HINTS & SOLUTIONS

EXERCISE - 1

Single Choice

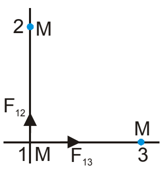
1. $F \propto \frac{1}{r^m}$; $F = \frac{C}{r^m}$

This force will provide the required centripetal force

Therefore

$$m\omega^2 r = \frac{C}{r^m} \; ; \; \omega^2 = \frac{C}{mr^{m+1}} \Rightarrow T = \frac{2\pi}{\omega} \Rightarrow T \propto r^{(m+1)/2}$$

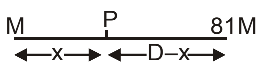
2. $$F_1 = F_2 = \frac{G(1)(1)}{(.2)^2} = \frac{6.67\times10^{-11}}{.04} = 1.67\times10^{-9}$$



$$\vec{F}_{net} = F_1(\hat{i}) + F_2(\hat{j}) = F(\hat{i}+\hat{j}) = 1.67\times10^{-9}(\hat{i}+\hat{j})$$

3. At P : $g = \frac{GM}{x^2} - \frac{G81M}{(D-x)^2} = 0$

$\Rightarrow$ $D - x = 9x;\ 10x = D$



$x = \frac{D}{10}$ from the Moon and $\frac{9D}{10}$ from the earth

4. $g' = g\left(1 - \frac{2h}{R}\right)$; $\frac{\Delta g}{g} = \frac{2h}{R}$

$$1 = 2\frac{h}{R} \Rightarrow \frac{h}{R} = \frac{1}{2} \; ; \; g' = g\left(1 - \frac{d}{R}\right)$$

$\frac{\Delta g'}{g} = \frac{d}{R} \Rightarrow \frac{h}{R}$ g decreases by 0.5%

5. $g = \frac{GM}{r^2}$

$\Rightarrow$ R is reduced to R/2 and the mass of the mars becomes 10 times

$$g_{mars} = \frac{4}{10} g_{earth} \text{ and } W_{mars} = \frac{4}{10} W_{earth} = 80N.$$

6. $t = \sqrt{\frac{2h}{g}} = 1$ sec; $t' = \sqrt{\frac{2h}{g'}} = \sqrt{6}\sqrt{\frac{2h}{g}} = \sqrt{6}$ sec

7. $g' = \frac{GM}{(R+h)^2} = \frac{g}{49}$; $w' = \frac{mg}{49} = \frac{10}{49} = 0.20\,N$

Apparent weight of the rotating satellite is zero because satellite is in free fall state.

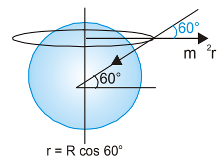
8. $$g = \frac{GM}{R^2} = \frac{G\rho\frac{4}{3}\pi R^3}{R^2} \Rightarrow g = \frac{4}{3}G\rho\pi R$$

$$\Rightarrow g \propto R \Rightarrow \frac{g}{g'} = \frac{R}{3R} \Rightarrow g' = 3g$$

9. $g' = g - \omega^2 r \cos 60$

$g' = g - \omega^2 R \cos^2 60$

$g' = 0,\ g = \omega^2 R \cos^2 60$



$$\sqrt{\frac{4g}{R}} = \omega,\ t = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{4g}} = \pi\sqrt{\frac{R}{g}}$$

10. By applying conservation of energy

$KE_i + PE_i = KE_f + PE_f$

$$\frac{1}{2}mv^2 - \frac{GM_em}{R} = 0 - \frac{GM_em}{2R}$$

$$\frac{1}{2}mv^2 = \frac{GM_em}{R}\left[-\frac{1}{2}+1\right]$$

$$\frac{1}{2}mv^2 = \frac{GM_em}{2R} \Rightarrow u = \sqrt{\frac{GM_e}{R}}$$

11. Acceleration of small body w.r.t. earth=g–(–2g) =3g

Now from second equation of motion

$$H = \frac{1}{2}(3g)t^2 \Rightarrow t = \sqrt{\frac{2H}{3g}}$$

OR

$$\bar{X}_{CM} = \frac{2mx_1 + m(x_2)}{2m+m} = \frac{2mH+0}{2m+m} = \frac{2H}{3}$$

$$\frac{H}{3} = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2H}{3g}}$$

12. Gravitational field inside the shell is zero. But the force on the man due to the point mass at the centre is

$$F_{New} = \frac{GMm}{3R^2} \; ; \; F_{old} = \frac{GMm}{R^2}$$

Change in force $= \frac{2GMm}{3R^2}$

13. Centre of gravity of the two particles.

$$X_{CG} = \frac{W_1X_1 + W_2X_2}{W_1 + W_2} = \frac{(0)(0)+(mg)(R)}{0+mg} = R$$

The centre of mass of the two particle system is at

$$X_{CM} = \frac{M(R)+m(0)}{2M} = \frac{R}{2}$$

14. $PE_i = -\frac{GM_em}{R} = -mgR$; $PE_f = -\frac{GM_em}{2R} = -\frac{mgR}{2}$

Increase in PE is $\frac{mgR}{2}$

15. $\int dV = -\int I_g.dx$; $\int_v^0 dV = -\int_r^\infty \frac{k}{x^3}dx$

$$0 - V = \left[-\frac{1}{2x^2}\right]_r^\infty \Rightarrow V = +\frac{k}{2r^2} \Rightarrow V = \frac{k}{2x^2}$$

16. $I_g = \frac{GM}{R^2}$, $V = -\frac{GM}{R}$,

$V = I_gR = 6 \times 8 \times 10^6 = 4.8 \times 10^7$

17. Equilibrium position of the neutral point from mass 'm' is

$$= \left(\frac{\sqrt{m}}{\sqrt{m}+\sqrt{M}}\right)d$$

$$V_1 = \frac{-Gm_1}{r_1};\ V_2 = \frac{-Gm_2}{r_2}$$

$$V_1 = \frac{-Gm}{\sqrt{m}d}(\sqrt{M}+\sqrt{m})\ ;\ V_2 = \frac{-GM}{\sqrt{M}d}(\sqrt{M}+\sqrt{m})$$

$$V_1 = \frac{-G}{d}\sqrt{m}(\sqrt{M}+\sqrt{m})\ ;\ V_2 = \frac{-G}{d}\sqrt{M}(\sqrt{M}+\sqrt{m})$$

$$V = \frac{-G}{d}(\sqrt{M}+\sqrt{m})^2$$

18. $v_e = \sqrt{\frac{2GM}{R}}$; $v'_e = \sqrt{\frac{2G2M}{R/2}} = 2\sqrt{\frac{2GM}{R}}$

$v_e = 2\,(11.2\text{ km/sec}) = 22.4\text{ km/sec}$

19. There will be no buoyant force on the moon. (Eventually balloon bursts)

20. There is no atmosphere on the moon.

21. To escape from the earth total energy of the body should be zero KE+ PE = 0

$$\frac{1}{2}mv^2 - \frac{GMm}{5R} = 0 \Rightarrow KEmin = \frac{mgR_e}{5}$$

22. Relative angular velocity when the particle are moving in same direction is

$$\omega_1 + \omega_2 \Rightarrow (\omega_1+\omega_2)t = 2\pi$$

$$\therefore\ \omega_2 = \frac{2\pi}{24}rad/sec\ ;\ \omega_1 = \frac{\pi}{6}$$

When the particles are moving in the same direction then angular velocity becomes

$$(\omega_1 - \omega_2) \Rightarrow (\omega_1-\omega_2)t = 2\pi$$

By substituting ω_1 and ω_2 in equation we get

23. $K.E. = +\frac{1}{2}\frac{GM_1M_2}{r}$

r= 2R for the first and r = 8R for the II[nd]

$$\frac{K.E_1}{K.E_2} = \left(\frac{1}{2R}\frac{8R}{1}\right) = 4:1$$

Similarly P.E. is $\Rightarrow -\frac{GM_1M_2}{R}$, $\frac{P.E_1}{P.E_2} = 4:1$

Put the ratio of $\frac{K.E}{P.E} = 2$

24. At points A, B and C, total energy is negative.

EXERCISE - 2

Part # I : Multiple Choice

1. $\Delta V = -E_g.dr$

Because field is uniform

$$\therefore\ 2 = -E_g.20 \Rightarrow E = -\frac{1}{10}\ ;\ \Delta V = +\frac{1}{10}[4] = \frac{2}{5}$$

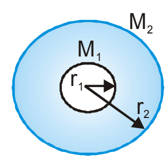
work done in taking a 5 kg body to height 4 m = m (change in gravitational potential)

$$= 5\left[\frac{2}{5}\right] \Rightarrow 2\text{ J}$$

2. Net force towards the centre $\Rightarrow m\omega^2(9R) = \frac{GMm}{(9R)^2}$

$$\Rightarrow \omega^2 = \frac{GM}{729R^3} \Rightarrow T = \frac{2\pi}{\omega} \Rightarrow 27\times 2\pi\sqrt{\frac{R}{g}}$$

3. when $r < r_1$, gravitational intensity is equal to 0



when $r > r_1$, gravitational intensity is equal to $\frac{GM_1}{r^2}$

when $r > r_2$, gravitational intensity is equal to $\frac{G(M_1+M_2)}{r^2}$

4. $\vec{F} = -\frac{dU}{dr} = \frac{d}{dr}(ax + 6y)$; $F_x = -a$ and $F_y = b$

$\vec{F} = -a\tilde{i} + b\tilde{j} \Rightarrow$ acceleration $= \frac{\sqrt{a^2 + b^2}}{m}$

5. $\int dV = -\int E.dr$, $\int dV = -\int \frac{k}{r} dr$

$v = -k \log r + c$ at $r = r_0$; $v = v_0$

$\Rightarrow$ $v_0 = -k \log r_0 + c \Rightarrow c = v_0 + k \log r_0$

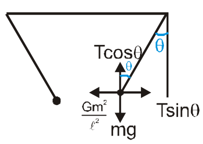
By substituting the value c from equation

$v = k \log \left(\frac{r_0}{r}\right) + V_0$

6. Gravitational field and the electrostatic field both are conservation in nature

8. $T \sin \theta = \frac{Gm^2}{\ell^2}$; $T \cos \theta = mg$

$\tan \theta = \frac{Gm}{g\ell^2}$; $\theta = \tan^{-1}\left(\frac{Gm}{g\ell^2}\right)$



10. By applying work energy theorem change in K.E. = work done by all the forces

$\Delta K.E. = W_g - W_{fr}; W_g > W_{fr}$

therefore KE_f increases due to the torque of the air resistance its angular momentum decreases therefore A,C

11. Both field and the potential inside the shell is non zero

12. **Case I**

$U_i + K_i = U_f + K_g$

$$\frac{-GM_em}{R} + \frac{1}{2}mv^2 = -\frac{GM_em}{R + h_1} + 0$$

$$\frac{-GM_em}{R} + \frac{1}{2}m\frac{2GM_e}{R^2}\frac{R}{3} = -\frac{GM_em}{R + h_1}$$

$$-\frac{1}{R} + \frac{1}{3R} = -\frac{1}{R + h_1} \Rightarrow h_1 = \frac{R}{2}$$

Case II

$U_i + K_i = U_f + K_g$

$$-\frac{GM_em}{R} + \frac{1}{2}mv^2 = -\frac{GM_em}{R + h_2} + 0$$

$$\frac{-GM_em}{R} + \frac{1}{2}m\frac{2GM_e}{R^2}R = -\frac{GM_em}{R + h_2}$$

$$-\frac{1}{R} + \frac{1}{2R} = -\frac{1}{R + h_2} \Rightarrow h_2 = R$$

Case III

$U_i + K_i = U_f + K_g$

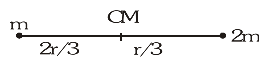
$$-\frac{GM_em}{R} + \frac{1}{2}m\frac{4GM_e}{R^2}\frac{R}{3} = -\frac{GM_em}{R + h_3}$$

$$-\frac{1}{R} + \frac{1}{3R} = -\frac{1}{R + h_3} \Rightarrow h_3 = 2R$$

13. F_{net} = force due for sphere + force due for cavity

$$= \frac{GMm}{R^3}\left(\frac{R}{2}\right) + 0 \Rightarrow \frac{GMm}{2R^2} = \frac{mg}{2}$$

15. Motion of m



$$m\omega^2\left(\frac{2r}{3}\right) = \frac{Gm(2m)}{r^2}$$

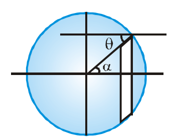
$$\Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{r^3}{3Gm}} \quad \therefore T \propto r^{3/2} \text{ and } T \propto m^{-1/2}$$

16. Pressing force by the particle on the wall of tunnel is and acceleration is mgsinθ.

Pressing force $=$ mgcosθ $\Rightarrow \frac{GMx}{R^3} \times \frac{R}{2x} \Rightarrow \frac{GM}{2R^2}$

Pressing force is independent from 'x' thus it is constant

$$g\sin\theta = \frac{GMx}{R^3}\sqrt{\frac{x^2 - \frac{R^2}{4}}{x^2}} = \frac{GM}{2R^3}\sqrt{4x^2 - R^2}$$



x is increases from $\frac{R}{2}$ to R, thus acceleration increases

17. Due to symmetry the gravitational field at the origin is zero. The equipotential line will take the shape of a circle in yz plane.

18. Gravitational potential $V = -\frac{GM}{R}$

(B) Gravitational field at the point x from the centre of the

$$\frac{GMx}{(R^2 + x^2)^{3/2}}$$

19. Gravitational field intensity $F = \frac{GMr}{R^3}$

Inside the sphere

$(F_1 \propto r_1, F_2 \propto r_2)$

$\frac{F_1}{F_2} = \frac{r_1}{r_2}$ of $r_1 < R$ & $r_2 < R$

Gravitational field intensity

$$I \propto \frac{1}{r^2} \text{ (Out side the sphere)}$$

$$\therefore \frac{F_1}{F_2} = \frac{r_2^2}{r_1^2} \text{ if } r_1 > R \text{ and } r_2 > R$$

20. Acceleration of the particle from the centre of the earth is directly proportional to the distance from the centre

$$\Rightarrow a = \frac{GMx}{R^3} \quad \Rightarrow a \propto x \quad \Rightarrow a = -\omega^2 x$$

Particle will perform oscillatory motion.

21. Gravitational potential due to hemisphere at the centre is V because distance of each mass particle from the centre O is R. If the distance between the point and mass is changed potential will also change

22. P.E. of the system is equal to $U_i = -\frac{3GMm}{2R}$

work done $\Rightarrow -\Delta U \Rightarrow -[U_f - U_i] \Rightarrow \quad U_i$

$$\Rightarrow -\frac{3GMm}{2R}$$

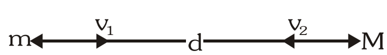
23. By energy conservation

$K_i + U_i = K_f + U_f$

$$0 - \frac{GMm}{2R} = \frac{1}{2}K\left(\frac{R}{2}\right)^2 - \frac{11GMm}{8R}$$

$$\frac{GMm}{R}\left[\frac{11}{8} - \frac{1}{2}\right] = \frac{1}{2}K\left(\frac{R^2}{4}\right) \Rightarrow \frac{7GMm}{8R} = \frac{KR^2}{8}$$

$$K = \frac{7GMm}{R^3}$$

24. 

Total energy of mass M will become zero, it will be escape $K + U = 0$

$$\frac{1}{2}Mv^2 - \frac{Gm_1m2}{d} - \frac{Gm_2m2}{d} = 0$$

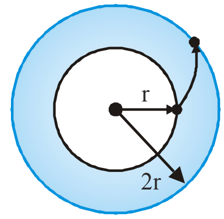
$$\frac{1}{2}MV^2 = \frac{GM2}{d}(M_1 + M_2) \Rightarrow V = \sqrt{\frac{4G}{d}(M_1 + M_2)}$$

Part # II : Assertion & Reason

1. A **2.** A **3.** A **4.** A **5.** C **6.** C
7. A **8.** A **9.** B **10.** C **11.** A

EXERCISE - 3

Part # I : Matrix Match Type

1. 

For (A) : $L' = mv'r' = m\left(\sqrt{\frac{GM}{2r}}\right)(2r) = \sqrt{2}\,mvr = \sqrt{2}\,L$

For (B) : Area of earth covered by satellite signal increases.

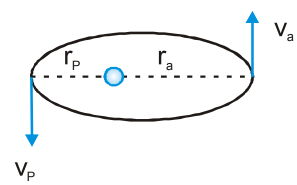
For (C) : Potential energy

$$U' = -\frac{GMm}{2r} = \frac{U}{2} \text{ and } -\frac{GMm}{2r} > -\frac{GMm}{r}$$

For (D): Kinetic energy

$$K' = \frac{1}{2}mv'^2 = \frac{K}{2} \Rightarrow K' < K$$

2. COAM : $mv_ar_a = mv_pr_p$



(A) At perigee $r_P < r_a$ $\therefore v_P > v_a$ (r)

(B) Distance from sun at the position of perigee decreases (q)

(C) Potential energy at perigee $U_P = -\frac{GMm}{r_P}$

$$U_P = -\frac{GMm}{r_P}$$

(D) Angular momentum remains same (p)

3. **(A)** Potential at A Potential at B

(B) We can not compare about gravitational field at A and at B

(C) At C and D, gravitational field and potential remains same

(D) As one moves from D to A, field decreases

4. **(A)** Kinetic energy in gravitational field increases if the total work done by all forces is positive.

(B) Potential energy in gravitational field increases as work done by gravitational force is negative.

(C) For mechanical energy in a gravitational field to increase, work done by external force should be non–zero.

Part # II : Comprehension

Comprehension # 1

1. $F = \dfrac{G2Mm}{(R+R_x)^2}$ $\Rightarrow$ $a = \dfrac{GM}{2R^2}$

$h = \dfrac{1}{2}at^2$ $\Rightarrow$ $\dfrac{4hR^2}{GM} = t^2$ $\Rightarrow$ $t = 2\sqrt{\dfrac{hR^2}{GM}}$

2. V at surface $= \sqrt{2as} = \sqrt{2\dfrac{GM}{2R^2}h} = \dfrac{\sqrt{GMh}}{R}$

If $a = 0$, $t_1 = \dfrac{R}{v} = \dfrac{R^2}{\sqrt{GMh}}$; but $a > 0$; $t < \dfrac{R^2}{\sqrt{GMh}}$

3. COME $\Rightarrow 0 - \dfrac{G(2M)m}{(2R+h)} = \dfrac{1}{2}mv^2 - \dfrac{GMm}{R} - \dfrac{GMm}{2R}$

$\Rightarrow v \cong \sqrt{\dfrac{GM}{R}}$

Comprehension # 2

1. By applying conservation of angular momentum

$mv_0R\cos\theta = m\,v\,(R+h)$

$v = \dfrac{v_0R\cos\theta}{R+h}$ $\left(\dfrac{R}{R+h} < 1\right)$ $\Rightarrow$ $v_0\cos\theta > v$

2. By applying conservation of energy

$\dfrac{1}{2}mv_0^2 - \dfrac{GM_em}{r} = \dfrac{1}{2}mv^2 - \dfrac{GM_em}{(R+h)}$

Solving above equation $h > \dfrac{v_0^2\sin^2\theta}{2g}$

Alternate : As height increases gravitational force decreases and hence the acceleration. Therefore height will be more than $H = \dfrac{v_0^2\sin^2\theta}{2g}$

Comprehension # 3

1. As the distance of the star is doubled the potential energy becomes half of the initial and the velocity of the particle will become $\dfrac{1}{\sqrt{2}}$ times of its initial value because K.E. = 1/2 P.E.

2. Its kinetic energy

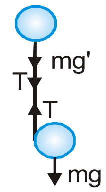
EXERCISE - 4

Subjective Type

1. $mg - T = ma$ ..... (i)

$T + mg' = ma$ ..... (ii)

By adding (i) and (ii)



$a = \dfrac{g+g'}{2}$; $T = \dfrac{m(g+g)}{2} - mg'$

$T = m\left(\dfrac{g-g'}{2}\right) = \dfrac{m}{2}\left[g - g\left[1 - \dfrac{2h}{R}\right]\right]$

$T = \dfrac{mg}{2}\dfrac{2\ell}{R} = \dfrac{mg\ell}{R} = \dfrac{GMm\ell}{R^3}$

2. $F_1 = \left|\vec{F}_{42} + \vec{F}_{41} + \vec{F}_{41}\right|$

$= \dfrac{G8}{4L^2} + \dfrac{G4}{2L^2}(2\cos 45°) = \dfrac{\left(2+2\sqrt{2}\right)G}{L^2}$

$F_2 = \left|\vec{F}_{24} + \vec{F}_{21} + \vec{F}_{21}\right| = \dfrac{G8}{4L^2} + \dfrac{G2}{2L^2}(2\cos 45)$

$= \dfrac{G}{L^2}\left(2+\sqrt{2}\right)$ $\Rightarrow$ $\dfrac{F_1}{F_2} = \sqrt{2}$

3. $F_1 = \dfrac{GMm}{4R^2}$

F_2 = force due to whole sphere – force due to cavity

$F_2 = \dfrac{GMm}{4R^2} - \dfrac{GMm}{18R^2}$ $\Rightarrow$ $\dfrac{7GMm}{36R^2}$ $\therefore$ $\dfrac{F_2}{F_1} = \dfrac{7}{9}$

4. Total mass of earth

$M = \dfrac{4}{3}\pi\left(\dfrac{R}{2}\right)^3\rho_1 + \dfrac{4}{3}\pi\left(R^3 - \dfrac{R^3}{8}\right)\rho_2$

$M = \dfrac{4\pi R^3}{24}(\rho_1 + 7\rho_2)$

Acceleration due to gravity at earth's surface $= \dfrac{GM}{R^2} = \dfrac{4\pi GR}{24}(\rho_1 + 7\rho_2)$

Acceleration due to gravity at depth $\dfrac{R}{2}$ from the surface $= \dfrac{G\left[\dfrac{4}{3}\pi\left(\dfrac{R}{2}\right)^3\rho_1\right]}{\left(\dfrac{R}{2}\right)^2} = \dfrac{4\pi G\rho_1 R}{6}$

Now according to question

$\dfrac{4\pi GR(\rho_1 + 7\rho_2)}{24} = \dfrac{4\pi GR\rho_1}{6}$ $\Rightarrow$ $\dfrac{\rho_1}{\rho_2} = \dfrac{7}{3}$

5. For the line $4y = 3x + 9$

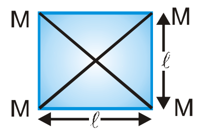
$4dy = 3dx;\ 4dy - 3\,dx = 0$ ....(i)

For work in the region,

$$dW = \vec{E}.\left(dx\tilde{i} + dy\tilde{j}\right) = \left(3\tilde{i} - 4\tilde{j}\right).\left(dx\tilde{i} + dy\tilde{j}\right)$$

$= 3dx - 4dy$ (from equation (i)) $= 0$

6. Potential at centre



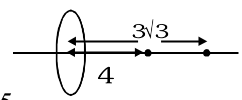
$$= \sum \frac{GM}{r} = \frac{-4GM}{r} = -\frac{4\sqrt{2}GM}{\ell}$$

Potential energy of the system

$$-\frac{4GM^2}{\ell} - \frac{2GM^2}{\sqrt{2}\ell} = -\frac{5.41GM^2}{\ell}$$

7. $$V_1 = \frac{GM}{(R^2 + x^2)^{1/2}} = \frac{G(5)}{(16+9)^{1/2}} = G$$

$$V_2 = \frac{GM}{(R^2 + x^2)^{1/2}}$$

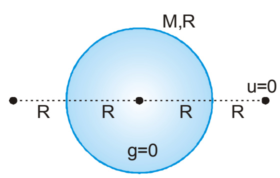


$$\Rightarrow \frac{G(5)}{(9+27)^{1/2}} \Rightarrow \frac{G5}{6}$$

work done $= m[V_2 - V_1] = \frac{G}{6} = 1.11 \times 10^{-11}$ Joule

8. Speed at surface = v (let)

COME



$$-\frac{GMm}{2R} + 0 = \frac{-GMm}{R} + \frac{1}{2}mv^2$$

$v = \sqrt{\frac{GM}{R}}$ ...(i)

Inside the shell $g = 0$

$$\therefore\ t = \frac{\text{distance}}{\text{speed}} = \frac{2R}{v} = 2\sqrt{\frac{R^3}{GM}}$$

9. No. of pairs for P.E. can be calculated by using

$$= {}^nC_2 = \frac{n(n-1)}{2} = 28$$

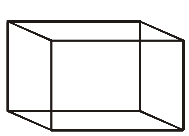
where n is the no. of mass particles.

out of 28 pairs :–

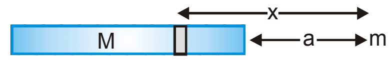
$12 \rightarrow$ sides of cube

$12 \rightarrow$ face diagonal

$12 \rightarrow$ body diagonal



11.

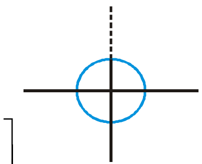


$U = \frac{GM_1M_2}{d}$ for the point Mass

$$dU = \frac{GmdM}{x} \Rightarrow U = -\int \frac{GMm}{\ell x}dx$$

$$U = -\frac{GMm}{\ell}\int_a^{\ell+a}\frac{dx}{x} = -\frac{GMm}{\ell}\ell n\left(\frac{\ell + a}{a}\right)$$

12. $$U_f = -\frac{GMm}{R(1+n)} \Rightarrow U_i = -\frac{GMm}{R}$$



$$\Delta U = U_f - U_i = -\frac{GMm}{R}\left[\frac{1}{1+n} - 1\right]$$

By applying energy conservation

$$\frac{1}{2}mv^2 = \frac{GMm}{R}\left[1 - \frac{1}{1+n}\right];\ v = \sqrt{\frac{2GM}{R}\left[1 - \frac{1}{1+n}\right]}$$

13. (i) Orbital velocity $v_0 = \sqrt{\frac{GM}{r}} = \frac{1}{2}\sqrt{\frac{2GM}{R}}$

$\Rightarrow r = 2R = R + h \quad \Rightarrow h = R = 6400$ km

(ii) COME : $-\frac{GMm}{2R} + 0 = -\frac{GMm}{R} + \frac{1}{2}mv^2$

$\Rightarrow v = \sqrt{\frac{GM}{R}} = \sqrt{gR} = 7.9184$ m/s

14. Net torque on the comet is zero then the angular momentum is conserved.

15. Let r = distance of apogee

COAM : $\sqrt{\frac{6GM}{5R}}\,R = vr$ ..... (iv)

COME : $-\frac{GMm}{R} + \frac{1}{2}m\left(\frac{6GM}{5R}\right)$

$= -\frac{GMm}{r} + \frac{1}{2}mv^2$ ..... (ii)

$\Rightarrow r = \frac{3R}{2}$ and $v = \sqrt{\frac{8GM}{15R}}$

Orbital speed at $r = \sqrt{\frac{GM}{r}} = \sqrt{\frac{2}{3}\frac{GM}{R}}$

$\therefore$ Increase in speed

$$= \sqrt{\frac{GM}{R}}\left[\sqrt{\frac{2}{3}} - \sqrt{\frac{8}{15}}\right]$$

16. Orbital velocity $v_0 = \sqrt{\dfrac{GM}{r_1}}$

After impulse $v_1 = \sqrt{k}v_0$

COAM : $mv_1r_1 = mv_2r_2$ ....(i)

COME : $\dfrac{-GMm}{r_1} + k\left(\dfrac{1}{2}mv_0^2\right)$

$= \dfrac{-GMm}{r_2} + \left(\dfrac{1}{2}mv_2^2\right)$ ...(ii)

Solving equation (i) and (ii) $\dfrac{r_2}{r_1} = \dfrac{k}{2-k}$

17. $v_1(H+R) = \dfrac{3}{2}\sqrt{\dfrac{GM}{2R}}R$

$v_1 = \left(\dfrac{3R}{R+H}\right)\sqrt{\dfrac{Gm}{8R}} \Rightarrow v_0^2 - \dfrac{GM}{2R} = v^2 - \dfrac{2GM}{(R+H)}$

$-\dfrac{GM}{2R} = \dfrac{9R^2}{(R+H)^2}\dfrac{GM}{8R} - \dfrac{2GM}{R+h}R$

$-1 = \dfrac{9R^2}{4(R+H)^2} - \dfrac{4R}{R+H}$

$\Rightarrow -1 = \dfrac{9R^2 - 16R^2 - 16RH}{4(R+H)^2}$

$4R^2 + 4H^2 + 8RH + 9R^2 - 16R^2 - 16RH = 0$

$4H^2 - 8RH - 3R^2 = 0$

$= \dfrac{8R \pm \sqrt{64R^2 + 48R^2}}{8} = R \pm R\sqrt{R^2 + \dfrac{3}{4}R^2}$

$= R \pm \dfrac{R}{2}\sqrt{7}$

18. COAM : $mv_1(2R) = mv_2(2R) \Rightarrow v_1 = 2v_2$ ..... (i)

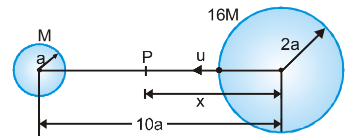
COME : $\dfrac{-GMm}{2R} + \dfrac{1}{2}mv_1^2 = -\dfrac{GMm}{4R} + \dfrac{1}{2}mv_2^2$ ..... (ii)

$\Rightarrow v_1 = \sqrt{\dfrac{2GM}{3R}}$

$\therefore$ Radius of curvature at perigee $= \dfrac{v_1^2}{g_1}$

$\Rightarrow R_p = \dfrac{2GM}{3R} \times \dfrac{4R^2}{GM} = \dfrac{8R}{3}$

19. Point P, where field is zero



$\Rightarrow \dfrac{GMm}{(10a-x)^2} = \dfrac{G(16M)m}{x^2} \Rightarrow x = 8a$

COME : $-\dfrac{G(16M)m}{2a} - \dfrac{GMm}{8a} + \dfrac{1}{2}mv^2$

$= -\dfrac{G(16M)m}{8a} - \dfrac{GMm}{2a} \Rightarrow v = \dfrac{3}{2}\sqrt{\dfrac{5GM}{a}}$

20. Loss of total energy $= |TE|_{final} - |TE|_{initial} = Ct$

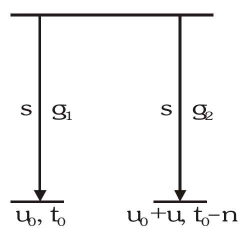
$\Rightarrow Ct = \dfrac{GM_sM_e}{2}\left(\dfrac{1}{R_e} - \dfrac{1}{R_S}\right)$

21. $s = \dfrac{1}{2}g_1t_0^2$ ...(i)

$s = \dfrac{1}{2}g_2(t_0-n)^2$ ...(ii)

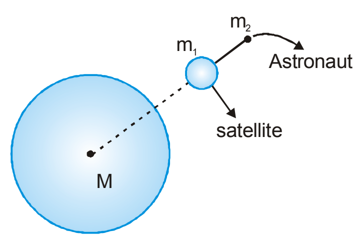
$u_0 = g_1t_0$ ...(iii)

$u_0 + u = g_2(t_0-n)$ ...(iv)



After solving we get $g_1g_2 = \left(\dfrac{u}{n}\right)^2$

22.



For satellite : $\dfrac{GMm_1}{R^2} - T = m_1\omega^2R$ ..... (i)

For astronaut : $\dfrac{GMm_2}{(R+r)^2} + T = m_2\omega^2(R+r)$ ......(ii)

Dividing eqn (i) by (ii) $\dfrac{gm_1 - T}{\dfrac{gm_2R^2}{(R+r)^2} + T} = \dfrac{m_1R\omega^2}{m_2(R+r)\omega^2}$

$\Rightarrow gm_1m_2\left[(R+r) - \dfrac{R^3}{(R+r)^2}\right] = T\left[m_1R + m_2(R+r)\right]$

$\Rightarrow T = \dfrac{m_2g}{R}\left[(R+r) - \dfrac{R^3}{(R+r)^2}\right]\left(\because \dfrac{m_2}{m_1} = 0\right)$

$$=m_2 g\left[\left(1+\frac{r}{R}\right)-\left(1-\frac{2r}{R}\right)\right]=\frac{3m_2 gr}{R}$$

$$=\frac{3\times100\times10\times64}{6400\times1000}=0.03\text{ N}$$

23. $v\frac{dv}{dh}=-\frac{GM}{(R+h)^2}$; $\int_{v_0}^{v} vdv=-\int\frac{gR^2}{(R+h)^2}dh$

$$\frac{v^2-v_0^2}{2}=\left(-gR^2\right)\left[\frac{-1}{R+h}\right]_0^h$$

$$\frac{v-v_0^2}{2}=+gR^2\left[\frac{1}{R+h}-\frac{1}{R}\right]$$

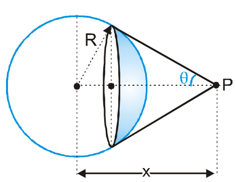
$$v_0^2-v^2=\frac{2gRh}{(R+h)}\ ;\ v_0^2-v^2=\frac{2gh}{\left(1+\frac{h}{R}\right)}$$

24. g below surface $g=\frac{GM}{R^3}(R-x)$

g above surface $g=\frac{GM}{(R+x)^2}$

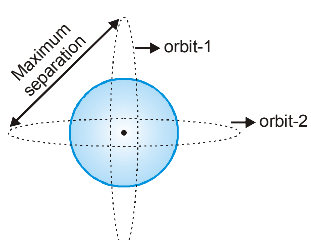
$$\Rightarrow\frac{GM}{R^3}(R-x)=\frac{GM}{(R+x)^2}\Rightarrow x=\left(\frac{\sqrt{5}-1}{2}\right)R\ \text{ or } 0$$

25. The area on earth surface in which satellite can not send message $=4\pi R^2\,[1-\cos\theta]$



$$=4\pi R^2\left[1-\frac{\sqrt{x^2-R^2}}{x}\right]$$

26. **(i)** Orbital velocity of each particle $v_0=\sqrt{\frac{GM}{r}}$



(ii) Maximum separation before collision $=\sqrt{2}\,r$

(iii) Relative velocity before collision

$$v_{Rel}=\sqrt{v_1^2+v_2^2}=\sqrt{\frac{2GM}{r}}$$

27. (i) At equator $m\omega^2R=\frac{GMm}{R^2}$

$$\Rightarrow\frac{2\pi}{T}=\omega=\sqrt{\frac{G\rho}{R^3}\cdot\frac{4\pi R^3}{3}}\quad\Rightarrow\quad T=\sqrt{\frac{3\pi}{G\rho}}$$

(ii) $T=\sqrt{\frac{3\pi}{6.63\times10^{-11}\times3000}}=1.9\text{ hr}$

28. Average velocity of satellite P,

$$W_P=\frac{v_P}{2R}=\frac{1}{2R}\sqrt{\frac{GM}{2R}}$$

Average velocity of satellite Q,

$$W_Q=\frac{v_Q}{3R}=\frac{1}{3R}\sqrt{\frac{GM}{3R}}$$

Least time when P and Q will be in same vertical line

$$=\frac{2\pi}{\omega_1}+\frac{2\pi}{\omega_2}=\frac{2\pi R^{3/2}6\sqrt{6}}{\sqrt{GM}\left(2\sqrt{2}+3\sqrt{3}\right)}$$

29. Let x = distance of the particle from the surface

Acceleration, $\frac{vdv}{dx}=\frac{GM}{R^3}(R-x)$

$$\Rightarrow\int_{v_e}^{v} vdv=\int_0^x g\left(1-\frac{x}{R}\right)dx$$

$$\Rightarrow v=\sqrt{v_e^2+2g\left(x-\frac{x^2}{2R}\right)}=\frac{dx}{dt}$$

$$\Rightarrow\int_0^t dt=\int_R^0\frac{dx}{\sqrt{2g\left[R+x-\frac{x^2}{2R}\right]}}$$

$$\Rightarrow t=\int_R^0\frac{dx}{\sqrt{\frac{g}{R}\left[3R^2-(x-R)^2\right]}}$$

Let $x-R=\sqrt{3}R\sin\theta$ $dx=\sqrt{3}R\cos\theta d\theta$

$$\therefore\ t=\sqrt{\frac{R}{g}}\int d\theta=\sqrt{\frac{R}{g}}\sin^{-1}\left(\frac{x-R}{\sqrt{3}R}\right)_0^R$$

$$\Rightarrow t=\sqrt{\frac{R}{g}}\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

30. (i) Necessary centripetal force = Gravitational force

$\Rightarrow \; M\omega^2 r = \dfrac{GM^2}{4r^2} \qquad \Rightarrow \; T = 4\pi\sqrt{\dfrac{r^3}{GM}}$

(iii) COME : KE + PE = 0

$\Rightarrow \; \dfrac{1}{2}mv^2 - \dfrac{2GMm}{r} = 0 \Rightarrow \; v = \sqrt{\dfrac{4GM}{r}} \; ; \left(r = \dfrac{d}{2}\right)$

31. Relative velocity when satellite revolving anticlockwise

$(\omega_1 + \omega_2)\, t = 2\pi \qquad \Rightarrow \left(\dfrac{4\pi}{3} + \dfrac{2\pi}{24}\right) t = 2\pi \; ; \; t = \dfrac{24}{17}$

If it moves in same direction $\left(\dfrac{4\pi}{3} - \dfrac{2\pi}{24}\right) t = 2\pi$

$\left(\dfrac{30\pi}{24}\right) t = 2\pi \Rightarrow \; t = \dfrac{24}{15} = 1.6$ hrs.

32. Range of throw is = 10m

$\dfrac{u^2}{g} = 10 \qquad \Rightarrow \; u^2 = 100 \; \Rightarrow \; u = 10$ m/s

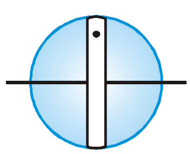
$v_e = \sqrt{\dfrac{2GM}{R}} \; \Rightarrow \; \dfrac{v_e}{v_{ep}} = \sqrt{\dfrac{M_e}{R_e} \times \dfrac{R_p}{M_p}}$

$\dfrac{v_e}{v_p} = \sqrt{\left(\dfrac{S_e}{S_p}\right)\left(\dfrac{R_e}{R_p}\right)^2}$ (S = density)

$\Rightarrow \; \dfrac{11.2}{10} = \dfrac{R_e}{R_p} \times \dfrac{1}{\sqrt{2}} \; \Rightarrow R_p = \dfrac{10}{11.2 \times \sqrt{2}} \times R_e$

$\Rightarrow \; \dfrac{10 \times 6.4 \times 10^6}{11.2 \times \sqrt{2}} = 40.42$ km

33. $F = \dfrac{GMmx}{R^3} = m\omega^2 x$



Particle will perform a oscillation

with angular speed $\omega^2 = \dfrac{GM}{R^3}$

$T_i = 2\pi\sqrt{\dfrac{R^3}{GM}} \; \Rightarrow \; t_1 = \dfrac{\pi}{2}\sqrt{\dfrac{R^3}{GM}}$

If acceleration is constant $g = \dfrac{GM}{R^2}$;

$S = \dfrac{1}{2}at^2 \qquad \Rightarrow \; R = \dfrac{1}{2}\dfrac{GM}{R^2}t^2$

$\Rightarrow \; t^2 = \dfrac{2R^3}{GM} \qquad \Rightarrow \; t_2 = \sqrt{\dfrac{2R^3}{GM}} \; ; \; \dfrac{t_1}{t_2} = \dfrac{\pi}{2\sqrt{2}}$

34. By applying conservation of linear momentum
m(v) – m(v) = 2mv'; v' = 0
Initially, energy of a satellite 'A' and 'B' is

$E_A = -\dfrac{GM_e m}{2R} \; ; \; E_B = -\dfrac{GM_e m}{2R}$

Total energy : $E_A + E_B = -\dfrac{GM_e m}{R}$

After collision velocity of satellite becomes zero then the K.E. = 0, therefore total mechanical energy becomes

$-\dfrac{GM_e 2M}{R}$

EXERCISE - 5

Part # I : AIEEE/JEE-MAIN

2. Required KE = $\dfrac{GMm}{R} = mgR$

3. Energy required = $U_f - U_i$

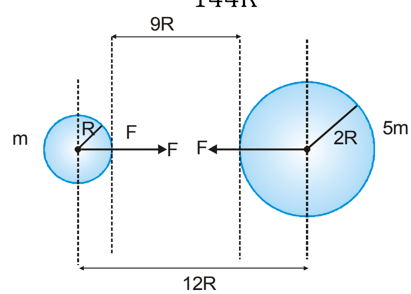
$= -\dfrac{GMm}{3R} + \dfrac{GMm}{2R} = -\dfrac{GMm}{R}\left(-\dfrac{1}{3} + \dfrac{1}{2}\right)$

$= \dfrac{GMm}{R}\left(-\dfrac{2+3}{6}\right) = \dfrac{GMm}{6R}$

4. $\because \quad T \propto R^{3/2}$

$\therefore \quad T_2 = T_1\left(\dfrac{R_2}{R_1}\right)^{3/2} = 5h(4)^{3/2} = 40h$

5. Force on M block $F = \dfrac{GM5m}{144R^2}$



$a = \dfrac{F}{m} = \dfrac{5GM}{144R^2}$

Force on 5 m block

$F = \dfrac{GM5m}{144R^2} \; \Rightarrow \; a = \dfrac{GM5m}{144R^2 \cdot 5m} = \dfrac{GM}{144R^2}$

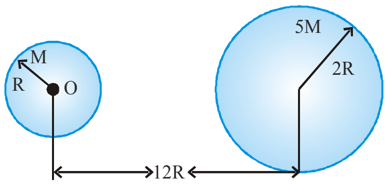
Relative acceleration

$a = \dfrac{5GM}{144R^2} + \dfrac{GM}{144R^2} = \dfrac{6GM}{144R^2} = \dfrac{GM}{24R^2}$

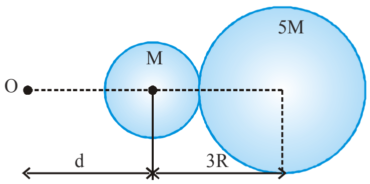
$$9R = \frac{1}{2} \times \frac{GM}{24R^2} \times t^2 \Rightarrow t^2 = \frac{18R \times 24R^2}{GM}$$

$$s = \frac{1}{2} \times \frac{5GM}{144R^2} \times \frac{18R \times 24R^2}{GM} = 7.5R$$

OR



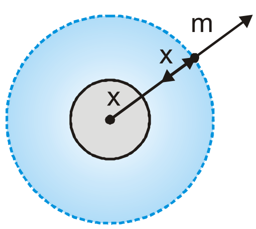
Initial position



Just before collision

$$\frac{M(0)+5M(12R)}{M+5M} = \frac{M(d)+5M(d+3R)}{M+5M} \Rightarrow d = 7.5R$$

6. $F = \frac{mv^2}{(R+x)}$



$$\frac{GMm}{(R+x)^2} = \frac{mv^2}{(R+x)}$$

$$v = \sqrt{\frac{GM}{(R+x)}} = \sqrt{\frac{GM}{R^2}\frac{R^2}{(R+x)}} = \sqrt{\frac{gR^2}{(R+x)}} = \left(\frac{gR^2}{R+x}\right)^{1/2}$$

8. $\Delta U = U_f - U_i = -\frac{GMm}{2R} + \frac{GMm}{R} = \frac{GMm}{2R} = \frac{mgR}{2}$

9. $F = \frac{GMm}{r^n} = m\omega^2 r$

$$\Rightarrow \omega = \sqrt{\frac{GM}{r^{n+1}}} \Rightarrow T = \frac{2\pi}{\omega} = \left(r^{\frac{n+1}{2}}\right)$$

11. $g_h = g\left(\frac{1-2h}{R}\right)$; $g_d = g\left(1-\frac{d}{R}\right)$

$$\Delta g_h = \Delta g_d \Rightarrow g\left(\frac{2h}{R}\right) = g\left(\frac{d}{R}\right) \Rightarrow 2h = d$$

12. Work done $= U_f - U_i = U_i = 0 - (-0)$

$$U_i = \frac{GM_1 m_2}{R} = \frac{6.67 \times 10^{-4} \times 100 \times 10 \times 10^{-1}}{10 \times 10^2}$$

$$= \frac{6.67 \times 10^{-11} \times 100 \times 10 \times 10^{-3}}{10 \times 10^{-2}} = 6.67 \times 10^{-10}$$

13. Electronic charge is independent from g, then the ratio will be equal for 1.

14. $v_e = \sqrt{\frac{GM}{R}}$ $v_e \propto \sqrt{\frac{M}{R}}$

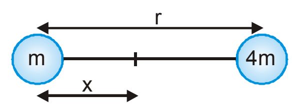
$$\frac{v_e}{v_e^1} = \sqrt{\frac{M_e}{R_e} \times \frac{R_p}{M_p}} \Rightarrow \sqrt{\frac{M_e}{R_e} \times \frac{R_e/10}{10Me}}$$

$$\frac{11}{v_e^1} = \frac{1}{10};\ v_e^1 = 110 \text{ m/s}$$

16. $g_h = \frac{g}{\left(1+\frac{h}{R}\right)^2} = \frac{g}{9} \Rightarrow \left(1+\frac{h}{R}\right)^2 = 9$

$$\Rightarrow 1+\frac{h}{R} = 3 \Rightarrow h = 2R$$

17.



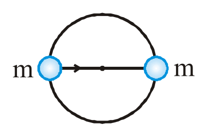
$$\frac{G \times m}{x^2} = \frac{G \times 4m}{(r-x)^2} \Rightarrow x = \frac{r}{3}$$

Potential at point the gravitational field is zero between the masses.

$$V = -\frac{3Gm}{r} - \frac{3 \times G \times 4m}{2r}$$

$$= -\frac{3Gm}{r}[1+2] = -\frac{9GM}{r}$$

18. $\frac{Gm^2}{4R^2} = \frac{mV^2}{R} = V = \sqrt{\frac{Gm}{4R}}$



19. $PE_i + KE_i = PE_f + KE_f$

$\Rightarrow$ $-mgR + KE_i = 0 + 0$

$KE_i = +mgR = 1000 \times 10 \times 6.4 \times 10^6$

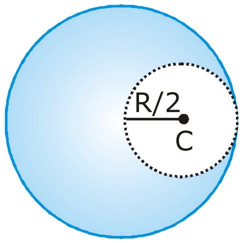
Work done $= 6.4 \times 10^{10}$ J

20. **(B)**

21. Solid sphere is of mass M, radius R.

Spherical portion removed have radius R/2, therefore its mass is M/8.

Potential at the centre of cavity $= V_{\text{solid sphere}} + V_{\text{removed part}}$



$$= \frac{-GM}{2R^3}\left[3R^2-\left(\frac{R}{2}\right)^2\right]+\frac{3G(M/8)}{2(R/2)}=\frac{-GM}{R}$$

22. $v_0=\sqrt{\frac{GM}{R+h}}=\sqrt{\frac{GM}{R}}=\sqrt{Rg}$; $\boxed{h \ll R}$

$$V_e=\sqrt{\frac{2GM}{R}}=\sqrt{2gR}$$

$$\frac{v_e}{v_0}=\sqrt{2}$$

Therefore increase in orbital velocity $=\left(\sqrt{2}-1\right)\sqrt{Rg}$

Part # II : IIT-JEE ADVANCED

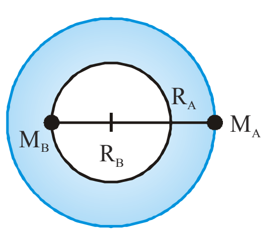
1. Force on satellite is always towards earth, therefore, acceleration of satellite S is always directed towards centre of the earth. Net torque of this gravitational force F about centre of earth is zero. Therefore, angular momentum (both in magnitude and direction) of S about centre of earth is constant throughout. Since the force F is conservative in nature, therefore mechanical energy of satellite remains constant. Speed of S is maximum when it is nearest to earth and minimum when it is farthest.

2. $T^2 \propto R^3$; with $R_e = 6400$ km,

$$\frac{T^2}{(24)^2}=\left(\frac{6400}{36000}\right)^3 \Rightarrow T \approx 1.7 \text{ hr}$$

For spy satellite R is slightly greater than R_e $\Rightarrow T_S > T$ $\therefore T_S = 2$hr

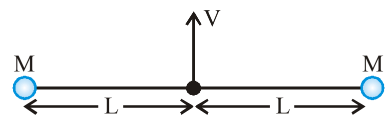
3. Figure shows a binary star system.



The gravitational force of attraction between the stars will provide the necessary centripetal forces. In this case angular velocity of both stars is the same. Therefore time period remains the same.

$$\left(\omega=\frac{2\pi}{T}\right)$$

4.



Total energy of m is conserved for escape velocity

$$K.E_f + P.E_f = K.E_i + P.E_i$$

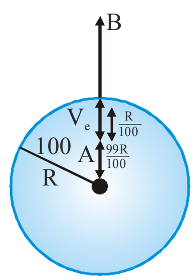
$$0+0=\frac{1}{2}mv^2+2\left[\frac{-GMm}{L}\right] \Rightarrow v=\sqrt{\frac{4GM}{L}}=2\sqrt{\frac{GM}{L}}$$

5. B

Subjective

1. Total energy at A = Total energy at B

$$(KE)_A + (PE)_A = (PE)_B$$



$$\frac{1}{2}m\times\frac{2GM}{R}+\left[\frac{-GMm}{2R^3}\left\{3R^2-\left(\frac{99R}{100}\right)^2\right\}\right]=-\frac{GMm}{R+h}$$

On solving we get h = 99.5 R

2. $\frac{g}{\left(1+\frac{h}{R_e}\right)^2}=\frac{8}{4}$; $1+\frac{h}{R_e}=2$ $\frac{h}{Re}=1$

h = Re

$$\frac{-GMe.m}{R_e}+\frac{1}{2}Mv^2=\frac{-GM_e m}{2R_e}$$

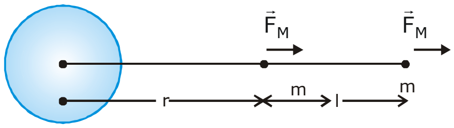
$$\frac{1}{2}m\upsilon^2=\frac{GM_e m}{2R_e} \; ; \; \upsilon=\sqrt{\frac{GM_e}{R_e}}$$

$$\therefore v_e=\sqrt{2}v_0 \Rightarrow v_e=\sqrt{2gR_e}$$

$$\Rightarrow v_e=\sqrt{2}v=\sqrt{N}v$$

$$\upsilon=\sqrt{gR_e}$$

N = 2

3. 

Writing the net force on system :

$$\frac{\frac{GMm}{(3l)^2}+\frac{GMm}{(4l)^2}}{2m}=\frac{\frac{GMm}{(3l)^2}-\frac{Gm}{(l)^2}}{m}$$

$$\Rightarrow \frac{GM}{9l^2}+\frac{M}{16l^2}=\frac{2M}{9l^2}-\frac{2m}{l^2}$$

$$\Rightarrow \frac{M}{16}-\frac{M}{9}=-2m$$

$$\Rightarrow \frac{9M-16M}{144}=-2m$$

$$\Rightarrow \frac{7M}{144}=-2m$$

$$\Rightarrow m=\frac{7M}{288}$$

$\therefore$ k = 7

MOCK TEST

1. Let the minimum speed imparted to the particle of mass m so that it just reaches surface of earth is v. Applying conservation of energy

$$\frac{1}{2}mv^2+\left(-\frac{3}{2}\frac{GM}{R}m\right)=-\frac{GM}{R}m+0$$

Solving we get $v=\sqrt{\frac{GM}{R}}$

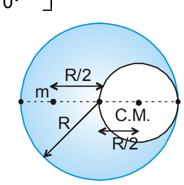
2. Gravitation field at mass 'm' due to full solid sphere

$$\vec{E}_1=\frac{\rho\vec{r}}{3\varepsilon_0}=\frac{\rho R}{6\varepsilon_0}....\left[\varepsilon_0=\frac{1}{4\pi G}\right]$$

Gravitational field at mass 'm' due to cavity ($-\rho$)

$$\vec{E}_2=\frac{(-\rho)(R/2)^3}{3\varepsilon_0 R^2}\ ....\ \left[\frac{\rho a^3}{3\varepsilon_0 r^2}\right]$$

$$=\frac{(-\rho)R^3}{24\varepsilon_0 R^2}=\frac{-\rho R}{24\varepsilon_0}$$

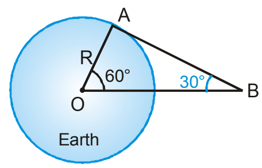


Net gravitational field $\vec{E}=\vec{E}_1+\vec{E}_2=\frac{\rho R}{6\varepsilon_0}-\frac{\rho R}{24\varepsilon_0}=\frac{\rho R}{8\varepsilon_0}$

Net force on 'm' $\to F=m\vec{E}=\frac{m\rho R}{8\varepsilon_0}$

Here $\rho=\frac{M}{4/3\pi R^3}$ & $\varepsilon_0=\frac{1}{4\pi G}$, then $F=\frac{3mg}{8}$

3. In ΔAOB : - $\cos 60°=\frac{R}{OB}$ $\Rightarrow$ OB = 2 R (where OB is orbital radius)



Here gravitational force will provide the required centripetal force.

Hence $\frac{GMm}{(OB)^2}=m(OB)\,\omega^2$

$$\Rightarrow \omega=\sqrt{\frac{GM}{(OB)^3}}=\sqrt{\frac{GM}{(2R)^3}}\ \Rightarrow\ \omega=\sqrt{\frac{GM}{8R^3}}$$

4. Given $8=\frac{2\pi}{\omega_1+\omega_2}=\frac{2\pi}{\frac{2\pi}{T_1}+\frac{2\pi}{T_2}}$,

T_1 = 24 hours for earth.

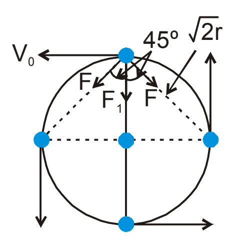
($\omega_1+\omega_2$ is the relative angular velocity for opposite direction)

$\Rightarrow$ T_2 = 12 hours (T_2 being the time period of satellite, it will remain same as the distance from the centre of the earth remains constant).

$\Rightarrow$ $T=\frac{2\pi}{\omega_2-\omega_1}=\frac{2\pi}{\frac{2\pi}{T_2}-\frac{2\pi}{T_1}}$ = 24 hours.($\omega_2-\omega_1$ is the relative angular velocity for same direction)

5. Centripetal force = net gravitational force

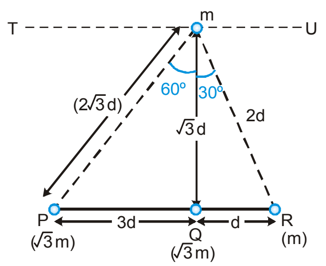
$$\frac{mv_0^2}{r}=2F\cos 45°+F_1=\frac{2Gm^2}{(\sqrt{2}r)^2}\frac{1}{\sqrt{2}}+\frac{Gm^2}{4r^2}$$



$$\frac{mv_0^2}{r}=\frac{Gm^2}{4r^2}[2\sqrt{2}+1]\Rightarrow\left(\frac{Gm(2\sqrt{2}+1)}{4r}\right)^{1/2}$$

6. In horizontal direction

$$\text{Net force} = \frac{G\sqrt{3}\,mm}{12\,d^2}\cos30^\circ - \frac{Gm^2}{4d^2}\cos60^\circ$$



$$= \frac{Gm^2}{8d^2} - \frac{Gm^2}{8d^2} = 0$$

in vertical direction

$$\text{Net force} = \frac{G\sqrt{3}\,m^2}{12\,d^2}\cos60^\circ + \frac{G\sqrt{3}\,m^2}{3\,d^2} + \frac{Gm^2}{4\,d^2}\cos30^\circ$$

$$= \frac{\sqrt{3}\,Gm^2}{24\,d^2} + \frac{\sqrt{3}Gm^2}{3\,d^2} + \frac{\sqrt{3}Gm^2}{8\,d^2}$$

$$= \frac{\sqrt{3}Gm^2}{d^2}\left[\frac{1+8+3}{24}\right] = \frac{\sqrt{3}Gm^2}{2d^2} \text{ along SQ}$$

7. $M_A = \sigma 4\pi R_A^{\,2}$, $M_B = \sigma 4\pi R_B^{\,2}$

where σ is surface density.

$$V_A = \frac{-GM_A}{R_A}, V_B = \frac{-GM_B}{R_B}$$

$$\frac{V_A}{V_B} = \frac{M_A}{M_B}\frac{R_B}{R_A} = \frac{\sigma 4\pi R_A^2}{\sigma 4\pi R_B^2}\frac{R_B}{R_A} = \frac{R_A}{R_B}$$

Given $\dfrac{V_A}{V_B} = \dfrac{R_A}{R_B} = \dfrac{3}{4}$, then $R_B = \dfrac{4}{3}R_A$

for New shell of mass M and radius R -

$M = M_A + M_B = \sigma 4\pi R_A^{\,2} + \sigma 4\pi R_B^{\,2}$

$\sigma 4\pi R^2 = \sigma 4\pi (R_A^{\,2} + R_B^{\,2})$

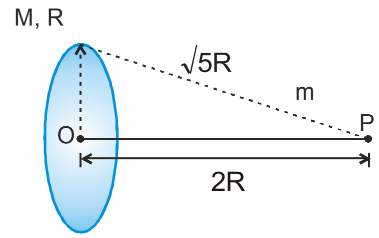
then $$\frac{V}{V_A} = \frac{M}{R}\frac{R_A}{M_A} = \frac{\sigma 4\pi [R_A^2 + R_B^2]}{(R_A^2 + R_B^2)^{1/2}}\frac{R_A}{\sigma 4\pi R_A^2}$$

$$= \frac{\sqrt{R_A^2 + R_B^2}}{R_A} = \frac{5}{3}$$

8. Gravitational potential at 'P'

$$V_P = \frac{-GM}{\sqrt{5}R}$$

Gravitational potential at 'O'



$$V_0 = -\frac{GM}{R}$$

work energy theorem

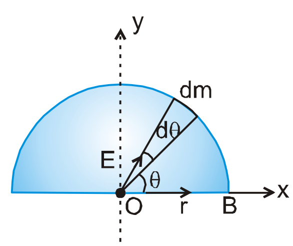
$W = \Delta K \Rightarrow m[V_P - V_O] = 1/2\ mv^2$

$$m\left[\frac{GM}{R} - \frac{GM}{\sqrt{5}R}\right] = \frac{1}{2}mv^2 \Rightarrow \sqrt{\frac{2GM}{R}\left[1 - \frac{1}{\sqrt{5}}\right]} = v$$

9. Let mass per unit length of wire, $\lambda = \dfrac{m}{\ell}$

and $\pi r = \ell$, $r = \dfrac{\ell}{\pi}$

mass of element, $dm = \lambda r d\theta$



then $$dE = \frac{Gdm}{r^2}$$

$$\int_0^\pi dE = \int_0^\pi \frac{G\lambda r d\theta}{r^2}(\hat{i}\cos\theta + \hat{j}\sin\theta)$$

$$E = \frac{G\lambda}{r}\left[\int_0^\pi \hat{i}\cos\theta\, d\theta + \int_0^\pi \hat{j}\sin\theta\, d\theta\right] = \frac{2G\lambda}{r}\hat{j} = \frac{2Gm}{\ell r}\hat{j}$$

$$= \frac{2Gm\pi}{\ell^2}\hat{j} \text{ (along y-axis)}$$

10. During total eclipse-
Total attraction due to sun and moon,

$$F_1 = \frac{GM_sM_e}{r_1^2} + \frac{GM_mM_e}{r_2^2}$$

When moon goes on the opposite side of earth.
Effective force of attraction,

$$F_2 = \frac{GM_sM_e}{r_1^2} - \frac{GM_mM_e}{r_2^2}$$

Change in force, $\Delta F = F_1 - F_2 = \frac{2GM_mM_e}{r_2^2}$

Change in acceleration of earth $\Delta a = \frac{\Delta F}{M_e} = \frac{2GM_m}{r_2^2}$

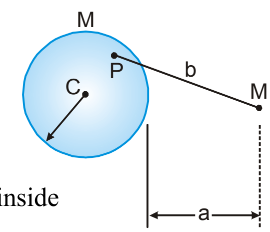
Average force on earth, $F_{av} = \frac{F_1 + F_2}{2} = \frac{GM_sM_e}{r_1^2}$

Average acceleration of earth, $a_{av} = \frac{F_{av}}{M_e} = \frac{GM_s}{r_1^2}$

%age change in acceleration

$$= \frac{\Delta a}{a_{av}} \times 100 = \frac{2GM_m}{r_2^2} \times \frac{r_1^2}{GM_s} \times 100 = 2\left(\frac{r_1}{r_2}\right)^2 \frac{M_m}{M_s} \times 100$$

11 At centre



$$V_c = -\frac{GM}{a} - \frac{GM}{2a};$$

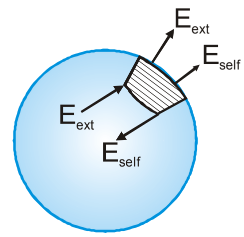
$E_c = \frac{GM}{(2a)^2}$; At any point P inside

$$V_P = -\frac{GM}{a} - \frac{GM}{b}$$

$E_P = \frac{GM}{b^2}$ {only due to outside mass M}

12. Consider a small area (shaded strip)
here E_{self} = Gravitational field due to this strip
and E_{ext} = Gravitational field due to the rest of spherical shell.
E_{in} = Gravitational field just inside the strip
E_{out} = Gravitational field just outside the strip
$E_{in} = E_{ext} - E_{self} = 0 \Rightarrow E_{ext} = E_{self}$

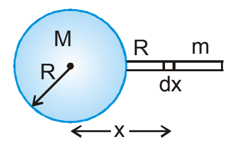
$$E_{out} = E_{ext} + E_{self} = \frac{GM}{R^2} \quad \Rightarrow \quad E_{ext} = \frac{GM}{2R^2}$$



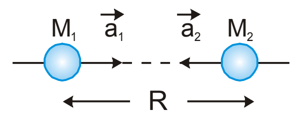
After the shaded area has been removed there is no E_{self} and only $E_{ext.}$

hence, $E_{net} = E_{ext} = \frac{GM}{2R^2}$

13. $F = \int_R^{2R} \frac{GM\left(\frac{m}{R}\right)dx}{x^2} = \frac{GMm}{2R^2}$



14. $a_1 = \frac{GM_1M_2}{R^2} / M_1 \qquad a_2 = \frac{GM_1M_2}{R^2} / M_2$



acceleration of M_1 w.r.t. M_2

$a_{rel.} = a_1 + a_2 = \frac{G(M_1 + M_2)}{R^2} = \frac{GM}{R^2}$.

15. $v = \frac{50}{100} V_e = \frac{1}{2}\sqrt{\frac{2GM}{R}}$
Applying energy conservation

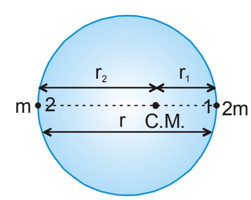
$$\Rightarrow -\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{(R+h)}$$

$$v^2 = \frac{2GM}{R} - \frac{2GM}{R+h}$$

$$\Rightarrow \frac{1}{4}.\frac{2GM}{R} = 2GM\left(\frac{1}{R} - \frac{1}{R+h}\right)$$

$$\Rightarrow \frac{1}{4R} = \frac{h}{R(R+h)} \quad \Rightarrow R + h = 4h \quad \Rightarrow h = R/3$$

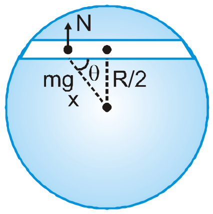
16. $r_2 = \frac{2mr}{m+2m} = \frac{2r}{3} \quad \Rightarrow T_2^2 = \frac{4\pi^2 r_2^3}{Gm}$



$$T_2^2 = \frac{32\pi^2 r^3}{27\,Gm} \Rightarrow T_2 \propto r^{3/2} \Rightarrow T_2 \propto m^{-1/2}$$

17. Net force towards centre of earth = $mg' = \dfrac{mgx}{R}$

Normal force $N = mg' \sin\theta$



Thus pressing force $N = \dfrac{mgx}{R}\dfrac{R}{2x}$

$N = \dfrac{Mg}{2}$ constant and independent of x.

Hence (B)

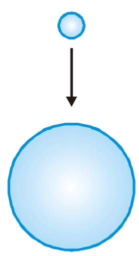
tangential force $F = ma = mg' \cos\theta$

$$Q = g' \cos\theta = \frac{gx}{R}\frac{\sqrt{\frac{R^2}{4} - x^2}}{x}$$

$$a = \frac{gx}{R}\sqrt{R^2 - 4x^2}$$

Curve is parabolic and at $x = \dfrac{R}{2}$, $a = 0$. Hence (C)

18. **(A)** It will fall because mg is acting on it towards the centre of planet and initial velocity is zero. It'll move in straight line.



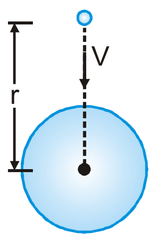
(C) Time of fall can be found by two methods :

I - Method : By energy conservation

$$\frac{1}{2}mv^2 - \frac{GMm}{r} = 0 - \frac{GMm}{R} \quad .... (1)$$

using this we get V = f(r). Now use

$$V = -\frac{dr}{dt} \Rightarrow f(r) = -\frac{dr}{dt}$$



$$\Rightarrow \int_R^{R'} \frac{dr}{f(r)} = -\int_0^t dt \;;$$

R' = radius of the planet.

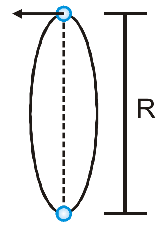
In the final expression

(or in the beginning itself) $R' \to 0$ $\{\because R >> R'\}$

you will get $t = \dfrac{T}{4\sqrt{2}}$

Here $\dfrac{GMm}{R^2} = m\left(\dfrac{2\pi}{T}\right)^2 R$

Note : This method is longer. If a student gets idea of solving the question only by this method then it is better to leave this question because it will consume more time.



II Method : Kepler's Law : $T^2 \propto r^3$.

Assume that the satellite moves in elliptical path with maximum and minimum distances from centre as R and R'.

$\because R >> R'$ $\therefore$ velocity at R is very small ($\simeq 0$). When it reaches R' then it touches the surface of the planet. This motion (from R to R') is almost, same as given in the question.

Now $\dfrac{T_1^2}{T_2^2} = \dfrac{r_1^3}{r_2^3}$, $T_1 = T, r_1 = R$

$$r_2 = \frac{R + R'}{2} \simeq \frac{R}{2} \qquad \therefore \; T_2 = \frac{T}{4\sqrt{2}}$$

19. Till the particle reaches the centre of planet, force on both bodies are in direction of their respective velocities, hence kinetic energies of both keep on increasing . After the particle crosses the centre of planet, forces on both are retarding in nature. Hence as the particle passes through the centre of the planet, sum of kinetic energies of both the bodies is maximum. Therefore statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

20. The total mechanical energy of the system after firing the rocket will increase by 10%.

Hence (B)

Note : $-0.9\,E_0 > -E_0$

21. Because the mechanical energy is negative, a decrease in magnitude is increase in energy.

$$\frac{E_{ell}}{E_{cir.}} = \frac{-\frac{GMm}{2a}}{-\frac{GMm}{2r}} = \frac{r}{a} \Rightarrow 0.9 = \frac{(6400 + 300)}{a} \Rightarrow a$$

$$= \frac{6.7 \times 10^4}{9} \text{ Km.}$$

22. Maximum distance from the centre of the Earth will occur when the spacecraft is at apogee thus

$$r_{max} = 2a - h - r = 2 \times \frac{6.7 \times 10^4}{9} - 6700$$

$$= \frac{7.37 \times 10^4}{9} \text{ km.}$$

$$h_{max} = r_{max} - R_E = \frac{7.37 \times 10^4}{9} - 6400$$

$$= \frac{1.61 \times 10^4}{9} \text{ km.}$$

23. Angular momentum of particle $= m(v_0 + v)$

$$a = \sqrt{\frac{5}{4}}\, mv_0 a \quad .... \ v_0 = \sqrt{\frac{GM_e}{a}}$$

Total energy of particle

$$= \frac{1}{2} m(v_0 + v)^2 - \frac{GMem}{a} = \frac{1}{2} \times \frac{5}{4} mv_0^{\ 2} - \frac{GM_e m}{a}$$

$$= \frac{5}{8} \frac{GM_e m}{a} - \frac{GM_e m}{a} = -\frac{3GM_e m}{a}$$

At any distance 'r'

$$\text{T.E.} = \frac{1}{2} mu^2 - \frac{GM_e m}{r}$$

but angular momentum conservation

$$mur = m\sqrt{\frac{5GM_e}{4a}}\, a \Rightarrow u = \sqrt{\frac{5}{4} \frac{GM_e a}{r^2}}$$

T.E. at any distance 'r'

$$= \frac{1}{2} m \frac{5}{4} \frac{GM_e a}{r^2} - \frac{GM_e m}{r}$$

but through conservation of total energy

$$= \frac{1}{2} m \frac{5}{4} \frac{GM_e a}{r^2} - \frac{GM_e m}{r} = -\frac{3GM_e m}{a}$$

on solving $3r^2 - 8ar + 5a^2 = 0$

$(r - a)(3r - 5a) = 0$

$r = a, \quad r = 5a/3$

minimum distance $= a$

maximum distance $= 5a/3$

24. **(A)** At centre of thin spherical shell $V \neq 0, E = 0$.

(B) At centre of solid sphere $V \neq 0$, $E = 0$.

(C) At centre of spherical cavity inside solid sphere $V \neq 0, E \neq 0$.

(D) At centre of two point masses $V \neq 0$, $E=0$.

25. Speed of the ball which can cross 10 m wide river is

$$R = \frac{V^2 \sin(2 \times 45^\circ)}{g} = 10,\ v = \sqrt{10g}$$

Let the radius of planet is 'R', then

$$\text{Mass of planet } M = \frac{4}{3} \pi R^3 \times 2\rho$$

$$= \frac{4}{3} \pi R^3 \times \frac{2 \times M_e}{4/3\pi R_e^3} = \frac{2M_e R^3}{R_e^3}$$

Escape velocity on planet

$$V = \sqrt{\frac{2GM}{R}} = \sqrt{10g}$$

$$\sqrt{\frac{2G \times 2 \times M_e R^3}{R_e^3 R}} = \sqrt{\frac{10GM_e}{R_e^2}} \ \ .... \left[g = \frac{GM_e}{R_e^2}\right]$$

$$2R = \sqrt{10R_e} \quad \Rightarrow \quad 2R = \sqrt{10 \times 6.4 \times 10^6}$$

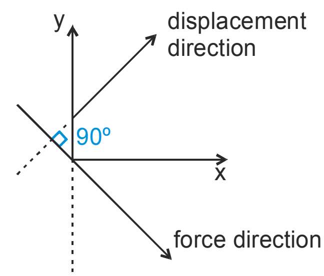
$$R = \frac{8 \times 10^3}{2} = 4 \times 10^3 \quad = \textbf{4 Km.}$$

26. Slope of displacement vector

$$m_1 = \frac{3}{4}$$

Slope of force vector

$$m_2 = -\frac{4}{3}$$



$$m_1 \times m_2 = \frac{3}{4} \times -\frac{4}{3} = -1$$

i.e. force and displacement directions are perpendicular. The work done is zero

27. Conserving angular momentum m.(V_1 cos60°).

$$4R = m.V_2.R \ ; \quad \frac{V_2}{V_1} = 2.$$

Conserving energy of the system

$$-\frac{GMm}{4R} + \frac{1}{2} mV_1^2 = -\frac{GMm}{R} + \frac{1}{2} mV_2^2$$

$$\frac{1}{2} V_2^{\ 2} - \frac{1}{2} V_1^{\ 2} = \frac{3}{4} \frac{GM}{R} \text{ or } V_1^{\ 2} = \frac{1}{2} \frac{GM}{R}$$

$$V_1 = \frac{1}{\sqrt{2}} \sqrt{64 \times 10^6} = \frac{8000}{\sqrt{2}} \text{ m/s} \quad \textbf{Ans. 8000}$$