AREA UNDER THE CURVE

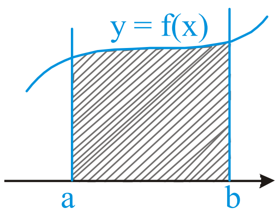
INTRODUCTION

The definite integral can be used to find the area between a graph curve & the 'x' axis or 'y' axis between two given of 'x' or 'y'. The area is called the area under the curve.

Area under the curve basically signifies the magnitude of the quantity that is obtained by the product of the quantities signified by the x and the y axis.

AREA INCLUDED BETWEEN THE CURVE y = f(x), X-AXIS AND THE ORDINATES x = a, x = b

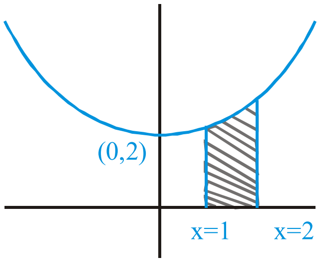
(A) Area bounded by the curve $y = f(x)$, the x-axis and the ordinates at $x = a$ and $x = b$ is given by $A = \int_a^b y\,dx$, where $y = f(x)$ lies above the x-axis and $b > a$. Here vertical strip of thickness dx is considered at distance x.



Graph of y = f(x)

Ex. Find the area enclosed between the curve $y = x^2 + 2$, x-axis, $x = 1$ and $x = 2$.

Sol. Graph of $y = x^2 + 2$



$$\text{Area} = \int_1^2 \left(x^2+2\right)dx = \left[\frac{x^3}{3}+2x\right]_1^2 = \frac{13}{3}$$

Ex. Find the area bounded by $y = \sec^2 x$, $x = \frac{\pi}{6}$, $x = \frac{\pi}{3}$ & x-axis

Sol. Area bounded $= \int_{\pi/6}^{\pi/3} y dx = \int_{\pi/6}^{\pi/3} \sec^2 x dx = [\tan x]_{\pi/6}^{\pi/3} = \tan\frac{\pi}{3} - \tan\frac{\pi}{6} = \sqrt{3} - \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$ sq.units.

Ex. Find area bounded by the curve $y = \ell n\, x + \tan^{-1} x$ and x-axis between ordinates x = 1 and x = 2.

Sol. $y = \ell n\, x + \tan^{-1}x$

Domain $x > 0$, $\frac{dy}{dx} = \frac{1}{x} + \frac{1}{1+x^2} > 0$

y is increasing and $x = 1, y = \frac{\pi}{4} \Rightarrow$ y is positive in [1, 2]

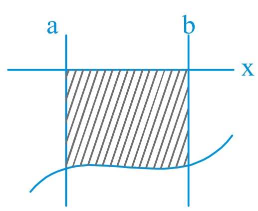
$\therefore$ Required area $= \int_1^2 (\ell n\, x + \tan^{-1} x)\, dx$

$= \left[x\, \ell n\, x - x + x \tan^{-1} x - \frac{1}{2} \ell n\,(1 + x^2)\right]_1^2$

$= 2\, \ell n\, 2 - 2 + 2 \tan^{-1}2 - \frac{1}{2}\, \ell n\, 5 - 0 + 1 - \tan^{-1} 1 + \frac{1}{2}\, \ell n\, 2$

$= \frac{5}{2}\, \ell n\, 2 - \frac{1}{2}\, \ell n\, 5 + 2 \tan^{-1}2 - \frac{\pi}{4} - 1$

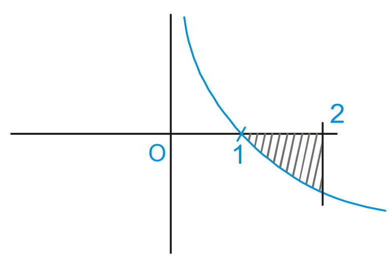
(B) If $y = f(x)$ lies completely below the x-axis then A is negative and we consider the magnitude only, i.e. $A = \left|\int_a^b y\, dx\right| = -\int_a^b f(x)\, dx$



Graph of y = f(x)

Ex. Find area bounded by $y = \log_{\frac{1}{2}} x$ and x-axis between x = 1 and x = 2

Sol. A rough graph of $y = \log_{\frac{1}{2}} x$ is as follows



Area $= -\int_1^2 \log_{\frac{1}{2}} x\, dx = -\int_1^2 \log_e x \cdot \log_{\frac{1}{2}} e\, dx$

$= -\log_{\frac{1}{2}} e \cdot [x \log_e x - x]_1^2$

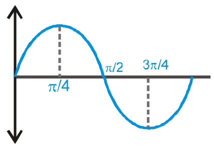
$= -\log_{\frac{1}{2}} e \cdot (2 \log_e 2 - 2 - 0 + 1)$

$= -\log_{\frac{1}{2}} e \cdot (2 \log_e 2 - 1)$

(C) If the curve crosses the x-axis at x = c and $f(x) \geq 0$ for $x \in [a, c]$ and $f(x) \leq 0$ for $x \in [c, b]$ $(a < c < b)$ then area bounded by curve y = f(x) and x-axis between x = a and x = b is $\int_a^c f(x)\, dx - \int_c^b f(x)\, dx = \int_a^c y dx + \left|\int_c^b y dx\right|$

Ex. Find the area bounded by the curve $y = \sin 2x$, x-axis and the lines $x = \dfrac{\pi}{4}$ and $x = \dfrac{3\pi}{4}$

Sol. Required area $= \int_{\pi/4}^{\pi/2} \sin 2x dx + \left|\int_{\pi/2}^{3\pi/4} \sin 2x dx\right| = \left(-\dfrac{\cos 2x}{2}\right)\Bigg|_{\pi/4}^{\pi/2} + \left|\left(-\dfrac{\cos 2x}{2}\right)\Bigg|_{\pi/2}^{3\pi/4}\right|$



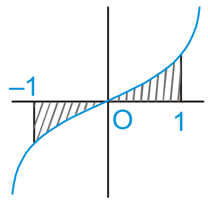
$$= -\frac{1}{2}[-1-0] + \left|\frac{1}{2}(0+(-1))\right| = 1 \text{ sq. unit}$$

Ex. Find the area bounded by $y = x^3$ and x- axis between ordinates $x = -1$ and $x = 1$

Sol. Required area $= \int_{-1}^{0} -x^3 dx + \int_{0}^{1} x^3 dx$

$$= \left[-\frac{x^4}{4}\right]_{-1}^{0} + \left[\frac{x^3}{4}\right]_{0}^{1}$$

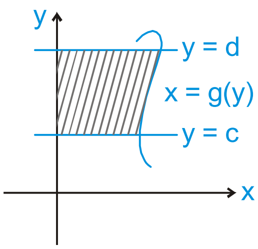
$$= 0 - \left(-\frac{1}{4}\right) + \frac{1}{4} - 0 = \frac{1}{2}$$



Graph of $y = x^3$

AREA INCLUDED BETWEEN THE CURVE x = g(y), Y-AXIS AND THE ABSCISSAS y = c, y = d

(A) If $g(y) \geq 0$ for $y \in [c,d]$ then area bounded by curve $x = g(y)$ and y–axis between abscissa $y = c$ and $y = d$ is $\int_{y=c}^{d} g(y)dy = \int_{c}^{d} x\, dy$



Graph of x = g(y)

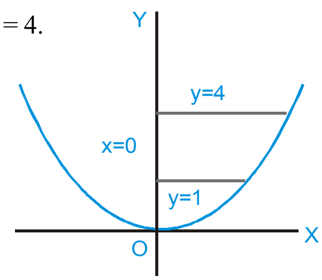
(B) If $g(y) \leq 0$ for $y \in [c,d]$ then area bounded by curve $x = g(y)$ and y–axis between abscissa $y = c$ and $y = d$ is $-\int_{y=c}^{d} g(y)dy = \left|\int_{c}^{d} x\, dy\right|$

Ex. Find the area in the first quadrant bounded by $y = 4x^2$, $x = 0$, $y = 1$ and $y = 4$.

Sol. Required area $= \int_1^4 x\, dy = \int_1^4 \dfrac{\sqrt{y}}{2} dy = \dfrac{1}{2}\left[\dfrac{2}{3}y^{3/2}\right]_1^4$

$$= \frac{1}{3}[4^{3/2} - 1] = \frac{1}{3}[8-1]$$

$$= \frac{7}{3} = 2\frac{1}{3} \text{ sq.units.}$$



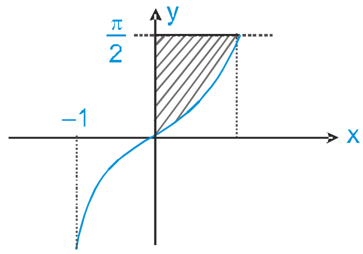
Ex. Find area bounded between $y = \sin^{-1}x$ and y–axis between $y = 0$ and $y = \frac{\pi}{2}$.

Sol. $y = \sin^{-1} x$

$\Rightarrow \quad x = \sin y$

Required area $= \int_0^{\frac{\pi}{2}} \sin y \, dy$

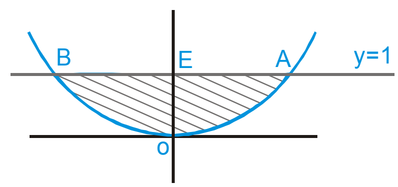
$= \left. -\cos y \right]_0^{\frac{\pi}{2}} = -(0-1) = 1$



Graph of $y = \sin^{-1}x$

Ex. Find the area bounded by the parabola $x^2 = y$ and line $y = 1$.

Sol. Graph of $y = x^2$



Required area is area OABO

$= 2$ area (OAEO)

$= 2\int_0^1 |x| \, dy = 2\int_0^1 \sqrt{y} \, dy = \frac{4}{3}$.

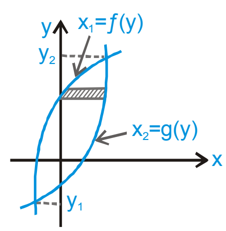
AREA ENCLOSED BETWEEN TWO CURVES

(A) Area bounded by two curves $y = f(x)$ & $y = g(x)$ such that $f(x) > g(x)$ is

$$A = \int_{x_1}^{x_2} (y_1 - y_2) dy$$

$$A = \int_{x_1}^{x_2} [f(x) - g(x)] dx$$

(B) In case horizontal strip is taken we have

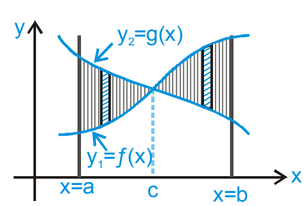


$$A = \int_{y_1}^{y_2} (x_1 - x_2) dy$$

$$A = \int_{y_1}^{y_2} [f(y) - g(y)] dy$$

(C) If the curves $y_1 = f(x)$ and $y_2 = g(x)$ intersect at $x = c$, then required area

$$A = \int_a^c (g(x) - f(x)) dx + \int_c^b (f(x) - g(x)) dx = \int_a^b |f(x) - g(x)| dx$$



❖ Required area must have all the boundaries indicated in the problem.

Ex. Find the area bounded by the curve $y = (x - 1)(x - 2)(x - 3)$ lying between the ordinates $x = 0$ and $x = 3$ and x-axis

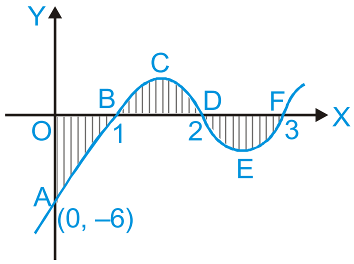
Sol. To determine the sign, we follow the usual rule of change of sign.

$y = +ve$ for $x > 3$

$y = -ve$ for $2 < x < 3$

$y = +ve$ for $1 < x < 2$

$y = -ve$ for $x < 1$.



$$\int_0^3 |y|\,dx = \int_0^1 |y|\,dx + \int_1^2 |y|\,dx + \int_2^3 |y|\,dx$$

$$= \int_0^1 -y\,dx + \int_1^2 y\,dx + \int_2^3 -y\,dx$$

Now let $F(x) = \int (x-1)(x-2)(x-3)\,dx$

$$= \int (x^3 - 6x^2 + 11x - 6)\,dx = \frac{1}{4}x^4 - 2x^3 + \frac{11}{2}x^2 - 6x.$$

$\therefore$ $F(0) = 0, F(1) = -\frac{9}{4}, F(2) = -2, F(3) = -\frac{9}{4}$.

Hence required Area $= -[F(1) - F(0)] + [F(2) - F(1)] - [F(3) - F(2)] = 2\frac{3}{4}$ sq.units.

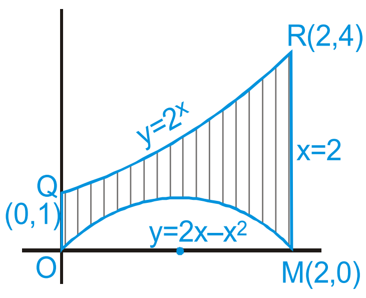
Ex. Find the area of the region bounded by $y = \sin x$, $y = \cos x$ and ordinates $x = 0$, $x = \pi/2$

Sol. $\displaystyle\int_0^{\pi/2} |\sin x - \cos x|\,dx$

$$\int_0^{\pi/4} (\cos x - \sin x)\,dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x)\,dx = 2(\sqrt{2} - 1)$$

Ex. Compute the area of the figure bounded by the straight lines $x = 0$, $x = 2$ and the curves $y = 2^x$, $y = 2x - x^2$.

Sol. Figure is self-explanatory $y = 2^x$, $(x - 1)^2 = -(y - 1)$



The required area $= \int_0^2 (y_1 - y_2)\,dx$

where $y_1 = 2^x$ and $y_2 = 2x - x^2 = \int_0^2 (2^x - 2x + x^2)\,dx$

$$= \left[\frac{2^x}{\ln 2} - x^2 + \frac{1}{3}x^3\right]_0^2 = \left(\frac{4}{\ln 2} - 4 + \frac{8}{3}\right) - \frac{1}{\ln 2} = \frac{3}{\ln 2} - \frac{4}{3} \text{ sq.units.}$$

Ex. Find the area enclosed by curve (graph) $y = x^2 + x + 1$ and its tangent at (1,3) between ordinates $x = -1$ and $x = 1$.

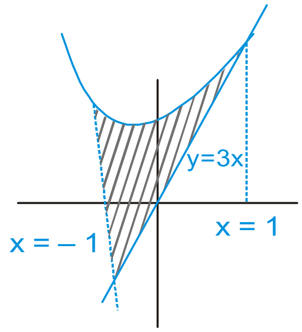
Sol. $\frac{dy}{dx} = 2x + 1$

$$\frac{dy}{dx} = 3 \text{ at } x = 1$$

Equation of tangent is

$$y - 3 = 3(x - 1)$$

$$y = 3x$$



Required area $= \int_{-1}^{1} (x^2 + x + 1 - 3x)\, dx$

$$= \int_{-1}^{1} (x^2 - 2x + 1)\, dx = \left.\frac{x^3}{3} - x^2 + x\right]_{-1}^{1}$$

$$= \left(\frac{1}{3} - 1 + 1\right) - \left(-\frac{1}{3} - 1 - 1\right) = \frac{2}{3} + 2 = \frac{8}{3}$$

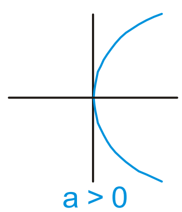
CURVE-TRACING

To find approximate shape of a curve, the following phrases are suggested :

(A) Symmetry

(i) Symmetry about x-axis

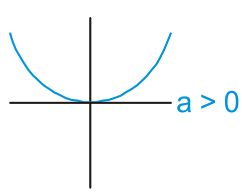
If all the powers of 'y' in the equation are even then the curve (graph) is symmetrical about the x-axis.



E.g. : $y^2 = 4ax$.

(ii) Symmetry about y-axis

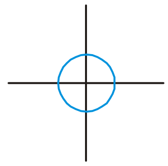
If all the powers of 'x' in the equation are even then the curve (graph) is symmetrical about the y-axis.



E.g. : $x^2 = 4ay$.

(iii) Symmetry about both axis

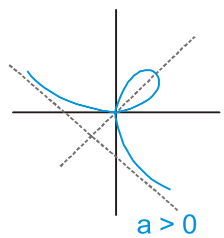
If all the powers of 'x' and 'y' in the equation are even, then the curve (graph) is symmetrical about the axis of 'x' as well as 'y' .



E.g. : $x^2 + y^2 = a^2$.

(iv) Symmetry about the line y = x

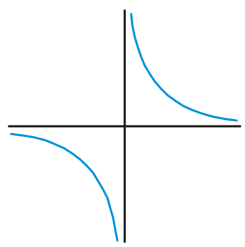
If the equation of the curve remain unchanged on interchanging 'x' and 'y', then the curve (graph) is symmetrical about the line $y = x$.



E.g. : $x^3 + y^3 = 3axy$.

(v) Symmetry in opposite quadrants

If the equation of the curve (graph) remain unaltered when 'x' and 'y' are replaced by '–x' and '–y' respectively, then there is symmetry in opposite quadrants.



E.g. : $xy = c^2$

(B) Find the points where the curve crosses the x-axis and the y-axis.

(C) Find $\frac{dy}{dx}$ and equate it to zero to find the points on the curve where you have horizontal tangents.

(D) Examine intervals when f(x) is increasing or decreasing

(E) Examine what happens to 'y' when $x \to \infty$ or $x \to -\infty$

Asymptotes

Asymptote(s) is (are) line (s) whose distance from the curve tends to zero as point on curve moves towards infinity along branch of curve.

(i) If $\lim\limits_{x\to a} f(x) = \infty$ or $\operatorname{Lt}\limits_{x\to a} f(x) = -\infty$, then $x = a$ is asymptote of $y = f(x)$

(ii) If $\lim\limits_{x\to\infty} f(x) = k$ or $\operatorname{Lt}\limits_{x\to-\infty} f(x) = k$ then $y = k$ is asymptote of $y = f(x)$

(iii) If $\lim\limits_{x\to\infty} \dfrac{f(x)}{x} = m_1$, $\operatorname{Lt}\limits_{x\to\infty} (f(x) - m_1x) = c_1$, then $y = m_1x + c_1$ is an asymptote (inclined to right).

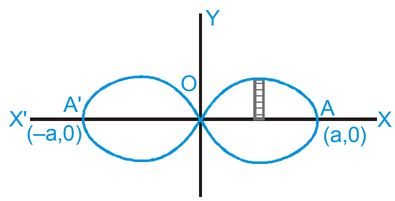
(iv) If $\lim\limits_{x\to-\infty} \dfrac{f(x)}{x} = m_2$, $\lim\limits_{x\to-\infty} (f(x) - m_2x) = c_2$, then $y = m_2x + c_2$ is an asymptote (inclined to left).

Ex. Find the area of a loop as well as the whole area of the curve $a^2y^2 = x^2(a^2 - x^2)$.

Sol. The curve is symmetrical about both the axes. It cuts x-axis at (0, 0), (–a, 0), (a, 0)

Area of a loop $= 2\int_0^a y\,dx = 2\int_0^a \dfrac{x}{a}\sqrt{a^2 - x^2}\,dx$

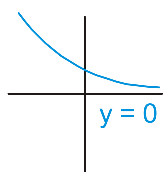
$$= -\frac{1}{a}\int_0^a \sqrt{a^2 - x^2}(-2x)dx = -\frac{1}{a}\left[\frac{2}{3}(a^2 - x^2)^{3/2}\right]_0^a = \frac{2}{3}a^2$$



Total area $= 2 \times \dfrac{2}{3}a^2 = \dfrac{4}{3}a^2$ sq.units.

Ex. Find asymptote of $y = e^{-x}$

Sol. $\lim\limits_{x\to\infty} y = \lim\limits_{x\to\infty} e^{-x} = 0$



Graph of $y = e^{-x}$

$\therefore$ $y = 0$ is asymptote.

Ex. Find the whole area included between the curve $x^2y^2 = a^2(y^2 - x^2)$ and its asymptotes.

Sol. **(i)** The curve is symmetric about both the axes (even powers of x & y)

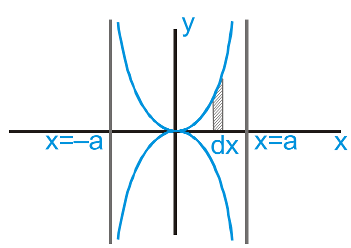
(ii) Asymptotes are $x = \pm a$

$$A = 4\int_0^a y\,dx$$

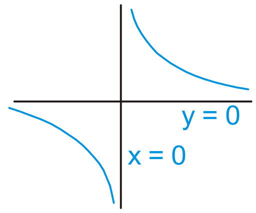
$$= 4\int_0^a \frac{ax}{\sqrt{a^2 - x^2}}dx$$

$$= 4a\left|-\sqrt{a^2 - x^2}\right|_0^a$$

$$= 4a^2$$



Ex. Find asymptotes of $xy = 1$ and draw graph.



Sol. $y = \dfrac{1}{x}$

$\lim\limits_{x\to 0} y = \lim\limits_{x\to 0} \dfrac{1}{x} = \infty \Rightarrow$ $x = 0$ is asymptote.

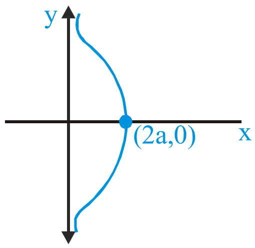
$\lim\limits_{x\to \infty} y = \lim\limits_{x\to \infty} \dfrac{1}{x} = 0 \Rightarrow$ $y = 0$ is asymptote.

Ex. Find the area bounded by the curve $xy^2 = 4a^2(2a-x)$ and its asymptote.

Sol. **(i)** The curve is symmetrical about the x-axis as it contains even powers of y.

(ii) It passes through (2a,0).

(iii) Its asymptote is $x = 0$, i.e., y-axis.



$$A = 2\int_0^{2a} y\,dx = 2\int_0^{2a} 2a\sqrt{\frac{2a-x}{x}}\,dx$$

Put $x = 2a\sin^2\theta$

$$A = 16a^2 \int_0^{\pi/2} \cos^2\theta\, d\theta$$

$$= 4\pi a^2$$

(i) If a function is known to be positive valued then graph is not necessary.

(ii) Most general formula for area bounded by curve $y = f(x)$ and x- axis between ordinates $x = a$ and $x = b$ is $\int_a^b |f(x)|\,dx$

(iii) If $y = f(x)$ does not change sign in [a, b], then area bounded by $y = f(x)$, x-axis between ordinates $x = a$, $x = b$ is $\left|\int_a^b f(x)\,dx\right|$

(iv) General formula for area bounded by curve $x = g(y)$ and y–axis between abscissa $y = c$ and $y = d$ is $\int_{y=c}^{d} |g(y)|\,dy$

(v) Area bounded by curves $y = f(x)$ and $y = g(x)$ between ordinates $x = a$ and $x = b$ is $\int_a^b |f(x) - g(x)|\,dx$.

(vi) If the curve is symmetric and suppose it has 'n' symmetric portions, then total area = n (Area of one symmetric portion).

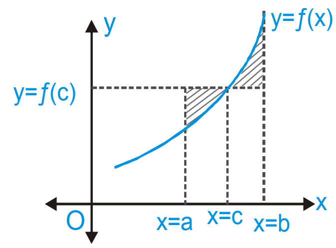
IMPORTANT POINTS

(A) Since area remains invariant even if the co-ordinate axes are shifted, hence shifting of origin in many cases proves to be very convenient in computing the area.

(B) If the equation of the curve is in parametric form, then $A = \int_{t=\alpha}^{t=\beta} y\dfrac{dx}{dt}.dt$ or $\int_{t=\gamma}^{t=\delta} x\dfrac{dy}{dt}.dt$, where α & β are values corresponding to values of x and γ & δ are values corresponding to values of y.

(C) If $y = f(x)$ is a monotonic function in (a, b), then the area bounded by the ordinates at $x = a$, $x = b$, $y = f(x)$ and $y = f(c)$ [where $c \in (a, b)$] is minimum when $c = \dfrac{a+b}{2}$.

Proof Let the function $y = f(x)$ be monotonically increasing.



Required area $A = \int_a^c [f(c) - f(x)]dx + \int_c^b [f(x) - f(c)]dx$

For minimum area, $\dfrac{dA}{dc} = 0$

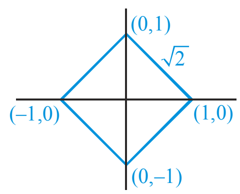
$\Rightarrow \quad [f'(c).c + f(c) - f'(c)a - f(c)] + [-f(c) - f'(c).b + f'(c).c + f(c)] = 0$

$\Rightarrow \quad f'(c)\left\{c - \dfrac{a+b}{2}\right\} = 0$

$\Rightarrow \quad c = \dfrac{a+b}{2} \qquad (\because f'(c) \neq 0)$

Ex. Find the area enclosed by $|x - 1| + |y + 1| = 1$.

Sol. Shift the origin to $(1, -1)$.



$X = x - 1 \qquad Y = y + 1$

$|X| + |Y| = 1$

Area $= \sqrt{2} \times \sqrt{2} = 2$ sq. units

Ex. Find the area bounded by x-axis and the curve given by $x = a\sin t$, $y = a\cos t$ for $0 \le t \le \pi$.

Sol. $\text{Area} = \int_0^\pi y\dfrac{dx}{dt}.dt = \int_0^\pi a\cos t(a\cos t)dt = \dfrac{a^2}{2}\int_0^\pi (1 + \cos 2t)dt = \dfrac{a^2}{2}\left|t + \dfrac{\sin 2t}{2}\right|_0^\pi = \dfrac{a^2}{2}|\pi| = \dfrac{\pi a^2}{2}$

Alternatively,

$\text{Area} = \int_0^\pi x\dfrac{dy}{dt}.dt = \left|\int_0^\pi a\sin t(-a\sin t)dt\right| = \dfrac{a^2}{2}\left|\int_0^\pi (\cos 2t - 1)dt\right| = \dfrac{a^2}{2}\left|\dfrac{\sin 2t}{2} - t\right|_0^\pi = \dfrac{\pi a^2}{2}$

Ex. Find the area of the figure bounded by one arc of the cycloid $x = a(t - \sin t)$, $y = a(1 - \cos t)$ and the x-axis.

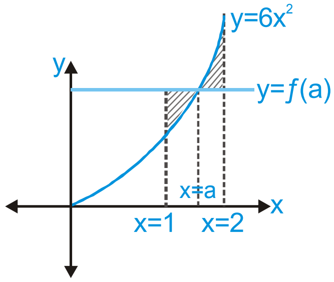
Sol. To find the points where an arc cuts x-axis

$a(1 - \cos t) = 0 \quad \Rightarrow \quad t = 0, \pi$

$\text{Area} = \int_0^\pi y\dfrac{dx}{dt}dt = \int_0^\pi a^2(1 - \cos t)^2 dt = a^2\left|\dfrac{3}{2}t - 2\sin t + \dfrac{\sin 2t}{4}\right|_0^\pi = a^2\left[\dfrac{3\pi}{2}\right] = \dfrac{3\pi a^2}{2}$

Ex. Find the value of 'a' for which area bounded by $x = 1$, $x=2$, $y=6x^2$ and $y=f(a)$ is minimum.

Sol. Let $b = f(a)$.



$$A = \int_1^a (b - 6x^2)dx + \int_a^2 (6x^2 - b)dx = \left|bx - 2x^3\right|_1^a + \left|2x^3 - bx\right|_a^2$$

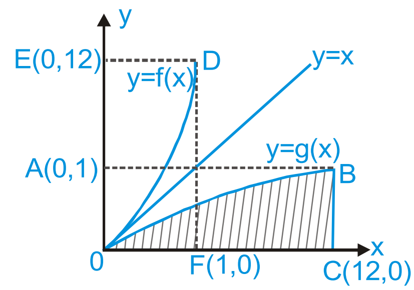
$$= 8a^3 - 18a^2 + 18$$

For minimum area $\dfrac{dA}{da} = 0$

$\Rightarrow \quad 24a^2 - 36a = 0 \quad \Rightarrow \quad a = 1.5$

Ex. If $y = g(x)$ is the inverse of a bijective mapping $f : R \to R$, $f(x) = 6x^5 + 4x^3 + 2x$, find the area bounded by g(x), the x-axis and the ordinate at $x = 12$.

Sol. $f(x) = 12$



$\Rightarrow \quad 6x^5 + 4x^3 + 2x = 12 \Rightarrow \quad x = 1$

$$\int_0^{12} g(x)dx = \text{area of rectangle OEDF} - \int_0^1 f(x)dx$$

$$= 1 \times 12 - \int_0^1 (6x^5 + 4x^3 + 2x)dx = 12 - 3 = 9 \text{ sq. units.}$$

USEFUL RESULTS

(A) Whole area of the ellipse, $x^2/a^2 + y^2/b^2 = 1$ is πab sq.units.

(B) Area enclosed between the parabolas $y^2 = 4ax$ & $x^2 = 4by$ is $16ab/3$ sq.units.

(C) Area included between the parabola $y^2 = 4ax$ & the line $y = mx$ is $8a^2/3m^3$ sq.units.

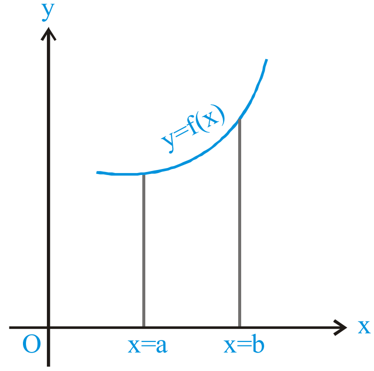
(D) The area of the region bounded by one arch of sin ax (or cosbx) and x-axis is 2/a sq.units.

(E) Average value of a function $y = f(x)$ over an interval $a \le x \le b$ is defined as : $y(av) = \dfrac{1}{b-a}\displaystyle\int_a^b f(x)dx$.

TIPS & FORMULAS

1 The area bounded by the curve y = f(x), the x-axis and the ordinates x = a & x = b is given by,

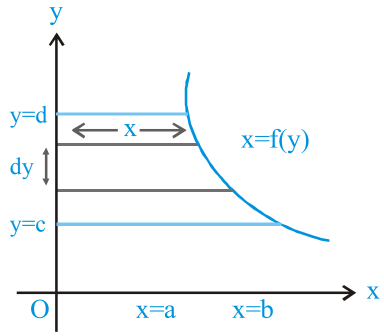
$$A = \int_a^b f(x)dx = \int_a^b y dx .$$



2. If the area is below the x-axis then A is negative. The convention is to consider the magnitude only i.e.

$A = \left|\int_a^b y\,dx\right|$ in this case.

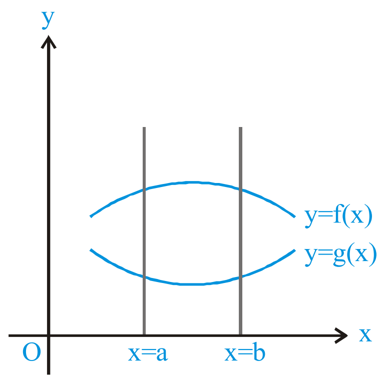
3. The area bounded by the curve x = f(y), y-axis & abscissa y = c, y = d is given by, Area $= \int_c^d x\,dy = \int_c^d f(y)dy$



4. Area between the curves y = f(x) & y = g(x) between the ordinates x = a & x = b is given by,

$$A = \int_a^b f(x)dx - \int_a^b g(x)dx$$

$$= \int_a^b [f(x) - g(x)]dx$$



5. Average value of a function y = f(x) w.r.t. x over an interval $a \le x \le b$ is defined as : $y(av) = \frac{1}{b-a}\int_a^b f(x)dx$.

6. **Curve Tracing**

The following outline procedure is to be applied in Sketching the graph of a function y = f(x) which is turn will be extremely useful to quickly and correctly evaluate the area under the curves.

(A) Symmetry : The symmetry of the curve is judged as follows :

- **(i)** If all the powers of y in the equation are even then the curve is symmetrical about the axis of x.
- **(ii)** If all the powers of x are even, the curve is symmetrical about the axis of y.
- **(iii)** If powers of x & y both are even, the curve is symmetrical about the axis of x as well as y.
- **(iv)** If the equation of the curve remains unchanged on interchanging x and y, then the curve is symmetrical about $y = x$.
- **(v)** If on interchanging the sign of x & y both the equation of the curve is unaltered then there is symmetry in opposite quadrants.

(B) Find dy/dx & equate it to zero to find the points on the curve where you have horizontal tangents.

(C) Find the points where the curve crosses the x-axis & also the y-axis.

(D) Examine if possible the intervals when f(x) is increasing or decreasing. Examine what happens to 'y' when $x \to \infty$ or $-\infty$.

7. Useful Results

(A) Whole area of the ellipse, $x^2/a^2 + y^2/b^2 = 1$ is π ab.

(B) Area enclosed between the parabolas $y^2 = 4ax$ & $x^2 = 4by$ is $16ab/3$.

(C) Area included between the parabola $y^2 = 4\,ax$ & the line $y = mx$ is $8a^2/3m^3$.

DCAM classes
Dynamic Classes for Academic Mastery