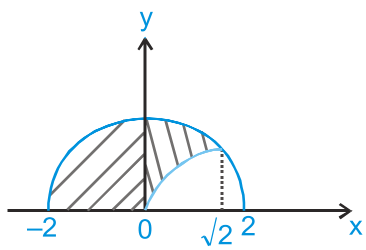
SOLVED EXAMPLES

SOLVED EXAMPLE

Ex. 1 The area enclosed by the curves $y = \sqrt{4-x^2}$, $y \geq \sqrt{2}\,\sin\left(\frac{x\pi}{2\sqrt{2}}\right)$ and x-axis is divided by y-axis in the ratio

Sol. $y = \sqrt{4-x^2}$, $y = \sqrt{2}\,\sin\left(\frac{x\pi}{2\sqrt{2}}\right)$



intersect at $x = \sqrt{2}$

Area of the left of y-axis is π

Area to the right of y-axis $= \int_0^{\sqrt{2}} \left(\sqrt{4-x^2} - \sqrt{2}\sin\frac{x\pi}{2\sqrt{2}}\right) dx$

$$= \left(\frac{x\sqrt{4-x^2}}{2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right)_0^{\sqrt{2}} + \frac{4}{\pi}\cos\left.\frac{x\pi}{2\sqrt{2}}\right|_0^{\sqrt{2}} = \left(1 + 2.\frac{\pi}{4}\right) + \frac{4}{\pi}(0-1)$$

$$= 1 + \frac{\pi}{2} - \frac{4}{\pi} = \frac{2\pi + \pi^2 - 8}{2\pi}$$

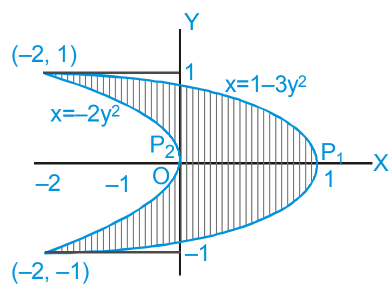
$\therefore$ ratio $= \dfrac{2\pi^2}{2\pi + \pi^2 - 8}$

Ex. 2 Compute the area of the figure bounded by the parabolas $x = -2y^2$, $x = 1 - 3y^2$.

Sol. Solving the equations $x = -2y^2$, $x = 1 - 3y^2$, we find that ordinates of the points of intersection of the two curves as $y_1 = -1$, $y_2 = 1$.

The points are (–2, –1) and (–2, 1).

The required area



$$2\int_0^1 (x_1 - x_2)\,dy = 2\int_0^1 [(1-3y^2) - (-2y^2)]dy$$

$$=2\int_0^1 (1-y^2)dy = 2\left[y - \frac{y^3}{3}\right]_0^1 = \frac{4}{3} \text{ sq.units.}$$

Ex. 3 The area cut off from a parabola by any double ordinate is k times the corresponding rectangle contained by the double ordinate and its distance from the vertex. Find the value of k ?

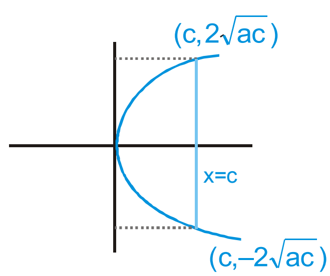
Sol. Consider $y^2 = 4ax$, $a > 0$ and $x = c$

Area by double ordinate $= 2\int_0^c 2\sqrt{a}\sqrt{x}\,dx = \frac{8}{3}\sqrt{a}\,c^{3/2}$

Area by double ordinate = k (Area of rectangle)

$$\frac{8}{3}\sqrt{a}\,c^{3/2} = k\,4\sqrt{a}\,c^{3/2}$$

$$k = \frac{2}{3}$$



Figure

Ex. 4 Find the area of the region common to the circle $x^2 + y^2 + 4x + 6y - 3 = 0$ and the parabola $x^2 + 4x = 6y + 14$.

Sol. Circle is $x^2 + y^2 + 4x + 6y - 3 = 0$

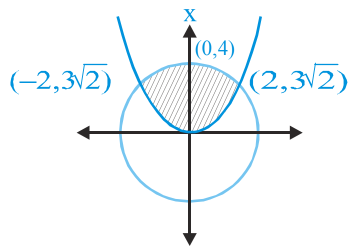
$\Rightarrow \quad (x+2)^2 + (y+3)^2 = 16$

Shifting origin to (–2,–3).

$X^2 + Y^2 = 16$

equation of parabola $\to (x+2)^2 = 6(y+3)$

$\Rightarrow \quad X^2 = 6Y$



Solving circle & parabola, we get $X = \pm 2\sqrt{3}$

Hence they intersect at $\left(-2\sqrt{3}, 2\right)$ & $\left(2\sqrt{3}, 2\right)$

$$A = 2\left[\int_0^2 \sqrt{6Y}\,dY + \int_2^4 \sqrt{16 - Y^2}\,dY\right]$$

$$= 2\left[\frac{2}{3}\sqrt{6}\left[Y^{3/2}\right]_0^2 + \left[\frac{1}{2}Y\sqrt{16 - Y^2} + \frac{16}{2}\sin^{-1}\frac{Y}{4}\right]_2^4\right] = \left(\frac{4\sqrt{3}}{3} + \frac{16\pi}{3}\right) \text{sq. units}$$

Ex. 5 If $f(x) = \sin x \ \forall\ x \in \left[0, \frac{\pi}{2}\right]$, $f(x) + f(\pi - x) = 2 \ \forall\ x \in \left(\frac{\pi}{2}, \pi\right]$ and $f(x) = f(2\pi - x)\ \forall\ x \in (\pi, 2\pi]$, then the area enclosed by $y = f(x)$ and x - axis is

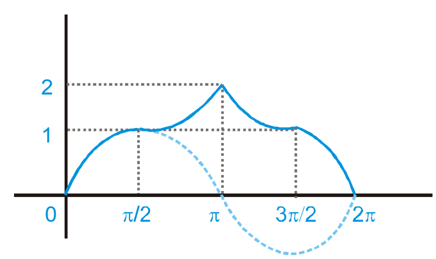
Sol. $f(x) = \sin x$

$f(x) + f(\pi - x) = 2$

$f(x) = 2 - f(\pi - x) = 2 - \sin(\pi - x) = 2 - \sin x \qquad x \in \left(\frac{\pi}{2}, \pi\right]$

$f(x) = f(2\pi - x)$

$\therefore \quad f(x + \pi) = f(\pi - x)$



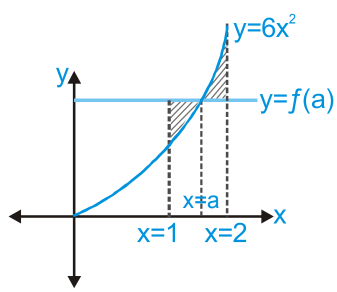
so curve is symmetric w.r.t. line $x = \pi$ for $(\pi, 2\pi]$

$f(x) = f(2\pi - x) = -\sin x$

$$\text{Area} = 2\left(\int_0^{\pi/2} \sin x\,dx \int_{\pi/2}^{\pi} (2 - \sin x)\,dx\right) = 2\left(1 + 2 \times \frac{\pi}{2} - 1\right) = 2\pi$$

Ex. 6 Find the value of 'a' for which area bounded by $x = 1$, $x = 2$, $y = 6x^2$ and $y = f(a)$ is minimum.

Sol. Let $b = f$ **(A)**.



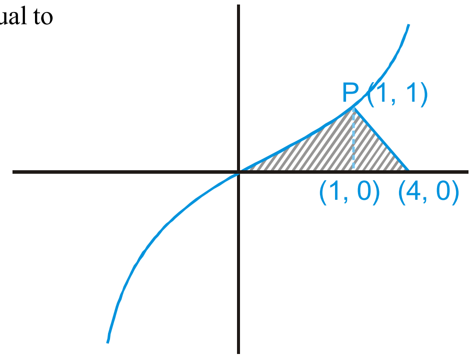
$$A = \int_1^a (b - 6x^2)dx + \int_a^2 (6x^2 - b)dx = \left|bx - 2x^3\right|_1^a + \left|2x^3 - bx\right|_a^2$$

$= 8a^3 - 18a^2 + 18$

For minimum area $\dfrac{dA}{da} = 0$

$\Rightarrow \quad 24a^2 - 36a = 0 \quad \Rightarrow \quad a = 1.5$

Ex. 7 The area enclosed by $y = x^3$, its normal at (1, 1) and x axis is equal to



Sol. $y = x^3,\ \frac{dy}{dx} = 3x^2$

$$\left(\frac{dy}{dx}\right)_{(1,1)} = 3$$

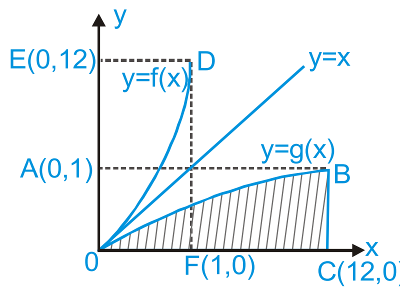
Normal at P (1, 1) is $y - 1 = -\frac{1}{3}(x-1)$

$3y + x = 4$ ..... **(1)**

So intersecting point of normal at x-axis is (4, 0)

$$\text{Area} = \int_0^1 x^3 dx + \frac{1}{2}(3 \times 1) = \left[\frac{x^4}{4}\right]_0^1 + \frac{3}{2} = \frac{7}{4}$$

Ex. 8 If y = g(x) is the inverse of a bijective mapping $f : R \to R$, $f(x) = 6x^5 + 4x^3 + 2x$, find the area bounded by g(x), the x-axis and the ordinate at x = 12.



Sol. $f(x) = 12$

$\Rightarrow \quad 6x^5 + 4x^3 + 2x = 12 \quad \Rightarrow \quad x = 1$

$$\int_0^{12} g(x)dx = \text{area of rectangle OEDF} - \int_0^1 f(x)dx$$

$$= 1 \times 12 - \int_0^1 (6x^5 + 4x^3 + 2x)dx = 12 - 3 = 9 \text{ sq. units.}$$

Ex. 9 For any real t, $x = \frac{1}{2}(e^t + e^{-t})$, $y = \frac{1}{2}(e^t - e^{-t})$ is point on the hyperbola $x^2 - y^2 = 1$. Show that the area bounded by the hyperbola and the lines joining its centre to the points corresponding to t_1 and $-t_1$ is t_1.

Sol. It is a point on hyperbola $x^2 - y^2 = 1$.

$$\text{Area (PQRP)} = 2\int_1^{\frac{e^{t_1}+e^{-t_1}}{2}} y\,dx = 2\int_1^{\frac{e^{t_1}+e^{-t_1}}{2}} \sqrt{x^2-1}\,dx$$

$$= 2\left[\frac{x}{2}\sqrt{x^2-1} - \frac{1}{2}\ell n(x + \sqrt{x^2-1})\right]_1^{\frac{e^{t_1}+e^{-t_1}}{2}}$$

$$= \frac{e^{2t_1} - e^{-2t_1}}{4} - t_1$$

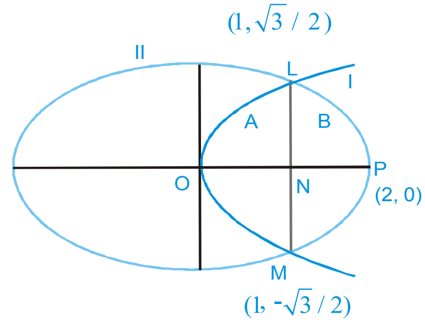
$$\text{Area of } \Delta OPQ = 2 \times \frac{1}{2}\left(\frac{e^{t_1}+e^{-t_1}}{2}\right)\left(\frac{e^{t_1}-e^{-t_1}}{2}\right)$$

$$= \frac{e^{2t_1} + e^{-2t_1}}{4}$$

$\therefore$ Required area = area ΔOPQ – area (PQRP)

$= t_1$

Ex. 10 Find the smaller of the areas bounded by the parabola $4y^2 - 3x - 8y + 7 = 0$ and the ellipse $x^2 + 4y^2 - 2x - 8y + 1 = 0$.



Sol. C_1 is $4(y^2 - 2y) = 3x - 7$

or $\quad 4(y-1)^2 = 3x - 3 = 3(x-1)$ .....**(i)**

Above is parabola with vertex at (1, 1)

C_2 is $(x^2 - 2x) + 4(y^2 - 2y) = -1$

or $\quad (x-1)^2 + 4(y-1)^2 = -1 + 1 + 4$

or $\quad \dfrac{(x-1)^2}{2^2} + \dfrac{(y-1)^2}{1^2} = 1$ ......**(ii)**

Above represents an ellipse with centre at (1, 1). Shift the origin to (1, 1) and this will not affect the magnitude of required area but will make the calculation simpler.

Thus the two curves are

$4Y^2 = 3X$ and $\dfrac{X^2}{2^2} + \dfrac{Y^2}{1} = 1$

They meet at $\left(1, \pm\dfrac{\sqrt{3}}{2}\right)$

Required area $= 2(A + B) = 2\left[\int Y_1 dX + \int Y_2 dX\right]$

$$= 2\left[\frac{\sqrt{3}}{2}\int_0^1 \sqrt{X}\,dX + \int_1^2 \frac{\sqrt{4 - X^2}}{2}\,dX\right] = \left[\frac{\sqrt{3}}{6} + \frac{2\pi}{3}\right] \text{ sq.units.}$$

Ex. 11 Find asymptotes of $y = x + \dfrac{1}{x}$ and sketch the curve (graph).

Sol. $\lim\limits_{x\to 0} y = \lim\limits_{x\to 0}\left(x + \dfrac{1}{x}\right) = +\infty$ or $-\infty$

$\Rightarrow \quad x = 0$ is asymptote.

$\lim\limits_{x\to 0} y = \lim\limits_{x\to 0}\left(x + \dfrac{1}{x}\right) = \infty$

$\Rightarrow \quad$ there is no asymptote of the type $y = k$

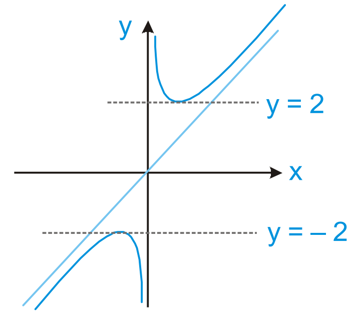
$\lim\limits_{x\to\infty} \dfrac{y}{x} = \lim\limits_{x\to\infty}\left(1 + \dfrac{1}{x^2}\right) = 1$

$\lim\limits_{x\to\infty} (y - x) = \lim\limits_{x\to\infty}\left(x + \dfrac{1}{x} - x\right) = \lim\limits_{x\to\infty} \dfrac{1}{x} = 0$

$\therefore \quad y = x + 0$

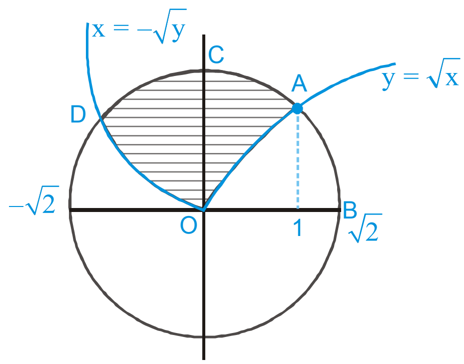
$\Rightarrow \quad y = x$ is asymptote.

A rough sketch is as follows



Ex. 12 Find the area bounded by the regions $y \geq \sqrt{x}$, $x > -\sqrt{y}$ & curve $x^2 + y^2 = 2$.

Sol. Common region is given by the diagram



If area of region OAB = λ

then area of OCD = λ

Because $y = \sqrt{x}$ & $x = -\sqrt{y}$

will bound same area with x & y axes respectively.

$y = \sqrt{x} \Rightarrow y^2 = x$

$x = -\sqrt{y} \Rightarrow x^2 = y$ and hence both the curves are

symmetric with respect to the line y = x

Area of first quadrant OBC = $\frac{\pi r^2}{4} = \frac{\pi}{2}$ $(\because\ r = \sqrt{2})$

Area of region OCA = $\frac{\pi}{2} - \lambda$

Area of shaded region = $(\frac{\pi}{2} - \lambda) + \lambda = \frac{\pi}{2}$ sq.units.

Ex. 13 The area of the figure bounded by the parabola $(y-2)^2 = x - 1$, the tangent to it at the point with the ordinate 3 and the x-axis is

Sol. The curve is $y^2 - 4y - x + 5 = 0$

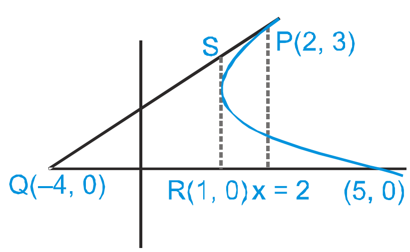
Equation of tangent at P(2, 3) is

$3y - 2(y+3) - \frac{1}{2}(x+2) + 5 = 0$

$y - 6 - \frac{1}{2}x - 1 + 5 = 0$

$x - 2y + 4 = 0$

if intersects x-axis at Q(–4, 0) and the line x = 1 at S$\left(1, \frac{5}{2}\right)$



$\therefore$ area of the $\Delta QRS = \frac{1}{2} \times 5 \times \frac{5}{2} = \frac{25}{4}$

$\therefore$ area of the bounded region = $\frac{25}{4} + \int_1^2 \left(\frac{x+4}{2} - \left(\sqrt{x-1} + 2\right)\right) dx + \int_1^5 \left(2 - \sqrt{x-1}\right) dx$

$$= \frac{25}{4} + \left(\frac{x^2}{4} - \frac{2}{3}(x-1)^{3/2}\right)\Bigg|_1^2 + \left(2x - \frac{2}{3}(x-1)^{3/2}\right)\Bigg|_1^5$$

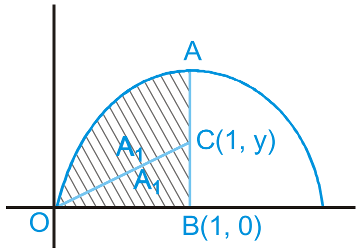
$$= \frac{25}{4} + 1 - \frac{2}{3} - \frac{1}{4} + 10 - \frac{16}{3} - 2$$

$$= 15 - 6 = 9$$

Ex. 14 Find the equation of line passing through the origin & dividing the curvilinear triangle with vertex at the origin, bounded by the curves $y = 2x - x^2$, $y = 0$ & $x = 1$ in two parts of equal areas.

Sol. Area of region OBA $= \int_0^1 (2x - x^2)dx$

$$= \left[x^2 - \frac{x^3}{3}\right]_0^1 = \frac{2}{3}$$



$\frac{2}{3} = A_1 + A_1 \quad \Rightarrow \quad A_1 = \frac{1}{3}$

Let pt. C has coordinates (1, y)

Area of $\Delta OCB = \frac{1}{2} \times 1 \times y = \frac{1}{3}$

$$y = \frac{2}{3}$$

C has coordinates $\left(1, \frac{2}{3}\right)$

Line OC has slope $m = \frac{\frac{2}{3} - 0}{1 - 0} = \frac{2}{3}$

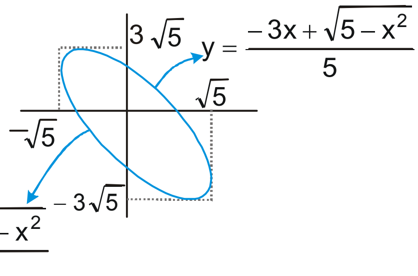
Equation of line OC is $y = mx \quad \Rightarrow \quad y = \frac{2}{3}x.$

Ex. 15 Find area contained by ellipse $2x^2 + 6xy + 5y^2 = 1$

Sol. $5y^2 + 6xy + 2x^2 - 1 = 0$

$$y = \frac{-6x \pm \sqrt{36x^2 - 20(2x^2 - 1)}}{10}$$

$$y = \frac{-3x \pm \sqrt{5 - x^2}}{5}$$



$\because$ y is real $\Rightarrow$ R.H.S. is also real.

$\Rightarrow \quad -\sqrt{5} \le x \le \sqrt{5}$

If $x = -\sqrt{5}$, $\quad y = 3\sqrt{5}$

If $x = \sqrt{5}$, $y = -3\sqrt{5}$

If $x = 0, \quad y = \pm\frac{1}{\sqrt{5}}$

If $y = 0, \quad x = \pm\frac{1}{\sqrt{2}}$

Required area $= \int_{-\sqrt{5}}^{\sqrt{5}} \left(\frac{-3x + \sqrt{5 - x^2}}{5} - \frac{-3x - \sqrt{5 - x^2}}{5}\right) dx$

$$= \frac{2}{5}\int_{-\sqrt{5}}^{\sqrt{5}} \sqrt{5 - x^2}\, dx \qquad = \frac{4}{5}\int_0^{\sqrt{5}} \sqrt{5 - x^2}\, dx$$

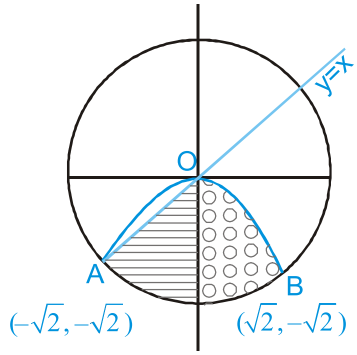
Put $x = \sqrt{5}\sin\theta : dx = \sqrt{5}\cos\theta\, d\theta$

L.L : $x = 0 \quad \Rightarrow \quad \theta = 0$

U.L : $x = \sqrt{5} \quad \Rightarrow \quad \theta = \frac{\pi}{2} \quad = \frac{4}{5}\int_{\theta=0}^{\frac{\pi}{2}} \sqrt{5 - 5\sin^2\theta}\ \sqrt{5}\cos\theta d\theta = 4\int_0^{\frac{\pi}{2}} \cos^2\theta d\theta = 4\ \frac{1}{2}\ \frac{\pi}{2} = \pi$

Ex. 16 Find the area bounded by the curves $x^2 + y^2 = 4$, $x^2 = -\sqrt{2}\,y$ and the line $x = y$, below x-axis.

Sol. Let C is $x^2 + y^2 = 4$, P is $y = -\frac{x^2}{\sqrt{2}}$ and L is $y = x$.



We have above three curves.

Solving P and C we get the points

$A(-\sqrt{2}, -\sqrt{2})$, $B(\sqrt{2}, -\sqrt{2})$

Also the line $y = x$ passes through $A(-\sqrt{2}, -\sqrt{2})$

$\therefore$ Required area = shaded + dotted

$$= \int_{-\sqrt{2}}^{0}(y_3 - y_1)dx + \int_{0}^{\sqrt{2}}(y_2 - y_1)dx$$

$$= \int_{-\sqrt{2}}^{0} x\,dx + \int_{0}^{\sqrt{2}} \frac{-x^2}{\sqrt{2}}dx - \int_{-\sqrt{2}}^{\sqrt{2}} \sqrt{4 - x^2}\,dx$$

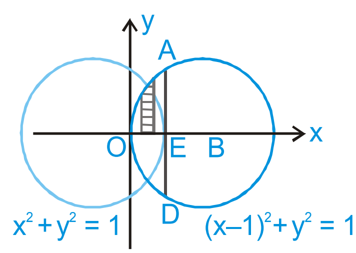
$$= \left[\frac{x^2}{2}\right]_{-\sqrt{2}}^{0} - \frac{1}{\sqrt{2}}\left[\frac{x^3}{3}\right]_{0}^{\sqrt{2}} - \left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_{0}^{\sqrt{2}}$$

$\therefore$ $|A| = \frac{3\pi + 16}{6}$ sq.units.

Ex. 17 The area of the region enclosed between the two circles $x^2 + y^2 = 1$ and $(x - 1)^2 + y^2 = 1$, is

Sol. Solving the given equation of circle, we get

$A \equiv \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$; $D \equiv \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$



Now area = 2[OBAO] = 2[area OEAO + EBAE]

$$= 2\left[\int_{0}^{x_E} \sqrt{[1-(x-1)^2]}\,dx + \int_{x_E}^{x_B} \sqrt{1-x^2}\,dx\right]$$

$$= 2\left[\int_{0}^{1/2} \sqrt{1-(x-1)^2}\,dx + \int_{1/2}^{1} \sqrt{1-x^2}\,dx\right] = \frac{2\pi}{3} - \frac{\sqrt{3}}{2} \text{ square units}$$

Ex. 18 Find the area contained between the two arms of curves $(y - x)^2 = x^3$ between $x = 0$ and $x = 1$.

Sol. $(y - x)^2 = x^3 \Rightarrow y = x \pm x^{3/2}$

For arm

$y = x + x^{3/2} \quad \Rightarrow \quad \frac{dy}{dx} = 1 + \frac{3}{2}x^{1/2} > 0 \quad x \geq 0.$

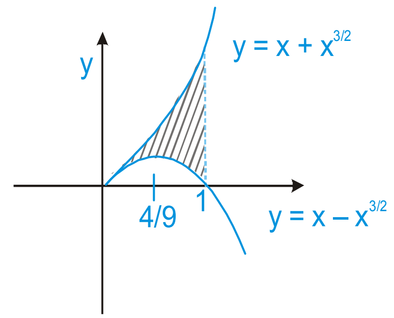
y is increasing function.

For arm

$y = x - x^{3/2} \quad \Rightarrow \quad \frac{dy}{dx} = 1 - \frac{3}{2}x^{1/2}$

$\frac{dy}{dx} = 0 \Rightarrow x = \frac{4}{9}, \frac{d^2y}{dx^2} = -\frac{3}{4}x^{-\frac{1}{2}} < 0$ at $x = \frac{4}{9}$

$\therefore$ at $x = \frac{4}{9}$, $y = x - x^{3/2}$ has maxima.



Required area $= \int_0^1 (x + x^{3/2} - x + x^{3/2})\,dx$

$= 2\int_0^1 x^{3/2}dx = \left.\frac{2\,x^{5/2}}{5/2}\right]_0^1 = \frac{4}{5}$

Ex. 19 Let A (m) be area bounded by parabola $y = x^2 + 2x - 3$ and the line $y = mx + 1$. Find the least area A(m).

Sol. Solving we obtain

$x^2 + (2 - m)x - 4 = 0$

Let α,β be roots $\Rightarrow \alpha + \beta = m - 2$, $\alpha\beta = -4$

$$A(m) = \left|\int_\alpha^\beta (mx + 1 - x^2 - 2x + 3)\,dx\right|$$

$$= \left|\int_\alpha^\beta (-x^2 + (m-2)x + 4)\,dx\right|$$

$$= \left|\left(-\frac{x^3}{3} + (m-2)\frac{x^2}{2} + 4x\right)_\alpha^\beta\right|$$

$$= \left|\frac{\alpha^3 - \beta^3}{3} + \frac{m-2}{2}(\beta^2 - \alpha^2) + 4(\beta - \alpha)\right|$$

$$= |\beta - \alpha|.\left|-\frac{1}{3}(\beta^2 + \beta\alpha + \alpha^2) + \frac{(m-2)}{2}(\beta + \alpha) + 4\right|$$

$$= \sqrt{(m-2)^2 + 16}\left|-\frac{1}{3}\left((m-2)^2 + 4\right) + \frac{(m-2)}{2}(m-2) + 4\right|$$

$$= \sqrt{(m-2)^2 + 16}\left|\frac{1}{6}(m-2)^2 + \frac{8}{3}\right|$$

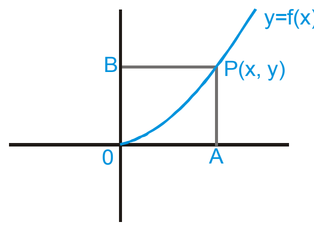
$$A(m) = \frac{1}{6}\left((m-2)^2 + 16\right)^{3/2}$$

Least $A(m) = \frac{1}{6}(16)^{3/2} = \frac{32}{3}$.

Ex. 20 A curve $y = f(x)$ passes through the origin and lies entirely in the first quadrant. Through any point P(x, y) on the curve, lines are drawn parallel to the coordinate axes. If the curve divides the area formed by these lines and coordinate axes in m : n, then show that $f(x) = cx^{m/n}$ or $f(x) = cx^{n/m}$ (c-being arbitrary).

Sol. Area (OAPB) = xy

$$\text{Area (OAPO)} = \int_0^x f(t)\,dt$$



$$\text{Area (OPBO)} = xy - \int_0^x f(t)\,dt$$

$$\frac{\text{Area (OAPO)}}{\text{Area (OPBO)}} = \frac{m}{n}$$

$$n\int_0^x f(t)\,dt = m\left(xy - \int_0^x f(t)\,dt\right)$$

$$n\int_0^x f(t)\,dt = mx\,f(x) - m\int_0^x f(t)\,dt$$

Differentiating w.r.t. x

$nf(x) = m\,f(x) + mx\,f'(x) - m\,f(x)$

$$\frac{f'(x)}{f(x)} = \frac{n}{m}\frac{1}{x}$$

$f(x) = cx^{n/m}$

similarly $f(x) = cx^{m/n}$

Ex. 21 The area bounded by $y = x^2 + 1$ and the tangents to it drawn from the origin is :-

Sol. The parabola is even function & let the equation of tangent is y= mx

Now we calculate the point of intersection of parabola & tangent

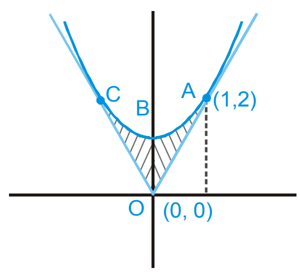
$mx = x^2 + 1$

$x^2 - mx + 1 = 0 \quad \Rightarrow \quad D = 0$

$\Rightarrow \quad m^2 - 4 = 0 \quad \Rightarrow \quad m = \pm 2$

Two tangents are possible $y = 2x$ & $y = -2x$

Intersection of $y = x^2 + 1$ & $y = 2x$ is $x = 1$ & $y = 2$



$$\text{Area of shaded region OAB} = \int_0^1 (y_2 - y_1)\,dx = \int_0^1 \left((x^2+1) - 2x\right)dx = \frac{1}{3} \text{ sq. units}$$

$$\text{Area of total shaded region} = 2\left(\frac{1}{3}\right) = \frac{2}{3} \text{ sq. units}$$

Ex. 22 **STATEMENT-1 :** The area bounded by the curve $|x| + |y| = a$ $(a > 0)$ is $2a^2$ and area bounded $|px + qy| + |qx - py| = a$, where $p^2 + q^2 = 1$, is also $2a^2$.

STATEMENT-2 : Since $\alpha x + \beta y = 0$ is perpendicular to $\beta x - \alpha y = 0$, we can take one as x-axis and another as y-axis and therefore the area bounded by $|\alpha x + \beta y| + |\beta x - \alpha y| = a$ is $2a^2$ for all $\alpha, \beta \in R, \alpha \neq 0, \beta \neq 0$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

Sol. Statement-1 Let $\frac{p}{\sqrt{p^2+q^2}}x+\frac{q}{\sqrt{p^2+q^2}}y=U$ and $\frac{q}{\sqrt{p^2+q^2}}x-\frac{p}{\sqrt{p^2+q^2}}y=V$

Then the axis get rotated through an angle θ, where $\cos\theta=\frac{p}{\sqrt{p^2+q^2}}$ and $\sin\theta=\frac{q}{\sqrt{p^2+q^2}}$

∴ the equation of the given curve becomes $|U|+|V|=a$

∴ the area bounded $=2a^2$.

∴ statement-1 is true

Statement-2 the equation of the curve is $|\alpha x+\beta y|+|\beta x-\alpha y|=a$ which is equivalent to

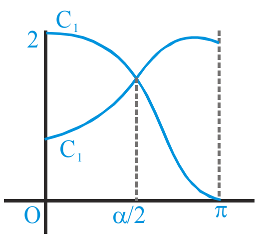
$$\left|\frac{\alpha}{\sqrt{\alpha^2+\beta^2}}x+\frac{\beta}{\sqrt{\alpha^2+\beta^2}}y\right|+\left|\frac{\beta}{\sqrt{\alpha^2+\beta^2}}x-\frac{\alpha}{\sqrt{\alpha^2+\beta^2}}y\right|=\frac{a}{\sqrt{\alpha^2+\beta^2}}$$

∴ area bounded $=\frac{2a^2}{\alpha^2+\beta^2}$

∴ statement-2 is false.

Ex. 23 Determination of unknown parameter

Consider the two curves $C_1: y=1+\cos x$ & $C_2: y=1+\cos(x-\alpha)$ for $\alpha\in\left(0,\frac{\pi}{2}\right); x\in[0,\pi]$. Find the value of α, for which the area of the figure bounded by the curves C_1, C_2 & $x=0$ is same as that of the figure bounded by C_2, $y=1$ & $x=\pi$.



Sol. $1+\cos x=1+\cos(x-\alpha)\Rightarrow x=\frac{\alpha}{2}$

$$A_1=\int_0^{\alpha/2}(1+\cos x)-(1+\cos(x-\alpha))dx$$

$$=|\sin x-\sin(x-\alpha)|_0^{\alpha/2}=2\sin\frac{\alpha}{2}-\sin\alpha$$

$1+\cos(x-\alpha)=1\Rightarrow x=\alpha+\frac{\pi}{2}$

$$A_2=\int_{\alpha+\frac{\pi}{2}}^{\pi}\left(1+\cos(x-\alpha)-1\right)dx=\left|\sin(x-\alpha)\right|_{\alpha+\frac{\pi}{2}}^{\pi}$$

$$=|\sin\alpha-1|=1-\sin\alpha$$

$A_1=A_2\Rightarrow 2\sin\frac{\alpha}{2}-\sin\alpha=1-\sin\alpha$

$\alpha=\frac{\pi}{3}$

24. Comprehension Type

Asymptotes are the tangents to the curve at infinity

To find the asymptotes of a curve we can use the following methods.

(A) Asymptote parallel to the x-axis is obtained by equating to zero, the coefficient of the highest power to x.

(B) Asymptote parallel to the y-axis is obtained by equating to zero, the coefficient of the highest power of y.

(C) Oblique Asymptote : $y=mx+c$

(i) Find ϕ_n (m) by putting x = 1 and y = m in the highest degree (n) terms of the equation similarly find $\phi_{n-1}(m)$.

(ii) Solve $\phi_n(m) = 0$ for m

(iii) Find c by the formula $c = -\dfrac{\phi_{n-1}(m)}{\phi'_n(m)}$ Using the value of m as obtained in **(ii)**

(iv) Obtain the equation of asymptote by putting these values of m and c in y = mx + c.

1. The equation of asymptotes of the curve $yx^2 - 4x^2 + x + 2 = 0$

(A) $y - 4 = 0$ and $x = 0$ **(B)** $y = 3$ and $x = 2$

(C) $y - 4 = 0$ and $x = 2$ **(D)** $y = 3$ and $x = 0$

Sol. $(y - 4)\, x^2 + x + 2 = 0$

the coefficient of the highest power of x i.e. x^2 is $y - 4 = 0$

$y - 4 = 0$ is the asymptote parallel to the x-axis.

The coefficient of the highest power of y is x, so x = 0 is also a asymptotes.

2. The equation of asymptotes of the curve $x^3 + y^3 - 3xy = 0$

(A) $y = x + 1$ **(B)** $y + x + 1 = 0$ **(C)** $y + x = 2$ **(D)** $y = 2x + 1$

Sol. $\phi_3(m) = 1 + m^3, \phi_2(m) = -3m$

Putting $\phi_3(m) = 0$ or $m^3 + 1 = 0$

or $(m + 1)(m^2 - m + 1) = 0$

$$m = -1,\ m = \frac{1 \pm \sqrt{1-4}}{2}$$

Only real value of m is – 1

Now we find c from the equation $c = -\dfrac{\phi_{n-1}(m)}{\phi'_n(m)}$

$$c = \frac{3m}{3m^2} = \frac{1}{m} = -1$$

On putting m = – 1 and c = – 1 in y = mx + c.

The equation of asymptote is $y = (-1)\,x + (-1)$ or $x + y + 1 = 0$

Exercise # 1

[Single Correct Choice Type Questions]

1. The area of the figure bounded by the curves $y = \ell nx$ & $y = (\ell nx)^2$ is -

(A) e + 1 **(B)** e – 1 **(C)** 3 – e **(D)** 1

2. Suppose y = f (x) and y = g(x) are two functions whose graphs intersect at the three points (0, 4), (2, 2) and (4, 0) with f(x) > g(x) for 0 < x < 2 and f(x) < g(x) for 2 < x < 4. If $\int_0^4 [f(x)-g(x)]dx = 10$ and $\int_2^4 [g(x)-f(x)]dx = 5$, the area between two curves for 0 < x < 2, is

(A) 5 **(B)** 10 **(C)** 15 **(D)** 20

3. The area bounded by the curve $x = a\cos^3 t$, $y = a\sin^3 t$ is

(A) $\frac{3\pi a^2}{8}$ **(B)** $\frac{3\pi a^2}{16}$ **(C)** $\frac{3\pi a^2}{32}$ **(D)** $3\pi a^2$

4. Let 'a' be a positive constant number. Consider two curves C_1: $y = e^x$, C_2 : $y = e^{a-x}$. Let S be the area of the part surrounding by C_1, C_2 and the y-axis, then $\lim_{a\to 0} \frac{S}{a^2}$ equals

(A) 4 **(B)** 1/2 **(C)** 0 **(D)** 1/4

5. Suppose y = f(x) and y = g(x) are two functions whose grahps intersect at three points (0, 4), (2, 2) and (4, 0) with f(x) > g(x) for 0 < x < 2 and f(x) < g(x) for 2 < x < 4.

If $\int_0^4 [f(x)-g(x)]dx = 10$ and $\int_2^4 [g(x)-f(x)]dx = 5$, the area between two curves for 0 < x < 2, is -

(A) 5 **(B)** 10 **(C)** 15 **(D)** 20

6. 3 points O(0, 0), P(a, a^2), Q(–b, b^2) (a > 0, b > 0) are on the parabola $y = x^2$. Let S_1 be the area bounded by the line PQ and the parabola and let S_2 be the area of the triangle OPQ, the minimum value of S_1/S_2 is

(A) 4/3 **(B)** 5/3 **(C)** 2 **(D)** 7/3

7. The area bounded by the curve $y = \frac{1}{x^2}$ and its asymptote from x = 1 to x = 3 is

(A) $\frac{1}{3}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{1}{6}$

8. The area of the region(s) enclosed by the curves $y = x^2$ and $y = \sqrt{|x|}$ is

(A) 1/3 **(B)** 2/3 **(C)** 1/6 **(D)** 1

9. The area of the closed figure bounded by y = x, y = –x & the tangent to the curve $y = \sqrt{x^2 - 5}$ at the point (3, 2) is -

(A) 5 **(B)** $2\sqrt{5}$ **(C)** 10 **(D)** $\frac{5}{2}$

10. The area bounded by the curve y = f(x), the x-axis & the ordinates x = 1 & x = b is (b – 1) sin (3b + 4). Then f(x) is :

(A) (x – 1) cos (3x + 4) **(B)** sin (3x + 4)

(C) sin (3x + 4) + 3 (x – 1) . cos (3x + 4) **(D)** none

11. The ratio in which the curve $y = x^2$ divides the region bounded by the curve; $y = \sin\left(\frac{\pi x}{2}\right)$ and the x–axis as x varies from 0 to 1, is :
(A) $2 : \pi$ (B) $1 : 3$ (C) $3 : \pi$ (D) $(6-\pi) : \pi$

12. A curve is such that the area of the region bounded by the co-ordinate axes, the curve & the ordinate of any point on it is equal to the cube of that ordinate. The curve represents
(A) a pair of straight lines (B) a circle
(C) a parabola (D) an ellipse

13. The area enclosed by the curves $y = \cos x$, $y = 1 + \sin 2x$ and $x = \frac{3\pi}{2}$ as x varies from 0 to $\frac{3\pi}{2}$, is -
(A) $\frac{3\pi}{2} - 2$ (B) $\frac{3\pi}{2}$ (C) $2 + \frac{3\pi}{2}$ (D) $1 + \frac{3\pi}{2}$

14. Area enclosed by the graph of the function $y = \ln^2 x - 1$ lying in the 4th quadrant is
(A) $\frac{2}{e}$ (B) $\frac{4}{e}$ (C) $2\left(e + \frac{1}{e}\right)$ (D) $4\left(e - \frac{1}{e}\right)$

15. The area bounded in the first quadrant between the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and the line $3x + 4y = 12$ is:
(A) $6(\pi - 1)$ (B) $3(\pi - 2)$ (C) $3(\pi - 1)$ (D) none

16. The area bounded by $y = 2 - |2 - x|$ & $y = \frac{3}{|x|}$ is :
(A) $\frac{4 + 3\ell n 3}{2}$ (B) $\frac{4 - 3\ell n 3}{2}$ (C) $\frac{3}{2} + \ln 3$ (D) $\frac{1}{2} + \ln 3$

17. Consider two curves $C_1 : y = \frac{1}{x}$ and $C_2 : y = \ell nx$ on the xy plane. Let D_1 denotes the region surrounded by C_1, C_2 and the line $x = 1$ and D_2 denotes the region surrounded by C_1, C_2 and the line $x = a$. If $D_1 = D_2$ then the value of 'a' -
(A) $\frac{e}{2}$ (B) e (C) $e - 1$ (D) $2(e - 1)$

18. The area bounded by the curve $y = f(x)$, the co-ordinate axes & the line $x = x_1$ is given by $x_1 \cdot e^{x_1}$. Therefore $f(x)$ equals :
(A) e^x (B) $x e^x$ (C) $xe^x - e^x$ (D) $x e^x + e^x$

19. If $f(x) = \sin x$, $\forall\ x \in \left[0, \frac{\pi}{2}\right]$, $f(x) + f(\pi - x) = 2$. $\forall\ x \in \left(\frac{\pi}{2}, \pi\right]$ and $f(x) = f(2\pi - x)$, $\forall\ x \in (\pi, 2\pi]$, then the area enclosed by $y = f(x)$ and x-axis is
(A) π (B) 2π (C) 2 (D) 4

20. Suppose $g(x) = 2x + 1$ and $h(x) = 4x^2 + 4x + 5$ and $h(x) = (fog)(x)$. The area enclosed by the graph of the function $y = f(x)$ and the pair of tangents drawn to it from the origin, is
(A) 8/3 (B) 16/3 (C) 32/3 (D) none

21. The area of the region for which $0 < y < 3 - 2x - x^2$ & $x > 0$ is -

(A) $\int_1^3 (3-2x-x^2)dx$ **(B)** $\int_0^3 (3-2x-x^2)dx$ **(C)** $\int_0^1 (3-2x-x^2)dx$ **(D)** $\int_1^3 (3-2x-x^2)dx$

22. The area bounded by the curves $y = -\sqrt{-x}$ and $x = -\sqrt{-y}$ where $x, y \le 0$

(A) cannot be determined

(B) is 1/3

(C) is 2/3

(D) is same as that of the figure bounded by the curves $y = \sqrt{-x}$; $x \le 0$ and $x = \sqrt{-y}$; $y \le 0$

23. The area bounded by the curve $y = f(x)$, x-axis and the ordinates $x = 1$ and $x = b$ is $(b - 1) \sin(3b + 4)$, $\forall\, b \in R$, then $f(x) =$

(A) $(x - 1) \cos(3x + 4)$ **(B)** $\sin(3x + 4)$

(C) $\sin(3x + 4) + 3(x - 1) \cos(3x + 4)$ **(D)** none of these

24. Area of the region enclosed between the curves $x = y^2 - 1$ and $x = |y| \sqrt{1-y^2}$ is

(A) 1 **(B)** 4/3 **(C)** 2/3 **(D)** 2

25. The area bounded by the curves $y = x(1 - \ell nx)$ and positive x-axis between $x = e^{-1}$ and $x = e$ is -

(A) $\left(\frac{e^2 - 4e^{-2}}{5}\right)$ **(B)** $\left(\frac{e^2 - 5e^{-2}}{4}\right)$ **(C)** $\left(\frac{4e^2 - e^{-2}}{5}\right)$ **(D)** $\left(\frac{5e^2 - e^{-2}}{4}\right)$

26. Area enclosed by the curves $y = \ln x$; $y = \ln|x|$; $y = |\ln x|$ and $y = |\ln|x||$ is equal to

(A) 2 **(B)** 4 **(C)** 8 **(D)** cannot be determined

27. Let $f:[0, \infty) \to R$ be a continuous and strictly increasing function such that $f^3(x) = \int_0^x t f^2(t)\,dt$, $\forall\, x \ge 0$. The area enclosed by $y = f(x)$, the x-axis and the ordinate at $x = 3$ is

(A) $\frac{3}{2}$ **(B)** $\frac{5}{2}$ **(C)** $\frac{7}{2}$ **(D)** none of htese

28. If $(a, 0)$; $a > 0$ is the point where the curve $y = \sin 2x - \sqrt{3} \sin x$ cuts the x-axis first, A is the area bounded by this part of the curve , the origin and the positive x-axis, then

(A) $4A + 8 \cos a = 7$ **(B)** $4A + 8 \sin a = 7$

(C) $4A - 8 \sin a = 7$ **(D)** $4A - 8 \cos a = 7$

29. Area of the curve $y^2 = (7 - x)(5 + x)$ above x–axis and between the ordinates $x = -5$ and $x = 1$ is

(A) 9π **(B)** 18π **(C)** 15π **(D)** none

30. A function $y = f(x)$ satisfies the differential equation, $\frac{dy}{dx} - y = \cos x - \sin x$, with initial condition that y is bounded when $x \to \infty$. The area enclosed by $y = f(x)$, $y = \cos x$ and the y-axis in the 1st quadrant is-

(A) $\sqrt{2} - 1$ **(B)** $\sqrt{2}$ **(C)** 1 **(D)** $\frac{1}{\sqrt{2}}$

Exercise # 2 Part # I [Multiple Correct Choice Type Questions]

1. Let 'a' be a positive constant number. Consider two curves $C_1 : y = e^x$, $C_2 : y = e^{a-x}$. Let S be the area of the part surrounding by C_1, C_2 and the y-axis, then -

(A) $\lim\limits_{a\to-\infty} S = 1$ **(B)** $\lim\limits_{a\to 0} \frac{S}{a^2} = \frac{1}{4}$

(C) Range of S is $[0,\infty)$ **(D)** S(a) is neither odd nor even

2. If $C_1 \equiv y = \frac{1}{1+x^2}$ and $C_2 \equiv y = \frac{x^2}{2}$ are two curves lying in the XY plane. Then -

(A) area bounded by curve C_1 and $y = 0$ is π **(B)** area bounded by C_1 and C_2 is $\frac{\pi}{2} - \frac{1}{3}$

(C) area bounded by C_1 and C_2 is $1 - \frac{\pi}{2}$ **(D)** area bounded by curve C_1 and x-axis is $\frac{\pi}{2}$

3. Consider $f(x) = \begin{cases} \cos x & 0 \le x < \frac{\pi}{2} \\ \left(\frac{\pi}{2} - x\right)^2 & \frac{\pi}{2} \le x < \pi \end{cases}$ such that f is periodic with period π, then

(A) The range of f is $\left[0, \frac{\pi^2}{4}\right)$

(B) f is continuous for all real x, but not differentiable for some real x

(C) f is continuous for all real x

(D) The area bounded by $y = f(x)$ and the X-axis from $x = -n\pi$ to $x = n\pi$ is $2n\left(1 + \frac{\pi^3}{24}\right)$ for a given $n \in N$

4. Area enclosed by the curve $y = \sin x$ between $x = 2n\pi$ to $x = 2(n+1)\pi$ is-

(A) $\int\limits_0^{2\pi} \sin x \, dx$ **(B)** $2\int\limits_0^{\pi} \sin x \, dx$ **(C)** $4\int\limits_0^{\pi/2} \sin x \, dx$ **(D)** 4

5. Let T be the triangle with vertices (0, 0), $(0, c^2)$ and (c, c^2) and let R be the region between $y = cx$ and $y = x^2$ where $c > 0$ then -

(A) Area (R) $= \frac{c^3}{6}$ **(B)** Area of R $= \frac{c^3}{3}$ **(C)** $\lim\limits_{c\to 0^+} \frac{\text{Area}(T)}{\text{Area}(R)} = 3$ **(D)** $\lim\limits_{c\to 0^+} \frac{\text{Area}(T)}{\text{Area}(R)} = \frac{3}{2}$

6. Let $f(x) = |x| - 2$ and $g(x) = |f(x)|$.

Now area bounded by x-axis and f(x) is A_1 and area bounded by x-axis and g(x) is A_2 then –

(A) $A_1 = 3$ **(B)** $A_1 = A_2$ **(C)** $A_2 = 4$ **(D)** $A_1 + A_2 = 8$

7. If (a, 0) & (b,0) [a,b > 0] are the points where the curve $y = \sin 2x - \sqrt{3}\sin x$ cuts the positive x-axis first & second time, A & B are the areas bounded by the curve & positive x-axis between x=0 to x=a and x = a to x=b respectively, then -

(A) $4A + 8\cos a = 7$ **(B)** $AB = \frac{1}{16}$ **(C)** $4A + 4B + 14\cos b = 0$ **(D)** $B - A = 4\cos a$

8. Consider the following regions in the plane :

$$R_1 = \{(x, y) : 0 \le x \le 1 \text{ and } 0 \le y \le 1\} \text{ and } R_2 = \{(x, y) : x^2 + y^2 \le 4/3\}$$

The area of the region $R_1 \cap R_2$ can be expressed as $\dfrac{a\sqrt{3} + b\pi}{9}$, where a and b are integers, then -

(A) $a = 3$ **(B)** $a = 1$ **(C)** $b = 1$ **(D)** $b = 3$

9. Consider the functions f (x) and g (x), both defined from $R \to R$ and are defined as $f(x) = 2x - x^2$ and $g(x) = x^n$ where $n \in N$. If the area between f (x) and g (x) is 1/2 then *n* is a divisor of

(A) 12 **(B)** 15 **(C)** 20 **(D)** 30

10. The area of the region of the plane bounded by $(|x|, |y|) \le 1$ & $xy \le \dfrac{1}{2}$ is -

(A) less then $4\ell n\, 3$ **(B)** $\dfrac{15}{4}$ **(C)** $2 + 2\ell n 2$ **(D)** $3 + \ell n 2$

11. For which of the following values of m, is the area of the region bounded by the curve $y = x - x^2$ and the line $y = mx$ equals to 9/2 ?

(A) –4 **(B)** –2 **(C)** 2 **(D)** 4

Part # II [Assertion & Reason Type Questions]

These questions contains, Statement I (assertion) and Statement II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.

(C) Statement-I is true, Statement-II is false.

(D) Statement-I is false, Statement-II is true.

1. **Statement-I :** The area of the curve $y = \sin^2 x$ from 0 to π will be more than that of curve $y = \sin x$ from 0 to π.

Statement-II : $t^2 > t$ if $t \in R - [0, 1]$.

2. **Statement-I :** $\int_0^{10\pi} |\cos x|\, dx = 20$

Statement-II : $\int_a^b f(x)\, dx \geq 0$, then $f(x) \geq 0, \forall x \in (a, b)$

3. **Statement-I :** The area bounded by the curves $y = x^2 - 3$ and $y = kx + 2$ is the least, if $k = 0$.

Statement-II : The area bounded by the curves $y = x^2 - 3$ and $y = kx + 2$ is $\sqrt{k^2 + 20}$.

4. **Statement-I :** Area bounded by $y = \tan x$, $y = \tan^2 x$ in between $x \in \left(0, \frac{\pi}{4}\right)$ is equal to $\left(\frac{\pi}{4} + \ell n\sqrt{2} - 1\right)$.

Statement-II : Area bounded by $y = f(x)$ and $y = g(x)$ $\{f(x) > g(x)\}$ between $x = a, x = b$ is $\int_a^b (f(x) - g(x))\, dx$.

$(b > a)$

5. Consider the two curves $y = x - \lambda x^2$ and $y = \frac{x^2}{\lambda}$, $(\lambda > 0)$.

Statement-I : The area bounded between the curves is maximum when $\lambda = 1$.

Statement-II : The area bounded between the curves is $\frac{\lambda^2}{(1+\lambda^2)^2}$ square units.

Exercise # 3 Part # I [Matrix Match Type Questions]

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with one or more statement(s) in **Column-II**.

1. Let f(x) = |x|, g(x) = |x – 1| and h(x) = |x + 1|.

	Column-I		Column-II
(A)	Area bounded by min (f(x), g(x)) and x-axis is	**(p)**	$\frac{1}{8}$ sq. unit
(B)	Area bounded by min (f(x), h(x)) and x-axis is	**(q)**	$\frac{1}{4}$ sq. unit
(C)	Area bounded by min ((f(x), g(x), h(x)) and x-axis is	**(r)**	$\frac{1}{2}$ sq. unit
(D)	Area bounded by min (f(x), g(x), h(x)) and $y = \frac{1}{2}$ is	**(s)**	$\frac{3}{4}$ sq. unit

2.

	Column – I		Column – II
(A)	Area bounded by region $0 \le y \le 4x - x^2 - 3$ is	**(p)**	32/3
(B)	Area of the region enclosed by $y^2 = 8x$ and $y = 2x$ is	**(q)**	1/2
(C)	The area bounded by $\|x\| + \|y\| \le 1$ and $\|x\| \ge 1/2$ is	**(r)**	8/3
(D)	Area bounded by $x \le 4 - y^2$ and $x \ge 0$ is	**(s)**	4/3

3.

	Column I		Column – II
(A)	The area bounded by the curve $x = 3y^2 - 9$ and the lines $x = 0$, $y = 0$ and $y = 1$ in square units is equal to	**(p)**	1
(B)	If a curve $f(x) = a\sqrt{x} + bx$, ($f(x) \ge 0\ \forall\ x \in [0, 9]$) passes through the point (1, 2) and the area bounded by the curve, line x = 4 and x-axis is 8 square unit, then 2a + b is equal to	**(q)**	4
(C)	The area enclosed between the curves $y = \sin^2 x$ and $y = \cos^2 x$ in the interval $0 \le x \le \pi$ in square units in equal to	**(r)**	8
(D)	The area bounded by the curve $y^2 = 16x$ and line $y = mx$ is $\frac{2}{3}$ square units, then m is equal to	**(s)**	5

Part # II [Comprehension Type Questions]

Comprehension # 1

Consider two curves $y=\frac{1}{x^2}$ and $y=\frac{1}{4(x-1)}$. 'a' is a number such that a > 2 & the reciprocal of the area of the figure bounded by the curves, the line x = 2 & x = a is a itself. 'b' is a number such that 1 < b < 2 & the area bounded by the two curves & the lines x = b & x = 2 is equal to $1-\frac{1}{b}$.

On the basis of above information, answer the following questions :

1. The value of $\ell na - \ell nb$ is -

(A) positive integer

(B) negative integer

(C) rational number of the form $\frac{p}{q}$, where p, q are co-prime & q > 1.

(D) irrational number

2. If $A=\begin{bmatrix}\ell n(a-1) & 0\\ 0 & \ell n(b-1)\end{bmatrix}$, then A^{-1} is -

(A) $-\frac{A}{4}$ **(B)** A **(C)** 4A **(D)** $\frac{A}{4}$

3. If z is a complex number such that $z = \ell n(a-1) + i\ell n(b-1)$ then arg(z) is -

(A) $\frac{\pi}{4}$ **(B)** $\frac{3\pi}{4}$ **(C)** $\frac{-\pi}{4}$ **(D)** $\frac{-3\pi}{4}$

Comprehension # 2

Let f(x) be a differentiable function, satisfying $f(0)=2$, $f'(0)=3$ and $f''(x)=f(x)$

On the basis of above information, answer the following questions :

1. Graph of $y=f(x)$ cuts x -axis at

(A) $x=-\frac{1}{2}\ell n5$ **(B)** $x=\frac{1}{2}\ell n5$ **(C)** $x=-\ell n5$ **(D)** $x=\ell n5$

2. Area enclosed by y = f(x) in the second quadrant is

(A) $3+\frac{1}{2}\ell n\sqrt{5}$ **(B)** $2+\frac{1}{2}\ell n5$ **(C)** $3-\sqrt{5}$ **(D)** 3

3. Area enclosed by $y=f(x)$, $y=f^{-1}(x)$, $x+y=2$ and $x+y=-\frac{1}{2}\ell n5$ is

(A) $8+\frac{1}{8}(\ell n5)^2$ **(B)** $8-2\sqrt{5}+\frac{1}{8}(\ell n5)^2$ **(C)** $2\sqrt{5}-\frac{1}{8}(\ell n5)^2$ **(D)** $8+2\sqrt{5}-\frac{1}{8}(\ell n5)^2$

Comprehension # 3

Five curves defined as follows : $C_1 : |x + y| \le 1$

$C_2 : |x - y| \le 1$

$C_3 : |x| \le \frac{1}{2}$

$C_4 : |y| \le \frac{1}{2}$

$C_5 : 3x^2 + 3y^2 = 1$

On the basis of above information, answer the following questions :

1. The area bounded by C_1 and C_2 which does not contain the area bounded by C_5, is -

(A) $2 - \frac{\pi}{4}$ **(B)** $2 - \frac{\pi}{6}$ **(C)** $2 - \frac{\pi}{3}$ **(D)** 2

2. That part of area of curve C_5 which does not contain points satisfying C_3 and C_4, is -

(A) $\frac{\pi}{3} - \frac{1}{2}$ **(B)** $\frac{\pi}{3} - 1$ **(C)** $\frac{\pi}{3} - \frac{1}{6}$ **(D)** $\frac{2\pi}{9} - \frac{1}{\sqrt{3}}$

3. That part of area which is bounded by C_1 and C_2 but not bounded by C_3 and C_4, is -

(A) 1 **(B)** $\frac{1}{2}$ **(C)** $\frac{1}{3}$ **(D)** none of these

Exercise # 4 [Subjective Type Questions]

1. Find the area of the region $\{(x,y): 0 \le y \le x^2+1,\ 0 \le y \le x+1,\ 0 \le x \le 2\}$.

2. For what value of 'a' is the area of the figure bounded by

$y = \frac{1}{x}$, $y = \frac{1}{2x-1}$, $x = 2$ & $x = a$ equal to $\ell n \frac{4}{\sqrt{5}}$?

3. Find the area enclosed between the curve $y = x^3 + 3$, $y = 0$, $x = -1$, $x = 2$.

4. A figure is bounded by the curves $y = \left|\sqrt{2}\sin\frac{\pi x}{4}\right|$, $y = 0$, $x = 2$ & $x = 4$. At what angles to the positive x-axis straight lines must be drawn through $(4, 0)$ so that these lines divide the figure into three parts of the same size.

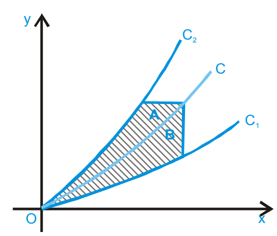
5. Consider the collection of all curve of the form $y = a - bx^2$ that pass through the point (2, 1), where a and b are positive constants. Determine the value of a and b that will minimise the area of the region bounded by $y = a - bx^2$ and x-axis. Also find the minimum area.

6. Compute the area of the figure bounded by straight lines $x = 0$, $x = 2$ and the curves $y = 2^x$ and $y = 2x - x^2$

7. The tangent to the parabola $y = x^2$ has been drawn so that the abscissa x_0 of the point of tangency belongs to the interval [1, 2]. Find x_0 for which the triangle bounded by the tangent, x-axis & the straight line $y = x_0^2$ has the greatest area.

8. Let C_1 & C_2 be two curves passing through the origin as shown in the figure. A curve C is said to "bisect the area" the region between C_1 & C_2, if for each point P of C, the two shaded regions A & B shown in the figure have equal areas. Determine the upper curve C_2, given that the bisecting curve C has the equation

$y = x^2$ & that the lower curve C_1 has the equation $y = x^2/2$.



9. Let $f(x) = \text{Maximum}\ \{x^2, (1-x)^2, 2x(1-x)\}$, where $0 \le x \le 1$. Determine the area of the region bounded by the curves $y = f(x)$, x– axis, $x = 0$ and $x = 1$.

10. For what value of 'a' is the area bounded by the curve $y = a^2x^2 + ax + 1$ and the straight lines $y = 0$, $x = 0$ & $x = 1$ the least ?

11. Show that the area bounded by the curve $y = \frac{\ell n x - c}{x}$, the x-axis and the vertical line through the maximum point of the curve is independent of the constant c. Also find the area.

12. Let $f(x) = \sqrt{\tan x}$. Show that area bounded by $y = f(x)$, $y = f(c)$, $x = 0$ and $x = a$, $0 < c < a < \frac{\pi}{2}$ is minimum when $c = \frac{a}{2}$

13. Consider the curve $y = x^n$ where $n > 1$ in the 1st quadrant. If the area bounded by the curve, the x-axis and the tangent line to the graph of $y = x^n$ at the point (1, 1) is maximum then find the value of n.

14. The line $3x + 2y = 13$ divides the area enclosed by the curve, $9x^2 + 4y^2 - 18x - 16y - 11 = 0$ in two parts. Find the ratio of the larger area to the smaller area.

15. **(i)** Find the area cut off between $x = 0$ and $x = 4 - y^2$.

(ii) Find the area of the region bounded by the curve $y^2 = 2y - x$ and the y-axis.

16. Let $f(x) = \text{Maximum } \{x^2, (1-x)^2, 2x(1-x)\}$, where $0 \le x \le 1$. Determine the area of the region bounded by the curves $y = f(x)$, x-axis, $x = 0$ & $x = 1$.

17. Let A_n be the area bounded by the curve $y = (\tan x)^n$ & the lines $x = 0$, $y = 0$ & $x = \pi/4$. Prove that for $n > 2$, $A_n + A_{n-2} = 1/(n-1)$ & deduce that $1/(2n+2) < A_n < 1/(2n-2)$.

18. **(i)** Draw graph of $y = (\tan x)^n$, $n \in N$, $x \in \left[0, \frac{\pi}{4}\right]$. Hence show $0 < (\tan x)^{n+1} < (\tan x)^n$, $x \in \left(0, \frac{\pi}{4}\right)$

(ii) Let A_n be the area bounded by the cunrve $y = (\tan x)^n$ and the lines $x = 0$, $y = 0$ and $x = \pi/4$. Prove that for $n > 2$, $A_n + A_{n-2} = 1/(n-1)$ and deduce that $1/(2n+2) < A_n < 1/(2n-2)$.

19. Find the area enclosed between the curves : $y = \log_e(x + e)$, $x = \log_e(1/y)$ & the x-axis.

20. A polynomial function f(x) satisfies the condition $f(x + 1) = f(x) + 2x + 1$. Find f(x) if $f(0) = 1$. Find also the equations of the pair of tangents from the origin on the curve $y = f(x)$ and compute the area enclosed by the curve and the pair of tangents.

21. Find the area bounded by the y-axis and the curve $x = e^y \sin \pi y$, $y = 0$, $y = 1$.

22. Find the value(s) of the parameter 'a' ($a > 0$) for each of which the area of the figure bounded by the straight line

$y = \dfrac{a^2 - ax}{1 + a^4}$ & the parabola $y = \dfrac{x^2 + 2ax + 3a^2}{1 + a^4}$ is the greatest.

23. Consider a square with vertices at (1, 1), (–1, 1), (–1, –1) & (1, –1). Let S be the region consisting of all points inside the square which are nearer to the origin than to any edge. Sketch the region S & find its area.

24. **(i)** If $f(x) = \min\{x + 1, \sqrt{1 - x}\}$, then find the value of $\int_{-1}^{1} \frac{12}{7} f(x)\, dx$.

(ii) Find the area of the region bounded by $y = \{x\}$ and $2x - 1 = 0$, $y = 0$, ({ } stands for fraction part)

25. Find the positive value of 'a' for which the parabola $y = x^2 + 1$ bisects the area of the rectangle with vertices (0, 0), (a, 0), $(0, a^2 + 1)$ and $(a, a^2 + 1)$.

26. Let f(x) be a continuous function given by $f(x) = \begin{cases} 2x & \text{for } |x| \le 1 \\ x^2 + ax + b & \text{for } |x| > 1 \end{cases}$. Find the area of the region in the third quadrant bounded by the curves, $x = -2y^2$ and $y = f(x)$ lying on the left of the line $8x + 1 = 0$

27. Find the area included between the parabolas $y^2 = x$ and $x = 3 - 2y^2$.

28. In what ratio does the x-axis divide the area of the region bounded by the parabolas $y = 4x - x^2$ and $y = x^2 - x$?

29. A tangent is drawn to the curve $x^2 + 2x - 4ky + 3 = 0$ at a point whose abscissa is 3. This tangent is perpendicular to $x + 3 = 2y$. Find the area bounded by the curve, this tangent and ordinate $x = -1$

30. Find the area bounded by $y = x + \sin x$ and its inverse between $x = 0$ and $x = 2\pi$.

Exercise # 5 Part # I [Previous Year Questions] [AIEEE/JEE-MAIN]

1. If the area bounded by the x-axis, curve y = f(x) and the lines x = 1, x = b is equal to $\sqrt{b^2+1} - \sqrt{2}$ for all b > 1, then f(x) is- **[AIEEE-2002]**

(1) $\sqrt{(x-1)}$ (2) $\sqrt{(x+1)}$ (3) $\sqrt{(x^2+1)}$ (4) $\frac{x}{\sqrt{1+x^2}}$

2. The area of the region bounded by the curves y = |x – 1| and y = 3 – |x| is - **[AIEEE-2003]**

(1) 6 sq. units (2) 2 sq. units (3) 3 sq. units (4) 4 sq. units

3. The area of the region bounded by the curves y = |x – 2|, x = 1, x = 3 and the x-axis is- **[AIEEE-2004]**

(1) 1 (2) 2 (3) 3 (4) 4

4. Area of the greatest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is - **[AIEEE-2005]**

(1) 2ab (2) ab (3) $\sqrt{ab}$ (4) $\frac{a}{b}$

5. The area enclosed between the curve $y = \log_e(x+e)$ and the cooordinate axes is- **[AIEEE-2005]**

(1) 1 (2) 2 (3) 3 (4) 4

6. The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines x = 4, y = 4 and the coordinate axes. If S_1, S_2, S_3 are respectively the areas of these parts numbered from top to bottom; then $S_1 : S_2 : S_3$ is -

(1) 1 : 2 : 1 (2) 1 : 2 : 3 (3) 2 : 1 : 2 (4) 1 : 1 : 1 **[AIEEE-2005]**

7. Let f(x) be a non-negative continuous function such that the area bounded by the curve y= f(x), x-axis and the ordinates $x = \frac{\pi}{4}$ and $x = \beta > \frac{\pi}{4}$ is $\left(\beta\sin\beta+\frac{\pi}{4}\cos\beta+\sqrt{2}\beta\right)$. Then $f\left(\frac{\pi}{2}\right)$ is - **[AIEEE-2005]**

(1) $\left(\frac{\pi}{4}+\sqrt{2}-1\right)$ (2) $\left(\frac{\pi}{4}-\sqrt{2}+1\right)$ (3) $\left(1-\frac{\pi}{4}-\sqrt{2}\right)$ (4) $\left(1-\frac{\pi}{4}+\sqrt{2}\right)$

8. The area enclosed between the curves $y^2 = x$ and y = |x| is- **[AIEEE-2007]**

(1) $\frac{2}{3}$ (2) 1 (3) $\frac{1}{6}$ (4) $\frac{1}{3}$

9. The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is equal to- **[AIEEE-2008]**

(1) $\frac{5}{3}$ (2) $\frac{1}{3}$ (3) $\frac{2}{3}$ (4) $\frac{4}{3}$

10. The area of the region bounded by the parabola $(y-2)^2 = x - 1$, the tangent to the parabola at the point (2, 3) and the x–axis is :- **[AIEEE-2009]**

(1) 9 (2) 12 (3) 3 (4) 6

11. The area bounded by the curves y = cos x and y = sin x between the ordinates x = 0 and $x = \frac{3\pi}{2}$ is -

(1) $4\sqrt{2} - 2$ **(2)** $4\sqrt{2} + 2$ **(3)** $4\sqrt{2} - 1$ **(4)** $4\sqrt{2} + 1$ **[AIEEE-2010]**

12. The area of the region enclosed by the curves y = x, x = e, $y = \frac{1}{x}$ and the positive x-axis is - **[AIEEE-2011]**

(1) $\frac{3}{2}$ square units **(2)** $\frac{5}{2}$ square units **(3)** $\frac{1}{2}$ square units **(4)** 1 square units

13. The area bounded by the curves $y^2 = 4x$ and $x^2=4y$ is:= **[AIEEE-2011]**

(1) 0 **(2)** $\frac{32}{3}$ **(3)** $\frac{16}{3}$ **(4)** $\frac{8}{3}$

14. The area bounded between the parabolas $x^2 = \frac{y}{4}$ and $x^2 = 9y$, and the straight line y = 2 is : **[AIEEE-2012]**

(1) $10\sqrt{2}$ **(2)** $20\sqrt{2}$ **(3)** $\frac{10\sqrt{2}}{3}$ **(4)** $\frac{20\sqrt{2}}{3}$

15. The area (in square units) bounded by the curves $y = \sqrt{x}$, 2y – x + 3 = 0, x-axis and lying in the first quadrant is **[JEE (Main)-2013]**

(1) 9 **(2)** 36 **(3)** 18 **(4)** $\frac{27}{4}$

16. The area of the region described by $A = \{(x, y) : x^2 + y^2 \leq 1 \text{ and } y^2 \leq 1 - x\}$ is **[Main 2014]**

(1) $\frac{\pi}{2}+\frac{4}{3}$ **(2)** $\frac{\pi}{2}-\frac{4}{3}$ **(3)** $\frac{\pi}{2}-\frac{2}{3}$ **(4)** $\frac{\pi}{2}+\frac{2}{3}$

17. The area (in sq. units) of the region described by $\{(x, y) : y^2 \leq 2x \text{ and } y \geq 4x - 1\}$ is **[Main 2015]**

(1) $\frac{15}{64}$ **(2)** $\frac{9}{32}$ **(3)** $\frac{7}{32}$ **(4)** $\frac{5}{64}$

18. The area (in sq. units) of the region $\{(x, y) : y^2 \geq 2x \text{ and } x^2 + y^2 \leq 4x, x \geq 0, y \geq 0\}$ **[Main 2016]**

(1) $\pi - \frac{8}{3}$ **(2)** $\pi - \frac{4\sqrt{2}}{3}$ **(3)** $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$ **(4)** $\pi - \frac{4}{3}$

Part # II [Previous Year Questions][IIT-JEE ADVANCED]

1. The triangle formed by the tangent to the curve $f(x) = x^2 + bx - b$ at the point (1 ,1) and the coordinates axes, lies in the first quadrant. If its area is 2, then the value of b is - **[JEE 2001]**

(A) –1 **(B)** 3 **(C)** –3 **(D)** 1

2. The area bounded by the curves y = |x| – 1 and y = –|x|+ 1 is - **[JEE 2002 (Screening)]**

(A) 1 **(B)** 2 **(C)** $2\sqrt{2}$ **(D)** 4

3. Find the area of the region bounded by the curves $y = x^2$, $y = |2 - x^2|$ and $y = 2$, which lies to the right of the line $x = 1$. **[JEE 2002, (Mains)]**

4. (A) The area of the quadrilateral formed by the tangents at the end points of latus recta to the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$, is-

(A) 27/4 sq. units **(B)** 9 sq. units **(C)** 27 sq. units **(D)** 27/2 sq. units

(B) The area bounded by the curves $y = \sqrt{x}$, $2y + 3 = x$ and x-axis in the 1^{st} quadrant is -

(A) 18 **(B)** 27/4 **(C)** 36 **(D)** 9 **[JEE 2003 (Screening)]**

5. (A) The area bounded by the angle bisectors of the lines $x^2 - y^2 + 2y = 1$ and the line $x + y = 3$, is -

(A) 2 **(B)** 3 **(C)** 4 **(D)** 6

(B) The area enclosed between the curves $y = ax^2$ and $x = ay^2$ $(a > 0)$ is 1 sq. unit, then the value of a is -

(A) $\frac{1}{\sqrt{3}}$ **(B)** $\frac{1}{2}$ **(C)** 1 **(D)** $\frac{1}{3}$ **[JEE 2004 (Screening)]**

6. The area bounded by the parabolas $y = (x + 1)^2$ and $y = (x - 1)^2$ and the line $y = 1/4$ is - **[JEE 2005 (Screening)]**

(A) 4 sq. units **(B)** 1/6 sq. units **(C)** 4/3 sq. units **(D)** 1/3 sq. units

7. Find the area bounded by the curves $x^2 = y$, $x^2 = -y$ and $y^2 = 4x - 3$. **[JEE 2005, (Mains)]**

8. If $\begin{bmatrix} 4a^2 & 4a & 1 \\ 4b^2 & 4b & 1 \\ 4c^2 & 4c & 1 \end{bmatrix} \begin{bmatrix} f(-1) \\ f(1) \\ f(2) \end{bmatrix} = \begin{bmatrix} 3a^2 + 3a \\ 3b^2 + 3b \\ 3c^2 + 3c \end{bmatrix}$, f(x) is a quadratic function and its maximum value occurs at a point

V. A is a point of intersection of $y = f(x)$ with x-axis and point B is such that chord AB subtends a right angle at V. Find the area enclosed by f(x) and chord AB. **[JEE 2005 (Mains)]**

9. Match the following -

(A) $\int_0^{\pi/2} (\sin x)^{\cos x}(\cos x \cot x - \ell n(\sin x)^{\sin x})dx$ **(p)** 1

(B) Area bounded by $-4y^2 = x$ and $x - 1 = -5y^2$ **(q)** 0

(C) Cosine of the angle of intersection of curves $y = 3^{x-1} \ln x$ and $y = x^x - 1$ is **(r)** $6\,\ell n\, 2$

(D) Let $\frac{dy}{dx} = \frac{6}{x+y}$, where $y(0) = 0$, then the value of y when $x + y = 6$ is **(s)** 4/3

[JEE 2006, 6M]

Paragraph for Question Nos. 10 to 12

Consider the functions defined implicitly by the equation $y^3 - 3y + x = 0$ on various intervals in the real line. If $x \in (-\infty, -2) \cup (2, \infty)$, the equation implicitly defines a unique real valued differentiable function $y = f(x)$. If $x \in (-2, 2)$, the equation implicitly defines a unique real valued differentiable function $y = g(x)$ satisfying $g(0) = 0$.

10. If $f(-10\sqrt{2}) = 2\sqrt{2}$, then $f''(-10\sqrt{2}) =$ **[JEE 2008]**

(A) $\frac{4\sqrt{2}}{7^3 3^2}$ **(B)** $-\frac{4\sqrt{2}}{7^3 3^2}$ **(C)** $\frac{4\sqrt{2}}{7^3 3}$ **(D)** $-\frac{4\sqrt{2}}{7^3 3}$

11. The area of the region bounded by the curve $y = f(x)$, the x-axis, and the lines $x = a$ and $x = b$, where $-\infty < a < b < -2$, is - **[JEE 2008]**

(A) $\int_a^b \frac{x}{3((f(x))^2 - 1)}dx + bf(b) - af(a)$ **(B)** $-\int_a^b \frac{x}{3\left((f(x))^2 - 1\right)}dx + bf(b) - af(a)$

(C) $\int_a^b \frac{x}{3\left((f(x))^2 - 1\right)}dx - bf(b) + af(a)$ **(D)** $-\int_a^b \frac{x}{3\left((f(x))^2 - 1\right)}dx - bf(b) + af(a)$

12. $\int_{-1}^{1} g'(x)\,dx =$ **[JEE 2008]**

(A) $2g(-1)$ **(B)** 0 **(C)** $-2g(1)$ **(D)** $2g(1)$

13. The area of the region between the curves $y = \sqrt{\frac{1+\sin x}{\cos x}}$ and $y = \sqrt{\frac{1-\sin x}{\cos x}}$ bounded by the lines $x = 0$ and $x = \frac{\pi}{4}$ is :- **[JEE 2008]**

(A) $\int_0^{\sqrt{2}-1} \frac{t}{(1+t^2)\sqrt{1-t^2}}dt$ **(B)** $\int_0^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}dt$

(C) $\int_0^{\sqrt{2}+1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}dt$ **(D)** $\int_0^{\sqrt{2}+1} \frac{t}{(1+t^2)\sqrt{1-t^2}}dt$

14. Area of the region bounded by the curve $y = e^x$ and lines $x = 0$ and $y = e$ is - **[JEE 2009]**

(A) $e - 1$ **(B)** $\int_1^e \ln(e+1-y)dy$ **(C)** $e - \int_0^1 e^x dx$ **(D)** $\int_1^e \ln y\,dy$

Paragraph for Question 15 to 17 **[JEE 2010]**

Consider the polynomial

$f(x) = 1 + 2x + 3x^2 + 4x^3$.

Let s be the sum of all distinct real roots of f(x) and let t = |s|.

15. The real number s lies in the interval

(A) $\left(-\frac{1}{4}, 0\right)$ **(B)** $\left(-11, -\frac{3}{4}\right)$ **(C)** $\left(-\frac{3}{4}, -\frac{1}{2}\right)$ **(D)** $\left(0, \frac{1}{4}\right)$

16. The area bounded by the curve y = f(x) and the lines x = 0, y = 0 and x = t, lies in the interval

(A) $\left(\frac{3}{4}, 3\right)$ **(B)** $\left(\frac{21}{64}, \frac{11}{16}\right)$ **(C)** (9, 10) **(D)** $\left(0, \frac{21}{64}\right)$

17. The function f'(x) is

(A) increasing in $\left(-t, -\frac{1}{4}\right)$ and decreasing in $\left(-\frac{1}{4}, t\right)$

(B) decreasing in $\left(-t, -\frac{1}{4}\right)$ and increasing in $\left(-\frac{1}{4}, t\right)$

(C) increasing in (–t, t)

(D) decreasing in (–t, t)

18.(A) Let the straight line x = b divide the area enclosed by $y = (1 - x)^2$, y = 0 and x = 0 into two parts $R_1(0 \le x \le b)$ d and $R_2(b \le x \le 1)$ such that $R_1 - R_2 = \frac{1}{4}$. Then b equals

(A) $\frac{3}{4}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{1}{4}$

(B) Let $f:[-1,2] \to [0,\infty)$ be a continuous function such that $f(x) = f(1-x)$ for all $x \in [-1,2]$. Let $R_1 = \int_{-1}^{2} x f(x)dx$, and R_2 be the area of the region bounded by y = f(x), x=–1, x=2, and the x-axis. Then - **[JEE 2011]**

(A) $R_1 = 2R_2$ **(B)** $R_1 = 3R_2$ **(C)** $2R_1 = R_2$ **(D)** $3R_1 = R_2$

19. The area enclosed by the curve y = sinx + cosx and y = |cosx – sinx| over the interval $\left[0, \frac{\pi}{2}\right]$ is

[JEE Ad. 2013]

(A) $4(\sqrt{2}-1)$ **(B)** $2\sqrt{2}(\sqrt{2}-1)$ **(C)** $2(\sqrt{2}+1)$ **(D)** $2\sqrt{2}(\sqrt{2}+1)$

20. For a point P in the plane, let d_1 (P) and d_2 (P) be the distances of the point P from the lines x – y = 0and x + y = 0 respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \le d_1(P) + d_2(P) \le 4$, is **[JEE Ad. 2014]**

21. Area of the region $\left\{(x,y) \in R^2 : y \ge \sqrt{|x+3|}, 5y \le x+9 \le 15\right\}$ is equal to **[JEE Ad. 2016]**

(A) $\frac{1}{6}$ **(B)** $\frac{4}{3}$ **(C)** $\frac{3}{2}$ **(D)** $\frac{5}{3}$

MOCK TEST

SECTION - I : STRAIGHT OBJECTIVE TYPE

1. One of the values of 'a' for which the area bounded by the curve $y = 8x^2 - x^5$, straight lines x = 1, x = a and x-axis is equal to $\frac{16}{3}$, is

(A) 1 **(B)** 2 **(C)** – 1 **(D)** $\frac{1}{2}$

2. The area enclosed by the curves $y = \sqrt{4-x^2}$, $y \geq \sqrt{2}\sin\left(\frac{x\pi}{2\sqrt{2}}\right)$ and x-axis is divided by y-axis in the ratio

(A) $\frac{\pi^2-8}{\pi^2+8}$ **(B)** $\frac{\pi^2-4}{\pi^2+4}$ **(C)** $\frac{\pi-4}{\pi-4}$ **(D)** $\frac{2\pi^2}{2\pi+\pi^2-8}$

3. The area of the region bounded by the curve $a^4y^2 = (2a - x)x^5$ is to that of the circle whose radius is a, is given by the ratio

(A) 4 : 5 **(B)** 5 : 4 **(C)** 2 : 3 **(D)** 3 : 2

4. Value of the parameter a such that the area bounded by $y = a^2x^2 + ax + 1$, co-ordinate axes and the line x = 1, attains it's least value, is equal to

(A) $-\frac{1}{4}$ **(B)** $-\frac{1}{2}$ **(C)** $-\frac{3}{4}$ **(D)** – 1

5. The area bounded by $y = x^2$, $y = [x + 1]$, $x \leq 1$ and the y-axis is

(A) 1/3 **(B)** 2/3 **(C)** 1 **(D)** 7/3

6. Area bounded by the curve $y = \ell n\, x + \tan^{-1}x$ and x– axis from x = 1 to x = 2, is

(A) $\frac{5}{2}\ell n\, 2 - \frac{1}{2}\ell n\, 5 + 2\tan^{-1}2 - \frac{\pi}{3} - 1$ **(B)** $\frac{5}{2}\ell n\, 2 - \frac{1}{2}\ell n\, 5 + 2\tan^{-1}2 - \frac{\pi}{4} + 1$

(C) $\frac{5}{2}\ell n\, 2 - \frac{1}{2}\ell n\, 5 + 2\tan^{-1}2 + \frac{\pi}{4} - 1$ **(D)** $\frac{5}{2}\ell n\, 2 - \frac{1}{2}\ell n\, 5 + 2\tan^{-1}2 - \frac{\pi}{4} - 1$

7. The area of the closed figure bounded by the curves y = cos x; $y = 1 + \frac{2}{\pi}x$ and $x = \frac{\pi}{2}$ is

(A) $\frac{\pi+4}{4}$ **(B)** $\frac{3\pi}{4}$ **(C)** $\frac{3\pi+4}{4}$ **(D)** $\frac{3\pi-4}{4}$

8. The area of the region enclosed between the two circles $x^2 + y^2 = 1$ and $(x - 1)^2 + y^2 = 1$, is

(A) $\left(\frac{2\pi}{3} - \sqrt{\frac{3}{4}}\right)$ **(B)** $\left(\frac{3\pi}{2} - \frac{\sqrt{3}}{2}\right)$ **(C)** $\left(\frac{3\pi}{2} + \frac{\sqrt{3}}{2}\right)$ **(D)** $\left(\frac{2\pi}{3} + \sqrt{\frac{3}{4}}\right)$

9. The area of the region bounded in first quadrant by $y = x^{1/3}$; $y = -x^2 + 2x + 3$; $y = 2x - 1$ and the axis of ordinates is :

(A) 12/55 **(B)** 55/12 **(C)** 32/55 **(D)** none

10. Which of the following statements are true/false –

S_1 : Area between $x^2 = 4by$ and $y^2 = 4ax$ is $\frac{16ab}{3}$

S_2 : Area enclosed by $|x| + |y| = 1$ is 1.

S_3 : Smaller area enclosed by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x}{a} + \frac{y}{b} = 1$ is $\frac{\pi ab}{4} - \frac{ab}{2}$

S_4 : Area enclosed by $y = [x]$ and $y = \{x\}$ is 1.

(A) TFTT **(B)** TTTT **(C)** TFTF **(D)** FFTT

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

11. The area bounded by a curve, the axis of co-ordinates & the ordinate of some point of the curve is equal to the length of the corresponding arc of the curve . If the curve passes through the point P (0 , 1) then the equation of this curve can be :

(A) $y = \frac{1}{2}(e^x - e^{-x} + 2)$ **(B)** $y = \frac{1}{2}(e^x + e^{-x})$

(C) $y = 1$ **(D)** $y = \frac{2}{e^x + e^{-x}}$

12. If $f(x) = 2^{\{x\}}$, where $\{x\}$ denotes the fractional part of x. Then which of the following is true ?

(A) f is periodic **(B)** $\int_0^1 2^{\{x\}} dx = \frac{1}{\ell n2}$ **(C)** $\int_0^1 2^{\{x\}} dx = \log_2 e$ **(D)** $\int_0^{100} 2^{\{x\}} dx = 100 \log_2 e$

13. Let T be the triangle with vertices (0, 0), (0, c^2) and (c, c^2) and let R be the region between $y = cx$ and $y = x^2$ where $c > 0$ then

(A) Area (R) = $\frac{c^3}{6}$ **(B)** Area of R = $\frac{c^3}{3}$

(C) $\lim_{c \to 0^+} \frac{\text{Area (T)}}{\text{Area (R)}} = 3$ **(D)** $\lim_{c \to 0^+} \frac{\text{Area (T)}}{\text{Area (R)}} = \frac{3}{2}$

14. Let $f(x) = \int_0^x |2t - 3|\, dt$, then f is

(A) continuous at x = 3/2 **(B)** continuous at x = 3

(C) differentiable at x = 3/2 **(D)** differentiable at x = 0

15. Consider the functions f (x) and g (x), both defined from $R \to R$ and are defined as $f(x) = 2x - x^2$ and $g(x) = x^n$ where $n \in N$. If the area between f (x) and g (x) is 1/2 then *n* is a divisor of

(A) 12 **(B)** 15 **(C)** 20 **(D)** 30

SECTION - III : ASSERTION AND REASON TYPE

16. **Statement-I :** The area bounded by the curve $|x| + |y| = a$ $(a > 0)$ is $2a^2$ and area bounded $|px + qy| + |qx - py| = a$, where $p^2 + q^2 = 1$, is also $2a^2$.

Statement-II : Since $\alpha x + \beta y = 0$ is perpendicular to $\beta x - \alpha y = 0$, we can take one as x-axis and another as y-axis and therefore the area bounded by $|\alpha x + \beta y| + |\beta x - \alpha y| = a$ is $2a^2$ for all $\alpha, \beta \in R, \alpha \neq 0, \beta \neq 0$.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

17. A curve C has the property that its initial ordinate of any tangent drawn is less than the abscissa of the point of tangency by unity.

Statement-I: Differential equation satisfying the curve is linear.

Statement-II: Degree of differential equation is one

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

18. **Statement-I :** Area bounded by $y = \tan x$, $y = \tan^2 x$ in between $x \in \left(0, \frac{\pi}{4}\right)$ is equal to $\left(\frac{\pi}{4} + \ell n\sqrt{2} - 1\right)$.

Statement-II : Area bounded by $y = f(x)$ and $y = g(x)$ $\{f(x) > g(x)\}$ between $x = a$, $x = b$ is $\int_a^b (f(x) - g(x))\,dx$. $(b > a)$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

19. C_1 is a circle of radius 2 touching x-axis and y-axis. C_2 is another circle of radius greater than 2 and touching both the axes as well as the circle C_1.

Statement-I : Radius of circle C_2 is $\sqrt{2}\left(\sqrt{2}+1\right)\left(\sqrt{2}+2\right)$.

Statement-II : Centres of both circles always lie on the line $y = x$.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

20. **Statement-I :** Area bounded by parabola $y = x^2 - 4x + 3$ and $y = 0$ is 4/3 sq. units.

Statement-II : Area bounded by curve $y = f(x) \geq 0$ and $y = 0$ between ordinates $x = a$ and $x = b$ $(b > a)$ is $\int_a^b f(x)\,dx$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

SECTION - IV : MATRIX - MATCH TYPE

21.

Column – I		**Column – II**	
(A)	The area bounded by the curve y = x + sin x and its inverse function between the ordinates x = 0 to x = 2π is 4s. Then the value of s is	**(p)**	0
(B)	The area bounded by $y = x\,e^{\|x\|}$ and lines $\|x\| = 1$, $y = 0$ is	**(q)**	1
(C)	The area bounded by the curves $y^2 = x^3$ and $\|y\| = 2x$ is	**(r)**	$\frac{16}{5}$
(D)	The smaller area included between the curves $\sqrt{x} + \sqrt{\|y\|} = 1$ and $\|x\| + \|y\| = 1$ is	**(s)**	$\frac{1}{3}$
		(t)	2

22.

Column – I		**Column – II**	
(A)	Area enclosed by $y = \|x\|$, $\|x\| = 1$ and $y = 0$ is	**(p)**	3
(B)	Area enclosed by the curve y = sinx, x = 0, x = π and y = 0 is	**(q)**	4
(C)	If the area of the region bounded by $x^2 \le y$ and $y \le x + 2$ is $\frac{k}{4}$, then k =	**(r)**	27
(D)	Area of the quadrilateral formed by tangents at the ends of latus rectum of ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$ is	**(s)**	18
		(t)	1

SECTION - V : COMPREHENSION TYPE

23. **Read the following comprehension carefully and answer the questions.**

Let f(x) be a differentiable function, satisfying $f(0) = 2$, $f'(0) = 3$ and $f''(x) = f(x)$

1. Graph of y = f(x) cuts x -axis at

(A) $x = -\frac{1}{2}\ell n5$ **(B)** $x = \frac{1}{2}\ell n5$ **(C)** $x = -\ell n5$ **(D)** $x = \ell n5$

2. Area enclosed by y = f(x) in the second quadrant is

(A) $3 + \frac{1}{2}\ell n\sqrt{5}$ **(B)** $2 + \frac{1}{2}\ell n5$ **(C)** $3 - \sqrt{5}$ **(D)** 3

3. Area enclosed by $y = f(x)$, $y = f^{-1}(x)$, $x + y = 2$ and $x + y = -\frac{1}{2}\ell n5$ is

(A) $8 + \frac{1}{8}(\ell n5)^2$ **(B)** $8 - 2\sqrt{5} + \frac{1}{8}(\ell n5)^2$ **(C)** $2\sqrt{5} - \frac{1}{8}(\ell n5)^2$ **(D)** $8 + 2\sqrt{5} - \frac{1}{8}(\ell n5)^2$

24. **Read the following comprehension carefully and answer the questions.**

Asymptotes are the tangents to the curve at infinity

To find the asymptotes of a curve we can use the following methods.

(A) Asymptote parallel to the x-axis is obtained by equating to zero, the coefficient of the highest power to x.

(B) Asymptote parallel to the y-axis is obtained by equating to zero, the coefficient of the highest power of y.

(C) Oblique Asymptote : $y = mx + c$

(i) Find ϕ_n (m) by putting $x = 1$ and $y = m$ in the highest degree (n) terms of the equation similarly find $\phi_{n-1}(m)$.

(ii) Solve $\phi_n(m) = 0$ for m

(iii) Find c by the formula $c = -\dfrac{\phi_{n-1}(m)}{\phi'_n(m)}$ Using the value of m as obtained in **(ii)**

(iv) Obtain the equation of asymptote by putting these values of m and c in $y = mx + c$.

1. The equation of asymptotes of the curve $yx^2 - 4x^2 + x + 2 = 0$

(A) $y - 4 = 0$ and $x = 0$ **(B)** $y = 3$ and $x = 2$

(C) $y - 4 = 0$ and $x = 2$ **(D)** $y = 3$ and $x = 0$

2. The equation of asymptotes of the curve $x^3 + y^3 - 3xy = 0$

(A) $y = x + 1$ **(B)** $y + x + 1 = 0$ **(C)** $y + x = 2$ **(D)** $y = 2x + 1$

3. The equation of asymptotes of the curve $y^2 = \dfrac{x^3}{(2-x)}$ is $ax + by + c = 0$, then the value of $|a + b + c|$ is

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4

25. **Read the following comprehension carefully and answer the questions.**

If $y = \int_{u(x)}^{v(x)} f(t)\,dt$, let us define $\dfrac{dy}{dx}$ in a different manner as $\dfrac{dy}{dx} = v'(x)\, f^2(v(x)) - u'(x)\, f^2(u(x))$ and the equation of the tangent at (a, b) as $y - b = \left(\dfrac{dy}{dx}\right)_{(a,b)} (x - a)$

1. If $y = \int_{x}^{x^2} t^2\,dt$, then equation of tangent at $x = 1$ is

(A) $y = x + 1$ **(B)** $x + y = 1$ **(C)** $y = x - 1$ **(D)** $y = x$

2. If $F(x) = \int_{1}^{x} e^{t^2/2}(1 - t^2)\,dt$, then $\dfrac{d}{dx}F(x)$ at $x = 1$ is

(A) 0 **(B)** 1 **(C)** 2 **(D)** – 1

3. If $y = \int_{x^3}^{x^4} \ell n t\,dt$, then $\lim\limits_{x \to 0^+} \dfrac{dy}{dx}$ is

(A) 0 **(B)** 1 **(C)** 2 **(D)** – 1

SECTION - VI : INTEGER TYPE

26. Find the area of the region bounded by $y = f(x)$, $y = |g(x)|$ and the lines $x = 0$, $x = 2$, where 'f', 'g' are continuous functions satisfying $f(x + y) = f(x) + f(y) - 8xy \ \forall \ x, y \in R$ and $g(x + y) = g(x) + g(y) + 3xy(x + y) \ x, y \in R$ also $f'(0) = 8$ and $g'(0) = -4$.

27. Find the area enclosed by the solution set of $[x] \cdot [y] = 2$.
Where $[\cdot]$ represent greatest integer function of x.

28. Let ABC be a triangle with vertices $A(6, 2(\sqrt{3} + 1))$, $B(4, 2)$ and $C(8, 2)$. If R be the region consisting of all these points and point P inside ΔABC which satisfy $d(P, BC) \geq \max. \{d(P, AB), d(P, AC)\}$
where $d(P, L)$ denotes the distance of the point P from the line L. Sketch the region R and find its area.

29 The area of the loop of the curve, $a y^2 = x^2 (a - x)$ is $\frac{\lambda a^2}{15}$, then find λ

30. Find the area of the region which is inside the parabola $y = -x^2 + 6x - 5$, out side the parabola $y = -x^2 + 4x - 3$ and left of the straight line $y = 3x - 15$.

ANSWER KEY

EXERCISE - 1

1. C **2.** C **3.** A **4.** D **5.** C **6.** A **7.** B **8.** B **9.** A **10.** C **11.** D **12.** C **13.** C
14. B **15.** B **16.** B **17.** B **18.** D **19.** B **20.** B **21.** C **22.** B **23.** C **24.** D **25.** B **26.** B
27. B **28.** A **29.** A **30.** A

EXERCISE - 2 : PART # I

1. ABCD **2.** AB **3.** AD **4.** BCD **5.** AC **6.** BCD **7.** ABCD **8.** AC **9.** BCD **10.** AD **11.** BD

PART - II

1. D **2.** C **3.** C **4.** A **5.** C

EXERCISE - 3 : PART # I

1. $A \to q$ $B \to q$ $C \to r$ $D \to s$ **2.** $A \to s$ $B \to s$ $C \to q$ $D \to p$ **3.** $A \to r$ $B \to s$ $C \to p$ $D \to q$

PART - II

Comprehension # 1 : 1. A **2.** D **3.** D **Comprehension # 2 : 1.** A **2.** C **3.** B
Comprehension # 3 : 1. C **2.** D **3.** A

EXERCISE - 5 : PART # I

1. 4 **2.** 4 **3.** 1 **4.** 1 **5.** 1 **6.** 4 **7.** 4 **8.** 3 **9.** 4 **10.** 1 **11.** 1 **12.** 1 **13.** 3
14. 4 **15.** 1 **16.** 1 **17.** 2 **18.** 1

PART - II

1. C **2.** B **3.** $\left(\frac{20}{3} - 4\sqrt{2}\right)$ sq. units **4. A.** C **B.** D **5. A.** A **B.** A **6.** D **7.** $\frac{1}{3}$ sq. units

8. $\frac{125}{3}$ sq. units **9.** **A** $\to$ p **B** $\to$ s **C** $\to$ p **D** $\to$ r **10.** B **11.** A **12.** D **13.** B **14.** BCD **15.** C

16. A **17.** B **18. A.** B **B.** C **19.** B **20.** 6 **21.** C

MOCK TEST

1. C **2.** D **3.** B **4.** C **5.** B **6.** D **7.** D **8.** A **9.** B **10.** A **11.** BC **12.** ABCD
13. AC **14.** ABD **15.** BCD **16.** C **17.** B **18.** A **19.** C **20.** B
21. $A \to t$ $B \to t$ $C \to r$ $D \to s$ **22.** $A \to t$ $B \to p$ $C \to s$ $D \to r$ **23. 1.** A **2.** C **3.** B **24. 1.** A

2. B **3.** B **25. 1.** C **2.** A **3.** A **26.** $\frac{4}{3}$ **27.** 4 **28.** $\frac{4\sqrt{3}}{3}$ **29.** 8 **30.** $\frac{73}{6}$