SOLVED EXAMPLES

Ex. 1 A coin is tossed three times, consider the following events.

A : 'no head appears'

B : 'exactly one head appears'

C : 'at least two heads appear'

Do they form a set of mutually exclusive and exhaustive events ?

Sol. The sample space of the experiment is

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

Events A, B and C are given by

A = {TTT}

B = {HTT, THT, TTH}

C = {HHT, HTH, THH, HHH}

Now,

$A \cup B \cup C$ = {TTT, HTT,THT,TTH,HHT,HTH,THH,HHH} = S

Therefore A,B and C are exhaustive events. Also, $A \cap B = \phi, A \cap C = \phi$ and $B \cap C = \phi$. Therefore, the events are pair-wise disjoint, i.e., they are mutually exclusive. Hence, A,B and C form a set of mutually exclusive and exhaustive events.

Ex. 2 A bag contains 5 red and 4 green balls. Four balls are drawn at random, then find the probability that two balls are of red and two balls are of green colour.

Sol. n(s) = the total number of ways of drawing 4 balls out of total 9 balls : ${}^9C_4$

A : Drawing 2 red and 2 green balls ; $n(A) = {}^5C_2 \times {}^4C_2$

$$\therefore \quad P(A) = \frac{n(A)}{n(s)} = \frac{{}^5C_2 \times {}^4C_2}{{}^9C_4} = \frac{\frac{5 \times 4 \times 4 \times 3}{2 \times 2}}{\frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2}} = \frac{10}{21}$$

Ex. 3 Three coins are tossed. Describe

(i) two events A and B which are mutually exclusive

(ii) three events A, B and C which are mutually exclusive and exhaustive.

(iii) two events A and B which are not mutually exclusive.

(iv) two events A and B which are mutually exclusive but not exhaustive.

(v) three events A, B and C which are mutually exclusive but not exhaustive.

Sol.

(i) A : “getting at least two heads” B : “getting at least two tails”

(ii) A : “getting at most one heads” B : “getting exactly two heads”
C : “getting exactly three heads”

(iii) A : “getting at most two tails” B : “getting exactly two heads”

(iv) A : “getting exactly one head” B : “getting exactly two heads”

(v) A : “ getting exactly one tail” B : “getting exactly two tails”
C : “getting exactly three tails”

Ex. 4 Words are formed with the letters of the word PEACE. Find the probability that 2 E's come together.

Sol. Total number of words which can be formed with the letters P, E, A, C, E $= \dfrac{5!}{2!} = 60$

Number of words in which 2 E's come together $= 4! = 24$ $\quad \therefore$ reqd. prob. $= \dfrac{24}{60} = \dfrac{2}{5}$

Ex. 5 A four digit number is formed using the digits 0, 1, 2, 3, 4 without repetition. Find the probability that it is divisible by 4.

Sol. Total 4 digit numbers formed $\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }$

$4 \times 4 \times 3 \times 2 = 96$

Each of these 96 numbers are equally likely & mutually exclusive of each other.

Now, A number is divisible by 4, if last two digits of the number is divisible by 4

Hence we can have

$\boxed{\ }\boxed{\ }\boxed{0}\boxed{4}$	$\rightarrow$	first two places can be filled in $3 \times 2 = 6$ ways
$\boxed{\ }\boxed{\ }\boxed{1}\boxed{2}$	$\rightarrow$	first two places can be filled in $2 \times 2 = 4$ ways
$\boxed{\ }\boxed{\ }\boxed{2}\boxed{0}$	$\rightarrow$	6 ways
$\boxed{\ }\boxed{\ }\boxed{2}\boxed{4}$	$\rightarrow$	4 ways
$\boxed{\ }\boxed{\ }\boxed{3}\boxed{2}$	$\rightarrow$	4 ways
$\boxed{\ }\boxed{\ }\boxed{4}\boxed{0}$	$\rightarrow$	6 ways
Total number of ways		30 ways

$$\text{Probability} = \frac{\text{favourable outcomes}}{\text{Total outcomes}} = \frac{30}{96} = \frac{5}{16}$$

Ex. 6 If the letters of INTERMEDIATE are arranged, then the odds in favour of the event that no two 'E's occur together, are -

Sol. $I \rightarrow 2, N \rightarrow 1, T \rightarrow 2, E \rightarrow 3, R \rightarrow 1, M \rightarrow 1, D \rightarrow 1, A \rightarrow 1$ (3'E's, Rest 9 letters)

First arrange rest of the letters $= \dfrac{9!}{2!\ 2!}$,

Now 3'E's can be placed by ${}^{10}C_3$ ways, so favourable cases $= \dfrac{9!}{2!\ 2!} \times {}^{10}C_3 = 3 \times 10!$

Total cases $= \dfrac{12!}{2!\,2!\,3!} = \dfrac{11}{2} \times 10!$; Non-favourable cases $= \left(\dfrac{11}{2} - 3\right) \times 10! = \dfrac{5}{2} \times 10!$

Odds in favour of the event $= \dfrac{3}{5/2} = \dfrac{6}{5}$

Ex. 7 If n positive integers taken at random are multiplied together, show that the probability that the last digit of the product being 5 is $\frac{5^n - 4^n}{10^n}$ and that the probability of the last digit being 0 is $\frac{10^n - 8^n - 5^n + 4^n}{10^n}$.

Sol. Let n positive integers be $x_1, x_2, \ldots, x_n$. Let $a = x_1.x_2....x_n$.

Let S be the sample space, since the last digit in each of the numbers, $x_1, x_2, \ldots, x_n$ can be any one of the digits 0, 1, 2, 3,...,9 (total 10)

$\therefore \quad n(S) = 10^n$

Let E_1 and E_2 be the events when the last digit in a is 1, 3, 5, 7 or 9 and 1, 3, 7 or 9 respectively

$\therefore \quad n(E_1) = 5^n$ and $n(E_2) = 4^n$

and let E be the event that the last digit in a is 5.

$n(E) = n(E_1) - n(E_2) = 5^n - 4^n$

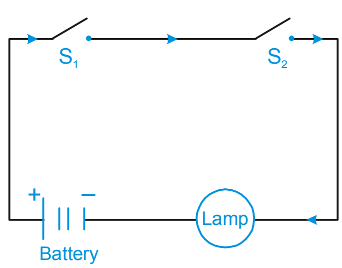
Hence required probability $P(E) = \frac{n(E)}{n(S)} = \frac{5^n - 4^n}{10^n}$

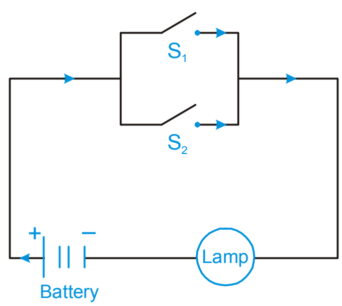
Second part : Let E_3 and E_4 be the events when the last digit in a is 1, 2, 3, 4, 6, 7, 8 or 9 and 0 respectively.

Then $\quad n(E_4) = n(S) - n(E_3) - n(E) = 10^n - 8^n - (5^n - 4^n) = 10^n - 8^n - 5^n + 4^n$

$\therefore \quad$ Required probability $P(E_4) = \frac{n(E_4)}{n(S)} = \frac{10^n - 8^n - 5^n + 4^n}{10^n}$

Ex. 8 If two switches S_1 and S_2 have respectively 90% and 80% chances of working. Find the probabilities that each of the following circuits will work.





Sol. Consider the following events :

A = Switch S_1 works,

B = Switch S_2 works,

We have,

$$P(A) = \frac{90}{100} = \frac{9}{10} \text{ and } P(B) = \frac{80}{100} = \frac{8}{10}$$

(i) The circuit will work if the current flows in the circuit. This is possible only when both the switches work together. Therefore,

Required probability

$= P(A \cap B) = P(A)\,P(B)$ $\quad$ [$\because$ A and B are independent events]

$$= \frac{9}{10} \times \frac{8}{10} = \frac{72}{100} = \frac{18}{25}$$

(ii) The circuit will work if the current flows in the circuit. This is possible only when at least one of the two switches S_1, S_2 works. Therefore,

Required Probability

$= P(A \cup B) = 1 - P(\bar{A})\,P(\bar{B})$ [$\because$ A, B are independent events]

$$= 1 - \left(1 - \frac{9}{10}\right)\left(1 - \frac{8}{10}\right) = 1 - \frac{1}{10} \times \frac{2}{10} = \frac{49}{50}$$

Ex. 9 If four cards are drawn at random from a pack of fifty-two playing cards, find the probability that at least one of them is an ace.

Sol. If A is a combination of four cards containing at least one ace (i.e. either one ace, or two aces, or three aces or four aces) then $\bar{A}$ is a combination of four cards containing no aces.

Now $$P(\bar{A}) = \frac{\text{Number of combinations of four cards with no aces}}{\text{Total number of combinations of four cards}} = {}^{48}C_4 / {}^{52}C_4 = 0.72$$

Using $P(A) + P(\bar{A}) = 1$ we have $P(A) = 1 - P(\bar{A}) = 1 - 0.72 = 0.28$

Ex. 10 A bag contains n white and n red balls. Pairs of balls are drawn without replacement until the bag is empty. Show that the probability that each pair consists of one white and one red ball is $2^n/({}^{2n}C_n)$.

Sol. Let S be the sample space & E be the event that each of the n pairs of balls drawn consists of one white and one red ball.

$\therefore$ $n(S) = ({}^{2n}C_2)\,({}^{2n-2}C_2)\,({}^{2n-4}C_2)\ldots({}^{4}C_2)({}^{2}C_2)$

$$= \left\{\frac{(2n)(2n-1)}{1.2}\right\}\left\{\frac{(2n-2)(2n-3)}{1.2}\right\}\left\{\frac{(2n-4)(2n-5)}{1.2}\right\}\ldots\ldots\left\{\frac{4.3}{1.2}\right\}\left\{\frac{2.1}{1.2}\right\}$$

$$= \frac{1.2.3.4\ldots.(2n-1)2n}{2^n} = \frac{2n!}{2^n}$$

and $n(E) = ({}^{n}C_1.{}^{n}C_1)\,({}^{n-1}C_1.{}^{n-1}C_1)\,({}^{n-2}C_1.{}^{n-2}C_1)\ldots({}^{2}C_1.{}^{2}C_1)\,({}^{1}C_1.{}^{1}C_1)$

$= n^2.(n-1)^2.(n-2)^2\ldots.2^2.1^2 = [1.2.3\ldots..(n-1)n]^2 = (n!)^2$

$\therefore$ Required Probability, $$P(E) = \frac{n(E)}{n(S)} = \frac{(n!)^2}{(2n)!/2^n} = \frac{2^n}{\frac{2n!}{(n!)^2}} = \frac{2^n}{{}^{2n}C_n}$$

Ex. 11 A speaks truth in 60% of the cases and b in 90% of the cases. In what percentage of cases are they likely to contradict each other in stating the same fact ?

Sol. Let E be the event that A speaks truth and F be the event that B speaks truth. Then E and F are independent events such that

$$P(E) = \frac{60}{100} = \frac{3}{5} \text{ and } P(F) = \frac{90}{100} = \frac{9}{10}$$

A and B will contradict each other in narrating the same fact in the following mutually exclusive ways:

(i) A speaks truth and B tells a lie i.e. $E \cap \overline{F}$

(ii) A tells a lie and B speaks truth lie i.e. $\overline{E} \cap F$

$\therefore$ P(A and B contradict each other)

$= P(\text{I or II}) = (\text{I} \cup \text{II}) \;=\; P[(E \cap \overline{F}) \cup (\overline{E} \cap F)]$

$= P(E \cap \overline{F}) + P(\overline{E} \cap F)$ $\quad$ [$\because E \cap \overline{F}$ and $\overline{E} \cap F$ are mutually exclusive]

$= P(E)\,P(\overline{F}) + P(\overline{E})\,P(F)$ $\quad$ [$\because$ E and F are indep.]

$$= \frac{3}{5} \times \left(1-\frac{9}{10}\right) + \left(1-\frac{3}{5}\right) \times \frac{9}{10} = \frac{3}{5} \times \frac{1}{10} + \frac{2}{5} \times \frac{9}{10} = \frac{21}{50}$$

Ex. 12 If two events A and B are such that $P(\overline{A}) = 0.3$, P(B) = 0.4 and $P(A\overline{B}) = 0.5$ then $P(B \mid (A \cup \overline{B}))$ equals -

(A) 1/2 **(B)** 1/3 **(C)** 1/4 **(D)** 1/5

Sol. We have $P(B \mid (A \cup \overline{B})) = \dfrac{P[B \cap (A \cup \overline{B})]}{P(A \cup \overline{B})} = \dfrac{P[(B \cap A) \cup (B \cap \overline{B})]}{P(A) + P(\overline{B}) - P(A \cap \overline{B})}$

$$= \frac{P(AB)}{P(A) + P(\overline{B}) - P(A\overline{B})} = \frac{P(A) - P(A\overline{B})}{P(A) + P(\overline{B}) - P(A\overline{B})} = \frac{0.7 - 0.5}{0.7 + 0.6 - 0.5} = \frac{0.2}{0.8} = \frac{1}{4}$$

Ex. 13 If A and B are two events such that $P(A \cup B) = \dfrac{3}{4}$, $P(A \cap B) = \dfrac{1}{4}$ and $P(A^c) = \dfrac{2}{3}$. Then find -

(i) P(A) **(ii)** P(B) **(iii)** $P(A \cap B^c)$ **(iv)** $P(A^c \cap B)$

Sol. $P(A) = 1 - P(A^c) = 1 - \dfrac{2}{3} = \dfrac{1}{3}$

$P(B) = P(A \cup B) + P(A \cap B) - P(A) = \dfrac{3}{4} + \dfrac{1}{4} - \dfrac{1}{3} = \dfrac{2}{3}$

$P(A \cap B^c) = P(A) - P(A \cap B) = \dfrac{1}{3} - \dfrac{1}{4} = \dfrac{1}{12}$

$P(A^c \cap B) = P(B) - P(A \cap B) = \dfrac{2}{3} - \dfrac{1}{4} = \dfrac{5}{12}$

Ex. 14 A family has three children. Event 'A' is that family has at most one boy, Event 'B' is that family has at least one boy and one girl, Event 'C' is that the family has at most one girl. Find whether events 'A' and 'B' are independent. Also find whether A, B, C are independent or not.

Sol. A family of three children can have

(i) All 3 boys **(ii)** 2 boys + 1 girl **(iii)** 1 boy + 2 girls **(iv)** 3 girls

(i) $P\,(3 \text{ boys}) = {}^3C_0 \left(\dfrac{1}{2}\right)^3 = \dfrac{1}{8}$ (Since each child is equally likely to be a boy or a girl)

(ii) $P\,(2 \text{ boys} + 1 \text{girl}) = {}^3C_1 \times \left(\dfrac{1}{2}\right)^2 \times \dfrac{1}{2} = \dfrac{3}{8}$ (Note that there are three cases BBG, BGB, GBB)

(iii) $P\,(1\text{ boy} + 2\text{ girls}) = {}^3C_2 \times \left(\frac{1}{2}\right)^1 \times \left(\frac{1}{2}\right)^2 = \frac{3}{8}$

(iv) $P\,(3\text{ girls}) = \frac{1}{8}$

Event 'A' is associated with **(iii)** & **(iv)**. Hence $P(A) = \frac{1}{2}$

Event 'B' is associated with **(ii)** & **(iii)**. Hence $P(B) = \frac{3}{4}$

Event 'C' is associated with **(i)** & **(ii)**. Hence $P(C) = \frac{1}{2}$

$P(A \cap B) = P$ **(iii)** $= \frac{3}{8} = P(A)\,.\,P(B)$. Hence A and B are independent of each other

$P(A \cap C) = 0 \neq P(A)\,.\,P(C)$. Hence A, B, C are not independent

Ex. 15 A pair of dice is rolled together till a sum of either 5 or 7 is obtained. Find the probability that 5 comes before 7.

Sol. Let E_1 = the event of getting 5 in a roll of two dice = {(1, 4), (2, 3), (3, 2), (4, 1)}

$\therefore \quad P(E_1) = \frac{n(E_1)}{n(S)} = \frac{4}{6 \times 6} = \frac{1}{9}$

Let $\quad E_2$ = the event of getting either 5 or 7

= {(1, 4), (2, 3), (3, 2), (4, 1), (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}

$\therefore \quad P(E_2) = \frac{n(E_2)}{n(S)} = \frac{10}{6 \times 6} = \frac{5}{18}$

$\therefore$ the probability of getting neither 5 nor 7 $= P(\bar{E}_2) = 1 - P(E_2) = 1 - \frac{5}{18} = \frac{13}{18}$

The event of getting 5 before 7 $= E_1 \cup (\bar{E}_2 E_1) \cup (\bar{E}_2 \bar{E}_2 E_1) \cup \ldots\ldots \text{ to } \infty$

$\therefore$ the probability of getting 5 before 7

$= P(E_1) + P(\bar{E}_2 E_1) + P(\bar{E}_2 \bar{E}_2 E_1) + \ldots \text{ to } \infty = P(E_1) + P(\bar{E}_2)P(E_1) + P(\bar{E}_2)P(\bar{E}_2)P(E_1) + \ldots \text{ to } \infty$

$$= \frac{1}{9} + \frac{13}{18}\cdot\frac{1}{9} + \frac{13}{18}\cdot\frac{13}{18}\cdot\frac{1}{9} + \ldots\ldots \text{ to } \infty = \frac{1}{9}\left[1 + \frac{13}{18} + \left(\frac{13}{18}\right)^2 + \ldots\ldots \text{ to } \infty\right] = \frac{1}{9}\cdot\frac{1}{1 - \frac{13}{18}} = \frac{1}{9}\cdot\frac{18}{5} = \frac{2}{5}$$

Ex. 16 A lot contains 50 defective and 50 non-defective bulbs. Two bulbs are drawn at random, one at a time, with replacement. The events A, B, C are defined as :

A = {The first bulb is defective}

B = {The second bulb is non-defective}

C = {The two bulbs are both defective or both non-defective}

Determine whether

(i) A, B, C are pairwise independent, **(ii)** A, B, C are independent.

Sol. We have $P(A)=\frac{50}{100}.1=\frac{1}{2}$; $P(B)=1.\frac{50}{100}=\frac{1}{2}$; $P(C)=\frac{50}{100}.\frac{50}{100}+\frac{50}{100}.\frac{50}{100}=\frac{1}{2}$

$A \cap B$ is the event that first bulb is defective and second is non-defective.

$\therefore$ $P(A \cap B)=\frac{1}{2}.\frac{1}{2}=\frac{1}{4}$

$A \cap C$ is the event that both bulbs are defective.

$\therefore$ $P(A \cap C)=\frac{1}{2}.\frac{1}{2}=\frac{1}{4}$

Similarly $P(B \cap C)=\frac{1}{4}$

Thus we have $P(A \cap B)=P(A).P(B)$; $P(A \cap C)=P(A).P(C)$; $P(B \cap C)=P(B).P(C)$

$\therefore$ A, B and C are pairwise independent.

There is no element in $A \cap B \cap C$

$\therefore$ $P(A \cap B \cap C)=0$

$\therefore$ $P(A \cap B \cap C) \neq P(A).P(B).P(C)$

Hence A, B and C are not mutually independent.

Ex. 17 If E_1 and E_2 are two events such that $P(E_1)=\frac{1}{4}$; $P(E_2)=\frac{1}{2}$; $P\left(\frac{E_1}{E_2}\right)=\frac{1}{4}$, then choose the correct options.

(i) E_1 and E_2 are independent **(ii)** E_1 and E_2 are exhaustive

(iii) E_1 and E_2 are mutually exclusive **(iv)** E_1 & E_2 are dependent

Also find $P\left(\frac{\overline{E}_1}{E_2}\right)$ and $\left(\frac{E_2}{\overline{E}_1}\right)$

Sol. Since $P\left(\frac{E_1}{E_2}\right)=P(E_1)$ $\Rightarrow$ E_1 and E_2 are independent of each other

Also since $P(E_1 \cup E_2)=P(E_1)+P(E_2)-P(E_1).P(E_2) \neq 1$

Hence events are not exhaustive. Independent events can't be mutually exclusive.

Hence only **(i)** is correct

Further since E_1 & E_2 are independent; E1 and $\overline{E}_2$ or $\overline{E}_1$, E_2 are $\overline{E}_1$, $\overline{E}_2$ are also independent.

Hence $P\left(\frac{\overline{E}_1}{E_2}\right)=P\left(\overline{E}_1\right)=\frac{3}{4}$ and $P\left(\frac{E_2}{\overline{E}_1}\right)=P(E_2)=\frac{1}{2}$

Ex. 18 If a fair coin is tossed 10 times, find the probability of getting

(i) exactly six heads **(ii)** atleast six heads **(iii)** atmost six heads

Sol. The repeated tosses of a coin are Bernoulli trials. Let X denotes the number of heads in an experiment of 10 trials.

Clearly, X has the binomial distribution with n = 10 and $p=\frac{1}{2}$

Therefore $P(X=x)={}^{n}C_{x}q^{n-x}p^{x},\ x=0,1,2,\ldots,n$

Here $\quad n = 10,\ p = \frac{1}{2},\ q = 1 - p = \frac{1}{2}$

Therefore $\quad P(X = x) = {}^{10}C_x \left(\frac{1}{2}\right)^{10-x} \left(\frac{1}{2}\right)^x = {}^{10}C_x \left(\frac{1}{2}\right)^{10}$

Now **(i)** $\quad P(X = 6) = {}^{10}C_6 \left(\frac{1}{2}\right)^{10} = \frac{10!}{6! \times 4! 2^{10}} = \frac{105}{512}$

(ii) $\quad P(\text{atleast six heads}) = P(X \ge 6) = P(X=6) + P(X=7) + P(X=8) + P(X=9) + P(X=10)$

$$= {}^{10}C_6 \left(\frac{1}{2}\right)^{10} + {}^{10}C_7 \left(\frac{1}{2}\right)^{10} + {}^{10}C_8 \left(\frac{1}{2}\right)^{10} + {}^{10}C_9 \left(\frac{1}{2}\right)^{10} + {}^{10}C_{10} \left(\frac{1}{2}\right)^{10}$$

$$= \left[\left(\frac{10!}{6! \times 4!}\right) + \left(\frac{10!}{7! \times 3!}\right) + \left(\frac{10!}{8! \times 2!}\right) + \left(\frac{10!}{9! \times 1!}\right) + \left(\frac{10!}{10!}\right)\right] \frac{1}{2^{10}} = \frac{193}{512}$$

(iii) $\quad P(\text{at most six heads}) = P(X \le 6)$

$= P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5) + P(X=6)$

$$= \left(\frac{1}{2}\right)^{10} + {}^{10}C_1 \left(\frac{1}{2}\right)^{10} + {}^{10}C_2 \left(\frac{1}{2}\right)^{10} + {}^{10}C_3 \left(\frac{1}{2}\right)^{10} + {}^{10}C_4 \left(\frac{1}{2}\right)^{10} + {}^{10}C_5 \left(\frac{1}{2}\right)^{10} + {}^{10}C_6 \left(\frac{1}{2}\right)^{10}$$

$$= \frac{848}{1024} = \frac{53}{64}$$

Ex. 19 A coin is tossed 7 times. Each time a man calls head. The probability that he wins the toss on more than three occasions is -

Sol. The man has to win at least 4 times.

$\therefore$ the required probability

$$= {}^7C_4 \left(\frac{1}{2}\right)^4 \cdot \left(\frac{1}{2}\right)^3 + {}^7C_5 \cdot \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^2 + {}^7C_6 \left(\frac{1}{2}\right)^6 \cdot \frac{1}{2} + {}^7C_7 \left(\frac{1}{2}\right)^7$$

$$= ({}^7C_4 + {}^7C_5 + {}^7C_6 + {}^7C_7) \cdot \frac{1}{2^7} = \frac{64}{2^7} = \frac{1}{2}.$$

Ex. 20 A pack of cards is counted with face downwards. It is found that one card is missing. One card is drawn and is found to be red. Find the probability that the missing card is red.

Sol. Let A be the event of drawing a red card when one card is drawn out of 51 cards (excluding missing card.) Let A_1 be the event that the missing card is red and A_2 be the event that the missing card is black.

Now by Bayes's theorem, required probability,

$$P(A_1/A) = \frac{P(A_1) \,.\, P(A/A_1)}{P(A_1) \,.\, P(A/A_1) + P(A_2) \,.\, P(A/A_2)} \qquad \text{......(i)}$$

In a pack of 52 cards 26 are red and 26 are black.

Now $P(A_1)$ = probability that the missing card is red = $\frac{^{26}C_1}{^{52}C_1} = \frac{26}{52} = \frac{1}{2}$

$P(A_2)$ = probability that the missing card is black = $\frac{26}{52} = \frac{1}{2}$

$P(A/A_1)$ = probability of drawing a red card when the missing card is red.

$= \frac{25}{51}$

[$\because$ Total number of cards left is 51 out of which 25 are red and 26 are black as the missing card is red]

Again $P(A/A_2)$ = Probability of drawing a red card when the missing card is black = $\frac{26}{51}$

Now from **(i)**, required probability, $P(A_1/A) = \frac{\frac{1}{2}\cdot\frac{25}{51}}{\frac{1}{2}\cdot\frac{25}{51}+\frac{1}{2}\cdot\frac{26}{51}} = \frac{25}{51}$

Ex. 21 In a factory which manufactures bolts, machines A, B and C manufacture respectively 25%, 35% and 40% of the bolts. Of their outputs, 5, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it is manufactured by the machine B ?

Sol. Let events B_1, B_2, B_3 be the following :

B_1 : the bolt is manufactured by machine A

B_2 : the bolt is manufactured by machine B

B_3 : the bolt is manufactured by machine C

Clearly, B_1, B_2, B_3 are mutually exclusive and exhaustive events and hence, they represent a partition of the sample space.

Let the event E be ' the bolt is defective'.

The event E occurs with B_1 or with B_2 or with B_3.

Given that $P(B_1) = 25\% = 0.25$, $P(B_2) = 0.35$ and $P(B_3) = 0.40$

Again $P(E|B_1)$ = Probability that the bolt drawn is defective given that it is manufactured by machine A = 5% = 0.05.

Similarly, $P(E|B_2) = 0.04$, $P(E|B_3) = 0.02$.

Hence, by Baye's Theorem, we have

$$P(B_2 \mid E) = \frac{P(B_2)P(E \mid B_2)}{P(B_1)P(E \mid B_1) + P(B_2)P(E \mid B_2) + P(B_3)P(E \mid B_3)}$$

$$= \frac{0.35 \times 0.04}{0.25 \times 0.05 + 0.35 \times 0.04 + 0.40 \times 0.02} = \frac{0.0140}{0.0345} = \frac{28}{69}$$

Ex. 22 In a test an examinee either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he makes a guess is $\frac{1}{3}$ and the probability that he copies the answer is $\frac{1}{6}$. The probability that his answer is correct given that he copied it, is $\frac{1}{8}$. Find the probability that he knew the answer to the question given that he correctly answered it.

Sol. Let A_1 be the event that the examinee guesses that answer; A_2 the event that he copies the answer and A_3 the event that he knows the answer. Also let A be the event that he answers correctly. Then as given, we have

$P(A_1) = \frac{1}{3}$, $P(A_2) = \frac{1}{6}$, $P(A_3) = 1 - \frac{1}{3} - \frac{1}{6} = \frac{1}{2}$

[We have assumed here that the events A_1, A_2 and A_3 are mutually exclusive and totally exhaustive.]

Now $P(A/A_1) = \frac{1}{4}$, $P(A/A_2) = \frac{1}{8}$ (as given)

Again it is reasonable to take the probability of answering correctly given that he knows the answer as 1, that is, $P(A/A_3) = 1$

We have to find $P(A_3/A)$.

By Baye's theorem, we have $P(A_3/A) = \dfrac{P(A_3)P(A/A_3)}{P(A_1)P(A/A_1) + P(A_2)P(A/A_2) + (A_3)P(A/A_3)}$

$$= \frac{(1/2).1}{(1/3)(1/4) + (1/6)(1/8) + (1/2).1} = \frac{24}{29}$$

Ex. 23 A purse contains four coins each of which is either a rupee or two rupees coin. Find the expected value of a coin in that purse.

Sol. Various possibilities of coins in the purse can be

	p_i	M_i	p_iM_i
(i) 4 one rupee coins	$\frac{1}{16}$	4	$\frac{4}{16}$
(ii) 3 one Rs. + 1 two Rs.	$\frac{4}{16}$	5	$\frac{20}{16}$
(iii) 2 one Rs. + 2 two Rs.	$\frac{6}{16}$	6	$\frac{36}{16}$
(iv) 1 one Rs. + 3 two Rs.	$\frac{4}{16}$	7	$\frac{28}{16}$
(iv) 4 two Rs.	$\frac{1}{16}$	8	$\frac{8}{16}$
			6 / -

Hence expected value is Rs. 6/-

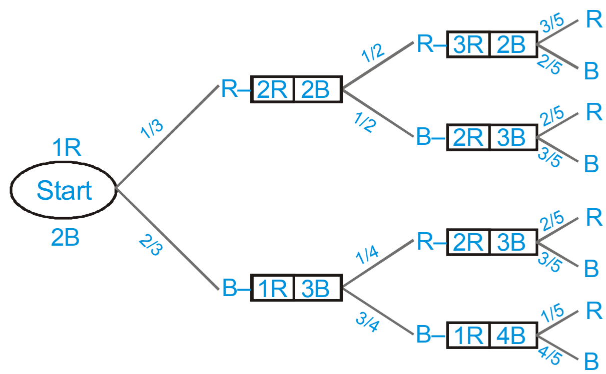
Ex. 24 A bag initially contains 1 red ball and 2 blue balls. A trial consists of selecting a ball at random, noting its colour and replacing it together with an additional ball of the same colour. Given that three trials are made, draw a tree diagram illustrating the various probabilities. Hence, or otherwise, find the probability that

(A) atleast one blue ball is drawn

(B) exactly one blue ball is drawn

(C) Given that all three balls drawn are of the same colour find the probability that they are all red.

Sol.



Calculations :

$$P(A) = 1 - P(RRR) = 1 - \frac{1}{3}.\frac{1}{2}.\frac{3}{5} = 1 - \frac{1}{10} = \frac{9}{10}$$

$$P(\text{exactly one Blue}) = \frac{1}{3}.\frac{1}{2}.\frac{2}{5} + \frac{1}{3}.\frac{1}{2}.\frac{2}{5} + \frac{2}{3}.\frac{1}{4}.\frac{2}{5} = \frac{1}{15} + \frac{1}{15} + \frac{1}{15} = \frac{3}{15} = \frac{1}{5}$$

$$P(C) = P\left(\frac{RRR}{(RRR \cup BBB)}\right) = \frac{P(RRR)}{P(RRR) + P(BBB)} = \frac{\frac{1}{3}.\frac{1}{2}.\frac{3}{5}}{\frac{1}{3}.\frac{1}{2}.\frac{3}{5} + \frac{2}{3}.\frac{3}{4}.\frac{4}{5}} = \frac{\frac{1}{10}}{\frac{1}{10} + \frac{4}{10}} = \frac{1}{5}$$

Ex. 25 The probabilities of three events A, B and C are P(A) = 0.6, P(B) = 0.4 and P(C) = 0.5. If $P(A \cup B) = 0.8$, $P(A \cap C) = 0.3$, $P(A \cap B \cap C) = 0.2$ and $P(A \cup B \cup C) \geq 0.85$, find $P(B \cap C)$.

Sol. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.8 = 0.6 + 0.4 - P(A \cap B)$

$\therefore \quad P(A \cap B) = 0.2$

Now $\quad P(A \cup B \cup C) = S_1 - S_2 + S_3 = (0.6 + 0.4 + 0.5) - (0.2 + P(B \cap C) + 0.3) + 0.2$

$= 1.5 - 0.3 - P(B \cap C)$

We know $0.85 \leq P(A \cup B \cup C) \leq 1$

or $\quad 0.85 \leq 1.2 - P(B \cap C) \leq 1$

$\therefore \quad 0.2 \leq P(B \cap C) \leq 0.35$

Ex. 26 A set A has n elements. A subset P of A is selected at random. Returning the element of P, the set Q is formed again and then a subset Q is selected from it. Find the probability that P and Q have no common elements.

Sol. The set P be the empty set, or one element set or two elements set .... or n elements set. Then the set Q will be chosen amongst the remaining n elements or n – 1 elements or n – 2 elements .... or no elements. The probability of P being an empty set is ${}^nC_0/2^n$, the probability of P being one element set is ${}^nC_1/2^n$ and in general, the probability of P being an r element set is ${}^nC_r/2^n$.

When the set P consisting of r elements is chosen from A, then the probability of choosing the set Q from amongst the remaining n – r elements is $2^{n-r}/2^n$. Hence the probability that P and Q have no common elements is given by

$$\sum_{r=0}^{n} \frac{{}^nC_r}{2^n}.\frac{2^{n-r}}{2^n} = \frac{1}{4^n}\sum_{r=0}^{n} {}^nC_r.2^{n-r} = \left(\frac{1}{4}\right)^n (1+2)^n = \left(\frac{3}{4}\right)^n \qquad \text{[By binomial theorem]}$$

Ex. 27 Difference between mean and variance of a Binomial variate is '1' and difference between their squares is '11'. Find the probability of getting exactly three success

Sol. Mean = np & variance = npq

therefore, $np - npq = 1$ ..... **(i)**

$n^2p^2 - n^2p^2q^2 = 11$ ..... **(ii)**

Also, we know that $p + q = 1$ ..... **(iii)**

Divide equation **(ii)** by square of **(i)** and solve, we get, $q = \frac{5}{6}$, $p = \frac{1}{6}$ & $n = 36$

Hence probability of '3' success $= {}^{36}C_3 \times \left(\frac{1}{6}\right)^3 \times \left(\frac{5}{6}\right)^{33}$

Ex. 28 Three persons A, B, C in order cut a pack of cards, replacing them after each cut, on the condition that the first who cuts a spade shall win a prize; find their respective chances.

Sol. Let p be the chance of cutting a spade and q the chance of not cutting a spade from a pack of 52 cards.

Then $p = \frac{{}^{13}C_1}{{}^{52}C_1} = \frac{1}{4}$ and $q = 1 - \frac{1}{4} = \frac{3}{4}$

Now A will win a prize if he cuts spade at 1st, 4th, 7th, 10th turns, etc. Note that A will get a second chance if A, B, C all fail to cut a spade once and then A cuts a spade at the 4th turn. Similarly he will cut a spade at the 7th turn when A, B, C fail to cut spade twice, etc.

Hence A's chance of winning the prize $= p + q^3p + q^6p + q^9p + \ldots\ldots = \frac{p}{1-q^3} = \frac{\frac{1}{4}}{1-\left(\frac{3}{4}\right)^3} = \frac{16}{37}$

Similarly B's chance $= (qp + q^4p + q^7p + \ldots\ldots..) = q(p + q^3p + q^6p + \ldots\ldots\ldots.) = \frac{3}{4}.\frac{16}{37} = \frac{12}{37}$

and C's chance $= \frac{3}{4}$ of B's chance $= \frac{3}{4}.\frac{12}{37} = \frac{9}{37}$

Ex. 29 **(A)** If p and q are chosen randomly from the set {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, with replacement, determine the probability that the roots of the equation $x^2 + px + q = 0$ are real.

(B) Each coefficient in the equation $ax^2 + bx + c = 0$ is determined by throwing an ordinary die. Find the probability that the equation will have equal roots.

Sol. **(A)** If roots of $x^2 + px + q = 0$ are real, then $p^2 - 4q \geq 0$ ..... **(i)**

Both p, q belongs to set S = {1, 2, 3, ... 10} when p = 1, no value of q from S will satisfy **(i)**

p = 2	q = 1 will satisfy	1 value
p = 3	q = 1, 2	2 value
p = 4	q = 1, 2, 3, 4	4 value
p = 5	q = 1, 2, 3, 4, 5, 6	6 value
p = 6	q = 1, 2, 3, 4, 5, 6, 7, 8, 9	9 value

For p = 7, 8, 9, 10 all the ten values of q will satisfy.

Sum of these selections is 1 + 2 + 4 + 6 + 9 + 10 + 10 + 10 + 10 = 62

But the total number of selections of p and q without any order is 10 × 10 = 100

Hence the required probability is = $\frac{62}{100} = 0.62$

(B) Roots equal $\Rightarrow$ $b^2 - 4ac = 0$

$\therefore \quad \left(\frac{b}{2}\right)^2 = ac \qquad$ ...... **(i)**

Each coefficient is an integer, so we consider the following cases :

b = 1 $\quad\therefore\quad \frac{1}{4} = ac$

No integral values of a and c

b = 2 $\quad$ 1 = ac $\quad\therefore\quad$ (1, 1)

b = 3 $\quad$ 9/2 = ac

No integral values of a and c

b = 4 $\quad$ 4 = ac $\quad\therefore\quad$ (1, 4), (2, 2), (4, 1)

b = 5 $\quad$ 25/2 = ac

No integral values of a and c

b = 6 $\quad$ 9 = ac $\quad\therefore\quad$ (3, 3)

Thus we have 5 favourable way for b = 2, 4, 6

Total number of equations is 6.6.6 = 216

$\therefore$ Required probability is $\frac{5}{216}$

Ex.30 A speaks the truth '3 times out of 4' and B speaks the truth '2 times out of 3'. A die is thrown. Both assert that the number turned up is 2. Find the probability of the truth of their assertion.

Sol. Let A and B be the events 'A speaks the truth' and 'B speaks the truth' repectively. Let C be the event 'the number turned up is not 2 but both agree to the same conclustion that the die has turned up 2'. Then

$P(A) = \frac{3}{4}$, $P(B) = \frac{2}{3}$ and $P(C) = \frac{1}{5} \times \frac{1}{5}$

There are two hypotheses

(i) the die turns up 2 **(ii)** the die does not turns up 2

Let these be the events E_1 and E_2 respectively, then

$P(E_1) = \frac{1}{6}$, $P(E_2) = \frac{5}{6}$ (a priori probabilities)

Now let E be the event 'the statement made by A and B agree to the same conclusion.

then $\quad P(E / E_1) = P(A) \,.\, P(B) = \frac{3}{4} \cdot \frac{2}{3} = \frac{1}{2}$

$$P(E / E_2) = P(\bar{A}) \,.\, P(\bar{B}) \,.\, P(C) = \frac{1}{4} \cdot \frac{1}{3} \cdot \frac{1}{25} = \frac{1}{300}$$

Thus $P(E) = P(E_1)\, P(E / E_1) + P(E_2)\, P(E / E_2) = \frac{1}{6} \times \frac{1}{2} + \frac{5}{6} \times \frac{1}{300} = \frac{31}{360}$

$$\therefore \quad P(E_1 / E) = \frac{P(E_1)\, P(E / E_1)}{P(E)} = \frac{30}{31}$$

Exercise # 1

[Single Correct Choice Type Questions]

1. If the integers m and n are chosen at random between 1 and 100, then the probability that a number of the form $7^m + 7^n$ is divisible by 5 equals -

(A) $\frac{1}{4}$ **(B)** $\frac{1}{7}$ **(C)** $\frac{1}{8}$ **(D)** $\frac{1}{49}$

2. If the letters a, A, b, B, c and C are arranged at random in a row. The probability that the lower case letters appear in alphabetical order, is

(A) $\frac{1}{6}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{1}{5}$ **(D)** $\frac{1}{30}$

3. A quadratic equation is chosen from the set of all quadratic equations which are unchanged by squaring their roots. The chance that the chosen equation has equal roots is -

(A) 1/2 **(B)** 1/3 **(C)** 1/4 **(D)** 2/3

4. In drawing of a card from a well shuffled ordinary deck of playing cards the events 'card drawn is spade' and 'card drawn is an ace' are

(A) mutually exclusive **(B)** equally likely

(C) forming an exhaustive system **(D)** none of these

5. 5 persons entered the lift cabin on the ground floor of an 8 floor building. Suppose that each of them independently and with equal probability, can leave the cabin at any other floor, starting from the first. The probability that all 5 persons leave at different floors is -

(A) $\left(\frac{5}{8}\right)^5$ **(B)** $\frac{{}^8C_5}{8^5}$ **(C)** $\frac{5!}{8^5}$ **(D)** $\frac{{}^8C_5\,5!}{8^5}$

6. A bag contains 5 balls, three red and two white. Balls are randomly removed one at a time without replacement until all the red balls are drawn or all the white balls are drawn. The probability that the last ball drawn is white, is

(A) $\frac{3}{10}$ **(B)** $\frac{5}{10}$ **(C)** $\frac{6}{10}$ **(D)** $\frac{7}{10}$

7. A determinant is chosen at random from the set of all determinant of order 2 with elements 0 or 1 only. The probability that the determinant chosen has the value non negative is -

(A) 3/16 **(B)** 6/16 **(C)** 10/16 **(D)** 13/16

8. Players A and B alternately toss a biased coin, with A going first. A wins if A tosses a Tail before B tosses a Head; otherwise B wins. If the probability of a head is p, the value of p for which the game is fair to both players, is

(A) $2-\sqrt{3}$ **(B)** $\frac{\sqrt{2}}{2}$ **(C)** $\sqrt{3}-1$ **(D)** $\frac{\sqrt{5}-1}{2}$

9. The chance that a 13 card combination from a pack of 52 playing cards is dealt to a player in a game of bridge, in which 9 cards are of the same suit, is

(A) $\dfrac{4\,.\,{}^{13}C_9\,.\,{}^{39}C_4}{{}^{52}C_{13}}$ **(B)** $\dfrac{4!\,.\,{}^{13}C_9\,.\,{}^{39}C_4}{{}^{52}C_{13}}$ **(C)** $\dfrac{{}^{13}C_9\,.\,{}^{39}C_4}{{}^{52}C_{13}}$ **(D)** none of these

10. The entries in a two-by-two determinant $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ are integers that are chosen randomly and independently, and, for each entry, the probability that the entry is odd is p. If the probability that the value of the determinant is even is 1/2, then the value of p, is

(A) $\dfrac{1}{3}$ **(B)** $\dfrac{1}{2}$ **(C)** $\dfrac{2}{3}$ **(D)** $\dfrac{\sqrt{2}}{2}$

11. There are ten prizes, five A's, three B's and two C's, placed in identical sealed envelopes for the top ten contestants in a mathematics contest. The prizes are awarded by allowing winners to select an envelope at random from those remaining. When the 8^{th} contestant goes to select the prize, the probability that the remaining three prizes are one A, one B and one C, is -

(A) 1/4 **(B)** 1/3 **(C)** 1/12 **(D)** 1/10

12. A die is thrown a fixed number of times. If probability of getting even number 3 times is same as the probability of getting even number 4 times, then probability of getting even number exactly once is

(A) $\dfrac{1}{4}$ **(B)** $\dfrac{3}{128}$ **(C)** $\dfrac{5}{64}$ **(D)** $\dfrac{7}{128}$

13. A license plate is 3 letters (of English alphabets) followed by 3 digits. If all possible license plates are equally likely, the probability that a plate has either a letter palindrome or a digit palindrome (or both), is -

(A) $\dfrac{7}{52}$ **(B)** $\dfrac{9}{65}$ **(C)** $\dfrac{8}{65}$ **(D)** none

14. Let A and B are two independent events with $P(A) = \dfrac{1}{2}$ and $P(A \cup B) = \dfrac{2}{3}$ then the probabilities of the events P(B), P(A/B) and $P(B^c/A)$ are in

(A) A.P. **(B)** G.P. **(C)** H.P. **(D)** None

15. 'A' and 'B' each have a bag that contains one ball of each of the colours blue, green, orange, red and violet. 'A' randomly selects one ball from his bag and puts it into B's bag. 'B' then randomly selects one ball from his bag and puts it into A's bag. The probability that after this process the contents of the two bags are the same, is -

(A) 1/2 **(B)** 1/3 **(C)** 1/5 **(D)** 1/6

16. If two subsets A and B of set S containing n elements are selected at random, then the probability that $A \cap B = \phi$ and $A \cup B = S$ is

(A) $\frac{1}{2}$ **(B)** $\frac{1}{2^n}$ **(C)** $\left(\frac{3}{4}\right)^4$ **(D)** $\frac{1}{3^n}$

17. Two cards are drawn from a well shuffled pack of 52 playing cards one by one. If

A : the event that the second card drawn is an ace and

B : the event that the first card drawn is an ace card.

then which of the following is true ?

(A) $P(A)=\frac{4}{17}$; $P(B)=\frac{1}{13}$ **(B)** $P(A)=\frac{1}{13}$; $P(B)=\frac{1}{13}$

(C) $P(A)=\frac{1}{13}$; $P(B)=\frac{1}{17}$ **(D)** $P(A)=\frac{16}{221}$; $P(B)=\frac{4}{51}$

18. Mr. A forgot to write down a very important phone number. All he remembers is that it started with 713 and that the next set of 4 digit involved are 1, 7 and 9 with one of these numbers appearing twice. He guesses a phone number and dials randomly. The odds in favour of dialing the correct telephone number, is

(A) 1 : 35 **(B)** 1 : 71 **(C)** 1 : 23 **(D)** 1 : 36

19. I alternatively toss a fair coin and a fair die until I either toss a head or throw a 2. If I toss the coin first, the probability that I throw a 2 before I toss a head, is

(A) 1/7 **(B)** 7/12 **(C)** 5/12 **(D)** 5/7

20. A & B throw with one dice for a stake of Rs. 99/- which is to be won by the player who first throws 4. If A has the first throw then their respective expectations of rupees are:

(A) 50 & 49 **(B)** 54 & 45 **(C)** 45 & 54 **(D)** none

21. A fair die is thrown 3 times. The chance that sum of three numbers appearing on the die is less than 11, is equal to

(A) $\frac{1}{2}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{5}{8}$

22. If an integer q is chosen at random in the interval $-10 \le q \le 10$, then the probability that the roots of the equation

$x^2 + qx + \frac{3q}{4} + 1 = 0$ are real is

(A) $\frac{16}{21}$ **(B)** $\frac{15}{21}$ **(C)** $\frac{14}{21}$ **(D)** $\frac{17}{21}$

23. Two cubes have their faces painted either red or blue. The first cube has five red faces and one blue face. When the two cubes are rolled simultaneously, the probability that the two top faces show the same colour is 1/2. Number of red faces on the second cube, is -

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4

24. An experiment resulting in sample space as $S = \{a, b, c, d, e, f\}$

with $P(A) = \frac{1}{16}$, $P(B) = \frac{1}{16}$, $P(C) = \frac{2}{16}$, $P(D) = \frac{3}{16}$, $P(e) = \frac{4}{16}$ and $P(f) = \frac{5}{16}$.

Let three events A, B and C are defined as $A = \{a, c, e,\}$, $B = \{c, d, e, f\}$ and $C = \{b, c, f\}$.

If $P(A/B) = p_1$, $P(B/C) = p_2$, $P(C/A^c) = p_3$ and $P(A^c/C) = p_4$, then the correct order sequance is

(A) $p_1 < p_3 < p_2 < p_4$ **(B)** $p_1 < p_4 < p_3 < p_2$ **(C)** $p_1 < p_3 < p_4 < p_2$ **(D)** $p_3 < p_1 < p_4 < p_2$

25. 7 persons are stopped on the road at random and asked about their birthdays. If the probability that 3 of them are born on Wednesday, 2 on Thursday and the remaining 2 on Sunday is $\frac{K}{7^6}$, then K is equal to -

(A) 15 **(B)** 30 **(C)** 105 **(D)** 210

26. An experiment results in four possible out comes S_1, S_2, S_3 & S_4 with probabilities p_1, p_2, p_3 & p_4 respectively. Which one of the following probability assignment is possbile.

[Assume S_1 S_2 S_3 S_4 are pair wise exclusive]

(A) $p_1 = 0.25, p_2 = 0.35, p_3 = 0.10, p_4 = 0.05$

(B) $p_1 = 0.40, p_2 = -0.20, p_3 = 0.60, p_4 = 0.20$

(C) $p_1 = 0.30, p_2 = 0.60, p_3 = 0.10, p_4 = 0.10$

(D) $p_1 = 0.20, p_2 = 0.30, p_3 = 0.40, p_4 = 0.10$

27. A box has four dice in it. Three of them are fair dice but the fourth one has the number five on all of its faces. A die is chosen at random from the box and is rolled three times and shows up the face five on all the three occasions. The chance that the die chosen was a rigged die, is -

(A) $\frac{216}{217}$ **(B)** $\frac{215}{219}$ **(C)** $\frac{216}{219}$ **(D)** none

28. Let a red die, a blue die, a green die and a white die are rolled once, the dice being fair. The outcomes on the red, blue, green and white die denote the numbers a, b, c and d respectively. Let E denotes the event that absolute value of $(a - 1)(b - 2)(c - 3)(d - 6) = 1$, then P(E) is

(A) $\frac{1}{324}$ **(B)** $\frac{1}{648}$ **(C)** $\frac{2}{324}$ **(D)** $\frac{1}{162}$

29. In a series of 3 independent trials the probability of exactly 2 success is 12 times as large as the probability of 3 successes. The probability of a success in each trial is:

(A) 1/5 **(B)** 2/5 **(C)** 3/5 **(D)** 4/5

30. Mr. Dupont is a professional wine taster. When given a French wine, he will identify it with probability 0.9 correctly as French and will mistake it for a Californian wine with probability 0.1. When given a Californian wine, he will identify it with probability 0.8 correctly as Californian and will mistake it for a French wine with probability 0.2. Suppose that Mr. Dupont is given ten unlabelled glasses of wine, three with French and seven with Californian wines. He randomly picks a glass, tries the wine and solemnly says : "French". The probability that the wine he tasted was Californian, is nearly equal to -

(A) 07.14 **(B)** 0.24 **(C)** 0.34 **(D)** 0.44

Exercise # 2

Part # I [Multiple Correct Choice Type Questions]

1. For two events A & B, which of the following is/are correct -

(A) $(A \cup B) \cap B = B$
(B) $(A \cup B)^C \cap B^C = (A \cup B)^C$
(C) $A^C \cap (A \cap B) \cap B^C = \phi$
(D) $(A^C \cap B) \cap (A \cup B) = B$

2. If E_1 and E_2 are two events such that $P(E_1)=1/4$, $P(E_2/E_1)=1/2$ and $P(E_1/E_2) = 1/4$ then -

(A) E_1 and E_2 are independent

(B) E_1 and E_2 are exhaustive

(C) E_2 is twice as likely to occur as E_1

(D) probabilities of the events $E_1 \cap E_2$, E_1 and E_2 are in G.P.

3. In throwing a die let A be the event 'coming up of an odd number', B be the event 'coming up of an even number', C be the event 'coming up of a number ≥ 4' and D be the event 'coming up of a number < 3', then

(A) A and B are mutually exclusive and exhaustive
(B) A and C are mutually exclusive and exhaustive
(C) A, C and D form an exhautive system
(D) B, C and D form an exhautive system

4. Before a race the chance of three runners A, B & C were estimated to be proportional to 5, 3 & 2 respectively but during the race A meets with an accident which reduces his chance to 1/3. If the respective chances of B and C are P(B) and P(C) then -

(A) $P(B) = \frac{2}{5}$
(B) $P(C) = \frac{4}{15}$
(C) $P(C) = \frac{2}{5}$
(D) $P(B) = \frac{4}{15}$

5. A box contains 11 tickets numbered from 1 to 11. Two tickets are drawn similtaneously at random.

Let E_1 denotes the event that the sum of the numbers on the tickets drawn is even

and E_2 denotes the event that the sum of the numbers on the tickets drawn is odd

Which of the following hold good ?

(A) $P(E_1/E_2) = P(E_2/E_1)$
(B) E_1 and E_2 are exhaustive
(C) $P(E_2) > P(E_1)$
(D) E_1 and E_2 are equally likely

6. If M & N are any two events, then which one of the following represents the probability of the occurance of exactly one of them ?

(A) $P(M) + P(N) - 2P(M \cap N)$
(B) $P(M) + P(N) - P(M \cap N)$
(C) $P(\overline{M}) + P(\overline{N}) - 2P(\overline{M} \cap \overline{N})$
(D) $P(M \cap \overline{N}) + P(\overline{M} \cap N)$

7. Let $0 < P(A) < 1$, $0 < P(B) < 1$ and $P(A \cup B) = P(A) + P(B) - P(A)\,P(B)$. Then -

(A) $P\left(\frac{B}{A}\right) = P(B) - P(A)$
(B) $P(A^C \cup B^C) = P(A^C) + P(B^C)$
(C) $P((A \cup B)^C) = P(A^C)\,P(B^C)$
(D) $P\left(\frac{A}{B}\right) = P(A)$

8. If A & B are two events such that $P(B) \neq 1$, B^C denotes the event complementary to B, then -

(A) $P(A/B^C) = \dfrac{P(A) - P(A \cap B)}{1 - P(B)}$ **(B)** $P(A \cap B) \geq P(A) + P(B) - 1$

(C) $P(A) >< P(A/B)$ according as $P(A/B^C) >< P(A)$ **(D)** $P(A/B^C) + P(A^C/B^C) = 1$

9. Events A and B satisfy $P(A) = 0.4$, $P(B) = 0.5$ and $P(A \cup B) = 0.8$. Which of the following statement(s) is/are correct?

(A) $P(\bar{A} \cap B) = \dfrac{2}{5}$ **(B)** $P(B/\bar{A}) = \dfrac{2}{3}$

(C) $P(A \cap B) < P(A) \cdot P(B)$ **(D)** P (exactly one of either A or B occurs) $= \dfrac{7}{10}$

10. Let $0 < P(A) < 1$, $0 < P(B) < 1$ & $P(A \cup B) = P(A) + P(B) - P(A).\ P(B)$, then:

(A) $P(B/A) = P(B) - P(A)$ **(B)** $P(A^C \cup B^C) = P(A^C) + P(B^C)$

(C) $P((A \cup B)^C) = P(A^C).\ P(B^C)$ **(D)** $P(A/B) = P(A)$

11. For any two events A & B in a sample space :

(A) $P\left(\dfrac{A}{B}\right) \geq \dfrac{P(A) + P(B) - 1}{P(B)}$, $P(B) \neq 0$ is always true

(B) $P\left(A \cap \bar{B}\right) = P(A) - P(A \cap B)$

(C) $P(A \cup B) = 1 - P\left(A^C\right) P\left(B^C\right)$, if A & B are independent

(D) $P(A \cup B) = 1 - P\left(A^C\right) P\left(B^C\right)$, if A & B are disjoint

12. A student appears for tests I, II & III. The student is successful if he passes either in tests I & II or tests I & III. The probabilities of the student passing in the tests I, II & III are p, q &
1/2 respectively. If the probability that the student is successful is 1/2, then:

(A) $p = 1, q = 0$ **(B)** $p = 2/3, q = 1/2$

(C) $p = 3/5, q = 2/3$ **(D)** there are infinitely many values of p & q.

13. A fair coin is tossed 99 times. If X is the number of times heads occur, then $P(X = r)$ is maximum when r is

(A) 49 **(B)** 50 **(C)** 51 **(D)** none of these

14. From a pack of 52 playing cards, face cards and tens are removed and kept aside then a card is drawn at random from the remaining cards. If

A : The event that the card drawn is an ace

H : The event that the card drawn is a heart

S : The event that the card drawn is a spade

then which of the following holds ?

(A) $9P(A) = 4P(H)$ **(B)** $P(S) = 4P(A \cap H)$

(C) $4P(H) = 3P(A \cup S)$ **(D)** $P(H) = 9P(A \cap S)$

15. Two real numbers, x & y are selected at random. Given that $0 \le x \le 1$; $0 \le y \le 1$. Let A be the event that $y^2 \le x$; B be the event that $x^2 \le y$, then -

(A) $P(A \cap B) = \dfrac{1}{3}$ **(B)** A & B are exhaustive events

(C) A & B are mutually exclusive **(D)** A & B are independent events.

16. If E and F are two complementry events defined on a sample space with $P(E) > 0$ and $P(F) > 0$, then

(A) E and F must be disjoint. **(B)** E and F may be equally likely.

(C) E and F must be exhaustive. **(D)** E and F can be independent.

17. A student has to match three historical events i.e. Dandi March, Quit India Movement and Mahatma Gandhi's assasination with the years 1948, 1930 and 1942 and each event happens in different years . The student has no knowledge of the correct answers and decides to match the events and years randomly. Let $E_i : (0 \le i \le 3)$ denote the event that the student gets exactly i correct answer, then

(A) $P(E_o) + P(E_3) = P(E_1)$ **(B)** $P(E_o) . P(E_1) = P(E_3)$

(C) $P(E_0 \cap E_1) = P(E_2)$ **(D)** $P(E_0) + P(E_1) + P(E_3) = 1$

18. A pair of fair dice having six faces numbered from 1 to 6 are thrown once, suppose two events E and F are defined as

E : Product of the two numbers appearing is divisible by 5.

F : At least one of the dice shows up the face one.

Then the events E and F are

(A) mutually exclusive **(B)** independent

(C) neither independent nor mutually exclusive **(D)** are equiprobable

19. For any two events A & B defined on a sample space,

(A) $P(A/B) \ge \dfrac{P(A) + P(B) - 1}{P(B)}$, $P(B) \ne 0$ is always true

(B) $P(A \cup \bar{B}) = P(A) - P(A \cap B)$

(C) $P(A \cup B) = 1 - P(A^c). P(B^c)$, if A & B are independent

(D) $P(A \cup B) = 1 - P(A^c). P(B^c)$, if A & B are disjoint

20. An unbiased coin is tossed n times. Let X denote the number of times head occurs. If P(X = 4), P (X = 5) and P(X = 6) are in AP, then the value of n can be

(A) 7 **(B)** 10 **(C)** 12 **(D)** 14

21. In a maths paper there are 3 sections A, B & C. Section A is compulsory. Out of sections B & C a student has to attempt any one. Passing in the paper means passing in A & passing in B or C. The probability of the student passing in A, B & C are p, q & 1/2 respectively. If the probability that the student is successful is 1/2 then, which of the following is false -

(A) $p = q = 1$ **(B)** $p = q = 1/2$ **(C)** $p = 1, q = 0$ **(D)** $p = 1, q = 1/2$

22. If A and B are two independent events such that $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{5}$, then -

(A) $P(A \cup B) = \frac{3}{5}$ **(B)** $P(A / B) = \frac{1}{2}$

(C) $P(A / A \cup B) = \frac{5}{6}$ **(D)** $P(A \cap B / A' \cup B') = 0$

23. The probabilities that a man makes a certain dangerous journey by car, motor cycle or on foot are 1/2, 1/6 and 1/3 respectively. The probabilities of an accident when he uses these means of transport are 1/5, 2/5 and 1/10 respectively. Which of the following statement(s) hold good ?

(A) The probability of an accident occurring in a single journey, is $1/5$.

(B) If an accident is known to have happened, the probability that the man was travelling by car, is $1/2$.

(C) If an accident is known to have happened, the probability that the man was travelling by motor cycle, is $1/5$.

(D) If an accident is known to have happened, the probability that the man was travelling on foot, is $1/3$.

24. If $\bar{E}$ & $\bar{F}$ are the complementary events of events E & F respectively & if $0 < P(F) < 1$, then -

(A) $P(E|F) + P(\bar{E}|F) = 1$ **(B)** $P(E|F) + P(E|\bar{F}) = 1$

(C) $P(\bar{E}|F) + P(E|\bar{F}) = 1$ **(D)** $P(E|\bar{F}) + P(\bar{E}|\bar{F}) = 1$

Part # II [Assertion & Reason Type Questions]

These questions contain, Statement I (assertion) and Statement II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I

(C) Statement-I is true, Statement-II is false

(D) Statement-I is false, Statement-II is true

1. **Statement - I** $P\left(\frac{(A \cap \bar{B})}{C}\right) = P\left(\frac{A}{C}\right) + P\left(\frac{A \cap B}{C}\right)$

Statement - II $P(A \cap \bar{B}) = P(A) - P(A \cap B)$

2. From a well shuffled pack of 52 cards. A card is drawn, outcome is noted and the card is replaced in the pack. 16 such trials were made. Let X denotes a binomial variate denoting the number of times Heart occurs.

Statement - I Mean and variance of X are 4 and 3 respectively.

Statement - II In a binomial probability distribution mean is always greater than variance.

3. **Statement - I :** If p is chosen at random in the closed interval [0, 5], then the probability that the equation

$$x^2 + px + \frac{1}{4}(p+2) = 0 \text{ has real root is } \frac{3}{5}.$$

Statement - II : If discriminant ≥ 0 then roots of the quadratic equation of real coefficient are always real.

4. **Statement - I** Let $f(x) = x^3 - ax^2 + bx + 6$ where $a, b \in \{1, 2, 3, 4, 5, 6\}$. The probability that f(x) is strictly increasing function is 4/9.

Statement - II If $y = g(x)$ is differentiable function in $(-\infty, \infty)$ then g(x) is strictly increasing provided $g'(x) \geq 0$ and $g'(x) = 0$ does not form an interval.

5. **Statement - I** If A and B are two independent events such that $P(A) \neq 0$, $P(B) \neq 0$, then A and B can not be mutually exclusive.

Statement - II For independent events A and B, we have P(A/B) = P(A) which is not so for mutually exclusive events.

6. A fair coin is tossed 3 times. Consider the events

A : first toss is head ; B : second toss is head ;

C : exactly two consecutive heads or exactly two consecutive tails

Statement - I A,B,C are independent events.

Statement - II A,B,C are pairwise independent.

7. Four children A, B, C and D have 1, 3, 5 and 7 identical unbiased dice respectively and roll them with the condition that one who obtains an even score, wins. They keep playing till some one or the other wins.

Statement - I All the four children are equally likely to win provided they roll their dice simultaneously.

Statement - II The child A is most probable to win the game if they roll their dice in order of A, B, C and D respectively.

8. **Statement - I** Since sample space of the experiment 'A coin is tossed if it turns up head, a die is thrown' is {(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), T}.

$\therefore$ Prob. of the event {(H, 1), (H, 2), (H, 5)} is $\frac{3}{7}$.

Statement - II If all the sample points in the sample space of an experiment are pair wise mutually exclusive, equally likely and exhaustive, then probability of an event E is defined as

$$P(E) = \frac{\text{number of sample points favourable to the event E}}{\text{Total number of sample points in the sample space}}$$

9. Consider (a, b), where a and b are respective outcomes in throwing an unbiased die twice.

Statement-I : If $\lim_{x \to 0}\left(\frac{a^x + b^x + 1}{3}\right)^{\frac{3}{x}} = 6$, then the probability that 'a' is a prime number is $\frac{1}{2}$.

Statement-II : If A & B are two events then probability of event B when event A has already happened is

$$P(B/A) = \frac{P(A \cap B)}{P(A)}.$$

10. **Statement - I** If $\frac{1+4P}{4}, \frac{1-P}{4}, \frac{1-2P}{4}$ are probabilities of three pair-wise mutually exclusive events, then the possible values of P belong to the set $\left[-\frac{1}{4}, \frac{1}{2}\right]$.

Statement - II If three events are pair wire mutually exclusive and exhaustive then sum of there probability is equal to 1.

Exercise # 3 Part # I [Matrix Match Type Questions]

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with one or more statement**(s)** in **Column-II**.

1. **Column-I** **Column-II**

(A) A natural number x is randomly selected from the set of first 100 natural numbers. The probability that it satisfies the inequality $x+\frac{100}{x}>50$ is — **(p)** $\frac{12}{17}$

(B) 5 different marbles are placed in 5 different boxes randomly. If each box can hold any number of marbles then the probability that exactly two boxes remain empty is — **(q)** $\frac{11}{20}$

(C) A letter is known to have come either from London or Clifton. On the postmark only the two consecutive letters ON are legible. The chance that it came from London is — **(r)** $\frac{12}{25}$

(D) There are three works, one consisting of 3 volumes, one of 4 and the other of one volume. They are placed on a shelf at random. If the chance that volumes of same works are all together is P_1 then $7P_1$ = — **(s)** $\frac{3}{20}$

2. **Column I** **Column II**

(A) A gambler has one rupee in his pocket. He tosses an unbiased normal coin unless either he is ruined or unless the coin has been tossed for a maximum of five times. If for each head he wins a rupee and for each tail he looses a rupee, then the probability that the gambler is ruined is — **(p)** $\frac{1}{2}$

(q) $\frac{4}{5}$

(B) The probability at least one of the events A and B occur is 0.6. If A and B occur simultaneously with probability 0.2, then $P(\overline{A})+P(\overline{B})$ is

(C) 3 firemen X, Y and Z shoot at a common target. The probabilities that X and Y can hit the target are 2/3 and 3/4 respectively. If the probability that exactly two bullets are found on the target is 11/24, then the proficiency of Z to hit the target is — **(r)** $\frac{6}{5}$

(s) $\frac{11}{16}$

3. **Column – I** **Column – II**

(A) If the probability that units digit in square of an even integer is 4 is p, then the value of 5p is — **(p)** 1

(B) If A and B are independent events and $P(A \cap B) = \frac{1}{6}$, $P(A) = \frac{1}{3}$, then $6P\left(\frac{B}{\bar{A}}\right) =$ — **(q)** 2

(C) One mapping is selected at random from all mappings of the set S = {1, 2, 3, ..., n} into itself. If the probability that the mapping is one-one and is $\frac{3}{32}$, then the value of n is — **(r)** 3

(D) A boy has 20% chance of hitting at a target. Let p denote the probability of hitting the target for the first time at the n^{th} trial. If p satisfies the inequality $625p^2 - 175p + 12 < 0$, then value of n is — **(s)** 4

4. In a certain town, 40% of the people have brown hair, 25% have brown eyes and 15% have both brown hair and brown eyes. A person is randomly selected from the town. Match the events of column I with their corresponding probabilities given in column II.

	Column I		Column II
(A)	If he has brown hair, the probability that he also has brown eyes, is	(p)	0.25
(B)	If he has brown eyes, the probability that he also has brown hair, is	(q)	0.375
(C)	The probability that he has neither brown hair nor brown eyes, is	(r)	0.60
		(s)	0.50

5. **Column – I** **Column – II**

(A) A pair of dice is thrown. If total of numbers turned up on both the dies is 8, then the probability that the number turn up on the second die is 5 is — **(p)** 5/16

(B) A box contains 4 white and 3 black balls. Two balls are drawn successively and is found that second ball is white, then the probability that first ball is also white is — **(q)** 1/3

(C) A biased coin with probability p, 0 < p < 1 of heads is tossed until a head appears for the first time. If the probability that the number of tosses required is even is 2/5, then p equals — **(r)** $\frac{1}{2}$

(D) A coin whose faces are marked 3 and 5 is tossed 4 times : what is the probability that the sum of the numbers thrown being less, than 15 ? — **(s)** $\frac{1}{5}$

6. An urn contain six red balls and four black balls. All ten balls are drawn from the urn, one by one and their colour is noted. Balls are not replaced once they have been drawn.

	Column-I		Column-II
(A)	Probability that first three balls are of same colour	**(p)**	$\frac{1}{5}$
(B)	Probability that last three balls are of same colour	**(q)**	$\frac{3}{5}$
(C)	If it is known that first three are of the same colour, then the probability that colour is red is	**(r)**	$\frac{4}{15}$
(D)	Probability that no two consecutive balls in first three draw are same is	**(s)**	$\frac{5}{6}$

Part # II [Comprehension Type Questions]

Comprehension # 1

Let S and T are two events defined on a sample space with probabilities

$P(S) = 0.5, P(T) = 0.69, P(S/T) = 0.5$

On the basis of above information, answer the following questions :

1. Events S and T are -

(A) mutually exclusive **(B)** independent

(C) mutually exclusive and independent **(D)** neither mutually exclusive nor independent

2. The value of P(S and T) -

(A) 0.3450 **(B)** 0.2500 **(C)** 0.6900 **(D)** 0.350

3. The value of P(S or T) -

(A) 0.6900 **(B)** 1.19 **(C)** 0.8450 **(D)** 0

Comprehension # 2

If A and B are two events, then probability that atleast one of them is selected is $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
For three events A, B, C the probability that atleast one of them is seleted is
$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$.

1. Probability that none of the three events (A, B, C) occurs

(A) $P(\bar{A} \cup \bar{B} \cup \bar{C}) - P(A) - P(B) - P(C) + P(A \cap B) + P(B \cap C) + P(C \cap A)$

(B) $P(A \cup B \cup C) - P(A) - P(B) - P(C) + P(A \cap B) + P(B \cap C) + P(C \cap A) - P(A \cap B \cap C)$

(C) $P(\bar{A}) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

(D) none of these

2. Probability that exactly two events occurs

(A) $P(A)+P(B)+P(C)-P(A\cup B)-P(B\cup C)-P(C\cup A)$

(B) $2(P(A)+P(B)+P(C))-P(A\cup B)-P(B\cup C)-P(C\cup A)$

(C) $P(A\cap B)+P(B\cap C)+P(C\cap A)-3\times P(A\cap B\cap C)$

(D) $P(\bar{A}\cap\bar{B}) + P(\bar{B}\cap\bar{C}) + P(\bar{C}\cap\bar{A})$

3. Probability that atmost two events happen

(A) $P(A\cap B)+P(B\cap C)+P(C\cap A)-3\times P(A\cap B\cap C)$

(B) $P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)$

(C) $P(\bar{A}) + P(\bar{B}) + P(\bar{C})$

(D) $P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)+P(A\cup B\cup C)^C$

Comprehension # 3

Three bags A, B and C are given, each containing 6 marbles. The first bag A has 5 black marbles and 1 white. The second bag B has 4 black marbles and 2 white marbles. The third bag C has 3 black marbles and 3 white marbles. Two marbles are drawn randomly one from each of two different bags (we do not know which bags) and found to be one white and the other black. Let P denote the probability that a marble drawn from the remaining bag is white.

1. The probability of drawing one white and one black marble from any two of the selected bags, is

(A) $\frac{25}{54}$ **(B)** $\frac{25}{162}$ **(C)** $\frac{27}{54}$ **(D)** None

2. If one white and one black marble has been drawn, the probability that bags A and B were selected, is

(A) $\frac{6}{25}$ **(B)** $\frac{7}{25}$ **(C)** $\frac{8}{25}$ **(D)** $\frac{9}{25}$

3. If $P=\frac{m}{n}$ (as a reduced fraction), then the value of $(m+n)$ equals

(A) 25 **(B)** 33 **(C)** 42 **(D)** 47

Comprehension # 4

A JEE as print estimates that he will be successful with an 80 percent chance if he studies 10 hours per day, with a 60 percent chance if he studies 7 hours per day and with a 40 percent chance if he studies 4 hours per day. She further believes that she will study 10 hours, 7 hours and 4 hours per day with probabilities 0.1, 0.2 and 0.7 respectively.

1. The chance she will be successfull is

(A) 0.28 **(B)** 0.38 **(C)** 0.48 **(D)** 0.58

2. Given that she is successful the chance she studied for 4 hours, is

(A) $\frac{6}{12}$ **(B)** $\frac{7}{12}$ **(C)** $\frac{8}{12}$ **(D)** $\frac{9}{12}$

3. Given that she does not achieve success, the chance the studied for 4 hour, is

(A) $\frac{18}{26}$ **(B)** $\frac{19}{26}$ **(C)** $\frac{20}{26}$ **(D)** $\frac{21}{26}$

Comprehension # 5

From a pack of 52 playing cards, all the face card are removed. From the remaining pack, cards are dealt one by one until an ace appears. Let P_1 denote the probability that exactly 10 cards are dealt before the first ace. If cards continue to be dealt until a second ace appears. Let P_2 denotes the probability that exactly 20 cards are dealt before the second ace. In case the first ace appears in one of the 20 cards then the probability is P_3.

1. The value of P_1 equals

(A) $\frac{(27)(28)(29)}{(37)(38)(39)}$ **(B)** $\frac{(27)(28)(29)}{(10)(37)(38)(39)}$

(C) $\frac{(30)(29)(28)(27)}{(40)(39)(38)(37)}$ **(D)** None

2. The value of P_2 equals

(A) $\frac{9}{(10)(13)(37)}$ **(B)** $\frac{9}{(13)(37)}$ **(C)** $\frac{18}{(13)(37)}$ **(D)** $\frac{6}{(13)(37)}$

3. The value of P_3 equals

(A) $2P_2$ **(B)** $10P_2$ **(C)** $11P_2$ **(D)** $20P_2$

Exercise # 4 [Subjective Type Questions]

1. Numbers are selected at random, one at a time, from the two digit numbers 00, 01, 02,....., 99 with replacement. An event E occurs if & only if the product of the two digits of a selected number is 18. If four numbers are selected, find the probability that the event E occurs at least 3 times.

2. A player tosses an unbiased coin and is to score two points for every head turned up and one point for every tail turned up. If P_n denotes the probability that his score is exactly n points, prove that

$$P_n - P_{n-1} = \frac{1}{2}(P_{n-2} - P_{n-1}),\ n \geq 3$$

Also compute P_1 and P_2 and hence deduce the probability that he scores exactly 4.

3. Before a race the chance of three runners A, B, C were estimated to be proportional to 5, 3, 2, but during the race A meets with an accident which reduces his chance to 1/3. What are the respective chance of B and C now ?

4. The chance of one event happening is the square of the chance of a 2^{nd} event, but odds against the first are the cubes of the odds against the 2^{nd}. Find the chances of each. (Assume that both events are neither sure nor impossible).

5. What is the probability that in a group of

(A) 2 people, both will have the same date of birth.

(B) 3 people, atleast 2 will have the same date of birth.

Assume the year to be ordinary consisting of 365 days.

6. A box contains 2 red and 3 blue balls. Two balls are drawn successively. If getting 'a red ball on first draw and a blue ball on second draw' is considered a success, then write the probability distribution of successes. It is given that the above experiment is performed 3 times, with replacement.

7. The odds that a book will be favourably reviewed by three independent critics are 5 to 2, 4 to 3 and 3 to 4 respectively. What is the probability that of the three reviews a majority will be favourable ?

8. In a tournament, team X, plays with each of the 6 other teams once. For each match the probabilities of a win, draw and loss are equal. If the probability that the team X, finishes with more wins than losses, can be expressed as rational $\frac{p}{q}$ in their lowest form, then find (p + q).

9. There are 5 brilliant students in class XI and 8 brilliant students in class XII. Each class has 50 students. The odds in favour of choosing the class XI are 2 : 3. One of the classes is chosen randomly and then a student is randomly selected. Find the probability of selecting a brilliant student.

10. Three shots are fired at a target in succession. The probabilities of a hit in the first shot is $\frac{1}{2}$, in the second $\frac{2}{3}$ and in the third shot is $\frac{3}{4}$. In case of exactly one hit, the probability of destroying the target is $\frac{1}{3}$ and in the case of exactly two hits $\frac{7}{11}$ and in the case of three hits is 1.0. Find the probability of destroying the target in three shots.

11. A, B are two inaccurate arithmeticians whose chances of solving a given question correctly are (1/8) and (1/12) respectively. They solve a problem and obtained the same result. If it is 1000 to 1 against their making the same mistake, find the chance that the result is correct.

12. Let A & B be two events defined on a sample space. Given P(A) = 0.4 ; P(B) = 0.80 and $P\left(\bar{A}/\bar{B}\right) = 0.10$. Then find

(i) $P\left(\bar{A} \cup B\right)$ & **(ii)** $P\left[\left(\bar{A} \cap B\right) \cup \left(A \cap \bar{B}\right)\right]$.

13. A cube painted red on all sides, is cut into 125 equal small cubes. A small cube when picked up is found to show red colour on one of its faces. Find the probability that two more faces also show red color.

14. Eight players $P_1, P_2, P_3, \ldots\ldots\ldots P_8$ play a knockout tournament. It is known that whenever the players P_i and P_j play the player P_i will win if $i < j$. Assuming that the players are paired at random in each round, what is the probability that the players P_4 reaches the final ?

15. 3 students {A, B, C} tackle a puzzle together and offers a solution upon which majority of the 3 agrees. Probability of A solving the puzzle correctly is p. Probability of B solving the puzzle correctly is also p. C is a dumb student who randomly supports the solution of either A or B. There is one more student D, whose probability of solving the puzzle correctly is once again, p. Out of the 3 member team {A, B, C} and one member team {D}, which one is more likely to solve the puzzle correctly.

16. In a multiple choice question there are 4 alternative answers of which 1, 2, 3 or all may be correct. A candidate will get marks in the question only if he ticks all the correct answer. The candidate decides to tick answers at random. If he is allowed upto 5 chances to answer the question, find the probability that he will get the marks in the question.

17. A plane is landing. If the weather is favourable, the pilot landing the plane can see the runway. In this case the probability of a safe landing is p_1. If there is a low cloud ceiling, the pilot has to make a blind landing by instruments. The reliability (the probability of failure free functioning) of the instruments needed for a blind landing is P. If the blind landing instruments function normally, the plane makes a safe landing with the same probability p_1 as in the case of a visual landing. If the blind landing instruments fail, then the pilot may make a safe landing with probability $p_2 < p_1$. Compute the probability of a safe landing if it is known that in K percent of the cases there is a low cloud ceiling. Also find the probability that the pilot used the blind landing instruments, if the plane landed safely.

18. There are two packs A and B of 52 playing cards. All the four aces from the pack A are removed whereas from the pack B, one ace, one king, one queen and one jack is removed. One of these two packs is selected randomly and two cards are drawn simultaneously from it, and found to be a pair (i.e. both have same rank e.g. two 9's or two king etc). If $Q = \frac{m}{n}$ (expressed in lowest form) denotes the probability that the pack A was selected, find (m + n).

19. Mr. Dupont is a professional tea taster. When given a high grade tea, he will identify it with probability 0.9 correctly as high grade and will mistake it for a low grade tea with probability 0.1 . When given a low grade tea, he will identify it with probability 0.8 correctly as low grade tea and will mistake it for a high grade tea with probability 0.2 . Suppose that Mr. Dupont is given ten unlabelled cups of tea, three with high grade and seven with low grade tea. He randomly picks a cup, tries the tea and solemnly says “high grade tea”. Find the probability that the tea he tasted was low grade tea.

20. A biased coin which comes up heads three time as often as tails is tossed. If it shows head, a chip is drawn from urn-I which contains 2 white chips and 5 red chips. If the coin comes up tail, a chip is drawn from urn-II which contains 7 white and 4 red chips. Given that a red chip was drawn, what is the probability that the coin came up head ?

21. An aircraft gun can take a maximum of four shots at an enemy's plane moving away from it. The probability of hitting the plane at first, second, third & fourth shots are 0.4, 0.3, 0.2 & 0.1 respectively. What is the probability that the gun hits the plane.

22. 2 hunters A & B shot at a bear simultaneously. The bear was shot dead with only one hole in its hide. Probability of A shooting the bear 0.8 & that of B shooting the bear is 0.4. The hide was sold for Rs. 280/-. If this sum of money is divided between A & B in a fair way, then find their respective shares.

23. 3 players A, B & C toss a coin cyclically in that order (that is A, B, C, A, B, C, A, B, ...) till a head shows. Let p be the probability that the coin shows a head. Let α, β & γ be respectively the probability that A, B and C gets the first head. Prove that $\beta = (1-p)\alpha$. Determine α, β & γ (in terms of p).

24. A person flips 4 fair coins and discards those which turn up tails. He again flips the remaining coin and then discards those which turn up tails. If $P = \frac{m}{n}$ (expressed in lowest form) denotes the probability that he discards atleast 3 coins, find the value of $(m + n)$.

25. An examination consists of 8 questions in each of which the candidate must say which one of the 5 alternatives is correct one. Assuming that the student has not prepared earlier chooses for each of the question any one of 5 answers with equal probability.

(A) prove that the probability that he gets more than one correct answer is $(5^8 - 3 \times 4^8) / 5^8$.

(B) find the probability that he gets correct answers to six or more questions.

Exercise # 5 Part # I [Previous Year Questions] [AIEEE/JEE-MAIN]

1. A problem in Mathematics is given to three students A, B, C and their respectively probability of solving the problem is $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$. Probability that the problem is solved is- **[AIEEE 2002]**

(1) $\frac{3}{4}$ (2) $\frac{1}{2}$ (3) $\frac{2}{3}$ (4) $\frac{1}{3}$

2. If A and B are events such that $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$, $P(\bar{A}) = \frac{2}{3}$, then $P(\bar{A} \cap B)$ is- **[AIEEE 2002]**

(1) $\frac{5}{12}$ (2) $\frac{3}{8}$ (3) $\frac{5}{8}$ (4) $\frac{1}{4}$

3. A die is tossed 5 times. Getting an odd number is considered a success. Then the variance of distribution of success is- **[AIEEE 2002]**

(1) $\frac{8}{3}$ (2) $\frac{3}{8}$ (3) $\frac{4}{5}$ (4) $\frac{5}{4}$

4. Five horses are in a race. Mr. A selects two of the horses at random and bets on them. The probability that Mr. A selected the winning horse is- **[AIEEE 2003]**

(1) $\frac{4}{5}$ (2) $\frac{3}{5}$ (3) $\frac{1}{5}$ (4) $\frac{2}{5}$

5. If $\frac{(1+3p)}{3}, \frac{(1-p)}{4}$ and $\frac{(1-2p)}{2}$ are the probabilities of three mutually exclusive events, then the set of all values of p is- **[AIEEE 2003]**

(1) $\frac{1}{3} \le p \le \frac{1}{2}$ (2) $\frac{1}{3} < p < \frac{1}{2}$ (3) $\frac{1}{2} \le p \le \frac{2}{3}$ (4) $\frac{1}{2} < p < \frac{2}{3}$

6. A speaks truth in 75% cases and B in 80% cases. What is the probability that they contradict each other in stating the same fact ? **[AIEEE 2004]**

(1) $\frac{7}{20}$ (2) $\frac{13}{20}$ (3) $\frac{3}{20}$ (4) $\frac{1}{5}$

7. A random variable X has the probability distribution :

X :	1	2	3	4	5	6	7	8
p(X) :	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the events E = {X is a prime number} and F = {X < 4}, the probability $P(E \cup F)$ is- **[AIEEE 2004]**

(1) 0.35 (2) 0.77 (3) 0.87 (4) 0.50

8. The mean and the variance of a binomial distribution are 4 and 2 respectively. Then the probability of 2 successes is- **[AIEEE 2004]**

(1) $\frac{128}{256}$ (2) $\frac{219}{256}$ (3) $\frac{37}{256}$ (4) $\frac{28}{256}$

9. Let A and B be two events such that $P\left(\overline{A \cup B}\right)=\frac{1}{6}$, $P(A \cap B)=\frac{1}{4}$ and $P(\overline{A})=\frac{1}{4}$, where $\overline{A}$ stands for complement of event A. Then events A and B are- **[AIEEE 2005]**

(1) mutually exclusive and independent **(2)** independent but not equally likely

(3) equally likely but not independent **(4)** equally likely and mutually exclusive

10. Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that all the three apply for the same house is- **[AIEEE 2005]**

(1) 7/9 **(2)** 8/9 **(3)** 1/9 **(4)** 2/9

11. Two aeroplanes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2, respectively. The second plane will bomb only if the first misses the target. the probability that the targert is hit by the second plane is :- **[AIEEE-2007]**

(1) 0.14 **(2)** 0.2 **(3)** 0.7 **(4)** 0.06

12. It is given that the events A and B are such that $P(A)=\frac{1}{4}$, $P(\frac{A}{B})=\frac{1}{2}$ and $P(\frac{B}{A})=\frac{2}{3}$. Then P(B) is **[AIEEE-2008]**

(1) $\frac{1}{6}$ **(2)** $\frac{1}{3}$ **(3)** $\frac{2}{3}$ **(4)** $\frac{1}{2}$

13. A die is thrown. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then $P\left(A \cup B\right)$ is **[AIEEE-2008]**

(1) $\frac{3}{5}$ **(2)** 0 **(3)** 1 **(4)** $\frac{2}{5}$

14. One ticket is selected at random from 50 tickets numbered 00, 01, 02, ..., 49. Then the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, equals **[AIEEE-2009]**

(1) 5/14 **(2)** 1/50 **(3)** 1/14 **(4)** 1/7

15. In a binomial distribution $B\left(n, p=\frac{1}{4}\right)$, if the probability of at least one success is greater than or equal to $\frac{9}{10}$, then n is greater than **[AIEEE-2009]**

(1) $\frac{9}{\log_{10}4-\log_{10}3}$ **(2)** $\frac{4}{\log_{10}4-\log_{10}3}$ **(3)** $\frac{1}{\log_{10}4-\log_{10}3}$ **(4)** $\frac{1}{\log_{10}4+\log_{10}3}$

16. An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have difference colours is **[AIEEE-2010]**

(1) $\frac{1}{3}$ **(2)** $\frac{2}{7}$ **(3)** $\frac{1}{21}$ **(4)** $\frac{2}{23}$

17. Four numbers are chosen at random (without replacement) from the set (1, 2, 3, ..., 20).

Statement–1 : The probability that the chosen numbers when arranged in some order will form an AP is $\frac{1}{85}$

Statement–2 : In the four chosen numbers form an AP, then the set of all possible values of common difference is $\{\pm1, \pm2, \pm3, \pm4, \pm5\}$. **[AIEEE-2010]**

(1) Statement–1 is true, Statement–2 is true; Statement–2 is a correct explanation for Statement–1.
(2) Statement–1 is true, Statement–2 is true; Statement–2 is not a correct explanation for statement–1.
(3) Statement–1 is true, Statement–2 is false.
(4) Statement–1 is false, Statement–2 is true.

18. If C and D are two events such that $C \subset D$ and P(D) ≠ 0, then the correct statement among the following is :- **[AIEEE-2011]**

(1) $P(C\,|\,D) < P(C)$ **(2)** $P(C\,|\,D) = \frac{P(D)}{P(C)}$ **(3)** $P(C|D) = P(C)$ **(4)** $P(C|D) \geq P(C)$

19. Consider 5 independent Bernoulli's trials each with probability of success p. If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then p lies in the interval **[AIEEE-2011]**

(1) $\left[0, \frac{1}{2}\right]$ **(2)** $\left(\frac{11}{12}, 1\right]$ **(3)** $\left(\frac{1}{2}, \frac{3}{4}\right]$ **(4)** $\left(\frac{3}{4}, \frac{11}{12}\right]$

20. Let A, B, C be pairwise independent events with P(C)> 0 and $P(A \cap B \cap C)=0$. Then $P(A^c \cap B^c|C)$ is equal to: **[AIEEE-2011]**

(1) $P(A^c) - P(B)$ **(2)** $P(A) - P(B^c)$ **(3)** $P(A^c) + P(B^c)$ **(4)** $P(A^c) - P(B^c)$

21. Three numbers are chosen at random without replacement from $\{1, 2, 3, ..., 8\}$. The probability that their minimum is 3, given that their maximum is 6, is : **[AIEEE-2012]**

(1) $\frac{2}{5}$ **(2)** $\frac{3}{8}$ **(3)** $\frac{1}{5}$ **(4)** $\frac{1}{4}$

22. A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is : **[JEE (Main)-2013]**

(1) $\frac{17}{3^5}$ **(2)** $\frac{13}{3^5}$ **(3)** $\frac{11}{3^5}$ **(4)** $\frac{10}{3^5}$

23. Let A and B be two events such that $P\left(\overline{A \cup B}\right) = \frac{1}{16}$, $P(A \cap B) = \frac{1}{4}$ and $P(\bar{A}) = \frac{1}{4}$, where $\bar{A}$ stands for the complement of the event A. Then the events A and B are : **[JEE (Main)-2014]**

(1) mutually exclusive and independent **(2)** equally likely but not independent.
(3) independent but not equally likely. **(4)** independent and equally likely.

24. If 12 identical balls are to be placed in 3 identical boxes, then the probability that one of the boxes contains exactly 3 balls is : **[JEE (Main)-2015]**

(1) $220\left(\frac{1}{3}\right)^{12}$ **(2)** $22\left(\frac{1}{3}\right)^{11}$ **(3)** $\frac{55}{3}\left(\frac{2}{3}\right)^{11}$ **(4)** $55\left(\frac{2}{3}\right)^{10}$

25. If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary ; then the position of the word SMALL is : **[JEE (Main)-2016]**

(1) 59^{th} **(2)** 52^{nd} **(3)** 58^{th} **(4)** 46^{th}

Part # II [Previous Year Questions][IIT-JEE ADVANCED]

1. A coin has probability 'p' of showing head when tossed. If is tossed 'n' times. Let p_n denote the probability that no two (or more) consecutive heads occur. Prove that , $p_1 = 1$, $p_2 = 1 - p^2$ & $p_n = (1-p)\,p_{n-1} + p\,(1-p)\,p_{n-2}$, for all $n \geq 3$. **[JEE 2000]**

2. **(A)** An urn contains 'm' white and 'n' black balls. A ball is drawn at random and is put back into the urn along with K additional balls of the same color as that of the ball drawn. A ball is again drawn at random. What is the probability that the ball drawn now is white. **[JEE 2001]**

(B) An unbiased die, with faces numbered 1, 2, 3, 4, 5, 6 is thrown n times and the list of n numbers showing up is noted. What is the probability that among the numbers 1, 2, 3, 4, 5, 6, only three numbers appear in the list. **[JEE 2001]**

3. A box contains N coins, m of which are fair and the rest are biased . The probability of getting a head when a fair coin is tossed is 1/2, while it is 2/3 when a biased coin is tossed. A coin is drawn from the box at random and is tossed twice. The first time it shows head and the second time it shows tail. What is the probability that the coin drawn is fair ? **[JEE 2002]**

4. **(A)** For a student to qualify , he must pass at least two out of three exams. The probability that he will pass the 1^{st} exam is p . If he fails in one of the exams then the probability of his passing in the next exam is $\frac{p}{2}$ otherwise it remains the same. Find the probability that he will qualify.

(B) A is targeting to B, B and C are targeting to A . Probability of hitting the target by A, B and C are $\frac{2}{3}, \frac{1}{2}, \frac{1}{3}$ respectively. If A is hit then find the probability that B hits the target and C does not. **[JEE 2003]**

5. **(A)** If A & B are two independent events, prove that $P(A \cup B).P(A' \cap B') \leq P(C)$, where C is an event defined as exactly one of A or (and) B occurs.

(B) A bag contains 12 red balls and 6 white balls. Six balls are drawn one by one without replacement of which atleast four balls are white. Find the probability that in the next two draws exactly one white ball is drawn. (Leave the answer in terms of ${}^nC_r$.)

(C) Three distinct numbers are selected from first hundred natural numbers, then probability of selected numbers divisible by both 2 and 3 is - **[JEE-2004]**

(A) $\frac{4}{25}$ **(B)** $\frac{4}{35}$ **(C)** $\frac{4}{55}$ **(D)** $\frac{4}{1155}$

6. **(A)** A fair dice is thrown until 1 comes, then probability that 1 comes in even number of trials is -

(A) 5/11 **(B)** 5/6 **(C)** 6/11 **(D)** 1/6

[JEE 2005]

(B) A person goes to office either by car, scooter, bus or train probability of which being $\frac{1}{7}, \frac{3}{7}, \frac{2}{7}$ and $\frac{1}{7}$ respectively. Probability that he reaches office late, if he takes car, scooter, bus or train is $\frac{2}{9}, \frac{1}{9}, \frac{4}{9}$ and $\frac{1}{9}$ respectively. Given that he reached office in time, then what is the probability that he travelled by a car.

[JEE 2005]

7. There are n urns each containing n + 1 balls such that the i^{th} urn contains i white balls and (n + 1 – i) red balls. Let u_i, be the event of selecting ith urn, i = 1, 2, 3,......,n and w denotes the event of getting a white ball

[JEE 2006]

(A) If $P(u_i) \propto i$, where i = 1, 2, 3,......,n, then $\lim\limits_{n\to\infty} P(w)$ is equal to -

(A) 1 **(B)** 2/3 **(C)** 3/4 **(D)** 1/4

(B) If $P(u_i) = c$ where c is a constant then $P(u_n/w)$ is equal to -

(A) $\frac{2}{n+1}$ **(B)** $\frac{1}{n+1}$ **(C)** $\frac{n}{n+1}$ **(D)** $\frac{1}{2}$

(C) If n is even and E denotes the event of choosing even numbered urn $\left(P(u_i) = \frac{1}{n}\right)$, then the value of P(w/E) is -

(A) $\frac{n+2}{2n+1}$ **(B)** $\frac{n+2}{2(n+1)}$ **(C)** $\frac{n}{n+1}$ **(D)** $\frac{1}{n+1}$

8. One Indian and four American men and their wives are to be seated randomly around a circular table. Then the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife is - **[JEE 2007]**

(A) $\frac{1}{2}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{2}{5}$ **(D)** $\frac{1}{5}$

9. Let E^c denote the complement of an event E. Let E,F,G be pairwise independent events with P(G) > 0 and $P(E \cap F \cap G) = 0$. Then $P(E^c \cap F^c|G)$ equals :- **[JEE 2007]**

(A) $P(E^c) + P(F^c)$ **(B)** $P(E^c) - P(F^c)$ **(C)** $P(E^c) - P(F)$ **(D)** $P(E) - P(F^c)$

10. Let $H_1, H_2, ..., H_n$ be mutually exclusive and exhaustive events with $P(H_i) > 0$, i = 1,2...,n. Let E be any other event with 0 < P (E) < 1.

Statement-I : $P(H_i|E) > P(E|H_i).P(H_i)$ for i = 1,2,...,n...

Statement-II : $\sum\limits_{i=1}^{n} P(H_i) = 1$ **[JEE 2007]**

(A) Statement-I is True, Statement-II is True ; Statement-II is a correct explanation for Statement-I.
(B) Statement-I is True, Statement-II is True ; Statement-II is NOT a correct explanation for Statement-I.
(C) Statement-I is True, Statement-II is False.
(D) Statement-I is False, Statement-II is True.

11. Consider the system of equations ax + by = 0, cx + dy = 0, where a, b, c, d $\in$ {0, 1}

Statement-I : The probability that the system of equations has a unique solution is $\frac{3}{8}$.

Statement-II : The probability that the system of equations has a solution is 1. **[JEE 2008]**

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.
(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.
(C) Statement-I is true, Statement-II is false.
(D) Statement-I is false, Statement-II is true.

12. An experiment has 10 equally likely outcomes. Let A and B be two non-empty events of the experiment. If A consists of 4 outcomes, the number of outcomes that B must have so that A and B are independent, is - **[JEE 2008]**

(A) 2, 4, or 8 **(B)** 3, 6 or 9 **(C)** 4 or 8 **(D)** 5 or 10

Comprehension : (Q.N. 13 to 15)

A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required.

13. The probability that X = 3 equals - **[JEE 2009]**

(A) $\frac{25}{216}$ **(B)** $\frac{25}{36}$ **(C)** $\frac{5}{36}$ **(D)** $\frac{125}{216}$

14. The probability that X $\geq$ 3 equals - **[JEE 2009]**

(A) $\frac{125}{216}$ **(B)** $\frac{25}{36}$ **(C)** $\frac{5}{36}$ **(D)** $\frac{25}{216}$

15. The conditional probability that X $\geq$ 6 given X > 3 equals - **[JEE 2009]**

(A) $\frac{125}{216}$ **(B)** $\frac{25}{216}$ **(C)** $\frac{5}{36}$ **(D)** $\frac{25}{36}$

16. Let ω be a complex cube root of unity with $\omega \neq 1$. A fair die is thrown three times. If r_1, r_2 and r_3 are the numbers obtained on the die, then the probability that $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$ is **[JEE 2010]**

(A) $\frac{1}{18}$ **(B)** $\frac{1}{9}$ **(C)** $\frac{2}{9}$ **(D)** $\frac{1}{36}$

17. A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is $\frac{3}{4}$. If the signal received at station B is green, then the probability that the original signal was green is - **[JEE 2010]**

(A) $\frac{3}{5}$ **(B)** $\frac{6}{7}$ **(C)** $\frac{20}{23}$ **(D)** $\frac{9}{20}$

Paragraph for Question 18 and 19

Let U_1 and U_2 be two urns such that U_1 contains 3 white and 2 red balls, and U_2 contains only 1 white ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from U_1 and put into U_2. However, if tail appears then 2 balls are drawn at random from U_1 and put into U_2. Now 1 ball is drawn at random from U_2.

18. The probability of the drawn ball from U_2 being white is -

(A) $\frac{13}{30}$ **(B)** $\frac{23}{30}$ **(C)** $\frac{19}{30}$ **(D)** $\frac{11}{30}$

19. Given that the drawn ball from U_2 is white, the probability that head appeared on the coin is -

(A) $\frac{17}{23}$ **(B)** $\frac{11}{23}$ **(C)** $\frac{15}{23}$ **(D)** $\frac{12}{23}$ **[JEE 2011]**

20. Let E and F be two independent events. The probability that exactly one of them occurs is $\frac{11}{25}$ and the probability of none of them occurring is $\frac{2}{25}$. If P(T) denotes the probability of occurrence of the event T, then - **[JEE 2011]**

(A) $P(E)=\frac{4}{5},\ P(F)=\frac{3}{5}$ **(B)** $P(E)=\frac{1}{5}, P(F)=\frac{2}{5}$

(C) $P(E)=\frac{2}{5}, P(F)=\frac{1}{5}$ **(D)** $P(E)=\frac{3}{5}, P(F)=\frac{4}{5}$

21. A ship is fitted with three engines E_1, E_2 and E_3. The engines function independently of each other with respective probabilities $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{4}$. For the ship to be operational at least two of its engines must function. Let X denote the event that the ship is operational and X_1, X_2, X_3 denotes respectively the events that the engines E_1, E_2 and E_3 are functioning. Which of the following is (are) true ? **[JEE 2012]**

(A) $P\left[X_1^c \mid X\right]=\frac{3}{16}$ **(B)** P[Exactly two engines of ship are functioning $\mid X]=\frac{7}{8}$

(C) $P[X \mid X_2]=\frac{5}{16}$ **(D)** $P[X \mid X_1]=\frac{7}{16}$

22. Four fair dice D_1, D_2, D_3 and D_4, each having six faces numbered 1, 2, 3, 4, 5 and 6 are rolled simultaneously. The probability that D_4 shows a number appearing on one of D_1, D_2 and D_3 is - **[JEE 2012]**

(A) $\frac{91}{216}$ **(B)** $\frac{108}{216}$ **(C)** $\frac{125}{216}$ **(D)** $\frac{127}{216}$

23. Let X and Y be two events such that $P(X \mid Y)=\frac{1}{2}, P(Y \mid X)=\frac{1}{3}$ and $P(X \cap Y)=\frac{1}{6}$. Which of the following is(are) correct ? **[JEE 2012]**

(A) $P(X \cup Y)=\frac{2}{3}$ **(B)** X and Y are independent

(C) X and Y are not independent **(D)** $P(X^C \cap Y)=\frac{1}{3}$

24. Four persons independently solve a certain problem correctly with probabilities $\frac{1}{2},\frac{3}{4},\frac{1}{4},\frac{1}{8}$. Then the probability that the problem is solved correctly by at least one of them is **[JEE Ad. 2013]**

(A) $\frac{235}{256}$ (B) $\frac{21}{256}$ (C) $\frac{3}{256}$ (D) $\frac{253}{256}$

25. Of the three independent events E_1, E_2 and E_3, the probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let the probability p that none of events E_1, E_2 or E_3 occurs satisfy the equations $(\alpha - 2\beta)\, p = \alpha\beta$ and $(\beta - 3\gamma)\, p = 2\beta\gamma$. All the given probabilities are assumed of lie in the interval (0,1).

Then $\frac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3} =$ **[JEE Ad. 2013]**

Paragraph for Question 26 and 27

A box B_1 contains 1 white ball, 3 red balls and 2 black balls. Another box B_2 contains 2 white balls, 3 red balls and 4 black balls. A third box B_3 contains 3 white balls, 4 red balls and 4 black balls.

26. If 2 balls are drawn (without replacement) from a randomly selected box and one of the balls is white and the other ball is red, the probability that these 2 balls are drawn from box B_2 is **[JEE Ad. 2013]**

(A) $\frac{116}{181}$ (B) $\frac{126}{181}$ (C) $\frac{65}{181}$ (D) $\frac{55}{181}$

27. If 1 ball is drawn from each of the boxes B_1, B_2 and B_3, the probability that all 3 drawn balls are of the same colour is

(A) $\frac{82}{648}$ (B) $\frac{90}{648}$ (C) $\frac{558}{648}$ (D) $\frac{566}{648}$

[JEE Ad.]

28. Three boys and two girls stand in a queue. The probability, that the number of boys ahead of every girl is at least one more than the number of girls ahead of her, is **[JEE Ad. 2014]**

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$

Comprehension

Box 1 contains three cards bearing numbers 1, 2, 3 ; box 2 contains five cards bearing numbers 1, 2, 3, 4, 5 ; and box 3 contains seven cards bearing numbers 1, 2, 3, 4, 5, 6, 7. A card is drawn from each of the boxes. Let x_i be the number on the card drawn from the i^{th} box, i = 1, 2, 3.

29. The probability that $x_1 + x_2 + x_3$ is odd, is **[JEE Ad. 2014]**

(A) $\frac{29}{105}$ (B) $\frac{53}{105}$ (C) $\frac{57}{105}$ (D) $\frac{1}{2}$

30. The probability that x_1, x_2, x_3 are in an arithmetic progression, is **[JEE Ad. 2014]**

(A) $\frac{9}{105}$ (B) $\frac{10}{105}$ (C) $\frac{11}{105}$ (D) $\frac{7}{105}$

31. The minimum number of times a fair coin needs to be tossed, so that the probability of getting at least two heads is at least 0.96 is. **[JEE Ad. 2015]**

Comprehension (32-33)

Let n_1 and n_2 be the number of red and black balls, respectively, in box I. Let n_3 and n_4 be the number of red and black balls, respectively, in box II.

32. One of the two boxes, box - I and box - II, was selected at random and a ball was drawn randomly out of this box. The ball was found to be red. If the probability that this red ball was drawn from box II is 1/3, then the correct option(s) with the possible values of n_1, n_2 and n_4 is (are) **[JEE Ad. 2015]**

(A) $n_1 = 3, n_2 = 3, n_3 = 5, n_4 = 15$ **(B)** $n_1 = 3, n_2 = 6, n_3 = 10, n_4 = 50$
(C) $n_1 = 8, n_2 = 6, n_3 = 5, n_4 = 20$ **(D)** $n_1 = 6, n_2 = 12, n_3 = 5, n_4 = 20$

33. A ball is drawn at random from box I and transffered to box. II. If the probability of drawing a red ball from box I, after this transfer, is 1/3, then the correct option(s) with the possible values of n_1 and n_2 is(are) **[JEE Ad. 2015]**

(A) $n_1 = 4$ and $n_2 = 6$ **(B)** $n_1 = 2$ and $n_2 = 3$
(C) $n_1 = 10$ and $n_2 = 20$ **(D)** $n_1 = 3$ and $n_2 = 6$

34. A computer producing factory has only two plants T_1 and T_2. Plant T_1 produces 20 % and plant T_2 produces 80 % of the total computers produced. 7 % of computers produced in the factory turn out to be defective. It is known that P (computer turn out to be defective given that it is produced in plant T_1) **[JEE Ad. 2016]**
= 10 P (computer turns out to be defective given that it is produced in plant T_2),
where P(E) denotes the probability of an event E. A computer produced in the factory is produced in plant T_2 is

(A) $\frac{36}{73}$ **(B)** $\frac{47}{79}$ **(C)** $\frac{78}{93}$ **(D)** $\frac{75}{83}$

Comprehension

Football teams T_1 and T_2 have to play two games against each other. It is assumed that the outcomes of the two games are independent. The probabilities of T_1 winning, drawing and losing a game against T_2 are $\frac{1}{2}, \frac{1}{6}$ and $\frac{1}{3}$, respectively. Each team gets 3 points for a win, 1 point for a draw and 0 point for loss in a game. Let X and Y denote the total points scored by teams T_1 and T_2 respectively, after two games.

35. $P(X > Y)$ is

(A) $\frac{1}{4}$ **(B)** $\frac{5}{12}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{7}{12}$

36. $P(X = Y)$ is

(A) $\frac{11}{36}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{13}{36}$ **(D)** $\frac{1}{2}$

MOCK TEST

SECTION - I : STRAIGHT OBJECTIVE TYPE

1. South African cricket captain lost the toss of a coin 13 times out of 14. The chance of this happening was

(A) $\frac{7}{2^{13}}$ **(B)** $\frac{1}{2^{13}}$ **(C)** $\frac{13}{2^{14}}$ **(D)** $\frac{13}{2^{13}}$

2. In a room there are 4 students each of which is equally likely to be a girl or a boy. 2 students have walked out from the room, first is found to be a boy and the second a girl. The probability that the remaining students are boys is

(A) $\frac{2}{7}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{3}{8}$

3. There are two urns. There are m white & n black balls in the first urn and p white & q black balls in the second urn. One ball is taken from the first urn & placed into the second. Now, the probability of drawing a white ball from the second urn is:

(A) $\frac{pm+(p+1)n}{(m+n)(p+q+1)}$ **(B)** $\frac{(p+1)m+pn}{(m+n)(p+q+1)}$ **(C)** $\frac{qm+(q+1)n}{(m+n)(p+q+1)}$ **(D)** $\frac{(q+1)m+qn}{(m+n)(p+q+1)}$

4. Let set 'A' has 7 elements and set B has 5 elements. If one function is selected from all possible defined functions from A to B then the probability that it is onto is

(A) $\frac{7!\times 2}{3\times 5^6}$ **(B)** $\frac{7!}{10\times 5^6}$ **(C)** $\frac{7!}{5^6}$ **(D)** $\frac{7!}{5^7}$

5. The probability that 4^{th} power of a positive integer ends in the digit 6 is:

(A) 10 % **(B)** 20 % **(C)** 25 % **(D)** 40 %

6. A bag contains 5 balls all of different colours (one of which is white) three persons A, B and C whose probabilities of speaking truth are $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{3}{4}$ respectively assert that a ball drawn from the bag is white, the probability of truth of their assertion, is

(A) $\frac{96}{97}$ **(B)** $\frac{24}{25}$ **(C)** $\frac{1}{20}$ **(D)** $\frac{6}{7}$

7. Let p be the probability that a man aged x years will die in a year time. The probability that out of 'n' men A_1, A_2, A_3,......, A_n each aged 'x' years. A_1 will die & will be the first to die is:

(A) $\frac{1-p^n}{n}$ **(B)** $\frac{p}{n}$ **(C)** $\frac{p\ (1-p)^{n-1}}{n}$ **(D)** $\frac{1-(1-p)^n}{n}$

8. Numbers are selected at random one at a time, from the numbers 00,01,02,...........,99 with replacement. An event E occurs if and only if the product of the digits of a selected number is 18. If four numbers are selected, the probability that the event E occurs 3 times is equal to :

(A) $\frac{37}{390625}$ **(B)** $\frac{24}{390625}$ **(C)** $\frac{96}{390625}$ **(D)** None of these

9. 5 girls and 10 boys sit at random in a row having 15 chairs numbered as 1 to 15, then the probability that end seats are occupied by the girls and between any two girls an odd number of boys sit is:

(A) $\frac{20\times10!\times5!}{15!}$ **(B)** $\frac{10\times10!\times5!}{15!}$ **(C)** $\frac{20\times10!\times30}{15!}$ **(D)** $\frac{10\times10!\times5!}{25!}$

10. **S_1 :** 5 letters are selected from the letters of word MISSISIPPI. The probability, that 3 of them are alike and other two are also alike but their kind is different from those of 3 alike, is $\frac{23}{{}^{10}C_5}$

S_2 : A submatrix of order 2×2 is randomly selected from a matrix of order 5×4. The probability that the rows of the selected matrix are adjacent rows of the parent matrix, is $\frac{3}{5}$

S_3 : A, B and C are three shooters. Their probabilities of hitting a target are $\frac{4}{5}, \frac{3}{4}$ and $\frac{2}{3}$ respectively. If all of them make an attempt on the target and it is found that two of them have missed out then the probability that A has hit the target is equal to $\frac{2}{5}$

(A) TFT **(B)** FFT **(C)** FTF **(D)** TTF

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

11. The probabilities of events, $A \cap B$, A, B & $A \cup B$ are respectively in A.P. with second term equal to the common difference. Therefore A & B are:

(A) mutually exclusive **(B)** independent

(C) such that one of them must occur **(D)** such that one is twice as likely as the other

12. Two real numbers, x & y are selected at random. Given that $0 \le x \le 1$; $0 \le y \le 1$. Let A be the event that $y^2 \le x$; B be the event that $x^2 \le y$, then:

(A) $P(A \cap B) = \frac{1}{3}$ **(B)** A & B are exhaustive events

(C) A & B are mutually exclusive **(D)** A & B are independent events.

13. In an experimental performance of a single throw of a pair of unbiased normal dice, three events E_1, E_2 & E_3 are defined as follows:

E_1: getting a prime numbered face on each dice

E_2: getting the same number on each dice

E_3: getting a sum total of dots on two dice equal to 8. Then:

(A) the events E_1, E_2 & E_3 are mutually exclusive

(B) the events E_1, E_2 & E_3 are not pairwise mutually exclusive

(C) the events E_1, E_2 are independent

(D) $P(E_3 | E_1) = 2/9$.

14. Cosider the cartesian plane R^2 and let X denote the subset of points for which both coordinates are integers. A coin of diameter $\frac{1}{2}$ is tossed randomly into the plane. The probability p that the coin covers a point of X satisfies :

(A) $P = \frac{\pi}{16}$ **(B)** $p < \frac{\pi}{3}$ **(C)** $p > \frac{\pi}{30}$ **(D)** $p = \frac{1}{4}$

15. Three numbers are chosen at random without replacement from {1, 2, 3,......, 10}. The probability that the minimum of the chosen numbers is 3 or their maximum is 7 is:

(A) $\frac{11}{40}$ **(B)** $\frac{9}{40}$ **(C)** $\frac{{}^{7}C_2 + {}^{6}C_2 - 3}{{}^{10}C_3}$ **(D)** none

SECTION - III : ASSERTION AND REASON TYPE

16. **Statement - I :** If P (A) > 0, then the event A is indepedent of itself if and only if P(A) is 1

Statement - II : Events A and B are said to be independent if and only if $P(A \cap B) = P(A).P(B)$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

17. **Statement - I :** $x + y + z = 5$, $xy + yz + zx = 3$ and $x + 2y \neq 5$ $(x, y, z \in R)$ then the probability for x is positive only is $\frac{13}{16}$.

Statement - II : If $x + y + z = 5$ and $xy + yz + zx = 3$ then maximum and minimum values of x, y and z are same.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

18. **Statement - I :** Two non-negative integers are chosen at random. The probability that the sum of their squares is divisible by 5 is 9/25.

Statement - II : If at the unit place in any number is zero then number is only divisible by 5.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

19. **Statement-I :** An integer is marked on each of 5 different tickets. The integers maked are out of 5 consecutive integers. If no two tickets bears equal numbers, then the probability that the numbers on three tickets at random are in A.P. is $\frac{2}{15}$.

Statement-II : An integer is marked on each of (2n + 1) different tickets. The integers maked are out of (2n + 1) consecutive integers. If no two tickets bears equal numbers, then the probability that the numbers on three tickets at random are in A.P. is $\frac{3n}{4n^2 - 1}$.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

20. **Statement-I :** In throwing of two dice, let the events A, B and C be 'the first dice shows an even number', 'the second dice shows an odd numbers' and 'both the dice show an odd numbers or both the dice show an even number' respectively. Then $P(A) = P(B) = P(C) = \frac{1}{2}$ and $P(A \cap B) = P(B \cap C) = P(C \cap A) = \frac{1}{4}$. Therefore A, B and C are mutually independent events

Statement-II : Three events A, B and C are mutually independent if and only if $P(A \cap B) = P(A) . P(B)$, $P(B \cap C) = P(B) . P(C)$, $P(C \cap A) = P(C) . P(A)$ and $P(A \cap B \cap C) = P(A) . P(B) . P(C)$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

SECTION - IV : MATRIX - MATCH TYPE

21. A bag contains some white and some black balls, all combinations being equally likely. The total number of balls in the bag is 12. Four balls are drawn at random from the bag without replacement. Now match the entries from the following two columns :

	Column - I		Column - II
(A)	Probability that all the four balls are black is equals to	**(p)**	$\frac{14}{33}$
(B)	If the bag contains 10 black and 2 white balls then the probability that all four balls are black is equal to	**(q)**	$\frac{1}{5}$
(C)	If all the four balls are black, then the probability, that the selected bag contains 10 black balls, is equal to	**(r)**	$\frac{70}{429}$
(D)	Probability that two balls are black and two are white	**(s)**	$\frac{13}{110}$
		(t)	$\frac{13}{165}$

22. **Column - I** | **Column - II**

	Column - I		Column - II
(A)	One ball is drawn from a bag containing 4 balls and is found to be white. The events that the bag contains "1 white", "2 white", "3 white" and "4 white" are equaly likely. If the probability that all the balls are white is $\frac{p}{15}$, then the value of p is	**(p)**	9
(B)	From a set of 12 persons, if the number of different selection of a committee, its chair person and its secretary (possibly same as chair person) is 13.2^{10} m, then value of m is	**(q)**	3
(C)	If x, y, z > 0 and x + y + z = 1, then the least value of $\frac{5x}{2-x}+\frac{5y}{2-y}+\frac{5z}{2-z}$ is	**(r)**	8
(D)	If $\sum_{k=1}^{12} 12k.\,{}^{12}C_k.{}^{11}C_{k-1}$ is equal to	**(s)**	6
	$\frac{12\times21\times19\times17\times...\times3}{11!}\times 2^{12}\times$ p, then the value of p is	**(t)**	12

SECTION - V : COMPREHENSION TYPE

23. **Read the following comprehension carefully and answer the questions.**

There are four boxes A_1, A_2, A_3 and A_4. Box A_i has i cards and on each card a number is printed, the numbers are from 1 to i. A box is selected randomly, the probability of selection of box A_i is $\frac{i}{10}$ and then a card is drawn. Let E_i represents the event that a card with number 'i' is drawn.

1. $P(E_1)$ is equal to

(A) $\frac{1}{5}$ **(B)** $\frac{1}{10}$ **(C)** $\frac{2}{5}$ **(D)** $\frac{1}{4}$

2. $P(A_3/E_2)$ is equal to

(A) $\frac{1}{4}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{3}$

3. Expectation of the number on the card is

(A) 2 **(B)** 2.5 **(C)** 3 **(D)** 3.5

24. Read the following comprehension carefully and answer the questions.

A player 'A' plays a game against a machine. At each round he deposits one rupee in a slot and then flips a coin which has a probability p of showing a head. If the flipped coin show head, he gets back the rupee he deposited and one more rupee from the machine, else he loses his rupee. Let A starts with 10 rupee coins and $q = 1 - p$ (the probability of showing a tail), then

1. The probability that he will be drained out with all of his rupee coins exactly at the eleventh round is -

(A) q^{11} **(B)** $1 - q^{11}$ **(C)** $pq^{10} + q^{11}$ **(D)** 0

2. The probability that all his money will be finished exactly at the twelth round is -

(A) q^{12} **(B)** $1 - q^{12}$ **(C)** ${}^{10}C_1\, pq^{11}$ **(D)** ${}^{12}C_2 p^2q^{10}$

3. The probability that he is left with no money by the 14^{th} round or earlier is -

(A) $q^{10}(1 + 10pq + 65p^2q^2)$ **(B)** $q^{14}(p^2q + 36pq + 7)$

(C) $q^{12} + 3pq^{13} + 3p^{13}q + p^{12}$ **(D)** $1 - {}^{10}C_1\, pq^{11} - {}^{10}C_2\, p^2q^{12}$

25. Read the following comprehension carefully and answer the questions.

Let $n = 10k + r$ when $k, r \in N$, $0 \le r \le 9$. A number a is chosen at random from the set $\{1, 2, ... n\}$ and let p_n denote the probability that $a^2 - 1$ is divisible by 10.

1. If $r = 0$, p_n equals

(A) $2k/n$ **(B)** $(k + 1)/n$ **(C)** $(2k + 1)/n$ **(D)** k / n

2. If $r = 9$, p_n equals

(A) $2k/n$ **(B)** $2(k + 1) / n$ **(C)** $(2k + 1)/n$ **(D)** k/n

3. If $1 \le r \le 8$, p_n equals

(A) $(2k - 1) / n$ **(B)** $(2k / n)$ **(C)** $(2k + 1) / n$ **(D)** k / n

SECTION - VI : INTEGER TYPE

26. A quadratic equation is chosen from the set of all quadratic equations which are unchanged by squaring their roots. Given that roots are real. Find the probability that the chosen equation has equal roots.

27. An urn contains 3 white balls, 5 black balls and 2 red balls. Two persons draw balls in turn, without replacement. The person who draw first a white ball wins the game. If a red ball is drawn, the game is a tie. Suppose A_1 = { the player who begins the game is the winner }, A_2 = { the second participant is the winner} & B = { the game is a tie }. If λ P(B) = 2, then find the value of λ.

28. A slip of paper is given to a person "A" who marks it with either a (+) ve or (–) ve sign, the probability of his writing a (+) ve sign being 1/3. "A" passes the slip to "B" who leave it alone or change the sign before passing it to "C". Similarly "C" passes on the slip to "D" and "D" passes on the slip to refree, who finds a plus sign on the slip. If it is known that B, C and D each change the sign with a probability of 2/3, then find the probability that "A" originally wrote a (+)ve sign.

29. A is a set containing n elements. A subset P of A is chosen at random. The set A is reconstructed by replacing the elements of the subset P. A subset Q of A is again chosen at random. The probability such that $P \cap Q$ contains 2 elements is $\frac{{}^{n}C_{a}.3^{n-b}}{4^{n}}$, then find a + b.

30. P makes a bet with Q of Rs. 8 to Rs. 120 that three races will be won by the three horses A, B, C against which the betting is 3 to 2, 4 to 1, and 2 to 1 respectively. The first race having been won by A, and it being known that the second race was won either by B, or by a horse D against which the betting was 2 to 1, find the value of P's expectation.

ANSWER KEY

EXERCISE - 1

1. A **2.** A **3.** A **4.** D **5.** D **6.** C **7.** D **8.** D **9.** A **10.** D **11.** A **12.** D **13.** A
14. A **15.** B **16.** B **17.** B **18.** A **19.** A **20.** B **21.** A **22.** D **23.** C **24.** C **25.** B **26.** D
27. C **28.** A **29.** A **30.** C

EXERCISE - 2 : PART # I

1. ABC **2.** ACD **3.** AC **4.** AB **5.** ABC **6.** ACD **7.** CD **8.** ABCD **9.** ABCD
10. CD **11.** ABC **12.** ABCD **13.** AB **14.** ACD **15.** AB **16.** ABC **17.** ABCD **18.** CD
19. AC **20.** AD **21.** ABC **22.** ABCD **23.** AB **24.** AD

PART - II

1. D **2.** B **3.** A **4.** A **5.** A **6.** B **7.** B **8.** D **9.** A **10.** B

EXERCISE - 3 : PART # I

1. **A** $\to$ q **B** $\to$ r **C** $\to$ p D $\to$ s **2.** **A** $\to$ s **B** $\to$ r **C** $\to$ p **3.** **A** $\to$ q **B** $\to$ r **C** $\to$ s D $\to$ r
4. **A** $\to$ q **B** $\to$ r **C** $\to$ s **5.** **A** $\to$ s **B** $\to$ r **C** $\to$ q D $\to$ p **6.** **A** $\to$ p **B** $\to$ q **C** $\to$ a D $\to$ r

PART - II

Comprehension # 1 : **1.** B **2.** A **3.** C
Comprehension # 2 : **1.** A **2.** C **3.** D
Comprehension # 3 : **1.** A **2.** B **3.** B
Comprehension # 4 : **1.** C **2.** B **3.** D
Comprehension # 5 : **1.** B **2.** A **3.** D

EXERCISE - 5 : PART # I

1. 1 **2.** 1 **3.** 4 **4.** 4 **5.** 1 **6.** 1 **7.** 2 **8.** 4 **9.** 2 **10.** 3 **11.** 1 **12.** 2 **13.** 3
14. 3 **15.** 3 **16.** 2 **17.** 3 **18.** 4 **19.** 1 **20.** 1 **21.** 3 **22.** 3 **23.** 3 **24.** 3 **25.** 3

PART - II

2. **(A)** $\frac{m}{m+n}$ **(B)** $\frac{{}^{6}C_{3}(3^{n}-3.2^{n}+3)}{6^{n}}$ **3.** $\frac{9m}{m+8N}$ **4.** **(A)** $2p^{2}-p^{3}$ **(B)** 1/2

5. **(B)** $\frac{{}^{10}C_{1}.\,{}^{2}C_{1}.\,{}^{6}C_{4}.\,{}^{12}C_{2}+{}^{11}C_{1}.\,{}^{1}C_{1}.\,{}^{6}C_{5}.\,{}^{12}C_{1}}{{}^{12}C_{2}\left[{}^{6}C_{4}.\,{}^{12}C_{2}+{}^{6}C_{5}.{}^{12}C_{1}+{}^{6}C_{6}.\,{}^{12}C_{0}\right]}$ **(C)** D **6.** **(A)** A **(B)** $\frac{1}{7}$

7. **(A)** B **(B)** A **(C)** B

8. C **9.** C **10.** D **11.** B **12.** D **13.** A **14.** B **15.** D **16.** C **17.** C **18.** B **19.** D **20.** AD
21. BD **22.** A **23.** AB **24.** A **25.** 6 **26.** D **27.** A **28.** A **29.** B **30.** C **31.** 8 **32.** AB **33.** CD
34. C **35.** B **36.** C

MOCK TEST

1. A **2.** B **3.** B **4.** A **5.** D **6.** A **7.** D **8.** C **9.** A
10. A **11.** AD **12.** AB **13.** ABD **14.** ABC **15.** AC **16.** A **17.** B **18.** C
19. D **20.** D
21. $A \to q$ $B \to p$ $C \to r$ $D \to q$ **22.** $A \to s$ $B \to t$ $C \to q$ $D \to s$
23. **1.** C **2.** B **3.** A **24.** **1.** D **2.** C **3.** A **25.** **1.** A **2.** B **3.** C
26. $\frac{1}{2}$ **27.** 5 **28.** $\frac{13}{41}$ **29.** 4 **30.** Rs. 8