HINTS & SOLUTIONS

EXERCISE - 1
Single Choice

2. There are 6 possible arrangements of a, b, c and only one of them is in alphabetical order.

Alternatively $\frac{{}^6C_3(\text{for } a,b,c)\cdot 3!}{6!} = \frac{1}{6}$

3. Let the roots of the quadratic equation be α, β

After squaring α^2, β^2

$\alpha\beta = (\alpha\beta)^2 \Rightarrow \alpha\beta(\alpha\beta - 1) = 0$

$\Rightarrow \alpha\beta = 0$ ..... **(i)**

$\Rightarrow \alpha\beta = 1$ ..... **(ii)**

Now $\alpha^2 + \beta^2 = \alpha + \beta$

$(\alpha+\beta)^2 - 2\alpha\beta = (\alpha+\beta)$

$\Rightarrow (\alpha+\beta)^2 - (\alpha+\beta) - 2\alpha\beta = 0$

$(\alpha+\beta)\{(\alpha+\beta) - 1\} = 0$ $(\because \alpha\beta = 0)$

$\alpha + \beta = 0$ ... (3)

$\alpha + \beta = 1$ ... (4)

solving (1) & (3)

$\alpha = 0, \beta = 0$

solving (1) & (4)

$\alpha(1-\alpha) = 0 \Rightarrow \alpha = 0, 1$

$\Rightarrow \beta = 1, 0$

solving (2) & (4)

$\alpha + \frac{1}{\alpha} = 1$

$\alpha^2 - \alpha + 1 = 0$

$(\alpha, \beta) \in (\omega, \omega^2)$

Hence sample space $\rightarrow (0, 0)\ (1, 1)\ (0, 1)\ (\omega, \omega^2)$

$\therefore P(A) = \frac{2}{4} = \frac{1}{2}$

6. $\left\langle \begin{matrix} WW \\ RRR \end{matrix} \right.$

$S = \{WW \text{ or } \underbrace{RWW}_{WRW} \text{ or } \underbrace{RRWW}_{\substack{WRRW \\ RWRW}}\}$

P(last drawn ball is white) $= \frac{2}{5}\cdot\frac{1}{4} + (2)\left(\frac{3}{5}\cdot\frac{2}{4}\cdot\frac{1}{3}\right) +$

$(3)\left(\frac{3}{5}\cdot\frac{2}{4}\cdot\frac{2}{3}\cdot\frac{1}{2}\right) = \frac{1}{10} + \frac{1}{5} + \frac{3}{10} = \frac{6}{10}$

7. $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc < 0$

P(E) $\rightarrow$ when determinant value is negative

a	d	b	c
0	0	1	1
0	1	1	1
1	0	1	1

$\therefore$ Probability will be $\rightarrow 1 - \frac{3}{16} = \frac{13}{16}$

8. $P(H) = p;\ P(T) = 1 - p$

A wins if A throws a Tail before B tosses a Head

$P(A) = P(T \text{ or } HTT \text{ or } HTHTT \text{ or } ......)$

[HTT $\Rightarrow$ A throws Head and B throws Tail and again A throws a Tail]

$\therefore \frac{1-p}{1-p(1-p)} = \frac{1}{2} \Rightarrow 1 - p + p^2 = 2 - 2p$

$\Rightarrow p^2 + p - 1 = 0$

$\Rightarrow p = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{\sqrt{5}-1}{2}$

10. Determinant = ad – bc

probability that randomly chosen product (xy) will be odd $= P(\text{both odd}) = p^2$

$\therefore$ Probability (xy) is even $= 1 - p^2$

Now (ad – bc) is even

$\Rightarrow$ both odd or both even $= p^4 + (1 - p^2)^2$

Hence $p^4 + (1-p^2)^2 = \frac{1}{2}$

put $p^2 = t;\ t^2 + (1-t)^2 = \frac{1}{2} \Rightarrow 2t^2 - 2t + \frac{1}{2} = 0$

$\Rightarrow 4t^2 - 4t + 1 = 0$

$(2t-1)^2 = 0 \Rightarrow t = \frac{1}{2};\quad \therefore p^2 = \frac{1}{2};$

Hence $p = \frac{1}{\sqrt{2}}$

13. letters Digits

$26 \times 26 \times 1 \quad 10 \times 10 \times 1$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{26\times 26\times 1}{(26)^3} + \frac{10\times 10\times 1}{(10)^3} - \frac{26\times 26\times 1}{(26)^3} . \frac{10\times 10\times 1}{(10)^3}$$

$$= \frac{1}{26} + \frac{1}{10} - \frac{1}{260} = \frac{7}{52}$$

14. We have $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\therefore \quad \frac{2}{3} = \frac{1}{2} + P(B) - \frac{1}{2}\, P(B) = \frac{1}{2} + \frac{1}{2} P(B)$$

$$P(B) = \frac{4}{3} - 1 = \frac{1}{3} = p_1$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)P(B)}{P(B)} = P(A) = \frac{1}{2} = p_2$$

$$\text{Again } P(B^c/A) = \frac{P(B^c \cap A)}{P(A)} = \frac{(1 - P(B))P(A)}{P(A)}$$

$$= 1 - \frac{1}{3} = \frac{2}{3} = p_3$$

$\frac{1}{3}, \frac{1}{2}, \frac{2}{3}$ are in A.P.

18. $n(S) = \underbrace{713\times\times\times}\; = 3\left(\frac{4!}{(2!)(1!)(1!)}\right) = 36$

$n(A) = 1; \; p = \frac{1}{36}$

odds in favour 1 : 35

19. H: tossing a Head, $P(H) = \frac{1}{2}$; A : event of tossing a 2 with die, $P(A) = \frac{1}{6}$

E: tossing a 2 before tossing a head

$$P(E) = P\left(\bar{H} \cap A \text{ or } \{(\bar{H} \cap \bar{A}) \text{ and } (\bar{H} \cap A)\} \text{ or } \ldots\ldots\right)$$

$$= \left(\frac{1}{2}\cdot\frac{1}{6}\right) + \left(\frac{1}{2}\cdot\frac{5}{6}\right)\cdot\left(\frac{1}{2}\cdot\frac{1}{6}\right) + \ldots\ldots$$

$$= \frac{1}{12} + \frac{5}{12}\cdot\frac{1}{12} + \ldots\ldots \infty$$

$$P(E) = \frac{\frac{1}{12}}{1 - \frac{5}{12}} = \frac{1}{7}$$

21. $n(S) = 216$

x_1: number appearing on first dice.

x_2: number appearing on second dice.

x_3: number appearing on third dice.

$x_1 + x_2 + x_3 \le 10 \quad (x_1, x_2, x_3 \in [1, 6])$

$\Rightarrow \quad x_1 + x_2 + x_3 \le 7$ (after giving 1 each to x_1, x_2, x_3)

$x_1 + x_2 + x_3 + X = 7$ (adding X as a false beggar)

Total number of solutions ${}^{(7+3)}C_3 = {}^{10}C_3 = 120$

Now, number of solutions when any one of x_1, x_2, x_3 takes the value 7 is $x_1 + x_2 + x_3 + X = 1$

$\Rightarrow \quad {}^{(1+3)}C_3 = {}^4C_3 = 4$

$\therefore$ total number of ways are ${}^{10}C_3 - {}^4C_3 \times {}^3C_1$

$= 120 - 12 = 108$

$\therefore$ required probability is $\frac{108}{216} = \frac{1}{2}$

24. Given $P(A) = \frac{1}{16}$; $P(B) = \frac{1}{16}$; $P(C) = \frac{2}{16}$;

$$P(D) = \frac{3}{16};\; P(e) = \frac{4}{16};\; P(f) = \frac{5}{16};$$

$$P(a, c, e) = P(A) = \frac{1}{16} + \frac{2}{16} + \frac{4}{16} = \frac{7}{16} \;;\; A^c = \{b, d, f\}$$

$$P(c, d, e, f) = P(B) = \frac{2}{16} + \frac{3}{16} + \frac{4}{16} + \frac{5}{16} = \frac{14}{16}$$

$$P(b, c, f) = P(C) = \frac{1}{16} + \frac{2}{16} + \frac{5}{16} = \frac{8}{16}$$

$$p_1 = P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(c,e)}{P(B)} = \frac{6}{14} = \frac{3}{7}$$

$$p_2 = P(B/C) = \frac{P(B \cap C)}{P(C)} = \frac{P(c,f)}{P(C)} = \frac{7}{8} = \frac{7}{8}$$

$$p_3 = P(C/A^c) = \frac{P(C \cap A^c)}{P(A^c)} = \frac{P(b,f)}{P(b,d,f)} = \frac{6}{9} = \frac{2}{3}$$

$$p_4 = P(A^c/C) = \frac{P(A^c \cap C)}{P(C)} = \frac{P(b,f)}{P(b,c,f)} = \frac{6}{8} = \frac{3}{4}$$

Hence $p_1 = \frac{72}{168}$; $p_2 = \frac{147}{168}$; $p_3 = \frac{112}{168}$; $p_4 = \frac{126}{168}$;

$p_1 < p_3 < p_4 < p_2 \Rightarrow$ **(C)**

25. Probability that 3 people out of 7 born on

Wednesday $= \dfrac{^7C_3}{7^3}$

Probability that 2 people out of remaining 4, born on

Thursday is $\dfrac{^4C_2}{7^2}$

Probability of remaining 2 born on Sunday is $\dfrac{^2C_2}{7^2}$

$\therefore$ required probability $= \dfrac{^7C_3}{7^3} \times \dfrac{^4C_2}{7^2} \times \dfrac{^2C_2}{7^2} = \dfrac{K}{7^6}$

$\Rightarrow$ $K = 30$

27. Events are defined as

E_1 = A rigged die is chosen

E_2 = A fair die is chosen

A = die shows 5 in all the three times

using Baye's Theorem :

$$P(E_1/A) = \frac{P(E_1)\ P(A/E_1)}{P(E_1)\ P(A/E_1) + P(E_2)\ P(A/E_2)}$$

$$= \frac{\frac{1}{4}\times(1)^3}{\frac{1}{4}\times(1)^3 + \frac{3}{4}\times\left(\frac{1}{6}\right)^3} = \frac{216}{219}$$

28. 'a' can take only one value i.e. 2 **Note:** absolute

'b' can be 1 or 3 i.e. two values

'c' can be 2 or 4 i.e. two values

and 'd' can take only one value i.e. 5

hence total favourable ways = 1 × 2 × 2 = 4

$n(S) = 6^4 = 1296$

$P(E) = \dfrac{4}{1296} = \dfrac{1}{324}$

EXERCISE - 2

Part # I : Multiple Choice

5. E_1 : Both even or both odd

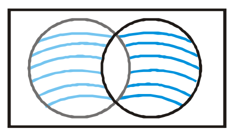
$$P(E_1) = \frac{^5C_2 + {}^6C_2}{^{11}C_2} = \frac{10+15}{55} = \frac{5}{11}$$

$\Rightarrow$ $P(E_2) = 1 - P(E_1) = \dfrac{6}{11}$

(i) $P(E_1/E_2) = \dfrac{P(E_1 \cap E_2)}{P(E_2)} = 0$

$\Rightarrow$ $P(E_2/E_1) = \dfrac{P(E_1 \cap E_2)}{P(E_1)} = 0$

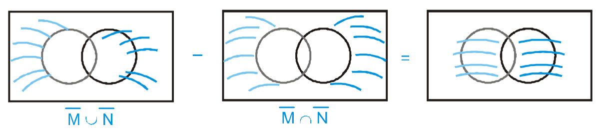
(ii) E_1 and E_2 exhaustive **(iii)** $P(E_2) > P(E_1)$

6. 

$P(M) + P(N) - 2P(M \cap N)$ **(A)**

$P(\bar{M}) + P(\bar{N}) - 2P(\bar{M} \cap \bar{N})$ **(C)**

$P(\bar{M} \cup \bar{N}) + P(\bar{M} \cup \bar{N})$ **(D)**



8. $P(B) \neq 1$

(A) $P\left(\dfrac{A}{B^C}\right) = \dfrac{P(AB^C)}{P(B^C)} = \dfrac{P(A) - P(A \cap B)}{1 - P(B)}$

(B) $1 \geq P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) \geq P(A) + P(B) - 1$

(D) $P\left(\dfrac{A}{B^C}\right) + \dfrac{P(A^C)}{P(B^C)}$

$$= \frac{P(A) - P(A \cap B)}{1 - P(B)} + \frac{P(A^C) - P(A^C \cap B)}{1 - P(B)}$$

$$= \frac{P(A) - P(A \cap B) + 1 - P(A) - P(B) + P(A \cap B)}{1 - P(B)} = 1$$

9. $P(A) + P(B) - P(A \cap B) = 0.8$

$\therefore$ $P(A \cap B) = 0.5 + 0.4 - 0.8 = 0.1$

$P(\bar{A}\cap B) = P(B) - P(A\cap B) = 0.5 - 0.1 = 0.4$

$\Rightarrow$ **(A)** is correct

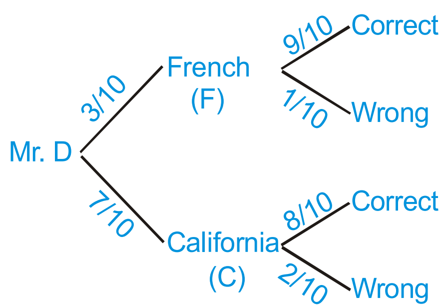
$$P(B/\bar{A}) = \frac{P(B\cap\bar{A})}{P(\bar{A})} = \frac{0.4}{1-P(A)} = \frac{0.4}{0.6} = \frac{2}{3}$$

$\Rightarrow$ **(B)** is correct

$P(A)\cdot P(B) = 0.2 \Rightarrow P(A\cap B) < P(A)\cdot P(B)$

$\Rightarrow$ **(C)** is correct

$P(\text{A or B but not both}) = 0.9 - 2\times 0.1 = 0.7$

10. 

$$P\left(\frac{C}{F}\right) = \frac{\frac{7}{10}\times\frac{2}{10}}{\frac{7}{10}\times\frac{2}{10}\times\frac{3}{10}\times\frac{9}{10}} = \frac{14}{41}$$

13. $$(p+q)^{99}\ r \le \frac{99+1}{1+\left|\frac{1/2}{1/2}\right|}$$

$$\Rightarrow r \le \frac{100}{2} \Rightarrow r \le 50$$

Terms 49 or 50 are highest

14. After removing face cards & tens remaning cards

$= 52 - 16 = 36$

$$P(A) = \frac{4}{36},\quad P(H) = \frac{9}{36},\ P(S) = \frac{9}{36}$$

$$P(A\cap S) = \frac{1}{36}$$

$$P(A\cap H) = \frac{1}{36}$$

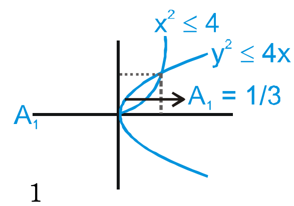
$$P(A\cup S) = \frac{12}{36}$$

15. $0 \le x \le 1, 0 \le y \le 1$

Let A be the event $y^2 \le x$

B the event $x^2 \le y$

total area = 1



$$P(A\cap B) = \frac{\text{Shaded area}}{\text{Total area}} = \frac{1}{3}$$

16. $E^c = F$; $E\cap E^c = \phi$; $P(E) + P(\text{not } E) = 1$; E and E^c can always be equally likely

17. $$P(E_0) = \frac{3!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}\right)}{3!} = \frac{1}{3} \Rightarrow P(E_1)$$

$$= \frac{{}^3C_1(1).\,2!\left(1-\frac{1}{1!}+\frac{1}{2!}\right)}{3!} = \frac{1}{2}$$

$$P(E_2) = \frac{{}^3C_2(1)^2.\,1!\left(1-\frac{1}{1!}\right)}{3!} = 0 \qquad \Rightarrow P(E_3)$$

$$= \frac{{}^3C_3(1)^3}{3!} = \frac{1}{6}$$

So $P(E_0) + P(E_3) = P(E_1)$

$P(E_0).P(E_1) = P(E_3)$ $\quad\because E_0\cap E_1 = \phi$

So $P(E_0\cap E_1) = 0$ $\quad$ So $P(E_0\cap E_1) = P(E_2)$

20.

$$\left.\begin{aligned} p(x=4) &= {}^nc_4\left(\frac{1}{2}\right)^n \\ p(x=5) &= {}^nc_5\left(\frac{1}{2}\right)^n \\ p(x=6) &= {}^nc_6\left(\frac{1}{2}\right)^n \end{aligned}\right]$$

$2\ {}^nc_5 = {}^nc_4 + {}^nc_6$

$4\ {}^nc_5 = {}^{n+1}c_5 + {}^{n+1}c_6$

$4\ {}^nc_5 = {}^{n+2}c_6 + {}^nc_6$

$$4.\frac{n!}{5!(n-5)!} = \frac{(n+2)!}{6!(n-4)!} \Rightarrow 4 = \frac{(n+2)(n+1)}{6(n-4)}$$

$\Rightarrow 24(n-4) = (n+2)(b+1)$

$n = 7, 14$

21. P(A) : denotes passing in A
P(B) : denotes passing in B
P(C) : denotes passing in C

$P(A) = p, P(B) = q, P(C) = \frac{1}{2}$

probability that student is successful $= \frac{1}{2}$

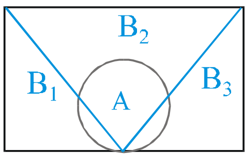
$= p.\frac{q}{2} + p.\frac{1}{2}.\frac{1}{2} = \frac{1}{2}$

$2pq + p = 2$

Now check options

23. B_1 = Journey by car
B_2 = Journey by motor cycle
B_3 = Journey on foot

$P(B_1) = \frac{1}{2}$; $P(B_2) = \frac{1}{6}$; $P(B_3) = \frac{1}{3}$



Let A = accident occurs

$P(A/B_1) = \frac{1}{5}$; $P(A/B_2) = \frac{2}{5}$; $P(A/B_3) = \frac{1}{10}$

$P(A) = P(A \cap B_1) + P(A \cap B_2) + P(A \cap B_3)$

$= P(B_1) \cdot P(A/B_1) + P(B_2) \cdot P(A/B_2) + P(B_3) \cdot P(A/B_3)$

$= \frac{1}{2}\cdot\frac{1}{5} + \frac{1}{6}\cdot\frac{2}{5} + \frac{1}{3}\cdot\frac{1}{10} \Rightarrow = \frac{3}{30} + \frac{2}{30} + \frac{1}{30} = \frac{6}{30} = \frac{1}{5}$

Ans. $\Rightarrow$ **(A) is correct**

$$P(B_1/A) = \frac{\frac{1}{2}\cdot\frac{1}{5}}{\frac{1}{5}} = \frac{1}{10}\cdot\frac{5}{1} = \frac{1}{2}$$

Ans. $\Rightarrow$ **(B) is correct**

$$P(B_2/A) = \frac{\frac{1}{6}\cdot\frac{2}{5}}{\frac{1}{5}} = \frac{2}{30}\cdot\frac{5}{1} = \frac{1}{3}$$

Ans. $\Rightarrow$ **(C) is incorrect**

$$P(B_3/A) = \frac{\frac{1}{3}\cdot\frac{1}{10}}{\frac{1}{5}} = \frac{1}{30}\cdot\frac{5}{1} = \frac{1}{6}$$

Ans. $\Rightarrow$ **(D) is incorrect**

Part # II : Assertion & Reason

1. **Statement-I :**

$$P\left(\frac{A \cap \bar{B}}{C}\right) = \frac{P((A \cap \bar{B}) \cap C)}{P(C)}$$

$$= \frac{P(A \cap C) - P((A \cap B) \cap C)}{P(C)}$$

Statement-II :

$P(A \cap \bar{B}) = P(A) - P(A \cap B)$

$\{\because A \cap \bar{B} = A - (A \cap B)\}$

2. Mean $= \mu = np = 16\left(\frac{1}{4}\right) = 4$

and variance $= \sigma^2 = npq = 16\left(\frac{1}{4}\right)\left(\frac{3}{4}\right) = 3$

3. **(A)** Here, n(s) = 1 length of the interval [0, 5] = 5; n(E) = length of the interval $\le$ [0, 5] in which P belongs such that the given equation has real roots.

Now $x^2 + Px + \frac{1}{4}(P + 2) = 0$ will have real roots

if $P^2 - 4.1.\frac{1}{4}(P + 2) \ge 0 \Rightarrow P^2 - P - 2 \ge 0$

$\Rightarrow (P + 1)(P - 2) \ge 0 \quad \Rightarrow P \le -1$ or $P \ge 2$

But $P \in [0, 5]$. So, E = [2, 5]

$\therefore$ n(E) = length of the interval [2, 5] = 3

$\therefore$ Required Probability $= \frac{3}{5}$

4. We have $f'(x) = 3x^2 - 2ax + b$

Now $y = f(x)$ is increasing

$\Rightarrow f'(x) \ge 0 \ \forall\ x$ and for $f'(x) = 0$ should not form an interval.

$\Rightarrow (2a)^2 - 4 \times 3 \times b \le 0 \quad \Rightarrow a^2 - 3b \le 0$

This is possible for exactly 16 ordered pairs

(a, b), $1 \le a, b \le 6$ namely

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6) & (4, 6)

Thus, required probability $= \frac{16}{36} = \frac{4}{9}$

7. If p_1, p_2, p_3, p_4 are the probabilities of success in a single throw for A, B, C and D then

$$P(A) = p_1 = \frac{1}{2}$$

$$P(B) = p_2 = \frac{1}{8} + {}^3C_1\left(\frac{1}{8}\right)$$

(All even + Exactly one even)

$$P(C) = p_3 = \frac{1}{2^5} + \frac{{}^5C_1}{2^5} + \frac{{}^5C_2}{2^5} = \frac{16}{2^5} = \frac{1}{2}$$

[(i) All even; (ii) one even & 4 odd
(iii) 3 even and 2 odd]

$$P(D) = p_4 = \underbrace{\frac{1}{2^7}}_{\text{all even}} + \underbrace{\frac{{}^7C_1}{2^7}}_{\text{exactly one even}} + \underbrace{\frac{{}^7C_3}{2^7}}_{\text{exactly 3 even}} + \underbrace{\frac{{}^7C_5}{2^7}}_{\text{exactly 5 even}}$$

$$= \frac{1+7+35+21}{2^7} = \frac{64}{2^7} = \frac{1}{2}$$

Hence probability of success is same for all in the single throw.

All equiprobable to win.

If they thow is succession i.e. A, B, C and D then

$$P(A) = P(S \text{ or } F\ F\ F\ F\ S \text{ or } \ldots\ldots\ldots)$$

$$= \frac{P(S)}{1-(P(F))^4} = \frac{1}{2}\cdot\frac{1}{1-\frac{1}{16}} = \left(\frac{1}{2}\right)\left(\frac{16}{15}\right) = \frac{8}{15}$$

$$P(B) = \frac{4}{15}; P(C) = \frac{2}{15}; P(D) = \frac{1}{15}$$

Hence both the statements are correct and S-2 is not the correct explanation.

10. We must have

$$0 \le \frac{1+4P}{4} \le 1, \quad 0 \le \frac{1-P}{4} \le 1 \text{ and } 0 \le \frac{1-2P}{4} \le 1$$

$$\Rightarrow -\frac{1}{4} \le P \le \frac{3}{4}, -3 \le P \le 1, -\frac{3}{2} \le P \le \frac{1}{2}$$

Again the events are pair-wise mutually exclusive so

$$0 \le \frac{1+4P}{4} + \frac{1-P}{4} + \frac{1-2P}{4} \le 1$$

$$\Rightarrow -3 \le P \le 1$$

Taking intersection of all four intervals of ‘P’

We get $-\frac{1}{4} \le P \le \frac{1}{2}$

EXERCISE - 3

Part # I : Matrix Match Type

1. **(B)** Selecting two box out of 5 which remain empty = ${}^5C_2$

Favourable ways = ${}^5C_2\,(3^5 - 3(2^5 - 2) - 3)$

Total ways = 5^5

$$\text{probability} = \frac{{}^5C_2(3^5 - 3(2^5 - 2) - 3)}{5^5} = \frac{12}{25}$$

(C) Let P(A) be the probability that the selected letters came from London

P(B) be the probability that the selected letters came from clifton

P(E) denotes the probability that ON is legible

$$P(A) = \frac{2}{5},\quad P(B) = \frac{1}{6}$$

$$P\left(\frac{A}{E}\right) = \frac{P(A).P\left(\frac{E}{A}\right)}{P(E)} = \frac{\frac{1}{2}.\frac{2}{5}}{\frac{1}{2}\left(\frac{2}{5}+\frac{1}{6}\right)} = \frac{12}{17}$$

2. **(A)** P(R) = P(T ; HTT ; HTHTT ; HHTTT) $\Rightarrow$ 11/16

(B) $P(A\cup B) = 0.6$; $P(A\cap B) = 0.2$ [JEE '87, 2]

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$\because$ $P(A) + P(B) = 0.6 + 0.2 = 0.8$

$P(\overline{A}) + P(\overline{B}) = 2 - 0.8 = 1.2 \Rightarrow$ (R)

(C) $P(X) = \frac{2}{3}$; $P(Y) = \frac{3}{4}$; $P(Z) = p$

E : exactly two bullets hit

$P(E) = P(XY\overline{Z}) + P(YZ\overline{X}) + P(ZX\overline{Y})$

$$\frac{11}{24} = \frac{2}{3}\cdot\frac{3}{4}(1-p) + \frac{3}{4}\cdot\frac{1}{3}p + p\frac{2}{3}\cdot\frac{1}{4} \Rightarrow p = \frac{1}{2}$$

3. **(A)** Even integers ends in 0, 2, 4, 6, 8. Square of an even integer ends in 4 only when the integer ends either in 2 or 8.

$\therefore$ probability = $\frac{2}{5}$

(B) $P(A\cap B) = \frac{1}{6} \Rightarrow P(A).P(B) = \frac{1}{6}$

$P(\overline{A}) = \frac{2}{3} \Rightarrow P(A) = \frac{1}{3} \Rightarrow P(B) = \frac{1}{2}$

$\therefore$ $6P\left(\frac{B}{\overline{A}}\right) = 6P(B) = 3$

(C) Total number of mapping = n^n.

Number of one-one mapping = ${}^nC_1\,.\,{}^{n-1}C_1 \,........\, {}^1C_1 = n!$

Hence the probability = $\frac{n!}{n^n} = \frac{3}{32} = \frac{4!}{4^4}$

Comparing, we get n = 4.

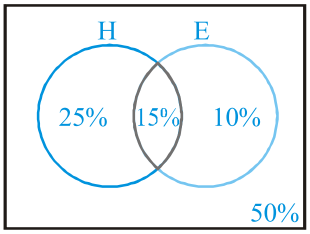
(D) $625p^2 - 175p + 12 < 0$ gives $p \in \left(\frac{3}{25}, \frac{4}{25}\right)$

$$\left(\frac{4}{5}\right)^{n-1}.\frac{1}{5} = p$$

$$\therefore\ \frac{3}{25} < \left(\frac{4}{5}\right)^{n-1}.\frac{1}{5} < \frac{4}{25} \text{ i.e. } \frac{3}{5} < \left(\frac{4}{5}\right)^{n-1} < \frac{4}{5}$$

value of n is 3

4. Interpret from the venn diagram

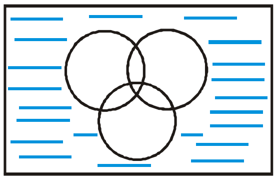


Part # II : Comprehension

Comprehension – 2

1. $P = 1 - P(A\cup B\cup C)$

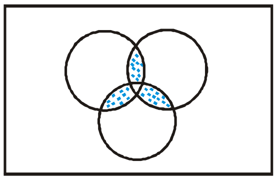
$= 1 - P(A) - P(B) - P(C) + P(A\cap C) + P(C\cap A) - P(A\cap B\cap C)$



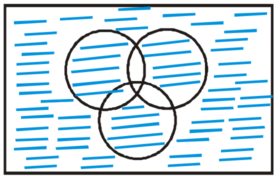
$= P(A\cap B\cap C)' - P(A) - P(B) - P(C) + P(A\cap B) + P(B\cap C) + P(C\cap A)$

$= P(\overline{A}\cup\overline{B}\cup\overline{C}) - P(A) - P(B) - P(C) + P(A\cap B) + P(B\cap C) + P(C\cap A)$

2.



3.



$= 1 - P(A \cup B \cup C)^c + P(A) + P(B) + P(C)$

$- \ P(A \cap B) - P(B \cap C) - P(C \cap A)$

Comprehension – 3

$\text{Bag A} \left\langle \begin{matrix} 1\,W \\ 5\,B \end{matrix} \right. ; \ \text{Bag B} \left\langle \begin{matrix} 2\,W \\ 4\,B \end{matrix} \right. ; \ \text{Bag C} \left\langle \begin{matrix} 3\,W \\ 3\,B \end{matrix} \right.$

Let E : Event of drawing 1 Black marble and 1 White marble from any 2 selected bags.

E_1 : Event of selecting the bags B & C

E_2 : Event of selecting the bags C & A

E_3 : Event of selecting the bags A & B

A : Event of drawing 1 White marble from bag A

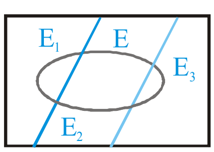
B : Event of drawing 1 White marble from bag B

C : Event of drawing 1 White marble from bag C

Now $E = (E \cap E_1) + (E \cap E_2) + (E \cap E_3)$

$$P(E \cap E_1) = P(E_1) \cdot P(E/E_1) = \frac{1}{3}\left(\frac{4 \cdot 3 + 2 \cdot 3}{6 \cdot 6}\right) = \frac{18}{108}$$

$$P(E \cap E_2) = P(E_2) \cdot P(E/E_2) = \frac{1}{3} \cdot \frac{3 \cdot 1 + 3 \cdot 5}{6 \cdot 6} = \frac{18}{108}$$



$$P(E \cap E_3) = P(E_3) \cdot P(E/E_3) = \frac{1}{3} \cdot \frac{5 \cdot 2 + 1 \cdot 4}{6 \cdot 6} = \frac{14}{108}$$

$\therefore \ P(E) = P(E \cap E_1) + P(E \cap E_2) + P(E \cap E_3)$

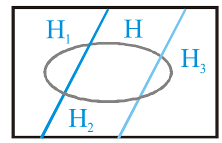
$$= \frac{18}{108} + \frac{18}{108} + \frac{14}{108} = \frac{50}{108} = \frac{25}{54} \qquad \textbf{Ans.(i)}$$

$$\therefore \ P(E_1/E) = \frac{P(E_1 \cap E)}{P(E)} = \frac{18/108}{50/108} = \frac{9}{25};$$

let $E_1/E = H_1$

$$P(E_2/E) = \frac{P(E_2 \cap E)}{P(E)} = \frac{18/108}{50/108} = \frac{9}{25};$$

let $E_2/E = H_2$



$$P(E_3/E) = \frac{P(E_3 \cap E)}{P(E)} = \frac{14/108}{50/108} = \frac{7}{25} \quad \textbf{Ans.(ii)};$$

let $E_3/E = H_3$

Let H : drawing 1 white marble from third bag

i.e. $P(H) \to P$

$P(H) = P(H \cap H_1) + P(H \cap H_2) + P(H \cap H_3)$

$P(H_1) \cdot P(H/H_1) + P(H_2) \cdot P(H/H_2) + P(H_3) \cdot P(H/H_3)$

$= P(H_1)\,P(A) + P(H_2)\,P(B) + P(H_3)\,P(C)$

$$= \frac{9}{25} \cdot \frac{1}{6} + \frac{9}{25} \cdot \frac{2}{6} + \frac{7}{25} \cdot \frac{3}{6} = \frac{48}{25 \cdot 6} = \frac{8}{25} = \frac{m}{n}$$

$\Rightarrow \ (m + n) = 33$ **Ans.(ii)**

Comprehension – 4

P (studies 10 hrs per day) = 0.1 = $P(B_1)$

P (studies 7 hrs per day) = 0.2 = $P(B_2)$

P (studies 4 hrs per day) = 0.7 = $P(B_3)$

A : successful

1. $P(A) = P(A \cap B_1) + P(A \cap B_2) + P(A \cap B_3)$

$$= \frac{1}{10} \times \frac{80}{100} + \frac{2}{10} \times \frac{60}{100} + \frac{7}{10} \times \frac{40}{100} = \frac{48}{100} = \frac{12}{25}$$

2. $$P(B_3/A) = \frac{P(B_3)\,.\,P(A/B_3)}{\Sigma\, P(B_i)\,.\,P(A/B_i)} = \frac{\frac{7}{10} \times \frac{40}{100}}{\frac{12}{25}} = \frac{7}{12}$$

3. $$P(B_3/\bar{A}) = \frac{P(B_3)\,.\,P(\bar{A}/B_3)}{\Sigma\, P(B_i)\,.\,P(\bar{A}/B_i)}$$

$$= \frac{\frac{7}{10}\times\frac{60}{100}}{\frac{1}{10}\times\frac{20}{100}+\frac{2}{10}\times\frac{40}{100}+\frac{7}{10}\times\frac{60}{100}}$$

$$= \frac{420}{520} = \frac{21}{26}$$

Comprehension – 5

$$52 \xrightarrow[40]{\text{face cards}} \left\langle \begin{array}{l} 4 \text{ aces} \\ 36 \text{ non aces} \end{array} \right.$$

(i) 10 cards are drawn before the 1^{st} ace – first 10 cards are all non aces and 11^{th} card is an ace.

$$\therefore \quad P_1 = \frac{{}^{36}C_{10}}{{}^{40}C_{10}} \cdot \frac{4}{30}$$

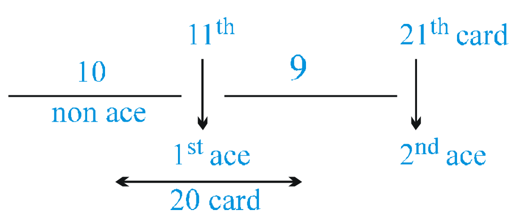
$$= \frac{(36)!}{(10!)(26!)} \frac{(30!)(10!)}{40!} \cdot \frac{4}{30}$$

$$P_1 = \frac{(30)(29)(28)(27)}{(40)(39)(38)(37)} \cdot \frac{4}{30} = \frac{(27)(28)(29)}{(10)(37)(38)(39)}$$

Ans.

(ii) Position of pack now

$$29 \left\langle \begin{array}{l} 3 \text{ aces} \\ 26 \text{ non aces} \end{array} \right.$$



$$P_2 = \frac{{}^{26}C_9}{{}^{29}C_9} \cdot \frac{3}{20} P_1$$

$$= \frac{26!}{(9!)(17!)} \cdot \frac{(9!)(20!)}{29!} \cdot \frac{3}{20} \cdot P_1$$

$$= \frac{(20)(19)(18)}{(29)(28)(27)}\left(\frac{(27)(28)(29)}{(37)(38)(39)} \cdot \frac{1}{10}\right)\frac{3}{20}$$

$$= \frac{(18)(3)}{(37)(39)} \frac{1}{20} = \frac{9}{(10)(13)(37)}$$ **Ans.**

(iii) $P_3 = 20P_2 = (20)\left(\frac{9}{(10)(13)(37)}\right) = \frac{18}{(13)(37)}$

EXERCISE - 4

Subjective Type

2. E_r = Scored exactly r points

$P(E_n) = P(E_{n-2}H \cup E_{n-1}T)$

$= P(E_{n-2})P(H) + P(E_{n-1})P(T)$

$$P_n = P_{n-2}\frac{1}{2} + \frac{1}{2}P_{n-1}$$

$$P_n - P_{n-1} = \frac{1}{2}(P_{n-2} - P_{n-1})$$

3. $P(A) = \dfrac{5}{10}$; $P(B) = \dfrac{3}{10}$; $P(C) = \dfrac{2}{10}$

after the race

$P'(A) = \dfrac{1}{3}$

$P'(B) + P'(C) = \dfrac{2}{3}$

That will increase probability of B & C in 3 : 2 respectively.

$\therefore\ P'(B) = \dfrac{2}{3} \times \dfrac{3}{5} = \dfrac{2}{5}$

$\therefore\ P'(C) = \dfrac{2}{3} \times \dfrac{2}{5} = \dfrac{4}{15}$

4. Let the first event A_1

Let the second event A_2

Let $P(A_1) = \dfrac{p^2}{q^2}$

$P(A_2) = \dfrac{p}{q}$

odds against seconds $= \dfrac{q-p}{p}$

odds against first $= \dfrac{q^2 - p^2}{p^2}$

$\Rightarrow \left(\dfrac{q-p}{p}\right)^3 = \dfrac{q^2 - p^2}{p^2}$

$\Rightarrow (q-p)^2 = (q+p)\,p$

$\Rightarrow q^2 = 3pq \Rightarrow \dfrac{p}{q} = \dfrac{1}{3}$

$P(A_1) = \dfrac{1}{9}$; $P(A_2) = \dfrac{1}{3}$

6. Bot – 2 R + 3B.

$$p\,(\text{Rad, blue}) = \frac{2}{5} \times \frac{3}{4} = \frac{3}{10} = p$$

x	0	1	2	3
p	${}^3C_0p^0(1-p)^3$	${}^3C_1p(1-p)^2$	${}^3C_2p^2(1-p)$	${}^3C_3p^3$

7. Let three independent critics A, B & C

Odd in favour for A is $\dfrac{5}{2}$ hence $P(A) = \dfrac{5}{7}$

Odd in favour for B is $\dfrac{4}{3}$ hence $P(B) = \dfrac{4}{7}$

Odd in favour for C is $\dfrac{3}{4}$ hence $P(C) = \dfrac{3}{7}$

Probability that majorty will be

favourable $= P(A)P(B)\,P(\bar{C}) + P(B).P(C).P(\bar{A}) +$

$P(C).P(A).P(\bar{B}) + P(A).P(B).P(C)$

$$= \frac{5}{7}\times\frac{4}{7}\times\frac{4}{7} + \frac{4}{7}\times\frac{3}{7}\times\frac{3}{7} + \frac{3}{7}\times\frac{5}{7}\times\frac{3}{7} + \frac{5}{7}\times\frac{4}{7}\times\frac{3}{7}$$

$$= \frac{209}{343}$$

8. By symmetry the probability of more wins than losses equals the probability of more losses than wins. We calculate the probability of the same number of wins and losses.

$\therefore\ P(L) = P(W) = P(D) = 1/3$

Case - I Probability of no wins and no losses

$$= P(D\,D\,D\,D\,D\,D) = \frac{1}{3^6}$$

Case-II Probability of 1 win, 1 loss and 4 draws

$$= P(W\,L\,D\,D\,D\,D) = \frac{6!}{4!}\cdot\frac{1}{3^6} = \frac{30}{3^6}$$

Case-III Probability of 2 wins, 2 losses and 2 draws

$$= P(W\,W\,L\,L\,D\,D) = \frac{6!}{2!2!2!}\cdot\frac{1}{3^6} = \frac{90}{3^6}$$

Case-IV Probability of 3 wins and 3 losses

$$= P(W\,W\,W\,L\,L\,L) = \frac{^6C_3}{3^6} = \frac{20}{3^6}.$$

Hence probability of the same number of wins or losses

$$= \frac{(1+30+90+20)}{729} = \frac{141}{729} = \frac{47}{243}.$$

Hence probability more wins than losses = probability more losses than wins

$$= \frac{1}{2}\left[1-\frac{47}{243}\right] = \frac{1}{2}\left[\frac{196}{243}\right] = \frac{98}{243}$$

$\Rightarrow \quad p+q=341$

10. A : Target hit in 1st shot
B : Target hit in 2nd shot
C : Target hit in 3rd shot
E_1 : destroyed in exactly one shot
E_2 : destroyed in exactly two shot
E_3 : destroyed in exactly three shot

$P(E_1) = P(E_1 A\bar{B}\bar{C} \cup E_1 \bar{A}B\bar{C} \cup E_1 \bar{A}\bar{B}C)$

$$= \frac{1}{3}\left[\frac{1}{2}.\frac{1}{3}.\frac{1}{4} + \frac{1}{2}.\frac{1}{3}.\frac{3}{4} + \frac{1}{2}.\frac{2}{3}.\frac{1}{4}\right] = \frac{1+3+2}{3.24} = \frac{1}{12}$$

$P(E_2) = P(E_2 \bar{A}BC \cup E_2 AB\bar{C} \cup E_3 A\bar{B}C)$

$$= \frac{7}{11}\left[\frac{1}{2}.\frac{2}{3}.\frac{3}{4} + \frac{1}{2}.\frac{2}{3}.\frac{1}{4} + \frac{1}{2}.\frac{1}{3}.\frac{3}{4}\right] = \frac{7.\ 11}{11.24} = \frac{7}{24}$$

$$P(E_3) = P(E_3 ABC) = 1.\frac{1}{2}.\frac{2}{3}.\frac{3}{4} = \frac{1}{4}$$

$P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3)$

$$= \frac{1}{12} + \frac{7}{24} + \frac{1}{4} = \frac{2+7+6}{24} = \frac{15}{24} = \frac{5}{8}$$

11. A : A solves correctly
B : B solves correctly
E : Commit same mistake
F : same result

$$P(AB/F) = \frac{P(AB)}{P(AB) + P(\bar{A}\bar{B}E)} = \frac{\frac{1}{8}.\frac{1}{12}}{\frac{1}{8}.\frac{1}{12} + \frac{7}{8}.\frac{11}{12}.\frac{1}{1001}}$$

$$= \frac{1001}{1078} = \frac{13}{14}$$

12. $P(\bar{A} \cup B) = P(\bar{A}) + P(B) - P(\bar{A} \cap B)$ ..... **(i)**

also $\dfrac{P(\bar{A} \cap \bar{B})}{P(\bar{B})} = 0.1 \quad \Rightarrow \quad P(\overline{A \cup B}) = 0.02$

$P(A \cup B) = 0.98$

$P(A \cap B) = 0.4 + 0.8 - 0.98$

$= 0.22$ ..... **(ii)**

Put (2) in (1)

$P(\bar{A} \cup B) = 0.6 + 0.8 - [P(B) - P(A \cap B)]$

$= 0.6 + 0.8 - (0.8 - 0.22) = 0.82$

(ii) $P(\bar{A} \cup B) + P(A \cap \bar{B})$

$= P(A) + P(B) - 2P(A \cap B)$

$= 0.4 + 0.8 - 2(0.22) = 0.76$

15. **(A)** : puzzle solved by A
(B) : puzzle solved by B
(D) : puzzle solved by D
(C) : support either A or B
(A) $= p$, $P(B) = p$, $P(D) = p$

If C supports A $\quad P(C) = \dfrac{1}{2}$

$$P(\bar{C}) = \frac{1}{2}$$

for team {A, B, C} $= P(A)\dfrac{1}{2} + P(B)\dfrac{1}{2}$

$$= \frac{p}{2} + \frac{p}{2} = p$$

which is equal to P(D)

$\Rightarrow$ both are equally likely.

16. $P(C) = \dfrac{1}{^4C_1 + {}^4C_4} = \dfrac{1}{2^4 - 1} = \dfrac{1}{15}$

P(correct) = 1 – P (all wrong)

$$= 1 - \frac{14}{15} \times \frac{13}{14} \times \frac{12}{13} \times \frac{11}{12} \times \frac{10}{11} = \frac{1}{3}$$

17. A : Weather is favourable

$\bar{A}$: Weather not good or low cloud
B : Reliability (instrument functions probability)
C : Safe landing

$P(C/A) = p_1. \qquad P(B) = P.\ P(C/B) = p_1$

$P(C/\bar{B}) = p_2.\ P(\bar{A}) = \dfrac{K}{100}$

$P(C) = P(A\,C \cup \bar{A}\ B\,C \cup \bar{A}\bar{B}C)$

$$= \left(1 - \frac{K}{100}\right) p_1 + \frac{K}{100}\,[P\,p_1 + (1-P)p_2]$$

$P((\bar{A}BC \cup \bar{A}\bar{B}C)/C)$

$$= \frac{\frac{K}{100}[Pp_1 + (1-P)p_2]}{\left(1-\frac{K}{100}\right)p_1 + \frac{K}{100}(Pp_1 + (1-P)p_2)}$$

18. Let B_1 : pack A was selected $\Rightarrow P(B_1) = \frac{1}{2}$;

Pack A $\xrightarrow{4 \text{ aces}}$ 48 cards in 12 different denominations

B_2 : pack B was selected $\Rightarrow P(B_2) = \frac{1}{2}$; Pack

B $\rightarrow$ 48 cards (A K Q J removed, 1 each) $\rightarrow$ 3 ace, 3 kings, 3 queen, 3 jack, 36 other cards

A : two cards drawn all of same rank.

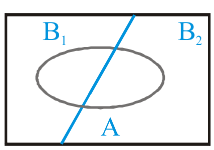
Now $A = (A \cap B_1) + (A \cap B_2)$

$\therefore \quad P(A) = P(A \cap B_1) + P(A \cap B_2)$

$= P(B_1)P(A/B_1) + P(B_2)P(A/B_2)$

$$P(A/B_1) = \frac{{}^{12}C_1 \cdot {}^4C_2}{{}^{48}C_2}$$

$$P(A/B_2) = \frac{{}^9C_1 \cdot {}^4C_2 + {}^4C_1 \cdot {}^3C_2}{{}^{48}C_2}$$



$$P(B_1/A) = \frac{P(A \cap B_1)}{P(A)}$$

$$= \frac{P(B_1)P(A/B_1)}{P(B_1)P(A/B_1) + P(B_2)P(A/B_2)}$$

$$= \frac{{}^{12}C_1 \cdot {}^4C_2}{{}^{12}C_1 \cdot {}^4C_2 + {}^9C_1 \cdot {}^4C_2 + {}^4C_1 \cdot {}^3C_2}$$

$$= \frac{(12)(6)}{(12)(6) + (9)(6) + (4)(3)} = \frac{12}{23} \Rightarrow m + n = 35$$

19. P(identify high grade tea correctly) $= \frac{9}{10}$

P (identify low grade tea correctly) $= \frac{8}{10}$

P (Given high grade tea) $= \frac{3}{10}$

P (Given low grade tea) $= \frac{7}{10}$

P (Low grade tea / says high grade tea)

$$= \frac{\frac{7}{10} \times \frac{2}{10}}{\frac{7}{10} \times \frac{2}{10} + \frac{3}{10} \times \frac{9}{10}} = \frac{14}{41}$$

21. Let the probability hitting the enemy plane in I, II, III & IV shots are denoted by $P(G_1)$, $P(G_2)$, $P(G_3)$ & $P(G_4)$

$$P(G_1) = \frac{4}{10}, P(G_2) = \frac{3}{10}, P(G_3) = \frac{2}{10}, P(G_4) = \frac{1}{10}$$

P(All four shots do not hit the plane)

$$= P(\bar{G}_1).P(\bar{G}_2).P(\bar{G}_3).P(\bar{G}_4)$$

$$= \frac{6}{10} \times \frac{7}{10} \times \frac{8}{10} \times \frac{9}{10} = \frac{189}{625}$$

so probability of hitting the plane

$$= 1 - \frac{189}{625} = \frac{436}{625}$$

22. $P(HA) = 0.8$; $P(HB) = 0.4$

A = Only one bullet in bear

B_1 = Shot by HA & missed by HB $= P(B_1)$

$= 0.8 \times 0.61$

B_2 = Shot by HB & missed by HA $= P(B_2) = 0.4 \times 0.2$

$$P(B_1/A) = \frac{P(A/B_1)P(B_1)}{P(A/B_1)P(B_1) + P(A/B_2)P(B_2)}$$

$$= \left(\frac{0.8 \times 0.6}{0.8 \times 0.6 + 0.2 \times 0.4}\right) = \frac{48}{48+8} = 240$$

$E_A = 280 \times P(B_1/A) \qquad E_B = E - E_A$

23. Let $q = 1 - p$ = probability of getting the tail. We have α = probability of A getting the head on tossing firstly

$= P(H_1 \text{ or } T_1T_2T_3H_4 \text{ or } T_1T_2T_3T_4T_5T_6H_7 \text{ or } ...)$

$= P(H) + P(H)\,P(T)^3 + P(H)P(T)^6 + ...$

$$= \frac{P(H)}{1 - P(T)^3} = \frac{p}{1 - q^3}$$

Also,

β = probability of B getting the head on tossing secondly

$= P(T_1H_2 \text{ or } T_1T_2T_3T_4H_5 \text{ or } T_1T_2T_3T_4T_5T_6T_7H_8 \text{ or } ...)$

$= P(H)P(T) + P(H)\,P(T)^4 + P(H)P(T)^7 + ...$

$= P(T)\,[P(H) + P(H)\,P(T)^3 + P(H)P(T)^6 + ...]$

$$= q\,\alpha = (1 - p)\,\alpha = \frac{p(1-p)}{1 - q^3}$$

Again we have $\alpha + \beta + \gamma = 1$

$$\Rightarrow \quad \gamma = 1 - (\alpha + \beta) = 1 - \frac{p + p(1-p)}{1 - q^3}$$

$$= 1 - \frac{p + p(1-p)}{1 - (1-p)^3} = \frac{1 - (1-p)^3 - p - p(1-p)}{1 - (1-p)^3}$$

$$= \frac{1 - (1-p)^3 - 2p + p^2}{1 - (1-p)^3} = \frac{p - 2p^2 + p^3}{1 - (1-p)^3}$$

Also, $\alpha = \dfrac{p}{1 - (1-p)^3}, \beta = \dfrac{p(1-p)}{1 - (1-p)^3}$

24. Let C_1, C_2, C_3, C_4 are coins.

4 coins tossed twice $\rightarrow$ each coin is tossed twice.

Let S : denotes the success that a coin is discarded

P(S) = 1 – coin is not discarded

$$= 1 - P(HH) = 1 - \frac{1}{4} = \frac{3}{4}$$

Hence S can take value 0, 1, 2, 3, 4

$P(S = 3 \text{ or } 4) \; = P(S = 3) + P(S = 4)$

$$= {}^4C_1\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right) + {}^4C_4\left(\frac{3}{4}\right)^4 = \left(\frac{3}{4}\right)^3\left(1 + \frac{3}{4}\right)$$

$$= \frac{(27)(7)}{256} = \frac{189}{256} = \frac{m}{n}$$

$\therefore \quad m + n = 445$

EXERCISE - 5

Part # I : AIEEE/JEE-MAIN

1. Probability problem is not solved by $A = 1 - \frac{1}{2} = \frac{1}{2}$

Probability problem is not solved by $B = 1 - \frac{1}{3} = \frac{2}{3}$

Probability problem is not solved by $C = 1 - \frac{1}{4} = \frac{3}{4}$

Probability of solving the problem = 1 – P (not solved by any body)

$$\therefore \; P = 1 - \frac{1}{2}.\frac{2}{3}.\frac{3}{4} = 1 - \frac{1}{4} = \frac{3}{4}$$

2. $P(A \cup B) = \frac{3}{4}, P(A \cap B) = \frac{1}{4}$

$$P(\bar{A}) = \frac{2}{3} \quad \Rightarrow \quad P(A) = \frac{1}{3}$$

$\therefore \quad P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$$\frac{1}{4} = \frac{1}{3} + P(B) - \frac{3}{4} \Rightarrow P(B) = \frac{2}{3}$$

$P(\bar{A} \cap B) = P(B) - P(A \cap B)$

$$= \frac{2}{3} - \frac{1}{4} = \frac{8-3}{12} = \frac{5}{12}.$$

3. Probability of getting odd $p = \frac{3}{6} = \frac{1}{2}$

Probability of getting others $q = \frac{3}{6} = \frac{1}{2}$

$$\text{Variance} = npq = 5.\frac{1}{2}.\frac{1}{2} = \frac{5}{4}$$

4. Out of 5 horses only one is the winning horse. The probability that Mr. A selected the losing horse $= \frac{4}{5} \times \frac{3}{4}$

$\therefore$ The probability that Mr. A selected the winning horse

$$= 1 - \frac{4}{5} \times \frac{3}{4} = \frac{2}{5}$$

7. E = {x is a prime number}

$P(E) = P(2) + P(3) + P(5) + P(7) = 0.62$

$F = (x < 4), P(F) = P(1) + P(2) + P(3) = 0.50$

$\therefore \quad P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$= 0.62 + 0.50 - 0.35 = 0.77$

8. $\left.\begin{aligned} np &= 4 \\ npq &= 2 \end{aligned}\right\} \Rightarrow q = \frac{1}{2}, p = \frac{1}{2}, n = 8$

$$P(X=2) = {}^8C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^6 = 28.\frac{1}{2^8} = \frac{28}{256}$$

10. For a particular house being selected,

Probability $= \frac{1}{3}$

Probability (all the persons apply for the same house)

$$= \left(\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}\right)3 = \frac{1}{9}.$$

14. Let A be the event that sum of digits is 8

exhaustive cases $\rightarrow {}^{50}C_1$

favourable cases $\rightarrow$ 08, 17, 26, 35, 44 $= {}^5C_1$

$$P(A) = \frac{{}^5C_1}{{}^{50}C_1}$$

Let B be the event that product of digits is zero

favourable cases $\rightarrow$

$\{00, 01, ---, 09, 10, 20, 30, 40\} = {}^{14}C_1$

$$\therefore \quad P(B) = \frac{{}^{14}C_1}{{}^{50}C_1}$$

$$\therefore \quad P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{1/{}^{50}C_1}{{}^{14}C_1/{}^{50}C_1} = \frac{1}{14}$$

15. The probability of at least one success

$$1 - \left(\frac{3}{4}\right)^n \geq \frac{9}{10}$$

$$\left(\frac{3}{4}\right)^n \leq \frac{1}{10}$$

$$n \geq \log_{3/4}\left(\frac{1}{10}\right)$$

$$n \geq \frac{-\log 10}{\log_{10} 3 - \log_{10} 4}$$

$$n \geq \frac{1}{\log_{10} 4 - \log_{10} 3}$$

16. Required probability $= \frac{{}^3C_1}{{}^9C_1}\ \frac{{}^4C_1}{{}^8C_1}\ \frac{{}^2C_1}{{}^7C_1}\ 3! = \frac{2}{7}$

17. Let terms of an AP

a, a + d, a + 2d, a + 3d

$\because$ a $\geq$ 1, a + 3d $\leq$ 20

$$3d \leq 19 \Rightarrow d \leq \frac{19}{3}$$

so d = ±1, ±2, ±3, ±4, ±5 and ±6

statement 2 is wrong

if d = 1

then a + 3d ≤ 20 similarly d = –1

a ≤ 17 so in this case also

so 17 cases will 17 cases will be there

be there

Total case for d = ±1 is 34

18. $P\left(\frac{C}{D}\right) = \frac{P(C\cap D)}{P(D)} = \frac{P(C)}{P(D)}$

$$P(D) = \frac{P(C)}{P\left(\frac{C}{D}\right)} \leq 1$$

$$P(C) \leq P\left(\frac{C}{D}\right)$$

$$P\left(\frac{C}{D}\right) \geq P(C)$$

19. at least one failure = 1 – all sucess

$$1 \geq 1 - p^5 \geq \frac{31}{32}$$

$$0 \leq p^5 \leq \frac{1}{32}$$

$$0 \leq p \leq \frac{1}{2}$$

$$p \in \left[0, \frac{1}{2}\right]$$

20. $P(A\cap B\cap C) = 0$

$$P\left(\frac{\bar{A}\cap\bar{B}}{C}\right) = \frac{P\{(\bar{A}\cap\bar{B})\cap C\}}{P(C)} = \frac{P(\bar{A}\cap\bar{B})P(C)}{P(C)}$$

$$= \frac{[1 - P(A) - P(B) + P(A)P(B)]P(C)}{P(C)}$$

$(\because P(A\cap B\cap C) = 0)$

$$= \frac{P(C) - P(A)P(C) - P(B)P(C)}{P(C)}$$

$= 1 - P(A) - P(B) = P(A^C) - P(B)$

21. Let Events A denotes the getting min No. is 3 & B denotes the max. no. is 6

$$P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}=\frac{{}^2C_1}{{}^5C_2}=\frac{2}{10}=\frac{1}{5}$$

Aliter

$$P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}=\frac{\frac{{}^4C_3-(2)}{{}^8C_3}}{\frac{{}^6C_3-{}^5C_3}{{}^8C_3}}=\frac{2}{10}=\frac{1}{5}$$

22. P(4correct) + P(5 correct)

$$={}^5C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)+\left(\frac{1}{3}\right)^5=\frac{11}{3^5}$$

25. A $\quad \frac{4}{2}=12$

L $\quad 4=24$

M $\quad \frac{4}{2}=12$

SA $\quad \frac{3}{2}=3$

SL $\quad 3=6$

Total 57

Next word is SMALL..

Part # II : IIT-JEE ADVANCED

1. p_n denotes the probability that no two (or more) consecutive heads occur

$\Rightarrow$ p_n denotes the probability that 1 or no head occur.

For n = 1, p_1 = 1 because in both cases we get less than two heads (H, T)

For n = 2

$p_2 = 1 - p$(two head simultaneously occur)

$= 1 - p(HH) = 1 - pp = 1 - p^2$

(probability of head is given as p not 1/2)

For $n \geq 3$, $p_n = p_{n-1}(1-p) + p_{n-2}(1-p)p$

$= (1-p)p_{n-1} + p(1-p)p_{n-2}$ Hence proved.

2. **(a)** Let $w_1 \to$ ball drawn in the first draw is white.

$b_1 \to$ ball drawn in the first draw is black.

$w_2 \to$ ball drawn in the second draw is white.

Then

$$P(w_2)=P(w_1).P(w_2/w_1)+P(b_1)P(w_2/b_1)$$

$$=\left(\frac{m}{m+n}\right)\left(\frac{m+k}{m+n+k}\right)+\left(\frac{n}{m+n}\right)\left(\frac{m}{m+n+k}\right)$$

$$=\frac{m(m+k)+mn}{(m+n)(m+n+k)}$$

$$=\frac{m(m+n+k)}{(m+n)(m+n+k)}=\frac{m}{m+n}$$

2. **(b)** Total number of favourable cases

$=(3^n-3.2^n+3).{}^6C_3$

$\Rightarrow$ required probability

$$=\frac{(3^n-3.2^n+3)\times{}^6C_3}{6^n}$$

5. **(a)** Here, $P(A\cup B).P(A'\cap B')$

$\Rightarrow$ $\{P(A)+P(B)-P(A\cap B)\}\ \{P(A').P(B')\}$

{Since A, B are independent $\Rightarrow$ A', B' are independent}

$\therefore$ $P(A\cup B).P(A'\cap B')$

$\leq\{P(A)+P(B)\}.\{P(A').P(B')\}$

$=P(A).P(A').P(B')+P(B).P(A').P(B')$ .....(i)

$\leq P(A).P(B')+P(B).P(A')$

{Since in (1), $P(A')\leq 1$ and $P(B')\leq 1$}

$\Rightarrow$ $P(A\cup B).P(A'\cap B')\leq P(A).P(B')+P(B).P(A')$

$\Rightarrow$ $P(A\cup B).P(A'\cap B')\leq P(C)$

{as $P(C)=P(A).P(B')+P(B).P(A')$}

5. **(b)** Using Baye's theorem;P(B/A)

$$=\frac{\sum_{i=1}^{3}P(A_i).P(B/A_i)}{\sum_{i=1}^{3}P(A_i)}$$

where A be the event at least 4 white balls have been drawn.

A_1 be the event exactly 4 white balls have been drawn.

A_2 be the event exactly 5 whitle balls have been drawn.

A_3 be the event exactly 6 white balls have been drawn

B be the event exactly 1 white ball is drawn from two draws.

$\therefore$ P(B/A)

$$=\frac{\frac{^{12}C_2.^6C_4}{^{18}C_6}.\frac{^{10}C_1.^2C_1}{^{12}C_2}+\frac{^{12}C_1.^6C_5}{^{18}C_6}.\frac{^{11}C_1.^1C_1}{^{12}C_2}}{\frac{^{12}C_2.^6C_4}{^{18}C_6}+\frac{^{12}C_1.^6C_5}{^{18}C_6}+\frac{^{12}C_0.^6C_6}{^{18}C_6}}$$

$$=\frac{(^{12}C_2.^6C_4.^{10}C_1.^2C_1)+(^{12}C_1.^6C_5.^{11}C_1.^1C_1)}{^{12}C_2(^{12}C_2.^6C_4+^{12}C_1.^6C_5+^{12}C_0.^6C_6)}$$

5. **(c)** As three distinct numbers are to be selected from first 100 natural numbers

$\Rightarrow$ $n(S)={}^{100}C_3$

$E_{(favourable\ events)}$ = All three of them are divisible by both 2 and 3.

$\Rightarrow$ divisible by 6 i.e., {6, 12, 18, ..., 96}

$n(E)={}^{16}C_3$

$$P(E)=\frac{16\times15\times14}{100\times99\times98}=\frac{4}{1155}$$

10. Statement I : If $P(H_i\cap E)=0$ for some i, then

$$P\left(\frac{H_i}{E}\right)=P\left(\frac{E}{H_i}\right)=0$$

If $P(H_i\cap E)\neq0$ for $\forall\ i=1,2,....,n$

Then $P\left(\frac{H_i}{E}\right)=\frac{P(H_i\cap E)}{P(H_i)}\times\frac{P(H_i)}{P(E)}$

$$=\frac{P\left(\frac{E}{H_i}\right)\times P(H_i)}{P(E)}>P\left(\frac{E}{H_i}\right)P(H_i)\quad[\text{as } 0<P(E)<1]$$

Hence statement I may not always be true.

Statement II : Clearly, $H_1\cup H_2\cup...\cup H_n=S$ (sample space)

$\Rightarrow$ $P(H_1)+P(H_2)+...+P(H_n)=1$

12. Let B have n number of outcomes.

so $P(B)=\frac{n}{10}$, $P(A)=\frac{4}{10}$

$$P(A\cap B)=\frac{4}{10}\times\frac{n}{10}=\frac{2n/5}{10}$$

$\Rightarrow$ $\frac{2n}{5}$ is an integer

$\Rightarrow$ n = 5 or 10

17. C : Correct signal is transmitted

$\bar{C}$: false signal is transmitted

G : Original signal is green

R : Original signal is red

K : Signal received at station B is green.

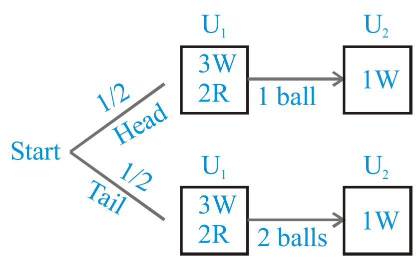
$$P(G/K)=\frac{P(G).P(K/G)}{P(K)}$$

$$=\frac{P(GCC)+P(G\bar{C}\bar{C})}{P(GCC)+P(G\bar{C}\bar{C})+P(RC\bar{C})+P(R\bar{C}C)}$$

$$=\frac{\frac{4}{5}\times\frac{3}{4}\times\frac{3}{4}+\frac{4}{5}\times\frac{1}{4}\times\frac{1}{4}}{\frac{4}{5}\times\frac{3}{4}\times\frac{3}{4}+\frac{4}{5}\times\frac{1}{4}\times\frac{1}{4}+\frac{1}{5}\times\frac{3}{4}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{4}\times\frac{3}{4}}$$

$$=\frac{40}{46}=\frac{20}{23}$$

Paragraph for Question 18 and 19



18. **(B)**
Required probability

$$=\frac{1}{2}\left(\frac{3}{5}.1+\frac{2}{5}.\frac{1}{2}\right)+\frac{1}{2}\left(\frac{^3C_2}{^5C_2}.1+\frac{^2C_2}{^5C_2}.\frac{1}{3}+\frac{^3C_1{}^2C_1}{^5C_2}.\frac{2}{3}\right)$$

$$=\frac{1}{2}\left(\frac{4}{5}\right)+\frac{1}{2}\left(\frac{3}{10}+\frac{1}{30}+\frac{2}{5}\right)=\frac{2}{5}+\frac{11}{30}=\frac{23}{30}$$

19. **(D)**
Required probability

$$=\frac{2/5}{2/5+11/30}\quad\text{(using Baye's theorem)}$$

$$=\frac{12}{23}$$

21. $P(X) = E_1E_2E_3 + E_1E_2\bar{E}_3 + E_1\bar{E}_2E_3 + \bar{E}_1E_2E_3$

$$= \frac{1}{2}\times\frac{1}{4}\times\frac{1}{4}+\frac{1}{2}\times\frac{1}{4}\times\frac{3}{4}+\frac{1}{2}\times\frac{3}{4}\times\frac{1}{4}+\frac{1}{2}\times\frac{1}{4}\times\frac{1}{4}$$

$\Rightarrow \quad P(X) = \frac{1}{4}$

$$P\left(\frac{X_1^c}{X}\right) = \frac{P\left(X_1^C \cap X\right)}{P(X)} = \frac{1/32}{1/4} = \frac{1}{8}$$

P(Exactly two engines are functioning |x)

$$= \frac{7/32}{1/4} = \frac{7}{8}$$

$$P\left(\frac{X}{X_2}\right) = \frac{P(X \cap X_2)}{P(X_2)} = \frac{5/32}{1/4} = \frac{5}{8}$$

$$P\left(\frac{X}{X_1}\right) = \frac{P(X \cap X_1)}{P(X_1)} = \frac{7/32}{1/2} = \frac{7}{16}$$

22. $1 - \frac{{}^6C_1.5^3}{6^4} = \frac{91}{216}$

23. $P(X \cap Y) = P(X), P(Y/X)$

$\Rightarrow \quad P(X) = \frac{1}{2}$

Also $P(X \cap Y) = P(Y).P(X/Y)$

$\Rightarrow \quad P(Y) = \frac{1}{3}$

$\Rightarrow \quad P(X \cap Y) = P(X).P(Y)$

$\Rightarrow$ X,Y are independent

$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

$$= \frac{1}{3}+\frac{1}{2}-\frac{1}{6} = \frac{2}{3}$$

$$P(X^C \cap Y) = P(Y) - P(X \cap Y) = \frac{1}{3}-\frac{1}{6} = \frac{1}{6}$$

$\Rightarrow$ (A, B) are correct

24. P(Problem is solved by at least one of them)

= 1 – P(solved by none)

$$= 1 - \left(\frac{1}{2}\times\frac{1}{4}\times\frac{3}{4}\times\frac{7}{8}\right)$$

$$= 1 - \frac{21}{256} = \frac{235}{256}$$

25. Let $P(E_1) = p_1$, $P(E_2) = p_2$, $P(E_3) = p_3$

given that $p_1(1 - p_2)(1 - p_3) = \alpha$ ......**(i)**

$p_2(1 - p_1)(1 - p_3) = \beta$ ......**(ii)**

$p_3(1 - p_1)(1 - p_2) = \gamma$ ......**(iii)**

and $(1 - p_1)(1 - p_2)(1 - p_3) = p$ ......**(iv)**

$$\Rightarrow \frac{p_1}{1-p_1} = \frac{\alpha}{p}, \frac{p_2}{1-p_2} = \frac{\beta}{p} \;\; \& \;\; \frac{p_3}{1-p_3} = \frac{\gamma}{p}$$

Also $\beta = \frac{\alpha p}{\alpha + 2p} = \frac{3\gamma p}{p - 2\gamma}$

$\Rightarrow \alpha p - 2\alpha\gamma = 3\alpha\gamma + 6p\gamma$

$\Rightarrow \alpha p - 6p\gamma = 5\alpha\gamma$

$$\Rightarrow \frac{p_1}{1-p_1} - \frac{6p_3}{1-p_3} = \frac{5p_1p_3}{(1-p_1)(1-p_3)}$$

$\Rightarrow p_1 - 6p_3 = 0$

$\Rightarrow \frac{p_1}{p_3} = 6$

Paragraph for Question 26 to 27

26. **(D)**

B_1: 1W, 3R, 2B

B_2: 2W, 3R, 4B

B_3: 3W, 4R, 5B

A = Total drawn balls are drawn & one is white, another is Red

$P(B_2|A)$ is to be determined

$P(B_2|A)$

$$= \frac{P(A \mid B_2)P(B_2)}{P(A \mid B_1)P(B_1) + P(A \mid B_2)P(B_2) + P(A \mid B_3)P(B_3)}$$

$P(B_1) = P(B_2) = P(B_3) = \frac{1}{3}$

$$P\left(\frac{A}{B_1}\right) = \frac{{}^1C_1 \times {}^3C_1}{{}^6C_2}$$

$$P(A \mid B_2) = \frac{{}^2C_1 \times {}^3C_1}{{}^9C_2}$$

$$P(A \mid B_3) = \frac{{}^3C_1 \times {}^4C_1}{{}^9C_2}$$

By putting the values

$$P(B_2 \mid A) = \frac{55}{181}$$

27. (A)

$$B_1\begin{cases}1W\\3R\\2B\end{cases}\qquad B_2\begin{cases}2W\\3R\\4B\end{cases}\qquad B_3\begin{cases}3W\\4R\\5B\end{cases}$$

Probability of 3 drawn balls of same colour

$$=\frac{1}{6}\times\frac{2}{9}\times\frac{3}{12}+\frac{3}{6}\times\frac{3}{9}\times\frac{4}{12}+\frac{2}{6}\times\frac{4}{9}\times\frac{5}{12}=\frac{82}{648}$$

34. (C)

$$P(T1)=\frac{1}{5}$$

$$P(T2)=\frac{4}{5}$$

$$P(D)=\frac{7}{100}$$

$$P\left(\frac{D}{T_1}\right)=10.P\left(\frac{D}{T_2}\right).\quad \text{Let}\quad P\left(\frac{D}{T_1}\right)=x$$

$$\text{Now},\ P(T_1)\times P\left(\frac{D}{T_1}\right)+P(T_2).P\left(\frac{D}{T_2}\right)=\frac{7}{100}$$

$$=\frac{1}{5}\times 10x+\frac{4}{5}\times x=\frac{7}{100}\Rightarrow x=\frac{1}{40}$$

$$\therefore\ P\left(\frac{T_2}{D}\right)=\frac{\frac{4}{5}\times\frac{39}{40}}{\frac{93}{100}}=\frac{78}{93}$$

35. $P(X>Y)=\left(\frac{1}{2}\times\frac{1}{2}\right)+\left(\frac{1}{2}\times\frac{1}{6}\right)+\left(\frac{1}{6}\times\frac{1}{2}\right)=\frac{5}{12}$

36. $P(X=Y)=\left(\frac{1}{2}\times\frac{1}{3}\times 2\right)+\left(\frac{1}{6}\times\frac{1}{6}\right)=\frac{13}{36}$

MOCK TEST

1. $P={}^{14}C_{13}\ \frac{1}{2}\left(\frac{1}{2}\right)^{14-13}=14\times\frac{1}{2^{13}}\ \frac{1}{2}=\frac{7}{2^{13}}$

2. (B)

Since the two boys came out are a girl and a boy, therefore remaining 2 are boys iff among the 4 students 3 are boys and 1 is a girl

$$\therefore\ \text{probability}={}^4C_3\left(\frac{1}{2}\right)^4=\frac{1}{4}$$

3. A ball from first urn can be drawn is two mannars ball is white or ball is black

$$P(w)=\frac{m}{m+n}\qquad P(B)=\frac{n}{m+n}$$

Let E $\to$ selecting a white ball from second urn after a ball from urn first has been placed into it

$P(E)=P(w)\,P(E/W)+P(B)\,P(E/B)$

$$=\frac{m}{m+n}\times\frac{p+1}{p+q+1}+\frac{n}{m+n}\ \frac{p}{p+q+1}$$

$$=\frac{m(p+1)+np}{(m+n)(p+q+1)}$$

4. (A)

Total number of functions from A to B $= n(S)=5^7$

total number of onto functions from A to B is

$$n(E)=\frac{7!}{3!4!}\times 5!+\frac{7!}{3!2!}\times\frac{1}{2!2!}\times 5!=\frac{7!\times 20}{6}$$

$$\therefore\ P(E)=\frac{n(E)}{n(S)}=\frac{7!\times 2}{3\times 5^6}$$

5. Last place can be occupied by $(0-9)$ 10 methods.

to get '6' at unit place of x^4 Last digit should be 2, 4 ,6 or 8 is 4 ways

$$\Rightarrow\ P=\frac{4}{10}\equiv 40\%$$

6. (A)

$$P(E)=\frac{\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\frac{1}{5}}{\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\frac{1}{5}+\frac{4}{5}\times\frac{1}{2}\times\frac{1}{3}\times\frac{1}{4}\left(\frac{1}{4}\right)^3}=\frac{96}{97}$$

7. P = P(1 person lies) + P(2 person lies) P(A 1 lies first / 2 person lied) + P(3 person lied) P(A 1 died first / 3 person lied)

$$= {}^{n}C_1\, pq^{n-1} \times \frac{1}{n} + {}^{n}C_1\, p^2 q^{n-2} \times \frac{1}{2} + {}^{n}C_2\, p^3 q^{n-3} \times \frac{1}{3} + \ldots\ldots$$

$$= pq^{n-1} + {}^{n-1}C_{r-1}\, p^2 q^{n-2}\frac{1}{2} + {}^{n}C_3\, p^3 q^{n-3}\frac{1}{3} + \ldots\ldots.$$

$$= pq^{n-1} + \sum_{r=2} {}^{n-1}C_{r-1}\, p^r q^{n-r}\frac{1}{r}$$

As $\dfrac{{}^{n-1}C_{r-1}}{r} = \dfrac{{}^{n}C_1}{n}$

$$P = Pq^{n-1} + \frac{1}{n}\sum_{r=2}^{n} {}^{n}C_r\, P^r q^{n-r}$$

$$\Rightarrow \quad P = pq^{n-1} + \frac{1}{n}(1 - {}^{n}C_0\, p^0 q^n - {}^{n}C_1\, P^1 q^{n-1})$$

$$P = Pq^{n-1} + \frac{1}{n}(1 - q^n - nPq^{n-1}) = \frac{1-(1-p)^n}{n}$$

8. (C)

Favourable cases are 29, 92, 36, 63

$$\therefore \quad \text{required probabiltiy} = {}^4C_3 \times \left(\frac{4}{100}\right)^3 \frac{96}{100}$$

$$= \frac{96}{390625}$$

9. n(S) = ways of sitting of 10 boys and 5 girls = 15 !

S_1..............S_a..................S_b................S_c...............S_5

x y z w

Girl Girl Girl Girl Girl

Let end seats are occupied by the girls & between first and second girl x boys are seated similarly between second and third y boys

.... so on then

$x + y + z + w = 10$

where x, y, z, w are (2k + 1) type

$2k_1 + 1 + 2k_2 + 1 + 2k_3 + 1 + 2k_4 + 1 = 10$

$\Rightarrow \quad k_1 + k_2 + k_3 + k_4 = 3 \quad$ where $k_i \geq 0$

number of solution are ${}^{3+4-1}C_{4-1} = {}^6C_3$

$$n(E) = \frac{{}^6C_3 \times 10! \times 5!}{15!}$$

10. (A)

S_1. $P = \dfrac{{}^4C_3 \times {}^3C_2 + {}^4C_3 \times {}^2C_2 + {}^3C_3 \times {}^4C_2 + {}^3C_3 \times {}^2C_2}{{}^{10}C_5}$

$$= \frac{4\times 3 + 4\times 1 + 1\times 6 + 1}{{}^{10}C_5} = \frac{23}{{}^{10}C_5}$$

S_2. Two adjacent row can be selected out of 5 rows in 4 ways. Total ways of selecting 2 rows is ${}^5C_2$ hence

$$P = \frac{4}{{}^5C_2} = \frac{2}{5}$$

S_3. Required probability

$$= \frac{P(A\,\bar{B}\,\bar{C})}{P(\bar{A}\,\bar{B}\,\bar{C}) + P(A\,\bar{B}\,\bar{C}) + P(\bar{A}\,B\,\bar{C}) + P(\bar{A}\,\bar{B}\,C)}$$

$$= \frac{\frac{4}{5}\cdot\frac{1}{4}\cdot\frac{1}{3}}{\frac{1}{5}\cdot\frac{1}{4}\cdot\frac{1}{3} + \frac{4}{5}\cdot\frac{1}{4}\cdot\frac{1}{3} + \frac{1}{5}\cdot\frac{3}{4}\cdot\frac{1}{3} + \frac{1}{5}\cdot\frac{1}{4}\cdot\frac{2}{3}}$$

$$= \frac{4}{10} = \frac{2}{5}$$

11. $P(A \cap B) = a,\ P(A) = a + d, \quad P(B) = a + 2d$

$P(A \cup B) = a + 3d$

also $\quad a + d = d \quad \Rightarrow \quad a = 0$

$\Rightarrow \quad P(A \cap B) = 0,\ P(A) = d,\ P(B) = 2d$

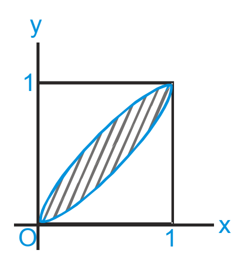
$P(A \cup B) = 3d$

12. (A, B)

Area of the shaded region

$$= \int_0^1 \left(\sqrt{x} - x^2\right) dx$$

$$= \left(\frac{2}{3}x^{3/2} - \frac{x^3}{3}\right)\Bigg|_0^1 = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$



Area of the square = 1

$\therefore$ Probability = 1/3

$A \cup B$ = whole of the region enclosed in

$0 \leq x \leq 1,\ 0 \leq y \leq 1$

$\therefore$ A and B are exhaustive events.

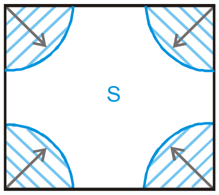
$$P(A)=\int_0^1 \sqrt{x}\,dx = \frac{2}{3}x^{\frac{3}{2}}\Big|_0^1 = \frac{2}{3},\ P(B)=\frac{1}{3}$$

$\therefore$ $P(A)\,P(B)=\frac{2}{9}\neq P(A\cap B)$

$\therefore$ A and B are not independent

14. (A, B, C)

Let S denote the set of points inside a squre with corners $(x, y), (x, y+1), (x+1, y), (x+1, y+1)$, x and y are integers. Clearly each of the four points belong to the set X.



Let P denote the set of points in S with distance less than $\frac{1}{4}$ from any corner point. P consists of four quarter circles each of radius $\frac{1}{4}$.

A coin, whose centre falls in S, will cover a point of X if and only if its centre falls in P.

hence, the required probability,

$$p=\frac{\text{area of } P}{\text{area of } S}=\frac{\pi\left(\frac{1}{4}\right)^2}{1\times 1}=\frac{\pi}{16}$$

15. n (S) = ways of selecting 3 number from 10 is ${}^{10}C_3$

n (E) $\rightarrow$ n $(A\cup B)$ where A $\rightarrow$ min. number chosen is 3

n (A) = ${}^{7}C_2$

B $\rightarrow$ max number chosen is 7

n (B) = ${}^{6}C_2$ also n $(A\cap B)={}^{3}C_1=3$

n (E) = ${}^{7}C_2+{}^{6}C_2-3$

16. (A)

$P(A\cap A)=P(A).P(A)=P(A)$

17. (B)

$Z=5-x-y$ $\quad\therefore\ xy+yz+zx=3$

$\Rightarrow$ $y^2+y(x-5)+(3+x^2-5x)=0$

$\because$ $y\in R$ $\quad\therefore\ D\geq 0$

$\Rightarrow$ $(3x-13)(x+1)\leq 0$

$\Rightarrow$ $x\in\left[-1,\frac{13}{3}\right]$

So Maximum value of $x=\frac{13}{3}$

and Minimum value of $x=-1$

Then required probability $=\dfrac{\frac{13}{3}-0}{\frac{13}{3}+1}=\dfrac{13}{16}$

18. (C)

Let the two non-negative integers be x and y

Then $x=5a+\alpha$ and $y=5b+\beta$

where $0\leq\alpha\leq 4, 0\leq\beta\leq 4$

Now $x^2+y^2=(5a+\alpha)^2+(5b+\beta)^2$

$=25(a^2+b^2)+10(a\alpha+b\beta)+\alpha^2+\beta^2$.

$\therefore$ x^2+y^2 is divisible by 5 if and only if 5 divides $\alpha^2+\beta^2$

The total number of ways of choosing α and $\beta=5\times 5=25$.

Further, $\alpha^2+\beta^2$ will be divisible by 5 if

$(\alpha_1\beta)\in\{(0,0),(1,2),(1,3),(2,1),(2,4),(3,1),(3,4),(4,2),(4,3)\}$

$\therefore$ Favourable number of ways of choosing α and $\beta=9$

$\therefore$ Required probability $=\frac{9}{25}$.

19. (D)

$2n+1=5$

$2n=4$

$n=2$; $P(E)=\dfrac{3n}{4n^2-1}=\dfrac{6}{15}=\dfrac{2}{5}$

For a, b, c in A.P. $a+c=2b$

$\Rightarrow$ a + c is even, so a and c both are even or both odd

So, a and c can be chosen in ${}^{n}C_2+{}^{n+1}C_2=n^2$ ways

$\therefore$ $P(E)=\dfrac{n^2}{{}^{(2n+1)}C_3}=\dfrac{n^2\times 3\times 2\times 1}{(2n+1)2n(2n-1)}=\dfrac{3n}{4n^2-1}$

20. (D)

Statement-2 true (by definition)

Statement-1 false

$\because$ $A\cap B\cap C=\phi$

21. (A) → q, (B) → p, (C) → r, (D) → q

Let E_i denotes the event that the bag contains i black and (12 – i) white balls (i = 0, 1, 2,........, 12) and A denotes the event that the four balls drawn are all black. Then

$$P(E_i) = \frac{1}{13}\ (i = 0, 1, 2, ..., 12)\ ;\ P\left(\frac{A}{E_i}\right) = 0$$

$$\text{for } i = 0, 1, 2, 3\ ;\ P\left(\frac{A}{E_i}\right) = \frac{^iC_4}{^{12}C_4} \text{ for } i \geq 4$$

(A) P(A) =

$$\sum_{i=0}^{12} P(E_i)\, P\left(\frac{A}{E_i}\right) = \frac{1}{13} \times \frac{1}{^{12}C_4}\left[{}^4C_4 + {}^5C_4 + ... + {}^{12}C_4\right]$$

$$= \frac{^{13}C_5}{13 \times {}^{12}C_4} = \frac{1}{5}$$

(B) Clearly, $P\left(\frac{A}{E_{10}}\right) = \frac{^{10}C_4}{^{12}C_4} = \frac{14}{33}$

(C) By Baye's theorem,

$$P\left(\frac{E_{10}}{A}\right) = \frac{P(E_{10})P\left(\frac{A}{E_{10}}\right)}{P(A)} = \frac{\frac{1}{13} \times \frac{14}{33}}{\frac{1}{5}} = \frac{70}{429}$$

(D) Let B denotes the probability of drawing 2 white and 2 black balls then

$$P\left(\frac{B}{E_i}\right) = 0 \text{ if } i = 0, 1 \text{ or } 11, 12$$

$$P\left(\frac{B}{E_i}\right) = \frac{^iC_2 \times {}^{12-i}C_2}{^{12}C_4} \text{ for } i = 2, 3, ..., 10$$

$$\therefore\ P(B) = \sum_{i=0}^{12} P(E_i)\ P\left(\frac{B}{E_i}\right) = \frac{1}{13} \times \frac{1}{^{12}C_4}\ [{}^2C_2 \times {}^{10}C_2 + {}^3C_2 \times {}^9C_2 + {}^4C_2 \times {}^8C_2 + ... + {}^{10}C_2 \times {}^2C_2]$$

$$= \frac{1}{13} \times \frac{1}{^{12}C_4}\ [2\{{}^2C_2 \times {}^{10}C_2 + {}^3C_2 \times {}^9C_2 + {}^4C_2 \times {}^8C_2 + {}^5C_2 \times {}^7C_2\} + {}^6C_2 \times {}^6C_2] = \frac{1}{13} \times \frac{1}{495}\ (1287) = \frac{1}{5}$$

22. (A) → (s), (B) → (t), (C) → (q), (D) → s

(A) Let E_1, E_2, E_3, E_4 be the events that the bag contains 1 white, 2 white, 3 white, 4 white ball respectively.

$$\text{let } P(E_1) = P(E_2) = P(E_3) = P(E_4) = \frac{1}{4}$$

let W be the event that the ball drawn is white.

Then

$$P(W) = \sum P(E_i)P(W/E_i) = \frac{1}{4}\left(\frac{1}{4} + \frac{2}{4} + \frac{3}{4} + \frac{4}{4}\right) = \frac{5}{8}$$

$$\text{Now } P(E_4/W) = \frac{P(E_4)P(W/E_4)}{P(W)} = \frac{1/4}{5/8} = \frac{2}{5}$$

$$\therefore\ \frac{2}{5} = \frac{p}{15} \quad \Rightarrow \quad p = 6$$

(B) $^{12}c_1 + {}^{12}c_2\,({}^2c_1 + 2.\,{}^2c_2) + {}^{12}c_3\,({}^3C_1 + 2\,{}^3C_2) + ... + {}^{12}c_{12}\,({}^{12}c_1 + 2.\,{}^{12}c_2)$

$$= ({}^{12}c_1 + 2\,{}^{12}c_2 + 3\,{}^{12}c_3 + ... + 12\,{}^{12}c_2) + 2\,({}^{12}c_2\,.\,{}^2c_2 + {}^{12}c_3\,.\,{}^3c_2 + ... + {}^{12}c_{12}\,.\,{}^{12}c_2)$$

$$= \sum_{r=1}^{12} r\ {}^{12}c_r + 12 \times 11 \times \sum_{r=2}^{12} {}^{10}c_{r-2}$$

$= 12 \times 2^{11} + 12 \times 11 \times 2^{10}$

$= 12 \times 2^{10}\,(2 + 11) = 13 \times 2^{10} \times 12$

$\therefore\ 13 \times 2^{10} \times 12 = 13 \times 2^{10} \times m$

$\therefore\ m = 12$

(C) $$\frac{5x}{2-x} + \frac{5y}{2-y} + \frac{5z}{2-z} = 5\left[\frac{x}{2-x} + \frac{y}{2-y} + \frac{z}{2-z}\right]$$

$$= 5\left[\frac{x-2+2}{2-x} + \frac{y-2+2}{2-y} + \frac{z-2+2}{2-z}\right]$$

$$= 5\left[-3 + 2\left[\frac{1}{2-x} + \frac{1}{2-y} + \frac{1}{2-z}\right]\right]$$

Now $2 - x + 2 - y + 2 - z = 5$

$$\therefore\ \frac{5}{3} \geq \frac{3}{\frac{1}{2-x} + \frac{1}{2-y} + \frac{1}{2-z}}$$

$$\text{ie}\quad \frac{1}{2-x} + \frac{1}{2-y} + \frac{1}{2-z} \geq \frac{9}{5}$$

$$\text{Hence}\ \frac{5x}{2-x} + \frac{5y}{2-y} + \frac{5z}{2-z} \geq 5\left[-3 + 2.\frac{9}{5}\right] = 3$$

$\therefore$ least value is 3.

(D) $\sum_{k=1}^{12} 12 .K. {}^{12}c_k . {}^{11}c_{k-1} = 12^2 \sum_{k=1}^{12} ({}^{11}c_{k-1})^2 = 12^2 . \frac{22!}{11!11!}$

$= 12. \frac{21.19.......3.}{11!} . 2^{12}. 6$

$\therefore$ p = 6

23.

1. (C)

$P(E_1) = \frac{1}{10} \times 1 + \frac{2}{10} \times \frac{1}{2} + \frac{3}{10} \times \frac{1}{3} + \frac{4}{10} \times \frac{1}{4} = \frac{2}{5}$.

2. (B)

$$P(A_3/E_2) = \frac{\frac{3}{10}\times\frac{1}{3}}{\frac{2}{10}\times\frac{1}{2}+\frac{3}{10}\times\frac{1}{3}+\frac{4}{10}\times\frac{1}{4}} = \frac{1}{3}$$

3. (A)

$\text{Expectation} = \frac{4}{10} \times 1 + \frac{3}{10} \times 2 + \frac{2}{10} \times 3 + \frac{1}{10} \times 4 = 2.$

24.

1. (D)

A can be drawn out only at even numbered round. Therefore A will not be drained out at the 11th round.

2. (C)

To finish at the 12th round he must have exactly 1 head in the first 10 rounds, and a tail at the 11^{th} and the 12 th round. The probability of this is ${}^{10}C_1\, pq^{11}$.

3. (A)

To drain out at the 14^{th} round , two cases arise

(i) He gets exactly 2 heads in the first 10 rounds

$\therefore$ probability in this case is ${}^{10}C_2\, p^2 q^8 . q^4 = 45p^2q^{12}$

(ii) He gets exactly 1 head in the first 10 rounds and then exactly one head at the next two rounds

$\therefore$ probability in this case is ${}^{10}C_1\, pq^9 . {}^2C_1\, pq . q^2$

$= 20p^2q^{12}$

Therefore required probability $= 65\, p^2q^{12}$

25. $n = 10k + r, \quad k, r \in N, 0 \le r \le 9$

unit place of a^2 will contain 0, 1, 4, 5, 6, 9 only.

$\therefore$ $a^2 - 1$ is divisible by 10 only if unit place of a^2 contain 1.

If unit place of a^2 is 1

then unit place of a will be 1 or 9.

1. (A)

n = 10 k + r

r = 0

n = 10 k	no. of a whose unit place is 1 or 9
$\therefore$ k = 1, n = 10	no. of a whose unit place is = 2
$\therefore$ k = 2, n = 20	no. of a whose unit place is = 4
$\therefore$ k = k, n = 10k	no. of a whose unit place is = 2k

$\therefore p_n = \frac{2k}{n}$

2. (B)

n = 10 k + 9

no. of a whose unit place is 1 or 9 = 2 (k + 1)

$\therefore p_n = \frac{2(k+1)}{n}$

3. (C)

$\therefore$ no. of a whose unit place is 1

or 9 = 2k + 1.

$\therefore p_n = \frac{2k+1}{n}$

26. Let quadratic equation is $ax^2 + bx + c = 0$

Since $\alpha + \beta = \alpha^2 + \beta^2$ & $\alpha\beta = \alpha^2\beta^2$

$\Rightarrow$ $\alpha\beta = 0$ or $\alpha\beta = 1$

$\Rightarrow$ $\alpha = 0$ or $\beta = 0$ or $\alpha\beta = 1$

If $\alpha = 0, \beta = \beta^2$

$\Rightarrow$ $\beta = 0$ or 1

$\Rightarrow$ roots are (0,0) (0,1)

If $\beta = 0 \quad \alpha = \alpha^2$

$\Rightarrow$ $\alpha = 0$ or 1

$\Rightarrow$ roots are (0,0) (1,0)

When $\alpha\beta = 1$ $\alpha + \beta = (\alpha + \beta)^2 - 2\alpha\beta$

$\Rightarrow$ $(\alpha + \beta) = (\alpha + \beta)^2 - 2$

$\Rightarrow$ $(\alpha + \beta)^2 - (\alpha + \beta) - 2 = 0$

$\Rightarrow$ $\alpha + \beta = 2$ or $\alpha + \beta = -1$

When $\alpha + \beta = 2$ we get $\alpha = \beta = 1$

When $\alpha + \beta = -1$ we get $\alpha + \frac{1}{\alpha} = -1$ give imaginary roots

$\Rightarrow$ roots are (0,0) (1,0) (0,1) (1,1)

$\Rightarrow$ $P = \frac{2}{4} = \frac{1}{2}$

27. (5)

$\Rightarrow$ $P(A_1) \quad = P(\omega) + P(BB\omega) + P(BBBB\omega)$

$$= \frac{3}{10} + \frac{5}{10}\cdot\frac{4}{9}\cdot\frac{3}{8} + \frac{5}{10}\cdot\frac{4}{9}\cdot\frac{3}{8}\cdot\frac{2}{7}\cdot\frac{3}{6}$$

$$= \frac{3}{10} + \frac{1}{12} + \frac{1}{12\times 7} = \frac{332}{840} = \frac{83}{210}$$

$\Rightarrow$ $P(A_2) \quad = (B\omega) + (BBB\omega) + P(BBBBB\omega)$

$$= \frac{5}{10}\cdot\frac{3}{9} + \frac{5}{10}\cdot\frac{4}{9}\cdot\frac{3}{8}\cdot\frac{3}{7} + \frac{5}{10}\cdot\frac{4}{9}\cdot\frac{3}{8}\cdot\frac{2}{7}\cdot\frac{1}{6}\cdot\frac{3}{5}$$

$$= \frac{1}{6} + \frac{1}{28} + \frac{1}{420} = \frac{86}{420} = \frac{43}{210}$$

$$P(B) = 1 - P(A_1) - P(A_2) = \frac{2}{5}$$

28. $$P\left(\frac{A+}{R+}\right) = \frac{P(A+).P\left(\frac{R+}{A+}\right)}{P(A+)\,.\,P\left(\frac{R+}{A+}\right) + P(A-)\,.\,P\left(\frac{R+}{A-}\right)}$$

$A+$: A wrotes + sign

$A-$: A wrotes – sign

$R+$: Refree got + sign

$P\left(\frac{R+}{A+}\right)$ = No change or two change and 1 will remain

same $= {}^3C_0\left(\frac{1}{3}\right)^3 + {}^3C_2\left(\frac{2}{3}\right)^2\cdot\frac{1}{3}$

$P\left(\frac{R+}{A+}\right)$ = one change or all three change the sign

$$= {}^3C_1\,\frac{2}{3}\left(\frac{1}{3}\right)^2 + {}^3C_3\left(\frac{2}{3}\right)^3 = \frac{13}{41}$$

29. (4)

Let $A = \{a_1, a_2, ... a_n\}$. For each $a_i \in A (1 \le i \le n)$ we have the following four cases ;

(i) $a_i \in P$ and $a_i \in Q$ **(ii)** $a_i \notin P$ and $a_i \in Q$

(iii) $a_i \in P$ and $a_i \notin Q$ **(iv)** $a_i \notin P$ and $a_i \notin Q$

Thus the total number of ways of choosing P and Q is 4^n

$P \cap Q$ contains exactly two element in $({}^nC_2)(3^{n-2})$.

$\therefore$ Probability of $P \cap Q$ contains two elements

$$= \frac{{}^nC_2.3^{n-2}}{4^n}$$