HINTS & SOLUTIONS

EXERCISE - 1

Single Choice

3. $\phi = \frac{r\theta}{\ell} = \frac{1\times 10^{-2}}{2}\times 0.8 = 0.004$ radian

5. Stress $= \frac{F}{A}$

for breaking the copper stress should be same i.e.

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow \frac{F}{AR^2} = \frac{F_2}{\pi 4R^2} \Rightarrow F_2 = 4F$$

6. Volume = constant $\Rightarrow$ A × L = constant

$$\Rightarrow A \propto \frac{1}{L};\ \Delta\ell = \frac{FL}{AY} \Rightarrow \Delta\ell \propto \frac{L}{A} \propto L^2$$

8. $\Delta\ell = \frac{FL}{AY} \Rightarrow \Delta\ell \propto \frac{L}{r^2}$

9. $\Delta\ell = \frac{FL}{AY} \Rightarrow \frac{\Delta\ell_1}{\Delta\ell_2} = \frac{L_1}{L_2}\times\frac{r_2^2}{r_1^2} = \frac{1}{2}\times\left(\frac{\sqrt{2}}{1}\right)^2 = 1$

10. $-\frac{\Delta V}{V} = \frac{0.004}{100}$ $\because$ $B = \frac{\Delta P}{\left(-\frac{\Delta V}{V}\right)} \Rightarrow \Delta P = B\left(-\frac{\Delta V}{V}\right)$

$$= 2100\times 10^6 \times \left(\frac{0.004}{100}\right) = 84 \text{ kpa}$$

11. $\Delta\ell = \frac{FL}{AY} = \frac{(1\times 10)\times 1.1}{1\times 10^{-6}\times 1.1\times 10^{11}} = 0.1$ mm

12. Increment in length due to own weight

$$\Delta\ell = \frac{mgL}{2AY} = \frac{\rho g L^2}{2Y} = \frac{1.5\times 9.8\times(8\times 10^{-2})^2}{2\times 5\times 10^8}$$

$= 9.6\times 10^{-11}$ m

13. $W = \frac{1}{2}F\,\Delta\ell = \frac{1}{2}\frac{F^2L}{AY} \Rightarrow W \propto L;\ \frac{W_1}{W_2} = \frac{\ell_1}{\ell_2} = \frac{1}{2}$

14. $K = \frac{\Delta P}{\left(-\frac{\Delta V}{V}\right)} = \frac{h\rho g}{\left(-\frac{\Delta V}{V}\right)} = \frac{200\times 10^3\times 9.8}{\left(\frac{0.1}{100}\right)}$

$= 19.6\times 10^8$ N/m^2

16. Increase in energy

$$= \frac{1}{2}\frac{F^2L}{AY} = \frac{(5\times 10)^2\times 0.2}{2\times 10^{-4}\times 10^{11}} = 2.5\times 10^{-5}\text{ J}$$

17. $F_{ex} = 2T\ell = 2\times 7.5\times 1.5 = 22.5$ N

18. Initial surface energy $= 2\times T\times 4\pi r^2 = 8\pi r^2 T$

Final surface energy $= 2\times T\times 4\pi(2r)^2 = 32\pi r^2 T$

So energy neeeded $= 32\pi r^2T - 8\pi r^2 T = 24\pi r^2 T$

19. $F_{ex} = 4\pi r T \Rightarrow T = \frac{F_{ex}}{4\pi r} = \frac{4}{4\pi\times 1} = \frac{1}{\pi}$ N/m

21. $\Delta SE = 4\pi R^2 T(n^{1/3}-1)$

$$= 4\pi\frac{D^2}{4}T[(27)^{1/3}-1] = 2\pi D^2 T$$

22. $P_{in} = P_{atm} + \frac{2T}{r}$

$$= 1.013\times 10^5 + \frac{2\times 70\times 10^{-3}}{10^{-3}} = 1.0144\times 10^5\text{ Pa}$$

23. $P_{excess_1} = \frac{4T}{R_1};\ P_{excess_2} = \frac{4T}{R_2}$

$$\Rightarrow \frac{(P_{excess})_1}{(P_{excess})_2} = \frac{R_2}{R_1} \Rightarrow \frac{R_2}{R_1} = \frac{1.01}{1.02} = \frac{1}{2}$$

So ratio of volume $\frac{v_1}{v_2} = \frac{R_1^3}{R_2^3} = \frac{8}{1}$

24. $r_{new} = \sqrt{r_1^2 + r_2^2} = \sqrt{3^2+4^2} = 5$ cm

25. $r_{common} = \frac{r_1 r_2}{r_2 - r_1}[r_2 > r_1]$ here $r_1 = r_2$, so $r_{common} = \infty$

29. $h = \frac{2T\cos\theta}{rdg} \Rightarrow \frac{h_1}{h_2} = \frac{T_1}{T_2}\times\frac{d_2}{d_1} = \frac{60}{50}\times\frac{0.6}{0.8} = \frac{9}{10}$

30. at moon $g' = \frac{g}{6}$, height $h \propto \frac{1}{g}$

31. Density of water at 4°C is maximum so water rises in capillary is minimum by $h = \frac{2T\cos\theta}{rdg}$

32. $g_{eff} = 0,\ h = \frac{2T\cos\theta}{rdg}$

so water rises maximum height i.e. length of the capillary = 30 cm

34. Mass of water M = volume × density = $\pi r^2 h\rho$

$\because$ hr = constant $\Rightarrow$ $M \propto r$ $\Rightarrow$ $\frac{M_2}{M_1} = \frac{2r}{r} = 2$

35. $\ell = \frac{h}{\cos\phi} = \frac{h}{\cos 45°} = \sqrt{2}h$

36. $h\rho g = \frac{2T}{r} \Rightarrow h = \frac{2T}{r\rho g} = \frac{2\times 75}{0.05\times 10^{-1}\times 1\times 1000} = 30$ cm

37. $F = \frac{2AT}{t} = \frac{2\times 10^{-2}\times 70\times 10^{-3}}{0.05\times 10^{-3}} = 28N$

38. Force on bottom = Pressure × area

$= h\rho g \times \left(\frac{\pi d^2}{4}\right)$ ...(i)

force on vertical surface = Pressure × area

$= \left(\frac{h\rho g}{2}\right)\times\left(\frac{2\pi dh}{2}\right) = \frac{h^2\rho g\times \pi d}{2}$ ...(ii)

$\because$ according to question $\Rightarrow h\rho g \times \frac{\pi d^2}{4} = \frac{h^2\rho g\times \pi d}{2}$

$\Rightarrow h = \frac{d}{2}$

39. Let mass of gold is m then mass of copper =210–m
upthrust = loss of weight
$= 210g - 198g \Rightarrow V_{in}\rho_w g = 12g \Rightarrow V_{in} = 12\ cm^3$
Total volume

$= \frac{m}{\rho_{gold}} + \frac{210-m}{\rho_{cu}} = 12 \Rightarrow \frac{m}{19.3} + \frac{210-m}{8.5} = 12$

$\Rightarrow$ m = 193.
So weight of gold = 193 g

40. Pressure on the wall = $\frac{h\rho g}{2}$

Net horizontal force = P × area = $\frac{h\rho g}{2}\times(h\sigma) = \frac{h^2\rho g\sigma}{2}$

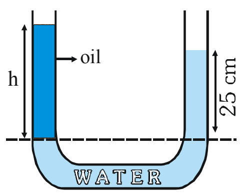
42. Total force = P × A = $\frac{h\rho g}{2}\times(h\times L)$

$= \frac{1\times 10^3\times 9.8}{2}\times(1\times 2) = 9.8\times 10^3$ N

43. Pressure at point A = Pressure at point B
$\Rightarrow h\rho_{oil}g = 25\ cm\ \rho_{water}g$

$\Rightarrow h = \frac{25\times 1}{0.8} = 31.25$ cm

$\Rightarrow$ height difference
$= 31.25 - 25 = 6.25$ cm



44. Barometer read atmospheric pressure.

45. Work = $\Delta PV = (3\times 10^5 - 1\times 10^5)\times 50000 = 10^{10}$J

47. $P_1V_1 = P_2V_2 \Rightarrow (P_{atm} + h\rho_w g)\frac{4}{3}\pi r^3 = P_{atm}\times\frac{4}{3}\pi(2r)^3$

$\Rightarrow h\rho_w g = 7\ P_{atm}$
$\because P_{atm} = H\rho_w g \Rightarrow h\rho_w g = 7H\rho_w g \Rightarrow h = 7H$

48. Upthrust = $V_{in}\rho_w g$ = 100 g-wt
weight of water and jar= weight + Th
= 700 + 100 = 800 g-wt

49. Weight = upthrust $\Rightarrow mg = (3\times 2\times 10^{-2})\times 10^3\times g$
$\Rightarrow$ m = 60 kg

50. Density of metal = $\frac{w_A}{w_A - w_w} = \frac{210}{210-180}$ = 7g/cm^3

density of liquid

$= \frac{w_A - w_L}{w_A - w_w} = \frac{210-120}{210-180} = \frac{90}{30} = 3$g/cm^3

51. In balanced condition Mg = Th $\Rightarrow 6g = \frac{V}{3}\rho_w g$ ...(i)

and $(6+m)g = V\rho_w g$ ...(ii)
from equation (i) and (ii) $18 = 6 + m \Rightarrow m = 12$ kg

53. Let mass of cube is m and side is a then
$(m+200)g = a^3\rho_w g$ ...(i)
$mg = a^2(a-2)\rho_w g$ ...(ii)
$\Rightarrow a^2(a-2)\rho_w + 200 = a^3\rho_w$
$\Rightarrow a^2 = 100 \Rightarrow a = 10$ cm

54. Reading of spring
$= Mg - Th = Mg - V_{in}\rho_w g$

$= 12 - \frac{1000\times 10^{-6}}{2}\times 10^3\times 10 = 7N$

58. For horizontal motion

$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$

$\Rightarrow 3\times 10^5 = 10^5 + \frac{1}{2}\times 10^3 V_2^2$
$\Rightarrow V_2^2 = 4\times 10^2 \Rightarrow V_2 = 20$ m/s

59. Force due to pressure difference = ΔP× A
In balanced condition = mg = ΔP× A

$\Rightarrow \Delta P = \frac{mg}{A} = \frac{3\times 10^4\times 10}{120} = 2.5$ kPa

60. $\frac{dV}{dt} = \frac{\pi\rho r^4}{8\eta\ell}$

62. Velocity of efflux $= \sqrt{2gh} = \sqrt{2 \times 10 \times 5} = 10\text{m/s}$

rate of flow $= Av = (1 \times 10^{-4}) \times 10 = 10^{-3}\ m^3/s$

63. Rate of flow $= Av = \pi r^2 \times \sqrt{2gh}$

$= 3.14 \times 1 \times \sqrt{2 \times 1000 \times 10} = 444\ cm^3/s$

64. $V_2^2 = V_1^2 + 2gh = (2)^2 + 2 \times 1000 \times 5.1 \times 10^{-1} = 1024$

$V_2 = 32$ cm/s

67. $m_1 = m_2 \Rightarrow V_1 d_1 = V_2 d_2$

Rate of flow $\dfrac{dV_1}{dt} = \dfrac{\pi \rho r^4}{8\eta\ell} \Rightarrow \dfrac{n_1}{n_2} = \dfrac{V_2 t_1}{V_1 t_2} = \dfrac{d_1 t_1}{d_2 t_2}$

68. Viscous force $= 6\pi\eta rv$

$= 6 \times 3.14 \times 18 \times 10^{-5} \times 0.3 \times 10^{-1} \times 10^2$

$= 101.73 \times 10^{-4}$ dyne

71. Radius of big drop $\Rightarrow R = (n)^{1/3} r = (2)^{1/3} r$

$v_T \propto r^2 \Rightarrow \dfrac{v'_T}{v_T} = \dfrac{R^2}{r^2} = (2)^{2/3} = 4^{1/3}$

$\Rightarrow v'_T = 4^{1/3} \times 5$ cm/s

EXERCISE - 2

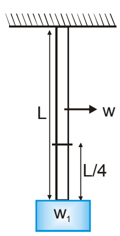
Part # I : Multiple Choice

1 Tension in wire at lowest position

$T = mg + m\omega^2 r$

So elongation $\Delta\ell = \dfrac{FL}{AY} = \dfrac{(mg + m\omega^2 L)L}{\pi r^2 Y}$

2. Stress $= \dfrac{F}{A} = \dfrac{\left(W_1 + \dfrac{W}{4}\right)}{S}$



3. Tension in wire $= mg = 10N$

so elongation $= \dfrac{F.L}{AY} = \dfrac{10 \times 3}{10^{-6} \times 10 \times 10^{10}} = 0.3$ mm

4. $\Delta\ell = \dfrac{FL}{AY} \Rightarrow \dfrac{\Delta\ell}{F/A} = \dfrac{L}{Y} =$ Slope of curve

$\Rightarrow \dfrac{L}{Y} = \dfrac{(4-2) \times 10^{-3}}{(8000-4000) \times 10^3} = \dfrac{1}{2} \times 10^{-9}$

$\because L = 1 \qquad \therefore Y = 2 \times 10^9\ N/m^2$

5. Spring constant of wire $= \dfrac{YA}{L}$

So effective spring constant

$= \dfrac{k_1 k_2}{k_1 + k_2} = \dfrac{k\dfrac{YA}{L}}{k + \dfrac{YA}{L}} = \dfrac{kYA}{kL + YA}$

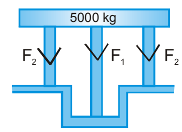
Time period $= 2\pi\sqrt{\dfrac{m}{k_{eff}}} = 2\pi\sqrt{\dfrac{m(kL + YA)}{kYA}}$

6. Surface tension does not depend on surface area.

7. $\because \Delta\ell_1 = \Delta\ell_2 \Rightarrow \dfrac{F_1 L_1}{A_1 Y_1} = \dfrac{F_2 L_2}{A_2 Y_2}$

$\Rightarrow \dfrac{F_1 \times 30 \times 10^{-2}}{16 \times 2 \times 10^6} = \dfrac{F_2 \times 20 \times 10^{-2}}{10 \times 10^6} \Rightarrow F_1 = \dfrac{32}{15} F_2$

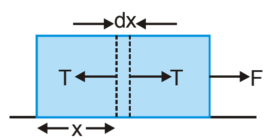
in balanced condition $F_1 + 2F_2 = 5000$ g



$\Rightarrow F_1 + \dfrac{2 \times 15}{32} F_1 = 5000\text{ g} \Rightarrow F_1 = 2580$ g

So stress in steel rod $= \dfrac{F_1}{A_1} = \dfrac{2580g}{16cm^2} = 161.2\text{kg/cm}^2$

8. Acceleration $a = \dfrac{F}{m}$



then tension in dx $= \dfrac{mx}{\ell} \times \dfrac{F}{m} = \dfrac{Fx}{\ell}$

Extension in dx element $= \dfrac{Tdx}{AY} = \dfrac{Fxdx}{AY\ell}$

total extension $\Delta\ell = \displaystyle\int_0^{\ell} \dfrac{Fxdx}{AY\ell} = \dfrac{F\ell}{2AY}$

9. In balanced condition $mg = 2\pi rT$

$\therefore \quad 2\pi r = \dfrac{mg}{T} = \dfrac{75 \times 10^{-4}}{6 \times 10^{-2}} = 12.5 \times 10^{-2}$ m

11. $\Delta h = h_1 - h_2 = \dfrac{2T}{r_1 dg} - \dfrac{2T}{r_2 dg} = \dfrac{2T}{dg}\left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right)$

$= \dfrac{2 \times 72}{1 \times 980}\left(\dfrac{2}{0.5} - \dfrac{2}{1}\right) = 0.293$cm

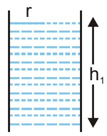
12. Potential energy

$$= mg\frac{H}{2} = (\pi r^2 h\rho)g\,\frac{H}{2} = \frac{\pi\rho g}{2}(rh)^2$$

according to Zurin law rH = constant $\Rightarrow u_1 = u_2$

13. For spring balance A $= Mg - Th = 2g - Th$
for balance B $= Mg + Th = 5g + Th$

14. For uniform radius tube in balanced condition



$$h_1 = \frac{2T}{r\rho g}$$

but weight of liquid in tapered tube is more than uniform tube of radius r then for balanced condition

$$h < h_1 \Rightarrow h < \frac{2T}{r\rho g}$$

16. Due to extra water, extra upthrust act on the steel ball so ball move up.

17. Acceleration of ball in water

$$= \frac{\text{net force}}{m} = \frac{Th - mg}{m} = \frac{V(d-D)g}{VD} = \frac{(d-D)g}{D}$$

Velocity at the surface $v = \sqrt{2ah} = \sqrt{2\frac{(d-D)}{D}gh}$

When ball come out from water then g act on the ball so height in air

$$h' = \frac{v^2}{2g} = \frac{2(d-D)gh}{D\times 2g} = \left(\frac{d}{D}-1\right)h$$

18. When the ball is pushed down, the water gains potential energy, whereas the ball loses potential energy. Hence, gain in potential energy of water

$$= (V\rho)rg - \left(\frac{V}{2}\rho\right)\left(\frac{3}{8}r\right)g$$

(When half of the spherical ball is immersed in water, rise of c.g. of displaced water $= \frac{3r}{8}$)

$$= V\rho rg\left(1 - \frac{3}{16}\right) = \frac{4}{3}\pi r^3\rho rg \times \frac{13}{16} = \frac{13}{12}\pi r^4\rho g$$

Loss in PE of ball $= V\rho' rg = \frac{4}{3}\pi r^4\rho' g$

Work done $= \frac{13}{12}\pi r^4\rho g - \frac{4}{3}\pi r^4\rho' g$

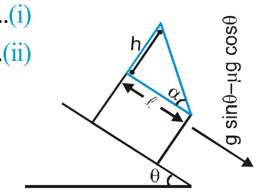
$$= \pi r^4\rho g\left[\frac{13}{12} - \frac{4}{3}\frac{\rho'}{\rho}\right]$$

$$= \pi r^4\rho g\left[\frac{13}{12} - \frac{4}{3}\times 0.5\right] = \frac{5}{12}\pi r^4\rho g$$

19. Let V_1 volume of the ball in the lower liquid then
$V\rho g = V_1\rho_2 g + (V - V_1)\rho_1 g$

$$\Rightarrow Vg(\rho-\rho_1) = V_1 g(\rho_2-\rho_1) \Rightarrow \frac{V_1}{V} = \frac{\rho-\rho_1}{\rho_2-\rho_1} = \frac{\rho_1-\rho}{\rho_1-\rho_2}$$

20. $\Delta P = \rho(g\sin\theta - \mu g\cos\theta)\ell$ .....(i)
$\Delta P = \rho g\cos\theta\, h$ ....(ii)
Both should be same



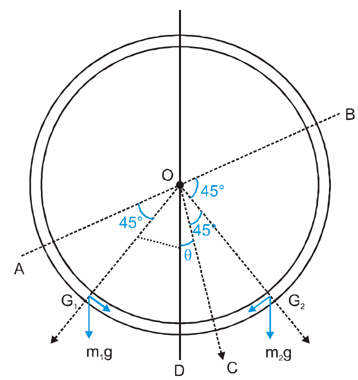
$$\frac{h}{\ell} = \tan\theta - \mu$$

$\Rightarrow \tan\phi = \tan\theta - \mu$

21. According to equation's of continuity $A_1v_1 = A_2v_2$

$$(\pi R^2)v = n(\pi r^2)v' \Rightarrow v' = \frac{v}{n}\left(\frac{R}{r}\right)^2$$

22. Given $\angle COD = Q$



G_1 & G_2 be the center of gravities of two liquids, then
$\angle AOC = 90° = \angle COB \Rightarrow \angle AOG_1 = 45°$
$\angle G_1OD = 45° - \theta \Rightarrow \angle COG_2 = 45°$
$\angle G_2OD = 45 + \theta$
Net torque about point O is zero
$\Rightarrow rm_1 g\sin(45° - \theta) = rm_2 g\sin(45 + \theta)$
$s v\sin(45 - \theta) = \sigma v\sin(45 + \theta)$

$$\frac{s}{\sigma} = \frac{\sin(45+\theta)}{\sin(45-\theta)}$$

$$\frac{s}{\sigma} = \frac{\sin 45\cos\theta + \cos 45\sin\theta}{\sin 45\cos\theta - \cos 45\sin\theta}$$

$$\frac{s-\sigma}{s+\sigma} = \frac{\cos\theta + \sin\theta - \cos\theta + \sin\theta}{\cos\theta + \sin\theta + \cos\theta - \sin\theta}$$

$$\frac{s-\sigma}{s+\sigma} = \tan\theta \Rightarrow \theta = \tan^{-1}\left(\frac{s-\sigma}{s+\sigma}\right)$$

23. Velocity of efflux of water $= \sqrt{2g\left(\frac{h}{2}\right)} = \sqrt{gh}$

Force due to ejected water

$$= \frac{dp}{dt} = \frac{dm}{dt}\ v = \rho(av)v = \rho av^2$$

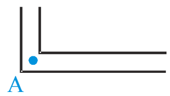
Torque of these forces about central line

$= F \times 2R + F \times 2R = 4\rho av^2 \times R = 4\rho\ aghR$

24. In pure rolling acceleration of the tube = 2a

$P_A = P_{atm} + \rho(2a)L$ (from horizontal)

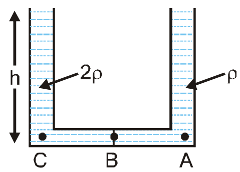
$P_A = P_{atm} + \rho gH$ (from vertical)



$$\Rightarrow\ a = \frac{gH}{2L}$$

25. From right Limb

$P_A = P_{atm} + h\rho g$



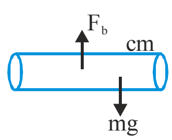
$$P_C = P_A + \rho a\left(\frac{\ell}{2}\right) + (2\rho)\,a\frac{\ell}{2}$$

$$= P_A + \frac{3}{2}\rho a\ell = P_{atm} + h\rho g + \frac{3}{2}\rho a\ell \quad \text{....(i)}$$

From left limb $P_C = P_{atm} + 2\rho gh$ ....(ii)

$$\Rightarrow\ P_{atm} + h\rho g + \frac{3}{2}\rho\ell a = P_{atm} + 2\rho gh \Rightarrow\ h = \frac{3a}{2g}\ell$$

26. Torque about CM $F_b \times \frac{\ell}{4} = I\alpha$



$$\Rightarrow \alpha = \frac{F_b\ell}{4I} = \frac{(\pi r^2\ell\rho g)\ell}{4I} = \frac{\pi r^2\ell^2\rho g}{4I}$$

27. Rate of flow = Av

Volume of water filled in tank in 15 s

$$V = \int_0^{15} A \times 10\left[1 - \sin\frac{\pi}{30}t\right]dt$$

$$= 10A\left[t + \frac{\cos\pi/t}{\pi/t}\right]_0^{15} = 10A\left[15 - \frac{30}{\pi}\right]$$

$$\text{height of water level} = \frac{V}{10A} = \left[15 - \frac{30}{\pi}\right]\text{m}$$

28. $v_0 = \sqrt{2gh},\ v = \sqrt{2g\frac{h}{\sqrt{2}}} = \frac{v_0}{\sqrt[4]{2}}$

29. As the cork moves up, the force due to buoyancy remains constant. As its speed increases, retarding force due to viscosity increase. The acceleration is variable, and hence the relation between velocity and time is not linear.

30. The free liquid surface between the plates is cylindrical and curved along one axis only so radius of curvature

$$r = \frac{d}{2} \text{ and } P_0 - P = \frac{s}{r} = \frac{2s}{d}$$

$$\Rightarrow\ P = P_0 - \frac{2s}{d}$$

32. $P_C - P_A = \ell\rho a$ and $P_B = P_C + h\,\rho g$

$P_B - P_A = h\rho g + \ell\rho a$

33. When the levels equalize then the height of the liquid in each arm $= \frac{h_1 + h_2}{2}$

Transferred length of liquid

$$= h_1 - \frac{h_1 + h_2}{2} = \frac{h_1 - h_2}{2}$$

$$\text{Transferred mass} = \left(\frac{h_1 - h_2}{2}\right)A\rho.$$

Loss in gravitational potential energy

$$= mgh = \left(\frac{h_1 - h_2}{2}\right)^2 A\rho g$$

Mass of the entire liquid $= (h_1 + h_2 + h)\,A\rho$

If this liquid moves with a velocity v then its

$$KE = \frac{1}{2}(h_1 + h_2 + h)A\rho v^2$$

$$\Rightarrow \left(\frac{h_1 - h_2}{2}\right)^2 A\rho g = \frac{1}{2}(h_1 + h_2 + h)\,A\rho v^2$$

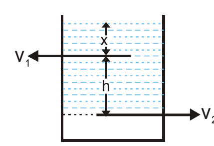
$$\Rightarrow\ v = \sqrt{\frac{g}{2(h_1 + h_2 + h)}}\,(h_1 - h_2)$$

35. $v_1 = \sqrt{2gx}$ and $v_2 = \sqrt{2g(x+h)}$.

Let cross section area of hole is a then rate of flow = av

force = v(avρ) = aρv²

$\therefore\ F_1 = a\rho v_1^2$ and $F_2 = a\rho v_2^2$



Net force

$= (F_2 - F_1) = a\rho\,(v_2^2 - v_1^2)$

$= a\rho(2g\,(x+h) - 2gx)$

$= 2a\rho gh$

37. In floating condition

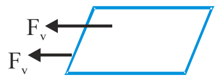
weight = upthrust

$$\Rightarrow \left(\frac{A}{5}L\right)Dg = \left(\frac{A}{5}\frac{L}{4}\right)2dg + \left(\frac{A}{5}\frac{3L}{4}\right)dg$$

$$\Rightarrow\ D = \frac{d}{2} + \frac{3d}{4} = \frac{5d}{4}$$

40. Net viscous force

$= 2F_v = 2\eta A \dfrac{dv}{dx}$



$\because$ $F = 1N \Rightarrow 1 = 2\eta \times (0.5) \times \dfrac{0.5}{1.25 \times 10^{-2}}$

$\Rightarrow \eta = 2.5 \times 10^{-2}$ kg–s/m^2

41. Viscous force = weight

$\eta A \dfrac{v}{t} = mg \sin\theta \Rightarrow \eta a^2 \dfrac{v}{t} = a^3 \rho g \sin\theta$

$\Rightarrow \eta = \dfrac{a\rho g t \sin\theta}{v}$

42. $v_T \propto (\rho_B - \rho_L)$

$\Rightarrow \dfrac{v'_T}{v_T} = \dfrac{(10.5 - 1.5)}{(19.5 - 1.5)} = \dfrac{9}{18} = \dfrac{1}{2}$

$\Rightarrow v'_T = \dfrac{v_T}{2} = \dfrac{0.2}{2} = 0.1$ m/s

46. The angle of contact at the free liquid surface inside the capillary tube will change such that the vertical component of the surface tension forces just balance the weight of the liquid column.

47. Tension in B $= T_B = \dfrac{mg}{3}$

Tension in A $= T_A = T_B + mg = \dfrac{4mg}{3}$

$\therefore$ $T_A = 4T_B$

Stress $= \dfrac{F}{A} = \dfrac{T}{\pi r^2}$

Wire breaks when stress

= Breaking stress for $r_A = r_B \Rightarrow s_A = 4s_B$

$\therefore$ A breaks before B

for $r_A = 2r_B \Rightarrow s_B = \dfrac{T_B}{\pi r_B^2}$

$s_A = \dfrac{T_A}{\pi r_A^2} = \dfrac{4T_B}{\pi (2r_B)^2}$

$\therefore$ stresses are equal so either A or B may break

50. If one surface is pushed down by x the other surface moves up by x.

Net unbalanced force on the liquid column $= 2xA\rho g$

mass of the liquid column $= \ell A\rho$

$\Rightarrow -2x A\rho g = (\ell A\rho)a \Rightarrow a = \left(-\dfrac{2g}{\ell}\right)x$

$\because$ $a = -\omega^2 x \Rightarrow \omega = \sqrt{\dfrac{2g}{\ell}} \Rightarrow T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{\ell}{2g}}$

Part # II : Assertion & Reason

1. A	2. D	3. A	4. B	5. A	6. A
7. A	8. A	9. A	10. C	11. A	12. C
13. D	14. A	15. A	16. A	17. A	18. A
19. C	20. A	21. A	22. D	23. A	24. A
25. A	26. A	27. B	28. A	29. A	30. C
31. C	32. A				

EXERCISE - 3

Part # I : Matrix Match Type

1. A $\to$ Q ; B $\to$ R ; C $\to$ R **2.** A $\to$ Q ; B $\to$ P ; C $\to$ Q

3. A $\to$ P ; B $\to$ Q ; C $\to$ Q **4.** A $\to$ P ; B $\to$ P, C $\to$ R

Part # II : Comprehension

Comprehension # 1

1. $F = \rho A(V_0 - 0)^2[1 - \cos 180°]$

$= 2\rho A v^2 = 2 \times 1000 \times 2 \times 10^{-4} \times 10 \times 10 = 40N$

2. $F = 2\rho A\,(V_0 - u)^2$ u = speed of cart

$m\dfrac{du}{dt} = 2\rho A(v_0 - u)^2;\quad \displaystyle\int_0^u \dfrac{du}{(V_0 - u)^2} = \dfrac{2\rho A}{m}\int_0^t dt$

$\left[\dfrac{2\rho A}{m} = \dfrac{2 \times 10^3 \times 2 \times 10^{-4}}{10} = \dfrac{4}{100}\right]$

$\left[\dfrac{1}{V_0 - u}\right]_0^u = \dfrac{2\rho A t}{m}$

$\dfrac{1}{V_0 - u} - \dfrac{1}{V_0} = \dfrac{2\rho A t}{m} = \dfrac{4t}{100}$ ...(i)

at t = 10 sec $\to \dfrac{1}{V_0 - u} = \dfrac{4}{10} + \dfrac{1}{10} = \dfrac{1}{2}$

$V_0 - u = 2$ u = 8 m/sec.

3. $F = 2\rho A(V_0 - u)^2$

$= 2 \times 10^3 \times 2 \times 10^{-4}(10-8)^2 = 2 \times 10^3 \times 2 \times 10^{-4} \times 4$

$a = \dfrac{F}{M} = 0.16$ m/sec^2

4. $\dfrac{1}{V_0 - u} - \dfrac{1}{V_0} = \dfrac{4t}{100} \Rightarrow \dfrac{1}{8} - \dfrac{1}{10} = \dfrac{4t}{100}$

$\Rightarrow \dfrac{2}{80} = \dfrac{4t}{100}$, t = 1.6 sec.

5. $F = 2\rho A(V_0 - u)^2$

$= 2 \times 10^3 \times 2 \times 10^{-4} \times 25 = 10N$

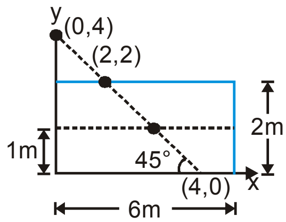
$P = F.u = 10 \times 5 = 50$ W.

Comprehension # 2

1. $\frac{dy}{dx} = \frac{a_x}{a_y + g} = \frac{g/2}{-g/2+g} = 1$

....(effective g will be $g - a = g/2$) $\theta = 45°$

2. As the slope of free surface is 45°.



Thus free surface passes through centre of box and having co-ordinates (2,2) at top of box.
Length of exposed top part = 6–2=4m.

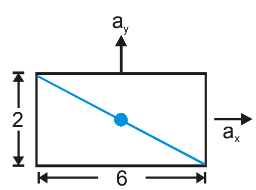
3. $P = P_a + \rho gh = 10^5 + 1000 \times 10 \times 1$
$= (10^5 + 10^4)\ N/m^2 = 0.11\ MPa$

4. $p = (10^5 + 10^3 \times 10 \times 4) N/m^2$
$= [0.1 + 0.04]\ MPa = 0.14\ MPa$

5. As maximum slope of free surface is 1/3 for the condition of non-exposure of bottom of box, then

$\frac{a_x}{a_y + g} = \frac{1}{3}$ as $a_x = g/2$, $3a_x = a_y + g$

$a_y = g/2$, thus g/2 upward.



Comprehension # 3

1. Equating the pressures at the same level of third liquid at the boundary of first and third liquids on left hand side.
Pressure on left hand side = pressure on right hand side
$\therefore P_0 + \rho(20)g = P_0 + 10(1.5)g + h(2\rho)g.$
Solving this equation, we get h = 2.5 cm

2. Rewriting the equation as
$P_0 + \rho(20)g = P_0 + 10(1.5)g + h(2\rho)g.$
From here we can see that h will decrease.

Comprehension#4

1. When the muscles of the heart relax, as they do during diastole, the heart is not exerting any force on the blood.

2. Volume flow rate
$\propto$ Pressure difference
$\propto$ (Radius of vessel)^4
If radius is increased by 10% volume flow rate would be increased by a factor $(1.1)^4 \approx 1.44$.

3. Gravitational potential energy

$$= \left(\frac{\text{energy}}{\text{volume}}\right) \times \text{volume} = (\rho gh)(\text{volume})$$

$\therefore PE = 1050 \times 9.8 \times 0.3 \times 8.0 \times 10^{-6} = 2.46 \times 10^{-2} J$

4. $W = mgh = (200 \times 10^{-6} \times 1050)(9.8)(0.5) \approx 1.0J$

5. $\text{Power} = \frac{\text{blood pressure} \times \text{volume of blood pumped}}{\text{time} - (\text{which blood is pumped})}$

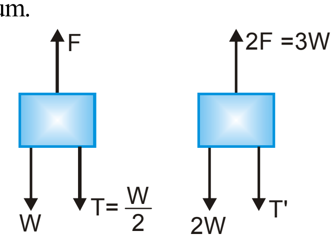
Factor by which power increased = 7 × 1.2 = 8.4, 20% increases means increase by a factor of 1.2.

Comprehension # 5

1. When the string is cut, tension becomes zero i.e., net upward force on the block becomes W/2 or net upward acceleration of the block will become g/2 or 5 m/s^2.

Now, $t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 2}{5}} = \frac{2}{\sqrt{5}} s$

2. If weight is doubled then obviously upthrust will also become two times, because weight can be increased only by increasing the volume by two times. When two out of three forces acting on the block have doubled then tension will also become two times to keep the block in equilibrium.



Before $F = W + \frac{W}{2} = \frac{3W}{2}$

After $3W = 2W + T'$ $\therefore T' = W = 2T$

When string is cut in second case, net upward acceleration will be $\frac{3W - 2W}{(2W/g)} = \frac{g}{2}$,

So time taken will not change.

Comprehension # 6

1. $Q \propto \frac{1}{\eta}$ when volume flow rate is multiplied by density, it becomes mass flow rate. Both rates are inversely proportional to η.

2. From $Q=\dfrac{\pi R^4(P_2-P_1)}{8\eta L}$

we have, $\eta=\dfrac{\pi R^4(P_2-P_1)}{8LQ}$

Substituting the value we get $\eta\approx 4\times10^{-3}\,\text{Pa}-\text{s}$

3. From $R_e=\dfrac{2\bar{v}\rho R}{\eta}$; $\bar{v}=\dfrac{\eta R_e}{2\rho R}$

Flow remains laminar till $R_e=2000$

$\therefore\ \bar{v}=\dfrac{4\times10^{-3}\times2000}{2\times1000\times8\times10^{-3}}=0.5$ m/s

4. $F=6\pi\eta rv=6\pi\times10^{-3}\times10^{-3}\times3=5.65\times10^{-5}$N

5. $6\pi\eta rv_T=mg$

$\therefore\ v_T=\dfrac{mg}{6\pi\eta r}=\dfrac{10^{-5}\times9.8}{6\pi\times10^{-3}\times10^{-3}}=5.2$ m/s

EXERCISE - 4

Subjective Type

1. (i) The area of the hysteresis loop is proportional to the energy dissipated by the material as heat when the material undergoes loading and unloding. A material for which the hysteresis loop has larger area would absrob more energy when subjected to vibrations. Therefore to absorb vibrations one would prefer rubber B.

(ii) Rubber A, to avoid excessive heating of the car tire.

2. (i) Material A has greater value of Young's modulus. Because slope of A is greater than B.

(ii) A material is more ductile because there is a large plastic deformation range between the elastic limit and the breaking point.

(iii) B material is more brittle because the plastic region between the elastic limit and breaking point is small.

(iv) Strength of a material is determined by the amount of stress required to cause fracture. Material A is stronger than material B.

3. Maximum stress $=\dfrac{F}{\text{Area}}=\dfrac{m(g+a)}{\pi r_{min}^2}$

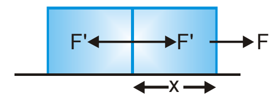
$\Rightarrow\ \dfrac{3}{\pi}\times10^8=\dfrac{900(9.8+2.2)}{\pi r_{min}^2}$

$\Rightarrow\ r_{min}=\sqrt{\dfrac{900\times12}{3\times10^8}}=6$mm

4. (a) $F'=m'a=\left(A\dfrac{L}{2}d\right)\left(\dfrac{F}{m}\right)$

$=\left(A\dfrac{L}{2}d\right)\left(\dfrac{dALg}{2\times ALd}\right)=\dfrac{ALdg}{4}$

Stress $=\dfrac{F'}{A}=\dfrac{Ldg}{4}$



$\because\ Y=\dfrac{\text{stress}}{\text{strain}}\Rightarrow$ strain $=\dfrac{\text{stress}}{Y}=\dfrac{Ldg}{4Y}$

5. $(\Delta\ell)_{steel}=\dfrac{FL}{AY}=\dfrac{(4+6)\times10\times1.5}{\pi(0.125\times10^{-2})^2\times2\times10^{11}}$

$=1.49\times10^{-4}$m

$(\Delta\ell)_{brass}=\dfrac{FL}{AY}=\dfrac{6\times10\times1}{\pi(0.125\times10^{-2})^2\times0.91\times10^{11}}$

$=1.31\times10^{-4}$ m

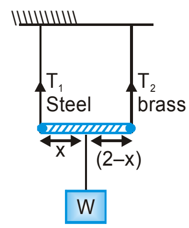
6. In equilibrium $mg=2T\ell\Rightarrow\pi r^2\ell\rho g=2T\ell$

$\Rightarrow\ r=\sqrt{\dfrac{2T}{\pi\rho g}}=\sqrt{\dfrac{2\times0.045}{3.14\times8.96\times10^3\times9.8}}=5.7$ mm

$\therefore$ diameter $=2r=1.14$ mm

7. For translatery equilibrium

$T_1+T_2=W$ ....(i)



for equal stress $\dfrac{T_1}{A_1}=\dfrac{T_2}{A_2}$

$\Rightarrow\ \dfrac{T_1}{T_2}=\dfrac{A_1}{A_2}=\dfrac{0.1\times10^{-4}}{0.2\times10^{-4}}=\dfrac{1}{2}$

$\Rightarrow T_2=2T_1$

from equation (i) $T_1+2T_1=W\Rightarrow T_1=\dfrac{W}{3}$, $T_2=\dfrac{2W}{3}$

for rotational equilibrium

$T_1x=T_2(2-x)\Rightarrow\dfrac{W}{3}x=\dfrac{2W}{3}(2-x)$

$\Rightarrow\ x=\dfrac{4}{3}$ m from steel wire

8. Compressive strength $=\dfrac{F_{max}}{\text{Area}}$

$\Rightarrow F_{max}=7.7\times10^8\times3.6\times10^{-4}=2.772\times10^5$N

$\because$ a pplied force $<F_{max}$ $\therefore$ bone will not break.

(ii) $\Delta\ell=\dfrac{FL}{AY}=\dfrac{3\times10^4\times20\times10^{-2}}{3.6\times10^{-4}\times1.5\times10^{10}}$

$=11.11\times10^{-4}=1.11$ mm

9. $\frac{4T}{r} = h\,\rho_{water}\,g \Rightarrow T = \frac{h\rho_{water}gr}{4}$

$= \frac{8\times10^{-1}\times1\times980\times0.35}{4} = 68.6$ dyne/cm

11. $P_1V_1 + P_2V_2 = PV$

$\Rightarrow \left(P+\frac{4T}{R_1}\right)\frac{4}{3}\pi R_1^3 + \left(P+\frac{4T}{R_2}\right)\frac{4}{3}\pi R_2^3$

$= \left(P+\frac{4T}{R}\right)\frac{4}{3}\pi R^3 \Rightarrow P\left(\frac{4}{3}\pi R_1^3+\frac{4}{3}\pi R_2^3-\frac{4}{3}\pi R^3\right)$

$= \frac{4T}{3}\left(\frac{4}{3}\pi R^2 - \frac{4}{3}\pi R_1^2 - \frac{4}{3}\pi R_2^2\right)$

$\because V = \frac{4}{3}\pi R^3 - \frac{4}{3}\pi R_1{}^3 - \frac{4}{3}\pi R_2{}^3$ and

$S = 4\pi R^2 - 4\pi R_1{}^2 - 4\pi R_2{}^2$

$\therefore P[-V] = \frac{4T}{3}[S] \Rightarrow 3PV + 4ST = 0$

12. When the tube is taken out, a convex meniscus is formed at the bottom then. The total upward force due to surface tension is

$F = 2\pi rT + 2\pi rT = 4\pi rT$

This balances the weight of water column of length H

$\Rightarrow 4\pi rT = (\pi r^2H)\rho g \Rightarrow H = \frac{4T}{r\rho g}$

but $h = \frac{2T}{r\rho g}$ therefore $H = 2h$

The length of the liquid column remaining $= 2h$

14. Pressure $= \frac{F}{A} = \frac{3000\times10}{425\times10^{-4}} = 7.06\times10^5$ Pa

13. In equilibrium $\frac{600\times10}{800\times10^{-4}} = \frac{F}{25\times10^{-4}} + h\rho g$

$\Rightarrow \frac{F}{25\times10^{-4}} = \frac{60}{8}\times10^4 - 8\times(0.75\times10^3)\times10$

$\frac{F}{25\times10^{-4}} = 1.5\times10^4 \Rightarrow F = 37.5$ N

15. $P_1V_1 = P_2V_2 \Rightarrow (P_{atm} + H\rho_w g)V = P_{atm}\times 2V$

$\Rightarrow H\rho_\omega g = P_{atm} = 76\text{cm}\times\rho_{Hg}\times g = 76\text{cm}\times13.6\,\rho_w g$

$\Rightarrow H = 1033.6$ cm $= 10.34$ m

16. Pressure an the water surface

$= \frac{Mg}{A} = \frac{3\times10}{\pi[16\times10^{-4}-1\times10^{-4}]}$

$= \frac{30\times10^4}{\pi\times15} = \frac{2}{\pi}\times10^4$ Pa

According to Pascal law = Pr = hρg

$\Rightarrow h = \frac{\Pr}{\rho g} = \frac{\frac{2}{\pi}\times10^4}{10^3\times10} = \frac{2}{\pi}$ m

Mass of water in the pipe

$= (\pi r^2h)\,\rho = \pi\times10^{-4}\times\frac{2}{\pi}\times10^{-3} = 0.2$ kg

mass of water in cylinder=750–200=550 g = 0.55

$\Rightarrow 0.55 = (\pi R^2H)\rho$

$\Rightarrow H = \frac{0.55}{\pi\times16\times10^{-4}\times10^3} = \frac{11}{32\pi}$ m

17. Initial potential energy $= m_1g\frac{h_1}{2} + m_2g\frac{h_2}{2}$

$= A\rho g\frac{h_1^2}{2} + A\rho g\frac{h_2^2}{2} = A\rho g\left[\frac{h_1^2+h_2^2}{2}\right]$

Final height $= \frac{h_1+h_2}{2}$

Final potential energy

$= mg\left(\frac{h_1+h_2}{4}\right) = A\rho g\left(\frac{h_1+h_2}{2}\right)^2$

Work done by = Initial PE – Final PE

$= A\rho g\left(\frac{h_1^2+h_2^2}{2}\right) - A\rho g\left(\frac{h_1+h_2}{2}\right)^2$

$= \frac{A\rho g}{4}(h_1-h_2)^2$

19. $\because h_w + 8\text{cm} + h_o = 22\text{ cm} + 22\text{ cm}$

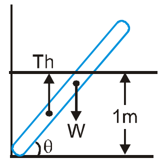
$\Rightarrow h_w + h_o = 36$ cm

$P_C = P_B \Rightarrow h_o\rho_o g = h_w\rho_w g \Rightarrow h_o\times0.8 = h_w\times1$

$\Rightarrow h_o = \frac{h_w}{0.8} = 1.25\,h_w \Rightarrow h_w + 1.25\,h_w = 36$

$\Rightarrow h_w = \frac{36}{2.25} = 16$cm so BE = 22 – 16 = 6cm

20. $\Rightarrow A\left(\frac{1}{\sin\theta}\right)\rho_w g \times \left(\frac{1}{2\sin\theta}\right)\cos\theta = mg\ (2\cos\theta)$



$\Rightarrow \frac{25\times10^{-4}\times10^{3}}{2\sin^2\theta} = 2.5\times2$

$\Rightarrow \sin^2\theta = \frac{1}{4} \Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \theta = 30°$

For minimum depth of water let water height is h

$\Rightarrow A\left(\frac{h}{\sin 90°}\right)\rho_w g \times \left(\frac{h}{2}\right) = mg\times2$

$\Rightarrow h^2 = \frac{2.5\times2}{25\times10^{-4}\times10^{3}\times\left(\frac{1}{2}\right)} \Rightarrow h = 2m$

21. $P_P = P_{atm} + h\rho g$

$= 1.013\times10^5 + 3\times800\times9.8 = 124.9\ KN/m^2$

$P_p = P_Q + (1.5+3)\rho g$

$\Rightarrow P_Q = 124.9\times10^3 - 4.5\times800\times9.8 = 89.5\ KN/m^2$

$P_R = P_Q = 89.5\ KN/m^2$

$P_s = P_R - (3+2.5)\rho g = 89.5\times10^3 - 5.5\times800\times9.8$

$= 46.4\ KN/m^2$

22. Specific gravity of block $= \frac{W_A}{W_A - W_W} = \frac{15N}{15-12} = 5$

24. When beaker half full with water then it float with completely immersed.

So weight = upthrust

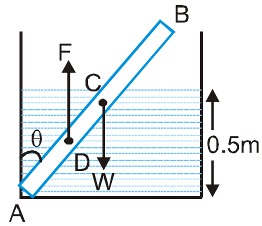
$\Rightarrow \left(390g + \frac{500}{2}\times1\times g\right) = V_{in}\times1\times g = 640\ cm^3$

So volume of glass beaker = 640–500 = 140 cm^3

density of beaker $= \frac{390}{140} = 2.78\ g/cm^3$

25. Let cross-section area of the plank is A then weight of plank W= (1 × A) 0.5 × g length of plank inside the water

$= \frac{0.5}{\cos\theta}$



So upthrust on the plank $= \left(\frac{0.5}{\cos\theta}\right) A\times1\times g$

torque about point A

$W\times AC\sin\theta = Th\times AD\sin\theta$

$(1\times A)\times0.5\times g\times0.5\sin\theta$

$= \left(\frac{0.5}{\cos\theta}\right) A\times1\times g\times\left[\left(\frac{1}{2}\right)\times\frac{0.5}{\cos\theta}\right]\sin\theta$

$\Rightarrow 1 = \frac{1}{2\cos^2\theta} \Rightarrow \cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45°$

26. Work done per unit volume by pressure

= change in energy

$= \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(h_2 - h_1)$

$= \frac{1}{2}\times10^3[(0.5)^2 - (1)^2] + 10^3\times10(5-2)$

$= -\frac{3}{8}\times10^3 + 30\times10^3 = 29.625\times10^3\ J/m^3$

work done per unit volume by gravity froce

$= \rho g(h_1 - h_2) = 10^3\times10(2-5) = -30\times10^3\ J/m^3$

27. $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

$\Rightarrow \frac{1}{2}\rho(v_2^2 - v_1^2) = P_1 - P_2 = h\rho g$

$v_2^2 = v_1^2 + 2gh$

$v_2 = \sqrt{(2)^2 + 2\times1000\times0.51} = 32\ cm/s$

28. (i) Reaction force

$= \frac{vdm}{dt} = v^2\frac{A}{100}\rho \Rightarrow m_o a = 2gh\times\frac{A}{100}\rho$

$\Rightarrow (A\times h\times\rho)a = \frac{2ghA\rho}{100} \Rightarrow a = \frac{2g}{100} = 0.2\ m/s^2$

(ii) $\frac{m_0}{4} = Ah'\rho \Rightarrow h' = \frac{m_0}{4A\rho}$

$v = \sqrt{2gh'} = \sqrt{2g\times\frac{m_0}{4A\rho}} = \sqrt{\frac{m_0 g}{2A\rho}}$

29. Let v' be the horizontal speed of water when it emerges from the nozzle then from equation of continuity

$$Av = av' \Rightarrow v' = \frac{Av}{a}$$

Let t be the time taken by the stream of water to strike the ground then vertical distance

$$h = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$$

$\Rightarrow$ horizontal distance

$$R = v'\sqrt{\frac{2h}{g}} = \frac{Av}{a}\sqrt{\frac{2h}{g}}$$

30. **(i)** Velocity of flow

$$= \sqrt{2gh} = \sqrt{2\times10\times3.6} = 6\sqrt{2}\ \text{m/s}$$

(ii) Rate of flow

$$= Av = \pi\left(\frac{4\times10^{-2}}{\sqrt{\pi}}\right)^2\times6\sqrt{2}$$

$$= 9.6\times\sqrt{2}\times10^{-3}\ \text{m}^3/\text{s}$$

(iii) Bernoulli's theorem between surface and A

$$P_{atm} = P + \frac{1}{2}\rho v^2 + \rho gh$$

$$\Rightarrow P = P_{atm} - \frac{1}{2}\rho v^2 - \rho gh$$

$$= 10^5 - \frac{1}{2}\times10^3(6\sqrt{2})^2 - 10^3\times10\times1.8$$

$$= 4.6\times10^4\ \text{N/m}^2$$

32. **(i)** $v = \sqrt{2gh}$ (Acc. to Torricellis law of efflux)

(ii) Reaction of out flowing liquid (F) = Mass coming out per second × velocity

$$F = v\left(\frac{dm}{dt}\right) \Rightarrow Ma = v\frac{dm}{dt} \Rightarrow (\rho A_2h)a = v\rho A_1v$$

$$\because \frac{dm}{dt} = \frac{d(\rho A_1x)}{dt} = \rho A_1\frac{dx}{dt} = \rho A_1v]$$

$$\Rightarrow A_2ha = v^2A_1 \Rightarrow A_2ha = 2ghA_1$$

$$[\because v = \sqrt{2gh}] \Rightarrow a = \frac{2gA_1}{A_2}$$

33. $v_A = \sqrt{2g\times\frac{h}{4}} = \sqrt{\frac{gh}{2}}$

$$(\text{Range})_A = v_A\times t = \sqrt{\frac{gh}{2}}\times\sqrt{2\times\frac{3h}{4g}} \quad \text{....(i)}$$

Bernoulli's theorem between surface and B

$$2\sigma g\frac{h}{2} + \sigma g\frac{h}{2} = \frac{1}{2}(2\sigma)v^2 + \left(2\sigma g\frac{h}{4}\right) \Rightarrow v = \sqrt{gh}$$

$$(\text{Range})_B = \sqrt{gh}\times\sqrt{\frac{2\times h}{4g}} \Rightarrow \frac{R_A}{R_B} = \frac{\sqrt{3}}{\sqrt{2}}$$

34. Velocity at surface = terminal velocity

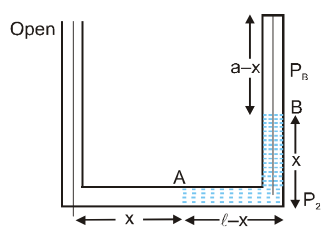
$$\Rightarrow \sqrt{2gh} = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$$

$$\sqrt{2gh} = \frac{2}{9}\times\frac{(3\times10^{-4})^2\times(10^4-10^3)\times9.8}{9.8\times10^{-6}} = 180$$

$$\Rightarrow h = \frac{(180)^2}{2g} = \frac{180\times180}{2\times9.8} = 1.65\times10^3\ m$$

35. $F = \eta A\frac{dv}{dx} = 1\times100\times10^{-4}\times\frac{7\times10^{-2}}{10^{-3}} = 0.7\text{N}$

37. $V = A(a-x)$



Final pressure, $P = \frac{P_0V_0}{V} = \frac{P_0a}{a-x}$ or pressure at B, P_2

$$= P + x\rho g = \frac{P_0a}{a-x} + x\rho g$$

force exerted by pressure difference is

$$f_1 = (P_B - P_A)s = (P_2 - P_0)s = \left(\frac{P_0x}{a-x} + x\rho g\right)s$$

Mass of horizontal arm A(B) of liquid is
$m = A(\ell - x)\rho$

$$r = x + \frac{\ell-x}{2} = \frac{\ell+x}{2}$$

$$\{A\rho(\ell-x)\}\left(\frac{\ell+x}{2}\right)\omega_0^2 = \left(\frac{P_0x}{a-x} + x\rho g\right)A$$

$x = 0.01$ m $\Rightarrow$ x = 1 cm
length of air column in sealed arm $(a-x) = 6-1 = 5$cm

38. Upthrust on the block

$$= \frac{2}{5}V \times 1500\left(g+\frac{g}{2}\right)+\frac{3}{5}V \times 1000 \times\left(g+\frac{g}{2}\right)$$

$= 1800 \times 10^{-3} \times 10 = 18N$

Weight of the block $= 10^{-3} \times 800 \times \left(g+\frac{g}{2}\right) = 12N$

So Tension in the string = Th–mg = 18–12 = 6N

39. **(i)** As for floating W = Th

$V\rho g = V_1 d_1 g + V_2 d_2 g$

or $L\left(\frac{A}{5}\right)\rho = \left(\frac{3}{4}L\right)\left(\frac{A}{5}\right)d + \left(\frac{1}{4}L\right)\left(\frac{A}{5}\right)2d$

i.e., $\rho = \frac{3}{4}d + \frac{2}{4}d = \frac{5}{4}d$

(ii) Total pressure $= p_0 +$ (weight of liquid + weight of solid) A i.e.,

$$P = P_0 + \frac{H}{2}dg + \frac{H}{2}2dg + \frac{5}{4}d \times \left(\frac{A}{5}\times L\right)\times g \times \frac{1}{A}$$

i.e. $P = P_0 + \frac{3}{2}H\,dg + \frac{1}{4}L\,dg = P_0 + \frac{1}{4}(6H+L)dg$

(b) **(i)** By Bernoulli theorem for a point just inside and outside the hole

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2$$

i.e., $P_0 + \frac{H}{2}dg + \left(\frac{H}{2}-h\right)2dg = P_0 + \frac{1}{2}(2d)v^2$

or $g(3H-4h) = 2v^2$ or $v = \sqrt{\frac{g}{2}(3H-4h)}$

(ii) As at the hole vertical velocity of liquid is zero so time taken by it to reach the ground,

$t = \sqrt{(2h/g)}$ So that

$$x = vt\sqrt{\frac{g}{2}(3H-4h)} \times \sqrt{\frac{2h}{g}} = \sqrt{h(3H-4h)}$$

(iii) For x to be maximum x^2 must be maximum,

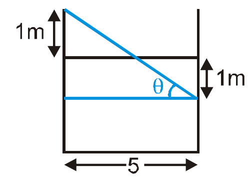
i.e., $\frac{d}{dh}(x^2) = 0$ or $\frac{d}{dh}(3Hh - 4h^2) = 0$

or $3H - 8h = 0$, i.e., $h = (3/8)H$

and $X_{max} = \sqrt{\frac{3H}{8}\left(3H - \frac{3}{2}H\right)} = \frac{3}{4}H$

41. **(i)** $\tan\theta = \frac{a}{g}$

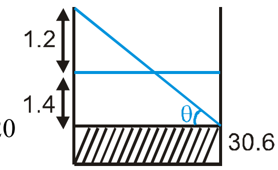
$a = 4\ m/s^2$



(ii) New a = 4.8

$\tan\theta = \frac{4.8}{10} \Rightarrow \frac{P}{5m} \Rightarrow 2.4$

= volume of water left



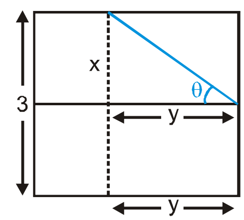
$\Rightarrow \frac{1}{2}\times(2.4)(5)\times 4 + 0.6\times 20$

$\Rightarrow 24 + 12 = 36\ m^3$

$V_f = 36\ m^3$; $V_i = 5\times 4\times 2 \Rightarrow 40\ m^3$

$\left(\frac{V_f - V_i}{V_i}\right)\times 100 = 10\%$

(iii) $\tan\theta = \frac{9}{10} = \frac{x}{4} = \frac{9}{10}$



$60 - \frac{20x^2}{9} = 40$

forces on front wall is 0

$\Rightarrow \int_0^3 \rho\left[\frac{5}{3} + x\tan\theta\right]9(4dx)$

$\Rightarrow 36\times 10^3\left[\left[\frac{5}{3}x\right] + \frac{9x^2}{20}\right] \Rightarrow 36\times 10^3 + \left[\frac{15}{3}+\frac{81}{2}\right]$

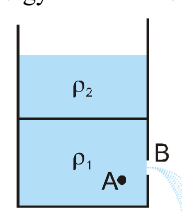
$\Rightarrow \left(\rho\times 9\times\frac{5}{3} + \rho g\frac{h}{2}\right)12 \Rightarrow (\rho\,15 + 15\rho) = 360\rho$

42. Pressure at A, $P_A = P_0 + h\rho_2 g + (h-y)\rho_1 g$

Pressure at B, $P_B = P_0$

According to Bernoulli's theorem,

pressure energy at A = pressure energy at B + kinetic energy at B



$\therefore P_A = P_B + \frac{1}{2}\rho_1 v^2$

$\therefore v = 4ms^{-1}$

$\therefore F = (Av\rho)(v-0) = A\rho v^2$

or F = 7.2N

Total mass of the liquid in the cylinder is

$m = Ah\rho_1 + Ah\rho_2 = 450$ kg

Limiting friction $= \mu mg = 45N$

$\therefore$ F < Limiting friction, therefore, minimum force required is zero.

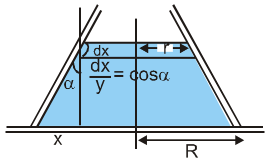
Consider free body diagram for maximum value of force.

Considering vertical forces, N = mg

Now considering horizontal forces,

$F_{max} = F + \mu N$ or $F_{max} = 52.2$ N

43. $W + N_v = \rho g h\, \pi R^2$



$$W = \int \rho g x (2\pi r)\frac{dx}{\cos\alpha}\sin\alpha$$

$$\int_0^h \rho g x 2h - (9 + x\tan\alpha)\frac{dy}{\cos\alpha}$$

$$\Rightarrow \rho g 2\pi \int_0^h ax\frac{dx}{\cos\alpha} + \int_0^h x^2\tan\alpha dx$$

$$\Rightarrow \rho g 2\pi\left[\frac{ah^2}{2\cos\alpha} + \frac{h^3}{3}\tan\alpha\right]$$

$$\Rightarrow \rho g 2\pi h^2\left[\frac{9R - h\sin\alpha}{2\cos\alpha} + \frac{h}{3}\tan\alpha\right]$$

$$\Rightarrow W = \rho g 2\pi h^2\left[\frac{R}{2\cos\alpha} - \frac{h}{6}\tan\alpha\right]$$

$$= \rho g \pi h^2\left[\frac{R}{\cos\alpha} - \frac{h}{3}\tan\alpha\right]$$

$$\Rightarrow \rho = \frac{W}{g\pi h^2\left[\frac{R}{\cos\alpha} - \frac{h}{3}\tan\alpha\right]}$$

44. Initially : $mg = f_B \Rightarrow mg = Vd_Lg = Ahd_Lg$

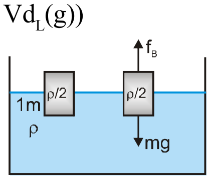
when pulled slightly up by x then

$f_{net} = mg - f_B = mg - A(h-x)d_Lg$

$= mg - Ahd_Lg + Axd_Lg \Rightarrow f_{net} = Axd_Lg$

force directly proportional to x therefore if will perform S.H.M.

(ii) $ma = (mg - Vd_L(g))$



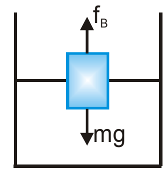
$$a = \left(g - \frac{Ad_Lxg}{A(H)d_m}\right) \Rightarrow a = g - \frac{2gx}{2}$$

$$\frac{d^2x}{dt^2} + gx + g \text{ can be compared with}$$

$$\frac{d^2x}{dt^2} + \omega^2 x + g = 0 \Rightarrow w = \sqrt{g}$$

$$T = \frac{2\pi}{\omega} \Rightarrow \text{and time required is} = T/2,\ t = 1 \text{ sec}$$

45. (i) $AH\, d_mg = Ahd_Lg$



$h = H\frac{d_m}{d_L}$; $f_{net} = mg - f_B$

$f_{net} = AHd_mg - Axgd_L$

If will perform SHM about its position

$$x = \frac{d_M}{d_L}H, \text{ with } \omega = \frac{d_Lg}{d_M}$$

$$f_{net} = (Ag)\, d_L\left[\frac{Hd_m}{d_L} - x\right]dx$$

$$d\omega = \int_0^{0.8H} f_{net}dx = Agd_L\left[\frac{Hd_m}{d_L}x - \frac{x^2}{2}\right]_0^{0.8H}$$

$$= Ag\left[H(0.8)x - \frac{x^2}{2}\right]_0^{0.8}$$

$$= Ag\left[H(0.8)(0.8)H - \frac{(0.8)^2H^2}{2}\right]_0^{0.8} = \frac{AgH^2dm^2}{2}$$

$A = 4000 \times 10^{-4}$; $g = 10, H = 50 \times 10^{-2}$;

$d_m = 8 \times 10^{+2}$

$$\omega = \frac{4000 \times 10^{-4} \times 10 \times 2.500 \times 10^{-4} \times .64}{2}$$

$\omega = .32 \times 10^4 \Rightarrow 32$ kgC

(ii) Particle starts oscillating in the fluid

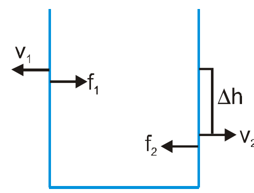
$\therefore$ Work done by person

$$= \text{Total energy of oscillation work } = \frac{1}{2}M\omega^2A^2$$

$$\Rightarrow \frac{1}{2}(AH)d_m\frac{d_L}{d_m}g\ H\left(1 - \frac{d_m}{L}\right)^2$$

$$\text{work} = \frac{1}{2}AH^2\left(1 - \frac{d_m}{d_L}\right)^2 g = 2\text{kg f-m}$$

46. $f_{net} = f_2 - f_1$



$$f_2 = \frac{dp}{dt} = v_2\frac{dm}{dt} = \rho s v_2^2$$

$$f_1 = \frac{dp}{dt} = v_1\frac{dm}{dt} = \rho s v_1^2$$

$f_{net} = \rho s(v_2^2 - v_1^2) = \rho s 2g(h_2 - h_1)$

$f_{net} \Rightarrow 0.51$ Newton

47. $mg\frac{b}{2}\sin\theta = f_B\left(b - \frac{x}{2}\right)\sin\theta$

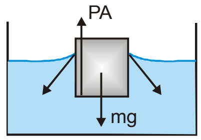
$$(Ab)\, d_mg\frac{b}{2} = (Ax)d_Lg\left(\frac{bx}{2}\right);\ \frac{5}{9}\frac{b^2}{2} = bx - \frac{x^2}{2}$$

By solving It $x = \dfrac{b}{3}$

48. $\vec{f}_g + \vec{f}_{S,T} = \rho_\omega ghA$

$mg + S(4a) = \rho_\omega gha^2$

$$h = \frac{mg + 4aS}{\rho_\omega ga^2}$$



49. We consider a ring element of radius r and thickness dr whose centre is at the centre of disc. The velocity of fluid at distance r from axis is $v = r\omega$

$$\therefore \quad \frac{dv}{dx} = \omega\frac{dr}{dx}$$

Where dx is the thickness of layer of liquid.

The area of the considered element is $dA = (2\pi\, rdr)$

$\therefore$ the viscous force on the considered element is

$$dF = \eta(2\pi\, rdr)\frac{dv}{dx}$$

Here, velocity gradient is

$$\frac{dv}{dx} = \frac{v}{h} = \frac{r\omega}{h}$$

$$\therefore \quad dF = \eta(2\pi\, r\, dr)\frac{r\omega}{h} = \frac{2\pi\eta\omega}{h}r^2dr$$

The power developed on the considered element by viscous force is

$$dP = v\, dF = (r\,\omega)\frac{2\pi\eta\omega}{h}r^2dr = \frac{2\pi\eta\omega^2}{h}r^3dr$$

$\therefore$ Total power developed due to viscous force is

$$P = 2\int_{r=0}^{r=R} dP \quad \text{(on both sides)}$$

$$= 2\int_0^R \frac{2\pi\eta\omega^2}{h}r^3dr = \frac{\pi\eta\omega^2R^4}{h}$$

$$= \frac{3.14 \times 0.08 \times 10^{-1} \times (60)^2 \times (10^{-1})^4}{1 \times 10^{-3}} = 9\text{W}$$

50. From diagram $r\cos\theta = \frac{d}{2} \Rightarrow r = \frac{d}{2\cos\theta}$

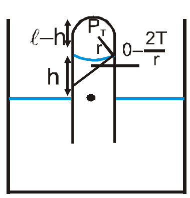
$P_0\ell A = P_T(A)(\ell - h)$

$$P_T = \frac{P_0\ell}{\ell - h};\ P_A = \left(\frac{P_0\ell}{\ell - h} - \frac{2T}{r}\right)$$

$$P_B = \frac{P_0\ell}{\ell - h} - \frac{2T}{r} + \rho gh = P_0$$

$$= \left(\frac{P_0h}{\ell - h} + \rho gh\right) = \frac{2T}{d}(2\cos\theta)$$

$$T = \left(\frac{P_0h}{\ell - h} + \rho gh\right)\frac{d}{4\cos\theta}$$



EXERCISE - 5

Part # I : AIEEE/JEE-MAIN

1. Elastic energy $= \frac{1}{2} \times F \times x$

$F = 200\text{ N}, x = 1\text{mm} = 10^{-3}\text{ m}$

$\therefore \quad E = \frac{1}{2} \times 200 \times 1 \times 10^{-3} = 0.1\text{ J}$

2. Work done $\frac{1}{2}kx^2 = \frac{1}{2}k\ell^2$ where ℓ is the total extensions. $= \frac{1}{2}(k\ell)\ell = \frac{1}{2}F\ell$

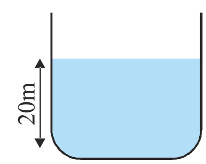
3. Energy density

$$= \frac{\text{Energy}}{\text{Volume}} = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

$$= \frac{1}{2}\text{Stress} \times \frac{\text{Stress}}{Y} = \frac{1}{2}\frac{S^2}{Y}$$

Energy density $= \frac{1}{2}\frac{S^2}{Y}$

5. Velocity of efflux through a small hole $= \sqrt{2gh}$ where h is the position of the small hole from the top of the vessel.



$$v_{\text{efflux}} = \sqrt{2 \times 10 \times 20} = 20\text{ m/s}$$

6. The viscous force experienced by the spherical ball is expressed as

$F = 6\pi\eta rv \Rightarrow f \propto r \Rightarrow F \propto v$

7. Excess pressure inside a soap bubble is $P = \frac{4T}{r}$

Air will flow from the bubble at high pressure to the bubble at lower pressure as $P \propto \frac{1}{r}$, hence bubble of smaller radius will be at higher pressure, hence air will flow from smaller to the bigger sphere.

8. Water will rise to the full length of capillary tube

9. $\frac{v_s}{v_g} = \frac{(\rho_s - \rho_\ell)}{(\rho_g - \rho_\ell)} \Rightarrow \text{vs} = 0.1\text{ m/s}$

10. $\rho_1Vg - \rho_2Vg = kv_T^2 \Rightarrow v_T = \sqrt{\frac{Vg(\rho_1 - \rho_2)}{k}}$

11. As liquid 1 floats above liquid 2, $\rho_1<\rho_2$
The ball is unable to sink into liquid 2, $\rho_3<\rho_2$
The ball is unable to rise over liquid 1, $\rho_1<\rho_3$
Thus $\rho_1<\rho_3<\rho_2$

12. Capillary rise $\dfrac{2T\cos\theta}{\rho g r}$.
As soap solution has lower T, h will be low

14. $\therefore Y=\dfrac{F/A}{\Delta\ell/\ell}$ $\quad\therefore F=\dfrac{YA^2\Delta\ell}{\ell A}$

$F=\dfrac{YA^2\Delta\ell}{v}$ here v = volume of wire

$F\propto A^2 \Rightarrow \dfrac{F_2}{F_1}=\left(\dfrac{A_2}{A_1}\right)^2=\left(\dfrac{3A}{A}\right)^2=9\Rightarrow F_2=9F$

15. In equilibrium ball will remain at the interface of water and oil.

16. According to equation of continuity

$A_1V_1=A_2V_2$ or $r_2=\sqrt{\dfrac{r_1^2v_1}{v_2}}$

Velocity of stream at 0.2 m below tap.
$V_2^2=V_1^2+2as=0.16+2\times10\times0.2=4.16$ m/s

$r_2=\sqrt{\dfrac{r_1^2v_1}{v_2}}=\sqrt{\dfrac{16\times10^{-6}\times0.4}{2}}=\sqrt{3.2}\times10^{-3}$ m

So diameter $=2\times\sqrt{3.2}\times10^{-3}$ m
$=2\times1.8\times10^{-3}=3.6\times10^{-3}$ m

17. $W=8\pi T[(r_2^2)-(r_1^2)]$
$=8\times\pi\times0.03\,[25-9]\times10^{-4}=\pi\times0.24\times16\times10^{-4}$
$=3.8\times10^{-4}\pi \quad =0.384\,\pi\,\text{mJ}\approx0.4\,\pi\,\text{mJ}$

18. By volume conservation

$\dfrac{4}{3}\pi R^3=2\left(\dfrac{4}{3}\pi r^3\right)\Rightarrow R=2^{1/3}r$

Surface energy E = T (A)
$=T(4\pi R^2)=T(4\pi\,2^{2/3}r^2)=2^{8/3}\pi r^2T$

19. Terminal velocity $V\propto\dfrac{d_b-d_\ell}{\eta}$

$\dfrac{V_1}{V_2}=\dfrac{7.8-1}{8.5\times10^{-4}}\times\dfrac{13.2}{7.8-1.2}$

$\dfrac{10}{V_2}=1.6\times10^4$

$V_2=\dfrac{10}{1.6\times10^4}=6.25\times10^{-4}$

20. weight = mg $=1.5\times10^{-2}$ N (given)
length $=\ell=30$ cm (given)
$=0.3$ m

$2T\ell=mg$

$T=\dfrac{mg}{2\ell}=\dfrac{1.5\times10^{-2}}{2\times0.3}=0.025$ N/m.

21. 1 **22.** 1 **23.** 1

24. $T=2\pi\sqrt{\dfrac{\ell}{g}}$; $T_M=2\pi\sqrt{\dfrac{\ell'}{g}}$

$\gamma=\dfrac{Mg/A}{\Delta\ell/\ell}\Rightarrow\dfrac{\ell'-\ell}{\ell}=\dfrac{Mg}{\gamma A}=\dfrac{\ell'}{\ell}=1+\dfrac{Mg}{\gamma A}$

Also:

$\dfrac{T_M}{T}=\sqrt{\dfrac{\ell'}{\ell}}\therefore T_M=T\left[1+\dfrac{Mg}{\gamma A}\right]^{1/2}$

$\Rightarrow\dfrac{T_M^2}{T^2}=1+\dfrac{Mg}{\gamma A}\Rightarrow\left[\dfrac{T_M^2}{T^2}-1\right]=\dfrac{Mg}{\gamma A}$

$\Rightarrow\dfrac{1}{\gamma}=\dfrac{A}{Mg}\left[\left(\dfrac{T_M}{T}\right)^2-1\right]$

Part # II : IIT-JEE ADVANCED

1. From equation of continuity $v_1A_1=v_2A_2$
and

$v_2^2-u^2=2gs$; $v_2^2-1=2\times10\times0.15\Rightarrow v_2=2$ m/s

Hence $A_2=\dfrac{v_1A_1}{v_2}=\dfrac{1\times10^{-4}}{2}=5\times10^{-5}\text{m}^2$

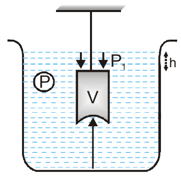
3. If we apply Newton's law to find the force exerted by the molecules on the walls of the container, we will have to apply a pseudo force (the frame of molecules is an accelerated frame). This pseudo force acting on gas molecules will act in opposite to the direction of motion of closed compartment. The result will be more pressure on the rear side and less pressure on the front side.

4. Equating the rate of flow

$\sqrt{(2gy)}\times L^2=\sqrt{(2g\times4y)}\pi R^2$

$\Rightarrow L^2=2\pi R^2\Rightarrow R=\dfrac{L}{\sqrt{2\pi}}$

5. According to Archimedes principle



Upthrust = Wt. of fluid displaced.

$\therefore F_{bottom} = F_{top} + V\ \rho\, g$

$= P_1 \times A + V \rho\, g$

$= (h\, \rho\, g) \times (\pi R^2) + V \rho\, g$

$= \rho\, g\, [\pi R^2 h + V]$

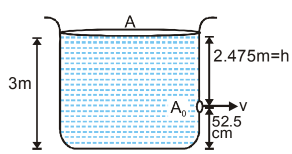
6. ℓ decreases as the block moves up. h will also decreases because when the coin is in the water it will displace equal volume of water, whereas when it is on the block an equal weight of water is displaced.

7. $Y = \dfrac{F}{A} \Big/ \dfrac{\Delta \ell}{\ell} = \dfrac{20 \times 1}{10^{-6} \times 10^{-4}} = 2 \times 10^{11}\, N/m^2$.

8. $K = \dfrac{\Delta P}{\left(-\dfrac{\Delta v}{v}\right)} = \dfrac{(1.165 - 1.01) \times 10^5}{10^{-3}} = 1.55 \times 10^5\, Pa$

9. The square of the velocity of flux

$$v^2 = \frac{2gh}{\sqrt{1 - \left(\dfrac{A_0}{A}\right)^2}} = \frac{2 \times 10 \times 2.475}{\sqrt{1 - (0.1)^2}} = 50\ m^2/s^2$$



11. $P_2 - P_{atm} = \dfrac{4T}{R_2}$, $P_1 - P_{atm} = \dfrac{4T}{R_1}$

Here $R_2 > R_1$. So $P_2 < P_1$

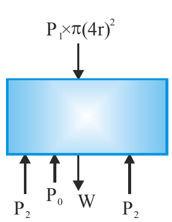
$\Rightarrow$ Air will flow from end 1 to end 2.

Comprehension

1. **(a)** Consider the equilibrium of wooden block.

Forces acting in the downward direction are

Weight of wooden cylinder



$= \pi(4r)^2 \times h \times \dfrac{\rho}{3} \times g = \pi \times 16r^2 \dfrac{h\rho}{3} g$

(b) Force due to pressure (P_1) created by liquid of height h_1 above the wooden block is

$= P_1 \times \pi(4r^2) = [P_0 + h_1 \rho g] \times \pi(4r)^2$

$= [P_0 + h_1 \rho g] \pi \times 16r^2$

Force acting on the upward direction due to pressure P_2 exerted from below the wooden block and atmospheric pressure is

$= P_2 \times \pi[(4r)^2 - (2r)^2] + P_0 \times (2r)^2$

$= [P_0 + (h_1 + h)\rho g] \times \pi \times 12r^2 + 4r^2 P_0$

At the verge of rising

$[P_0 + (h_1 + h)\rho g]\, \pi \times 12r^2 + 4r^2 P_0$

$= \pi \times 10r^2 h \times \dfrac{\rho}{3} g + [P_0 + h_1 \rho g] \times \pi \times 16r^2$

$\Rightarrow 12h_1 + 12h = \dfrac{16h}{3} + 16h_1 \Rightarrow \dfrac{5h}{3} = h_1$

2. **(b)** Again considering equilibrium of wooden block.

Total Downward force = Total force upwards

Wt. of block + force due to atmospheric pressure = Force due to pressure of liquid + Force due to atmospheric pressure

$\pi(16r^2) \dfrac{\rho}{3} + g + P_0 \pi \times 16r^2$

$= [h_2 \rho g + P_0]\, \pi\, [16 - 4r^2] + P_0 \times 4r^2$

$\pi(16r^2) h \dfrac{\rho}{g} g = h_2 \rho g \times \pi \times 12r^2$

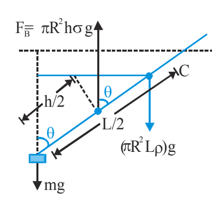
$\Rightarrow 16 \dfrac{h}{3} = 12h_2 \Rightarrow \dfrac{4}{9} h = h_2$

3. **(a)** When the height h_2 of water level is further decreased, then the upward force acting on the wooden block decreases. The total force downward remains the same. This difference will be compensated by the normal reaction by the tank wall on the wooden block. Thus the block does not moves up and remains at its original position.

Subjective

1. For the wooden stick-mass system to be in stable equilibrium the center of gravity of stick-mass system should be lower than the center of buoyancy. Also in equilibrium the centre of gravity (G) and the center of buoyancy (B) lie in the same vertical axis.

The above condition 1 will be satisfied if the mass is towards the lower side of the stick as shown in the figure. The two forces will create a torque which will bring the stick-



mass system in the vertical position of the stable equilibrium. Let ℓ be the length of the stick immersed in the liquid.

$OB = \frac{\ell}{2}$. Let $OG = y$

For vertical equilibrium $F_G = F_B \Rightarrow (M + m)\,g = F_B$

$\Rightarrow \pi R^2 L\rho g + mg = \pi R^2 \ell \sigma g$

$$\ell = \frac{\pi R^2 L\rho + m}{\pi R^2 \sigma} \qquad ...(i)$$

Now using the concept of centre of mass to find y.

Then $y = \frac{My_1 + my_2}{M + m}$

Since mass m is at O the origin $\therefore y_2 = 0$

$$\therefore\ y = \frac{M(L/2) + m \times O}{M + m} = \frac{ML}{2(M+m)}$$

$$= \frac{(\pi R^2 L\rho)L}{2(\pi R^2 L\rho + m)} \qquad ...(ii)$$

Therefore for stable equilibrium $\frac{\ell}{2} > y$

$$\therefore\ \frac{\pi R^2 L\rho + m}{2\pi R^2 \sigma} > \frac{(\pi R^2 L\rho)L}{2(\pi R^2 L\rho + m)}$$

$\Rightarrow m \geq \pi R^2 L(\sqrt{\rho\sigma} - \rho)$

$\therefore$ Minimum value of m is $\pi R^2 L(\sqrt{\rho\sigma} - \rho)$

2. **(i)** As the pressure exerted by liquid A on the cylinder is radial and symmetric. The force due to this pressure cancels out and the net value is zero.

(ii) For equilibrium

Buoyant force= weight of the body

$\Rightarrow h_A\rho_A Ag + h_B\rho_B Ag = (h_A + h + h_B)A\,\rho_C g$

(where ρ_c = density of cylinder)

$$h = \left(\frac{h_A\rho_A + h_B\rho_B}{\rho_c}\right) - (h_A + h_B) = 0.25\text{ cm}$$

(iii) $a = \frac{F_{Buoyant} - Mg}{M}$

$$= \left[\frac{h_A\rho_A + \rho_B(h + h_B) - (h + h_A + h_B)\rho_C}{\rho_C(h + h_A + h_C)}\right]g = \frac{g}{6}\text{ upwards}$$

3. When the force due to excess pressure in the bubble equals the force of air striking at the bubble, the bubble will detach from the ring

$$\therefore\ \rho A v^2 = \frac{4T}{r} \times A \ \Rightarrow\ r = \frac{4T}{\rho v^2}$$

4. When the tube is not there, using Bernoulli's theorem

$$P + P_0 + \frac{1}{2}\rho v_1^2 + \rho g H = \frac{1}{2}\rho v_0^2 + P_0$$

$$\Rightarrow\ P + \rho g H = \frac{1}{2}\rho\left(v_0^2 - v_1^2\right)$$

But according to equation of continuity

$$A_1 v_1 = A_2 v_2 \text{ or } v_1 = \frac{A_2 v_0}{A_1}$$

$$\therefore\ P + \rho g H = \frac{1}{2}\rho\left[v_0^2 - \left(\frac{A_2}{A_1}v_0\right)^2\right]$$

$$P + \rho g H = \frac{1}{2}\rho v_0^2\left[1 - \left(\frac{A_2}{A_1}\right)^2\right]$$

Here $P + \rho g H = \Delta P$

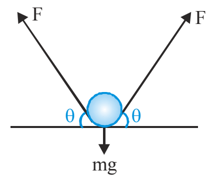
According to Poisseuille's equation

$$Q = \frac{\pi(\Delta P)a^4}{8\eta\ell} \qquad \therefore\ \eta = \frac{\pi(\Delta P)a^4}{8Q\ell}$$

$$\therefore\ \eta = \frac{\pi(P + \rho g H)a^4}{8Q\ell} = \frac{\pi}{8Q\ell} \times \frac{1}{2}\rho v_0^2\left[1 - \left(\frac{A_2}{A_1}\right)^2\right] \times a^4$$

where $\frac{A_2}{A_1} = \frac{b^2}{D^2}$

5. The free body diagram of wire is given below.



If ℓ is the length of wire, then for equilibrium

$2F\sin\theta = W$.

$F = S \times \ell \Rightarrow 2S \times \ell \times \sin\theta = \lambda \times \ell \times g$

or $S = \frac{\lambda g}{2\sin\theta}$ also $\sin\theta = y/a$

$$\therefore\ S = \frac{\lambda g}{2y/a} = \frac{a\lambda g}{2y} \Rightarrow \text{Surface tension } S = \frac{a+g}{2y}$$

6. From law of continuous $A_1v_1 = A_2v_2$

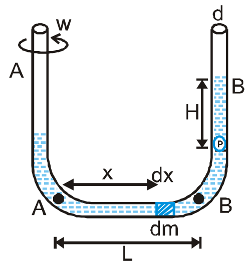
$$\Rightarrow\ v_2 = \frac{\pi \times (4\times10^{-3})^2 \times 0.25}{\pi \times (1\times10^{-3})^2} = 4\text{m/s}$$

and $x = v \times t = v \times \sqrt{\frac{2h}{g}} = 2\text{m}$

7. Weight of liquid of height H

$$= \frac{\pi d^2}{4} \times H \times \rho \times g \qquad \text{....(i)}$$

Let us consider a mass dm situated at a distance x from A as shown in the figure.



The centripetal force required for the mass to rotate $= (dm)\, x\omega^2$

$\therefore$ The total centripetal force required for the mass of length

L to rotate $= \int_0^L (dm) x\omega^2$

Here, $dm = \rho \times \frac{\pi d^2}{4} \times dx$

$\therefore$ Total centripetal force

$$= \int_0^L \left(\rho \times \frac{\pi d^2}{4} \times dx\right) \times x\omega^2$$

$$= \rho \times \frac{\pi d^2}{4} \times \omega^2 \int_0^L x dx = \rho \times \frac{\pi d^2}{4} \times \omega^2 \times \frac{L^2}{2} \qquad \text{...(ii)}$$

This centripetal force is provided by the weight of liquid of height H.

From (i) and (ii)

$$\frac{\pi d^2}{4} \times H \times \rho \times g = \rho \times \frac{\pi d^2}{4} \times \frac{\omega^2 \times L^2}{L} \; ; \; H = \frac{\omega^2 L^2}{2g}$$

Integer Type

1. $(P_{in})_A = \frac{4S}{r_A} + P_0 = \frac{4 \times .04}{0.02} + 8 = 16\ N/m^2$

$(P_{in})_B = \frac{4S}{r_B} + P_0 = \frac{4 \times .04}{.04} + 8 = 12\ N/m^2$

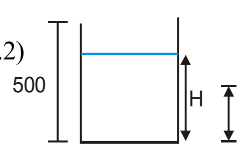
$$n_A = \frac{(P_{in})_A V_A}{RT} \; ; \; \frac{n_B}{n_A} = \frac{(P_{in})_B}{(P_{in})_A} \times \left(\frac{r_B}{r_A}\right)^3 = 6$$

2. $(500 - H)\, P_0 = 300\, (P_0 - rg \times 0.2)$

$(0.5 - H) \times 10^5 = 0.3\, [10^5 - 10^4 \times 0.2)$

$0.5 - H = 0.294$

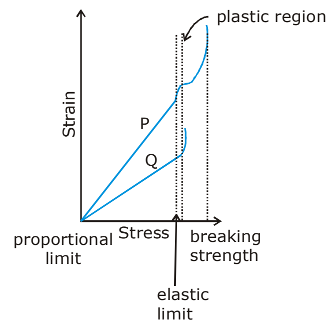
$H = 206$ mm



Multiple Choice Questions

1. The maximum stress is called the breaking strength (stress) or tensile strength.

The materials of the wire which break as soon as stress is increased beyond elastic limit are called brittle. While the materials of the wire, which have a good plastic range are called ductile.



2. $(dm)\, g = dP.\, dA$

$$(\rho.dA)\, dr.\, \frac{\rho r(4\pi G)}{3} = dp.\, dA$$

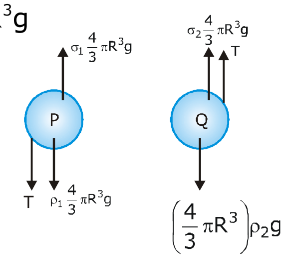
$$\int_p^0 dP = \frac{4\pi G\rho^2}{3} \int_r^R r dr$$

$$p = \frac{4\pi G\rho^2}{3} \cdot \left[\frac{\rho^2}{2}\right]_r^R \Rightarrow p \propto (R^2 - r^2)$$

3 $\sigma_1 \frac{4}{3}\pi R^3 g + T = \rho_1 \frac{4}{3}\pi R^3 g$

$\Rightarrow \left(\sigma_1 - \rho_1\right)\frac{4}{3}\pi R^3 g = T$

$\sigma_2 \frac{4}{3}\pi R^3 g + T = \rho_2 \frac{4}{3}\pi R^3 g$

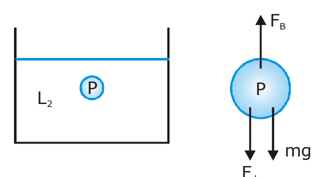


$\Rightarrow \left(\sigma_2 - \rho_2\right)\frac{4}{3}\pi R^3 g = T$

$\sigma_1 - \rho_1 = \rho_2 - \sigma_2$

$\sigma_2 - \rho_1 = \rho_2 - \sigma_1$

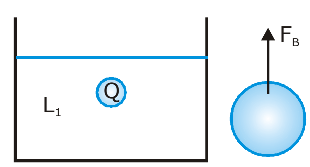
$\sigma_1 - \rho_2 = \rho_1 - \sigma_2$



$\sigma_2 \frac{4}{3}\pi R^3 g = \rho_1 \frac{4}{3}\pi R^3 g + \sigma\pi\eta_2 R v_P$

$$\frac{V_P}{V_Q} = \frac{\sigma_2 - \rho_1}{\sigma_1 - \rho_2}$$

$$V_T = \frac{2}{9} r^2 \frac{(\rho - \sigma) g}{\eta}$$



σ = density of fluid

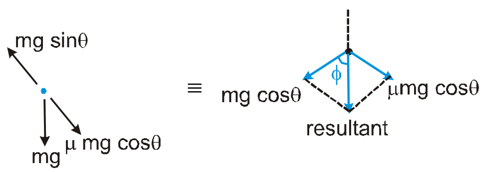
ρ = density of object

$$\frac{V_P}{V_Q} = \frac{(\rho_1 - \rho_2)}{\eta_2}\frac{\eta_1}{(\rho_2 - \sigma_1)} = \frac{\eta_1}{\eta_2}$$

$\because \quad \sigma_2 - \rho_1 = \rho_2 - \sigma_1$

MOCK TEST

1. figure shows forces acting on a 'particle' on the surface, with respect to vessel.



(mg sin θ & μ mg cos θ are pseudo forces).

$\tan\phi = \mu \therefore \quad \phi = \tan^{-1}\mu$.

φ is angle between normal to the inclined surface and the resultant force. The same angle will be formed between the surface of water & the inclined surface.

{∵ free surface is ⊥ to the resultant force acting on it.}

2. Velocity of efflux of water $(v) = \sqrt{2g\left(\frac{h}{2}\right)} = \sqrt{gh}$

force on ejected water = Rate of change of momentum of ejected water.

$= \rho\,(av)\,(v) \quad = \rho\, av^2$

Torque of these forces about central line

$= (\rho a v^2)\, 2R \, . \, 2 \qquad = 4\rho a v^2 R = 4\rho\, agh\, R$

3. Let ρ_S, ρ_L be the density of silver and liquid. Also m and V be the mass and volume of silver block.

∴ Tension in string = mg – buoyant force

$T = \rho_S Vg - \rho_L Vg = (\rho_S - \rho_L)\, Vg$

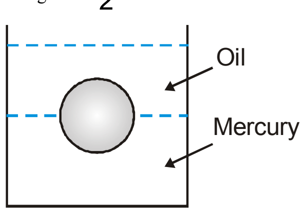
Also $V = \frac{m}{\rho_S}$

$$\therefore \quad T = \left(\frac{\rho_S - \rho_L}{\rho_S}\right) mg = \frac{(10-0.72)\times 10^3}{10\times 10^3} \times 4 \times 10$$

$= 37.12$ N.

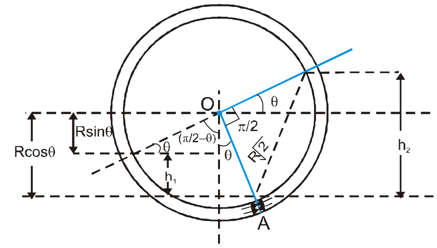
4. Weight = Buoyant force

$$V\rho_m g = \frac{V}{2}\rho_{Hg} g + \frac{V}{2}\rho_{oil} g$$



$$\rho_m = \frac{\rho_{Hg} + \rho_{oil}}{2} = \frac{13.6 + 0.8}{2} = \frac{14.4}{2} = 7.2$$

5 Pressure at 'A' from both side must balance. Figure is self–explanatory.

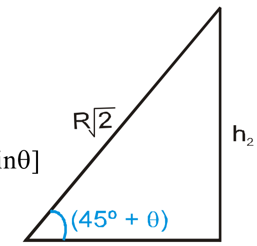


$\sigma\, h_2 g = \rho h_1 g$

$\sigma \sqrt{2}R \sin(45^\circ + \theta) =$

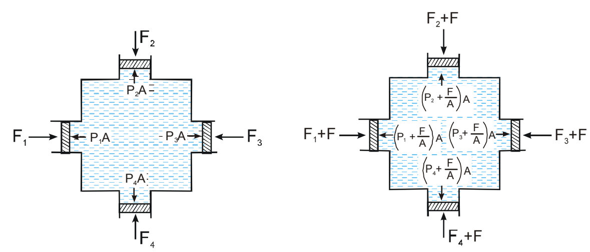
$\rho R\,[\cos\theta - \sin\theta]$

$\sigma\,[\cos\theta + \sin\theta] = \rho[\cos\theta - \sin\theta]$



$$\tan\theta = \frac{\rho - \sigma}{\rho + \sigma}$$

6. The four piston are initially in equilibrium. As additional force F is applied to each piston, the pressure in fluid at each point must be increased by $\frac{F}{A}$ so that each piston retains state of equilibrium.



Thus the increment in pressure at each point is

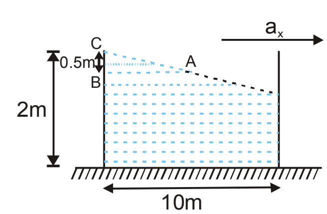
$\Delta P = \frac{F}{A}$ (by Pascal's law)

7. Increasing the temperature of water from 2^0C to 3^0C increases its density while decreases the density of iron.

Hence the bouyant force increases.

8. $v = u + a_x t$, $a_x = \frac{v}{t}$

$$\tan\theta = \frac{a_x}{g} = \frac{v}{tg} = \frac{0.5}{5}$$



(in triangle ABC)

$$\Rightarrow \quad t = \frac{10 \times 20}{10} = 20 \text{ sec.}$$

9. For the given situation, liquid of density 2ρ should be behind that of ρ.

From right limb :

$P_A = P_{atm} + \rho gh$

$P_B = P_A + \rho a \frac{\ell}{2} = P_{atm} + \rho gh + \rho a \frac{\ell}{2}$

$P_C = P_B + (2\rho) a \frac{\ell}{2} = P_{atm} + \rho gh + \frac{3}{2}\rho a \ell$ .... (1)

But from left limb :

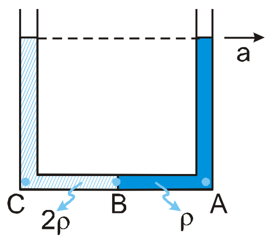
$P_C = P_{atm} + (2\rho) gh$ .... (2)

From (1) and (2) :

$P_{atm} + \rho gh + \frac{3}{2}\rho a \ell$

$= P_{atm} + 2\rho gh$

$\Rightarrow$ $h = \frac{3a}{2g}\ell$ **Ans.**



10. No sliding $\Rightarrow$ pure rolling

Therefore, acceleration of the tube = 2a (since COM of cylinders are moving at 'a')

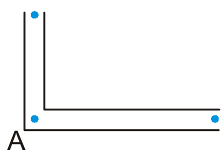
$P_A = P_{atm} + \rho(2a) L$

(From horizontal limb)

Also ; $P_A = P_{atm} + \rho g H$

(From vertical limb)

$\Rightarrow$ $a = \frac{gH}{2L}$ **Ans.**



11. Pressure at (1) :

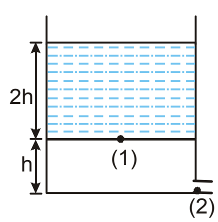
$P_1 = P_{atm} + \rho g (2h)$

Applying Bernoulli's theorum between points (1) and (2)

$[P_{atm} + 2\rho gh] + 2\rho g(h) + \frac{1}{2}(2\rho)(0)^2$

$= P_{atm} + (2\rho) g(0) + \frac{1}{2}(2\rho) v^2$

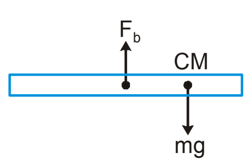
$\Rightarrow v = 2\sqrt{gh}$ **Ans.**



12. Torque about CM :

$F_b \cdot \frac{\ell}{4} = I\alpha$

$\Rightarrow$ $\alpha = \frac{1}{I}(\pi r^2)(\ell)(\rho)(g). \frac{\ell}{4}$



$\alpha = \frac{\pi r^2 \ell^2 g\rho}{4I}$ 'α' will be same for all points.

Hence (B).

13. by dimensional analysis, (c) is the only correct answer.

14. $V_0 = \sqrt{2gh}$ $V_2 = \sqrt{2g\frac{h}{\sqrt{2}}} = \frac{V_0}{\sqrt[4]{2}}$

15. Pressure at all points in stream will be atmospheric.

16. Volume of water filled in tank in

t = 15 sec.

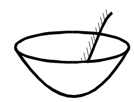
$V = \int_0^{15} A \times 10[1 - \sin\frac{\pi}{30}t]dt$

$V = 10A[t + \left[\frac{\cos\pi/30t}{\pi/30}\right]_0^{15}$

$V = 10[15 - \frac{30}{\pi}]$ A

$h = \frac{V}{10A} = \left[15 - \frac{30}{\pi}\right]$ m

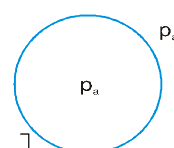
17. Figure shows one of the legs of the mosquito landing upon the water surface.



Therefore, T. $2\pi a \times 8$ = W = weight of the mosquito.

18. Inside pressure must be $\frac{4T}{r}$ greater than outside pressure in bubble. This excess pressure is provided by charge on bubble.

$\frac{4T}{r} = \frac{\sigma^2}{2\varepsilon_0}$



$\frac{4T}{r} = \frac{Q^2}{16\pi^2 r^4 \times 2\varepsilon_0}$ .... $\left[\sigma = \frac{Q}{4\pi r^2}\right]$

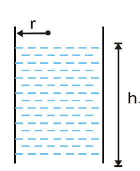
$Q = 8\pi r\sqrt{2rT\varepsilon_0}$

19. The force exerted by film on wire or thread depends only on the nature of material of the film and not on its surface area. Hence the radius of circle formed by elastic thread does not change.

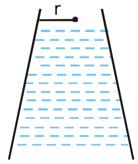
20. As weight of liquid in capillary is balanced by surface tension,

then $T \times 2\pi r = \pi r^2 h_1 \rho g$ (for uniform r radius tube)

$h_1 = \frac{2T}{r\rho g}$

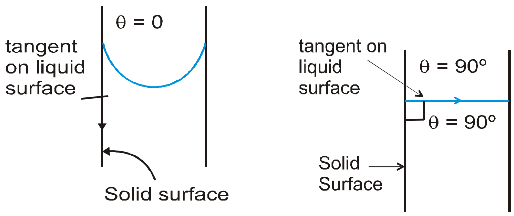


but weight of liquid in tapered tube is more than uniform tube of radius r, then in order to balance $h < h_1$



$$h < \frac{2T}{r\rho g}$$

21. For hemispherical shape – For flat surface -

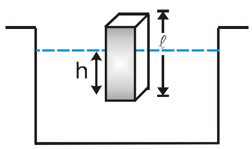


22. Balancing the force :

$T.4a \cos 120^o + \ell \rho a^2 g = a^2 h \rho g$

$T.2a = a^2 \rho g (\ell - h)$

$$(\ell - h) = \frac{2T}{a\rho g}$$

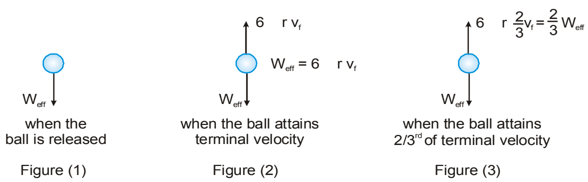


23. Viscous force = mg sin θ

$\therefore\ \eta A \dfrac{v}{t} = mg \sin\theta$ or $\eta a^2 \dfrac{v}{t} = a^3 \rho g \sin\theta$

$$\eta = \frac{t\rho g \sin\theta a}{v}$$

24.



When the ball is just released, the net force on ball is W_{eff} (= mg – buoyant force)

The terminal velocity 'v_f' of the ball is attained when net force on the ball is zero.

$\therefore$ Viscous force $6\pi\eta r\, v_f = W_{eff}$

When the ball acquires $\dfrac{2}{3}$ rd of its maximum velocity v_f

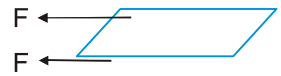
the viscous force is $= \dfrac{2}{3} W_{eff}$.

Hence net force is $W_{eff} - \dfrac{2}{3} W_{eff} = \dfrac{1}{3} W_{eff}$

$\therefore$ required acceleration is $= \dfrac{a}{3}$

25. Velocity gradient $= \dfrac{0.5 \times 2}{2.5 \times 10^{-2}}$

Also, $F = 2\eta A \dfrac{dv}{dz} = 2 \times \eta \times (0.5)\ \dfrac{0.5}{1.25 \times 10^{-2}}$



$\Rightarrow\ \eta = 2.5 \times 10^{-2}$ kg – sec/m^2

26. From continuity equation, velocity at cross-section (1) is more than that at cross-section (2).

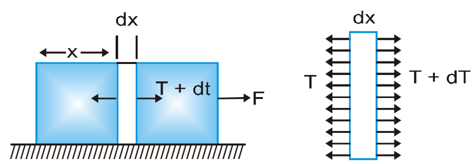
Hence ; $P_1 < P_2$ Hence (A)

27. $\Delta l = \dfrac{F\ell}{AY} \Rightarrow \dfrac{\Delta\ell}{(F/A)} = \dfrac{\ell}{Y}$ = slope of curve

$$\frac{\ell}{Y} = \frac{(4-2)\times 10^{-3}}{4000 \times 10^3}$$

Given $l = 1m \rightarrow Y = \dfrac{4000 \times 10^3}{2 \times 10^{-3}} = 2 \times 10^9$ N/m^2

28.



Acceleration $a = F/m$

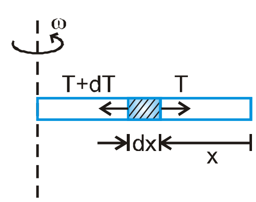
then $T = \dfrac{mx}{\ell} \times \dfrac{F}{m} = \dfrac{Fx}{\ell}$

Extension in 'dx' element $- d\delta = \dfrac{Tdx}{AY} = \dfrac{Fxdx}{\ell AY}$

Total extension $\delta = \displaystyle\int_0^\ell \frac{Fxdx}{\ell AY} = \frac{F\ell}{2AY}$

29. $dT = dm(\ell - x)\omega^2 \quad dT = \dfrac{m}{\ell}.dx(\ell - x)\omega^2$

$$\Rightarrow \int_0^T dT = \int_0^{\ell/2} \frac{m\omega^2}{\ell}(\ell - x)\,dx$$



$$= \frac{m\omega^2}{\ell}\left[\ell x - \frac{x^2}{2}\right]_0^{\ell/2}$$

$$= \frac{m\omega^2}{\ell}\left[\frac{\ell^2}{2} - \frac{\ell^2}{8}\right]$$

$\therefore$ Tension at mid point is :

$$T = \frac{3}{8} m\ell\omega^2 \Rightarrow \text{stress} = \frac{3m\ell\omega^2}{8A}$$

$$\Rightarrow \text{strain} = \frac{3m\ell\omega^2}{8AY}$$

Alternatively

Tension at mid point can be found by using $F_{ext} = ma_{cm}$

$$T = \frac{m}{2} \cdot \left(\omega^2 \frac{3\ell}{4}\right) = \frac{3}{8} m\omega^2 \ell$$

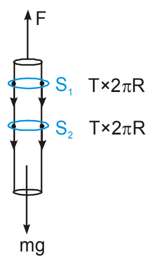
30. $\rho gh \pi r^2 = 2\pi r S \cos\theta$

$$\Rightarrow r = \frac{2S\cos\theta}{\rho g h} = \frac{2\times1\times0.5}{10^3\times10\times10} = 10^{-6}\ \text{m}$$

31. The free body diagram of the capillary tube is as shown in the figure. Net force F required to hold tube is

F = force due to surface tension at cross-section $(S_1 + S_2)$ + weight of tube.

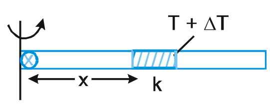
$= (2\pi RT + 2\pi RT) + mg = 4\pi RT + mg$



Free body daigram of capillary tube

32. $-\int_T^0 \Delta T = \int_0^\ell \frac{m}{\ell} dx\omega^2 x \Rightarrow T = \frac{m}{\ell}\omega^2 \frac{x^2}{2}$

$$\Rightarrow Y = \frac{F\ell}{A\Delta\ell} \quad \Delta\ell = \frac{F\ell}{Ay}$$



$$\Delta\ell = \frac{\frac{m}{\ell}\frac{\omega^2 x^2}{2} dx}{AY} \Rightarrow \Delta\ell = \frac{m}{\ell}\frac{\omega^2\ell^3}{6AY}$$

$$\Rightarrow \Delta\ell = \frac{\rho\omega^2\ell^3}{6y}$$

$\Delta\ell \propto \omega^2 \Rightarrow \omega_2 = 2\omega_1$

33. The change in length of rod due to increase in temperature in absence of walls is

$\Delta\ell = \ell\,\alpha\,\Delta T = 1000 \times 10^{-4} \times 20\ \text{mm} = 2\ \text{mm}$

But the rod can expand upto 1001 mm only.

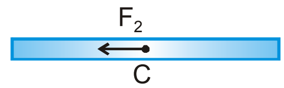
At that temperature its natural length is = 1002 mm.

$\therefore$ compression = 1mm

$\therefore$ mechanical stress $= Y\frac{\Delta\ell}{\ell} = 10^{11} \times \frac{1}{1000} = 10^8\ \text{N/m}^2$

34. The force F_1 causes extension in rod.

F_2 causes compression in left half of rod and an equal extension in right half of rod. Hence F_2 does not effectively change length of the rod.



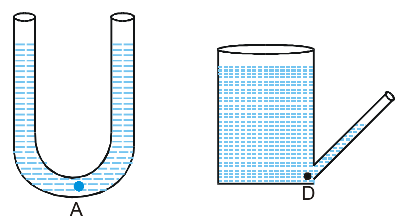
35. The maximum horizontal distance from the vessel comes from hole number 3 and 4

$v = \sqrt{2gh} \rightarrow$ h is height of hole from top.

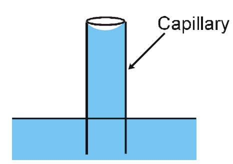
horizontal distance $x = vt = \sqrt{2gh}\sqrt{\frac{2(H-h)}{g}}$ x

$$= 2\sqrt{h(H-h)}$$

36. The pressure at any point can never have different values. Hence (A) & (D) are not possible. (Calculate the pressures at points A & D from both their left and right)



In case of insufficient length of capillary tube the shape of meniscus is as below :



(Stationary fluid)

37. Since the net bouyant force on the brick completely submerged in water is independent of its depth below the water surface, the man will have to exert same force on both the bricks. Hence Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

38. Tension at a point on rod of (length L) at a distance x from point of application of force is

$$T = F\left(1 - \frac{x}{L}\right) \text{ in both cases.}$$

Hence weight has no effect on tension in situation of figure (ii).

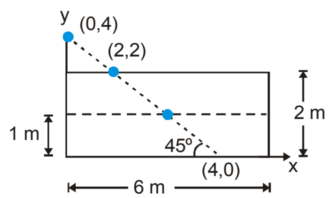
Extension in rod occurs due to force acting at any point on the rod. In certain cases when net force acts at the centre of rod like weight, extension due to this force may not occur like the given case.

39. $\frac{dy}{dx} = \frac{a_x}{a_y + g} = \frac{g/2}{-g/2 + g} = 1$

.... (effective g will be $g - a = g/2$]

$\theta = 45^o$

40. As the slope of free surface is 45^o. Thus free surface passes through centre of box and having co–ordinates (2,2) at top of box. Thus length of exposed top part- = 6 – 2 = 4 m.

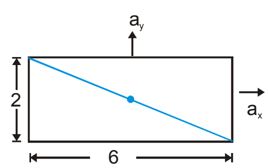


41. $P = P_A + \rho g_{eff} h = 10^5 + 1000 \times (10/2) \times 1 = 0.105$ MPa

42. $p = (10^5 + 10^3 \times 10/2 \times 4)$ N/m^2 = [0.1 + 0.02] MPa = 0.12 MPa

43. As maximum slope of free surface is $\frac{1}{3}$ for the condition of non–exposure of bottom of box, then

$\frac{a_x}{a_y + g} = \frac{1}{3}$



as $a_x = g/2 \Rightarrow 3a_x = a_y + g$

$a_y = g/2$, thus g/2 upward.

44. $F = \rho A(V_0 - 0)^2 [1 - \cos 180^o]$

$= 2\rho Av^2 = 2 \times 1000 \times 2 \times 10^{-4} \times 10 \times 10 = 40$ N

45. $F = 2\rho A (V_0 - u)^2$ u = speed of cart

$m\frac{du}{dt} = 2\rho A (v_0 - u)^2 \Rightarrow \int_0^u \frac{du}{(V_0 - u)^2} = \frac{2\rho A}{m}\int_0^t dt$

$\left[\frac{2\rho A}{m} = \frac{2\times 10^3 \times 2 \times 10^{-4}}{10} = \frac{4}{100}\right]$

$\left[\frac{1}{V_0 - u}\right]_0^u = \frac{2\rho At}{m}$

$\frac{1}{V_0 - u} - \frac{1}{V_0} = \frac{2\rho At}{m} = \frac{4t}{100}$ ....(1)

at t = 10 sec. $\rightarrow \frac{1}{V_0 - u} = \frac{4}{10} + \frac{1}{10} = \frac{1}{2}$

$V_0 - u = 2$ u = 8 m/sec.

46. $F = 2\rho A (V_0 - u)^2 = 2 \times 10^3 \times 2 \times 10^{-4} (10 - 8)^2$
$= 2 \times 10^3 \times 2 \times 10^{-4} \times 4 = 1.6$ N

$a = \frac{F}{M} = 0.16$ m/sec^2

47. From equation (1)

$\frac{1}{V_0 - u} - \frac{1}{V_0} = \frac{4t}{100}$

$\frac{1}{8} - \frac{1}{10} = \frac{4t}{100} \Rightarrow \frac{2}{80} = \frac{4t}{100}$, $t = \frac{10}{16}$ sec.

48. $F = 2\rho A(V_0 - u)^2 = 2 \times 10^3 \times 2 \times 10^{-4} \times 25 = 10$ N

$P = F.u = 10 \times 5 = 50$ W.

49. Pressure varies with height $\Rightarrow$ P = ρgh
and is horizontal with acceleration $\Rightarrow$ P = $\rho \ell a$
so on (A) ρgh part is zero while average of ρax is

$\left[\frac{0 + \rho\ell a}{2}\right][\ell^2] = \frac{\ell\rho a}{2}(\ell^2) = \frac{(\rho\ell^3)}{2} a = \frac{ma}{2}$

In (B) $\rho\ell a$ part is zero while average of ρgx is

$\left[\frac{0 + \rho g\ell}{2}\right]\left[\ell^2\right] = \frac{\rho g}{2}(\ell^3) = \frac{\rho(\ell^3)}{2}(g) = \frac{mg}{2}$

Similarly for other part.

50 **(A)** On ABCD avg pressure = $\left[\frac{0 + \rho_1 gh}{2}\right]$

So $F = \left[\frac{\rho_1 gh}{2}\right][\ell h] = \frac{\rho_1 gh^2 \ell}{2}$

(B) No contact of ρ_2 and not any pressure on ABCD due to ρ_2

(C) On CDEF due to ρ_1, at every point pres sure is $\rho_1 gh$ so average is also $\rho_1 gh$,

so $F = (\rho_1 gh)(h\ell) = \rho_1 gh^2\ell$

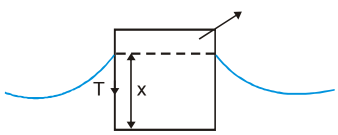
(D) On CDEF due to ρ_1 constant but ρ_1 is variable so average is ρ_1 will be taken.

$\left[\rho_1 gh + \left\{\frac{0 + \rho_2 gh}{2}\right\}\right][h\ell]$

51. Downward force = Buoyant force

$Mg + 4Ta = a^2 x\rho g$

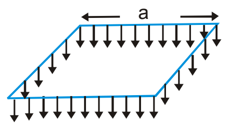
$$x = \frac{Mg + 4Ta}{a^2 \rho g}$$



$$= \frac{20 \times 10^{-3} \times 10 + 4 \times 70 \times 10^{-3} \times 30 \times 10^{-2}}{9 \times 10^{-4} \times 10^{3} \times 10}$$

$$= \frac{0.2 + 84}{9} = \frac{2084}{9} = 2315 \text{ m}$$

$= 2315$ cm



Ans. $\left[x = \dfrac{mg + 4aT}{a^2 \rho g}\right] = 23$ cm

52. Thickness of annular space $= \dfrac{20.0628 - 20}{2}$

$= .0314$ cm

In steady state, gravitational force = viscous force

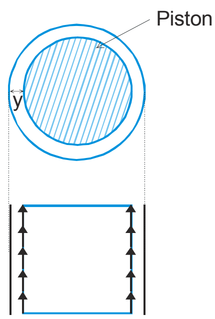
$$mg = \eta A \frac{v}{y}$$

$$1 \times 10 = 10 \times 10^{-1} \times 2\pi r l \frac{v}{y}$$

$$1 \times 10 = 10 \times 10^{-1} \times 2 \times 3.14 \times 10$$

$$\times 10^{-2} \times 20 \times 10^{-2} \frac{v}{.0314 \times 10^{-2}}$$

$1 = 40v$



$$v = \frac{1}{40} = 0.025 \text{ m/sec.} = 2.5 \text{ cm/sec.}$$

53. Due to rotation, Let the shift of liquid is x cm.

Let cross–section area of tube = A

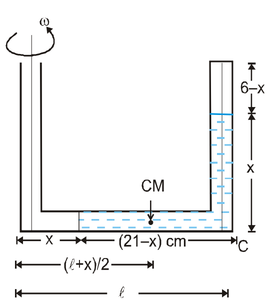
In the right limb for compressed air -

$p_1 v_1 = p_2 v_2$

$p_0 A \times 6 = p_2 A (6 - x)$

$$p_2 = \frac{6p_0}{6 - x} \quad \text{..............(1)}$$

Force at the corner 'C' of right limb climb due to liquid above,



$$F_1 = \left[\frac{6p_0}{6 - x} + x\rho g\right] A$$

Mass of the liquid in horizontal arm

$m = \rho (l - x)A$

It is rotated about left limb, then centripeted force

$F_2 = m\,\omega_0^2 r$

$$= \rho (l - x)A\, \omega_0^2 \frac{\ell + x}{2} = \frac{\rho A \omega_0^2}{2} (l^2 - x^2)$$

But $F_1 = F_2$

$$\frac{\rho A \omega_0^2 (\ell^2 - x^2)}{2} = \left[\frac{6p_0}{6 - x} + x\rho g\right] A$$

$$= \frac{10^3 \times 100 \times (21^2 - x^2) \times 10^{-4}}{2}$$

$$= \left[\frac{6 \times 10500}{(6 - x)} + x \times 10^3 \times 10 \times 10^{-2}\right]$$

On solving x = 1 cm

then length of air column = 6 – 1 = 5 cm

54. Maximum stress lies in stepped bar in the portion of lesser area (5 cm²)

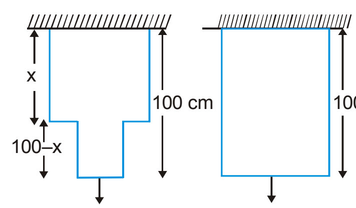
For the stress σ in lesser area, the stress in larger

cross–section $= \dfrac{\sigma A/2}{A} = \dfrac{\sigma}{2}$

Strain energy of stepped bar –

$$= \frac{\sigma^2}{2y} \times 5 \times (100 - x) + \left(\frac{\sigma}{2}\right)^2 \frac{1}{2y} \times 10 \times x$$

$$= \frac{\sigma^2}{2y} [500 - 5x + 2.5x] = \frac{\sigma^2}{2y} [500 - 2.5x]$$



Strain energy of uniform bar $= \dfrac{\sigma^2}{2y} \times 10 \times 100$

As per given condition

$$\frac{\sigma^2}{2y} [500 - 2.5x] = \frac{40}{100} \times \frac{\sigma^2}{2y} \times 10 \times 100$$

$500 - 2.5x = 400$

$2.5x = 100$

x = 40 cm **Ans.**