	**4.** A	**5.** C	**6.** C	**7.** A	**8.** D	**9.** D	**10.** A	**11.** B	**12.** C	**13.** D
14. A	**15.** D	**16.** A	**17.** A	**18.** B	**19.** A	**20.** C	**21.** B	**22.** C	**23.** B	**24.** D	**25.** D	**26.** C
27. B	**28.** B	**29.** A	**30.** A	**31.** B	**32.** A	**33.** A	**34.** C	**35.** B	**36.** C	**37.** D	**38.** B	**39.** C
40. C												

EXERCISE - 3

1. 2	**2.** 2	**3.** 3	**4.** 3	**5.** 2