SOLVED EXAMPLES

Ex.1 Pendulum A is a physical pendulum made from a thin rigid and uniform rod whose length is ℓ. One end of this rod is attached to the ceiling by a frictionless hinge so that rod is free to swing back and forth. Pendulum B is a simple pendulum whose length is also ℓ. The ratio $\frac{T_A}{T_B}$ for small angular oscillations-

(A) $\sqrt{\frac{3}{2}}$ **(B)** $\sqrt{\frac{2}{3}}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{3}{2}$

Sol. $T_A = 2\pi\sqrt{\frac{I}{mg\ell}} = 2\pi\sqrt{\frac{m\ell^2/3}{mg\ell/2}} = 2\pi\sqrt{\frac{2\ell}{3g}}$ & $T_B = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow \frac{T_A}{T_B} = \sqrt{\frac{2}{3}}$

Ex.2 A simple pendulum has time period 2s. The point of suspension is now moved upward according to relation $y = (6t - 3.75\,t^2)$m where t is in second and y is the vertical displacement in upward direction. The new time period of simple pendulum will be

(A) 2s **(B)** 1s **(C)** 4s **(D)** None of these

Sol. $T_1 = 2s = 2\pi\sqrt{\frac{\ell}{g}}$, $T_2 = 2\pi\sqrt{\frac{\ell}{g'}}$ where $g' = g + \frac{d^2y}{dt^2} = 10 - 7.5 = 2.5 = \frac{g}{4} \Rightarrow T_2 = 2\pi\sqrt{\frac{\ell}{g/4}} = 2 \times 2 = 4s$

Ex.3 A point particle of mass 0.1 kg is executing SHM of amplitude 0.1 m. When the particle passes through the mean position, its KE is 8×10^{-3} J. Find the equation of motion of this particle if the initial phase of oscillation is 45° .

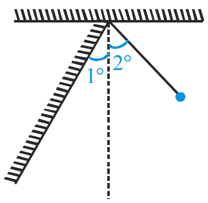
(A) $y = 0.1 \cos(3t + \pi/4)$ **(B)** $y = 0.1 \sin(6t + \pi/4)$

(C) $y = 0.1 \sin(4t + \pi/4)$ **(D)** $y = 0.1 \cos(4t + \pi/4)$

Sol. At mean position KE = $\frac{1}{2}m\omega^2a^2 = 8 \times 10^{-3} \Rightarrow \omega = 4$rad/sec

Now let equation of SHM be $y = 0.1 \sin(4t + \phi)$. At t = 0, $\phi = 45° = \frac{\pi}{4}$. Therefore $y = 0.1\sin\left(4t + \frac{\pi}{4}\right)$

Ex.4 A simple pendulum of length 1m is allowed to oscillate with amplitude 2°. It collides elastically with a wall inclined at 1° to the vertical. Its time period will be : (use $g = \pi^2$)



(A) 2/3 sec **(B)** 4/3 sec

(C) 2 sec **(D)** None of these

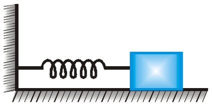
Sol. Time period for half part : $T = 2\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{1}{g}} = \frac{2\pi}{\pi} = 2\,\text{sec}$.

So 2° part will be covered in a time $t = \frac{T}{2} = 1$ sec.

For the left 1° part : $\theta = \theta_0 \sin(\omega t) \Rightarrow 1° = 2° \sin\left(\frac{2\pi}{T} \times t\right) \Rightarrow \frac{1}{2} = \sin\left(\frac{2\pi}{2} \times t\right) \Rightarrow \frac{\pi}{6} = \pi \times t \Rightarrow t = 1/6$ sec.

Total time $= \frac{T}{2} + 2t \Rightarrow 1 + 2 \times \frac{1}{6} = 1 + \frac{1}{3} = \frac{4}{3}$ sec.

Ex.5 Spring of spring constant 1200 Nm^{-1} is mounted on a smooth frictionless surface and attached to a block of mass 3 kg. Block is pulled 2 cm to the right and released. The angular frequency of oscillation is



(A) 5 rad/sec **(B)** 30 rad/sec **(C)** 10 rad/sec **(D)** 20 rad/sec

Sol. Angular frequency $\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{1200}{3}} = 20 \text{ rad / sec}$

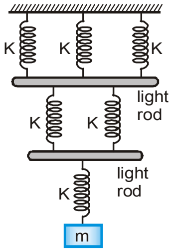
Ex.6 The time period of oscillation of a mass m suspended from a spring is 2 seconds. If another 2 kg mass is also suspended along with m the time period increases by 1 second. The value of mass m is

(A) 2 kg **(B)** 1 kg **(C)** 1.6 kg **(D)** 2.6 kg

Sol. **Ans. (C)**

Here $2\pi\sqrt{\frac{m}{k}} = 2s$ and $2\pi\sqrt{\frac{m+2}{k}} = 3s \Rightarrow \frac{3}{2} = \sqrt{\frac{m+2}{m}} \Rightarrow \frac{9}{4} = \frac{m+2}{m}$ which yields m = 1.6 kg

Ex.7 The time period of small oscillations of mass m :-



(A) $2\pi\sqrt{\frac{m}{6K}}$ **(B)** $2\pi\sqrt{\frac{11m}{6K}}$

(C) $2\pi\sqrt{\frac{6m}{11K}}$ **(D)** $2\pi\sqrt{\frac{m}{K}}$

Sol. Equivalent spring constant $\frac{1}{K_{eq}} = \frac{1}{3K} + \frac{1}{2K} + \frac{1}{K} \Rightarrow K_{eq} = \frac{6K}{11}$ $\therefore T = 2\pi\sqrt{\frac{m}{K_{eq}}} = 2\pi\sqrt{\frac{11m}{6K}}$

Ex.8 Values of the acceleration $\ddot{x}$ of a particle moving in simple harmonic motion as a function of its displacement x are given in the table below.

	16	8	0	–8	–16
x (mm)	–4	–2	0	2	4

The period of the motion is

(A) $\frac{1}{\pi}$s **(B)** $\frac{2}{\pi}$s **(C)** $\frac{\pi}{2}$s **(D)** π s

Sol. In SHM $a = -\omega^2 x$. So $16 = -\omega^2(-4) \Rightarrow \omega = 2 \Rightarrow$ Time period $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi s$

Ex.9 The angle made by the string of a simple pendulum with the vertical depends upon time as $\theta = \frac{\pi}{90} \sin \pi t$. Find the length (in m) of the pendulum if $g = \pi^2 m/s^2$

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4

Sol. Here $T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow \frac{\ell}{g} = \frac{1}{\pi^2}$ But $g = \pi^2$ therefore $\ell = 1$ m

Ex.10 The time period of a particle executing SHM is T. After a time T/6 after it passes its mean position, its :

(A) displacement will be one-half of its amplitude

(B) velocity will be one-half of its maximum velocity

(C) kinetic energy = 1/3 (potential energy)

(D) acceleration will be $\frac{\sqrt{3}}{2}$ times of its maximum acceleration

Sol. Let $x = A\sin\omega t$ where $\omega = \frac{2\pi}{T}$

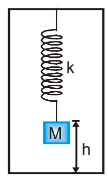
For **(A)** : At t = $x = A\sin\left(\frac{2\pi}{T}\right)\left(\frac{T}{6}\right) = A\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}A$, $x = A\sin\left(\frac{2\pi}{T}\right)\left(\frac{T}{6}\right) = A\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}A$

For **(B)** : $v = \frac{dx}{dt} = A\omega\cos\omega t$ At $t = \frac{T}{6}$, $v = A\omega\cos\left(\frac{2\pi}{T}\right)\left(\frac{T}{6}\right) = A\omega\cos\frac{\pi}{3} = \frac{A}{2}$

For **(C)** : $KE = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{v_{max}}{2}\right)^2 = \frac{1}{4}(KE)_{max} = \frac{1}{4}(TE)$ & $PE = TE - KE = \frac{3}{4}TE \Rightarrow KE = \frac{1}{3}(PE)$

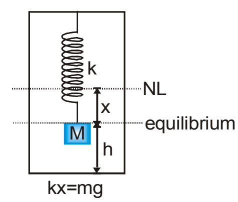
For **(D)** : Acceleration $a = \frac{dv}{dt} = -A\omega^2\sin\omega t = -\omega^2 x$

Ex.11 A block is in equilibrium at rest in a lift as shown in figure. Now lift starts accelerating. The minimum magnitude of acceleration of lift for which block will touch the floor of lift, is



(A) $\frac{kh}{M}$ **(B)** $\frac{2kh}{M}$ **(C)** $\frac{kh}{2M}$ **(D)** $\frac{3}{2}\frac{kh}{M}$

Sol. From work energy theorem $W_g + W_{pseudo} + W_{spring} = \Delta KE$



$\Rightarrow Mgh + Mah + \frac{1}{2}kx^2 - \frac{1}{2}k(h+x)^2 = 0$ where $Mg = kx$

$\Rightarrow Mgh + Mah - \frac{1}{2}kh^2 - khx = 0 \quad \therefore a = \frac{kh}{2M}$

Ex.12 Time period of a spring mass system can be changed by

(A) changing the mass

(B) accelerating the point of suspension of the block

(C) cutting the spring (i.e. changing the length of the spring)

(D) immersing the mass in a liquid

Sol. Time period of spring mass system $T = 2\pi\sqrt{\frac{M}{K}}$ Which can be changed by **(A)**, **(C)** & **(D)**

Ex.13 A particle moves along the Z-axis according to the equation $z = 5 + 12\cos\left(2\pi t + \frac{\pi}{2}\right)$, where z is in cm and t is in seconds. Select the correct alternative (s)-

(A) The motion of the particle is SHM with mean position at z = 5cm

(B) The motion of the particle is SHM with extreme position at z = –7cm and z = +17 cm.

(C) Amplitude of SHM is 13 cm

(D) Amplitude of SHM is 12 cm

Sol. $z = 5 + 12\cos\left(2\pi t + \frac{\pi}{2}\right) = 5 - 12\sin(2\pi t) \Rightarrow (z-5) = -12\sin 2\pi t \Rightarrow$ SHM with mean position z = 5cm

Extreme position $z_{max} = 5 + 12 = 17$ cm, $z_{min} = 5 - 12 = -7$ cm Amplitude of SHM = 12 cm

Ex.14 The potential energy of a particle of mass 100 g, moving along the x-axis, is given by $U = 5x(x-4)$ J, where x is in metre. Select correct alternative(s).

(A) The particle execute SHM with mean position at x = 4 m

(B) The particle execute SHM with mean position at x = 2 m

(C) The particle execute SHM with time period $\pi/5$ second

(D) The particle execute SHM with time period $\pi/10$ second

Sol. $U = 5x^2 - 20x \Rightarrow F = -\frac{dU}{dx} = -10x + 20 = -10(x-2)$

Acceleration $a = \frac{F}{m} = -100(x-2)$ So $\omega^2 = 100 \Rightarrow \omega = 10$

Time period $T = \frac{2\pi}{\omega} = \frac{2\pi}{10} = \frac{\pi}{5}$ sec

Ex.15 The motion of a particle is represented by the function "$A\sin^2\omega t + B\cos^2\omega t + C\sin\omega t\cos\omega t$"

where x, ω and t represent displacement, angular frequency and time respectively. The constants A, B and C are related to the amplitudes of displacement. Select **correct** alternative(s).

(A) The motion of particle is SHM when A=B.

(B) The motion of particle is SHM when A=B and C=0.

(C) If $B = \frac{C}{2} = -A$, then the amplitude of SHM is $B\sqrt{2}$.

(D) If $A=B=\frac{C}{2}$, then the axis of vibration of SHM shifts by a distance B towards +x axis.

Sol. If A= B then $x = A + C\sin\omega t\cos\omega t = A + \frac{C}{2}\sin 2\omega t \Rightarrow$ SHM

If A = B & C = 0 then x = A $\Rightarrow$ along a straight line.

If $B = \frac{C}{2} = -A$ then $x = B\cos 2\omega t + B\sin 2\omega t \Rightarrow$ amplitude $= B\sqrt{2}$

If $A = B = \frac{C}{2}$ then $x = B + B\sin 2\omega t \Rightarrow$ Axis of vibration of SHM shifts by a distance B towards + x-axis.

Ex.16 For a SHM with given angular frequency, two arbitrary initial conditions are necessary and sufficient to determine the motion completely. These initial conditions may be

(A) Amplitude and initial phase

(B) Amplitude and total energy of oscillation

(C) Initial phase and total energy of oscillation

(D) Initial position and initial velocity

Sol. $x = A\sin(\omega t + \phi)$ As ω is given, to describe the motion completely we need A and ϕ.

Ex.17 to 19

X_1 and X_2 are two points on the path of a particle executing SHM in a straight line, at which its velocity is zero. Starting from a certain point X ($X_1X < X_2X$) the particle crosses this point again at successive intervals of 2s and 4s with a speed of 5 m/s.

17. The time period of SHM is

(A) 6 s **(B)** 3s **(C)** 4s **(D)** 2s

18. The amplitude of SHM is

(A) $10\sqrt{3}$ m **(B)** $\frac{10}{\sqrt{3}\pi}$ m **(C)** $\frac{10\sqrt{3}}{\pi}$ m **(D)** can't be determined

19. The maximum speed of the particle is

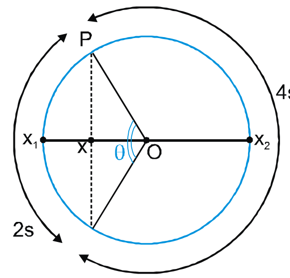
(A) $\frac{10\sqrt{3}}{\pi}$ m/s **(B)** $10\sqrt{3}$ m/s **(C)** $\frac{\sqrt{3}}{10}$ m/s **(D)** $\frac{10}{\sqrt{3}}$ m/s

Sol.

17. 

Time period = 2 + 4 = 6s

18. The given SHM can be represented in circular motion



In shown diagram $\theta = \left(\frac{2\pi}{6}\right)(2) = \frac{2\pi}{3}$

Let OX = x_0 and amplitude = OP=A

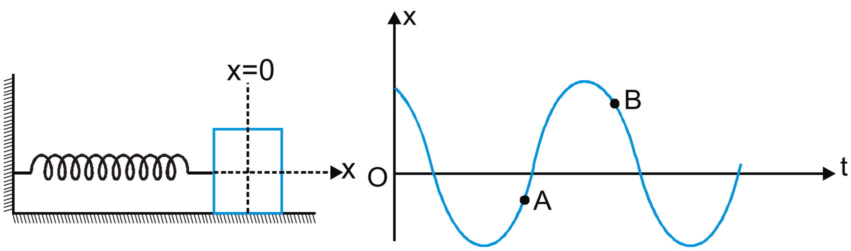
Now $\frac{x_0}{A} = \cos\frac{\theta}{2}$ so $x_0 = \frac{A}{2}$

As $v = \omega\sqrt{A^2 - X_0^2}$ So $5 = \frac{2\pi}{6}\sqrt{A^2 - \frac{A^2}{4}}$ $\Rightarrow A = \frac{10\sqrt{3}}{\pi}$m

19. Maximum speed = $A\omega = \left(\frac{10\sqrt{3}}{\pi}\right)\left(\frac{2\pi}{6}\right) = \frac{10}{\sqrt{3}}$ m/s

Ex. 20 to 22

A 100 g block is connected to a horizontal massless spring of force constant 25.6 N/m. The block is free to oscillate on a horizontal frictionless surface. The block is displaced 3 cm from the equilibrium position, and at t = 0, it is released from rest at x = 0. The position-time graph of motion of the block is shown in figure.



20. When the block is at position A on the graph, its

(A) position and velocity both are negative. **(B)** position and velocity both are positive.

(C) position is negative and velocity is positive. **(D)** position is positive and velocity is negative.

21. When the block is at position B on the graph, its

(A) position and velocity both are negative. **(B)** position and velocity both are positive.

(C) position is negative and velocity is positive. **(D)** position is positive and velocity is negative.

22. Let us now make a slight change to the initial conditions. At t=0, let the block be released from the same position with an initial velocity v_i = 64 cm/s. Position of the block as a function of time can be expressed as

(A) x = 5 sin16t **(B)** x =5sin(16t+37°) **(C)** x =5sin(16t–37°) **(D)** x =5cos(16t+37°)

Sol.

20. At position A, x is negative and $\frac{dx}{dt}$ is positive.

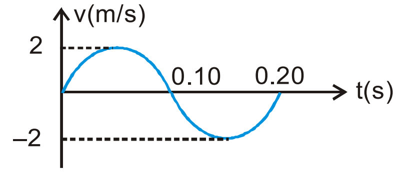
21. At position B, x is positive and $\frac{dx}{dt}$ is negative.

22. Let $x = A\sin(16t + \phi)$ $v = A\omega\cos(16t + \phi)$ where $\omega^2 = \frac{K}{m} = \frac{25.6}{0.1} = 256 \Rightarrow \omega = 16$ rad/sec

At $t = 0$, $3 = A\sin\phi$ & $64 = 16A\cos\phi \Rightarrow \tan\phi = \frac{3}{4} \Rightarrow \phi = 37°$ Also $A = 5$ cm

Therefore equation of SHM $x = 5\sin(16t + 37°)$

Ex.23 A simple harmonic oscillator consists of a block attached to a spring with k = 200 N/m. The block slides on a frictionless horizontal surface, with equilibrium point x = 0. A graph of the block's velocity v as a function of time t is shown (use $\pi^2 = 10$)



	Column I		**Column II**
(A)	The block's mass in kg	**(P)**	–0.20
(B)	The block's displacement at t = 0 in meters	**(Q)**	–200
(C)	The block's acceleration at t = 0.10 s in m/s^2	**(R)**	+0.20
(D)	The block's maximum kinetic energy in joule	**(S)**	+4.0
		(T)	+200

Sol. $v_m = A\omega \Rightarrow A = \frac{v_m}{\omega} = \frac{2\pi}{2\pi} \times (0.2) = 0.20$ m; $T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow m = \frac{T^2 k}{4\pi^2} = 0.2$ kg

At t = 0.1, acceleration is maximum $\Rightarrow -\omega^2 A = -200$ m/s^2

Maximum energy $= \frac{1}{2} m v_m^2 = 4$J or $\frac{1}{2} kA^2 = E_{max} = \frac{1}{2} \times 200 \times 0.04 = 4$ J

Ex.24 The speed (v) of a particle moving along a straight line, when it is at a distance (x) from a fixed point on the line, is given by : $v^2 = 144 - 9x^2$.

	Column I		**Column II**
(A)	Motion is simple harmonic of period	**(P)**	$\frac{2\pi}{3}$ units
(B)	Maximum displacement from the fixed point is	**(Q)**	12 units
(C)	Maximum velocity of the particle	**(R)**	27 units
(D)	Magnitude of acceleration at a distance 3 units from the fixed point is	**(S)**	4 unit
		(T)	16 units

Sol. **For (A) :** $v^2 = 144 - 9x^2 \Rightarrow 2v\frac{dv}{dx} = 0 - 18x \Rightarrow a = -9x \Rightarrow \omega^2 = 9 \Rightarrow \omega = 3$

Time period $T = \frac{2\pi}{\omega} = \frac{2\pi}{3}$ units

For (B) : $\because v^2 \geq 0 \quad \therefore 144 - 9x^2 \geq 0 \Rightarrow x^2 \leq 16 \Rightarrow x \leq 4$

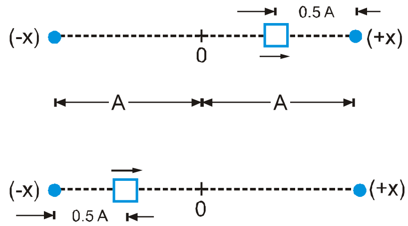
$\Rightarrow$ Amplitude = 4 units

For (C) : Maximum velocity $= A\omega = (4)(3) = 12$ units

For (D) : At x = 3 units, $a = -9x = -27$ units

Exercise # 1 [Single Correct Choice Type Questions]

1. Two bodies performing S.H.M. have same amplitude and frequency. Their phases at a certain instant are as shown in the figure. The phase difference between them is

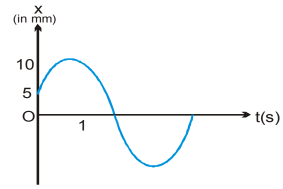


(A) $\frac{11}{6}\pi$ **(B)** π **(C)** $\frac{5}{3}\pi$ **(D)** $\frac{3}{5}\pi$

2. The equation of motion of a particle of mass 1 g is $\frac{d^2x}{dt^2} + \pi^2 x = 0$ where x is displacement(in m)from mean position. The frequency of oscillation is (in Hz) :

(A) $\frac{1}{2}$ **(B)** 2 **(C)** $5\sqrt{10}$ **(D)** $\frac{1}{5\sqrt{10}}$

3. The figure shows the displacement time graph of a particle executing S.H.M. If the time period of oscillation is 2 s the equation of motion of its SHM is



(A) $x = 10\sin(\pi t + \pi/3)$
(B) $x = 10\sin \pi t$
(C) $x = 10\sin(\pi t + \pi/6)$
(D) $x = 10 \sin (2\pi t + \pi/6)$

4. A small mass executes linear S.H.M. about O with amplitude 'a' and period 'T'. Its displacement from O at time T/8 after passing through O is

(A) a/8 **(B)** $\frac{a}{2\sqrt{2}}$ **(C)** a/2 **(D)** $\frac{a}{\sqrt{2}}$

5. Two particle executes S.H.M. of same amplitude and frequency along the same straight line. They pass one another when going in opposite directions, each time their displacement is half of their amplitude. The phase difference between them is :–

(A) 30° **(B)** 60° **(C)** 90° **(D)** 120°

6. A particle performing S.H.M. is found at its equilibrium at t = 1 s and it is found to have a speed of 0.25 m/s at t = 2 s. If the period of oscillation is 6s Calculate amplitude of oscillation

(A) $\frac{3}{2\pi}$ m **(B)** $\frac{3}{4\pi}$ m **(C)** $\frac{6}{\pi}$ m **(D)** $\frac{3}{8\pi}$ m

7. A particle executes S.H.M. along a straight line with mean position x = 0, period 20 s and amplitude 5 cm. The shortest time taken by the particle to go from x = 4 cm to x = –3cm is

(A) 4 s **(B)** 7 s **(C)** 5 s **(D)** 6 s

8. Two particles A and B perform SHM along the same straight line with the same amplitude 'a', same frequency 'f' and same equilibrium position 'O'. The greatest distance between them is found to be 3a/2. At some instant of time they have the same displacement from mean position. What is this displacement?

(A) a/2 **(B)** $a\sqrt{7}/4$ **(C)** $\sqrt{3}/a2$ **(D)** 3a/4

9. A particle executes S.H.M. in a straight line. In the first second starting from rest it travels a distance 'a' and in the next second a distance 'b' in the same direction. The amplitude of S.H.M. will be

(A) $\frac{2a^2}{3a-b}$ (B) a – b (C) 2a – b (D) a / b

10. A particle executes SHM with time period T and amplitude A. The maximum possible average velocity in time T/4 is

(A) $\frac{2A}{T}$ (B) $\frac{4A}{T}$ (C) $\frac{8A}{T}$ (D) $\frac{4\sqrt{2}A}{T}$

11. A particle is subjected to two mutually perpendicular simple harmonic motions such that its x and y coordinates are given by: $x = 2\sin\omega t$; $y = 2\sin\left(\omega t + \frac{\pi}{4}\right)$. The path of the particle will be

(A) an ellipse (B) a straight line (C) a parabola (D) a circle

12. The period of a particle is 8s. AT t = 0 it is at the mean position. The ratio of distance covered by the particle in first second and second will be-

(A) $\frac{\sqrt{2}-1}{\sqrt{2}}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{1}{\sqrt{2}-1}$ (D) $[\sqrt{2}-1]$

13. The P.E. of an oscillating particle at rest position is 15 J and its average K.E. is 5 J. The total energy of particle at any instant will be-

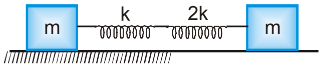
(A) 10 J (B) 20 J (C) 25 J (D) 5 J

14. The time taken by a particle performing S.H.M. to pass from point A to B where its velocities are same is 2 seconds . After another 2 seconds it returns to B . The time period of oscillation is (in seconds) :

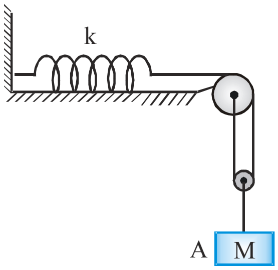
(A) 2 (B) 4 (C) 6 (D) 8

15. A system is shown in the figure. The time period for small oscillations of the two blocks will be :-

(A) $2\pi\sqrt{\frac{3m}{k}}$ (B) $2\pi\sqrt{\frac{3m}{2k}}$ (C) $2\pi\sqrt{\frac{3m}{4k}}$ (D) $2\pi\sqrt{\frac{3m}{8k}}$



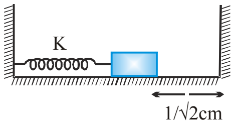
16. Block A in the figure is released from the rest when the extension in the spring is x_0. The maximum downward displacement of the block.



(A) $\frac{Mg}{2k} - x_0$ (B) $\frac{Mg}{2k} + x_0$

(C) $\frac{2Mg}{k} - x_0$ (D) $\frac{2Mg}{k} + x_0$

17. A block of mass 0.9 kg attached to a spring of force constant K is compressed by $\sqrt{2}$ cm and the block is at a distance $\frac{1}{\sqrt{2}}$ cm from the wall. When the block is released, it makes elastic collision with the wall and its period of motion is 0.2 s. The value of K is (take $\pi^2=10$)



(A) 100 Nm^{-1} (B) 10 Nm^{-1} (C) 0.1 Nm^{-1} (D) 1 Nm^{-1}

18. A horizontal spring is connected to a mass M. It executes simple harmonic motion. When the mass M passes through its mean position, an object of mass m is put on it and the two move together. The ratio of frequencies before and after will be-

(A) $\left(1+\frac{m}{M}\right)^{1/2}$ **(B)** $\left(1+\frac{m}{M}\right)$ **(C)** $\left(\frac{M}{M+m}\right)^{1/2}$ **(D)** $\left(\frac{M}{M+m}\right)$

19. The length of a spring is α when a force of 4 N is applied on it and the length is β when 5 N force is applied. Then the length of spring when 9 N force is applied is-

(A) $5\beta - 4\alpha$ **(B)** $\beta - \alpha$ **(C)** $5\alpha - 4\beta$ **(D)** $9(\beta - \alpha)$

20. Two simple pendulums, having periods of 2s and 3s respectively, pass through the mean position simultaneously at a particular instant. They may be in phase after an interval of :

(A) 5s **(B)** 3s **(C)** 1s **(D)** None of the above

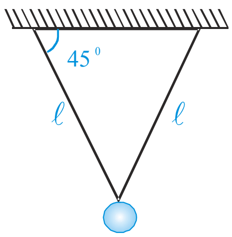
21. A pendulum is suspended in a lift and its period of oscillation when the lift is stationary is T_0. What must be the acceleration of the lift for the period of oscillation of the pendulum to be $T_0/2$?

(A) 2g downward **(B)** 2g upward **(C)** 3g downward **(D)** 3g upward

22. The frequency of a simple pendulum is n oscillations per minute while that of another is (n + 1) oscillations per minute. The ratio of length of first pendulum to the length of second is-

(A) $\frac{n}{n+1}$ **(B)** $\left(n+\frac{1}{n}\right)^2$ **(C)** $\left(\frac{n+1}{n}\right)^2$ **(D)** $\left(\frac{n}{n+1}\right)^2$

23. Time period of small oscillation (in a vertical plane normal to the plane of strings) of the bob in the arrangement shown will be



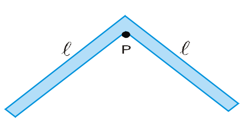
(A) $2\pi\sqrt{\frac{\ell}{g}}$ **(B)** $2\pi\sqrt{\frac{\ell}{\sqrt{2}g}}$

(C) $2\pi\sqrt{\frac{\sqrt{2}\ell}{g}}$ **(D)** $2\pi\sqrt{\frac{2\ell}{g}}$

24. The distance of point of a compound pendulum from its centre of gravity is ℓ, the time period of oscillation relative to this point is T. If $g = \pi^2$, the relation between ℓ and T will be :–

(A) $\ell^2 - \left[\frac{T^2}{4}\right]\ell + k^2 = 0$ **(B)** $\ell^2 + \left[\frac{T^2}{4}\right]\ell + k^2 = 0$ **(C)** $\ell^2 - \left[\frac{T^2}{4}\right]\ell - k^2 = 0$ **(D)** $\ell^2 + \left[\frac{T^2}{4}\right]\ell - k^2 = 0$

25. A system of two identical rods (L-shaped) of mass m and length ℓ are resting



on a peg P as shown in the figure. If the system is displaced in its plane by a small angle θ, find the period of oscillations

(A) $2\pi\sqrt{\frac{\sqrt{2}\ell}{3g}}$ **(B)** $2\pi\sqrt{\frac{2\sqrt{2}\ell}{3g}}$ **(C)** $2\pi\sqrt{\frac{2\ell}{3g}}$ **(D)** $3\pi\sqrt{\frac{\ell}{3g}}$

26. A man of mass 60 kg standing on a plateform executing S.H.M. in the vertical plane . The displacement from the mean position varies as $y = 0.5 \sin(2\pi ft)$. The minimum value of f, for which the man will feel weightlessness at the highest point is : (y is in metres)

(A) $\frac{g}{4\pi}$ **(B)** $4\pi g$ **(C)** $\frac{\sqrt{2g}}{2\pi}$ **(D)** $2\pi\sqrt{2g}$

27. A moving particle of mass has one-dimensional potential energy $U(x) = ax^2 + bx^4$, where 'a' and 'b' are positive constants. The angular frequency of small oscillations about the minima of the potential energy is equal to

(A) $\pi\sqrt{\frac{a}{2b}}$ **(B)** $2\sqrt{\frac{a}{m}}$ **(C)** $\sqrt{\frac{2a}{m}}$ **(D)** $\sqrt{\frac{a}{2m}}$

28. A heavy brass-sphere is hung from a spiral spring and it executes vertical vibrations with period T. The ball is now immersed in nonviscous liquid with a density one-tenth that of brass. When set into vertical vibrations with the sphere remaining inside the liquid all the time, the period will be-

(A) $\left[\frac{9}{10}\right]T$ **(B)** $T\sqrt{\left(\frac{10}{9}\right)}$ **(C)** Unchanged **(D)** $T\sqrt{\left(\frac{9}{10}\right)}$

29. A particle executes SHM on a line 8 cm long. Its K.E. and P.E. will be equal when its distance from the mean position is

(A) 4 cm **(B)** 2 cm **(C)** $2\sqrt{2}$ cm **(D)** $\sqrt{2}$ cm

30. A particle performs S.H.M. of amplitude A with angular frequency w along a straight line. When it is at a distance $\frac{\sqrt{3}}{2}$ A from mean position, its kinetic energy gets increased by an amout $\frac{1}{2}m\omega^2A^2$ due to an impulsive force. Then its new amplitude becomes-

(A) $\frac{\sqrt{5}}{2}A$ **(B)** $\frac{\sqrt{3}}{2}A$ **(C)** $\sqrt{2}\,A$ **(D)** $\sqrt{5}\,A$

31. Displacement of a particle is $x = 3\sin 2t + 4\cos 2t$, the amplitude and the maximum velocity will be :–

(A) 5, 10 **(B)** 3, 2 **(C)** 4, 2 **(D)** 3, 8

32. The total energy of a vibrating particle in SHM is E. If its amplitude and time period are doubled, its total energy will be :–

(A) 16E **(B)** 8E **(C)** 4E **(D)** E

33. The distance between the point of suspension and the centre of gravity of a compound pendulum is ℓ and the radius of gyration about the horizontal axis through the centre of gravity is k, then its time period will be

(A) $2\pi\sqrt{\frac{\ell+k}{g}}$ **(B)** $2\pi\sqrt{\frac{\ell^2+k^2}{\ell g}}$ **(C)** $2\pi\sqrt{\frac{\ell+k^2}{g}}$ **(D)** $2\pi\sqrt{\frac{2k}{\ell g}}$

34. The phase of a particle in SHM at time t is $\frac{13\pi}{6}$. The following inference is drawn from this

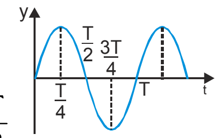
(A) the particle is at $x = \frac{a}{2}$ and moving in + X-direction

(B) the particle is at $x = \frac{a}{2}$ and moving in –X-direction

(C) the particle is at $x = -\frac{a}{2}$ and moving in + X-direction

(D) the particle is at $x = -\frac{a}{2}$ and moving in –X-direction

35. The graph shows the variation of displacement of a particle executing S.H.M. with time. We inference from this graph that:–



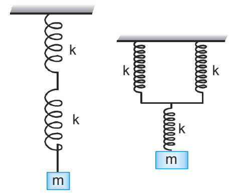
(A) the force is zero at time $\frac{3T}{4}$

(B) the velocity is maximum at time $\frac{T}{2}$

(C) the acceleration is maximum at time T

(D) the P.E. is equal to half of total energy at time $\frac{T}{2}$

36. Some springs are combined in series and parallel arrangement as shown in the figure and a mass m is suspended from them. The ratio of their frequencies will be



(A) 1 : 1 (B) 2 : 1 (C) $\sqrt{3}$: 2 (D) 4 : 1

37. The time period of an oscillator is 8 sec. The phase difference from t = 2 sec to t = 4 sec will be :–

(A) π (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{4}$ (D) 2π

38. The magnitude of average acceleration in half time period in a simple harmonic motion is

(A) $\frac{2A\omega^2}{\pi}$ (B) $\frac{A\omega^2}{2\pi}$ (C) $\frac{A\omega^2}{\sqrt{2}\pi}$ (D) zero

39. The acceleration due to gravity at height R above the surface of the earth is $\frac{g}{4}$. The periodic time of a simple pendulum in an artificial satellite at this height will be :–

(A) $T = 2\pi\sqrt{\frac{2l}{g}}$ (B) $T = 2\pi\sqrt{\frac{l}{2g}}$ (C) zero (D) infinity

40. A particle of mass m executing SHM makes f oscillation per second. The difference of its kinetic energy when at the centre, and when at a distance x from the centre is

(A) $\pi^2 f^2 x^2 m$ (B) $2\pi^2 f^2 x^2 m$ (C) $\frac{1}{2}\pi^2 f^2 x^2 m$ (D) $f^2 x^2 m$

41. A particle performs S.H.M. with time period T. The time taken by the particle to move from half the amplitude to the maximum displacement is

(A) $\frac{T}{2}$ (B) $\frac{T}{4}$ (C) $\frac{T}{6}$ (D) $\frac{T}{8}$

42. A particle is performing S.H.M. with acceleration $a = 8\pi^2 - 4\pi^2 x$ where x is coordinate of the particle w.r.t. the origin.The parameters are in S.I. units. The particle is at rest at $x = -2$ at $t = 0$.

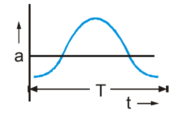
(A) coordinate of the particle w.r.t. origin at any time t is $2 - 4\cos 2\pi t$

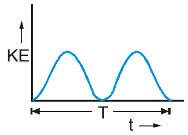
(B) coordinate of the particle w.r.t. origin at any time t is $-2 + 4\sin 2\pi t$

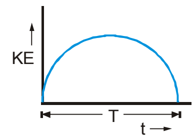
(C) coordinate of the particle w.r.t. origin at any time t is $-4 + 2\cos 2\pi t$

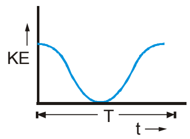
(D) the coordinate cannot be found because mass of the particle is not given.

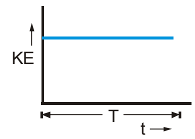
43. Acceleration a and time period T of a body in S.H.M. is given by a curve shown below. Then corresponding graph between kinetic energy KE and time t is correctly represented by



(A)


(B)


(C)


(D)


44. Vertical displacement of a plank with a body of mass 'm' on it is varying according to law $y = \sin\omega t + \sqrt{3}\cos\omega t$. The minimum value of ω for which the mass just breaks off the plank and the moment it occurs first after t = 0 are given by : (y is positive vertically upwards)

(A) $\sqrt{\frac{g}{2}}, \sqrt{\frac{2\pi}{6g}}$
(B) $\sqrt{\frac{g}{2}}, \frac{2\pi}{3\sqrt{g}}$

(C) $\sqrt{\frac{g}{2}}, \frac{\pi\sqrt{\pi}}{3\sqrt{g}}$
(D) $\sqrt{2g}, \sqrt{\frac{2\pi}{2g}}$

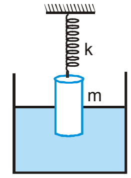
45. An oscillation is described by the equation $x = A \sin 2\pi\gamma_1 t$ where A changes with time according to the law $A = A_0 (1+\cos 2\pi\gamma_2 t)$ where A_0 is constant. Find the ratio of frequencies of harmonic oscillations forming oscillation

(A) $\gamma_1 : \gamma_2 : (\gamma_1 - \gamma_2)$
(B) $\gamma_1 : (\gamma_1 - \gamma_2) : (\gamma_1 + \gamma_2)$

(C) $\gamma_1 : \gamma_2 : (\gamma_2 - \gamma_1)$
(D) $\gamma_1 : \gamma_2 : (\gamma_1 + \gamma_2)$

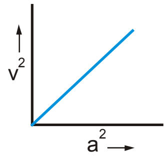
Exercise # 2 Part # I [Multiple Correct Choice Type Questions]

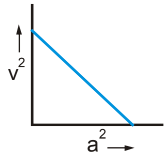
1. A uniform cylinder of mass m and length ℓ having area of cross-section a is suspended lengthwise with the help of a massless spring of constant k. The cylinder is half submerged in a liquid of density ρ. A small push and release makes it vibrate with small amplitude. The frequency of oscillation is

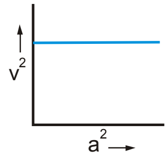


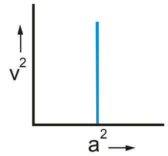
(A) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ **(B)** $\frac{1}{2\pi}\sqrt{\frac{ka\rho g}{m}}$ **(C)** $\frac{1}{2\pi}\sqrt{\frac{m+a\rho g}{k}}$ **(D)** $\frac{1}{2\pi}\sqrt{\frac{k+a\rho g}{m}}$

2. A mass M is performing linear simple harmonic motion, then correct graph for acceleration 'a' and corresponding linear velocity 'v' is

(A) 

(B) 

(C) 

(D) 

3. Two identical springs are fixed at one end and masses 1kg and 4kg are suspended at their other ends . They are both stretched down from their mean position and let go simultaneously. If they are in the same phase after every 4 seconds then the springs constant k is

(A) $\pi\frac{N}{m}$ **(B)** $\pi^2\frac{N}{m}$ **(C)** $2\pi\frac{N}{m}$ **(D)** given data is insufficient

4. A mass of 0.2kg is attached to the lower end of a massless spring of force-constant 200 N/m, the upper end of which is fixed to a rigid support. Which of the following statements is/are true?

(A) In equilibrium, the spring will be stretched by 1cm.

(B) If the mass is raised till the spring is unstretched state and then released, it will go down by 2cm before moving upwards.

(C) The frequency of oscillation will be nearly 5 Hz.

(D) If the system is taken to the moon, the frequency of oscillation will be the same as on the earth.

5. A cylindrical block of density ρ is partially immersed in a liquid of density 3ρ. The plane surface of the block remains parallel to the surface of the liquid. The height of the block is 60 cm. The block performs SHM when displaced from its mean position. [Use $g = 9.8\ m/s^2$]

(A) the maximum amplitude is 20 cm. **(B)** the maximum amplitude is 40 cm

(C) the time period will be $2\pi/7$ seconds **(D)** none

6. A particle moves in the x-y plane according to the equation, $\vec{r} = (\tilde{i}+2\tilde{j})A\cos\omega t$. The motion of the particle is-

(A) on a straight line **(B)** on an ellipse **(C)** periodic **(D)** simple harmonic

7. A particle is subjected to two simple harmonic motions along x and y directions according to, $x = 3\sin 100\pi t$; $y = 4\sin 100\pi t$.

(A) Motion of particle will be on ellipse traversing it in clockwise direction

(B) Motion of particle will be on a straight line with slope 4/3

(C) Motion will be simple harmonic motion with amplitude 5

(D) Phase difference between two motions is $\pi/2$

8. A horizontal plank has a rectangular block placed on it. The plank starts oscillating vertically and simple harmonically with an amplitude of 40 cm. The block just loses contact with the plank when the latter is at momentary rest. Then :

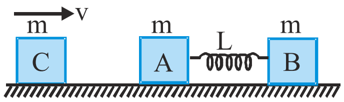
(A) the period of oscillation is $\left(\frac{2\pi}{5}\right)$

(B) the block weighs double of its weight, when the plank is at one of the positions of momentary rest

(C) the block weighs 0.5 times its weight on the plank halfway up

(D) the block weighs 1.5 times its weight on the plank halfway down

9. Two block A and B each of mass m are connected by a massless spring of natural length L and spring constant k. The blocks are initially resting on a smooth horizontal floor with the spring at its natural length as shown in fig. A third identical block C, also of mass m, moving on the floor with a speed v along the line joining A and B, and collides elastically with A. Then-



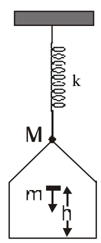
(A) The kinetic energy of the A-B system, at maximum compression of the spring, is zero.

(B) The kinetic energy of A-B system, at maximum compression of the spring is $\frac{mv^2}{4}$

(C) The maximum compression of the spring is $v\sqrt{\frac{m}{k}}$

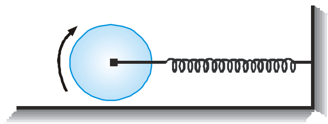
(D) The maximum compression of the spring is $v\sqrt{\frac{m}{2k}}$

10. A cage of mass M hangs from a light spring of force constant k. A body of mass m falls from height h inside the cage and sticks to its floor. The amplitude of oscillations of the cage will be-



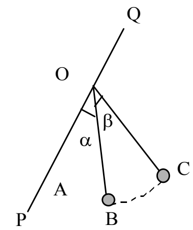
(A) $\left(\frac{2mgh}{k}\right)^{1/2}$ **(B)** $\left(\frac{k}{2mgh}\right)^{1/2}$ **(C)** $\frac{mg}{k}$ **(D)** $\left(\frac{mg}{k}\right)^{1/2}$

11. A solid cylinder of mass M attached to a massless spring of force constant k is placed on a horizontal surface in such a way that cylinder can roll without slipping. If the system is released from the stretched position of the spring, then the period will be-



(A) $2\pi\sqrt{\frac{M}{k}}$ **(B)** $2\pi\sqrt{\frac{3M}{2k}}$ **(C)** $2\pi\sqrt{\frac{M}{2k}}$ **(D)** $2\pi\sqrt{\frac{2M}{3k}}$

12. A ball is suspended by a thread of length L at the point O on the wall PQ which is inclined to the vertical through an angle α. The thread with the ball is now displaced through a small angle β away from the vertical and the wall. If $\beta < \alpha$, then the time period of oscillation of the pendulum will be-



(A) $2\pi\sqrt{\frac{L}{g}}$

(B) $2\pi\sqrt{\frac{L}{g}}\left[\pi + 2\sin^{-1}\left(\frac{\alpha}{\beta}\right)\right]$

(C) $2\pi\sqrt{\frac{L}{g}}\left[\frac{\pi}{2} + \sin^{-1}\left(\frac{\alpha}{\beta}\right)\right]$

(D) None of the above

13. The amplitude of a particle executing SHM about O is 10 cm . Then :
(A) when the kinetic energy is 0.64 times of its max. kinetic energy its displacement is 6 cm from O
(B) when the displacement is 5 cm from O its kinetic energy is 0.75 times its maximum kinetic energy
(C) Its total energy of SHM at any point is equal to its maximum kinetic energy
(D) Its speed is half the maximum speed when its displacement is half the maximum displacement

14. In the above problem, the frequency of oscillations of the cage will be-

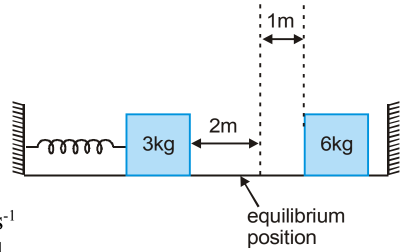
(A) $\frac{1}{2\pi}\left[\frac{k}{m}\right]^{1/2}$ **(B)** $\frac{1}{2\pi}\left[\frac{k}{M}\right]^{1/2}$ **(C)** $\frac{1}{2\pi}\left[\frac{k}{M+m}\right]^{1/2}$ **(D)** $\frac{1}{2\pi}\left[\frac{m}{k}\right]^{1/2}$

15. The displacement of a particle varies according to the relation $x = 3 \sin 100t + 8 \cos^2 50t$. Which of the following is/are correct about this motion .
(A) the motion of the particle is not S.H.M.
(B) the amplitude of the S.H.M. of the particle is 5 units
(C) the amplitude of the resultant S.H. M. is $\sqrt{73}$ units
(D) the maximum displacement of the particle from the origin is 9 units .

16. The angular frequency of a spring block system is ω_0 . This system is suspended from the ceiling of an elevator moving downwards with a constant speed v_0. The block is at rest relative to the elevator. Lift is suddenly stopped. Assuming the downwards as a positive direction, choose the wrong statement :

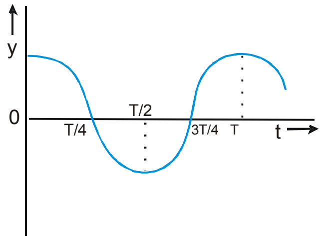
(A) the amplitude of the block is $\frac{v_0}{\omega_0}$
(B) the initial phase of the block is π
(C) the equation of motion for the block is $\frac{v_0}{\omega_0} \sin \omega_0 t$
(D) the maximum speed of the block is v_0

17. Two blocks of masses 3 kg and 6 kg rest on a horizontal smooth surface. The 3 kg block is attached to a spring with a force constant k = 900 Nm^{-1} which is compressed 2 m from beyond the equilibrium position. The 6 kg block is at rest at 1 m from mean position. 3 kg mass strikes the 6 kg mass and the two stick together.



(A) velocity of the combined masses immediately after the collision is 10 ms^{-1}
(B) velocity of the combined masses immediately after the collision is 5 ms^{-1}
(C) amplitude of the resulting oscillation is $\sqrt{2}$ m
(D) amplitude of the resulting oscillation is $\sqrt{5}$ /2m.

18. The displacement-time graph of a particle executing SHM is shown.



Which of the following statements is/are true?
(A) The velocity is maximum at t = T/2
(B) The acceleration is maximum at t = T
(C) The force is zero at t = 3T/4
(D) The potential energy equals the oscillation energy at t = T/2.

19. A disc of mass 3 m and a disc of mass m are connected by massless spring of stiffness k. The heavier disc is placed on the ground with the spring vertical and lighter disc on top. From its equilibrium position, the upper disc is pushed down by a distance δ and released. Then
(A) if δ >3mg/k, the lower disc will bounce up
(B) if δ =2 mg/k, maximum normal reaction from ground on lower disc = 6 mg
(C) if δ =2 mg/k, maximum normal reaction from ground on lower disc = 4 mg
(D) if δ > 4mg/k, the lower disc will bounce up

Part # II [Assertion & Reason Type Questions]

In each of the following questions, a statement of Assertion (A) is given followed by a corresponding statement of Reason (R) just below it . Of the statements mark the correct answer as

(A) Statement–1 is True, Statement–2 is True ; Statement–2 is a correct explanation for Statement–1

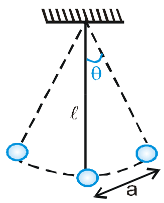
(B) Statement–1 is True, Statement–2 is True ; Statement–2 is NOT a correct explanation for Statement–1

(C) Statement–1 is True, Statement–2 is False.

(D) Statement–1 is False, Statement–2 is True.

1. **Statement–1 :** Pendulum clocks go slow in summer and fast in winter.
Statement–2 : The length of the pendulum used in clock increases in summer.

2. **Statement–1 :** The motion of a simple pendulum is simple harmonic only for $a << \ell$.



Statement–2 : Motion of a simple pendulum is SHM for small angular displacement.

3. **Statement–1 :** SHM is not a periodic motion.
Statement–2 : Periodic motion does not repeat its position after certain interval of time.

4. **Statement–1 :**Any oscillatory motion cannot be treated as simple harmonic.
Statement–2: Even under larger amplitude restoring force should be proportional to displacement for being classi fied as SHM.

5. **Statement–1 :** In compound pendulum, if suspension point and centre of oscillation are mutually interchange, then no change in time period is obtained.
Statement–2: Length of equivalent simple pendulum remains same in both the case.

6. **Statement–1 :** When a girl sitting on a swing stands up, the periodic time of the swing will increase.
Statement–2: In standing position of a girl, the length of the swing will decrease.

7. **Statement–1:** Mechanical energy of a particle executing SHM is E. Maximum KE of particle may be greater than E.
Statement–2 : Potential energy of a system may be negative.

Exercise # 3 Part # I [Matrix Match Type Questions]

1. A particle of mass 2 kg is moving on a straight line under the action of force $F = (8 - 2x)$ N. The particle is released at rest from x = 6 m. For the subsequent motion (All the values in the right column are in their S.I. units.)

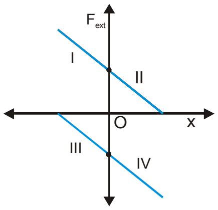
	Column- I		Column-II
(A)	Equilibrium position is at x =	(P)	$\pi/4$
(B)	Amplitude of S.H.M is	(Q)	$\pi/2$
(C)	Time taken to go directly from x = 2 m to x = 4 m	(R)	4
(D)	Energy of S.H.M. is	(S)	6
(E)	Phase constant of S.H.M. assuming equation of the form $A\sin(\omega t + \phi)$	(T)	2

2. In $y = A\sin wt + A\sin\left(\omega t + \frac{2\pi}{3}\right)$

	Column-I		Column-II
(A)	Motion	(P)	is periodic but not SHM
(B)	Amplitude	(Q)	is SHM
(C)	Initial phase	(R)	A
(D)	Maximum velocity	(S)	$\pi/3$
		(T)	$\omega A/2$
		(U)	None

3. A block is executing SHM on a rough horiozntal surface under the action of an external variable force. The force is plotted against the position x of the particle from the mean position.

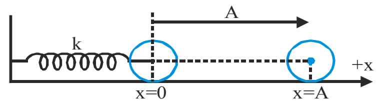
	Column I		Column II
(A)	x positive, v positive	(P)	I
(B)	x positive, v negative	(Q)	II
(C)	x negative, v positive	(R)	III
(D)	x negative, v negative	(S)	IV



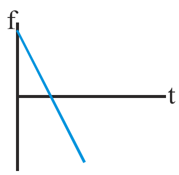
Part # II [Comprehension Type Questions]

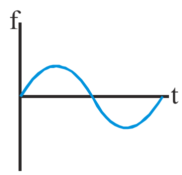
Comprehension # 1

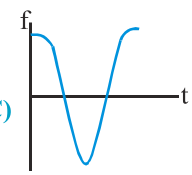
In case of pure rolling a = Rα, where a is the linear acceleration and α the angular acceleration. A disc of mass m and radius R is attached with a spring of force constant k at its centre as shown in figure. At x = 0, spring is unstretched. The disc is moved to x = A and then released. There is no slipping between disc and ground. Let f be the force of friction on the disc from the ground.

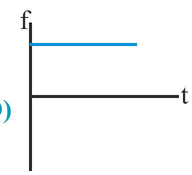


1. f versus t (time) graph will be as :

(A) 

(B) 

(C) 

(D) 

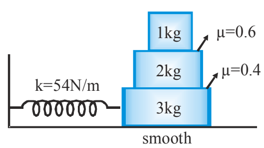
2. In the problem if k = 10 N/m, m = 2 kg, R = 1 m and A = 2 m. Find linear speed of the disc at mean position:

(A) $\sqrt{\frac{40}{3}}$ m/s **(B)** $\sqrt{20}$ m/s **(C)** $\sqrt{\frac{10}{3}}$ m/s **(D)** $\sqrt{\frac{50}{3}}$ m/s

Comprehension # 2

Angular frequency in SHM is given by $\omega = \sqrt{\frac{k}{m}}$. Maximum acceleration in SHM is $\omega^2 A$ and maximum value of friction between two bodies in contact is μN, where N is the normal reaction between the bodies.

1. In the figure shown, what can be the maximum amplitude of the system so that there is no slipping between any of the blocks ?



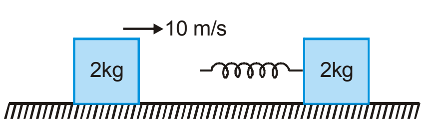
(A) $\frac{2}{7}$m **(B)** $\frac{3}{4}$m **(C)** $\frac{4}{9}$m **(D)** $\frac{10}{3}$m

2. Now the value of k, the force constant is increased then the maximum amplitude calculated in above question will:-

(A) remain same **(B)** increase **(C)** decrease **(D)** data in insufficient

Comprehension # 3

A 2 kg block moving with 10 m/s strikes a spring of constant π^2 N/m attached to 2kg block at rest kept on a smooth floor.



1. The time for which rear moving block remain in contact with spring will be

(A) $\sqrt{2}$ s **(B)** $\frac{1}{\sqrt{2}}$ s **(C)** 1 s **(D)** $\frac{1}{2}$ s

2. In the above question, the velocity of the rear 2 kg block after it separates from the spring will be

(A) 0 m/s **(B)** 5 m/s **(C)** 10 m/s **(D)** 7.5 m/s

Comprehension # 4

Two particles A and B are performing SHM along x and y-axis respectively with equal amplitude and frequency of 2 cm and 1 Hz respectively. Equilibrium positions of the particles A and B are at the coordinates (3 cm, 0) and (0, 4 cm) respectively. At t = 0, B is at its equilibrium position and moving towards the origin, while A is nearest to the origin and moving away from the origin.

1. Equation of motion of particle A can be written as-

(A) $x = (2 \text{ cm}) \cos 2\pi t$ **(B)** $x = (3 \text{ cm}) - (2 \text{ cm}) \cos 2\pi t$

(C) $x = (2 \text{ cm}) \sin 2\pi t$ **(D)** $x = (3 \text{ cm}) - (2 \text{ cm}) \sin 2\pi t$

2. Equation of motion of particle B can be written as-

(A) $y = (2 \text{ cm}) \cos 2\pi t$ **(B)** $y = (4 \text{ cm}) - (2 \text{ cm}) \cos 2\pi t$

(C) $y = (2 \text{ cm}) \sin 2\pi t$ **(D)** $y = (4 \text{ cm}) - (2 \text{ cm}) \sin 2\pi t$

3. Minimum and maximum distance between A and B during the motion is-

(A) $\sqrt{5}$ cm and $\sqrt{61}$ cm **(B)** 3 cm and 7 cm

(C) 1 cm and 5 cm **(D)** 9 cm and 16 cm

Comprehension # 5

An object of mass m is moving in uniform circular motion in xy-plane. The circle has radius R and object is moving clockwise around the circle with speed v. The motion is projected onto the x-axis where it appears as simple harmonic motion according to $x(t) = R\cos(\omega t + \phi)$. The motion starts from point of coordinates (0, R).

1. In this projection ω is-

(A) v/R **(B)** m^2/R **(C)** R/v **(D)** None of these

2. In this projection ϕ is-

(A) $\pi/2$ **(B)** π **(C)** $3\pi/2$ **(D)** 0

Exercise # 4 [Subjective Type Questions]

1. Find the resulting amplitude A' and the phase of the vibrations δ

$$S = A\cos(\omega t) + \frac{A}{2}\cos\left(\omega t + \frac{\pi}{2}\right) + \frac{A}{2}\cos(\omega t + \pi) + \frac{A}{8}\cos\left(\omega t + \frac{3\pi}{2}\right) = A'\cos(\omega t + \delta).$$

2. A particle executing simple harmonic motion completes 1200 oscillations per minute and passes through the mean position with a velocity of $3.14\,ms^{-1}$. Determine the maximum displacement of the particle from its mean position. Also obtain the displacement equation of the particle if its displacement be zero at the instant t = 0.

3. A particle is executing SHM given by $x = A\sin(\pi t + \phi)$. The initial displacement of particle is 1 cm and its initial velocity is π cm/sec. Find the amplitude of motion and initial phase of the particle.

4. Two particles A and B execute SHM along the same line with the same amplitude a and same frequency about same equilibrium position O. If the phase difference between them is $\phi = 2\sin^{-1}(0.9)$, then find the maximum distance between the two.

5. The shortest distance travelled by a particle executing S.H.M. from mean position in 2 seconds is equal to $\left(\frac{\sqrt{3}}{2}\right)$ times of its amplitude. Determine its time period.

6. A particle is oscillating in a straight line about a centre of force O , towards which when at a distance x the force is mn^2x where m is the mass , n a constant . The amplitude is a = 15 cm . When at a distance $\frac{a\sqrt{3}}{2}$ from O the particle receives a blow in the direction of motion which generates a velocity na . If the velocity is away from O, find the new amplitude.

7. The particle executing SHM in a straight line has velocities 8 m/s, 7 m/s, 4 m/s at three points distance one metre from each other. What will be the maximum velocity of the particle?

8. A body executing S.H.M. has its velocity 10 cm/s and 7 cm/s when its displacement from the mean position are 3 cm and 4 cm respectively. Calculate the length of the path.

9. The displacement of a particle varies with time as $x = (12\sin\omega t - 16\sin^3\omega t)$ cm. If its motion is SHM, find its maximum acceleration.

10. Potential Energy (U) of a body of unit mass moving in a one-dimension conservative force field is given by, $U = (x^2 - 4x + 3)$. All units are in S.I.

(i) Find the equilibrium position of the body.

(ii) Show that oscillations of the body about this equilibrium position is simple harmonic motion & find it time period.

(iii) Find the amplitude of oscillations if speed of the body at equilibrium posiiton is $2\sqrt{6}$ m/s

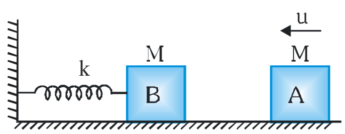
11. A point particle of mass 0.1 Kg is executing SHM with amplitude of 0.1m. When the particle passes through the mean position, its kinetic energy is 8×10^{-3} J. Obtain the equation of motion of this particle if the initial phase of oscillation is 45°.

12. A body of mass 1 kg suspended by an ideal spring oscillates up and down. The amplitude of oscillation is 1 metre and the periodic time is 1.57 second. Determine.

(i) Maximum speed of body. **(ii)** Maximum kinetic energy.

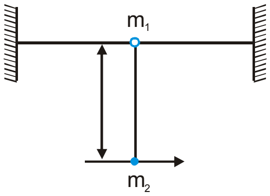
(iii) Total energy. **(iv)** Force constant of the spring.

13. In the figure shown, the block A collides with the block B and after collision they stick together. Calculate the amplitude of resultant vibration.



14. A body of mass 1 kg is suspended from a weightless spring having force constant 600 N/m. Another body of mass 0.5 kg moving vertically upwards hits the suspended body with a velocity of 3.0 m/s and get embedded in it. Find the frequency of oscillations and amplitude of motion.

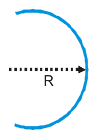
15. A small ring of mass m_1 is connected by a string of length ℓ to a small heavy bob of mass m_2. The ring is free to move (slide) along a fixed smooth horizontal wire. The bob is given a small displacement from its equilibrium position at right angles to string. Find period of small oscillations.



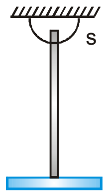
16. A block of mass 1kg hangs without vibrating at the end of a spring with a force constant 1 N/m attached to the ceiling of an elevator. The elevator is rising with an upward acceleration of g/4. The acceleration of the elevator suddenly ceases. What is the amplitude of the resulting oscillations?

17. Calculate the time period of a uniform square plate of side 'a' if it is suspended through a corner.

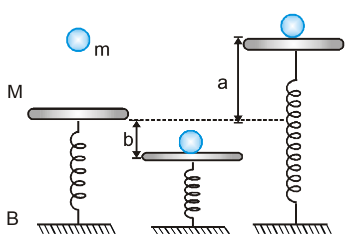
18. A half ring of mass m , radius R is hanged at its one end in verticle plane and is free to oscillate in its plane. Find oscillations frequency of the half ring.



19. Two identical rods each of mass m and length L, are rigidly joined and then suspended in a vertical plane so as to oscillate freely about an axis normal to the plane of paper passing through 'S' (point of suspension). Find the time period of such small oscillations.



20. A mass M is in static equilibrium on a massless vertical spring as shown in the figure. A ball of mass m dropped from certain height sticks to the mass M after colliding with it. The oscillations they perform reach to height 'a' above the original level of scales & depth 'b' below it.

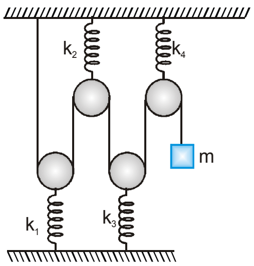


(i) find the constant of force of the spring

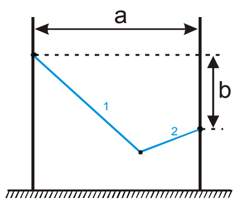
(ii) find the oscillation frequency.

(iii) what is the height above the initial level from which the mass m was dropped?

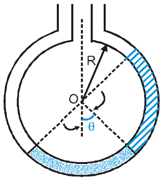
21. In the arrangement as shown in fig., pulleys are small and springs are ideal. k_1, k_2, k_3 and k_4 are force constants of the springs. Calculate period of small vertical oscillations of block of mass m.



22. One rope of a swing is fixed above the other rope by b. The distance between the poles of the swing is a. The lengths ℓ_1 and ℓ_2 of the ropes are such that $\ell_1^2 + \ell_2^2 = a^2 + b^2$ (Fig.) Determine the period T of small oscillations of the swing, neglecting the height of the swining person in comparison with the above lengths.

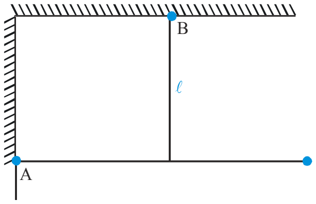


23. Two non-viscous, incompressible and immiscible liquids of densities ρ and 1.5ρ are poured into the two limbs of a circular tube of radius R and small cross-section kept fixed in a vertical plane as shown in figure. Each liquid occupies one-fourth the circumference of the tube.

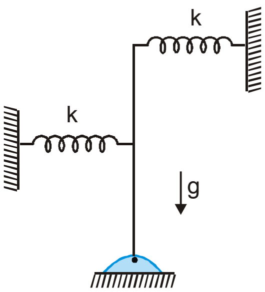


(i) Find the angle θ that the radius to the interface makes with the vertical in equilibrium position.
(ii) If the whole liquid column is given a small displacement from its equilibrium position, show that the resulting oscillations are simple harmonic. Find the time period of these oscillations.

24. A weightless rigid rod with a load at the end is hinged at point A to the wall so that it can rotate in all directions. The rod is kept in the horizontal position by a vertical inextensible thread of length ℓ, fixed at its midpoint. The load receives a momentum in the direction perpendicular to the plane of the figure. Determine the period T of small oscillations of the system.



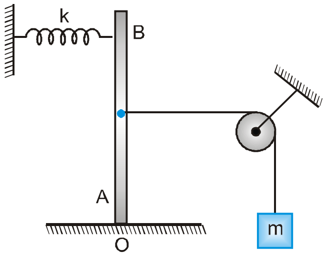
25. In the figure shown, the spring are connected to the rod at one end and at the midpoint. The rod is hinged at its lower end.



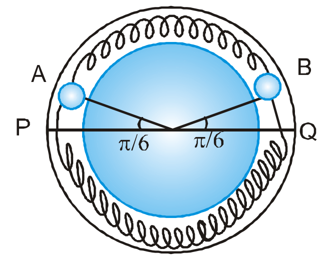
(i) Find the minimum value of k for rotational SHM of the rod (Mass m, length L)
(ii) If k = mg/L then find the angular frequency of oscillations of the rod.

26. A lift operator hung an exact pendulum clock on the lift wall in a lift in a building to know the end of the working day. The lift moves with an upward & downward accelerations during the same time (according to a stationary clock), the magnitudes of the acceleration remaining unchanged. Will the operator work for more or less than required time .

27. A massless rod rigidly fixed at O . A string carrying a mass m at one end is attached to point A on the rod so that OA = a . At another point B (OB = b) of the rod , a horizontal spring of force constant k is attached as shown . Find the period of small vertical oscillations of mass m around its equilibrium position



28. Two identical balls A and B each of mass 0.1 kg are attached to two identical massless springs. The spring mass system is constrained to move inside a rigid smooth pipe in the form of a circle as in fig. The pipe is fixed in a horizontal plane. The centres of the ball can move in a circle of radius 0.06 m. Each spring has a natural length $0.06\ \pi$m and force constant 0.1 N/m. Initially both the balls are displaced by an angle of $\theta = \pi/6$ radian with respect to diameter PQ of the circle and released from rest

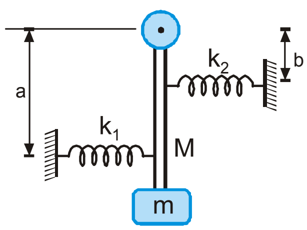


(i) calculate the frequency of oscillation of the ball B.

(ii) what is the total energy of the system

(iii) find the speed of the ball A when A and B are at the two ends of the diameter PQ.

29. A rod of mass M and length L is hinged at its one end and carries a block of mass m at its other end. A spring of force constant k_1 is installed at distance a from the hinge and another of force constant k_2 at a distance b as shown in the figure. If the whole arrangement rests on a smooth horizontal table top. Find the frequency of vibration.



Exercise # 5 Part # I [Previous Year Questions] [AIEEE/JEE-MAIN]

1. If a spring has time period T, and is cut into n equal parts, then the time period of each part will be - **[AIEEE-2002]**

(1) $T\sqrt{n}$ **(2)** $\frac{T}{\sqrt{n}}$ **(3)** nT **(4)** T

2. In a simple harmonic oscillator, at the mean position- **[AIEEE-2002]**

(1) kinetic energy is minimum, potential energy is maximum
(2) both kinetic and potential energies are maximum
(3) kinetic energy is maximum, potential energy is minimum
(4) both kinetic and potential energies are minimum

3. A child swinging on a swing in sitting position, stands up, then the time period of the swing will- **[AIEEE-2002]**

(1) increase **(2)** decrease **(3)** remain same
(4) increase if the child is long and decrease if the child is short

4. A mass M is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes SHM of time period T. If the mass is increased by m, the time period becomes 5T/3, then the ratio of $\frac{m}{M}$ is- **[AIEEE-2003]**

(1) $\frac{3}{5}$ **(2)** $\frac{25}{9}$ **(3)** $\frac{16}{9}$ **(4)** $\frac{5}{3}$

5. The displacement of a particle varies according to the relation $x = 4(\cos \pi t + \sin \pi t)$. The amplitude of the particle is- **[AIEEE-2003]**

(1) –4 **(2)** 4 **(3)** $4\sqrt{2}$ **(4)** 8

6. A body executes simple harmonic motion. The potential energy (PE), the kinetic energy (KE) and total energy (TE) are measured as function of displacement x. Which of the following statements is true ? **[AIEEE-2003]**

(1) KE is maximum when x = 0 **(2)** TE is zero when x = 0
(3) KE is maximum when x is maximum **(4)** PE is maximum when x = 0

7. Two particles A and B of equal masses are suspended from two massless springs of spring constants k_1 and k_2, respectively. If the maximum velocities, during oscillations are equal, the ratio of amplitudes of A and B is-

(1) $\sqrt{k_1 / k_2}$ **(2)** k_1/k_2 **(3)** $\sqrt{k_2 / k_1}$ **(4)** k_2/k_1 **[AIEEE-2003]**

8. The bob of a simple pendulum executes simple harmonic motion in water with a period t, while the period of oscillation of the bob is t_0 in air. Neglecting frictional force of water and given that the density of the bob is (4/3) × 1000 kg/m^3. What relationship between t and t_0 is true? **[AIEEE-2004]**

(1) $t = t_0$ **(2)** $t = t_0/2$ **(3)** $t = 2t_0$ **(4)** $t = 4t_0$

9. A particle at the end of a spring executes simple harmonic motion with a period t_1, while the corresponding period for another spring is t_2. If the period of oscillation with the two springs in series is T, then- **[AIEEE-2004]**

(1) $T = t_1 + t_2$ **(2)** $T^2 = t_1^2 + t_2^2$ **(3)** $T^{-1} = t_1^{-1} + t_2^{-1}$ **(4)** $T^{-2} = t_1^{-2} + t_1^{-2}$

10. The total energy of a particle, executing simple harmonic motion is- **[AIEEE-2004]**

(1) $\propto x$ **(2)** $\propto x^2$ **(3)** independent of x **(4)** $\propto x^{1/2}$

11. A particle of mass m is attached to a spring (of spring constant k) and has a natural angular frequency ω_0. An external force F(t) proportional to $\cos\omega t (\omega \neq \omega_0)$ is applied to the oscillator. The time displacement of the oscillator will be proportional to- **[AIEEE-2004]**

(1) $\frac{m}{\omega_0^2 - \omega^2}$ **(2)** $\frac{1}{m(\omega_0^2 - \omega^2)}$ **(3)** $\frac{1}{m(\omega_0^2 + \omega^2)}$ **(4)** $\frac{m}{(\omega_0^2 + \omega^2)}$

12. A particle of mass 0.3 kg is subjected to a force $F = -kx$ with $k = 15$ N/m. What will be its initial acceleration, if it is released from a point 20 cm away from the origin ? **[AIEEE-2005]**

(1) 3 m/s^2 **(2)** 15 m/s^2 **(3)** 5 m/s^2 **(4)** 10 m/s^2

13. The function $\sin^2(\omega t)$ represents- **[AIEEE-2005]**

(1) a periodic, but not simple harmonic, motion with a period $2\pi/\omega$
(2) a periodic, but not simple harmonic, motion with a period π/ω
(3) a simple harmonic motion with a period $2\pi/\omega$
(4) a simple harmonic motion with a period π/ω

14. Two simple harmonic motions are represented by the equations $y_1 = 0.1 \sin\left(100\pi t + \frac{\pi}{3}\right)$ and $y_2 = 0.1 \cos\pi t$. The phase difference of the velocity of particle 1, with respect to the velocity of particle 2 is- **[AIEEE-2005]**

(1) $\frac{-\pi}{6}$ **(2)** $\frac{\pi}{3}$ **(3)** $\frac{-\pi}{3}$ **(4)** $\frac{\pi}{6}$

15. If a simple harmonic motion is represented by $\frac{d^2x}{dt^2} + \alpha x = 0$, its time period is **[AIEEE-2005]**

(1) $\frac{2\pi}{\alpha}$ **(2)** $\frac{2\pi}{\sqrt{\alpha}}$ **(3)** $2\pi\alpha$ **(4)** $2\pi\sqrt{\alpha}$

16. The bob of a simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscillating bob gets suddenly unplugged. During observation, till water is coming out, the time period of oscillation would- **[AIEEE-2005]**

(1) first increase and then decrease to the original value
(2) first decrease and then increase to the original value
(3) remain unchanged
(4) increase towards a saturation value

17. The maximum velocity of a particle, executing simple harmonic motion with an amplitude 7 mm,is 4.4 m/s. The period of oscillation is- **[AIEEE-2006]**

(1) 0.01 s **(2)** 10s **(3)** 0.1 s **(4)** 100 s

18. Starting from the origin a body oscillates simple harmonically with a period of 2s. After what time will its kinetic energy be 75% of the total energy ? **[AIEEE-2006]**

(1) $\frac{1}{6}$ s **(2)** $\frac{1}{4}$ s **(3)** $\frac{1}{3}$ s **(4)** $\frac{1}{12}$ s

19. The displacement of an object attached to a spring and executing simple harmonic motion is given by $x = 2 \times 10^{-2} \cos \pi t$ metre. The time at which the maximum speed first occurs is- **[AIEEE-2007]**

(1) 0.5 s **(2)** 0.75 s **(3)** 0.125 s **(4)** 0.25 s

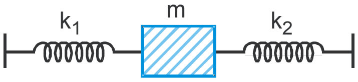
20. A point mass oscillates along the x-axis according to the law $x = x_0 \cos(wt - \pi/4)$ If the acceleration of the particle is written as- $a = A \cos(\omega t + \delta)$, then- **[AIEEE-2007]**

(1) $A = x_0, \delta = -\pi/4$ **(2)** $A = x_0\omega^2, \delta = \pi/4$ **(3)** $A = x_0\omega^2, \delta = -\pi/4$ **(4)** $A = x_0\omega^2, \delta = 3\pi/4$

21. A 2 kg block slides on a horizontal floor with a speed of 4 m/s. It strikes a uncompressed spring and compress it till the block is motionless. The kinetic friction force is 15 N and spring constant is 10,000 N/m. The spring compresses by- **[AIEEE-2007]**

(1) 5.5 cm **(2)** 2.5 cm **(3)** 11.0 cm **(4)** 8.5 cm

22. Two springs, of force constants k_1 and k_2, are connected to a mass m as shown. The frequency of oscillation of the mass is f. If both k_1 and k_2 are made four times their original values, the frequency of oscillation becomes **[AIEEE-2007]**



(1) f/2 **(2)** f/4 **(3)** 4f **(4)** 2f

23. A particle of mass m executes simple harmonic motion with amplitude a and frequency ν. The average kinetic energy during its motion from the position of equilibrium to the end is- **[AIEEE-2007]**

(1) $\pi^2 ma^2\nu^2$ **(2)** $\frac{1}{4}ma^2\nu^2$ **(3)** $4\pi^2 ma^2\nu^2$ **(4)** $2\pi^2 ma^2\nu^2$

24. If x, v and a denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period T, then, which of the following does not change with time ? **[AIEEE-2009]**

(1) $aT + 2\pi v$ **(2)** $\frac{aT}{v}$ **(3)** $a^2T^2 + 4\pi^2v^2$ **(4)** $\frac{aT}{x}$

25. A mass M, attached to a horizontal spring, executes S.H.M. with amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\frac{A_1}{A_2}\right)$ is :- **[AIEEE-2011]**

(1) $\left(\frac{M}{M+m}\right)^{1/2}$ **(2)** $\left(\frac{M+m}{M}\right)^{1/2}$ **(3)** $\frac{M}{M+m}$ **(4)** $\frac{M+m}{M}$

26. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x-axis. Their mean position is separated by distance $X_0(X_0 > A)$. If the maximum separation between them is $(X_0 + A)$, the phase difference between their motion is :- **[AIEEE-2011]**

(1) $\frac{\pi}{4}$ **(2)** $\frac{\pi}{6}$ **(3)** $\frac{\pi}{2}$ **(4)** $\frac{\pi}{3}$

27. A wooden cube (density of wood 'd') of side 'ℓ' floats in a liquid of density 'ρ' with its upper and lower surfaces horizontal. If the cube is pushed slightly down and released, it performs simple harmonic motion of period 'T'. Then, 'T' is equal to :- **[AIEEE-2011]**

(1) $2\pi\sqrt{\frac{\ell\rho}{(\rho-d)g}}$ **(2)** $2\pi\sqrt{\frac{\ell d}{\rho g}}$ **(3)** $2\pi\sqrt{\frac{\ell\rho}{dg}}$ **(4)** $2\pi\sqrt{\frac{\ell d}{(\rho-d)g}}$

28. If a spring of stiffness 'k' is cut into two parts 'A' and 'B' of length $\ell_A : \ell_B = 2 : 3$, then the stiffness of spring 'A' is given by :- **[AIEEE-2011]**

(1) $\frac{5}{2}k$ **(2)** $\frac{3k}{5}$ **(3)** $\frac{2k}{5}$ **(4)** k

29. If a simple pendulum has significant amplitude (up to a factor of 1/e of original) only in the period between t = 0 s to t = τs, then τ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity, with 'b' as the constant of proportionality, the average life time of the pendulum is (assuming damping is small) in seconds :- **[AIEEE-2012]**

(1) $\frac{2}{b}$ **(2)** $\frac{0.693}{b}$ **(3)** b **(4)** $\frac{1}{b}$

30. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M. The piston and the cylinder have equal cross sectional area A. When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is P_0. The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency. **[AIEEE-2013]**

(1) $\frac{1}{2\pi}\frac{A\gamma P_0}{V_0 M}$ **(2)** $\frac{1}{2\pi}\frac{V_0 M P_0}{A^2\gamma}$ **(3)** $\frac{1}{2\pi}\sqrt{\frac{A^2\gamma P_0}{MV_0}}$ **(4)** $\frac{1}{2\pi}\sqrt{\frac{MV_0}{A\gamma P_0}}$

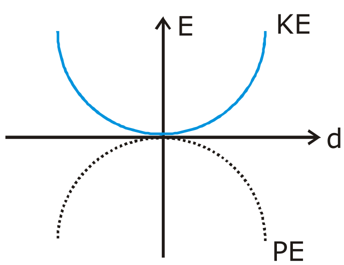
31. A particle moves with simple harmonic motion in a straight line. In first τ s, after starting from rest it travels a distance a, and in next τ s it travels 2a, in same direction, then : **[JEE (Main) -2014]**

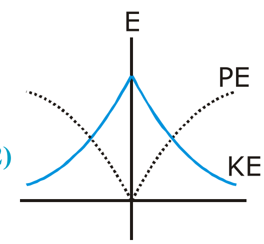
(1) amplitude of motion is 4a **(2)** time period of oscillations is 6τ
(3) amplitude of motion is 3a **(4)** time period of oscillations is 8τ

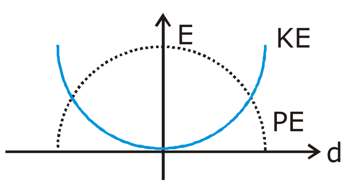
32. A pipe of length 85 cm is closed from one end. Find the number of possible natural oscillations of air column in the pipe whose frequencies lie below 1250 Hz. The velocity of sound in air is 340 m/s. **[JEE (Main) -2014]**

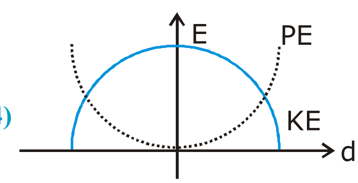
(1) 6 **(2)** 4 **(3)** 12 **(4)** 8

33. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly? (graphs are schematic and not drawn to scale) **[JEE (Main) -2015]**

(1)


(2)


(3)


(4)


34. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1s resolution. The accuracy in the determination of g is : **[JEE (Main) -2015]**

(1) 1% **(2)** 5% **(3)** 2% **(4)** 3%

35. A student measures the time period of 100 oscillations of a simple pendulum four times. The data set is 90 s, 91 s, 95 s and 92 s. If the minimum division in the measuring clock is 1 s, then the reported mean time should be :

(1) 92 ± 5.0 s **(2)** 92 ± 1.8 s **(3)** 92 ± 3 s **(4)** 92 ± 2 s **[JEE (Main) -2016]**

36. A particle performs simple harmonic motion with amplitude A. Its speed is troubled at the instant that it is at a distance $\frac{2A}{3}$ from equilibrium position. The new amplitude of the motion is : **[JEE (Main) -2016]**

(1) 3A **(2)** $A\sqrt{3}$ **(3)** $\frac{7A}{3}$ **(4)** $\frac{A}{3}\sqrt{41}$

Part # II [Previous Year Questions][IIT-JEE ADVANCED]

MCQ's one correct answers

1. A particle of mass m is executing oscillation about the origin on the x-axis. Its potential energy is $U(x) = k|x|^3$, where k is a positive constant. If the amplitude of oscillation is a, then its time period T is :-**[IIT-JEE 1998]**

(A) proportional to $\frac{1}{\sqrt{a}}$ **(B)** independent of a **(C)** proportional to $\sqrt{a}$ **(D)** proportional to $a^{3/2}$

2. A spring of force constant k is cut into two pieces such that one piece is double the length of the other. Then the long piece will have a force constant of :- **[IIT-JEE 1999]**

(A) $\frac{2}{3}k$ **(B)** $\frac{3}{2}k$ **(C)** 3k **(D)** 6k

3. A particle free to move along the x-axis has potential energy given by : $U(x) = k[1 - \exp(-x^2)]$ for $-\infty \le x \le +\infty$, where k is a positive constant of appropriate dimensions. Then :- **[IIT-JEE 1999]**

(A) at points away from the origin, the particle is in unstable equilibrium

(B) for any finite non-zero value of x, there is a force directed away from the origin

(C) if its total mechanical energy is $\frac{k}{2}$, it has its minimum kinetic energy at the origin

(D) for small displacements from x = 0, the motion is simple harmonic.

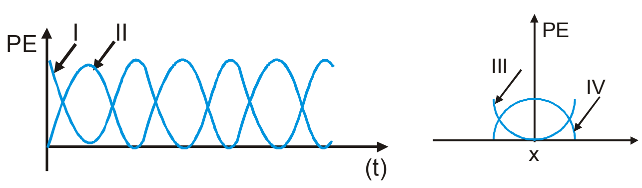
4. The period of oscillation of simple pendulum of length L suspended from the roof of the vehicle which moves without friction, down an inclined plane of inclination α, is given by :- **[IIT-JEE 2000]**

(A) $2\pi\sqrt{\frac{L}{g\cos\alpha}}$ **(B)** $2\pi\sqrt{\frac{L}{g\sin\alpha}}$ **(C)** $2\pi\sqrt{\frac{L}{g}}$ **(D)** $2\pi\sqrt{\frac{L}{g\tan\alpha}}$

5. A particle executes simple harmonic motion between x = –A and x = +A. The time taken for it to go from O to $\frac{A}{2}$ is T_1 and to go from $\frac{A}{2}$ to A is T_2, then :- **[IIT-JEE 2001]**

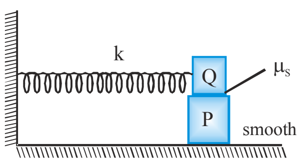
(A) $T_1 < T_2$ **(B)** $T_1 > T_2$ **(C)** $T_1 = T_2$ **(D)** $T_1 = 2T_2$

6. For a particle executing SHM the displacement x given by $x = A\cos\omega t$. Identify the graph which represents the variation of potential energy (PE) as a function of time t and displacement x :- **[IIT-JEE 2003]**



(A) I, III **(B)** II, IV **(C)** II, III **(D)** I, IV

7. A block P of mass m is placed on a frictionless horizontal surface. Another block Q of same mass is kept on P and connected to the wall with the help of a spring of spring constant k as shown in the figure, μ_s is the coefficient of friction between P and Q. The block move together performing SHM of amplutude A. The maximum value of the friction force between P and Q is **[IIT-JEE 2004]**

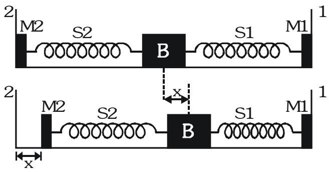


(A) kA **(B)** $\frac{kA}{2}$ **(C)** zero **(D)** $\mu_s mg$

8. A simple pendulum has time period T_1. The point of suspension is now moved upward according to the relation $y = Kt^2$, ($K = 1\ m/s^2$) where y is the vertical displacement. The time period now becomes T_2. The ratio of $\frac{T_1^2}{T_2^2}$ is : ($g = 10\ m/s^2$) **[IIT-JEE 2005]**

(A) $\frac{6}{5}$ **(B)** $\frac{5}{6}$ **(C)** 1 **(D)** $\frac{4}{5}$

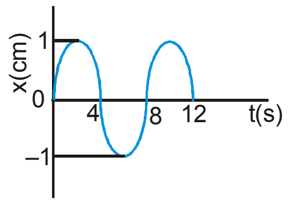
9. A block (B) is attached to two unstretched springs S_1 and S_2 with spring constants k and 4k, respectively (see figure I). The other ends are attached to identical supports M_1 and M_2 not attached to the walls. The springs and supports have negligible mass. .There is no friction anywhere. The block B is displaced towards wall 1 by a small distance x (figure II) and released. **[IIT-JEE 2008]**



The block returns and moves a maximum distance y towards wall 2. displacements x and y are measured with respect to the equilibrium position of the block B. The ratio $\frac{y}{x}$ is

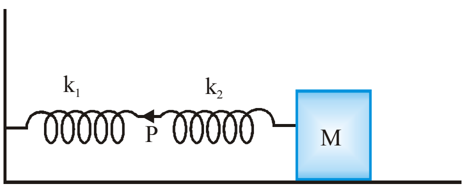
(A) 4 **(B)** 2 **(C)** $\frac{1}{2}$ **(D)** $\frac{1}{4}$

10. The x-t graph of a particle undergoing simple harmonic motion is shown below. The acceleration of the particle at $t = 4/3$ s is **[IIT-JEE 2009]**



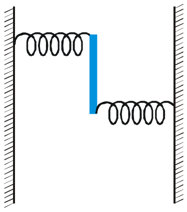
(A) $\frac{\sqrt{3}}{32}\pi^2 cm/s^2$ **(B)** $\frac{-\pi^2}{32} cm/s^2$ **(C)** $\frac{\pi^2}{32} cm/s^2$ **(D)** $-\frac{\sqrt{3}}{32}\pi^2 cm/s^2$

11. The mass M shown in the figure oscillates in simple harmonic motion with amplitude A. The amplitude of the point P is : **[IIT-JEE 2009]**



(A) $\frac{k_1 A}{k_2}$ **(B)** $\frac{k_2 A}{k_1}$ **(C)** $\frac{k_1 A}{k_1 + k_2}$ **(D)** $\frac{k_2 A}{k_1 + k_2}$

12. A uniform rod of length L and mass M is pivoted at the centre. Its two ends are attached to two springs of equal spring constant k. The springs are fixed to rigid supports as shown in the figure, and the rod is free to oscillate in the horizontal plane. The rod is gently pushed through a small angle θ in one direction and released. The frequency of oscillation is :- **[IIT-JEE 2009]**



(A) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ **(B)** $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$ **(C)** $\frac{1}{2\pi}\sqrt{\frac{6k}{M}}$ **(D)** $\frac{1}{2\pi}\sqrt{\frac{24k}{M}}$

MCQ's one or more than one correct answers

1. Three simple harmonic motions in the same direction having the same amplitude and same period are superposed. If each differ in phase from the next by 45°, then the :- **[IIT-JEE 1999]**

(A) resultant amplitude is $(1 + \sqrt{2})a$

(B) phase of the resultant motion relative to the first is 90°

(C) energy associated with the resulting motion is $(3 + 2\sqrt{2})$ times the energy associated with any single motion

(D) resulting motion is not simple harmonic

2. Function : $x = A\sin^2\omega t + B\cos^2\omega t + C \sin\omega t \cos\omega t$ represents SHM :- **[IIT-JEE 2006]**

(A) for any value of A, B and C (except C = 0)

(B) if $A = -B$, $C = 2B$, amplitude $= |B\sqrt{2}|$

(C) if $A = B$; $C = 0$

(D) if $A = B$; $C = 2B$, amplitude $= |B|$

3. A particle of mass m is attached to one end of a mass-less spring of force constant k, lying on a frictionless horizontal plane. The other end of the spring is fixed. The particle starts moving horizontally from its equilibrium position at time t = 0 with an initial velocity u_0. When the speed of the particle is 0.5 u_0, it collides elastically with a rigid wall. After this collision :- **[IIT-JEE 2013]**

(A) the speed of the particle when it returns to its equilibrium position is u_0

(B) the time at which the particle passes through the equilibrium position for the first time is $t=\pi\sqrt{\frac{m}{k}}$

(C) the time at which the maximum compression of the spring occurs is $t = \frac{4\pi}{3}\sqrt{\frac{m}{k}}$

(D) the time at which the particle passes through the equilibrium position for the second time is $t = \frac{5\pi}{3}\sqrt{\frac{m}{k}}$

4. In an experiment to determine the acceleratin due to gravity g, the formula used for the time period of a periodic motion nis $T = 2\pi\sqrt{\frac{7(R-T)}{5g}}$. The values of R and r are measured to be (60 ± 1) mm and (10 ± 1) mm, respectively. In five successive measurements, the time period is found to be 0.52 s, 0.56 s, 0.57 s, 0.54, and 0.59 s. The least count of the watch used for the measurement of time period is 0.014 s. Which of the following statment(s) is(are) ture?
(A) The error in the measurement of r is 10% **[IIT-JEE 2016]**
(B) The error in the measurement of T is 3.57%
(C) The error in the measurement of T is 2%
(D) The error in the determined value of g is 11%

5. A block with mass M is connected by a massless spring with stiffness constant k to a rigid wall and moves without friction on a horizontal surface. The block oscillates with small amplitude A about an equlibrium position x_0. Consider two cases: (i) when the block is at x_0 ; and (ii) when the block is at $x = x_0 + A$. In both the cases, a particle with mass m(<M) is softly placed on the block after which they stick to each other. Which of the following statement(s) is(are) true about the motion after the mass m is placed on the mass M? **[IIT-JEE 2016]**

(A) The amplitude of oscillation in the first case changes by a factor of $\sqrt{\frac{M}{m+M}}$, whereas in the second case it remains unchanged
(B) The final time period of oscillation in both the cases is same
(C) The total energy decreases in both the cases
(D) The instattneous speed at x_0 of the combined masses decreases in both the cases

Match the column

1. Column I describes some situations in which a small object moves. Column II describes some characteristics of these motions. Match the situations in column I with the characteristics in column II. **[IIT-JEE 2007]**

Column I	Column II
(A) The object moves on the x-axis under a conservative force in such a way that its "speed"and "positive" satisfy $v = c_1\sqrt{c_2 - x^2}$, where c_1 and c_2 are positive constants.	**(P)** The object executes a simple harmonic motion.
(B) The object moves on the x-axis in such a way that its velocity and its displacement from the origin satisfy $v = -kx$, where k is a positive constant.	**(Q)** The object does not change its direction.
(C) The object is attached to one end of a mass-less spring of a given spring constant. The other end of the spring is attached to the ceiling of an elevator. Initially everything is at rest. The elevator starts going upwards with a constant acceleration 'a'. The motion of the object is observed from the elevator during the period it maintains this acceleration.	**(R)** The kinetic energy of the object keeps on decreasing
(D) The object is projected from the earth's surface vertically upwards with a speed $2\sqrt{\frac{GM_e}{R_e}}$, where M_e is the mass of the earth and R_e is the radius of the earth. Neglect forces from objects other than the earth.	**(S)** The object can change its direction only once.

2. Column I gives a list of possible set of parameters measured in some experiments. The variations of the parameters in the form of graphs are shown in Column II. Match the set of parameters given Column I with the graphs given in Column II. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS. **[IIT-JEE 2008]**

Column I

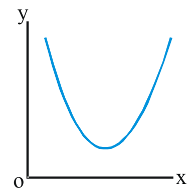
(A) Potential energy of a simple pendulum (y axis) as a function of displacement (x axis)

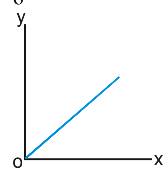
(B) Displacement (y axis) as a function of time (x axis) for a one dimensional motion at zero or constant acceleration when the body is moving along the positive x-direction.

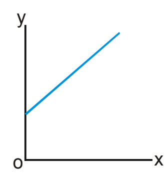
(C) Range of a projectile (y axis) as a function of its velocity (x axis) when projected at a fixed angle.

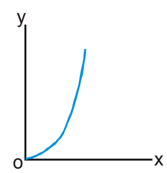
(D) The square of the time period (y axis) of a simple pendulum as a function of its length (x axis)

Column II

(P)


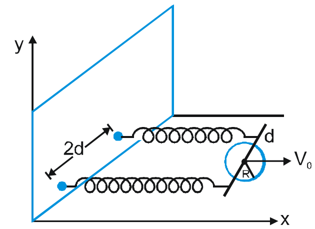
(Q)


(R)


(S)


Comprehension

A uniform thin cylindrical disk of mass M and radius R is attached to two identical massless springs of spring constant k which are fixed to the wall as shown in the figure. The springs are attached to the axle of the disk symmetrically on either side at a distance d from its centre. The axle is massless and both the springs and the axle are in a horizontal plane. The unstrethced length of each spring is L. The disk is initially at its equilibrium position with its centre of mass (CM) at a distance L from the wall. The disk rolls without slipping with velocity $\vec{V}_0 = V_0\tilde{i}$. The coefficient of friction is μ. **[IIT-JEE 2008]**



1. The net external force acting on the disk when its centre of mass is at displacement x with respect to its equilibrium position is

(A) $-kx$ **(B)** $-2kx$ **(C)** $-\frac{2kx}{3}$ **(D)** $-\frac{4kx}{3}$

2. The centre of mass of the disk undergoes simple harmonic motion with angular frequency ω equal to

(A) $\sqrt{\frac{k}{M}}$ **(B)** $\sqrt{\frac{2k}{M}}$ **(C)** $\sqrt{\frac{2k}{3M}}$ **(D)** $\sqrt{\frac{4k}{3M}}$

3. The maximum value of V_0 for which the disk will roll without slipping is

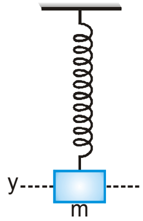
(A) $\mu g\sqrt{\frac{M}{k}}$ **(B)** $\mu g\sqrt{\frac{M}{2k}}$ **(C)** $\mu g\sqrt{\frac{3M}{k}}$ **(D)** $\mu g\sqrt{\frac{5M}{2k}}$

Subjective Questions

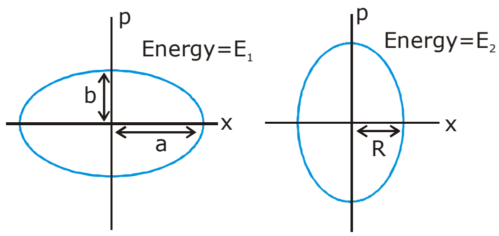
1. Two masses m_1 and m_2 connected by a light spring of natural length ℓ_0 is compressed completely and tied by a string. This system while moving with a veloicty v_0 along +ve x-axis pass through the origin at t=0. At this position the string snaps. Position of mass m_1 at time t is given by the equation $x_1(t) = v_0t - A(1-\cos\omega t)$. Calculate:(i) position of the particle m_2 as a functionof time. (ii) ℓ_0 in terms of A. **[IIT-JEE 2003]**

2. A solid sphere of radius R is floating in a liquid of density ρ with half of its volume submerged. If the sphere is slightly pushed and released, it starts performing simple harmonic motion. Find the frequency of these oscillations. **[IIT-JEE 2004]**

3. A mass m is undergoing SHM in the vertical direction about the mean position y_0 with amplitude A and angular frequency ω. At a distance y from the mean position, the mass detaches from the spring. Assume that the spring contracts and does not obstruct the motion of m. Find the distance y. (measured from the mean position) such that the height h attained by the block is maximum ($A\omega^2 > g$). **[IIT-JEE 2005]**



4. Two independent harmonic oscillators of equal mass are oscillating about the origin with angular frequencies ω_1 and ω_2 and have total energies E_1 and E_2, respectively. The variations of their moments p with positions x are shown in the figures. If $a/b = n^2$ and $a/R = n$, then the correct equation(s) is (are) **[IIT-JEE 2015]**



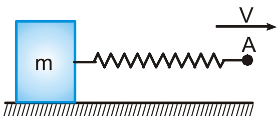
(A) $E_1\omega_2 = E_2\omega_2$ **(B)** $\frac{\omega_2}{\omega_1} = n^2$ **(C)** $\omega_1\omega_2 = n^2$ **(D)** $\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$

5. Four harmonic waves of equal frequencies and equal intensities I_0 have phase angles 0, $\pi/3$, $2\pi/3$ and π. when they are superposed, the intensity of the resulting wave is nI_0. The value of n is. **[IIT-JEE 2015]**

MOCK TEST

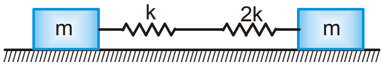
SECTION - I : STRAIGHT OBJECTIVE TYPE

1. A block of mass ' m ' is attached to a spring in natural length of spring constant ' k ' . The other end A of the spring is moved with a constant velocity v away from the block . Find the maximum extension in the spring.



(A) $\frac{1}{4}\sqrt{\frac{mv^2}{k}}$ **(B)** $\sqrt{\frac{mv^2}{k}}$ **(C)** $\frac{1}{2}\sqrt{\frac{mv^2}{k}}$ **(D)** $2\sqrt{\frac{mv^2}{k}}$

2. A system is shown in the figure. The time period for small oscillations of the two blocks will be.



(A) $2\pi\sqrt{\frac{3m}{k}}$ **(B)** $2\pi\sqrt{\frac{3m}{2k}}$ **(C)** $2\pi\sqrt{\frac{3m}{4k}}$ **(D)** $2\pi\sqrt{\frac{3m}{8k}}$

3. Two particles P and Q describe S.H.M. of same amplitude a, same frequency f along the same straight line from the same mean positon. The maximum distance between the two particles is a$\sqrt{2}$. The phase difference between the particle is:

(A) zero **(B)** $\pi/2$ **(C)** $\pi/6$ **(D)** $\pi/3$

4, The time taken by a particle performing SHM on a straight line to pass from point A to B where its velocities are same is 2 seconds. After another 2 seconds it returns to B. The time period of oscillation is (in seconds):

(A) 2 **(B)** 4 **(C)** 6 **(D)** 8

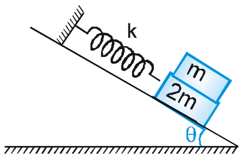
5. A particle performs S.H.M. on x–axis with amplitude A and time period T. The time taken by the particle to travel a distance A/5 starting from rest is:

(A) $\frac{T}{20}$ **(B)** $\frac{T}{2\pi}\cos^{-1}\left(\frac{4}{5}\right)$ **(C)** $\frac{T}{2\pi}\cos^{-1}\left(\frac{1}{5}\right)$ **(D)** $\frac{T}{2\pi}\sin^{-1}\left(\frac{1}{5}\right)$

6. A rod of length ℓ is in motion such that its ends A and B are moving along x-axis and y-axis respectively. It is given that $\frac{d\theta}{dt}$ = 2 rad/s always. P is a fixed point on the rod. Let M be the projection of P on x-axis. For the time interval in which θ changes from 0 to $\frac{\pi}{2}$, choose the correct statement,

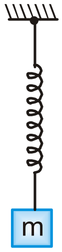
(A) The acceleration of M is always directed towards right
(B) M executes SHM
(C) M moves with constant speed
(D) M moves with constant acceleration

7. The coefficient of friction between blocks of mass m and 2m is $\mu = 2\tan\theta$. There is no friction between block of mass 2m and inclined plane. The maximum amplitude of two block system for which there is no relative motion between the blocks.



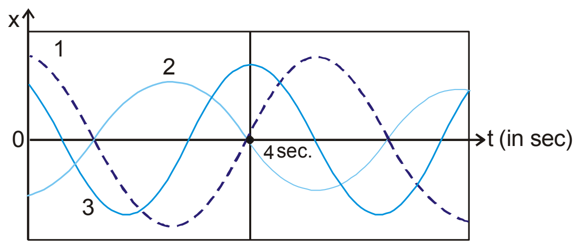
(A) $g\sin\theta\sqrt{\frac{k}{m}}$ **(B)** $\frac{mg\sin\theta}{k}$ **(C)** $\frac{3mg\sin\theta}{k}$ **(D)** None of these

8. A block of mass 'm' is suspended from a spring and executes vertical SHM of time period T as shown in figure. The amplitude of the SHM is A and spring is never in compressed state during the oscillation. The magnitude of minimum force exerted by spring on the block is



(A) $mg - \frac{4\pi^2}{T^2}mA$ **(B)** $mg + \frac{4\pi^2}{T^2}mA$ **(C)** $mg - \frac{\pi^2}{T^2}mA$ **(D)** $mg + \frac{\pi^2}{T^2}mA$

9. Graph shows the x(t) curves for three experiments involving a particular spring- block system oscillating in SHM. The kinetic energy of the system is maximum at t = 4 sec. for the situation :



(A) 1 **(B)** 2 **(C)** 3 **(D)** Same in all

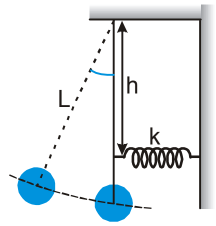
10. A particle performs S.H.M. of amplitude A along a straight line. When it is at a distance $\frac{\sqrt{3}}{2}$ A from mean position, its kinetic energy gets increased by an amount $\frac{1}{2}m\omega^2A^2$ due to an impulsive force. Then its new amplitude becomes:

(A) $\frac{\sqrt{5}}{2}$ A **(B)** $\frac{\sqrt{3}}{2}$ A **(C)** $\sqrt{2}$ A **(D)** $\sqrt{5}$ A

11. A horizontal spring–block system of mass 2kg executes S.H.M. When the block is passing through its equilibrium position, an object of mass 1kg is put on it and the two move together. The new amplitude of vibration is (A being its initial amplitude):

(A) $\sqrt{\frac{2}{3}}$ A **(B)** $\sqrt{\frac{3}{2}}$ A **(C)** $\sqrt{2}$ A **(D)** $\frac{A}{\sqrt{2}}$

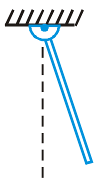
12. A pendulum of length L and bob of mass M has a spring of force constant k connected horizontally to it at a distance h below its point of suspension. The rod is in equilibrium in vertical position. The rod of length L used for vertical suspension is rigid and massless. The frequency of vibration of the system for small values of θ is :



(A) $\frac{1}{2\pi L}\sqrt{gL+\frac{kh}{m}}$ **(B)** $\frac{1}{2\pi L}\sqrt{\frac{mgL+k}{m}}$

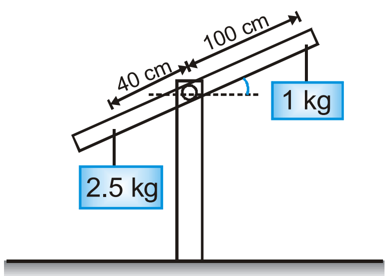
(C) $2\pi\sqrt{\frac{mL^2}{mgL+kh}}$ **(D)** $\frac{1}{2\pi L}\sqrt{gL+\left(\frac{kh^2}{m}\right)}$

13. A metre stick swinging in vertical plane about a fixed horizontal axis passing through its one end undergoes small oscillation of frequency f_0. If the bottom half of the stick is cut off, then its new frequency of small oscillation would become:



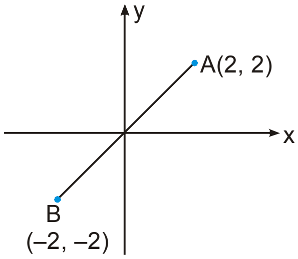
(A) f_0 **(B)** $\sqrt{2}\,f_0$ **(C)** $2f_0$ **(D)** $2\sqrt{2}\,f_0$

14. A straight rod of negligible mass is mounted on a frictionless pivot and masses 2.5 kg and 1 kg are suspended at distances 40 cm and 100 cm respectively from the pivot as shown. The rod is held at an angle θ with the horizontal and released.



(A) The rod executes periodic motion about horizontal position after the release.
(B) The rod remains stationary after the release.
(C) The rod comes to rest in vertical position with 2.5 kg mass at the lowest point
(D) The rod executes periodic motion about vertical position after the release.

15. A particle of mass m = 2 kg executes SHM in xy-plane between points A and B as shown in figure, under action of force $\vec{F} = F_x\hat{i} + F_y\hat{j}$. Minimum time taken by particle to move from A to B is 1 sec. At t = 0 the particle is at x = 2 and y = 2. Then F_x as function of time t is



(A) $-4\pi^2 \sin \pi t$ **(B)** $-4\pi^2 \cos \pi t$
(C) $4\pi^2 \cos \pi t$ **(D)** None of these

16. Two masses m_1 and m_2 ($m_1 > m_2$) are suspended by two springs vertically and are in equilibrium, extensions in the springs were same. Both the masses are displaced in the vertical direction by same distance and released. In subsequent motion T_1, T_2 are their time periods and E_1, E_2 are the energies of oscillations respectively then:

(A) $T_1 = T_2$; $E_1 < E_2$ **(B)** $T_1 > T_2$; $E_1 > E_2$
(C) $T_1 < T_2$; $E_1 > E_2$ **(D)** $T_1 = T_2$; $E_1 > E_2$

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

17. A particle is subjected to two simple harmonic motions along x and y directions according to, $x = 3 \sin 100\pi t$; $y = 4 \sin 100\pi t$.

(A) Motion of particle will be on ellipse traversing it in clockwise direction.

(B) Motion of particle will be on a straight line with slope 4/3.

(C) Motion will be a simple harmonic motion with amplitude 5.

(D) Phase difference between two motions is $\pi/2$.

18. The speed v of a particle moving along a straight line, when it is at a distance (x) from a fixed point of the line is given by

$$v^2 = 108 - 9x^2$$

(all quantities are in cgs units) :

(A) the motion is uniformly accelerated along the straight line

(B) the magnitude of the acceleration at a distance 3cm from the fixed point is 27 cm/sec^2

(C) the motion is simple harmonic about the given fixed point.

(D) the maximum displacement from the fixed point is 4 cm.

19. A horizontal plank has a rectangular block placed on it. The plank starts oscillating vertically and simple harmonically with an amplitude of 40 cm. The block just loses contact with the plank when the latter is at momentary rest. Then :

(A) the period of oscillation is $\left(\frac{2\pi}{5}\right)$

(B) the block weighs double its weight, when the plank is at one of the positions of momentary rest.

(C) the block weighs 0.5 times its weight on the plank halfway up

(D) the block weighs 1.5 times its weight on the plank halfway down

(E) the block weighs its true weight on the plank when the latter moves fastest

20. A 20 gm particle is subjected to two simple harmonic motions

$$x_1 = 2 \sin 10t,$$

$$x_2 = 4 \sin \left(10t + \frac{\pi}{3}\right).$$ where x_1 & x_2 are in metre & t is in sec.

(A) The displacement of the particle at t = 0 will be $2\sqrt{3}$ m.

(B) Maximum speed of the particle will be $20\sqrt{7}$ m/s.

(C) Magnitude of maximum acceleration of the particle will be $200\sqrt{7}$ m/s^2.

(D) Energy of the resultant motion will be 28 J.

21. The position of a particle w.r.t. origin varies according to the relation $x = 3 \sin 100t + 8 \cos^2 50t$. Which of the following is/are correct about this motion.

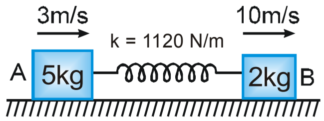
(A) the motion of the particle is not S.H.M.

(B) the amplitude of the S.H.M. of the particle is 5 units

(C) the amplitude of the resultant S.H. M. is $\sqrt{73}$ units

(D) the maximum displacement of the particle from the origin is 9 units.

22. Two blocks A (5kg) and B(2kg) attached to the ends of a spring constant 1120N/m are placed on a smooth horizontal plane with the spring undeformed. Simultaneously velocities of 3m/s and 10m/s along the line of the spring in the same direction are imparted to A and B then



(A) when the extension of the spring is maximum the velocities of A and B are zero.
(B) the maximum extension of the spring is 25cm.
(C) the first maximum compression occurs $3\pi/56$ seconds after start.
(D) maximum extension and maximum compression occur alternately.

SECTION - III : ASSERTION AND REASON TYPE

23. **Assertion :** Kinetic energy of SHM at mean position is equal to potential energy at ends for a particle moving in SHM.
Reason : Total energy in SHM is conserved.
(A) If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.
(B) If both Assertion and Reason are true, but Reason is not correct explanation of the Assertion.
(C) if Assertion is true, but the Reason is false.
(D) if Assertion is false, but the Reason is true.

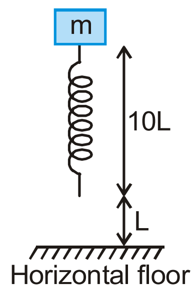
24. **Statement-1 :** A SHM may be assumed as composition of many SHM's.
Statement-2 : Superposition of many SHM's (along same line) of same frequency will be a SHM.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

25. **Statement-1 :** A particle is moving along x-axis. The resultant force F acting on it is given by $F = -ax - b$. Where a and b are both positive constants. The motion of this particle is not SHM.
Statement-2 : In SHM resultant force must be proportional to the displacement from mean position.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

SECTION - IV : COMPREHENSION TYPE

Comprehension - 1

A small block of mass m is fixed at upper end of a massless vertical spring of spring constant $K = \frac{4mg}{L}$ and natural length '10L'. The lower end of spring is free and is at a height L from fixed horizontal floor as shown in the figure. The spring is initially unstressed and the spring-block system is released from rest from shown position.



26. At the instant speed of block is maximum, the magnitude of force exerted by spring on the block is

(A) $\frac{mg}{2}$ (B) mg (C) Zero (D) None of these

27. As the block is coming down, the maximum speed attained by the block is

(A) $\sqrt{gL}$ **(B)** $\sqrt{3gL}$ **(C)** $\frac{3}{2}\sqrt{gL}$ **(D)** $\sqrt{\frac{3}{2}gL}$

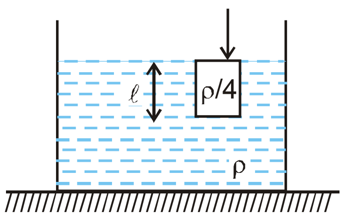
28. Till the block reaches its lowest position for the first time, the time duration for which the spring remains compressed is

(A) $\pi\sqrt{\frac{L}{2g}} + \sqrt{\frac{L}{4g}}\sin^{-1}\frac{1}{3}$ **(B)** $\frac{\pi}{4}\sqrt{\frac{L}{g}} + \sqrt{\frac{L}{4g}}\sin^{-1}\frac{1}{3}$

(C) $\pi\sqrt{\frac{L}{2g}} + \sqrt{\frac{L}{4g}}\sin^{-1}\frac{2}{3}$ **(D)** $\frac{\pi}{2}\sqrt{\frac{L}{2g}} + \sqrt{\frac{L}{4g}}\sin^{-1}\frac{2}{3}$

Comprehension - 2

A large tank of cross-section area A contains liquid of density ρ . A cylinder of density $\rho / 4$ and length ℓ and cross- section area a (a <<A) is kept in equilibrium by applying an external vertically downward force as shown in the figure. The cylinder is just submerged in liquid. At t = 0 the external force is removed instantaneously. Assume that water level in the tank remains constant.



29. The acceleration of cylinder immediately after the external force is removed is

(A) g **(B)** 2g **(C)** 3g **(D)** zero

30. The speed of the cylinder when it reaches its equlibrium position is

(A) $\frac{1}{2}\sqrt{g\ell}$ **(B)** $\frac{3}{2}\sqrt{g\ell}$ **(C)** $\sqrt{2g\ell}$ **(D)** $2\sqrt{g\ell}$

31. After its release at t = 0, the time taken by cylinder to reach its equilibrium position for the first time is

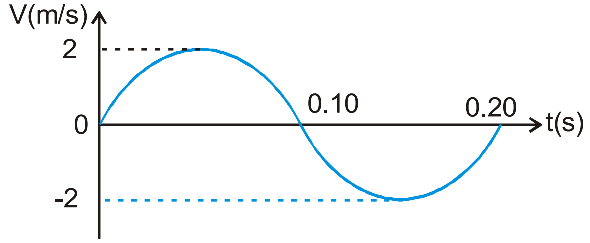
(A) $\frac{\pi}{8}\sqrt{\frac{\ell}{g}}$ **(B)** $\frac{\pi}{3}\sqrt{\frac{\ell}{g}}$ **(C)** $\frac{\pi}{4}\sqrt{\frac{\ell}{g}}$ **(D)** $\frac{\pi}{2}\sqrt{\frac{\ell}{g}}$

SECTION - V : MATRIX - MATCH TYPE

32. A particle of mass 2 Kg is moving on a straight line under the action of force F = (8 - 2x) N. The particle is released at rest from x = 6 m. For the subsequent motion match the following (All the values in the right cloumn are in their S.I. units.)

Column I	Column II
(A) Equilibrium postion is at x =	**(P)** $\pi/4$
(B) Amplitude of S.H.M is	**(Q)** $\pi/2$
(C) Time taken to go directly from x = 2 to x = 4	**(R)** 4
(D) Energy of S.H.M. is	**(S)** 6
	(T) 2

33. A simple harmonic oscillator consists of a block attached to a spring with k = 200 N/m. The block slides on a frictionless horizontal surface, with equilibrium point x = 0. A graph of the block's velocity v as a function of time t is shown. Correctly match the required information in the column I with the values given in the column II. (use $\pi^2 = 10$)

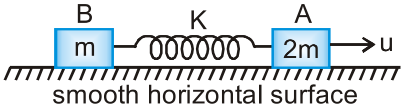


Column I	Column II
(A) The block's mass in kg	(P) – 0.20
(B) The block's displacement at t = 0 in metres	(Q) 0.0
(C) The block's acceleration at t = 0.10 s in m/s^2	(R) 0.20
(D) The block's maximum kinetic energy in Joule	(S) 4.0
	(T) – 200

34. In each situation of column-I, the x-coordinate of a particle moving along x-axis is given in terms of time t. (ω is a positive constant). Match the equation of motion given in column-I with the type of motion given in column-II.

Column-I	Column-II
(A) $\sin \omega t - \cos \omega t$	(P) SHM
(B) $\sin^3 \omega t$	(Q) Periodic
(C) $\sin \omega t + \sin 3\omega t + \sin 5\omega t$	(R) Periodic but not SHM
(D) $\exp(-\omega^2 t^2)$	(S) Non periodic
	(T) SHM but not periodic

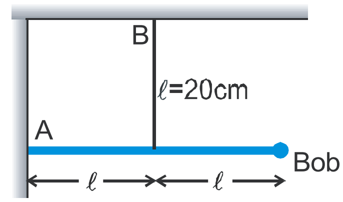
35. Two blocks A and B of mass m and 2m respectively are connected by a massless spring of spring constant K. This system lies over a smooth horizontal surface. At t = 0 the block A has velocity u towards right as shown in the figure while the speed of block B is zero and the length of spring is equal to its natural length at that instant. In each situation of column I, certain statements are given and corresponding results are given in column II.



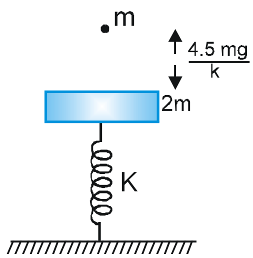
Column I	Column II
(A) The velocity of block A	(P) can never be zero
(B) The velocity of block B	(Q) may be zero at certain instants of time
(C) The kinetic energy of system of two blocks	(R) is minimum at maximum compression of spring
(D) The potential energy of spring	(S) is maximum at maximum extension of spring
	(T) is maximum at maximum compression of spring

SECTION - VI : INTEGER TYPE

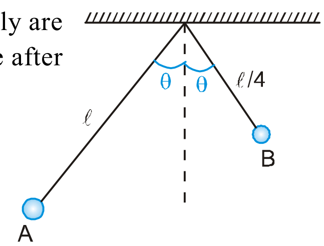
36. A weightless rigid rod of length 40 cm with a small iron bob at the end is hinged at point A to the wall so that it can rotate in all directions. The rod is kept in the horizontal position by a vertical inextensible string of length 20 cm, fixed at its mid point. The bob is displaced slightly, perpendicular to the plane of the rod and string. Find period of small oscillations of the system in the form $\frac{\pi X}{10}$sec. and fill value of X. ($g = 10\ m/s^2$)



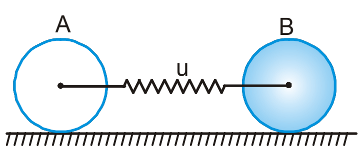
37. In the figure shown a plate of mass 60gm is at rest and in equilibrium. A particle of mass m = 30gm is released from height $\frac{4.5mg}{k}$ from the plate. The particle sticks to the plate. Neglecting the duration of collision find time from the collision of the particle and the plate to the moment when the spring has maximum compression. Spring has force constant 100 N/m. Calculate value of time in the form $X\pi$ ms (millisecond) and fill value of X.



38. Two simple pendulums A and B having lengths ℓ and $\ell/4$ respectively are released from the position as shown in the figure. Calculate the time after which the release of the two strings become parallel for the first time is $\frac{\pi}{x}\sqrt{\frac{l}{g}}$ then x is . Angle θ is very small



39. Two spheres A and B of the same mass m and the same radius are placed on a rough horizontal surface. A is a uniform hollow sphere and B is uniform solid sphere. A and B can roll without sliding on the floor. They are also tied centrally to a light spring of spring constant k as shown in figure. A and B are released when the extension in the spring is x_0. the amplitude of SHM of the spheres A is $\frac{x.x_0}{46}$ then x is



ANSWER KEY

EXERCISE - 1

1. C **2.** A **3.** C **4.** D **5.** D **6.** A **7.** C **8.** B **9.** A **10.** D **11.** A **12.** C **13.** C
14. D **15.** C **16.** A **17.** A **18.** A **19.** A **20.** B **21.** D **22.** C **23.** B **24.** A **25.** B **26.** C
27. B **28.** B **29.** C **30.** C **31.** A **32.** D **33.** B **34.** A **35.** B **36.** C **37.** B **38.** A **39.** D
40. B **41.** C **42.** A **43.** A **44.** A **45.** B

EXERCISE - 2 : PART # I

1. D **2.** B **3.** B **4.** A,B,C,D **5.** A,C **6.** C,D **7.** B,C **8.** A,B,C,D **9.** B,D
10. A **11.** B **12.** A **13.** A,B,C **14.** C **15.** B,D **16.** B **17.** A,C **18.** B,C,D **19.** B,D

PART # II

1. A **2.** A **3.** D **4.** B **5.** A **6.** D **7.** A

EXERCISE - 3 : PART # I :

1. A →R; B →T ; C →Q ; D → R ; E → Q **2.** A→ Q ; B →R ; C→S, D →U **3.** A →Q ; B →S ; C → P ; D → R

PART # II

Comprehension #1 : **1.** C **2.** A
Comprehension #2 : **1.** C **2.** C
Comprehension #3: **1.** C **2.** A
Comprehension #4 : **1.** B **2.** D **3.** B
Comprehension #5 : **1.** A **2.** C

EXERCISE - 4

1. $\frac{3\sqrt{5}}{8}$ A , $\tan^{-1}\left(\frac{1}{2}\right)$ **2.** 0.025 m, y =0.025 sin $(40\pi t)$ m **3.** $\sqrt{2}$ cm, $\frac{\pi}{4}$ rad **4.** 1.8 a **5.** 12 s **6.** $15\sqrt{3}$ cm

7. $\sqrt{65}$ m/s **8.** 9.52 cm **9.** $36\omega^2$ **10. (i)** 2m **(ii)** $\sqrt{2}\pi$ s **(iii)** $2\sqrt{3}$ m **11.** y = 0.1 sin $\left[4t+\frac{\pi}{4}\right]$ **12. (i)** 4 ms^{-1} **(ii)** 8J **(iii)** 8J **(iv)** 16 N/m **13.** $\sqrt{\frac{M}{2k}}\,u$ **14.** $\frac{10}{\pi}$ Hz, $\frac{5\sqrt{37}}{6}$ cm **15.** $2\pi\sqrt{\frac{m_1\ell}{(m_1+m_2)g}}$ **16.** 2.45 m **17.** $2\pi\sqrt{\frac{2\sqrt{2}a}{3g}}$

18. $\frac{1}{2\pi}\sqrt{\frac{g\sqrt{1+\frac{4}{\pi^2}}}{2R}}$ **19.** $2\pi\sqrt{\frac{17L}{18g}}$ **20. (i)** $\frac{2mg}{b-a}$ **(ii)** $\frac{1}{2\pi}\sqrt{\frac{2mg}{(b-a)(M+m)}}$ **(iii)** $\left(\frac{M+m}{m}\right)\left(\frac{ab}{b-a}\right)$

21. $\frac{1}{4\left[\frac{1}{k_1}+\frac{1}{k_2}+\frac{1}{k_3}+\frac{1}{k_4}\right]}$ **22. (i)** $\theta=\tan^{-1}\left(\frac{1}{5}\right)$, **(ii)** $2\pi\sqrt{\frac{1.803R}{g}}$ **23.** $2\pi\sqrt{\frac{\ell_1\ell_2}{g}}$ **24.** $2\pi\sqrt{\frac{2\ell}{g}}$

25. (i) $k=\frac{2mg}{5L}$ **(ii)** $\frac{3}{2}\sqrt{\frac{k}{m}}$ **26.** $\sqrt{\frac{4x}{a}}\left[\sqrt{\frac{g}{g-a}}-\sqrt{\frac{g}{g+a}}\right]$ where x is the total distance travelled **27.** $2\pi\frac{a}{b}\sqrt{\frac{m}{k}}$ **28. (i)** $\frac{1}{\pi}$ Hz, **(ii)** $4\pi^2\times10^{-5}$ J **(iii)** $2\pi\times10^{-2}$ ms^{-1} **29.** $\frac{1}{2\pi}\sqrt{\frac{k_1a^2+k_2b^2}{L^2(m+\frac{M}{3})}}$

EXERCISE - 5 : PART # I

1. 2 **2.** 3 **3.** 2 **4.** 3 **5.** 3 **6.** 1 **7.** 3 **8.** 3 **9.** 2 **10.** 3 **11.** 2 **12.** 4 **13.** 2
14. 1 **15.** 2 **16.** 1 **17.** 1 **18.** 1 **19.** 1 **20.** 4 **21.** 1 **22.** 4 **23.** 1 **24.** 3,4 **25.** 2 **26.** 4
27. 2 **28.** 1 **29.** 1 **30.** 3 **31.** 2 **32.** 1 **33.** 4 **34.** 4 **35.** 4 **36.** 3

PART # II

MCQ's with one correct answer

1. A **2.** B **3.** D **4.** A **5.** A **6.** A **7.** B
8. A **9.** C **10.** D **11.** D **12.** C

MCQ's one or more than one correct

1. A,C **2.** A,B,D **3.** A,D **4.** A **5.** A,B,D

Match the column

1. $A \rightarrow P ; B \rightarrow Q,R ; C \rightarrow P ; D \rightarrow Q,R$
2. $A \rightarrow P ; B \rightarrow Q,R,S ; C \rightarrow S ; D \rightarrow Q$

Comprehension Based questions

1. D **2.** D **3.** C

Subjective Questions

1. (i) $v_0 t + A\frac{m_1}{m_2}(1-\cos\omega t)$ **(ii)** $\left(\frac{m_1}{m_2}+1\right)A$ **2.** $\frac{1}{2\pi}\sqrt{\frac{3g}{2R}}$ **3.** $\frac{mg}{k}=\frac{g}{\omega^2}<a$ **4.** B,D **5.** 3

MOCK TEST

1. B **2.** C **3.** B **4.** D **5.** B **6.** B **7.** C **8.** A **9.** A **10.** C **11.** A **12.** D **13.** B
14. B **15.** B **16.** D **17.** B, C, D **18.** B, C **19.** A, B, C, D, E **20.** A, B, C, D **21.** B, D **22.** B, C, D
23. D **24.** A **25.** D **26.** B **27.** C **28.** B **29.** C **30.** B **31.** C **32.** $A \rightarrow R ; B \rightarrow T ; C \rightarrow Q ; D \rightarrow R$
33. $A \rightarrow R ; B \rightarrow P ; C \rightarrow T ; D \rightarrow S$ **34.** $A \rightarrow P,Q ; B \rightarrow Q,R ; C \rightarrow Q,R ;$ D,S
35. $A \rightarrow P ; B \rightarrow Q ; C \rightarrow P,R ; D \rightarrow Q,S,T$ **36.** 4 **37.** 20 **38.** 3 **39.** 21