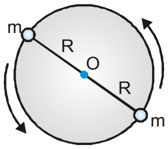
SOLVED EXAMPLES

Ex. 1 Two particles of equal mass m go round a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is

(A) $\frac{1}{2R}\sqrt{\frac{1}{Gm}}$ **(B)** $\sqrt{\frac{Gm}{2R}}$ **(C)** $\frac{1}{2}\sqrt{\frac{Gm}{R}}$ **(D)** $\sqrt{\frac{4Gm}{R}}$

Sol. Centripetal force provided by the gravitational force of attraction between two particles i.e. $\frac{mv^2}{R} = \frac{Gm \times m}{(2R)^2} \Rightarrow v = \frac{1}{2}\sqrt{\frac{Gm}{R}}$



Ex. 2 The escape velocity for a planet is v_e. A particle starts from rest at a large distance from the planet, reaches the planet only under gravitational attraction, and passes through a smooth tunnel through its centre. Its speed at the centre of the planet will be-

(A) $\sqrt{1.5}v_e$ **(B)** $\frac{v_e}{\sqrt{2}}$ **(C)** v_e **(D)** zero

Sol. From mechanical energy conservation $0 + 0 = \frac{1}{2}mv^2 - \frac{3GMm}{2R} \Rightarrow v = \sqrt{\frac{3GM}{R}} = \sqrt{1.5}\ v_e$

Ex. 3 A particle is projected vertically upwards the surface of the earth (radius R_e) with a speed equal to one fourth of escape velocity. What is the maximum height attained by it from the surface of the earth ?

(A) $\frac{16}{15}R_e$ **(B)** $\frac{R_e}{15}$ **(C)** $\frac{4}{15}R_e$ **(D)** None of these

Sol. From conservation of mechanical energy $\frac{1}{2}mv^2 = \frac{GMm}{R_e} - \frac{GMm}{R}$

Where R = maximum distance from centre of the earth Also $v = \frac{1}{4}v_e = \frac{1}{4}\sqrt{\frac{2GM}{R_e}}$

$$\Rightarrow \frac{1}{2}m \times \frac{1}{16} \times \frac{2GM}{R_e} = \frac{GMm}{R_e} - \frac{GMm}{R}$$

$$\Rightarrow R = \frac{16}{15}R_e \Rightarrow h = R - R_e = \frac{R_e}{15}$$

Ex. 4 A mass 6×10^{24} kg (= mass of earth) is to be compressed in a sphere in such a way that the escape velocity from its surface is 3×10^8 m/s (equal to that of light). What should be the radius of the sphere?

(A) 9 mm **(B)** 8 mm **(C)** 7 mm **(D)** 6 mm

Sol. As, $v_e = \sqrt{\left(\frac{2GM}{R}\right)}$, $R = \left(\frac{2GM}{v_e^2}\right)$, $\therefore\ R = \frac{2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{(3 \times 10^8)^2} = 9 \times 10^{-3}\,m = 9\ mm$

Ex. 5 Calculate the mass of the sun if the mean radius of the earth's orbit is 1.5×10^8 km and $G = 6.67 \times 10^{-11}\ N \times m^2/kg^2$.

(A) $M \simeq 2 \times 10^{30}$ kg **(B)** $M \simeq 3 \times 10^{30}$ kg **(C)** $M \simeq 2 \times 10^{15}$ kg **(D)** $M \simeq 3 \times 10^{15}$ kg

Sol. In case of orbital motion as $v = \sqrt{\left(\frac{GM}{r}\right)}$ So $T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{GM}}$, i.e., $M = \frac{4\pi^2 r^3}{GT^2}$

$M = \frac{4\times\pi^2\times(1.5\times10^{11})^3}{6.67\times10^{-11}\times(3.15\times10^7)^2}$ [as T = 1 year = 3.15×10^7s] i.e., $M \simeq 2\times10^{30}$ kg

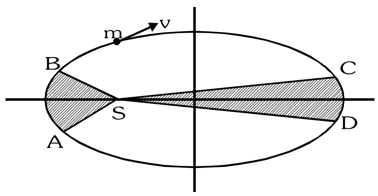
Ex. 6 Gravitational potential difference between a point on surface of planet and another point 10 m above is 4 J/kg. Considering gravitational field to be uniform, how much work is done in moving a mass of 2 kg from the surface to a point 5m above the surface?

(A) 4 J **(B)** 5 J **(C)** 6 J **(D)** 7 J

Sol. Gravitational field $g = -\frac{\Delta V}{\Delta x} = -\left(\frac{-4}{10}\right) = \frac{4}{10} J/kg\ m$

Work done in moving a mass of 2 kg from the surface to a point 5 m above the surface,

$W = mgh = (2kg)\left(\frac{4}{10}\frac{J}{kgm}\right)(5m) = 4J$

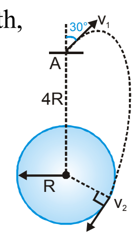


Ex. 7 The figure shows elliptical orbit of a planet m about the sun S. The shaded area SCD is twice the shaded area SAB. If t_1 be the time for the planet to move from C to D and t_2 is the time to move from A to B, then :

(A) $t_1 = t_2$ **(B)** $t_1 = 8t_2$ **(C)** $t_1 = 4t_2$ **(D)** $t_1 = 2t_2$

Sol. From Kepler's law : Areal velocity = constant so $\frac{\text{Area SCD}}{t_1} = \frac{\text{Area SAB}}{t_2} \Rightarrow t_1 = 2t_2$

Ex. 8 A particle is projected from point A, that is at a distance 4R from the centre of the Earth, with speed v_1 in a direction making 30° with the line joining the centre of the Earth and point A, as shown. Find the speed v_1 of particle (in m/s) if particle passes grazing the surface of the earth. Consider gravitational interaction only between these two.



(use $\frac{GM}{R} = 6.4\times10^7\ m^2/s^2$)

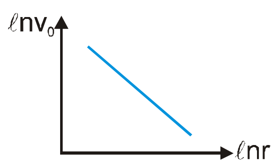
(A) $\frac{8000}{\sqrt{2}}$ **(B)** 800 **(C)** $800\sqrt{2}$ **(D)** None of these

Sol. Conserving angular momentum : $m(v_1\cos60°)4R = mv_2R \Rightarrow \frac{v_2}{v_1} = 2.$

Conserving energy of the system : $-\frac{GMm}{4R} + \frac{1}{2}mv_1^2 = -\frac{GMm}{R} + \frac{1}{2}mv_2^2$

$\frac{1}{2}v_2^2 - \frac{1}{2}v_1^2 = \frac{3}{4}\frac{GM}{R} \Rightarrow v_1^2 = \frac{1}{2}\frac{GM}{R} \Rightarrow v_1 = \frac{1}{\sqrt{2}}\sqrt{64\times10^6} = \frac{8000}{\sqrt{2}} m/s$

Ex. 9 If the law of gravitation be such that the force of attraction between two particles vary inversely as the 5/2th power of their separation, then the graph of orbital velocity v_0 plotted against the distance r of a satellite from the earth's centre on a log-log scale is shown alongside. The slope of line will be-



(A) $-\frac{5}{4}$ **(B)** $-\frac{5}{2}$ **(C)** $-\frac{3}{4}$ **(D)** –1

Sol. $\dfrac{mv_0^2}{r} = \dfrac{GMm}{r^{5/2}} \Rightarrow v_0 = \dfrac{\sqrt{GM}}{r^{3/4}} \Rightarrow \ell n v_0 = \ell n\sqrt{GM} - \dfrac{3}{4}\ell nr$

Ex. 10 Two point objects of masses m and 4m are at rest at an infinite separation. They move towards each other under mutual gravitational attraction. If G is the universal gravitational constant, then at a separation r

(A) the total mechanical energy of the two objects is zero **(B)** their relative velocity is $\sqrt{\dfrac{10Gm}{r}}$

(C) the total kinetic energy of the objects is $\dfrac{4Gm^2}{r}$ **(D)** their relative velocity is zero

Sol. By applying law of conservation of momentum 

$mv_1 - 4mv_2 = 0 \quad \Rightarrow \quad v_1 = 4v_2$

By applying conservation of energy $\dfrac{1}{2}mv_1^2 + \dfrac{1}{2}4mv_2^2 = \dfrac{Gm4m}{r} \Rightarrow 10mv_2^2 = \dfrac{G4m^2}{r} \Rightarrow v_2 = 2\sqrt{\dfrac{Gm}{10r}}$

$\therefore$ Total kinetic energy $= \dfrac{4Gm^2}{r}$; Relative velocity for the particle $\Rightarrow v_{rel} = |\vec{v}_1 - \vec{v}_2| = 5v_2 = \sqrt{\dfrac{10Gm}{r}}$

Or

Mechanical energy of system = 0 = constant. By using reduced mass concept

$\dfrac{1}{2}\mu v_{rel}^2 = \dfrac{Gm(4m)}{r}$ where $\mu = \dfrac{(m)(4m)}{m+4m} = \dfrac{4}{5}m \Rightarrow v_{rel} = \sqrt{\dfrac{10Gm}{r}}$

Also total KE of system $= \dfrac{G(m)(4m)}{r} = \dfrac{4Gm^2}{r}$

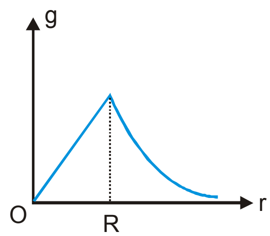
Ex. 11 Which of the following statements are true about acceleration due to gravity?

(A) 'g' decreases in moving away from the centre if r > R

(B) 'g' decreases in moving away from the centre if r < R

(C) 'g' is zero at the centre of earth

(D) 'g' decreases if earth stops rotating on its axis



Sol. Variation of g with distance

variation of g with ω : $g' = g - \omega^2 R\cos^2\lambda$

If ω=0 then g will not change at poles where $\cos\lambda = 0$.

Ex. 12 An astronaut, inside an earth satellite experiences weightlessness because:

(A) he is falling freely

(B) no external force is acting on him

(C) no reaction is exerted by floor of the satellite

(D) he is far away from the earth surface

Sol. As astronaut's acceleration = g so he is falling freely. Also no reaction is exerted by the floor of the satellite.

Ex. 13 If a satellite orbits as close to the earth's surface as possible

(A) its speed is maximum

(B) time period of its revolution is minimum

(C) the total energy of the 'earth plus satellite' system is minimum

(D) the total energy of the 'earth plus satellite' system is maximum

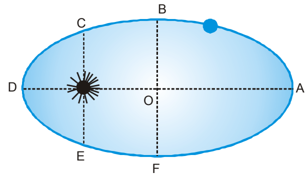
Sol. **For (A) :** orbital speed $v_0 = \sqrt{\dfrac{GM}{r}}$

For (B) : Time period of revolution $T^2 \propto r^3$

For (C/D): Total energy $= -\dfrac{GMm}{2r}$

Ex. 14 A planet is revolving around the sun is an elliptical orbit as shown in figure. Select correct alternative (S)

(A) Its total energy is negative at D.
(B) Its angular momentum is constant
(C) Net torque on planet about sun is zero
(D) Linear momentum of the planet is conserved

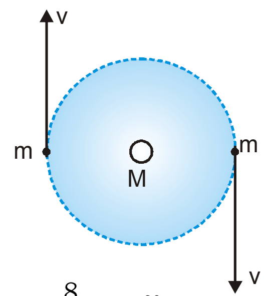


Sol. **For (A)** : For bounded system, the total energy is always negative.
For (B) : For central force field, angular momentum is always conserved
For (C) :For central force field, torque = 0.
For (D) : In presence of external force, linear momentum is not conserved.

Ex. 15 to 17

A triple star system consists of two stars, each of mass m, in the same circular orbit about central star with mass $M = 2 \times 10^{30}$ kg. The two outer stars always lie at opposite ends of a diameter of their common circular orbit. The radius of the circular orbit is $r = 10^{11}$ m and the orbital period of each star is 1.6×10^7 s.

[Take $\pi^2 = 10$ and $G = \frac{20}{3} \times 10^{-11}$ Nm²kg⁻²]



15. The mass m of the outer stars is

(A) $\frac{16}{15}\times 10^{30}$ kg **(B)** $\frac{11}{8}\times 10^{30}$ kg **(C)** $\frac{15}{16}\times 10^{30}$ kg **(D)** $\frac{8}{11}\times 10^{30}$ kg

16. The orbital velocity of each star is

(A) $\frac{5}{4}\sqrt{10}\times 10^{3}$ m/s **(B)** $\frac{5}{4}\sqrt{10}\times 10^{5}$ m/s **(C)** $\frac{5}{4}\sqrt{10}\times 10^{2}$ m/s **(D)** $\frac{5}{4}\sqrt{10}\times 10^{4}$ m/s

17. The total mechanical energy of the system is

(A) $-\frac{1375}{64}\times 10^{35}$ J **(B)** $-\frac{1375}{64}\times 10^{38}$ J **(C)** $-\frac{1375}{64}\times 10^{34}$ J **(D)** $-\frac{1375}{64}\times 10^{37}$ J

Sol.

15. F_{mm} = Gravitational force between two outer stars $= \frac{Gm^2}{4r}$

F_{mM} = Gravitational force between central star and outer star $= \frac{GmM}{r^2}$

For circular motion of outer star, $\frac{mv^2}{r} = F_{mm} + F_{mM} \quad \therefore \; v^2 = \frac{G(m+4M)}{4r}$

T = period of orbital motion $= \frac{2\pi r}{v} \quad \therefore \; m = \frac{16\pi^2 r^3}{GT^2} - 4M = \left(\frac{150}{16} - 8\right)10^{30} = \frac{11}{8}\times 10^{30}$ kg

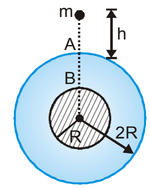
16. $T = \frac{2\pi r}{v} \Rightarrow v = \frac{2\pi r}{T} = \frac{(2)(\sqrt{10})(10^{11})}{1.6\times 10^{7}} = \frac{5}{4}\sqrt{10}\times 10^{4}$ m/s

17. Total mechanical energy = K.E. + P.E.

$$= 2\left(\frac{1}{2}mv^2\right) - \frac{2GMm}{r} - \frac{Gm^2}{2r} = m\left[\frac{G(4M+m)}{4r} - \frac{2GM}{r} - \frac{Gm}{2r}\right] = -\frac{Gm}{r}\left[M + \frac{m}{4}\right]$$

$$= -\left(\frac{20}{3}\times 10^{-11}\right)\left(\frac{11}{8}\times 10^{30}\right)\times \frac{1}{10^{11}}\left(2\times 10^{30} + \frac{11}{32}\times 10^{30}\right) = -\frac{1375}{64}\times 10^{38}\text{ J}$$

Ex. 18 to 20



A solid sphere of mass M and radius R is surrounded by a spherical shell of same mass M and radius 2R as shown. A small particle of mass m is released from rest from a height h (<<R) above the shell. There is a hole in the shell.

18. In what time will it enter the hole at A :–

(A) $2\sqrt{\dfrac{hR^2}{GM}}$ **(B)** $\sqrt{\dfrac{2hR^2}{GM}}$ **(C)** $\sqrt{\dfrac{hR^2}{GM}}$ **(D)** none of these

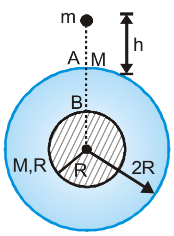
19. What time will it take to move from A to B ?

(A) $= \dfrac{4R^2}{\sqrt{GMR}}$ **(B)** $> \dfrac{4R^2}{\sqrt{GMR}}$ **(C)** $< \dfrac{4R^2}{\sqrt{GMR}}$ **(D)** none of these

20. With what approximate speed will it collide at B ?

(A) $\sqrt{\dfrac{2GM}{R}}$ **(B)** $\sqrt{\dfrac{GM}{2R}}$ **(C)** $\sqrt{\dfrac{3GM}{2R}}$ **(D)** $\sqrt{\dfrac{GM}{R}}$

Sol.



18. $a = \dfrac{GM}{(2R)^2} + \dfrac{GM}{(2R)^2} = \dfrac{GM}{2R^2}$ $\quad \therefore t = \sqrt{\dfrac{2 \times h}{a}} = \sqrt{\dfrac{2 \times h \times 2R^2}{GM}} = 2\sqrt{\dfrac{hR^2}{GM}}$

19. Given that (h<<R), so the velocity at A' is also zero.

We can see here that the acceleration always increases from 2R to R and its value must be greater than

$a = \dfrac{GM}{4R^2}(\text{at A}) \quad \therefore \; t < \dfrac{v}{a} \Rightarrow t < \sqrt{\dfrac{GM}{R}} \times \dfrac{4R^2}{GM} \Rightarrow t < \dfrac{4R^2}{\sqrt{GMR}}$

20. Given that (h<<R), so the velocity at A' is also zero.

Loss in PE = gain in KE $\quad \therefore \; \dfrac{GMm}{2R} = \dfrac{1}{2}mv^2 \Rightarrow v = \sqrt{\dfrac{GM}{R}}$

Ex. 21 Imagine a light planet revolving around a very massive star in a circular orbit of radius R. If the gravitational force of attraction between the planet and the star is proportional to $R^{-5/2}$, then match the following

	Column I		**Column II**
(A)	Time period of revolution is proportional to	**(P)**	R^0
(B)	Kinetic energy of planet is proportional to	**(Q)**	$R^{7/4}$
(C)	Orbital velocity of planet is proportional to	**(R)**	$R^{-1/2}$
(D)	Total mechanical energy of planet is proportional to	**(S)**	$R^{-3/2}$
		(T)	$R^{-3/4}$

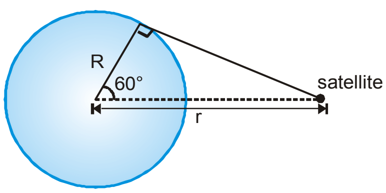
Sol. According to question $F = \frac{C}{R^{5/2}}$ where C is a constant so $\frac{mv^2}{R} = \frac{C}{R^{5/2}} \Rightarrow mv^2 \propto \frac{1}{R^{3/2}} \Rightarrow KE \propto R^{-3/2}$

Also $v \propto R^{-3/4}$ and $PE \propto R^{-3/2}$. Total mechanical energy = KE + PE $\propto R^{-3/2}$

Time period $T = \frac{2\pi R}{v} \propto R^{7/4}$

Ex. 22 A satellite is launched in the equatorial plane in such a way that it can transmit signals upto 60° latitude on the earth. The orbital velocity of the satellite is found to be $\sqrt{\frac{GM}{\alpha R}}$. Find the value of α.

Sol. $r\cos 60° = R \Rightarrow r = 2R$ orbital velocity $v_0 = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{2R}} \Rightarrow \alpha = 2$



Ex. 23 An artificial satellite (mass m) of a planet (mass M) revolves in a circular orbit whose radius is n times the radius R of the planet. In the process of motion, the satellite experiences a slight resistance due to cosmic dust. Assuming the force of resistance on satellite to depend on velocity as $F=av^2$ where 'a' is a constant, calculate how long the satellite will stay in the space before it falls onto the planet's surface.

Sol. Air resistance $F = -av^2$, where orbital velocity $v = \sqrt{\frac{GM}{r}}$

r = the distance of the satellite from planet's centre $\Rightarrow F = -\frac{GMa}{r}$

The work done by the resistance force $dW = Fdx = Fvdt = \frac{GMa}{r}\sqrt{\frac{GM}{r}}dt = \frac{(GM)^{3/2}a}{r^{3/2}}dt$ ....**(i)**

The loss of energy of the satellite = dE

$$\therefore \frac{dE}{dr} = \frac{d}{dr}\left[-\frac{GM\ m}{2r}\right] = \frac{GMm}{2r^2} \Rightarrow dE = \frac{GMm}{2r^2}dr \quad ...\textbf{(ii)}$$

Since $dE = -dW$ (work energy theorem) $-\frac{GMm}{2r^2}dr = \frac{(GM)^{3/2}}{r^{3/2}}dt$

$$\Rightarrow t = -\frac{m}{2a\sqrt{GM}}\int_{nR}^{R}\frac{dr}{\sqrt{r}} = \frac{m\sqrt{R}\left(\sqrt{n}-1\right)}{a\sqrt{GM}} = \left(\sqrt{n}-1\right)\frac{m}{a\sqrt{gR}}$$

Exercise # 1

[Single Correct Choice Type Questions]

1. If the gravitational force were to vary inversely as m[th] power of the distance, then the time period of a planet in circular orbit of radius r around the Sun will be proportional to

(A) $r^{-3m/2}$ **(B)** $r^{3m/2}$ **(C)** $r^{m+1/2}$ **(D)** $r^{(m+1)/2}$

2. Three identical point masses, each of mass 1 kg lie in the x–y plane at points (0, 0) (0, 0.2m) and (0.2m,0). The gravitational force on the mass at the origin is :–

(A) $1.67 \times 10^{-11} (\tilde{i} + \tilde{j})N$ **(B)** $3.34 \times 10^{-10} (\tilde{i} + \tilde{j})N$ **(C)** $1.67 \times 10^{-9} (\tilde{i} + \tilde{j})N$ **(D)** $3.34 \times 10^{-10} (\tilde{i} - \tilde{j})N$

3. If the distance between the centres of Earth and Moon is D and mass of Earth is 81 times that of Moon. At what distance from the centre of Earth gravitational field will be zero ?

(A) $\frac{D}{2}$ **(B)** $\frac{2D}{3}$ **(C)** $\frac{4D}{5}$ **(D)** $\frac{9D}{10}$

4. Weight of a body of mass m decreases by 1% when it is raised to height h above the Earth's surface. If the body is taken to a depth h in a mine, then its weight will :–

(A) decrease by 0.5% **(B)** decrease by 2% **(C)** increase by 0.5% **(D)** increase by 1%

5. The radius of Earth is about 6400 km and that of mars is 3200 km. The mass of the Earth is 10 times the mass of mars. An object weight 200 N on the surface of Earth. Its weight on the surface of mars will be :–

(A) 80 N **(B)** 40 N **(C)** 20 N **(D)** 8 N

6. A stone drop from height 'h' reaches to Earth surface in1 sec. If the same stone taken to Moon and drop freely then it will reaches from the surface of the Moon in the time (The 'g' of Moon is 1/6 times of Earth):–

(A) $\sqrt{6}$ second **(B)** 9 second **(C)** $\sqrt{3}$ second **(D)** 6 second

7. An object weighs 10 N at the north pole of the Earth. In a geostationary satellite distance 7R from the centre of the Earth (of radius R), the true weight and the apparent weight are–

(A) 0 N, 0 N **(B)** 0.2 N, 0 **(C)** 0.2 N, 9.8 N **(D)** 0.2 N, 0.2 N

8. Imagine a new planet having the same density as that of Earth but it is 3 times bigger than the Earth in size. If the acceleration due to gravity on the surface of Earth is g and that on the surface of the new planet is g', then

(A) $g' = 3g$ **(B)** $g' = \frac{g}{9}$ **(C)** $g' = 9g$ **(D)** $g' = 27\, g$

9. The rotation of the Earth having radius R about its axis speeds upto a value such that a man at latitude angle 60^0 feels weightless. The duration of the day in such case will be

(A) $8\pi\sqrt{\frac{R}{g}}$ **(B)** $8\pi\sqrt{\frac{g}{R}}$ **(C)** $\pi\sqrt{\frac{R}{g}}$ **(D)** $4\pi\sqrt{\frac{g}{R}}$

10. A body attains a height equal to the radius of the Earth when projected from Earth' surface. The velocity of the body with which it was projected is :–

(A) $\sqrt{\frac{GM_e}{R}}$ **(B)** $\sqrt{\frac{2GM_e}{R}}$ **(C)** $\sqrt{\frac{5}{4}\frac{GM_e}{R}}$ **(D)** $\sqrt{\frac{3GM_e}{R}}$

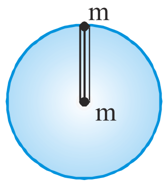
11. A small body of superdense material, whose mass is twice the mass of the Earth but whose size is very small compared to the size of the Earth, starts from rest at a height H << R above the Earth's surface, and reach the Earth's surface in time t. Then t is equal to

(A) $\sqrt{2H/g}$ **(B)** $\sqrt{H/g}$ **(C)** $\sqrt{2H/3g}$ **(D)** $\sqrt{4H/3g}$

12. A man of mass m starts falling towards a planet of mass M and radius R. As he reaches near to the surface, he realizes that he will pass through a small hole in the planet. As he enters the hole, he seen that the planet is really made of two pieces a spherical shell of negligible thickness of mass $\frac{2M}{3}$ and a point mass $\frac{M}{3}$ at the centre. Change in the force of gravity experienced by the man is :

(A) $\frac{2}{3}\frac{GMm}{R^2}$ **(B)** 0 **(C)** $\frac{1}{3}\frac{GMm}{R^2}$ **(D)** $\frac{4}{3}\frac{GMm}{R^2}$

13. Find the distance between centre of gravity and centre of mass of a two particle system attached to the ends of a light rod. Each particle has same mass. Length of the rod is R, where R is the radius of Earth



(A) R **(B)** R/2 **(C)** zero **(D)** R/4

14. If the gravitational acceleration at surface of Earth is g, then increase in potential energy in lifting an object of mass m to a height equal to the radius R of Earth will be :–

(A) $\frac{mgR}{2}$ **(B)** 2mgR **(C)** mgR **(D)** $\frac{mgR}{4}$

15. The gravitational field due to a mass distribution is $E = \frac{K}{x^3}$ in the x–direction. (K is a constant). Taking the gravitational potential to be zero at infinity, its value at the distance x is :–

(A) $\frac{K}{x}$ **(B)** $\frac{K}{2x}$ **(C)** $\frac{K}{x^2}$ **(D)** $\frac{K}{2x^2}$

16. The intensity of gravitational field at a point situated at a distance 8000 km from the centre of Earth is 6.0 newton / kg. The gravitational potential at that point in newton – meter/kg will be :–

(A) 6 **(B)** 4.8×10^7 **(C)** 8×10^5 **(D)** 4.8×10^2

17. Two bodies of masses m and M are placed at distance d apart. The gravitational potential (V) at the position where the gravitational field due to them is zero V is :–

(A) $V = -\frac{G}{d}(m + M)$ **(B)** $V = -\frac{G}{d}$ **(C)** $V = -\frac{GM}{d}$ **(D)** $V = -\frac{G}{d}(\sqrt{m} + \sqrt{M})^2$

18. Escape velocity of a body from the surface of Earth is 11.2 km/sec. from the Earth surface. If the mass of Earth becomes double of its present mass and radius becomes half of its present radius, then escape velocity will become

(A) 5.6 km/sec **(B)** 11.2 km/sec **(C)** 22.4 km/sec **(D)** 44.8km/sec

19. Gravitation on Moon is $\frac{1}{6}$th of that on Earth. When a balloon filled with hydrogen is released on Moon then this

(A) will rise with an acceleration less then $\frac{g}{6}$ **(B)** will rise with acceleration $\frac{g}{6}$

(C) will fall down with an acceleration less than $\frac{5g}{6}$ **(D)** will fall down with acceleration $\frac{g}{6}$

20. The atmospheric pressure and height of barometer column is 10^5 P_a and 760mm respectively on the Earth surface. If the barometer is taken to the Moon then column height will be :–

(A) zero **(B)** 76 mm **(C)** 126.6 mm **(D)** 760 mm

21. A body of mass m is situated at distance $4R_e$ above the Earth's surface, where R_e is the radius of Earth how much minimum energy be given to the body so that it may escape :–

(A) mgR_e **(B)** $2mgR_e$ **(C)** $\frac{mgR_e}{5}$ **(D)** $\frac{mgR_e}{16}$

22. A satellite is seen after each 8 hours over equator at a place on the Earth when its sense of rotation is opposite to the Earth. The time interval after which it can be seen at the same place when the sense of rotation of Earth & satellite is same will be:

(A) 8 hours **(B)** 12 hours **(C)** 24 hours **(D)** 6 hours

23. Two identical satellites are at the heights R and 7R from the Earth's surface. Then which of the following statement is incorrect :– (r = radius of the Earth)

(A) Ratio of total energy of both is 5

(B) Ratio of kinetic energy of both is 4

(C) Ratio of potential energy of both 4

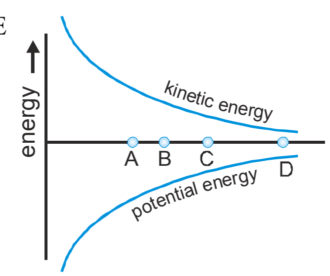
(D) Ratio of total energy of both is 4 and ratio of magnitude of potential to kinetic energy is 2

24. A hollow spherical shell is compressed to half its radius. The gravitational potential at the centre

(A) Increases **(B)** Decreases **(C)** Remains same

(D) During the compression increases then returns at the previous value

25. Potential energy and kinetic energy of a two particle system are shown by KE and PE. respectively in figure. This system is bound at :



(A) Only point A **(B)** Only point D

(C) Points A, B, and C **(D)** All points A, B, C and D

Exercise # 2 Part # I [Multiple Correct Choice Type Questions]

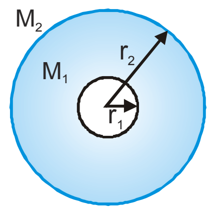
1. Gravitational potential difference between surface of a planet and a point situated at a height of 20m above its surface is 2 joule/kg. If gravitational field is uniform, then the work done in taking a 5kg body of height 4 meter above surface will be :–

(A) 2J **(B)** 20J **(C)** 40J **(D)** 10J

2. One projectile after deviating from its path starts moving round the Earth in a circular path of radius equal to nine times the radius of Earth R. Its time period will be :–

(A) $2\pi\sqrt{\frac{R}{g}}$ **(B)** $27 \times 2\pi\sqrt{\frac{R}{g}}$ **(C)** $\pi\sqrt{\frac{R}{g}}$ **(D)** $0.8 \times 3\pi\sqrt{\frac{R}{g}}$

3. Two concentric shells of masses M_1 and M_2 are having radii r_1 and r_2. Which of the following is the correct expression for the gravitational field at a distance r :–



(A) $\frac{G(M_1 + M_2)}{r^2}$, for $r < r_1$ **(B)** $\frac{G(M_1 + M_2)}{r^2}$, for $r < r_2$

(C) $\frac{GM_2}{r^2}$, for $r_1 < r < r_2$ **(D)** $\frac{GM_1}{r^2}$, for $r_1 < r < r_2$

4. The potential energy of a body of mass m is U = ax + by the magnitude of acceleration of the body will be :–

(A) $\frac{ab}{m}$ **(B)** $\left(\frac{a+b}{m}\right)$ **(C)** $\frac{\sqrt{a^2+b^2}}{m}$ **(D)** $\frac{a^2+b^2}{m}$

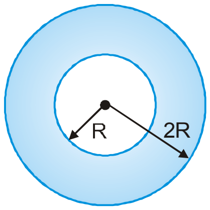
5. In a certain region of space, the gravitational field is given by $-\frac{k}{r}$, where r is the distance and k is a constant. If the gravitational potential at $r = r_0$ be V_0, then what is the expression for the gravitational potential (V) :–

(A) $v = k \log\left(\frac{r}{r_0}\right) + V_0$ (B) $v = k \log\left(\frac{r_0}{r}\right)$ (C) $v = k \log\left(\frac{r_0}{r}\right) + V_0$ (D) $v = k \log\left(\frac{r_0}{r}\right) - V_0$

6. If there were a smaller gravitational effect, which of the following forces do you think would alter in some respect.

(A) Viscous force **(B)** Archimedes uplift **(C)** Electrostatic force **(D)** Magnetic force

7. There is a concentric hole of radius R in a solid sphere of radius 2R. Mass of the remaining portion is M. What is the gravitational potential at centre ?



(A) $-\frac{5GM}{7R}$ **(B)** $-\frac{7GM}{14R}$

(C) $-\frac{3GM}{7R}$ **(D)** $-\frac{9GM}{14R}$

8. Two metallic balls of mass m are suspended by two strings of length L. The distance between upper ends is ℓ. The angle at which the string will be inclined with vertical due to attraction, is (m << M, where M is the mass of Earth):–

(A) $\tan^{-1}\frac{Gm}{g\ell^2}$ **(B)** $\tan^{-1}\frac{Gm}{gL^2}$ **(C)** $\tan^{-1}\frac{Gm}{g\ell}$ **(D)** $\tan^{-1}\frac{Gm}{gL}$

9. Select the correct alternative :–

(A) The gravitational field inside a spherical cavity, within a spherical planet must be non zero and uniform

(B) When a body is projected horizontally at an appreciable large height above the Earth, with a velocity less than for a circular orbit, it will fall to the Earth along a parabolic path

(C) A body of zero total mechanical energy placed in a gravitational field will escape the field

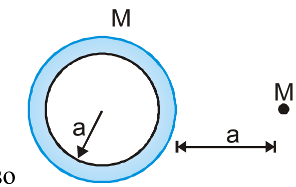
(D) Earth's satellite must be in equatorial plane

10. When a satellite in a circular orbit around the Earth enters the atmospheric region, it encounters small air resistance to its motion. Then

(A) Its kinetic energy increases **(B)** Its kinetic energy decreases

(C) Its angular momentum about the Earth decreases **(D)** Its period of revolution around the Earth increases

11. A particle of mass M is at a distance a from surface of a thin spherical shell of equal mass and having radius a



(A) Gravitational field and potential both are zero at centre of the shell

(B) Gravitational field is zero not only inside the shell but at a point outside the shell also

(C) Inside the shell, gravitational field alone is zero

(D) Neither gravitational field nor gravitational potential is zero inside the shell

12. Three particles are projected vertically upward from a point on the surface of the Earth with velocities $\sqrt{2gR/3}$, $\sqrt{gR}$, $\sqrt{4gR/3}$ respectively where R is the radius of the Earth and g is the acceleration due to gravity on the surface of the Earth. The maximum heights attained are respectively h_1, h_2, h_3.

(A) $h_1 : h_2 = 2 : 3$ **(B)** $h_2 : h_3 = 3 : 4$ **(C)** $h_1 : h_3 = 1 : 4$ **(D)** $h_2 = R$

13. A cavity of radius R/2 is made inside a solid sphere of radius R. The centre of the cavity is located at a distance R/2 from the centre of the sphere. The gravitational force on a particle of mass 'm' at a distance R/2 from the centre of the sphere on the joining both the centres of sphere and cavity is (opposite to the centre of cavity). [Here $g = GM/R^2$, where M is the mass of the sphere]

(A) $\frac{mg}{2}$ **(B)** $\frac{3mg}{8}$ **(C)** $\frac{mg}{16}$ **(D)** None of these

14. For a satellite to be geo–stationary, which of the following are essential conditions?

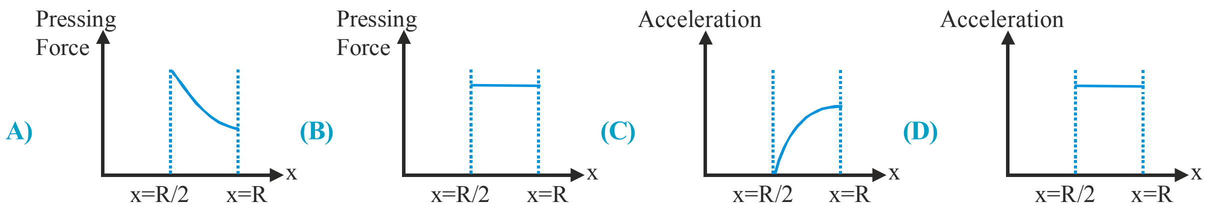
(A) It must always be stationed above the equator **(B)** It must be rotated from west to east

(C) It must be about 36000 km above the Earth **(D)** Its orbit must be circular, and not elliptical

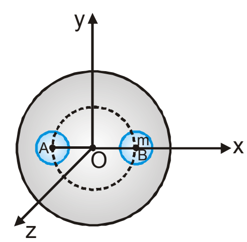
15. A double star is a system of two stars of masses m and 2m, rotating about their centre of mass only under their mutual gravitational attraction. If r is the separation between these two stars then their time period of rotation about their centre of mass will be proportional to :

(A) $r^{3/2}$ **(B)** r **(C)** $m^{1/2}$ **(D)** $m^{-1/2}$

16. A tunnel is dug along a chord of the Earth at a perpendicular distance R/2 from the Earth's centre. The wall of the tunnel may be assumed to be frictionless. A particle is released from one end of the tunnel. The pressing force by the particle on the wall, and the acceleration of the particle varies with x (distance of the particle from the centre) according to :



17. A solid sphere of uniform density and radius 4 units is located with its centre at the origin O of coordinates. Two spheres of equal radii 1 unit, with their centres at A (–2, 0, 0) and B (2, 0, 0) respectively, are taken out of the solid leaving behind spherical cavities as shown in figure. Then :–



(A) The gravitational field due to this object at the origin is zero

(B) The gravitational field at the point B (2, 0, 0) is zero

(C) The gravitational potential is the same at all points of circle $y^2+z^2=36$

(D) The gravitational potential is the same at all points on the circle $y^2+z^2=4$

18. Mark the correct statement/s :–

(A) Gravitational potential at curvature centre of a thin hemispherical shell of radius R and mass M is equal to $\frac{GM}{R}$

(B) Gravitational field strength at point lying on the axis of a thin, uniform circular ring of radius R and mass M is equal to $\frac{GMx}{\left(R^2+x^2\right)^{3/2}}$ where x is distance of that point from centre of the ring

(C) Newton's law of gravitation for gravitational force between two bodies is applicable only when bodies have spherically symmetric distribution of mass

(D) None of these

19. The magnitudes of the gravitational field at distance r_1 and r_2 from the centre of a uniform, sphere of radius R and mass M are F_1 and F_2 respectively. then :–

(A) $\frac{F_1}{F_2}=\frac{r_1}{r_2}$ if $r_1 < R$ and $r_2 < R$

(B) $\frac{F_1}{F_2}=\frac{r_2^2}{r_1^2}$ if $r_1 > R$ and $r_2 > R$

(C) $\frac{F_1}{F_2}=\frac{r_1^3}{r_2^3}$ if $r_1 < R$ and $r_2 < R$

(D) $\frac{F_1}{F_2}=\frac{r_1^2}{r_2^2}$ if $r_1 < R$ and $r_2 < R$

20. Suppose a smooth tunnel is dug along a straight line joining two points on the surface of the Earth and a particle is dropped from rest at its one end. Assume that mass of Earth is uniformly distributed over its volume. Then

(A) The particle will emerge from the other end with velocity $\sqrt{\frac{GM_e}{2R_e}}$ where M_e and R_e are Earth's mass and radius respectively

(B) The particle will come to rest at centre of the tunnel because at this position, particle is closest to Earth centre.

(C) Potential energy of the particle will be equal to zero at centre of tunnel if it is along tunnel's diameter

(D) Acceleration of the particle will be proportional to its distance from midpoint of the tunnel

21. Gravitational potential at the centre of curvature of a hemispherical bowl of radius R and mass M is V :–

(A) Gravitational potential at the centre of curvature of a thin uniform wire of mass M, bent into a semicircle of radius R, is also equal to V

(B) In part (A) if the same wire is bent into a quarter of a circle then also the gravitational potential at the centre of curvature will be V

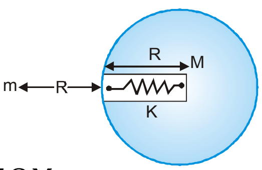
(C) In part (A) if the wire mass is non uniformly distributed along its length audit is bent into a semicircle of radius R, gravitational potential at the centre is V

(D) None of these

22. A particle of mass m is transferred from the centre of the base of a uniform solid hemisphere of mass M and radius R to infinity. The work performed in the process by the gravitational force exerted on the particle by the hemisphere is

(A) $\frac{GMm}{R}$ **(B)** $-\frac{1}{2}\frac{GMm}{R}$ **(C)** $-\frac{3}{2}\frac{GMm}{R}$ **(D)** $-\frac{3}{4}\frac{GMm}{R}$

23. A small ball of mass 'm' is released at a height 'R' above the Earth surface, as shown in the figure. If the maximum depth of the ball to which it goes is R/2 inside the Earth through a narrow grove before coming to rest momentarily. The grove, contain an ideal spring of spring constant K and natural length R, the value of K is (R is radius of Earth and M mass of Earth)



(A) $\frac{3GMm}{R^3}$ **(B)** $\frac{6GMm}{R^3}$ **(C)** $\frac{9GMm}{R^3}$ **(D)** $\frac{7GMm}{R^3}$

24. A planet is revolving around the Sun in an elliptical orbit. Its closest distance from the Sun is r_{min}. The farthest distance from the Sun is r_{max}. If the orbital angular velocity of the planet when it is nearest to the Sun is ω, then the orbital angular velocity at the point when it is at the farthest distance from the Sun is–

(A) $\left(\sqrt{\frac{r_{min}}{r_{max}}}\right)\omega$ **(B)** $\left(\sqrt{\frac{r_{max}}{r_{min}}}\right)\omega$ **(C)** $\left(\frac{r_{max}}{r_{min}}\right)^2\omega$ **(D)** $\left(\frac{r_{min}}{r_{max}}\right)^2\omega$

25. Masses and radii of Earth and Moon are M_1, M_2 and R_1, R_2 respectively. The distance between their centre is 'd'. The minimum velocity given to mass 'M' from the mid point of line joining their centre so that it will escape :–

(A) $\sqrt{\frac{4G(M_1+M_2)}{d}}$ **(B)** $\sqrt{\frac{4G}{d}\frac{M_1M_2}{(M_1+M_2)}}$ **(C)** $\sqrt{\frac{2G}{d}\left(\frac{M_1+M_2}{M_1M_2}\right)}$ **(D)** $\sqrt{\frac{2G}{d}(M_1+M_2)}$

26. A satellite is in a circular orbit very close to the surface of a planet. At some point it is given an impulse along its direction of motion, causing its velocity to increase n times. It now goes into an elliptical orbit. The maximum possible value of n for this to occur is

(A) 2 **(B)** $\sqrt{2}$ **(C)** $\sqrt{2}+1$ **(D)** $\frac{1}{\sqrt{2}-1}$

Part # II [Assertion & Reason Type Questions]

These questions contains, Statement 1 (assertion) and Statement 2 (reason).

(A) Statement–1 is True, Statement–2 is True ; Statement–2 is a correct explanation for Statement–1

(B) Statement–1 is True, Statement–2 is True ; Statement–2 is not a correct explanation for Statement–1

(C) Statement–1 is True, Statement–2 is False.

(D) Statement–1 is False, Statement–2 is True.

1. **Statement – I** The acceleration of a particle near the Earth surface differs slightly from the gravitational acceleration $a_g = GM/R^2$.

Statement – II The Earth is not a uniform sphere and because the Earth rotates.

2. **Statement – I** Two satellites A & B are in the same orbit around the Earth, B being behind A. B cannot overtake A by increasing its speed.

Statement – II It will then go into a different orbit.

3. **Statement – I** Kepler's law of areas is equivalent to the law of conservation of angular momentum.

Statement – II Areal velocity $\frac{dA}{dt} = \frac{L}{2m}$ = constant

4. **Statement – I** A person feels weightlessness in an artificial satellite of the Earth. However a person on the Moon (natural satellite) feels his weight.

Statement – II Artificial satellite is a freely falling body and on the Moon surface, the weight is mainly due to Moon's gravitational attraction.

5. **Statement – I** A satellite is moving in a circular orbit of the Earth. If the gravitational pull suddenly disappears, then it moves with the same speed tangential to the original orbit.

Statement – II The orbital speed of a satellite increases with the increase in radius of the orbit.

6. **Statement – I** Pendulum clock stops working on the spaceship.

Statement – II Pendulum of the pendulum clock falls down on the spaceship.

7. **Statement – I** Moon has no atmosphere.

Statement – II Due to less gravity Moon is unable to retain its atmosphere.

8. **Statement – I** Moon cannot be used as a satellite for communication.

Statement – II Moon doesn't move in the equatorial plane of the Earth.

9. **Statement – I** Escape velocity is independent of the angle of projection.

Statement – II Escape velocity for vertical projection from the surface of Earth is $\sqrt{\frac{2GM}{R}}$.

10. **Statement – I** When a planet approaches the point which is farthest from the Sun, its orbital speed decreases.

Statement – II Work done on the planet by the gravitational force exerted by the Sun is negative.

11. **Statement – I** The plane of the orbit of an artificial satellite must contain the centre of the Earth.

Statement – II For an artificial satellite, the necessary centripetal force is provided by gravity.

Exercise # 3

Part # I [Matrix Match Type Questions]

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with **one or more** statement(s) in **Column-II**.

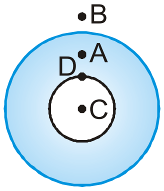
1. In elliptical orbit of a planet, as the planet moves from apogee position to perigee position,

	Column–I		Column–II
(A)	Speed of planet	(P)	Remains same
(B)	Distance of planet from centre of Sun	(Q)	Decreases
(C)	Potential energy	(R)	Increases
(D)	Angular momentum about centre of Sun	(S)	Can not say

2. A satellite is in a circular equatorial orbit of radius 7000 km around the Earth. If it is transferred to a circular orbit of double the radius

	Column I		Column II
(A)	Angular momentum	(P)	Increases
(B)	Area of Earth covered by satellite signal	(Q)	Decreases
(C)	Potential energy	(R)	Becomes double
(D)	Kinetic energy	(S)	Becomes half

3. Two concentric spherical shells are as shown in figure. :



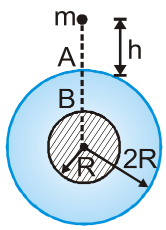
	Column–I		Column–II
(A)	Potential at A	(P)	greater than B
(B)	Gravitational field at A	(Q)	less than B
(C)	As one moves from C to D	(R)	potential remains constant
(D)	As one moves from D to A	(S)	gravitational field decreases
		(T)	None

4.

	Column–I		Column–II
(A)	Kinetic energy of a particle in gravitational field is increasing	(P)	work done by gravitational force should be positive
(B)	Potential energy of a particle in gravitational field is increasing	(Q)	work done by external force should be non zero
(C)	Mechanical energy of a particle in gravitational field is increasing	(R)	work done by gravitational force should be negative
		(S)	can not say anything

Part # II [Comprehension Type Questions]

Comprehension # 1



A solid sphere of mass M and radius R is surrounded by a spherical shell of same mass M and radius 2R as shown. A small particle of mass m is released from rest from a height h (<<R) above the shell. There is a hole in the shell.

1. In what time will it enter the hole at A :–

(A) $2\frac{\sqrt{hR^2}}{GM}$ (B) $\sqrt{\frac{2hR^2}{GM}}$ (C) $\sqrt{\frac{hR^2}{GM}}$ (D) None of these

2. What time will it take to move from A to B ?

(A) $= \frac{R^2}{\sqrt{GMh}}$ **(B)** $> \frac{R^2}{\sqrt{GMh}}$ **(C)** $< \frac{R^2}{\sqrt{GMh}}$ **(D)** None of these

3. With what approximate speed will it collide at B ?

(A) $\sqrt{\frac{2GM}{R}}$ **(B)** $\sqrt{\frac{GM}{2R}}$ **(C)** $\sqrt{\frac{3GM}{2R}}$ **(D)** $\sqrt{\frac{GM}{R}}$

Comprehension # 2

When a particle is projected from the surface of Earth, its mechanical energy and angular momentum about centre of Earth at all time are constant.

1. A particle of mass m is projected from the surface of Earth with velocity v_0 at angle θ with horizontal. Suppose h be the maximum height of particle from surface of Earth and v its speed at that point then v is :–

(A) $v_0 \cos\theta$ **(B)** $> v_0 \cos\theta$ **(C)** $< v_0 \cos\theta$ **(D)** Zero

2. Maximum height h of the particle is :–

(A) $= \frac{v_0^2 \sin^2\theta}{2g}$ **(B)** $> \frac{v_0^2 \sin^2\theta}{2g}$ **(C)** $< \frac{v_0^2 \sin^2\theta}{2g}$

(D) Can be greater than or less than $\frac{v_0^2 \sin^2\theta}{2g}$

Comprehension # 3

Two stars bound together by gravity orbit each other because of their mutual attraction. Such a pair of stars is referred to as a binary star system. One type of binary system is that of a black hole and a companion star. The black hole is a star that has collapsed on itself and is so missive that not even light rays can escape its gravitational pull. Therefore when describing the relative motion of a black hole and a companion star, the motion of the black hole can be assumed negligible compared to that of the companion.

The orbit of the companion star is either elliptical with the black hole at one of the foci or circular with the black hole at the centre. The gravitational potential energy is given by U = – GmM/r, where G is the universal gravitational constant, m is the mass of the companion star, M is the mass of the block hole, and r is the distance between the centre of the companion star and the centre of the black hole. Since the gravitational force is conservative, the companion star's total mechanical energy is a constant. Because of the periodic nature of the orbit, there is a simple relation between the average kinetic energy <K> of the companion star and its average potential energy <U>. In particular,

$< K > = < – U/2 >$.

Two special points along the orbit are single out by astronomers. Parigee is the point at which the companion star is closest to the black hole, and apogee is the point at which is the farthest from the black hole.

1. For circular orbits the potential energy of the companion star is constant throughout the orbit. If the radius of the orbit doubles, what is the new value of the velocity of the companion star ?

(A) It is 1/2 of the old value **(B)** It is $1/\sqrt{2}$ of the old value

(C) It is the same as old value **(D)** It is double the old value

2. Which of the following prevents the companion star from leaving its orbit and falling into the black hole ?

(A) The centripetal force **(B)** The gravitational force

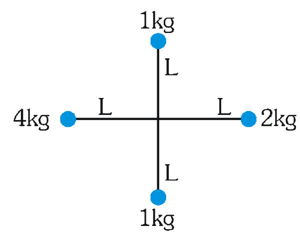
(C) The companion star's potential energy **(D)** The companion star's kinetic energy

Exercise # 4

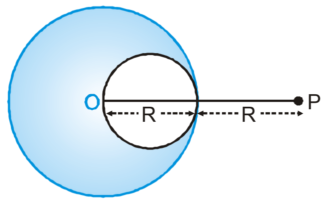
[Subjective Type Questions]

1. Two point–like objects, each with mass m, are connected by a massless rope of length ℓ. The objects are suspended vertically near the surface of Earth, so that one object is hanging below the other. Then the objects are released. Show that the tension in the rope is $T = \frac{GMm\ell}{R^3}$ where M is the mass of the Earth and R is its radius. [ℓ <<R]

2. In the given figure, L = 1 metre, if total gravitational force on 4 kg mass is $\vec{F}_1$ and on 2 kg mass is $\vec{F}_2$, then find out the ratio of $\left|\frac{\vec{F}_1}{\vec{F}_2}\right|$.



3. A solid sphere of uniform density and radius R applies a gravitational force of attraction equal to F_1 on a particle placed at P, distance 2R from the centre of the sphere. A spherical cavity of radius R/2 is now made in the sphere as shown in figure. The sphere with cavity now applies a gravitational force F_2 on the same particle placed at P. Find the ratio of F_2/F_1 .



4. The Earth may be regarded as a spherically shaped uniform core of density ρ_1 and radius $\frac{R}{2}$ surrounded by a uniform shell of thickness $\frac{R}{2}$ and density ρ_2. Find the ratio of $\frac{\rho_1}{\rho_2}$ if the value of acceleration due to gravity is the same at surface as at depth $\frac{R}{2}$ from the surface.

5. The gravitational field in a region is given by $\vec{E} = (3\hat{i} - 4\hat{j})$ N/kg. Find out the work done (in joule) in displacing a particle by 1 m, along the line $4y = 3x + 9$:

6. Find the potential energy of a system of four particles placed at the vertices of a square of side ℓ. Also obtain the potential at the centre of the square.

7. A particle of mass 1 kg is placed at a distance of 4 m from the centre and on the axis of a uniform ring of mass 5 kg and radius 3 m. Calculate the work done to increase the distance of the particle from 4 m to $3\sqrt{3}$ m.

8. A thin spherical shell of total mass M and radius R is held fixed. There is a small hole in the shell. A mass m is released from rest a distance R from the hole along a line that passes through the hole and also through the centre of the shell. This mass subsequently moves under the gravitational force of the shell. How long does the mass take to travel from the hole to the point diametrically opposite.

9. Find the potential energy of a system of eight particles placed at the vertices of a cube of side L. Neglect the self energy of the particles.

10. Two small dense stars rotate about their common centre of mass as a binary system with the period 1 year for each. One star is of double the mass of the other and the mass of the lighter one is $\frac{1}{3}$ of the mass of the Sun. Find the distance between the stars if distance between the Earth & the Sun is R.

11. Find the potential energy of the gravitational interaction of a point mass of mass m and a thin uniform rod of mass M and length ℓ , if they are located along a straight line such that the point mass is at a distance 'a' from one end. Also find the force of their interaction.

12. A body of mass m be projected vertically upward from the surface of the Earth so as to reach a height nR above the Earth's surface. Calculate :

(i) The increase in its potential energy **(ii)** The velocity with which the body must be projected

13. An artificial satellite is moving in a circular orbit around the Earth with a speed equal to half the magnitude of escape velocity from the Earth.

(i) Determine the height of the satellite above the Earth's surface.

(ii) If the satellite is stopped suddenly in its orbit and allowed to fall freely on the Earth, find the speed with which it hits and surface of Earth. Given M = mass of Earth & R = Radius of Earth

14. A comet orbits the Sun in a highly elliptical orbit. Does the comet have a constant

(i) linear speed, **(ii)** angular speed, **(iii)** angular momentum, **(iv)** kinetic energy, **(v)** potential energy, **(vi)** total energy throughout its orbit ? Neglect any mass loss of the comet when it comes very close to the Sun.

15. A satellite of mass m is in an elliptical orbit around the Earth of mass M (M>>m). The speed of the satellite at its nearest point to the Earth (perigee) is $\sqrt{\frac{6GM}{5R}}$ where R = its closest distance to the Earth. It is desired to transfer this satellite into a circular orbit around the Earth of radius equal its largest distance from the Earth. Find the increase in its speed to be imparted at the apogee (farthest point on the elliptical orbit).

16. A small satellite revolves around a heavy planet in a circular orbit. At certain point in its orbit a sharp impulse acts on it and instantaneously increases its kinetic energy to 'k' (<2) times without change in its direction of motion. Show that in its subsequent motion the ratio of its maximum and minimum distances from the planet is $\frac{k}{2-k}$, assuming the mass of the satellite is negligibly small as compared to that of the planet.

17. A body is launched from the Earth's surface a angle $\alpha = 30^0$ to the horizontal at a speed $v_0 = \sqrt{\frac{1.5GM}{R}}$. Neglecting air resistance and Earth's rotation, find **(i)** the height to which the body will rise. **(ii)** The radius of curvature of trajectory at its top point.

18. The minimum and maximum distances of a satellite from the centre of the Earth are 2R and 4R respectively, where R is the radius of Earth and M is the mass of the Earth. Find radius of curvature at the point of minimum distance.

19. Distance between the centres of two starts is 10a. The masses of these stars are M and 16M and their radii a and 2a respectively. A body of mass m is fired straight from the surface of the larger star towards the surface of the smaller star. What should be its minimum initial speed to reach the surface of the smaller star ? Obtain the expression in terms of G, M and a.

20. A satellite of mass M_S is orbiting the Earth in a circular orbit of radius R_S. It starts losing energy slowly at a constant rate C due to friction. If M_e and R_e denote the mass and radius of the Earth respectively, show that the satellite falls on the Earth in a limit time t given by $t = \frac{GM_S M_e}{2C}\left(\frac{1}{R_e} - \frac{1}{R_S}\right)$

21. A particle takes n seconds less and acquires a velocity u m/sec. higher at one place than at another place in falling through the same distance. Calculate the product of the acceleration due to gravity at these two places.

22. A cord of length 64 m is used to connect a 100 kg astronaut to spaceship whose mass is much larger than that of the astronaut. Estimate the value of the tension in the cord. Assume that the spaceship is orbiting near Earth surface. Assume that the spaceship and the astronaut fall on a straight line from the Earth centre. The radius of the Earth is 6400 km.

23. A rocket starts vertically upwards with speed v_0. Show that its speed v at a height h is given by

$$v_0^2 - v^2 = \left[(2gh)/\left(1+\frac{h}{R}\right)\right]$$

Where R is the radius of the Earth. Hence deduce the maximum height reached by a rocket fired with speed equal to 90% of escape velocity.

24. Calculate the distance from the surface of the Earth at which above and below the surface, acceleration due to gravity is the same.

25. A remote sensing satellite is revolving in an orbit of radius x on the equator of Earth. Find the area on Earth surface in which satellite can not send message.

26. A body moving radially away from a planet of mass M, when at distance r from planet, explodes in such a way that two of its many fragments move in mutually perpendicular circular orbits around the planet. What will be

(i) Then velocity in circular orbits ?

(ii) Maximum distance between the two fragments before collision and

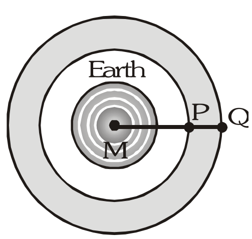
(iii) Magnitude of their relative velocity just before they collide ?

27. The fastest possible rate of rotation of a planet is that for which the gravitational force on material at the equator barely provides the centripetal force needed for the rotation. (Why?)

(i) Show then that the corresponding shortest period of rotation is given by $T = \sqrt{\frac{3\pi}{G\rho}}$ where ρ is the density of the planet, assumed to be homogeneous.

(ii) Evaluate the rotation period assuming a density of 3.0 gm/cm^3, typical of many planets, satellites, and asteroids. No such object is found to be spinning with a period shorter than found by this analysis.

28. A satellite P is revolving around the Earth at a height h= radius of Earth (R) above equator. Another satellite Q is at a height 2h revolving in opposite direction. At an instant the two are at same vertical line passing through centre of sphere. Find the least time of after which again they are in this situation.



29. Assume that a tunnel is dug across the Earth (radius = R) passing through its centre. Find the time a particle takes to reach centre of Earth if it is projected into the tunnel from surface of Earth with speed needed for it to escape the gravitational field of Earth.

30. In a particular double star system, two stars of mass 3.22×10^{30} kg each revolve about their common centre of mass, 1.12×10^{11} m away.

(i) Calculate their common period of revolution, in years.

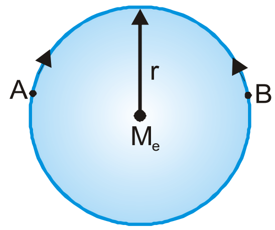
(ii) Suppose that a meteoroid (small solid particle in space) passes through this centre of mass moving at right angles to the orbital plane of the stars. What must its speed be if it is to escape from the gravitational field of the double star ?

31. A satellite close to the Earth is in orbit above the equator with a period of rotation of 1.5 hours. If it is above a point P on the equator at some time, it will be above P again after time...........

32. Ajay can throw a ball at a speed on Earth which can cross a river of width 10 m. Ajay reaches on an imaginary planet whose mean density is twice of the Earth. Find out the maximum possible radius of planet so that if Ajay throws the ball at same speed it may escape from planet. Given radius of Earth = 6.4×10^6 m:

33. A particle takes a time t_1 to move down a straight tunnel from the surface of Earth to its centre. If gravity were to remain constant this time would be t_2. Calculate the ratio $\frac{t_1}{t_2}$

34. Consider two satellites A and B of equal mass m, moving in the same circular orbit of radius r around the Earth E but in opposite sense of rotation and therefore on a collision course (see figure).



(i) In terms of G, M_e, m and r find the total mechanical energy $E_A + E_B$ of the two satellite plus Earth system before collision.

(ii) If the collision is completely inelastic so that wreckage remains as one piece of tangled material (mass =2m), find the total mechanical energy immediately after collision.

(iii) Describe the subsequent motion of the wreckage.

Exercise # 5 Part # I [Previous Year Questions] [AIEEE/JEE-MAIN]

1. If suddenly the gravitational force of attraction between earth and a satellite revolving around it becomes zero, then the satellite will- **[AIEEE - 2002]**

(A) continue to move in its orbit with same velocity

(B) move tangentially to the original orbit with same velocity

(C) become stationary in its orbit

(D) move towards the earth

2. The kinetic energy needed to project a body of mass m from the earth's surface (radius R) to infinity is- **[AIEEE - 2002]**

(A) $\frac{mgR}{2}$ **(B)** 2mgR **(C)** mgR **(D)** $\frac{mgR}{4}$

3. Energy required to move a body of mass m from an orbit of radius 2R to 3R is- **[AIEEE - 2002]**

(A) $GMm/12R^2$ **(B)** $GMm/3R^2$ **(C)** GMm/8R **(D)** GMm/6R

4. The time period of a satellite of earth is 5 hours. If the separation between the centre of earth and the satellite is increased to 4 times the previous value, the new time period will become- **[AIEEE - 2003]**

(A) 10 h **(B)** 80 h **(C)** 40 h **(D)** 20 h

5. Two spherical bodies of mass M and 5M and radii R and 2R respectively are released in free space with initial separation between their centres equal to 12 R. If they attract each other due to gravitational force only, then the distance covered by the smaller body just before collision is- **[AIEEE - 2003]**

(A) 2.5 R **(B)** 4.5 R **(C)** 7.5 R **(D)** 1.5 R

6. A satellite of mass m revolves around the earth of radius R at a height x from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is- **[AIEEE - 2004]**

(A) gx **(B)** $\frac{gR}{R-x}$ **(C)** $\frac{gR^2}{R+x}$ **(D)** $\left(\frac{gR^2}{R+x}\right)^{1/2}$

7. The time period of an earth satellite in circular orbit is independent of- **[AIEEE - 2004]**

(A) the mass of the satellite

(B) radius of its orbit

(C) both the mass and radius of the orbit

(D) neither the mass of the satellite nor the radius of its orbit

8. If g is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass m raised from the surface of the earth to a height equal to the radius R of the earth, is-**[AIEEE - 2004]**

(A) 2mgR **(B)** $\frac{1}{2}$ mgR **(C)** $\frac{1}{4}$ mgR **(D)** mgR

9. Suppose the gravitational force varies inversely as the n^{th} power of distance. Then the time period of a planet in circular orbit of radius R around the sun will be proportional to- **[AIEEE - 2004]**

(A) $R^{\left(\frac{n+1}{2}\right)}$ **(B)** $R^{\left(\frac{n-1}{2}\right)}$ **(C)** R^n **(D)** $R^{\left(\frac{n-2}{2}\right)}$

10. Average density of the earth- **[AIEEE - 2005]**

(A) does not depend on g **(B)** is a complex function of g

(C) is directly proportional to g **(D)** is inversely proportional to g

11. The change in the value of g at a height h above the surface of the earth is the same as at a depth d below the surface of earth. When both d and h are much smaller than the radius of earth, then which one of the following is correct ? **[AIEEE-2005]**

(A) $d = \frac{h}{2}$ **(B)** $d = \frac{3h}{2}$ **(C)** $d = 2h$ **(D)** $d = h$

12. A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them, to take the particle far away from the sphere (you may take $G = 6.67 \times 10^{-11}\ Nm^2/kg^2$) **[AIEEE - 2005]**

(A) 13.34×10^{-10} J **(B)** 3.33×10^{-10} J **(C)** 6.67×10^{-9} J **(D)** 6.67×10^{-10} J

13. If g_E and g_M are the accelerations due to gravity on the surfaces of the earth and the moon respectively and if Millikan's oil drop experiment could be performed on the two surfaces, one will find the ratio $\frac{\text{electronic charge on the moon}}{\text{electronic charge on the earth}}$ to be- **[AIEEE - 2007]**

(A) 1 **(B)** zero **(C)** g_E/g_M **(D)** g_M/g_E

14. A planet in a distance solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 km s^{-1}, the escape velocity from the surface of the planet would be **[AIEEE - 2008]**

(A) 1.1 km s^{-1} **(B)** 11 km s^{-1} **(C)** 110 km s^{-1} **(D)** 0.11 km s^{-1}

15. This quqestion contains statement-1 and statement -2 of the four choices given after the statements, choose the one that best describes the two statements. **[AIEEE - 2008]**

Statement 1: For a mass M kept at the centre of a cube of side 'a', the flux of gravitational field passing through its sides is 4π GM.

Statement 2 : If the direction of a field due to a point source is radial and its dependence on the distance 'r' from the source is given as $\frac{I}{r^2}$, its flux through a closed surface depends only on the strength of the source enclosed by the surface and not on the size or shape of the surface.

(A) Statement –1 is false, Statement –2 is true

(B) Statement–1 is true, Statement–2 is true; Statement–2 is a correct explanation for Statement–1

(C) Statement–1 is true, Statement–2 is true; Statement–2 is not a correct explanation for Statement–1

(D) Statement–1 is true, Statement–2 is false

16. The height at which the acceleration due to gravity becomes $\frac{g}{9}$ (where g = the acceleration due to gravity on the surface of the earth) in terms of R, the radius of the earth, is :- **[AIEEE - 2009]**

(A) $\frac{R}{2}$ **(B)** $\sqrt{2}R$ **(C)** 2R **(D)** $\frac{R}{\sqrt{2}}$

17. Two bodies of masses m and 4m are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is :- **[AIEEE - 2011]**

(A) $-\frac{6Gm}{r}$ **(B)** $-\frac{9Gm}{r}$ **(C)** zero **(D)** $-\frac{4Gm}{r}$

18. Two particles of equal mass 'm' go around a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle with respect to their centre of mass is:- **[AIEEE-2011]**

(A) $\sqrt{\frac{Gm}{R}}$ **(B)** $\sqrt{\frac{Gm}{4R}}$ **(C)** $\sqrt{\frac{Gm}{3R}}$ **(D)** $\sqrt{\frac{Gm}{2R}}$

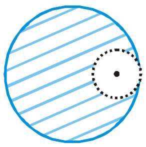
19. The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of 'g' and 'R' (radius of earth) are 10 m/s^2 and 6400 km respectively. The required energy for this work will be :- **[AIEEE-2012]**

(A) 6.4×10^{10} Joules **(B)** 6.4×10^{11} Joules **(C)** 6.4×10^{8} Joules **(D)** 6.4×10^{9} Joules

20. Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is : **[JEE (Main) -2014]**

(A) $\sqrt{\frac{Gm}{R}\left(1+2\sqrt{2}\right)}$ **(B)** $\frac{1}{2}\sqrt{\frac{Gm}{R}\left(1+2+\sqrt{2}\right)}$ **(C)** $\sqrt{\frac{Gm}{R}}$ **(D)** $\sqrt{2\sqrt{2}\frac{Gm}{R}}$

21. From a solid sphere of mass M and radius R, a spherical portion of radius R/2 is removed, as shown in the figure. Taking gravitational potential V = 0 at r = ∞, the potential at the centre of the cavity thus formed is :
(G = gravitational constant) **[JEE (Main) -2015]**



(A) $\frac{-2GM}{3R}$ **(B)** $\frac{-2GM}{R}$ **(C)** $\frac{-GM}{2R}$ **(D)** $\frac{-GM}{R}$

22. A satellite is revolving in a circular orbit at a height 'h' from the earth's surface (radius of earth R ; h<<R). The minimum increase in its orbital velocity required, so that the satellite could escape from the earth's gravitational field, is close to:(Neglect the effect of atmosphere.) **[JEE (Main) -2016]**

(A) $\sqrt{gR}$ **(B)** $\sqrt{gR/2}$ **(C)** $\sqrt{gR}(\sqrt{2}-1)$ **(D)** $\sqrt{2gR}$

Part # II [Previous Year Questions][IIT-JEE ADVANCED]

MCQs With One Correct Answer

1. A satellite S is moving in an elliptical orbit around the Earth. The mass of the satellite is very small compared to the mass of the Earth :– **[IIT-JEE 1998]**

(A) The acceleration of S always directed towards the centre of the Earth

(B) The angular momentum of S about the centre of the Earth changes in direction, but its magnitude remain constant

(C) The total mechanical energy of S varies periodically with time

(D) The linear momentum of S remains constant in magnitude

2. A geostationary satellite orbits around the Earth in a circular orbit of radius 36,000 km. Then, the time period of a spy satellite orbiting a few hundred km (600 km) above the Earth's surface (R_e= 6400 km) will approximately be :– **[IIT-JEE 2002]**

(A) $\frac{1}{2}$ h **(B)** 1h **(C)** 2h **(D)** 4h

3. A double star system consists of two stars A and B which have time period T_A and T_B. Radius R_A and R_B and mass M_A and M_B. Choose the correct option :– **[IIT-JEE 2006]**

(A) If $T_A > T_B$ then $R_A > R_B$ **(B)** If $T_A > T_B$ then $M_A > M_B$

(C) $\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{R_A}{R_B}\right)^3$ **(D)** $T_A = T_B$

4. Two bodies, each of mass M, are kept fixed with a separation 2L. A particle of mass m is projected from the midpoint of the line joining their centres, perpendicular to the line. The gravitational constant is G. The correct statement(s) is (are) :- **[IIT-JEE 2013]**

(A) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $4\sqrt{\frac{GM}{L}}$

(B) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $2\sqrt{\frac{GM}{L}}$

(C) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $\sqrt{\frac{2GM}{L}}$

(D) The energy of the mass m remains constant.

5. A planet of radius $R = \frac{1}{10} \times$ (radius of Earth) has the same mass density as Earth. Scientists dig a well of depth $\frac{R}{5}$ on it and lower a wire of the same length and of linear mass density 10^{-3} kgm^{-1} into it. If the wire is not touching anywhere, the force applied at the top of the wire by a person holding it in place is (take the radius of Earth = 6×10^6 m and the acceleration due to gravity of Earth is 10 ms^{-2}) **[IIT-JEE 2014]**

(A) 96 N **(B)** 108 N **(C)** 120 N **(D)** 150 N

Assertion-Reason

1. **Statement – 1 :** An astronaut in an orbiting space station above the Earth experiences weightlessness.

Statement – 2 : An object moving around the Earth under the influence of Earth's gravitational force is in a state of 'free-fall'. **[IIT-JEE 2008]**

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 is Not a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

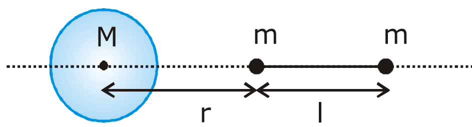
Subjective Questions

1. There is a crater of depth $\frac{R}{100}$ on the surface of the Moon (radius R). A projectile is fired vertically upward from the crater with velocity, which is equal to the escape velocity v from the surface of the Moon. Find the maximum height attained by the projectile.

2. A bullet is fired vertically upwards with velocity v from the surface of a spherical planet. When it reaches its maximum height, its acceleration due to the planet's gravity is $1/4^{th}$ of its value at the surface of the planet. If the escape velocity from the planet is $v_{esc} = v\sqrt{N}$, then the value of N is (ignore energy loss due to atmosphere)

[IIT-JEE 2015]

3. A large spherical mass M is fixed at one position and two identical point masses m are kept on a line passing through the centre of M (see figure). The point masses are connected by a rigid massless rod of length l and this assembly is free to move along the line connecting them. All three masses interact only through their mutual gravitational interaction. When the point mass nearer to M is at a distance r = 3l from M, the tension in the rod is zero for m = k $\left(\frac{M}{288}\right)$. The value of k is - **[IIT-JEE 2015]**



MOCK TEST

SECTION - I : STRAIGHT OBJECTIVE TYPE

1. A tunnel is dug along the diameter of the earth (Radius R & mass M). There is a particle of mass 'm' at the centre of the tunnel. The minimum velocity given to the particle so that it just reaches to the surface of the earth is :

(A) $\sqrt{\frac{GM}{R}}$ **(B)** $\sqrt{\frac{GM}{2R}}$ **(C)** $\sqrt{\frac{2GM}{R}}$

(D) it will reach with the help of negligible velocity.

2. A cavity of radius R/2 is made inside a solid sphere of radius R. The centre of the cavity is located at a distance R/2 from the centre of the sphere. The gravitational force on a particle of mass 'm' at a distance R/2 from the centre of the sphere on the line joining both the centres of sphere and cavity is (opposite to the centre of cavity). [Here $g = GM/R^2$, where M is the mass of the sphere]

(A) $\frac{mg}{2}$ **(B)** $\frac{3\,mg}{8}$ **(C)** $\frac{mg}{16}$ **(D)** none of these

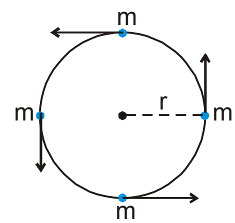
3. A satellite is launched in the equatorial plane in such a way that it can transmit signals upto 60º latitude on the earth. The angular velocity of the satellite is :

(A) $\sqrt{\frac{GM}{8R^3}}$ **(B)** $\sqrt{\frac{GM}{2R^3}}$ **(C)** $\sqrt{\frac{GM}{4R^3}}$ **(D)** $\sqrt{\frac{3\sqrt{3}GM}{8R^3}}$

4. A satellite is seen after each 8 hours over equator at a place on the earth when its sense of rotation is opposite to the earth. The time interval after which it can be seen at the same place when the sense of rotation of earth & satellite is same will be :

(A) 8 hours **(B)** 12 hours **(C)** 24 hours **(D)** 6 hours

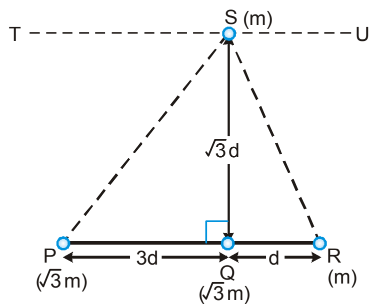
5. Four similar particles of mass m are orbiting in a circle of radius r in the same angular direction because of their mutual gravitational attractive force. Velocity of a particle is given by



(A) $\left[\frac{Gm}{r}\left(\frac{1+2\sqrt{2}}{4}\right)\right]^{\frac{1}{2}}$ **(B)** $\sqrt[3]{\frac{Gm}{r}}$

(C) $\sqrt{\frac{Gm}{r}\left(1+2\sqrt{2}\right)}$ **(D)** $\left[\frac{1}{2}\frac{Gm}{r}\left(\frac{1+\sqrt{2}}{2}\right)\right]^{\frac{1}{2}}$

6. Three particles P, Q and R are placed as per given figure. Masses of P, Q and R are $\sqrt{3}$ m, $\sqrt{3}$ m and m respectively. The gravitational force on a fourth particle 'S' of mass m is equal to



(A) $\frac{\sqrt{3}\,GM^2}{2d^2}$ in ST direction only

(B) $\frac{\sqrt{3}\,Gm^2}{2d^2}$ in SQ direction and $\frac{\sqrt{3}\,Gm^2}{2d^2}$ in SU direction

(C) $\frac{\sqrt{3}\,Gm^2}{2d^2}$ in SQ direction only

(D) $\frac{\sqrt{3}\,Gm^2}{2d^2}$ in SQ direction and $\frac{\sqrt{3}\,Gm^2}{2d^2}$ in ST direction

7. The gravitational potential of two homogeneous spherical shells A and B of same surface density at their respective centres are in the ratio 3 : 4. If the two shells coalesce into single one such that surface density remains same, then the ratio of potential at an internal point of the new shell to shell A is equal to :

(A) 3 : 2 **(B)** 4 : 3 **(C)** 5 : 3 **(D)** 5 : 6

8. A point P lies on the axis of a fixed ring of mass M and radius R, at a distance 2R from its centre O. A small particle starts from P and reaches O under gravitational attraction only. Its speed at O will be

(A) zero **(B)** $\sqrt{\frac{2GM}{R}}$ **(C)** $\sqrt{\frac{2GM}{R}(\sqrt{5}-1)}$ **(D)** $\sqrt{\frac{2GM}{R}(1-\frac{1}{\sqrt{5}})}$

9. Gravitational field at the centre of a semicircle formed by a thin wire AB of mass m and length ℓ is :

(A) $\frac{Gm}{\ell^2}$ along +x axis **(B)** $\frac{Gm}{\pi\ell^2}$ along +y axis

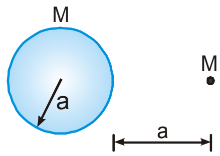
(C) $\frac{2\pi Gm}{\ell^2}$ along + x axis **(D)** $\frac{2\pi Gm}{\ell^2}$ along + y axis

10. The percentage change in the acceleration of the earth towards the sun from a total eclipse of the sun to the point where the moon is on a side of earth directly opposite to the sun is

(A) $\frac{M_s}{M_m}\frac{r_2}{r_1}\times 100$ **(B)** $\frac{M_s}{M_m}\left(\frac{r_2}{r_1}\right)^2\times 100$ **(C)** $2\left(\frac{r_1}{r_2}\right)^2\frac{M_s}{M_m}\times 100$ **(D)** $\left(\frac{r_1}{r_2}\right)^2\frac{M_m}{M_s}\times 100$

M_S = mass of the sun, M_M = mass of the moon, r_1 = earth sun distance, r_2 = earth moon distance.

11. A particle of mass M is at a distance 'a' from surface of a thin spherical shell of uniform equal mass and having radius a.



(A) Gravitational field & potential both are zero at centre of the shell

(B) Gravitational field is zero not only inside the shell but at a point outside the shell also

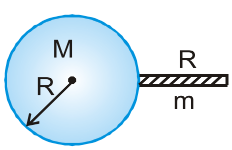
(C) Inside the shell, gravitational field alone is zero

(D) Neither gravitational field nor gravitational potential is zero inside the shell.

12. A small area is removed from a uniform spherical shell of mass M and radius R. Then the gravitational field intensity near the hollow portion is

(A) $\frac{GM}{R^2}$ **(B)** $\frac{GM}{2R^2}$ **(C)** $\frac{3GM}{2R^2}$ **(D)** Zero

13. A uniform thin rod of mass m and length R is placed normally on surface of earth as shown. The mass of earth is M and its radius is R. Then the magnitude of gravitational force exerted by earth on the rod is



(A) $\frac{GMm}{2R^2}$ **(B)** $\frac{GMm}{4R^2}$ **(C)** $\frac{4GMm}{9R^2}$ **(D)** $\frac{GMm}{8R^2}$

14. Two particles of combined mass M, placed in space with certain separation, are released. Interaction between the particles is only of gravitational nature and there is no external force present. Acceleration of one particle with respect to the other when separation between them is R, has a magnitude :

(A) $\frac{GM}{2R^2}$ **(B)** $\frac{GM}{R^2}$

(C) $\frac{2GM}{R^2}$ **(D)** not possible to calculate due to lack of information

15. Maximum height reached by a rocket fired with a speed equal to 50% of the escape velocity from earth's surface is :

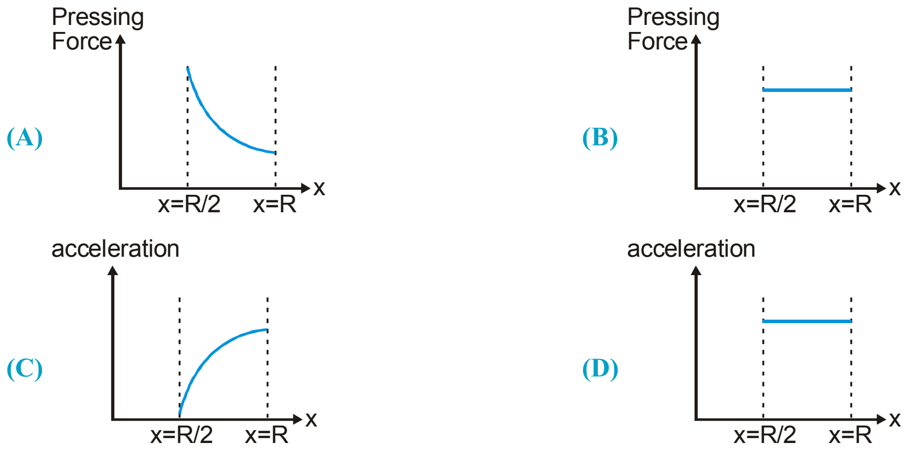
(A) R/2 **(B)** 16R/9 **(C)** R/3 **(D)** R/8

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

16. A double star is a system of two stars of masses m and 2m, rotating about their centre of mass only under their mutual gravitational attraction. If r is the separation between these two stars then their time period of rotation about their centre of mass will be proportional to

(A) $r^{3/2}$ **(B)** r **(C)** $m^{1/2}$ **(D)** $m^{-1/2}$

17. A tunnel is dug along a chord of the earth at a perpendicular distance R/2 from the earth's centre. The wall of the tunnel may be assumed to be frictionless. A particle is released from one end of the tunnel. The pressing force by the particle on the wall, and the acceleration of the particle varies with x (distance of the particle from the centre) according to :



18. A satellite revolves around a planet in circular orbit of radius *R* (much larger than the radius of the planet) with a time period of revolution *T*. If the satellite is stopped and then released in its orbit (Assume that the satellite experiences gravitational force due to the planet only).

(A) It will fall on the planet

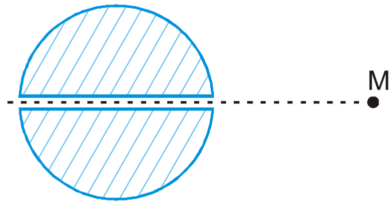
(B) The time of fall of the satellite is nearly $\frac{T}{\sqrt{8}}$

(C) The time of fall of the satellite on the planet is nearly $\frac{\sqrt{2}T}{8}$

(D) It cannot fall on the planet so time of fall of the satellite is meaningless

SECTION - III : ASSERTION AND REASON TYPE

19. **Statement-1 :** In free space a uniform spherical planet of mass M has a smooth narrow tunnel along its diameter. This planet and another superdense small particle of mass M start approaching towards each other from rest under action of their gravitational forces. When the particle passes through the centre of the planet, sum of kinetic energies of both the bodies is maximum.



Statement-2 : When the resultant of all forces acting on a particle or a particle like object (initially at rest) is constant in direction, the kinetic energy of the particle keeps on increasing.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

SECTION - IV : COMPREHENSION TYPE

Comprehension # 1

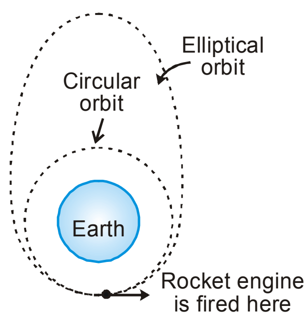
Changing from a circular to An elliptical orbit

Let us identify the system as the spacecraft and the Earth but not the portion of the fuel in the spacecraft that we use to change the orbit. In a given orbit, the mechanical energy of the spacecraft – Earth system is given by $E = -\dfrac{GMm}{2r}$.

This energy includes the kinetic energy of the spacecraft and the potential energy associated with the gravitational force between the spacecraft and the Earth. If the rocket engines are fired, the thrust force moves the spacecraft through a displacement. As a result, the mechanical energy of the spacecraft – Earth system increases.

The spacecraft has a new higher energy but is constrained to be in an orbit that includes the original starting point. It can not be in a higher energy circular orbit having a larger radius because this orbit would not contain the starting point. The only possibility is that the orbit is elliptical as shown in the figure.

$$E = -\frac{GMm}{2a}$$



20. If the spacecraft-earth system had initial energy ($-E_0$), then the total mechanical energy of the system after firing the rocket will be :

(A) $-1.1\ E_0$ **(B)** $-0.9\ E_0$ **(C)** $-E_0$ **(D)** None of these

21. Semimajor axis of the new elliptical orbit is

(A) $\dfrac{6.7\times10^4}{9}$ Km **(B)** $\dfrac{6.4\times10^4}{9}$ Km **(C)** $\dfrac{7.1\times10^4}{9}$ km **(D)** $\dfrac{6.1\times10^4}{9}$ Km

22. Maximum height of the spacecraft above the surface of the Earth will be :

(A) $\dfrac{1.06\times10^4}{9}$ km. **(B)** $\dfrac{0.61\times10^4}{9}$ km **(C)** 300 km **(D)** $\dfrac{1.61\times10^4}{9}$ km

SECTION - V : MATRIX - MATCH TYPE

23. A satellite is revolving around the earth in a circular orbit of radius 'a' with velocity v_0. A particle is projected from the satellite in forward direction with relative velocity $v = \left[\sqrt{\frac{5}{4}} - 1\right] v_0$. During subsequent motion of particle, match the following :

Column-I		Column-II	
(A)	Particle of total energy	(P)	$-\frac{3GM_e m}{a}$
(B)	Minimum distance of particle from the earth	(Q)	$-\frac{5}{8}\frac{GM_e m}{a}$
(C)	Maximum distance of particle from the earth	(R)	5a/3
		(S)	2a/3
		(T)	a

24. Let V and E denote the gravitational potential and gravitational field respectively at a point due to certain uniform mass distribution described in four different situations of column-I. Assume the gravitational potential at infinity to be zero. The value of E and V are given in column-II. Match the statement in column-I with results in column-II.

Column-I	Column-II
(A) At centre of thin spherical shell	(P) $E = 0$
(B) At centre of solid sphere	(Q) $E \neq 0$
(C) A solid sphere has a non-concentric spherical cavity. At the centre of the spherical cavity	(R) $V \neq 0$
(D) At centre of line joining two point masses of equal magnitude	(S) $V = 0$

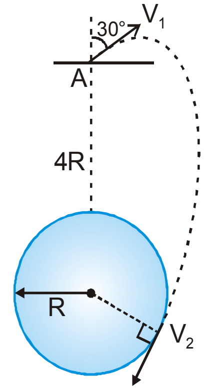
SECTION - VI : INTEGER TYPE

25. Ravi can throw a ball at a speed on earth which can cross a river of width 10 m. Ravi reaches on an imaginary planet whose mean density is twice of the earth. If maximum possible radius of planet so that if Ravi throws the ball at same speed it may escape from planet is x km. then x is. (Given radius of earth = 6.4×10^6 m.)

26. The gravitational field in a region is given by $\vec{E} = (3\hat{i} - 4\hat{j})$ N/kg. Find out the work done (in joule) in displacing a particle by 1 m along the line $4y = 3x + 9$.

27. A particle is projected from point A, that is at a distance 4R from the centre of the earth, with speed V_1 in a direction making 30° with the line joining the centre of the earth and point A, as shown in the figure. Find the speed V_1 if particle passes grazing the surface of the earth. Consider gravitational interaction only between these two. (use $\frac{GM}{R} = 6.4 \times 10^7$ m²/s²)

Express your answer in the form of $\frac{X}{\sqrt{2}}$ m/s and fill value of X.



ANSWER KEY

EXERCISE - 1

1. D **2.** C **3.** D **4.** A **5.** A **6.** A **7.** B **8.** A **9.** C **10.** A **11.** C **12.** A **13.** B
14. A **15.** D **16.** B **17.** D **18.** C **19.** D **20.** A **21.** C **22.** C **23.** A **24.** B **25.** C

EXERCISE - 2 : PART # I

1. A **2.** B **3.** D **4.** C **5.** D **6.** B **7.** D **8.** A **9.** C **10.** AC **11.** D **12.** CD **13.** B
14. A,B,C,D **15.** A,D **16.** B,C **17.** A,C,D **18.** B,C **19.** A,B **20.** D **21.** A,C **22.** C **23.** D
24. D **25.** A **26.** B

PART # II

1. A **2.** A **3.** A **4.** A **5.** C **6.** C **7.** A **8.** A **9.** B **10.** C **11.** A

EXERCISE - 3 : PART # I

1. A → R ; B → Q ; C → Q ; D → P
2. A → P ; B → P ; C → P,S ; D → Q,S
3. A → Q ; B → T ; C → r ; D → S
4. A → S ; B → R ; C → Q

PART # II

Comprehension #1 : **1.** A **2.** C **3.** D
Comprehension #2 : **1.** C **2.** B
Comprehension #3: **1.** B **2.** D

EXERCISE - 4

2. $\sqrt{2}$

3. $\frac{7}{9}$

4. $\frac{7}{3}$

5. 0

6. $\frac{-5.41Gm^2}{\ell}$, $\frac{-4\sqrt{2}Gm}{\ell}$

7. 1.11×10^{-11} Joule

8. $2\sqrt{\frac{R^3}{GM}}$

9. $\frac{-4GM^2}{L}\left[3+\frac{3}{\sqrt{2}}+\frac{1}{\sqrt{3}}\right]$

10. R

11. $-\frac{GMm}{\ell}\log_e\left(1+\frac{\ell}{a}\right)$

12. **(i)** $\left[\frac{n}{n+1}\right]mgR$ **(ii)** $\sqrt{\left(\frac{n}{n+1}\right)2gR}$

13. **(i)** 6400 km **(ii)** 7.9 km/s

14. All quantities vary over an orbit except angular momentum and total energy

15. $\sqrt{\frac{GM}{R}}\left[\sqrt{\frac{2}{3}}-\sqrt{\frac{8}{15}}\right]$

17. **(i)** $h=\left(\frac{\sqrt{7}}{2}+1\right)R$ **(ii)** 1.13 R

18. $\frac{8R}{3}$

19. $\frac{3}{2}\sqrt{\frac{5GM}{a}}$

21. $\left(\frac{u}{n}\right)^2$

22. 3×10^{-2}N

23. 4.26 R

24. $\left(\frac{\sqrt{5}-1}{2}\right)R$

25. $\left[1-\frac{\sqrt{x^2-R^2}}{x}\right]4\pi R^2$

26. **(i)** $\sqrt{\frac{GM}{r}}$ **(ii)** $r\sqrt{2}$ **(iii)** $\sqrt{\frac{2GM}{r}}$

27. **(ii)** 1.9 h

28. $\frac{2\pi R^{3/2}6\sqrt{6}}{\sqrt{GM}\left(2\sqrt{2}+3\sqrt{3}\right)}$

29. $T=\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)\sqrt{\frac{R_e}{g}}$

30. **(i)** $T = 4\pi\sqrt{\dfrac{r^3}{Gm}}$ **(ii)** $v=\sqrt{\dfrac{4Gm}{r}}$, $r=\dfrac{d}{2}$

31. 1.6 hours if it is rotating from west to east, $\dfrac{24}{17}$ hours if it is rotating east to west.

32. 4 Km **33.** $\dfrac{\pi}{2\sqrt{2}}$ **34. (i)** $\dfrac{-GmM_e}{r}$, **(ii)** $-\dfrac{2GmM_e}{r}$ **(iii)** free fall

EXERCISE - 5 : PART # I

1. 2 **2.** 3 **3.** 4 **4.** 3 **5.** 3 **6.** 4 **7.** 1 **8.** 2 **9.** 1 **10.** 1 **11.** 3 **12.** 4 **13.** 1
14. 3 **15.** 2 **16.** 3 **17.** 2 **18.** 2 **19.** 1 **20.** 2 **21.** 4 **22.** 3

PART # II

MCQ's with one or more than one correct answer **1.** A **2.** C **3.** D **4.** B, D **5.** B

Assertion-Reasoning **1.** A

Subjective Questions **1.** 99.5 R **2.** 2 **3.** 7

MOCK TEST

1.	A	**2.**	B	**3.**	A	**4.**	C	**5.**	A	**6.**	C	**7.**	C
8.	D	**9.**	D	**10.**	C	**11**	D	**12.**	B	**13.**	A	**14.**	B
15.	C	**16.**	A,D	**17.**	B, C	**18.**	A, C	**19.**	A	**20.**	B	**21.**	A
22.	D	**23.**	$A \to P$; $B \to T$; $C \to R$	**24.**	$A \to P,R$; $B \to P,R$; $C \to Q,R$; $D \to P,R$							**25.**	4
26.	0	**27.**	8000										