HINTS & SOLUTIONS

EXERCISE - 1

Single Choice

1. $\lambda = 100\text{m}, v = 25\text{m/s}, T = \frac{1}{f} = \frac{\lambda}{v} = \frac{100}{25} = 4\text{ s}$

2. Waves on surface of water are combination of longitudinal and transverse waves.

3. $\lambda = \frac{v}{f} = \frac{600/2}{500} = \frac{3}{5}\text{m}$

$\therefore$ Number of waves $= \frac{600}{\lambda} = \frac{600 \times 5}{3} = 1000$

4. $y_1 = a \sin \omega t$

$y_2 = a \cos \omega t = a \sin(\omega t + \pi/2)$

y_1 lags y_2 behind by phase $\frac{\pi}{2}$.

5. $f = \frac{2\text{waves}}{1\text{sec}} = 2\text{Hz}; \lambda = 5\text{m}$ $\quad\therefore v = f\lambda = 10\text{m/s}$

6. $y = 4\cos^2\left(\frac{t}{2}\right)\sin 1000t$

$= 2\cos\frac{t}{2}\left[\sin\left(1000+\frac{1}{2}\right)t + \sin\left(1000-\frac{1}{2}\right)t\right]$

$= \sin 1001t + \sin 1000t + \sin 1000t + \sin 999t$

$= \sin 1001t + 2\sin 1000t + \sin 999t$

7. $y_1 = \cos kx \sin \omega t = \frac{1}{2}[\sin(\omega t + kx) + \sin(\omega t - kx)]$

(Stationary Wave)

$y_3 = \cos^2(k\alpha + \omega t) = \frac{1}{2}[1 + \cos 2(kx + \omega t)]$

(Progressive Wave)

8. y_1 propagates in +x-axis and y_2 along –ve x-axis.

9. $y = y_0 \sin 2\pi\left(ft - \frac{x}{\lambda}\right)$; $v = y_0(2\pi f)\cos 2\pi\left(ft - \frac{x}{\lambda}\right)$

$\therefore v_{max} = y_0 2\pi f ; v_{wave} = f\lambda$

Given : $v_{max} = 4v_{wave} \Rightarrow y_0(2\pi f) = 4(f\lambda) \Rightarrow \lambda = \frac{\pi y_0}{2}$

10. $I = \frac{2\pi^2\rho A^2 v}{T^2} \Rightarrow I \propto \left(\frac{A}{T}\right)^2$

$\Rightarrow \frac{I_1}{I_2} = \left(\frac{A_1}{A_2}\frac{T_2}{T_1}\right)^2 = \left(\frac{1}{2}\times\frac{2}{1}\right)^2 = 1:1$

11. $y_1 = 10\sin\left(3\pi t + \frac{\pi}{3}\right)$; $y_2 = 5\left(\sin 3\pi t + \sqrt{3}\cos 3\pi t\right)$

$= 10\left(\frac{1}{2}\sin 3\pi t + \frac{\sqrt{3}}{2}\cos 3\pi t\right) = 10\sin\left(3\pi t + \frac{\pi}{3}\right)$

$\therefore A_1/A_2 = 10/10 = 1:1$

12. $y_1 = 0.25\cos(2\pi t - 2\pi x)$ $f\lambda$ = const.

$f = f/2 \Rightarrow \lambda = 2\lambda$

$y_2 = 2 \times 0.25\cos\left(\frac{2\pi t}{2} + \frac{2\pi x}{2}\right) = 0.5\cos(\pi t + \pi x)$

13. $a = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\frac{\pi}{2}} = \sqrt{a_1^2 + a_2^2}$

14. $I \propto A^2$ and $I \propto \frac{1}{2}\pi RL$

$\frac{I_1}{I_2} = \frac{R_2}{R_1} = \frac{A_1^2}{A_2^2} \Rightarrow \frac{A_1}{A_2} = \sqrt{\frac{R_2}{R_1}} = \sqrt{\frac{25}{9}} = 5:3$

15. $v \propto \sqrt{\text{Tension}} \Rightarrow v = \sqrt{kx}$

$v' = \sqrt{k(1.5x)} \Rightarrow \frac{v'}{v} = \sqrt{1.5} = 1.22 \Rightarrow v' = 1.22\text{ v}$

16. $\frac{I_1}{I_2} = \left(\frac{A_1}{A_2}\right)^2 = \frac{9}{1} \Rightarrow \frac{A_1}{A_2} = \frac{3}{1}$

$\therefore \frac{I_{max}}{I_{min}} = \left(\frac{A_1+A_2}{A_1-A_2}\right)^2 = \left(\frac{\frac{A_1}{A_2}+1}{\frac{A_1}{A_2}-1}\right)^2 = \left(\frac{3+1}{3-1}\right)^2 = \left(\frac{4}{2}\right)^2 = 4:1$

17. $v = \frac{\text{coeff of }\omega}{\text{coeff of }k} = \frac{30}{1} = 30\text{ m/s} = \sqrt{\frac{T}{\mu}}$

$\Rightarrow T = \mu \times 900 = 1.3 \times 10^{-4} \times 900 = 0.12\text{ N}$

18. $f_1 = f_2$ (m $\rightarrow$ no. of loops in steel wire n $\rightarrow$ no. of loops in aluminium wire)

$\Rightarrow \frac{m}{2L_1}\sqrt{\frac{T}{\rho_1A_1}} = \frac{n}{2L_2}\sqrt{\frac{T}{\rho_2A_2}}$

$\Rightarrow \frac{m}{2\times 60}\sqrt{\frac{80}{7800\times10^{-6}}} = \frac{n}{2\times 45}\sqrt{\frac{80}{2600\times3\times10^{-6}}}$

$\Rightarrow \frac{m}{n} = \frac{4}{3}$ (minimum)

$\therefore f = \frac{m}{2L_1}\sqrt{\frac{T}{\rho_1A_1}} = \frac{4}{2\times0.6}\sqrt{\frac{80}{7800\times10^{-6}}} = 337.5\text{Hz}$

19. $\Delta L = \dfrac{TL}{AY} \Rightarrow T = \dfrac{\Delta LAY}{L} = \dfrac{L\alpha\theta AY}{L} = \alpha\,\theta\,Ay$

$\mu = dA \quad \therefore\; v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{\alpha\theta AY}{dA}} = 70\text{ m/s}$

20. The right end will shoot up on the wire.

21. $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{\mu xg}{\mu}} = \sqrt{gx} \Rightarrow v^2 = gx$

(symmetrical about x)

22. The equation represents a progressive wave moving along x-axis of single frequency.

24. $y_{res} = y + y' = 0$ (at $x = 0$) $\Rightarrow a\sin(kx - \omega t) + y' = 0$
put $x = 0$ and get $y_{res} = 0$

25. $L = 5\dfrac{\lambda}{2} \Rightarrow 10 = 5\dfrac{\lambda}{2} \Rightarrow \lambda = 4\text{m} \;\therefore\; f = \dfrac{v}{\lambda} = \dfrac{20}{4} = 5\text{Hz}$

26. Distance between position having 3 nodes and 2 antinodes = wavelength = 1.21 Å

27. $f = \dfrac{1}{2L}\sqrt{\dfrac{T}{\mu}}; \quad f' = \dfrac{2}{2L}\sqrt{\dfrac{T}{\mu}\left(1 - \dfrac{1}{2\rho}\right)} \Rightarrow f' = f\sqrt{\dfrac{2\rho - 1}{2\rho}}$

28. $n_1\ell_1 = n_2\ell_2 = n_3\ell_3 = n\ell$

$\Rightarrow \ell_1 + \ell_2 + \ell_3 = \ell \quad \dfrac{n\ell}{n_1} + \dfrac{n\ell}{n_2} + \dfrac{n\ell}{n_3} = \ell \Rightarrow \dfrac{1}{n} = \dfrac{1}{n_1} + \dfrac{1}{n_2} + \dfrac{1}{n_3}$

29. $d = vt = 330 \times 5.5 = 1815\text{ m}$

30. $7\lambda = 0.14 ; \lambda = 0.02$

$f = \dfrac{v}{\lambda} = \dfrac{3 \times 10^8}{0.02} = 1.5 \times 10^{10}\text{ Hz}$

31. $v = f_1\lambda_1 = f_2\lambda_2 \Rightarrow 512 \times 4L = f_2 \times 2L \Rightarrow f_2 = 1024\text{ Hz}$

32. $v = \sqrt{\dfrac{\gamma RT}{M}} \Rightarrow M_{H_2+O_2} < M_{O_2} \;\therefore\; V_{H_2+O_2} > V_{O_2}$

33. $f_{air} = f_{water} \Rightarrow \lambda_{air} = \dfrac{v_{air}}{f_{air}} = \dfrac{330}{60 \times 10^3} = 5.5\text{mm}$

34. $f_1 = f_2,\; \dfrac{v}{\lambda_1} = \dfrac{v}{\lambda_2} \Rightarrow \lambda_1 = 4L_1 = \lambda_2 = \dfrac{2}{3}L_2$

$\Rightarrow \dfrac{L_1}{L_2} = \dfrac{2}{3 \times 4} = \dfrac{1}{6}$

35. $v = f_1\lambda_1 = f_2\lambda_2 \Rightarrow f \times 2L = f' \times 2L \Rightarrow f' = f$

36. $v = f\lambda \Rightarrow 333 = 333\,\lambda \Rightarrow \lambda = 1$
= Length of pipe in second harmonic.

37. $f_1\lambda_1 = v$

$f_1\left(\dfrac{4L}{3}\right) = v \Rightarrow f_1 = \dfrac{3v}{4L} \;;\; f_2(2L) = v \Rightarrow f_2 = \dfrac{v}{2L}$

Given that $f_1 - f_2 = 100$

$\Rightarrow \dfrac{3v}{4L} - \dfrac{v}{2L} = 100 \Rightarrow \dfrac{v}{2L} = 200\text{Hz}$

38. $\lambda = \dfrac{v}{f} = \dfrac{330}{330} = 1\text{m}$

$L = \lambda/4, 3\lambda/4, 5\lambda/4.... = 25\text{cm}, 75\text{cm}, 125\text{cm}$
Minimum length of water column = 120 – 75 = 45cm

39. $v = f\lambda\; (\lambda/4 = L) \Rightarrow 336 = 20 \times 4L \Rightarrow L = 4.2\text{ m}$

40. $f_1 = \dfrac{500\pi}{2\pi} = 250 ;\; f_2 = \dfrac{506\pi}{2\pi} = 253$

$\therefore\; \Delta f = 3\text{ s}^{-1} = 3 \times 60\text{ min}^{-1} = 180\text{ min}^{-1}$

41. $\dfrac{\lambda}{4} = L + 0.6\,r_1$ (closed organ pipe)

$\lambda = L + 1.2\,r_1$ (open organ pipe)
$\Rightarrow 4(L + 0.6\,r_1) = (L + 1.2r_2) \Rightarrow r_2 - 2r_1 = 2.5\,L$

42. For sonometer wire
$n \times 100 = (n+1) \times 95 \Rightarrow n$ = no of harmonics $\Rightarrow n = 19$

$\therefore\; f = 19\left(\dfrac{L}{2}\right) + 4 = 20\left(\dfrac{L}{2}\right) - 4 \Rightarrow L = 16$

$\Rightarrow f = 20\left(\dfrac{L}{2}\right) - 4 = 156\text{ Hz}$

43. If $f_B > f_A$; $f_B = 260$ Hz ; If $f_B < f_A$; $f_B = 252$ Hz

44. $2f = f \times 15 \times 8 \Rightarrow f = 120\text{ Hz}$

45. $v = f_1 \times 50 = f_2 \times 51$

$\Rightarrow f_1 - f_2 = \dfrac{v}{50} - \dfrac{v}{51} = 0.1 \Rightarrow v = 255\text{ m/s}$

46. $\lambda_1 = 2L ; \lambda_2 = 2(L - y)$

$\Delta f = f_2 - f_1 = \dfrac{v}{\lambda_2} - \dfrac{v}{\lambda_1} = \dfrac{v}{2}\left[\dfrac{1}{L - y} - \dfrac{1}{L}\right]$

$\Rightarrow \dfrac{vy}{2(L - y)L} \simeq \dfrac{vy}{2L^2}$

47. $\boxed{C_1}$ A $\quad \boxed{C_2}$ B

f_1 256 Hz $\quad f_2$ 262 Hz
$2(f_1 - 256) = (f_1 - 262) \Rightarrow f_1 = 250\text{Hz}$
$2(f_2 - 256) = (262 - f_2) \Rightarrow f_2 = 258\text{ Hz}$

48. $dB = 10\log\frac{I_2}{I_1} \Rightarrow 20 = 10\log\frac{I_2}{I_1}$

$\Rightarrow I_2 = 100\,I_1$ (taking antilog)

49. $dB = 10\log\frac{I_2}{I_1} = 10\log\frac{400}{20} = 10\log 20$

$= 10(1+0.3) = 13dB$

50. $\Delta f = f_{app} - f_o = f_o\left(\frac{v}{v-v_s}\right) - f_o$

$$\frac{\Delta f}{f_o} = \frac{v}{v-v_s} - 1 = \frac{v_s}{v-v_s} = \frac{2.5}{100}$$

$$\Rightarrow \frac{v_s}{v-v_s} = \frac{1}{40} \Rightarrow v_s = \frac{v}{41} \simeq 8\,m/s$$

51. $f = f_o\left(\frac{v}{v-v_{obs}}\right)$

(observer is approaching)

$$= 450\left(\frac{330}{330-33}\right) = 500\,Hz$$

52. $f_{min} = f_o\left(\frac{v}{v+\omega R}\right) = 385\left(\frac{340}{340+20\times 0.5}\right) = 374\,Hz$

53. $f_B = f_A\left(\frac{v+v_B}{v+v_A}\right) = 450\left(\frac{330+10}{330+30}\right) = 425\,Hz$

EXERCISE - 2

Part # I : Multiple Choice

1. $\frac{2\pi}{\lambda} = 10\pi \Rightarrow \lambda = 0.2\,m$

Node occurs at $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4} = 0.05\,m,\ 0.15\,m, ...;$

Antinode occurs at $x = \frac{\lambda}{2}, \lambda, 3\frac{\lambda}{2}$ .....= 0.1m,0.2m,

0.3m,......; wave speed $v = \frac{50\pi}{10\pi} = 5$ m/s

2. No. of wave striking the surface $f = f\left(\frac{c+v}{c}\right)$

freqeuncy of the reflected wave

$$f' = f\left(\frac{c}{c-v}\right) = f\left(\frac{c+v}{c-v}\right)$$

Wavelength of the reflected wave

$$\lambda = \frac{c}{f'} = \frac{c}{f}\left(\frac{c-v}{c+v}\right)$$

3. y=0 at x=0.

This can be statisfied by the term $\sin\left(\frac{n\pi x}{L}\right)$

4. For closed tube

$$\frac{\lambda}{4} = \frac{V}{4f} = L \text{ and } \frac{\lambda'}{4} = \frac{V+v}{4f'} = L+\ell \Rightarrow f' = \frac{V+v}{4(L+\ell)}$$

5. $f_1 - f_2 = 5 \Rightarrow \frac{v}{2\times 16} - \frac{v}{2\times 16.2} = 5$

$$\Rightarrow \frac{v\times 0.2}{2\times 16\times 16.2} = 5 \Rightarrow f_1 = \frac{v}{2\times 16} = \frac{5\times 16.2}{0.2} = 405Hz$$

$$\text{and } f_2 = \frac{v}{2\times 16.2} = \frac{5\times 16}{0.2} = 400Hz$$

6. $f = f_o\left(\frac{v+gt}{v}\right) \Rightarrow \frac{df}{dt} = f_o\left(0+\frac{g}{v}\right)$

$$\Rightarrow \frac{1000}{30} = \frac{1000\times 10}{v} \Rightarrow v = 300 \text{ m/s}$$

7. $y = \sin(\omega t - kx + \phi) \Rightarrow v = \omega\cos(\omega t - kx + \phi)$

at $t=0, x=0, y=-0.5, v>0 \Rightarrow \phi = -\frac{\pi}{6}$

$$\therefore\ y = \sin\left(\omega t - kx - \frac{\pi}{6}\right)$$

8. Let wave equation be

$$z = e^{-[x-v(t-t_0)]^2}$$

At $t = 0$, $x + vt_0 = x + 2 \Rightarrow vt_0 = 2$

At $t = 1s$, $x - v(1-t_0) = x - 2 \Rightarrow v = +4$ m/s

9. $y = 2mm\sin\left(2\pi x - 100\pi t + \frac{\pi}{3}\right)$

$$\Rightarrow 0 = 2\sin\left(2\pi x - 100\pi t + \frac{\pi}{3}\right)$$

$$\Rightarrow n\pi = 8\pi - 100\pi t + \frac{\pi}{3}\ (n=0,1,2,3,....)$$

$$\Rightarrow t_{min} = \frac{\frac{25\pi}{3} - n\pi}{100\pi} = \frac{\pi}{3}/100\pi = \frac{1}{300}\text{ s}$$

10. In the stationary waves, the particles in the alternate loops are out of phase.

11. $\mu = \frac{\lambda}{L}x$ (at a distance 'x' from free end)

$$\therefore\ T = \int_0^x \mu dx(g+2g) = \frac{3\lambda gx^2}{2L}$$

$$\therefore\ v_{wave} = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{3\lambda gx^2}{2L\left(\frac{\lambda x}{L}\right)}} = \sqrt{\frac{3xg}{2}}$$

$$\Rightarrow\ v^2 = \frac{3xg}{2} \Rightarrow 2v\frac{dv}{dx} = \frac{3g}{2} \Rightarrow a = 3g/4$$

(constant everywhere)

Now $S = ut + \frac{1}{2}at^2 \Rightarrow L = 0 + \frac{3g}{8}t^2 \Rightarrow t = \sqrt{8L/3g}$

12. Total energy

$$= \frac{1}{2}\mu\omega^2A^2 = 2\pi^2f_n^2\mu A^2 = 2\pi^2n^2f_1^2\mu A^2$$

(f_1 = fundamental frequency

f_n = Frequency of n^{th} harmonic

$$\langle KE \rangle = \langle PE \rangle = \frac{TE}{2}$$

14. $f = \frac{\sqrt{\gamma RT}}{2L\sqrt{M}} = \frac{k}{L\sqrt{M}}$, $f_A = \frac{k}{L\sqrt{2}}$

$f_B = \frac{2k}{L\sqrt{32}}$, $f_C = \frac{3k}{2L\sqrt{28}}$

$f_D = \frac{3k}{L\sqrt{44}}$, $\therefore\ f_C/f_D = \sqrt{\frac{11}{28}}$

13. $\frac{3\pi}{2} = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{4}{3}m$

Pr. Amp. $= P_0\cos\left(\frac{3\pi x}{2}\right) = P_0$ at $x = 0$

and $-P_0$ at $x = 2/3 = P_0$ at $x = 4/3$

$\therefore$ Pipe may be closed at $x = 0$ and open at $x = \frac{2}{3}$ m.

17. $\lambda = \frac{300m/s}{25Hz} = 12m.$

Separation between A and B = 6 m = $\lambda/2$

19. Comparing with the equation

$$y = 2A\sin\left(\frac{n\pi x}{L}\right)\cos(\omega t)$$

$2A = 2mm$ or $A = 1mm$

$\frac{n\pi x}{L} = 6.28x = 2\pi x$ or $L = \frac{n}{2}$ m. For $n = 1$, $L = 0.5$ m.

20. Let a = initial amplitude due to S_1 and S_2 each.
$I_0 = k(4a^2)$, where k is a constant After reduction of power of S_1, amplitude due to $S_1 = 0.6a$.
Due to superposition
$a_{max} = a + 0.6a = 1.6a$, and
$a_{min} = a - 0.6a = 0.4a$
$I_{max}/I_{min} = (a_{max}/a_{min})^2 = (1.6a/0.4a) = 16$

Part # II : Assertion & Reason

1. A	2. A	3. B	4. A	5. A	6. A
7. A	8. A	9. A	10. C	11. A	12. A
13. A	14. A	15. A	16. C	17. A	18. A
19. A	20. A	21. C			

EXERCISE - 3

Part # I : Matrix Match Type

1. $A \to P, Q$; $B \to S$; $C \to P,R$; $D \to S$
2. $A \to Q$; $B \to P$; $C \to P$
3. $A \to Q$; $B \to P$; $C \to R$
4. $A \to Q$; $B \to R$; $C \to S$; $D \to P$
5. $A \to R$; $B \to P$; $C \to Q$; $D \to T$

Part # II : Comprehension

Comprehension#1

1. $\frac{2\pi}{\lambda}\Delta = 2\pi n$ $(n = 0,1,2,3,.....)$ $\Rightarrow \frac{2\pi}{\lambda}(\pi R) = 2\pi n$

$$\Rightarrow \lambda = \frac{\pi R}{n} = \pi R, \frac{\pi R}{2}, \frac{\pi R}{3}, \frac{\pi R}{4} ....$$

2. $\frac{2\pi}{\lambda}\Delta = (2n+1)\pi$ (n=0,1,2,....)

$$\Rightarrow \frac{2\pi}{\lambda}(\pi R) = (2n+1)\pi$$

$$\Rightarrow \lambda = \frac{2\pi R}{2n+1} = 2\pi R, \frac{2\pi R}{3}, \frac{2\pi R}{5}.....$$

3. $I_{max} = \left(\sqrt{\frac{I_0}{2}} + \sqrt{\frac{I_0}{2}}\right)^2 = 2I_0$

4. λ_{max} to produce maxima at $D = \pi R$

5. λ_{max} to produce minima at $D = 2\pi R$

Comprehension#2

1. $v_{\uparrow}$ = Point a

2. $v_{\downarrow}$ = Points c,d,e

3. $v_{=0}$ = points b,f

4. v_{max} = points 0,d,h

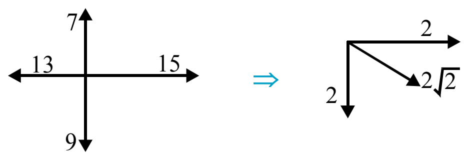
5. $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{w+\mu xg}{\mu}} = \sqrt{\dfrac{w}{\mu} + xg}$

$\Rightarrow \quad v^2 = \dfrac{w}{\mu} + xg \Rightarrow 2v\dfrac{dv}{dx} = g \Rightarrow a = \dfrac{g}{2}$

EXERCISE - 4

Subjective Type

1.



$A = 2\sqrt{2} = 2.828\text{ mm} \simeq 2.83\text{ mm}$

2. $h = \dfrac{1}{2}gt_1^2 \Rightarrow t_1 = \sqrt{\dfrac{2h}{g}}$ and $h = vt_2,\ t_2 = \dfrac{h}{v} = \dfrac{300}{340}$

$\therefore\ t = t_1 + t_2 = 8.707$ sec

3. $I \propto \left(\dfrac{A}{T}\right)^2 \Rightarrow \dfrac{I_1}{I_2} = \left(\dfrac{A_1}{A_2}\right)^2\left(\dfrac{T_2}{T_1}\right)^2 = \left(\dfrac{2}{1}\right)^2 \times \left(\dfrac{1}{2}\right)^2 = 1$

4. $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{90\times 9.8}{8\times 10^{-3}}} = 105\sqrt{10}$ m/s

$f = 256$ Hz; $A = 5\text{cm} = 0.05$ m

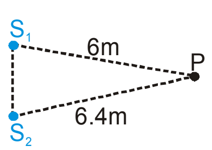
Equation of the wave

$\therefore\ y = A\sin(\omega t - kx)$

$= 0.05\sin\left(2\pi ft - \dfrac{2\pi f}{v}x\right)$

$= 0.05\sin(1609t - 4.84x)$

5.



$\dfrac{2\pi}{\lambda}\Delta = (2n+1)\pi$

for destructive interference $\dfrac{2\pi}{v}f(6.4-6) = (2n+1)\pi$

$\Rightarrow f = \dfrac{(2n+1)v}{0.8} = 400(2n+1)$ $(v = 320$ m/s$)$

$= 400$ Hz, 1200Hz, 2000 Hz, 2800 Hz, 3600 Hz, 4400 Hz

6. $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{2\times 10}{\dfrac{4.5\times 10^{-3}}{2.25}}} = 100\text{m/s}$

$t = \dfrac{L}{v} = \dfrac{2}{100} = 0.02$ sec

7. For wire, $\Delta L = L\alpha\theta = \dfrac{FL}{AY} \Rightarrow F = AY\alpha\theta$

$\therefore\ f = \dfrac{1}{2L}\sqrt{\dfrac{F}{\mu}} = \dfrac{1}{2\times 1}\sqrt{\dfrac{10^{-6}\times 2\times 10^{11}\times 1.21\times 10^{-5}\times 20}{0.1}}$

$= 11$ Hz

8. $V_{bottom} = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{2\times 10}{1/2}} = 2\sqrt{10}$ m/s

$V_{top} = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{8\times 10}{1/2}} = 4\sqrt{10}$ m/s

$\Rightarrow f = \dfrac{v_{top}}{\lambda_{top}} = \dfrac{v_{bottom}}{\lambda_{bottom}} \Rightarrow \lambda_{top} = \dfrac{4\sqrt{10}}{2\sqrt{10}}\times 0.06 = 0.12\text{m}$

9. $v = \sqrt{\dfrac{1\times 10}{10^{-3}/10^{-2}}} = 10\text{m/s},\ t = \dfrac{L}{v} = \dfrac{0.5}{10} = 0.05$ sec

10. **(i)** $(y_{max})_{x=5cm} = 4\sin\left(\dfrac{\pi x}{15}\right) = 4\sin\left(\dfrac{5\pi}{15}\right) = 2\sqrt{3}$

(ii) $\dfrac{2\pi}{\lambda} = \dfrac{\pi}{15} \Rightarrow \lambda = 30\text{cm}$

$\therefore$ Position of nodes $= 15$cm, 30cm.........

(iii) $v = -4(96\pi)\sin\left(\dfrac{\pi x}{15}\right)\sin(96\pi t)$

at $x = 7.5$ cm, $t = 0.25$ sec

$v = -4\times 96\pi\sin\left(\dfrac{7.5\pi}{15}\right)\sin\left(\dfrac{96\pi}{4}\right) = 0$

(iv) $Y = 4\sin\left(\dfrac{\pi x}{15}\right)\cos(96\pi t)$

$= 2\sin\left(\dfrac{\pi x}{15} - 96t\right) + 2\sin\left(\dfrac{\pi x}{15} + 96\pi t\right)$

11. $y = 5\sin\left(\frac{\pi x}{3}\right)\cos(40\pi t)$

$= 2.5\sin\left(40\pi t + \frac{\pi x}{3}\right) - 2.5\sin\left(40\pi t - \frac{\pi x}{3}\right)$

(i) Equation of incident wave

$y_1 = 2.5\sin\left(40\pi t + \frac{\pi x}{3}\right)$

Equation of reflected wave

$y_2 = -2.5\sin\left(40\pi t - \frac{\pi x}{3}\right)$

(ii) $\frac{2\pi}{\lambda} = \frac{\pi}{3} \Rightarrow \lambda = 6\text{cm}$

$\therefore$ Distance between adjacent nodes = 3cm

(iii) $v = -5(40\pi)\sin\left(\frac{\pi x}{3}\right)\sin(40\pi t)$

$= -200\pi\sin\left(\frac{\pi \times 1.5}{3}\right)\sin\left(40\pi \times \frac{9}{8}\right) = 0$

12. $\lambda_{air} = \frac{340}{10^6} = 3.4 \times 10^{-4}\,\text{m}$, $\lambda_{water} = \frac{1486}{10^6} = 1.49 \times 10^{-3}\,\text{m}$

13. $v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{4000 \times 10^6}{1000}} = 2000\,\text{m/s}$

15. Let the pipe resonates in n^{th} & $(n+1)^{th}$ harmonic

$\Rightarrow (n+1)\,1944 = n(2592)$

$\Rightarrow n = 4 \quad \therefore L = \frac{324}{1296} = 0.25\text{ m}$

16. $\frac{\lambda}{2} = 1 \Rightarrow \lambda = 2\text{m}$

$v = f\lambda = 2.53 \times 10^3 \times 2 = 5.06 \times 10^3\text{ m/s}$

18. $\Delta f = 305 - 300 = 5\text{Hz}$

(i) $\therefore$ Total beats produced in 5s = 5 × 5 = 25

(ii) Time interval in which max intensity becomes minimum $= \frac{1}{2} \times \frac{1}{\Delta f} = \frac{1}{2} \times \frac{1}{5} = 0.1\text{ sec}$

19. $F_{string} > F_{pipe}$

$F_{string} = \frac{2}{2L}\sqrt{\frac{T}{\mu}} = 4\sqrt{\frac{T}{10^{-2}}} = 40\sqrt{T}$

$F_{pipe} = \frac{v}{\lambda} = \frac{320}{4 \times 0.4} = 200\text{ Hz}$

$\Rightarrow 40\sqrt{T} - 200 = 8 \Rightarrow T = 27.04\text{ N}$

20. Frequency reaching the wall $f_1' = f_0\left(\frac{v}{v - v_s}\right)$

Frequency received by the observer

$f_1 = f_1'\left(\frac{v + v_s}{v}\right) = f_0\left(\frac{v + v_s}{v - v_s}\right)$

$\therefore$ Beat frequency

$\Delta f = f_0\left(\frac{v + v_s}{v - v_s}\right) - f_0 = 256\left(\frac{330 + 5}{330 - 5}\right) - 256$

$= 7.87\text{ Hz}$

21. The frequency of B < frequency of A

$\Rightarrow f_A - f_B = 5 \Rightarrow 427 - f_B = 5 \Rightarrow f_B = 422\text{ Hz}$

22. $\Delta f = f_0\left(\frac{v}{v - v_S}\right) = 180\left(\frac{330}{330 - 60}\right) = 220\text{Hz}$

23. $f_{observer} = f_0\left(\frac{v + v_w - v_0}{v + v_w}\right) = 700\left(\frac{340 + 10 - 10}{340 + 10}\right)$

$= 680\text{ Hz}$

24. $\Delta f = f_0\left(\frac{v}{v - v_S}\right) - f_0\left(\frac{v}{v + v_S}\right) = 3$

$\Rightarrow 3 = 340\left[\frac{330}{330 - v_S} - \frac{330}{330 + v_S}\right] \Rightarrow v_S = 1.5\text{m/s}$

25. $\Delta f_1 = f_0\left(\frac{v}{v - v_S}\right) = 440\left(\frac{330}{330 - 20 \times 1.5}\right) = 484\text{Hz}$

$f_2 = f_0\left(\frac{v}{v + v_S}\right) = 440\left(\frac{330}{330 + 20 \times 1.5}\right) = 403.3\text{Hz}$

26. Frequency received by submarine

$f_1 = f_1 = f_0\left(\frac{v}{v - v_s}\right)$

Frequency reflected by submarine,

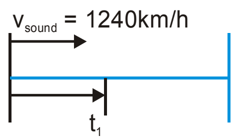
$f_2 = f_1\left(\frac{v + v_s}{v}\right) = f_0\left(\frac{v + v_s}{v - v_s}\right) = 140\left(\frac{1450 + 100}{1450 + 100}\right)$

$= 45.93\text{ kHz}$

27. **(i)** $f_{Hill} = f_0\left(\frac{v+v_w}{v+v_w-v_s}\right)$

$$=580\left(\frac{1200+40}{1200+40-40}\right)=599\text{ Hz}$$

(ii)



$1240\,t_1 = 1 \Rightarrow t_1 = \frac{1}{1240}$ hr

where t_1 = time the sound to reach the hill

Let t_2 = time for the echo to reach the train

v_{echo} = speed of echo

$= 1200 - 40 = 1160$ km/hr

$\therefore\ v_{train} + v_{echo} = d - v_{train}\,t_1$

$\Rightarrow (40 + 1160)\,t_2 = 1 - \frac{40}{1240} = \frac{1200}{1240}$

$\Rightarrow\ t_2 = \frac{1}{1240}$ hr

$\therefore$ Distance from the hill where echo reaches the train

$= d - v(t_1 + t_2) = 1 - \frac{40\times 2}{1240} = 0.935$ km

Frequency reaching the hill

$$f_1' = f_0\left(\frac{v+v_w}{v+v_w-v_t}\right)$$

Frequency of echo

$$f_1 = f_1'\left(\frac{v-v_w+v_t}{v-v_w}\right)$$

$$= f_0\left(\frac{v+v_w}{v+v_w-v_t}\right)\left(\frac{v-v_w+vt}{v-v_w}\right)$$

$$=580\times\frac{1240}{1200}\times\frac{1200}{1160}=620\text{ Hz}$$

28. $\mu = \frac{1\text{gm}}{10\text{cm}} = \frac{10^{-3}\text{kg}}{0.1\text{m}} = 10^{-4}$ kg/m, $T = 64$ N

$v = \sqrt{\frac{T}{\mu}} \Rightarrow f_0 = \frac{v}{2L} = \frac{1}{2\times 0.1}\sqrt{\frac{64}{10^{-4}}} = 5\times 8\times 100$

$= 4000$ m/s $\Rightarrow v_{string} - v_{fork} = 1$

$\Rightarrow 4000 - 4000\left(\frac{300}{300-v}\right) = 1 \Rightarrow v = 0.073$ m/s

29. $f_{max} = f_0\left(\frac{v+v_0}{v+v_s}\right) = 340\left(\frac{340+2\pi\left(\frac{5}{\pi}\right)6}{340-10\times 3}\right) = 438.7$ Hz

$$f_{min} = f_0\left(\frac{v-v_0}{v+v_s}\right) = 340\left(\frac{340-60}{340+30}\right) = 257.3\text{ Hz}$$

30. $f_{guard} = f_o\left(\frac{v+v_s}{v+v_s}\right) = f_o$

31. **(i)** For the particle P

$\frac{\partial y}{\partial t} = -v\left(\frac{\partial y}{\partial x}\right) \Rightarrow +20\sqrt{3} = -v(\sqrt{3})$

$\Rightarrow$ v = –20 cm/s

(along negative x-axis)

(ii) Equation of wave

$y = A\sin(\omega t + kx + \phi)$

at $t = 0$, $x = 0$, $y = 2\sqrt{2}$, $A = 4$

$\Rightarrow\ 4 = 2\sqrt{2}\ \sin\phi \Rightarrow \phi = \frac{\pi}{4}, \lambda = 5.5 - 1.5 = 4$ cm

$f = \frac{v}{\lambda} = \frac{20\text{cm/s}}{4\text{cm}} = 5\text{Hz}$

$\therefore\ y = 4\sin\left(10\pi t + \frac{\pi x}{2} + \frac{\pi}{4}\right)$

(iii) Energy carried in one wavelength

$E = \frac{1}{2}\mu A^2\omega^2\lambda$

$= \frac{1}{2}\times\frac{50}{1000}\times(4\times 10^{-2})^2\times(10\pi)^2\times\frac{4}{100}$

$= 16\pi^2\times 10^{-5}$ J

32. $I = 4I_0\cos^2\theta$ where $\theta = (\omega_1 - \omega_2)t = 10^3 t$

(i) For successive maxima

$\Delta t = \frac{2\pi}{10^3} = 6.28\times 10^{-3}$ sec

(ii) For detection of sound

$2A^2 = 4A^2\cos^2\theta$

$\Rightarrow \cos\theta = \pm\frac{1}{\sqrt{2}} \Rightarrow\ \theta = -\frac{\pi}{4}, \frac{\pi}{4}$

$\Rightarrow\ 10^3 t = 2\left(\frac{\pi}{4}\right) = \frac{\pi}{2}$

$\Rightarrow\ t = \frac{\pi}{2}\times 10^{-3} = 1.57\times 10^{-3}$ s

33. Frequency reaching the wall $f_1' = f_0\left(\frac{v}{v-v_b}\right)$

Frequency reaching the motorist after reflection from wall

$$f_1 = f_1'\left(\frac{v+v_m}{v}\right) = f_0\left(\frac{v+v_m}{v-v_b}\right)$$

Frequency directly reaching the motorist

$$f_2 = f_0\left(\frac{v+v_m}{v+v_b}\right) \quad \therefore \text{ Beat frequency}$$

$$\Delta f = f_1 - f_2 = f_0(v+v_m)\left(\frac{2v_b}{v^2-v_b^2}\right)$$

34. For air $\frac{\lambda_1}{2} = L_1 \Rightarrow \lambda_1 = 2L_1$

$v_1 = 330 \Rightarrow v_1 = f\lambda \Rightarrow 330 = 500\,(2L_1) \Rightarrow L_1 = 33$ cm

For CO_2 $\frac{\lambda_2}{4} = L_2 \Rightarrow \lambda_2 = 4L_2$, $v_2 = 264$

$\Rightarrow v_2 = f\lambda_2 \Rightarrow 264 = 500\,(4L_2) \Rightarrow L_2 = 13.2$ cm.

35. Amplitude of reflected wave

$$A_r = A_i\left(\frac{k_2-k_1}{k_2+k_1}\right) = 2\left(\frac{25-50}{25+50}\right)\times 10^{-3}$$

$= 0.667 \times 10^{-3} = 6.67\times 10^{-4}$

Amplitude of transmitted wave

$$A_t = \left(\frac{2k_1}{k_1+k_2}\right)A_i = \frac{2\times 50\times 2}{50+25}\times 10^{-3} = 2.67\times 10^{-3}$$

$\therefore$ Equation of reflected wave

$y_r = 6.67 \times 10^{-4} \cos \pi\,(2x + 50\,t)$

Equation of transmitted wave

$y_t = 2.67 \times 10^{-3} \cos \pi(x - 50t)$

37. **(i)** $\lambda = \frac{v}{f} = \frac{330}{200} = 1.65$ m, $\frac{d}{\lambda} = \frac{4}{1.65} = 2.4242$

At infinity, path difference = 0

As the man approaches, the path difference changes as $0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, 2\lambda$

$\therefore$ Hence only minima will appear to the man.

(ii) For $\Delta = \frac{\lambda}{2} = \sqrt{d^2+x_1^2} - x_1$ and $\frac{3\lambda}{2} = \sqrt{d^2+x_2^2} - x_2$

$\Rightarrow x_1 = 9.28$ m, $x_2 = 1.99$ m

38. **(i)** Combination of waves producing standing wave : $Z_1 + Z_2$

(ii) Combination of waves producing a wave travelling along x = y line : $Z_1 + Z_3$

(iii) Position of nodes in case (i) $x = (2n+1)\frac{\pi}{2k}$

case (ii) $x - y = (2n+1)\frac{\pi}{k}$

39. $\frac{2\pi}{\lambda} = \frac{\pi x}{10} \Rightarrow \lambda = 20$cm

(i) Total no of wavelength $= \frac{L}{\lambda} = \frac{100}{20} = 5$

$\therefore$ Number of loops formed $= 2\times 5 = 10$

(ii) Maximum displacement at $x = \frac{5}{3}$

$$A = 6\sin\left(\frac{\pi}{10}\times\frac{5}{3}\right) = 3\text{cm}$$

(iii) $y = 6\sin\left(\frac{\pi x}{10}\right)\cos(100\,\pi t)$

$= 6\sin(10\,\pi x)\cos(100\,\pi t)$

$v = 6\pi \sin(10\,\pi x)\sin(100\,\pi t)$ m/s

$$\therefore KE_{max} = \int_0^1 \frac{1}{2}\mu v^2 dx = \frac{1}{2}\mu\int_0^4 [6\pi\sin(10\pi x)]^2\,dx$$

where $\sqrt{\frac{T}{\mu}} = \frac{100\pi}{10\pi} = 10 = \mu = 0.4 \;\therefore KE_{max} = 36$J

(iv) $y = 6\sin\left(\frac{\pi x}{10}\right)\cos(100\pi t) = y_1 + y_2$

$\Rightarrow y_1 = 3\sin\left(\frac{\pi x}{10} - 100\pi t\right)$, $y_2 = 3\sin\left(\frac{\pi x}{10} + 100\pi t\right)$

40. $v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{2\times 10^{11}}{8000}} = 5000\,m/s$

$\frac{5}{2}\lambda = L \Rightarrow \lambda = \frac{2L}{5} = \frac{2\times 1}{5} = 0.4$ m

$\therefore\; k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.4} = 5\pi$

$\Rightarrow\; \omega = 2\pi f = \frac{2\pi v}{\lambda} = \frac{2\pi\times 5000}{0.4} = 2{,}5000\,\pi$

(i) Equation of the wave

$= 2\times 10^{-6}\cos(5\pi x)\sin(25{,}00\,\pi t)$

(ii) $y_1 = 10^{-6}\sin(25000\pi t - 5\pi x)$

$y_2 = 10^{-6}\sin(25000\,\pi t + 5\pi x)$

42. Amplitude after reflection

$$A_r = A_i\left(\frac{k_2 - k_1}{k_2 + k_1}\right) = 0.3\left(\frac{2.5-5}{2.5+5}\right) = -0.1$$

Amplitude after transmission

$$A_t = A_i\left(\frac{2k_1}{k_1 + k_2}\right) = 0.2 \text{ cm}$$

EXERCISE - 5

Part # I : AIEEE/JEE-MAIN

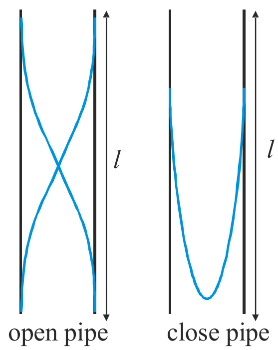
1. The fundamental frequency for an open pipe is

$$f = \frac{v}{\lambda}$$

$$\ell = \frac{\lambda}{2}; \lambda = 2\ell$$

$$f_{open} = \frac{v}{2\ell}$$

$$\ell = \frac{\lambda}{4}$$



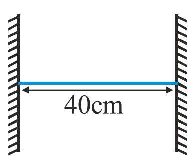
$$\lambda = 4\ell\,;\ f_{closed} = \frac{v}{\lambda} = \frac{v}{4\ell}$$

$$\frac{f_{open}}{f_{closed}} = \frac{f_A}{f_B} = \frac{v}{2\ell}\times\frac{4\ell}{v} = 2:1$$

4. As the string is tied between two rigid supports hence there will be nodes at both ends. The longest wavelength for nodes at both ends will be the one for which)

$$\ell = \frac{\lambda}{2} \Rightarrow \lambda = 2\ell$$

$$\lambda_{longest} = 2\times 40 = 80 \text{ cm}$$



5. $y = 10^{-4}\sin\left(600t - 2x + \frac{\pi}{3}\right)$

On comparing the given equation with the general equation of wave, we get

$$y = y_0 \sin(\omega t - kx + \phi)$$

$\omega = 600$; $k = 2$

Wave speed $= \frac{\omega}{k} = \frac{600}{2} = 300$ m/s

6. The frequency of the vibrating string with respect to tuning fork is either (256+5) Hz or (256–5) Hz

$$f_{wire} = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

On increasing tension; the beat frequency decreases to 2Hz, so probable frequency of the wire with respect to fork none is

$$256 \begin{cases} 256 \\ 258 \end{cases}$$

But on increasing the tension of wire, the frequency of the wire must have to increase. So, if the original frequency of the wire is assumed to be 261 then it reduces to 258 whereas if it is assumed to be 251 it has increased to 254. As we were expecting increase, so the correct frequency of the piano wire is (256–5) Hz.

7. If the frequency of fork 1 is 200 Hz then probable frequencies of fork 2 is either 196 Hz or 204 Hz.

As on attaching some tape on fork 2, be at frequency increases, this is possible only if the frequency of fork 2 is 196 Hz.

8. Given that $v_{observer} = \frac{v_{sound}}{5}$

Applying Doppler's effect, we get

$$f' = f\left[\frac{v + v_0}{v}\right];\ f' = f\left[\frac{6v/5}{v}\right] = \frac{6}{5};\ \frac{f'}{f} = \frac{6}{5}$$

$$\frac{f' - f}{f} = \frac{6}{5} - 1 = \frac{1}{5} = 20\%$$

9. $\because n' = \left(\frac{v}{v - v_s}\right)n$

$$\therefore 10000 = \left(\frac{300}{300 - v_s}\right)(9500)$$

$$\Rightarrow 300 - v_s = \frac{300\times 9500}{10000} = 285 \Rightarrow v_s = 15 \text{ ms}^{-1}$$

10. Intensity change in decibel

$$= 10\log\frac{I_2}{I_1} = 20 \Rightarrow \log\frac{I_2}{I_1} = 2 \Rightarrow \frac{I_2}{I_1} = 10^2 = 100$$

11. $n = \frac{1}{4x}\sqrt{\frac{\gamma RT}{M}},\ xn = \frac{1}{4}\sqrt{\frac{\gamma RT}{M}},\ x \propto \sqrt{T}$

12. $y = 0.005 \cos(\alpha x - \beta t)$

comparing the equation with the standard form,

$$y = A\cos\left[\left(\frac{x}{\lambda} - \frac{t}{T}\right)2\pi\right]$$

$\Rightarrow$ $2\pi/\lambda = \alpha$ and $2\pi/T = \beta$

$\Rightarrow$ $\alpha = 2\pi/0.08 = 25.00\pi$ and $\beta = \pi$

13.

	A	B	C
	$\nu - 1$	ν	$\nu + 1$

Between A & B $\quad$ 1 b/s $\quad$ 1

B & C $\quad$ 1 b/s $\quad$ 1

C & A $\quad$ 2 b/s $\quad$ $\frac{1}{2}$ $\quad$ $\frac{2}{2}$

$\Rightarrow$ 2 b/s

14. $n' = \dfrac{v - v_0}{v} n = \dfrac{94}{100} n$

from (iii) eq^n. of motion $v^2 = u^2 + 2as$

$\Rightarrow$ $v_0^2 = 0 + 2as \Rightarrow v_0 = \sqrt{2as}$

$$\Rightarrow \frac{94}{100} n = \left(\frac{v - \sqrt{2as}}{v}\right) n \Rightarrow s = 98 \text{ m.}$$

15. $y = 0.2 \sin\left[2\pi\left(\dfrac{t}{0.04} - \dfrac{x}{0.50}\right)\right]$

$$v = \sqrt{\frac{T}{m}} = \frac{\omega}{k} \Rightarrow \sqrt{\frac{T}{0.04}} = \frac{\frac{1}{0.04}}{\frac{1}{0.50}}$$

$$T = \left(\frac{0.50}{0.04}\right)^2 \times 0.04 = 6.25 \text{ N}$$

16. $y(x,t) = e^{-[\sqrt{a}x + \sqrt{b}t]^2}$

$v = \omega/K = \dfrac{\sqrt{b}}{\sqrt{a}}$ in –ve x direction.

17. $y_1(x, t) = 2a \sin(wt - kx)$

$y_2(x, t) = a \sin(2wt - 2kx)$

But Intensity

$$I = 2\pi^2 n^2 a^2 \rho v \Rightarrow \frac{I_1}{I_2} = \left(\frac{2a}{a} \times \frac{n}{2n}\right)^2 = \frac{1}{1}$$

Intensity depends on frequency and amplitude

So statement-1 is true statement-2 is false

18. $y_1 = A\sin(wt - kx)$ & $y_2 = A\sin(wt + kx)$

By superposition principle

$y = y_1 + y_2$

$= A\sin(wt - kx) + A\sin(wt + kx)$

$= 2A \sin wt \cos kx$

Amplitude $= 2A \cos kx$

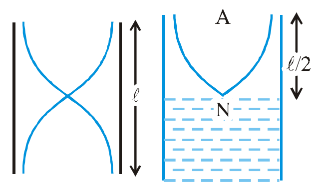
At nodes displacement is minimum

$2A\cos kx = 0 \Rightarrow \cos kx = 0$

$$kx = (2n+1)\frac{\pi}{2} \Rightarrow \frac{2\pi}{2} x = (2n+1)\frac{\pi}{2}$$

$x = (2n+1)\dfrac{\pi}{4}$ where $n = 0, 1, 2....$

19.



$$n_0 = \frac{v}{2\ell} \quad n_c = \frac{v}{4(\ell/2)} = \frac{v}{2\ell}$$

20. Fundamental frequency

$$f = \frac{V}{2\ell} = \frac{1}{2 \times 1.5}\sqrt{\frac{T}{eq}} = \frac{1}{3}\sqrt{\frac{y \times \text{strain} \times S}{\rho S}}$$

($S \rightarrow$ cross – section Area)

$$= \frac{1}{3}\sqrt{\frac{2.2 \times 10^{11} \times \frac{1}{100}}{7.7 \times 10^3}} = 178.2 \text{ Hz}$$

21. $f_1 = 1000\left(\dfrac{320}{300 - 20}\right) = 1066$ Hz

$f_2 = 1000\left(\dfrac{320}{300 + 20}\right) = 941$ Hz

$\therefore$ Change is $\simeq$ 12%

22. $t = 2\sqrt{\dfrac{\ell}{g}} = 2\sqrt{2}$ second

Part # II : IIT-JEE ADVANCED

1. Mass per unit length of the string,

$$m = \frac{10^{-2}}{0.4} = 2.5 \times 10^{-2} \text{kg/m}$$

$\therefore$ Velocity of wave in the string.

$$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{1.6}{2.5 \times 10^{-2}}} \Rightarrow v = 8 \text{m/s}$$

For constructive interference between successive pulses

$$\Delta t_{min} = \frac{2\ell}{v} = \frac{(2)(0.4)}{8} = 0.10 \text{s}$$

(After two reflections, the wave pulse is in same phase as it was produced, since in one reflection its phase changes by π, and if at this moment next identical pulse is produced, then constructive interference will be obtained.)

2. $$f_1 = f\left(\frac{v}{v - v_s}\right) \Rightarrow f_1 = f\left(\frac{340}{340 - 34}\right) = f\left(\frac{340}{306}\right)$$

and $$f_2 = f\left(\frac{340}{340 - 17}\right) = f\left(\frac{340}{323}\right) \quad \therefore \frac{f_1}{f_2} = \frac{323}{306} = \frac{19}{18}$$

3. Fundamental frequency is given by $v = \dfrac{1}{2\ell}\sqrt{\dfrac{T}{\mu}}$

(with both the ends fixed)

$\therefore$ Fundamental frequency $v \propto \dfrac{1}{\ell\sqrt{\mu}}$

[for same tension in both strings]

where μ = mass per unit length of wire= ρ.A

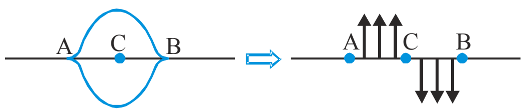
$$(\rho = \text{density}) = \rho(\pi r^2) \Rightarrow \sqrt{\mu} \propto r \quad \therefore v \propto \frac{1}{r\ell}$$

$$\therefore \frac{v_1}{v_2} = \left(\frac{r_2}{r_1}\right)\left(\frac{\ell_2}{\ell_1}\right) = \left(\frac{r}{2r}\right)\left(\frac{2L}{L}\right) = 1$$

4. Energy $E \propto (\text{amplitude})^2 (\text{frequency})^2$

Amplitude (A) is same in both the cases, but frequency 2ω, in the second case is two times the frequency (ω) in the first case. Therefore $E_2 = 4E_1$

5. After two seconds both the pulses will move 4 cm towards each other. So, by their superposition, the resultant displacement at every point will be zero. Therefore, total energy will be purely in the form of kinetic. Half of the particles will be moving upwards and half of downwards.



6. Using the formula $f' = f\left(\dfrac{v + v_0}{v}\right)$

we get, $$5.5 = 5\left(\frac{v + v_A}{v}\right) \quad \text{...(i)}$$

and $$6.0 = 5\left(\frac{v + v_B}{v}\right) \quad \text{...(ii)}$$

Here
v = speed of sound
v_A = speed of train A
v_B = speed of train B

Solving equations (i) and (ii) $\dfrac{v_B}{v_A} = 2$

7. Let f_0 = frequency of tuning fork.

$$f_0 = \frac{5}{2\ell}\sqrt{\frac{9g}{\mu}}$$

(μ = mass per unit length of wire) $= \dfrac{3}{2\ell}\sqrt{\dfrac{Mg}{\mu}}$

Solving this, we get M = 25 kg
In the first case frequency corresponds to fifth harmonic while in the second case it corresponds to third harmonic.

8. The motorcyclist observes no beats. So, the apparent frequency observed by him from the two sources must be equal.

$$f_1 = f_2 \therefore 176\left(\frac{330 - v}{330 - 22}\right) = 165\left(\frac{330 + v}{330}\right)$$

Solving this equation, we get v = 22m/s

9. Let $\Delta\ell$ be the end correction.
Given that, fundamental tone for a length 0.1m= first overtone for the length 0.35m.

$$\frac{v}{4(0.1 + \Delta\ell)} = \frac{3v}{4(0.35 + \Delta\ell)}$$

Solving this equation
we get $\Delta\ell = 0.025\text{m} = 2.5 \times 10^{-2}$ m

10. The frequency is a characteristic of source. It is independent of the medium.

11. $f_c = f_0$ (both first overtone)

$$\Rightarrow 3\left(\frac{v_c}{4L}\right) = 2\left(\frac{v_0}{2\ell_0}\right)$$

$$\therefore \ell_0 = \frac{4}{3}\left(\frac{v_0}{v_c}\right)L = \frac{4}{3}\sqrt{\frac{\rho_1}{\rho_2}}L \quad \text{as } v \propto \frac{1}{\sqrt{\rho}}$$

12. The frequency is a characteristic of source. It is independent of the medium.

13. $f_1 = \frac{v}{\ell}$ (2nd harmonic of open pipe)

$f_2 = n\left(\frac{v}{4\ell}\right)$ (n^{th} harmonic of closed pipe)

Here, n is odd and $f_2 > f_1$

It is possible when n=5 because with n = 5

$$\Rightarrow f_2 = \frac{5}{4}\left(\frac{v}{\ell}\right) = \frac{5}{4}f_1$$

14. The question is incomplete, as speed of sound is not given. Let us assume speed of sound as 330 m/s. Then, method will be as under.

$$\frac{\lambda}{2} = (63.2\text{–}30.7)\text{cm or } \lambda = 0.65 \text{ m}$$

$\therefore$ speed of sound observed

$v_0 = f\lambda = 512 \times 0.65 = 332.8$ m/s

$\therefore$ Error is calculting velocity of sound

$= 2.8\text{m/s} = 280$ cm/s

15. $f \propto v \propto \sqrt{T}$

$f_{AB} = 2f_{CD}$

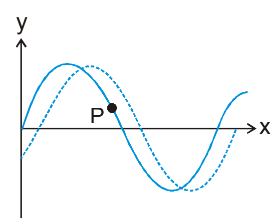
$\therefore T_{AB} = 4T_{CD}$ ....(i)

Further $\Sigma\tau_p = 0$

$\therefore T_{AB}(x) = T_{CD}(\ell - x)$

$\Rightarrow 4x = \ell - x$ (as $T_{AB} = 4T_{CD}$) $\Rightarrow x = \ell/5$

16. Take y=Asin(ωt–kx)



So $v_P = \frac{\partial y}{\partial t} = -A\omega\cos(\omega t - kx)$

$$\Rightarrow v_P = \omega\sqrt{A^2 - y^2} = \left(\frac{2\pi v}{\lambda}\right)\sqrt{A^2 - y^2}$$

$$= \frac{2\pi(10 \times 10^{-2})}{0.5}\left(\sqrt{(10)^2 - (5)^2}\right) \times 10^{-2}$$

$$= \frac{\sqrt{3}\pi}{50}\ \text{ms}^{-1}$$

17. $\frac{1}{2L}\sqrt{\frac{T}{m}} = \frac{3v}{4\ell}$

where $v = 340\ \text{ms}^{-1}$, $\ell = 75$ cm $= 0.75$ m

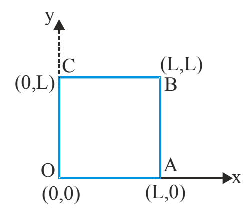
Now according to given condition

$$n - \frac{1}{2L}\sqrt{\frac{T}{m}} = 4 \quad \text{So } n = \frac{1}{2L}\sqrt{\frac{T}{m}} + 4 = \left(\frac{3v}{4\ell} + 4\right)$$

$$= \frac{3}{4} \times \frac{340}{0.75} + 4 = 344 \text{ Hz.}$$

MCQ

1. Since, the edges are clamped, displacement of the edges u(x,y) = 0 for



Line OA i.e.	y = 0	$0 \le x \le L$
AB i.e.	x = L	$0 \le y \le L$
BC i.e.	y = L	$0 \le x \le L$
OC i.e.	x = 0	$0 \le y \le L$

The above conditions are satisfied only i nalternatives (b) and (c).

Note that u(x,y)=0, for all four values e.g. in alternative (d), u(x,y)=0 for y=0, y=L but it is not zero for x=0 or x=L, Similarly in option (a) u(x,y) = 0 at x = L, y = L but it is not zero for x=0 or y=0 while in options (b) and (c), u(x,y) = 0 for x = 0, y = 0 x = L and y = L.

2. Maximum speed of any point on the string $= a\omega$
$= a(2\pi f)$

$$\therefore \quad = \frac{v}{10} = \frac{10}{10} = 1 \qquad \text{(Given : v=10m/s)}$$

$$\therefore \quad 2\pi af = 1$$

$$\therefore \quad f = \frac{1}{2\pi a} \;\Rightarrow\; a = 10^{-3}\text{m} \quad \text{(Given)}$$

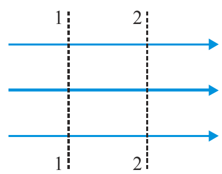
$$\therefore \quad f = \frac{1}{2\pi \times 10^{-3}} = \frac{10^3}{2\pi}\text{Hz}$$

Speed of wave $v = f\lambda$

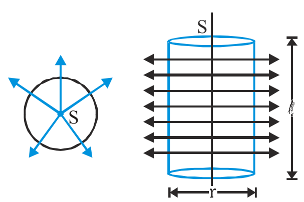
$$\therefore \quad (10\text{ m/s}) = \left(\frac{10^3}{2\pi}s^{-1}\right)\lambda$$

$$\therefore \quad \lambda = 2\pi \times 10^{-2}\text{m}$$

3. For a plane wave intensity (energy crossing per unit area per unit time) is constant at all points.



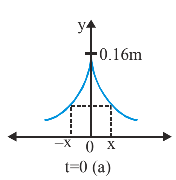
But for a spherical wave, intensity at a distance r from a point source of power P (energy transmitted per unit time) is given by :



$$I = \frac{P}{4\pi r^2} \quad \text{or} \quad I \propto \frac{1}{r^2}$$

4. The shape of pulse at x=0 and t=0 would be as shown, in figure (a).

$$y(0,0) = \frac{0.8}{5} = 0.16\text{m}$$



From the figure it is clear that $y_{max} = 0.16$m
Pulse will be symmetric (Symmetry is checked about y_{max}) if at t = 0

$$y(x) = y(-x)$$

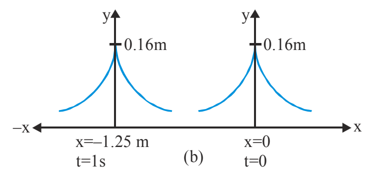
From the given equation and

$$\left.\begin{aligned} y(x) &= \frac{0.8}{16x^2+5} \\ y(-x) &= \frac{0.8}{16x^2+5} \end{aligned}\right\} \text{at } t=0 \Rightarrow y(x) = y(-x)$$

Therefore, pulse is symmetric.

Speed of pulse

At t=1s and x=–1.25 m



value of y is again 0.16 m, i.e., pulse has travelled a distance of 1.25 m in 1s in negative x- direction or we can say that the speed of pulse is 1.25 m/s and it is travelling in negative x-direciton. Therefore, it will travel a distance of 2.5m in 2s. The above statement can be better understood from figure (b).

5. In case of sound wave, y can represent pressure and displacement, while in case of an electromagnetic wave it represents electric and magnetic fields.

6. Standing waves can be produced only when two similar type of waves (same frequency and speed, but amplitude may be different) travel in opposite directions.

7. B **8.** ACD **9.** ABC

Comprehension#1

1. In one second number of maximas is called the beat frequency.

Hence, $f_b = f_1 - f_2 = \dfrac{100\pi}{2\pi} - \dfrac{92\pi}{2\pi} = 4\text{Hz}$

2. Speed of wave

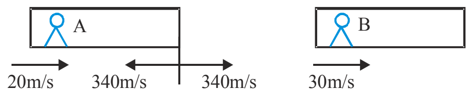
$$v = \frac{\omega}{R} = v = \frac{100\pi}{0.5\pi} \Rightarrow \frac{92\pi}{0.46\pi} = 200 \text{ m/s}$$

3. At x=0, $y = y_1 + y_2 = 2A\cos 96\pi t \cos 4\pi t$
Frequency of cos $(96\pi t)$ function is 48Hz and that of cos $(4\pi f)$ function is 2 Hz.
In one second cos function becomes zero at 2f times, where f is the frequency. Therefore, first function will become zero at 96 times and the second at 4 times. But second will not overlap with first. Hence, net y will become zero 100 times in 1s.

Comprehension#2

1. $v_{SA} = 340 + 20 = 360$ m/s

$v_{SB} = 340 - 30 = 310$ m/s



2. For the passengers in train A, there is no relative motion between source and observer, as both are moving with velocity 20m/s. Therefore, there is no change in observed frequencies and correspondingly there is no change in their intensities.

3. For the passengers in train B, observer is receding with velocity 30m/s and source is approaching with velocity 20m/s.

$$\therefore \quad f_1' = 800\left(\frac{340-30}{340-20}\right) = 775\text{Hz}$$

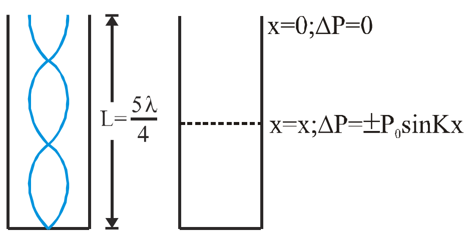
and $$f_2' = 1120\left(\frac{340-30}{340-20}\right) = 1085\text{Hz}$$

$\therefore$ Spread of frequency $= f_2' - f_1' = 310$ Hz

Subjective

1. **(i)** Frequency of second overtone of the closed pipe

$$= 5\left(\frac{v}{4L}\right) = 440$$



$$\therefore \; L = \frac{5v}{4\times 440}\text{m}$$

Substituting v= speed of sound in air =330m/s

$$L = \frac{5\times 330}{4\times 440} = \frac{15}{16}\text{m}$$

$$\lambda = \frac{4L}{5} = \frac{4\left(\frac{15}{16}\right)}{5} = \frac{3}{4}\text{m}$$

(ii) Open end is displacement antinode. Therefore, it would be a pressure node or at x=0; ΔP=0

Pressure amplitude at x = x, can be written as $\Delta P = \pm \Delta P_0 \sin kx$

where $$k = \frac{2\pi}{\lambda} = \frac{2\pi}{3/4} = \frac{8\pi}{3}\text{m}^{-1}$$

Therefore, pressure amplitude at

$$x = \frac{L}{2} = \frac{15/16}{2}\text{m or }(15/32)\text{ m will be}$$

$$\Delta P = \pm P_0 \sin\left(\frac{8\pi}{3}\right)\left(\frac{15}{32}\right) = \pm \Delta P_0 \sin\left(\frac{5\pi}{4}\right)$$

$$\Rightarrow \quad \Delta P = \pm\frac{\Delta P_0}{\sqrt{2}}$$

(iii) Open end is a pressure node i.e. $\Delta P = 0$

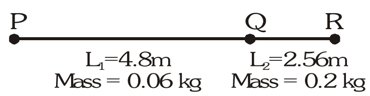
Hence, $P_{max} = P_{min} =$ Mean pressure (P_0)

(iv) Closed end is a displacement node or pressure antinode.

Therefore, $P_{max} = P_0 + \Delta P_0$ and $P_{min} = P_0 - \Delta P_0$

2. Amplitude of incident wave

$A_i = 3.5$ cm



Tension $T = 80$N

Mass per unit length of wire PQ is

$$m_1 = \frac{0.06}{4.8} = \frac{1}{80}\text{kg/m}$$

and mass per unit length of wire QR is

$$m_2 = \frac{0.2}{2.56} = \frac{1}{12.8}\text{kg/m}$$

(i) Speed of wave in wire PQ is

$$v_1 = \sqrt{\frac{T}{m_1}} = \sqrt{\frac{80}{1/80}} = 80\text{m/s}$$

and speed of wave in wire QR is

$$v_2 = \sqrt{\frac{T}{m_2}} = \sqrt{\frac{80}{1/128}} = 32\text{ m/s}$$

$\therefore$ Time taken by the wave pulse to reach from P to R is

$$t = \frac{4.8}{V_1} + \frac{2.56}{V_2} = \left(\frac{4.8}{80} + \frac{2.56}{32}\right)\text{s} \Rightarrow t = 0.14\text{s}$$

(ii) The expressions for reflected and transmitted amplitudes (A_r and A_t) in terms of v_1, v_2 and A_i are as follows :

$$A_r = \frac{v_2 - v_1}{v_2 + v_1} A_i \text{ and } A_t = \frac{2v_2}{v_1 + v_2} A_i$$

Substituting the values, we get

$$A_r = \left(\frac{32 - 80}{32 + 80}\right)(3.5) = -1.5 \text{ cm}$$

i.e., the amplitude of reflected wave will be 1.5cm. Negative sign of A_r indicates that there will be a phase change of π in reflected wave. Similarly.

$$A_t = \left(\frac{2 \times 32}{32 + 80}\right)(3.5) = 2.0\text{cm}$$

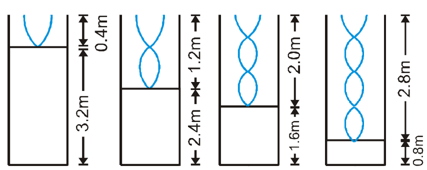
i.e., the amplitude of transmitted wave will be 2.0cm

3. Speed of sound $v = 340$ m/s

Let ℓ_0 be the length of air column corresponding to the fundamental frequency.

$$\frac{v}{4\ell_0} = 212.5$$

$$\Rightarrow \ell_0 = \frac{v}{4(212.5)} = \frac{340}{4(212.5)} = 0.4 \text{ m}$$



In closed pipe only odd harmonics are obtained. Now let $\ell_1, \ell_2, \ell_3 \ell_4$, etc., be the lengths corresponding to the 3rd harmonic, 4th harmonic, 7th harmonic etc. Then,

$$3\left(\frac{v}{4\ell_1}\right) = 212.5 \Rightarrow \ell_1 = 1.2\text{m}$$

$$5\left(\frac{v}{4\ell_2}\right) = 212.5 \Rightarrow \ell_2 = 2.0 \text{ m}$$

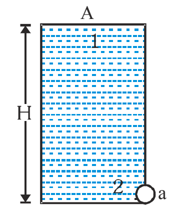
$$\text{and } 7\left(\frac{v}{4\ell_3}\right) = 212.5 \Rightarrow \ell_3 = 2.8\text{m}$$

$$9\left(\frac{v}{3\ell_4}\right) = 212.5 \Rightarrow \ell_4 = 3.6\text{m}$$

or heights of water level are (3.6–0.4)m, (3.6 –1.2)m, (3.6 – 2.0) m,(3.6 – 2.8)m.

$\therefore$ Heights of water level are 3.2m, 2.4m, 1.6m and 0.8m.

Let A and a be the area of cross-sections of the pipe and hole respectively. Then



$A = \pi(2 \times 10^{-2})^2 = 1.26 \times 10^{-3}\ \text{m}^2$

and $a = \pi(10^{-3})^2 = 3.14 \times 10^{-6}\ \text{m}^2$

Velocity of efflux $v = \sqrt{2gH}$

Continuity equaiton at 1 and 2 gives :

$$a\sqrt{2gH} = A\left(\frac{-dH}{dt}\right)$$

$\therefore$ Rate of fall of water level in the pipe.

$$\left(-\frac{dH}{dt}\right) = \frac{a}{A}\sqrt{2gH}$$

Substituting the values we get

$$\frac{-dH}{dt} = \frac{3.14 \times 10^{-6}}{1.26 \times 10^{-3}}\sqrt{2 \times 10 \times H}$$

$$\Rightarrow -\frac{dH}{dt} = (1.11 \times 10^{-2})\sqrt{H}$$

Between first two resonances, the water level falls from 3.2m to 2.4m.

$$\therefore \frac{dH}{\sqrt{H}} = -(1.11 \times 10^{-2})dt \Rightarrow \int_{3.2}^{2.4} \frac{dH}{\sqrt{H}} = -(1.11 \times 10^{-2})\int_0^t dt$$

$$\Rightarrow 2[\sqrt{2.4} - \sqrt{3.2}] = -(1.11 \times 10^{-2}).t \Rightarrow t \approx 43\text{s}$$

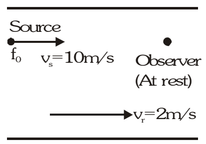
4. Velocity of sound in water is

$$v_w = \sqrt{\frac{\beta}{\rho}} = \sqrt{\frac{2.088 \times 10^9}{10^3}} = 1445\text{m/s}$$

Frequency of sound in water will be

$$f_0 = \frac{v_w}{\lambda_w} = \frac{1445}{14.45 \times 10^{-3}}\text{Hz} \Rightarrow f_0 = 10^5 \text{ hz}$$

(i) Frequency of sound detected by receiver (observer) at rest would be



$$f_1 = f_0\left(\frac{v_w + v_r}{v_w + v_r - v_s}\right) = (10^5)\left(\frac{1445+2}{1445+2-10}\right)\text{Hz}$$

$f_1 = 1.0069 \times 10^5$ Hz

(ii) Velocity of sound in air is

$$v_a = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{(1.4)(8.31)(20+273)}{28.8\times 10^{-3}}} = 344\text{m/s}$$

$\therefore$ Frequency does not depend on the medium. Therefore, frequency in air is also $f_0 = 10^5$ Hz

$\therefore$ Frequency of sound detected by receiver (observer) in air would be

$$f_2 = f_0\left(\frac{v_a - w}{v_a - w - v_s}\right) = 10^5\left[\frac{344-5}{344-5-10}\right]\text{Hz}$$

$f_2 = 1.0304 \times 10^5$ Hz

5. **(i)** Frequency of second harmonic in pipe A = frequency of third harmonic in pipe B

$$\therefore\ 2\left(\frac{v_A}{2\ell_A}\right) = 3\left(\frac{v_B}{4\ell_B}\right)$$

$$\Rightarrow \frac{v_A}{v_B} = \frac{3}{4}\ (\text{as } \ell_A = \ell_B) \Rightarrow \frac{\sqrt{\frac{\gamma_A RT_A}{M_A}}}{\sqrt{\frac{\gamma_B RT_B}{M_B}}} = \frac{3}{4}$$

$$\Rightarrow \sqrt{\frac{\gamma_A}{\gamma_B}}\sqrt{\frac{M_B}{M_A}} = \frac{3}{4}\ (\text{as } T_A = T_B)$$

$$\therefore \frac{M_A}{M_B} = \frac{\gamma_A}{\gamma_B}\left(\frac{16}{9}\right) = \left(\frac{5/3}{7/5}\right)\left(\frac{16}{9}\right)$$

$$\left(\gamma_A = \frac{5}{3} \text{ and } \gamma_B = \frac{7}{5}\right)$$

$$\Rightarrow \frac{M_A}{M_B} = \left(\frac{25}{21}\right)\left(\frac{16}{9}\right) = \frac{400}{189}$$

(ii) Ratio of fundamental frequency in pipe A and in pipe B is :

$$\frac{f_A}{f_B} = \frac{v_A/2\ell_A}{v_B/2\ell_B} = \frac{v_a}{v_B} \qquad (\text{as } \ell_A = \ell_B)$$

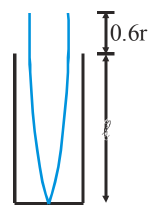
$$= \frac{\sqrt{\frac{\gamma_A RT_A}{M_A}}}{\sqrt{\frac{\gamma_B RT_B}{M_B}}} = \sqrt{\frac{\gamma_A}{\gamma_B}\cdot\frac{M_B}{M_A}}\ (\text{as } T_A = T_B)$$

Substituting $\frac{M_B}{M_A} = \frac{189}{400}$ from part (i), we get

$$\frac{f_A}{f_B} = \sqrt{\frac{25}{21}\times\frac{189}{400}} = \frac{3}{4}$$

6. Fundamental frequency

$$f = \frac{v}{4(\ell + 0.6r)}$$

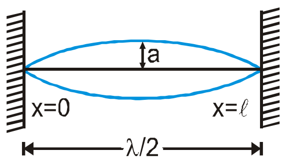


$\therefore$ Speed of sound $v = 4f(\ell + 0.6r)$

$\Rightarrow$ $v = (4)(480)[(0.16) + (0.6)(0.025)] = 336$m/s

7. $\ell = \frac{\lambda}{2} \Rightarrow \lambda = 2\ell,\ k = \frac{2\pi}{\lambda} = \frac{\pi}{\ell}$

The amplitude at a distance x from x=0 is given by A = a sin kx



Total mechanical energy at x of length dx is

$$dE = \frac{1}{2}(dm)A^2\omega^2 = \frac{1}{2}(\mu dx)(a\sin kx)^2(2\pi f)^2$$

$\Rightarrow$ $dE = 2\pi^2\mu f^2 a^2 \sin^2 kx\, dx$

Here, $f = \frac{v^2}{\lambda^2} = \frac{\left(\frac{T}{\mu}\right)}{(4\ell^2)}$ and $k = \frac{\pi}{\ell}$

Substituting these values in equation (i) and integrating it from x=0 to x=ℓ, we get total energy of string

$$E = \frac{\pi^2 a^2 T}{4\ell}$$

8. From the relation $f' = f\left(\frac{v}{v \pm v_s}\right)$

we have $2.2 = f\left[\frac{300}{300 - v_T}\right]$ ...(i)

and $1.8 = f\left[\frac{300}{300 + v_T}\right]$ ...(ii)

Here, $v_T = v_s$ = velocity of source/train

Solving equation (i) and (ii), we get $v_T = 30$m/s

9. Maximum particle velocity

$\omega A = 3$m/s ...(i)

Maximum particle acceleration

$\omega^2 A = 90$m/s^2 ...(ii)

Velocity of wave $\frac{\omega}{k} = 20$m/s ...(iii)

From equation (i), (ii) and (iii), we get ω =30 rad/s

$A = 0.1$m and $k = 15$m^{-1}

$\therefore$ Equation of waveform should be

$y = A\sin(\omega t + kx + \phi)$

$y = (0.1\text{m})\sin[(30\text{ rad/s})t + (1.5\text{m}^{-1})x + \phi]$

10. L = 20 cm; m = 1 gm

$$\mu = \frac{m}{L} = \frac{1}{20}\text{gm/cm} = \frac{1}{20} \times \frac{10^{-3}}{10^{-2}}\text{g/m}$$

$$\mu = \left(\frac{1}{200}\right)\text{kg/m} ;\ T = 0.5\text{ N}$$

$$v = \sqrt{\frac{0.5}{\left(\frac{1}{200}\right)}} = 10\text{ m/s};\ f = 100\text{ Hz}$$

$$\lambda = \frac{v}{f} = \frac{10}{100} = \frac{1}{10}\text{m} \Rightarrow \frac{\lambda}{2} = \frac{1}{20}\text{m} = 5\text{ cm}$$

MOCK TEST : STRING WAVE

1. As wave has been reflected from a rarer medium, therefore there is no change in phase. Hence equation for the reflected waves can be written as

$y = 0.5A\sin(-kx - \omega t + \theta) = -0.5A\sin(kx + \omega t - \theta)$

2. Substituting x = 0 we have given wave y = A sin ωt at x = 0 other should have y = – A sin ωt equation so displacement may be zero at all the time Hence (B) is correct option.

3. $f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$ $\qquad \mu = \rho\,\pi r^2$

If radius is doubled and length is doubled, mass per unit length will become four times. Hence

$$f' = \frac{1}{2 \times 2\ell}\sqrt{\frac{2T}{4\mu}} = \frac{f}{2\sqrt{2}}$$

4. $\lambda = 2\ell = 3$m

Equation of standing wave

(As x = 0 is taken as a node)

y = 2A sin kx cos ωt

y = A as amplitude is 2A.

A = 2A sin kx

$kx = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

$\frac{2\pi}{\lambda}x = \frac{\pi}{6} \Rightarrow x_1 = \frac{1}{4}$m

and $\frac{2\pi}{\lambda}.x = \frac{\pi}{2} + \frac{\pi}{3} \Rightarrow x_2 = 1.25\text{ m} \Rightarrow x_2 - x_1 = 1$m

5. Given $\omega = 3\pi$

$\therefore\ f = \frac{\omega}{2\pi} = 1.5,$

Also $\Delta x = 1.0$ cm

Given, $\phi = \frac{2\pi}{\lambda}\Delta x \Rightarrow \frac{\pi}{8} = \frac{2\pi}{\lambda} \times 1$

$\Rightarrow \lambda = 16\text{ cm} \Rightarrow v = f\lambda = 16 \times 1.5 = 24$ cm/sec

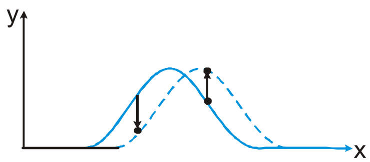
6. $\frac{I_1}{I_2} = \frac{a_1^2 f_1^2}{a_2^2 f_2^2} = \frac{(3)^2(8)^2}{(2)^2(12)^2} = 1$

7. $y(x, t = 0) = \frac{6}{x^2}$ then $y(x,t) = \frac{6}{(x - 2t)^2}$

$\Rightarrow \frac{\partial y}{\partial t} = \frac{24}{(x - 2t)^3}$ at x = 2, t = 2

$V_y = \frac{24}{(-2)^3} = -3$ m/s.

8. Dotted shape shows pulse position after a short time interval. Direction of the velocities are decided according to direction of displacements of the particles.



9. $R_A = \frac{V}{V_A}$, $R_B = \frac{V}{V_B}$ as $V_A > V_B , R_A < R_B$

10. $P = \frac{1}{2}\mu\omega^2A^2V$ using $V = \sqrt{\frac{T}{\mu}}$

$P = \frac{1}{2}\omega^2 A^2 \sqrt{T\mu}$

$\omega = \sqrt{\frac{2P}{A^2\sqrt{T\mu}}}$ $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{2P}{A^2\sqrt{T\mu}}}$

using the given data, we get $f = 30$ Hz.

11. In figure, 'C' reaches the position where 'A' already reaches after $wt = \frac{\pi}{2}$

and 'A' reaches the position where 'B' already reaches after $wt = \frac{\pi}{2}$

Hence (B).

12. $384 = \frac{nv}{2\ell}$ ...(i)

$288 = \frac{mv}{2\ell}$ ...(ii)

from equation (i) & (ii)

$\left(\frac{n}{m}\right) = \left(\frac{4}{3}\right)$

so n = 4

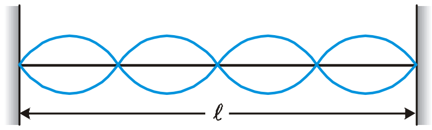
from equation (i)

$384 = \frac{4v}{2 \times 3/4} = \frac{10v}{3}$

v = 144 m/s

13. For a string vibrating in its n^{th} overtone ($(n + 1)^{th}$ harmonic)

$y = 2A \sin\left(\frac{(n+1)\pi x}{L}\right) \cos\omega t$



For x = , 2A = a and n = 3;

y = cos ωt = a. sin cos ωt = – a.cos ωt

i.e. at x = ; the amplitude is .

14. In a sonometer,

$\nu \propto \sqrt{T}$ $\therefore \frac{\nu_1}{\nu_2} = 2 = \sqrt{\frac{T_1}{T_2}} \Rightarrow T_2 = \frac{T_1}{4}$

∴ % change will be : × 100 = × 100 = 75% **Ans.**

15. For waves along a string :

$\upsilon \propto \sqrt{T} \Rightarrow \lambda \propto \sqrt{T}$

Now, for 6 loops : $3\lambda_1 = L \Rightarrow \lambda_1 = L/3$

& for 4 loops : $2\lambda_2 = L \Rightarrow \lambda_2 = L/2$

$\Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{2}{3} \Rightarrow T_2 = \frac{9}{4} x T_1 = \frac{9}{4} \times 36 = 81$ N.**Ans.**

16. Velocity of sound is inversely proportional to the square root of density of the medium.

i.e. $V\sqrt{\rho}$ = constant $\Rightarrow \frac{V_1}{V_2} = \sqrt{\frac{\rho_2}{\rho_1}} = \sqrt{\frac{2\rho}{\rho}} = \sqrt{2}$ **Ans.**

17. $y = \log\frac{x^2 - t^2}{x - t} = \log(x + t)$

$(\because \log a - \log b = \log\frac{a}{b})$

$\frac{\partial y}{\partial x} = \frac{1}{(x+t)} \Rightarrow \frac{\partial^2 y}{\partial x^2} = -\frac{1}{(x+t)^2}$ and

$\frac{\partial y}{\partial t} = \frac{(\partial x/\partial t)}{(x+t)} = \frac{v}{(x+t)}$

$\frac{\partial^2 y}{\partial t^2} = -\frac{v^2}{(x+t)^2} \Rightarrow \frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$

Which is the general form of wave equation.

18. At t = 2 second, the position of both pulses are separately given by fig.(a) and fig. (b); the superposition of both pulses is given by fig. (c)

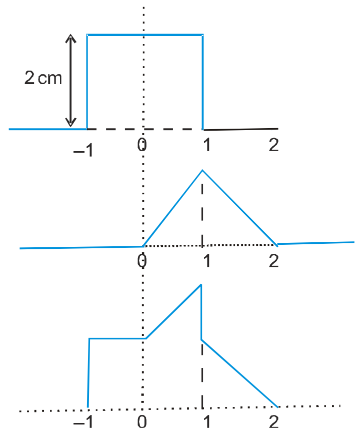


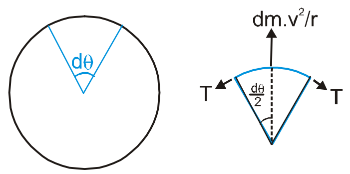
Fig. (a)

Fig. (b)

Fig. (c)

19. (B)

19. $dm.\omega^2 R = 2T \sin\dfrac{d\theta}{2}$



$$\mu R d\theta\, \omega^2 R = 2T\frac{d\theta}{2}$$

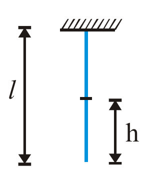
$$\Rightarrow \mu\omega^2 R^2 = T \Rightarrow v_w = \sqrt{\frac{T}{\mu}} = \sqrt{\omega^2 R^2} = \omega R$$

Also speed of string is ωR

$\therefore$ The velocity of disturbance w.r.t. ground $= \omega R + \omega R = 2\omega R$.

20. Let ℓ be the length of rope. Then tension in the string at height h will be :

$$T = \frac{m}{\ell}hg$$



$$u = \sqrt{\frac{T}{\mu}}$$

Here, μ = mass per unit length $= \dfrac{m}{\ell}$

$\therefore$ $u = \sqrt{gh}$ or $u^2 = gh$

i.e., u versus h graph is a parabola.

21. Let a_i and a_r be the amplitudes of incident and reflected wave. Then

$$\frac{a_i + a_r}{a_i - a_r} = n\,(\text{Given}) \quad \therefore \frac{a_r}{a_i} = \left(\frac{n-1}{n+1}\right)$$

$\therefore$ Fraction of energy reflected is

$$\frac{E_r}{E_i} = \left(\frac{a_r}{a_i}\right)^2 = \left(\frac{n-1}{n+1}\right)^2$$

22. $f_0 = \dfrac{v}{2\ell}$

Now beat frequency

$$= f_1 - f_2 = \frac{v}{2\left(\frac{\ell}{2} - \Delta\ell\right)} - \frac{v}{2\left(\frac{\ell}{2} + \Delta\ell\right)}$$

$$= \frac{v}{2}\left[\frac{1}{\frac{\ell}{2} - \Delta\ell} - \frac{1}{\frac{\ell}{2} + \Delta\ell}\right]$$

$$= (f_0\ell)\left[\frac{2}{\ell - 2\Delta\ell} - \frac{2}{\ell + 2\Delta\ell}\right] = 2f_0\ell\left[\frac{1 + 2\Delta\ell - \ell + 2\Delta\ell}{\ell^2 - 4(\Delta\ell)^2}\right]$$

$$\approx 2f_0\ell\left(\frac{4\Delta\ell}{\ell^2}\right) \approx \frac{8f_0\Delta\ell}{\ell}$$

23. $v = \sqrt{T/\mu}$ or $v \propto \dfrac{1}{\mu}$ $\quad 1 \to RP,\ 2 \to PQ,\ 3 \to QS.$

Here μ is mass per unit length.

$$\mu_1 = \frac{0.1}{2} = 0.05\ \text{kg/m} \quad \mu_2 = \frac{0.1}{3} = 0.067\ \text{kg/m and}$$

$$\mu_3 = \frac{0.15}{4} = 0.0375\ \text{kg/m}$$

$\mu_3 < \mu_1 < \mu_2$

$\therefore$ $\mu_3 < \mu_1 < \mu_2$

Between string RP and PQ, medium of string PQ is denser. Therefore, wave-2 will suffer a phase change of π. Between string PQ and QS, medium of string PQ is denser. Therefore wave 4 will not suffer any phase change.

24. Speed of wave in wire

$$V = \sqrt{\frac{T}{\rho A}} = \sqrt{\frac{Y\Delta\ell}{\ell}A \times \frac{1}{\rho A}} = \sqrt{\frac{Y\Delta\ell}{\ell\rho}}$$

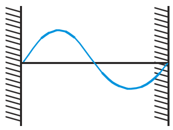
Maximum time period means minimum frequency ; that means fundamental mode.

$$f = \frac{V}{\lambda} = \frac{V}{2\ell}$$

$\therefore \quad T = \frac{2\ell}{V} = 2\ell\sqrt{\frac{\ell\rho}{Y\Delta\ell}} = \frac{1}{35}$ second **Ans.**

$\therefore \quad (f = 35\ Hz)$

and; frequency of first overtone



$= \frac{V}{\ell} = 70\ Hz.$

25. $y = 2A \sin kx. \sin\omega t$

$V_y = \frac{dy}{dt} = 2A \sin kx. \cos\omega t$

$V_y = 0 \Rightarrow t = T/4, 3T/4 \qquad \left(T = \frac{2\pi}{\omega}\right)$

(2 times in one time period)

26. In standing waves, particles may have phase differences only 0 or π.

27. $\frac{\lambda}{4} = 0.1 \Rightarrow \lambda = 0.4$ m

from graph $\Rightarrow$ T = 0.2 sec.

and amplitude of standing wave is 2A = 4 cm.

Equation of the standing wave

$y(x, t) = -2A \cos\left(\frac{2\pi}{0.4}x\right).\sin\left(\frac{2\pi}{0.2}t\right)$ cm

$y\ (x = 0.05,\ \ t = 0.05) = -2\sqrt{2}$ cm

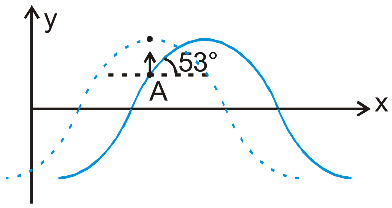
$y(x = 0.04, t = 0.025) = -2\sqrt{2}\ \cos 36°$

speed $= \frac{\lambda}{T} = 2$ m/sec.

$V_y = \frac{dy}{dt} = -2A \times \frac{2\pi}{0.2} \cos\left(\frac{2\pi x}{0.4}\right).\cos\left(\frac{2\pi t}{0.2}\right) V_y$

$= (x = \frac{1}{15}$ m, $t = 0.1) = 20\pi$ cm/sec.

28. As shown in the curve, if wave is moving along–



x axis, V_p is positive. $\frac{V_p}{V_w} = -\tan\theta$

$|-\tan\theta\ | > 1 \qquad \therefore V_p > V_w.$

29. As $f_1 : f_2 : f_3 = 3 : 5 : 7$, string is fixed at one end, Its fundamental frequency is $f_0 = \frac{f_1}{3} = \frac{105}{3} = 35$ Hz

30. Every small segment is acted upon by forces from both sides of it hence energy is not conserved, rather it is transmitted by the element.

31. Two waves moving in uniform string with uniform tension shall have same speed and may be moving in opposite directions. Hence both waves may have velocities in opposite direction. Hence statement-1 is false

32. (False) at node v = 0, at antinode Tension $\perp$ to velocity $\therefore$ at the points power = 0 $\quad (P = \vec{F}.\vec{V})$

At other points $P \neq 0$.

33. $\mu = \frac{1.2}{2} = 0.6$ kg/m

f = 5 Hz

$\lambda = 2\ell = 4$m

$V = f\lambda = 5 \times 4 = 20$ m/s **Ans. 11.34**

using $v = \sqrt{\frac{T}{\mu}} \Rightarrow T = 20^2 \times 0.6 = 240$ N **Ans. 11.33**

$\left(\frac{\partial y}{\partial t}\right)_{max} = 3.14$ m/s

$(2A)\omega = 3.14$

Amplitude $2A = \frac{3.14}{2 \times (3.14) \times 5} = 0.1$ m

Equation of standing wave is

$y = (0.1) \sin(\pi/2)x \sin(10\pi)t$ **Ans. 11.35**

36. The equation of wave moving in negative x-direction, assuming origin of position at x = 2 and origin of time (i.e. initial time) at t = 1 sec.

$y = 0.1 \sin(4\pi(t-1) + 8(x-2))$

Shifting the origin of position to left by 2m, that is, to x = 0. Also shifting the origin of time backwards by 1 sec, that is to t = 0 sec.

$y = 0.1 \sin(4\pi(t-1) + 8(x-2))$

37. As given the particle at x = 2 is at mean position at t = 1 sec.

$\therefore$ its velocity $v = \omega A = 4\pi \times 0.1 = 0.4\pi$ m/s.

38. Time period of oscillation $T = \frac{2\pi}{\omega} = \frac{2\pi}{4\pi} = \frac{1}{2}$ sec.

Hence at t = 1.125 sec, that is, at $\frac{T}{4}$ seconds after t = 1 second, the particle is at rest at extreme position. Hence instantaneous power at x = 2 at t = 1.125 sec is zero.

39. **(A) – p,q,r,t ; (B) – p,q,s ; (C) – p,r,s,t ; (D) – p,s**

40. **(A)** Number of loops (of length $\lambda/2$) will be even or odd and node or antinode will respectively be formed at the middle.

Phase difference between two particle in same loop will be zero and that between two particles in adjacent loops will be π.

(B) and **(D)** Number of loops will not be integral. Hence neither a node nor an antinode will be formed in in the middle.

Phase difference between two particle in same loop will be zero and that between two particles in adjacent loops will be π.

41. for refraction medium changes so frequency does not change beacuse frequency depends on sources.

for reflection medium does not change so speed of wave and wave length does not change

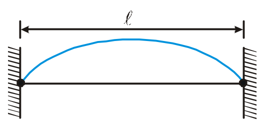
refraction from rarer to denser medium wave length decreases beacuses density of denser medium high but frequency does not change.

reflection from denser medium speed and frequency does not change beause medium is same but due to reflection change of π. takes place

42 $\mu = \dfrac{3.2\,\text{gm}}{40\text{ cm}} = \dfrac{3.2\times10^{-3}}{40\times10^{-2}} = \dfrac{3.2}{40} = \dfrac{32}{4000}$ kg/m

$\ell = \dfrac{\lambda}{2}$

$\Rightarrow \lambda = 2\ell$ ....(1)



$$f = \frac{v}{\lambda} = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$$

$$\Rightarrow \frac{1000}{64} = \frac{1}{2\times40\times10^{-2}}\sqrt{\frac{T}{32/4000}}$$

$$\Rightarrow \left[\frac{1000}{64}\times2\times40\times10^{-2}\right]^2 \frac{32}{4000}$$

$$= T\ \frac{10000}{64}\times\frac{32}{4000} = T \Rightarrow T = \frac{10}{8}\ \text{N}$$

$$\Rightarrow y = \frac{\dfrac{10/8}{10^{-6}}}{\dfrac{.05\times10^{-2}}{40\times10^{-2}}} = \frac{10^7}{8}\ \frac{40}{(.05)} = 10^9\ \text{N/m}^2.$$

43. $\mu = Kx = \dfrac{dM}{dx}$

$$\int_O^M dM = \int_O^\ell Kx\,dx \text{ and } K = \frac{2M}{\ell^2}$$

$$V = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{F}{Kx}} = \frac{dx}{dt} \quad \int_O^\ell \sqrt{x}\,dx = \sqrt{\frac{F}{K}}\int_o^t dt$$

$$\therefore t = \sqrt{\frac{4\ell^3}{9}.\frac{K}{f}} = \sqrt{\frac{4\ell^3}{9F}.\frac{2m}{\ell^2}} = \sqrt{\frac{8M\ell}{9F}} = \sqrt{\frac{8\times45\times1.5}{9\times15}} = 2.$$

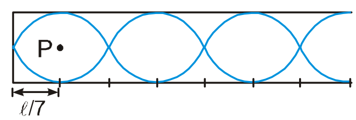
44. The magnitude of phase difference between the points separated by distance 10 metres

$= k\times10 = [10\pi\times0.]\times10 = \pi$

MOCK TEST : SOUND WAVE

1. The figure shows variation of displacement of particles in a closed organ pipe for 3rd overtone.

For third overtone $\ell = \dfrac{7\lambda}{4}$ or $\lambda = \dfrac{4\ell}{7}$ or $\dfrac{\lambda}{4} = \dfrac{\ell}{7}$



Hence the amplitude at P at a distance $\dfrac{\ell}{7}$ from closed end is 'a' because there is an antinode at that point

Alternate

Because there is node at x = 0 the displacement amplitude as function of x can be written as A = a sin

$kx = a\sin\dfrac{2\pi}{\lambda}x$

For third overtone $\ell = \dfrac{7\lambda}{4}$ or $\lambda = \dfrac{4\ell}{7}$

$\therefore A = a\sin\dfrac{7\pi}{2\ell}\ \dfrac{\ell}{7} = a\sin\dfrac{\pi}{2} = a \quad \text{at } x = \dfrac{\ell}{7} \Rightarrow A = a$

2. When a sound wave gets reflected from a rigid boundary, the particles at the boundary are unable to vibrate. Thus, a reflected wave is generated which interferes with the oncoming wave to produce zero displacement at the rigid boundary. At these points (zero displacement), the pressure variation is maximum. Thus, a reflected pressure wave has the same phase as the incident wave.

3. After a time t, velocity of observer V_0 = at

$\therefore\ f_0 = \left(\frac{V + V_0}{V}\right) f_s = \left(\frac{V + at}{V}\right) f_s$, which is a straight line graph of positive slope.

4. $\left[\left(\frac{v}{v - v_s}\right) - \left(\frac{v}{v + v_s}\right)\right] f_0 = 2$ Hz

$v_s = 0.5$ m/s

5. For a stationary observer between wall and source,

freq. from direct source $= \left(\frac{v}{v - v_s}\right) f_0$

frq. from reflected sound $= \left(\frac{v}{v - v_s}\right) f_0$.

So no beats will be heard.

6. **(B)** $dB = 10 \log\left(\frac{I}{I_0}\right) = 10 \log\left(\frac{K/r^2}{I_0}\right) = 10 [\log (K?)$

$- 2 \log r]$ $\left(K' = \frac{K}{I_0}\right)$

$dB_1 = 10 (\log K' - 2 \log r_1)$

$dB_2 = 10 (\log K' - 2 \log r_2)$

$3 = dB_1 - dB_2 = 20 \log\left(\frac{r_2}{r_1}\right)$

$(0.3) = \log\left(\frac{r_2}{r_1}\right)^2 \Rightarrow \left(\frac{r_1}{r_2}\right) = \frac{1}{\sqrt{2}}$

7. $\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = 25 \Rightarrow a_1 + a_2 = 5\,(a_1 - a_2)$

$\frac{a_1}{a_2} = \frac{3}{2} \Rightarrow \frac{I_1}{I_2} = \left(\frac{a_1}{a_2}\right)^2 = \frac{9}{4}$

8. $f_1 \lambda_1 = f_2 \lambda_2$ (in same medium)

$(300)(1) = (f_2)(1.5)$

$200 \text{ Hz} = f_2$

9. $v_{max} = \omega A = (2\pi f) A = (2\pi)(440)(10^{-6})$

$= 2.76 \times 10^{-3}$ m/sec.

10. Apparent frequency

$n' = n \frac{(u + v_w)}{(u + v_w - v_s \cos 60°)} = \frac{510\,(330 + 20)}{330 + 20 - 20 \cos 60°}$

$= 510 \times \frac{350}{340} = 525$ Hz **Ans.**

11 λ_i = wavelength of the incident sound

$= \frac{10u - \frac{u}{2}}{f} = \frac{19u}{2f}$

f_i = frequency of the incident sound $= \frac{10u - u}{10u - \frac{u}{2}} f = \frac{18}{19} f$

$= f_r$ = frequency of the reflected sound

λ_r = wavelength of the reflected sound $= \frac{10u + u}{f_r}$

$= \frac{11u}{18f} \times 19 = \frac{11 \times 19}{18} \cdot \frac{u}{f}$

$\frac{\lambda_i}{\lambda_r} = \frac{19u}{2f} \times \frac{18f}{11 \times 19u} = \frac{\mathbf{9}}{\mathbf{11}}$ **Ans.**

12. For minimum, $\Delta x = (2n - 1)\frac{\lambda}{2}$

The maximum possible path difference = distance between the sources = 3m.

For no minimum $\frac{\lambda}{2} > 3$

$\lambda > 6$

$\therefore\ f = \frac{V}{\lambda} < \frac{330}{6} = 55.$

$\therefore$ If $f < 55$ Hz, no minimum will occur.

13. The speed of sound in air is $v = \sqrt{\frac{\gamma RT}{M}}$

$\frac{\gamma}{M}$ of H_2 is greatest in the given gases, hence speed of sound in H_2 shall be maximum.

14. As $y = A_b \sin(2\pi n_{av} t)$

where $A_b = 2A\cos(2\pi n_A t)$

where $n_A = \frac{n_1 - n_2}{2}$

15. For interference at A : S_2 is behind of S_1 by a distance of $100\lambda + \frac{\lambda}{4}$.(equal to phase difference $\frac{\pi}{2}$). Further S_2 lags S_1 by $\frac{\pi}{2}$. Hence the waves from S_1 and S_2 interfere at A with a phase difference of $200.5\,\pi + 0.5\pi$
$= 201\pi \Rightarrow \pi$
Hence the net amplitude at A is 2a – a = a
For interference at B : S_2 is ahead of S_1 by a distance of $100\lambda + \frac{\lambda}{4}$.(equal to phase difference $\frac{\pi}{2}$). Further S_2 lags S_1 by $\frac{\pi}{2}$.
Hence waves from S_1 and S_2 interfere at B with a phase difference of $200.5\,\pi - 0.5\pi = 200\pi \Rightarrow 0\pi$.
Hence the net amplitude at A is 2a + a = 3a

Hence $\left(\frac{I_A}{I_B}\right) = \left(\frac{a}{3a}\right)^2 = \frac{1}{9}$

16. To get beat frequency 1, 2, 3, 5, 7, 8, it is possible when other three tuning fork have frequencies 551, 553, 558, etc.

17. $V_s = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{10^{11}}{10.0 \times 10^4}} = 10^3$ m/sec.

$t = \frac{2\ell}{V} = \frac{2 \times 100}{1000} = 0.2$ sec **Ans.**

18. $\xi = A \sin (kx - \omega t)$

$P_{ex} = -B\frac{d\xi}{dx} = -BAk \cos (kx - \omega t)$

amplitude of $P_{ex} = BAk = (5 \times 10^5)(10^{-4})\left(\frac{2\pi}{0.2}\right)$

$= 5\pi \times 10^2$ Pa
So correct ans is (D)

19. Fundamental frequency of wire $(f_{wire}) = \frac{v}{2\ell}$

(A) 

$f = \frac{v}{4\ell}, \frac{3v}{4\ell}, \frac{5v}{4\ell}$ cannot match with f_{wire}

(B)  $f = \frac{v}{2(2\ell)}, \frac{2v}{2(2\ell)}, \frac{3v}{2(2\ell)}$ its second harmonic $\frac{2v}{2(2\ell)}$ matches with f_{wire}.

(C)  $, f = \frac{v}{2(\ell/2)}, \frac{2v}{2(\ell/2)}$ cannot match with f_{wire}

(D)  $f = \frac{v}{4(\ell/2)}, \frac{3v}{4(\ell/2)}$ ..... cannot match with f_{wire}

20 $v_s = 4$ km/sec

$v_p = \sqrt{\frac{y}{\rho}} = \sqrt{\frac{12.8 \times 10^{10}}{2000}} = 8000$ m/sec. = 8 km/sec

$\frac{\ell}{v_s} - \frac{\ell}{v_p} = 3$ min $= 3 \times 60$ sec.

$\frac{\ell}{4} - \frac{\ell}{8} = 3 \times 60 \Rightarrow \ell = 1440$ km

21. Towards right wavelength gets compressed, towards left, wavelength gets expended

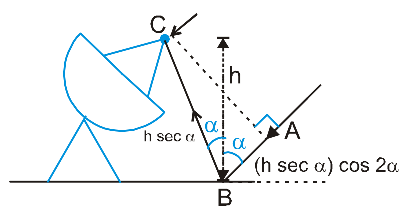
22. x_1 and x_2 are in successive loops of std. waves.
So, $\phi_1 = \pi$

and $\phi_2 = K(\Delta x) = K\left(\frac{3\pi}{2K} - \frac{\pi}{3K}\right) = \frac{7\pi}{6} = \frac{\phi_1}{\phi_2} = \frac{6}{7}$

23. $\ell_1 + \varepsilon = \frac{v}{4f_0} \Rightarrow \ell_2 + \varepsilon = \frac{3v}{4f_0} \Rightarrow \ell_3 + \varepsilon = \frac{5v}{4f_0}$

Solving get $\ell_3 = 2\ell_2 - \ell_1$

24. radio wave are electromagnetic wave. So it get extra phase after reflection
total path difference = AB + BC + $\lambda/2$
$= \lambda$ for maxima
$h \sec \alpha \;\cos 2\alpha + h \sec \alpha = \lambda/2$
$h \sec \alpha\;(2\cos^2 \alpha) = \lambda/2$



$h = \frac{\lambda}{2\cos\alpha}$

25. If detector moves x distance ,
distance from direct sound increases by x and distance from reflected sound decreases by x so path difference created = 2x

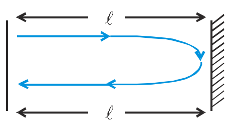
$2(0.14) = 14\lambda = 14\ c/f$

$$f = \frac{14 \times 3 \times 10^8}{0.14 \times 2} = 1.5 \times 10^{10}\ \text{Hz.}$$

26. Drumming frequency = 40 cycle/min = 40 cycle/ 60 sec

$$\text{Drumming time period} = \frac{1}{f} = \frac{60\ \text{sec}}{40\ \text{cycle}} = \frac{3}{2}\ \text{sec/cycle}$$

(time duration between consecutive drumming)
During this time interval, if sound goes to mountain and comes back then echo will not be heard distinctly.



$$\frac{3}{2} = \frac{2\ell}{v} \qquad \text{...... (1)}$$

Now if he moves 90 m. This situation arises at

$$t = 60\ \text{cycle/min},\ T = \frac{1}{f} = 1\ \text{sec/cycle}$$

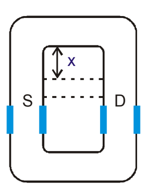
$$\text{so for this case } 1 = \frac{2(\ell - 90)}{v} \qquad \text{...... (2)}$$

Solving equation (1) and (2)
set $\ell = 270$ m
$v = 360$ m/sec.

27. $P_0 = B.K.S_0 = B\left(\frac{2\pi}{\lambda}\right)S_0 \Rightarrow P_0 \propto \frac{1}{\lambda}$

Thus, pressure amplitude is highest for minimum wavelength, other parameters B and S_0 being same for all. From given graphs.
$\lambda_3 < \lambda_2 < \lambda_1$. Hence (B).

28. Path difference introduced due to displacement of tube = 2x = 10 cm due to one wavelength change maxima / minima will be attained once hence for 10 maxima's



path difference $\Delta P = 10\ \lambda = 10$ cm so $\lambda = 1$ cm.

29. $x = x_0\sin(\omega t + \phi) = x_0\sin\omega t\cos\phi + x_0\cos\omega t\sin\phi$.
Comparing with given equation.

Thus $x_0\cos\phi = 3$ and $x_0\sin\phi = 4$

Dividing we get $\tan\phi = \frac{4}{3}$ or $\phi = 53°$

$x_1 = 4\cos\omega t = 4\sin(\omega t + 90°)$
$\Delta\theta = 90° - 53° = 37°$

30. The wavelength of sound source $= \frac{330}{110} = 3$ metre.

The phase difference betwen interfering waves at P is

$$= \Delta\phi = \frac{2\pi}{\lambda}(S_2P - S_1P) = \frac{2\pi}{3}(5-4) = \frac{2\pi}{3}$$

$\therefore$ Resultant intensity at

$$P = I_0 + 4I_0 + 2\sqrt{I_0}\sqrt{4I_0}\cos\frac{2\pi}{3} = 3I_0$$

31. This problem is a Doppler effect analogy
where $f = 20$/min, $v = 300$ m/min and $v_s = 30$ m/min

$$\therefore \quad f' = f\left(\frac{v}{v - v_s}\right) = (20)\left(\frac{300}{300-30}\right) = 22.22\ \text{min}^{-1}$$

32. $f = \left(\frac{v + v_0}{v}\right)f_0 = f_0 + \frac{f_0 v_0}{v} \qquad v_0 = gt$

$$\therefore \quad f = f_0 + \left(\frac{f_0 g}{v}\right)t$$

i.e., f-t graph is a straight line of slope $\frac{f_0 g}{v}$

or $\frac{f_0 g}{v} = \text{slope}$

or $v = \frac{f_0 g}{\text{slope}} = \frac{(10^3)(10)}{\left(\frac{10^3}{30}\right)} = 300$ m/s

33. Let Δl be the end correction.

Given that, fundamental tone for a length 0.1m = first overtone for the length 0.35 m.

$$\frac{v}{4(0.1 + \Delta l)} = \frac{3v}{4(0.35 + \Delta l)}$$

Solving this equation, we get $\Delta l = 0.025\text{m} = 2.5$ cm

34. When the skater is approaching the observer.

$f_1 = f\left(\dfrac{v}{v - v_s}\right) > f$ and constant

When it receds from the observer.

$f_2 = f\left(\dfrac{v}{v + v_s}\right) < f$ and constant.

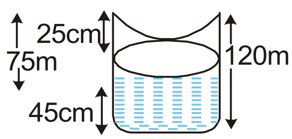
35. As $V = \nu\lambda$

$\lambda = \dfrac{V}{\nu} = \dfrac{340}{340} = 1\text{m}$

first Resonance depth (from upper end)

$R_1 = \dfrac{\lambda}{4} = \dfrac{1}{4}\ \text{m} = 25\ \text{cm}$

$\therefore\ R_2 = \dfrac{3\lambda}{4} = \dfrac{3}{4}\text{m} = 75\ \text{cm}$



$\therefore\ R_3 = \dfrac{5\lambda}{4} = \dfrac{5}{4}\text{m} = 125\ \text{cm}$

i.e. third resonance does not establish

Now Water is poured,

$\therefore$ Minimum length of water column to have the resonance = 45 cm

$\therefore$ Distance between two successive nodes $= \dfrac{\lambda}{2} = \dfrac{1}{2}$ m

= 50 cm

& maximum length of water column to create resonance

i.e. 120 – 25 = 95 cm.

36. $n = \dfrac{1}{2\ell}\sqrt{\dfrac{T}{\mu}}$

on increasing or decreasing (T_1 & T_2) significantly we can get result of higher beats.

37. Doppler formula for sound a wave is not symmetric w.r.t speed of source and speed of observer.

38. Propagation of sound in air is an adiabatic process.

39. $f_{1i} = f_{1r} = \dfrac{v}{v - v_c} f \Rightarrow f_{2i} = f_{2r} = \dfrac{v}{v + v_c} f$

Now, for driver

$f_{dr1} = \dfrac{v + v_c}{v} f_{1r}$ and $f_{dr2} = \dfrac{v - v_c}{v} f_{2r}$

So, beat frequency $= |f_{dr1} - f_{dr2}|$

$= \left|\dfrac{v + v_c}{v} f_{1r} - \dfrac{v - v_c}{v} f_{2r}\right|$

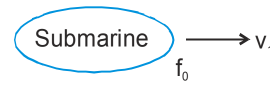
$= \left\{\dfrac{(v + v_c)^2 - (v - v_c)^2}{(v + v_c)(v - v_c)}\right\} f = \left(\dfrac{4vv_c}{v^2}\right) f = \left(\dfrac{4v_c}{v}\right) f$.

40. $\lambda_1 = \dfrac{v + v_c}{f} \Rightarrow \lambda_2 = \dfrac{v - v_c}{f}$

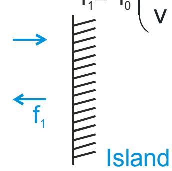
$\lambda_1 - \lambda_2 = \dfrac{2v_c}{f} \Rightarrow \lambda_1 + \lambda_2 = \dfrac{2v}{f} \Rightarrow \dfrac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} = \dfrac{v_c}{v}$.

41. $f'' = f_0\left(\dfrac{v + v_1}{v - v_1}\right)$, v = 1050

$\Rightarrow \left[\dfrac{f'' - f_0}{f_0} = 0.1\right]$



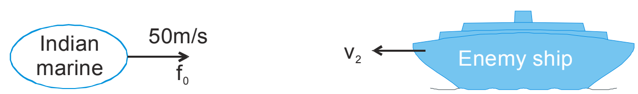
$f_1 = f_0\left(\dfrac{v}{v - v_1}\right)$



$\dfrac{f'' - f_0}{f_0} = \dfrac{2V_1}{V - V_1} = 0.1$

v_1 = 50 m/sec.

42. $f'' = f'\left(\dfrac{v + 50}{v - v_2}\right)$



$f' = f_0\left(\dfrac{v + v_2}{v - 50}\right)$

$f'' = f_0\left(\dfrac{(v + v_2)(v + 50)}{(v - v_2)(v - 50)}\right) = 1.21 f_0$[21% greater then sent waves]

get v_2 = 50 m/sec toward Indian submarine

43. $\lambda' = \dfrac{\text{v wrt to observer}}{f'} = \dfrac{v + v_2}{f_0\dfrac{(v + v_2)}{(v - 50)}} = \dfrac{v - 50}{f_0}$

$\lambda'' = \dfrac{v + 50}{f_0\dfrac{(v + v_2)(v + 50)}{(v - v_2)(v - 50)}} = \dfrac{(v - v_2)(v - 50)}{f_0(v + v_2)}$

$\dfrac{\lambda'}{\lambda''} = \dfrac{v + v_2}{v - v_2} = \dfrac{1050 + 50}{1050 - 50} = 1.1$

44. $v = \sqrt{\dfrac{B}{\rho}} \Rightarrow 1050 = \sqrt{\dfrac{B}{1000}}$ $\quad B \approx 10^9\ N/m^2$

45. At $t = 0$, $y = 10^{-2} \sin 2\pi\left(\dfrac{50}{17}x\right)$

Change in pressure will be maximum where $y = 0$ at $t = 0$,

$\dfrac{2\pi \times 50}{17}\ x = 0, \pi, 2\pi....$ or $\quad x = 0, 0.17\,m, 0.34m...$

46. $v = \dfrac{\omega}{k} = \sqrt{\dfrac{B}{\rho}}$ $\qquad \therefore\ B = \rho\left(\dfrac{\omega}{k}\right)^2$

$\therefore\ (\Delta P)_0 = BAK = \dfrac{\rho\omega^2}{K^2}\ AK = \dfrac{\rho A\omega^2}{K}$

Substituting the values, we get

$$(\Delta P)_0 = \frac{10^{-2} \times 10^{-2} \times (2\pi \times 1000)^2}{(2\pi \times 50/17)} = 21.36\ N/m^2$$

47. $A \to R,\ B \to Q,\ C \to P,S,\ D \to T$

48. **(A)** $y = 4 \sin(5x - 4t) + 3\cos(4t - 5x + \pi/6)$
is super position of two coherent waves moving in positive direction, so their equivalent will be an another travelling wave.

(B) $y = 10 \cos\left(t - \dfrac{x}{330}\right) \sin(100)\left(t - \dfrac{x}{330}\right)$ lets check at any point, say at $x = 0$,

$y = (10 \cos t) \sin(100\,t)$

at any point amplitude is changing sinusoidally. so this is equation of beats.

(C) $y = 10 \sin(2\pi x - 120t) + 10\cos(120t + 2\pi x) =$ superposition of two coherent waves travelling in opposite direction. $\Rightarrow$ equation of standing waves.

(D) $y = 10 \sin(2\pi x - 120\,t) + 8\cos(118t - 59/30\pi x) =$ superposition of two waves whose frequencies are slightly different

$(\omega_1 = 120, \omega_2 = 118) \Rightarrow$ equation of Beats.

49. $3 = 3.\dfrac{\lambda}{2} \Rightarrow \lambda = 2\ m$

$P_m = 100\ N/m^2,\ V = 330\ m/s,\ \rho_0 = 1\ kg/m^3$

$P_m = B\ s_0\ k = \rho_0\ v^2\ s_0\ \dfrac{2\pi}{\lambda} \Rightarrow s_0 = \dfrac{\lambda P_m}{\rho_0 v^2 2\pi}$

$= \dfrac{2 \times 100}{1 \times 330 \times 330 \times 2\pi}$ $\quad s_0 = \dfrac{1}{1089\ \pi}\ m$

50. $3 = 3.\dfrac{\lambda}{2} \Rightarrow \lambda = 2\ m$

$P_m = 100\ N/m^2,\ V = 330\ m/s,\ \rho_0 = 1\ kg/m^3$

$B = -\dfrac{dp}{dv/v} = \dfrac{dP}{d\rho/\rho}$

$[\because\ m = \rho\, v \Rightarrow O = \dfrac{d\rho}{\rho} + \dfrac{dv}{v}]$

$d\rho = \dfrac{\rho\,.\,dp}{B} \Rightarrow (d\rho)_{max} = \dfrac{\rho}{B}\ (dp)_{max} = \dfrac{\rho P_m}{B}$

$(d\rho)_{max} = \dfrac{\rho.P_m}{\rho v^2} = \dfrac{1}{1089}\ kg/m^3$

51. Let the velocities of car 1 and car 2 be V_1 m/s and V_2 m/s.

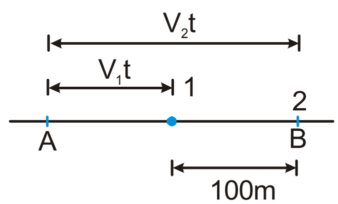
$\therefore$ Apparent frequencies of sound emitted by car 1 and car 2 as detected at end point are

$f_1 = f_0 \dfrac{V}{V - V_1},\quad f_2 = f_0 \dfrac{V}{V - V_2} \Rightarrow 330$

$= 300\ \dfrac{330}{330 - V_1},\ 360 = 300\ \dfrac{330}{330 - V_2}$

$\Rightarrow V_1 = 30$ m/s and $V_2 = 55$ m/s.

The distance between both the cars just when the 2nd car reach and point B (as shown in figure is)



$100m = V_2t - V_1t \Rightarrow t = 4$ sec.

52. $\lambda_{air} = \dfrac{V_{air}}{f} = \dfrac{330}{1000} = 0.33\ m$

$V_{water} = \sqrt{\dfrac{\beta}{\rho}} = \sqrt{\dfrac{2.25 \times 10^9}{1000}} = 1.5 \times 10^3 = 1500$

$\lambda_{water} = \dfrac{1500}{1000} = 1.5\ m$

$\lambda_{water} - \lambda_{air} = 1.5 - 0.33 = 1.17m.$