HINTS & SOLUTIONS

EXERCISE - 1
Single Choice

1. Since product of any r consecutive integers is divisible by r! and not divisible by r+1!.

So given product of 4 consecutive integers is divisible by 4! or 24.

2. Let three consecutive natural numbers are n, n+1, n+2,

$P(n) = (n)^3 + (n+1)^3 + (n+2)^3$

$P(1)=1^3 + 2^3 + 3^3=36$, which is divisible by 2 and 9

$P(2) = (2)^3 + (3)^3 + (4)^3 = 99$, which is divisible by 9 (not by 2).

Hence P(n) is divisible 9 $\forall$ n $\in$ N.

3. Let $P(n) = \frac{n^7}{7} + \frac{n^5}{5} + \frac{2n^3}{3} - \frac{n}{105}$

$P(1) = \frac{1}{7} + \frac{1}{5} + \frac{2}{3} - \frac{1}{105} = 1$ (integer)

$P(2) = 2^4\left(\frac{8}{7} + \frac{2}{5} + \frac{1}{3}\right) - \frac{2}{105} = 15$ (integer) etc.

Hence P(n) is an integer.

4. Let $P(n) = 10^n + 3.4^{n+2} + \lambda$ is divisible by 9 $\forall n \in N$

$P(1)=10+3.4^3+\lambda=202+\lambda=207+(\lambda-5)$

Which is divisible by 9 if $\lambda = 5$

9. Let $p(n) = n^2 + n = n(n + 1)$ is an odd integer since the product of two consecutive integers is always even. So hear principle of induction is not applicable.

10. Let $p(n) = 3^{4n+2} + 5^{2n+1}$

Here $P(1) = 3^6 + 5^3 = 9^3 + 5^3 = 14 \times 61$

Which is multiple of 14 but not of 16, 18 and 20.

14. $T_n = 1 + a + a^2 + \text{-------} + a^{n-1} = \frac{1-a^n}{1-a}$

$S_n = \Sigma T_n = \frac{1}{(1-a)}[\Sigma 1 - \Sigma a^n]$

$= \frac{1}{(1-a)}[n - (a + a^2 + a^3 + \text{----} a^n)]$

$= \frac{1}{(1-a)}\left[n - \frac{a(1-a^n)}{(1-a)}\right] = \frac{n}{1-a} - \frac{a(1-a^n)}{(1-a)^2}$

16. Since $x^n + y^n$ is divisible $(x + y)$ if n is odd.

Here $2n - 1$ is odd $\forall$ n $\in$ N.

18. n^{th} term of the given series

$$T_n = \frac{\frac{n}{2}\cdot\frac{n+1}{2}}{\Sigma n^3} = \frac{\frac{1}{4}n(n+1)}{\frac{1}{4}n^2(n+1)^2} = \frac{1}{n(n+1)} = \left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$$\therefore S_n = \left[\left(1-\frac{1}{2}\right) + \left(\frac{1}{2}-\frac{1}{3}\right) + \left(\frac{1}{3}-\frac{1}{4}\right) + \text{----} + \left(\frac{1}{n}-\frac{1}{n+1}\right)\right]$$

$$= 1 - \frac{1}{n+1} = \frac{n}{n+1}$$

25. Let $P(n) = 7^{2n} - 48n - 1$

$P(1) = 7^2 - 48.1 - 1 = 0$,

which is divisible by for all n $\in$ N

$P(2) = 7^4 - 48.2 - 1 = 2304$,

which is divisible by 2304 not by 25, 26 and 1234.

26. By Theorem-II

28. Let n is a positive integer.

$P(n) = n^3 - n$

$P(1) = 0$, which is divisible by for all n $\in$ N

$P(2) = 6$, which is divisible by 6 (not by 4 and 9)

30. Let $P(n): \quad n^p - n$

when $p = 2$

$P(n) = \quad n^2 - n$

$P(1) = \quad 0 \quad$ which is divisible all $n \in N$

$P(2) = \quad 2 \quad$ which is divisible by 2

$P(3) = \quad 6 \quad$ which is divisible by 2

Hence P(n) is divisible by 2 when n is greater than 1.

32. Let $P(n) = \cos\theta \,.\, \cos 2\theta \,.\, \cos 4\theta$ ----- $\cos 2^{n-1}\theta$

$$P(1) = \cos\theta = \frac{2\sin\theta\cos\theta}{2\sin\theta} = \frac{\sin 2\theta}{2\sin\theta}$$

$$P(2) = \cos\theta\cos 2\theta = \frac{2(2\sin\theta\cos\theta)\cos 2\theta}{4\sin\theta}$$

$$= \frac{2\sin 2\theta\cos 2\theta}{4\sin\theta}$$

$$= \frac{\sin 4\theta}{4\sin\theta} = \frac{\sin 2^2\theta}{2^2\sin\theta}$$

Clearly, $P(n) = \dfrac{\sin 2^n\theta}{2^n\sin\theta}$

34. Here $\quad T_n = n(n+1)^2$

$\therefore \quad S_n = \Sigma T_n = \Sigma n^3 + 2\Sigma n^2 + \Sigma n$

$$= \frac{n^2(n+1)^2}{4} + 2.\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$= \frac{1}{12}n(n+1)(n+2)(3n+5)$$

36. Let n^{th} term of the series is T_n and

$S_n = 4 + 14 + 30 + 52 + 80 + 114 +$ ---- $+ T_n$..**(i)**

$S_n = \quad 4 + 14 + 30 + 52 + 80 +$ ---- $+ T_n$ ..**(ii)**

Subtract **(ii)** from **(i)**

$0 = (4 + 10 + 16 + 22 + 28 + 34 +$ ---- n terms$) - T_n$

$T_n = 4 + 10 + 16 + 22 +$ ---- n terms

$$= \frac{n}{2}[2 \times 4 + (n-1)6] = n(3n+1) = 3n^2 + n$$

38. Let $p(n) = n^3 + (n+1)^3 + (n+2)^3$,

$p(A) = 36, p(B) = 99$ both are divisible by 99

Let it is true for $n = k$

$k^3 + (k+1)^3 + (k+2)^3 = 9q \quad ; q \in I$

adding $9k^2 + 27k + 27$ both sides

$k^3 + (k+1)^3 + (k+2)^3 + 9k^2 + 27k + 27 = 9q + 9k^2 + 27k + 27$

$(k+1)^3 + (k+2)^3 + (k+3)^3 = 9r \,; r \in I$

39. Let $P(n) = 11^{n+2} + 12^{2n+1}$

$P(1) = 11^3 + 12^3 = 23 \times 133$, which is divisible by 133 but not by 113 and 123.

EXERCISE - 2
Subjective Type

2. (i) Given statement is true for $n = 1$

(ii) Let us assume that the statement is true for $n = k$

i.e. $1+2+3+.........+k < \frac{1}{8}(2k+1)^2$

$\Rightarrow\ 1+2+3+.........+k = \frac{1}{8}(2k+1)^2 - \lambda$ where $\lambda \in R^+$

(iii) For $n = k+1$,

$$1+2+.........+k+k+1 = \frac{(2k+1)^2}{8} + (k+1) - \lambda$$

$$= \frac{(2k+3)^2}{8} - \lambda < \frac{(2k+3)^2}{8}$$

So the result is true for $n = k+1$

Hence by principle of mathematical induction the statement is true for all $n \in N$

3. $P(1) : 1^3 + 1$ is divisible by 3
$P(4) : 4^3 + 4$ is divisible by 3

5.

(i) Given statement is true for $n = 1$

(ii) Let us assume that the statement is true for $n = k$

i.e. $k(k+1)(k+2) = 6\lambda$

(iii) For $n = k+1$,

$$(k+1)(k+2)(k+3) = k(k+1)(k+2) + 3(k+1)(k+2)$$

$= 6\lambda + 3(k+1)(k+2) =$ multiple of 6 as $(k+1)(k+2)$ is even

So the result is true for $n = k+1$

Hence by principle of mathematical induction the statement is true for all $n \in N$

6. Let $P(n)$; $\sin\theta + \sin 2\theta + ...+ \sin n\theta$

$$= \sin\left(\frac{n+1}{2}\right)\theta \sin\frac{n\theta}{2} \operatorname{cosec}\frac{\theta}{2}$$

P(A) is true

Let P(k) is also true

$\sin\theta + \sin 2\theta + ...+ \sin k\theta$

$$= \sin\left(\frac{k+1}{2}\right)\theta \sin\frac{k\theta}{2} \operatorname{cosec}\frac{\theta}{2}$$

add $\sin(k+1)\theta$ both sides

$$\sin\theta + \sin 2\theta + ...+ \sin k\theta + \sin(k+1)\theta = \sin\left(\frac{k+1}{2}\right)\theta$$

$$\sin\frac{k\theta}{2} \operatorname{cosec}\frac{\theta}{2} + \sin(k+1)\theta$$

$$= \sin\left(\frac{k+1}{2}\right)\theta \left[\frac{\sin\frac{k\theta}{2} + 2\cos\left(\frac{k+1}{2}\right)\theta \sin\frac{\theta}{2}}{\sin\frac{\theta}{2}}\right]$$

$$= \sin\left(\frac{k+1}{2}\right)\theta \left[\frac{\frac{\sin k\theta}{2} + \frac{\sin(k+2)\theta}{2} - \frac{\sin k\theta}{2}}{\sin\frac{\theta}{2}}\right]$$

$$= \sin\left(\frac{k+1+1}{2}\right)\theta \sin\left(\frac{k+1}{2}\right)\theta\ .\operatorname{cosec}\frac{\theta}{2}$$

$\Rightarrow$ $P(k+1)$ is true

8.

(i) Given statement is true for $n = 1$

(ii) Let us assume that the statement is true for $n = k$

i.e. $1.3 + 2.3^2 + 3.3^3 + ...+ k.3^k = \frac{(2k-1)3^{k+1} + 3}{4}$

(iii) For $n = k+1$,

L.H.S. $= 1.3 + 2.3^2 + 3.3^3 + ...+ k.3^k + (k+1)\,3^{k+1}$

$$= \frac{(2k-1)3^{k+1} + 3}{4} + (k+1)\,3^{k+1} = \frac{(2k+1)3^{k+2} + 3}{4}$$

= R.H.S.

so by principle of mathematical induction the statement is true for all $n \in N$

11. (i) Given statement is true for $n = 1$

(ii) Let us assume that the statement is true for $n = k$

i.e. $2k + 7 = (k+3)^2 - \lambda$ where $\lambda \in R^+$

(iii) For $n = k+1$,

$$2(k+1) + 7 = 2k + 7 + 2 = (k+3)^2 - \lambda + 2$$

$$= (k+4)^2 - 2k - \lambda - 5 < (k+4)^2$$

So the result is true for $n = k+1$

Hence by principle of mathematical induction the statement is true for all $n \in N$

EXERCISE - 3

Part # I : AIEEE/JEE-MAIN

1. S(K) = 1 + 3 + 5 + ... + (2K – 1) = 3 + K^2

S(1) is not true

let S(K) is true then

S(K + 1) = 1 + 3 + 5 + ... + (2K – 1)+(2K + 1)

= S(K) + (2K + 1)

= 3 + K^2 + 2K + 1 = 3 + $(K + 1)^2$

Hence S(K) $\Rightarrow$ S(K + 1)

2. Since $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ----$

n terms = $\frac{n(n+1)^2}{2}$, when n is even

When n is odd the n^{th} term of series will be n^2 in this case, (n – 1) is even

so for finding sum of first (n – 1) terms of the series, we replacing n by (n – 1) in the given formula.

So sum of first (n – 1) terms = $\frac{(n-1)n^2}{2}$

Hence sum of n terms of the series

= (the sum of (n – 1) terms + the n^{th} term)

$$= \frac{(n-1)n^2}{2} + n^2 = \frac{(n+1)n^2}{2}$$

3. $$A^2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$$

$$\therefore \quad A^n = \begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix}$$

Now $nA-(n-1)I=\begin{bmatrix} n & 0 \\ n & n \end{bmatrix}-\begin{bmatrix} n-1 & 0 \\ 0 & n-1 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix}=A^n$

4. $\because \sqrt{n(n+1)} < \sqrt{(n+1)(n+1)}$

i.e. $\sqrt{n(n+1)} < n + 1 \quad \forall\ n \in N$

Hence statement-2 is true.

For n = 2 given result is true.

let it is true for n = K $\in$ N, K $\geq$ 2 then

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{K}} > \sqrt{K}$$

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{K}} + \frac{1}{\sqrt{K+1}} > \sqrt{K} + \frac{1}{\sqrt{K+1}}$$

$$= \frac{\sqrt{K(K+1)}+1}{\sqrt{K+1}} > \frac{\sqrt{KK}+1}{\sqrt{K+1}} = \sqrt{K+1}$$

($\because$ by statement-2 $\sqrt{n(n+1)} < n+1 \Rightarrow \sqrt{n} < \sqrt{n+1}$)

$$\Rightarrow \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{K+1}} > \sqrt{K+1}$$

Hence statement-1 is true for every natural number $n \geq 2$.

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