SOLVED EXAMPLES

Ex. 1 Evaluate $\int \frac{\cos^3 x}{\sin^2 x + \sin x} dx$

Sol. $I = \int \frac{(1-\sin^2 x)\cos x}{\sin x(1+\sin x)} dx = \int \frac{1-\sin x}{\sin x} \cos x \, dx$

Put sinx = t $\Rightarrow$ cosx dx = dt

$\Rightarrow$ $I = \int \frac{1-t}{t} dt = \ell n\,|\,t\,| - t + c = \ell n\,|\sin x| - \sin x + c$

Ex. 2 Evaluate $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} . \frac{1}{x} dx$

Sol. Put $x = \cos^2\theta$

$\Rightarrow$ $dx = -2\sin\theta \cos\theta \, d\theta$

$\Rightarrow$ $I = \int \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} . \frac{1}{\cos^2\theta}(-2\sin\theta\cos\theta)d\theta = -\int 2\tan\frac{\theta}{2}\tan\theta \, d\theta$

$= -4\int \frac{\sin^2(\theta/2)}{\cos\theta} d\theta = -2\int \frac{1-\cos\theta}{\cos\theta} d\theta = -2\ell n\,|\sec\theta - \tan\theta| + 2\theta + c$

$= -2\ell n \left| \frac{1+\sqrt{1-x}}{x} \right| + 2\cos^{-1}\sqrt{x} + c$

Ex. 3 Evaluate : $\int x \, \ell n(1+x) \, dx$

Sol. Let $I = \int x \, \ell n(1+x) \, dx$

$= \ell n\,(x+1) . \frac{x^2}{2} - \int \frac{1}{x+1} . \frac{x^2}{2} dx$

$= \frac{x^2}{2} \ell n\,(x+1) - \frac{1}{2} \int \frac{x^2}{x+1} dx = \frac{x^2}{2} \ell n\,(x+1) - \frac{1}{2} \int \frac{x^2-1+1}{x+1} dx$

$= \frac{x^2}{2} \ell n\,(x+1) - \frac{1}{2} \int \left(\frac{x^2-1}{x+1} + \frac{1}{x+1} \right) dx$

$= \frac{x^2}{2} \ell n\,(x+1) - \frac{1}{2} \int \left((x-1) + \frac{1}{x+1} \right) dx$

$= \frac{x^2}{2} \ell n\,(x+1) - \frac{1}{2} \left[\frac{x^2}{2} - x + \ell n\,|x+1| \right] + C$

Ex. 4 Evaluate $\int \frac{(x^2-1)dx}{(x^4+3x^2+1)\tan^{-1}\left(x+\frac{1}{x}\right)}$

Sol. The given integral can be written as

$$I = \int \frac{\left(1-\frac{1}{x^2}\right)dx}{\left[\left(x+\frac{1}{x}\right)^2+1\right]\tan^{-1}\left(x+\frac{1}{x}\right)}$$

Let $\left(x+\frac{1}{x}\right) = t$. Differentiating we get $\left(1-\frac{1}{x^2}\right)dx = dt$

Hence $I = \int \frac{dt}{(t^2+1)\tan^{-1} t}$

Now make one more substitution $\tan^{-1}t = u$. Then $\frac{dt}{t^2+1} = du$ and $I = \int \frac{du}{u} = \ell n\,|u| + c$

Returning to t, and then to x, we have

$$I = \ell n\,|\tan^{-1} t| + c = \ell n\left|\tan^{-1}\left(x+\frac{1}{x}\right)\right| + c$$

Ex. 5 Evaluate : $\int \frac{x}{1+\sin x}dx$

Sol. Let $I = \int \frac{x}{1+\sin x}dx = \int \frac{x(1-\sin x)}{(1+\sin x)(1-\sin x)}dx$

$$= \int \frac{x(1-\sin x)}{1-\sin^2 x}dx = \int \frac{x(1-\sin x)}{\cos^2 x}dx = \int x\sec^2 x dx - \int x\sec x\tan x dx$$

$$= \left[x\int \sec^2 x dx - \int\left\{\frac{dx}{dx}\int \sec^2 x dx\right\}dx\right] - \left[x\int \sec x\tan x dx - \int\left\{\frac{dx}{dx}\int \sec x\tan x dx\right\}dx\right]$$

$$= \left[x\tan x - \int \tan x dx\right] - \left[x\sec x - \int \sec x dx\right]$$

$$= [x\tan x - \ell n\,|\sec x|] - [x\sec x - \ell n\,|\sec x + \tan x|] + c$$

$$= x(\tan x - \sec x) + \ell n\left|\frac{(\sec x+\tan x)}{\sec x}\right| + c = \frac{-x(1-\sin x)}{\cos x} + \ell n\,|1+\sin x| + c$$

Ex. 6 Evaluate : $\int e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx$

Sol. Given integral $= \int e^x\left(\frac{1-2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}}\right)dx$

$$= \int e^x\left(\frac{1}{2}\operatorname{cosec}^2\frac{x}{2} - \cot\frac{x}{2}\right)dx = -e^x\cot\frac{x}{2} + C$$

Ex. 7 The value of $\int e^x\left(\frac{x^4+2}{(1+x^2)^{5/2}}\right)dx$ is equal to -

Sol. Let $I = \int e^x\left(\frac{x^4+2}{(1+x^2)^{5/2}}\right)dx = \int e^x\left(\frac{1}{(1+x^2)^{1/2}}+\frac{1-2x^2}{(1+x^2)^{5/2}}\right).dx$

$$= \int e^x\left(\frac{1}{(1+x^2)^{1/2}}-\frac{x}{(1+x^2)^{3/2}}+\frac{x}{(1+x^2)^{3/2}}+\frac{1-2x^2}{(1+x^2)^{5/2}}\right)dx$$

$$= \frac{e^x}{(1+x^2)^{1/2}}+\frac{xe^x}{(1+x^2)^{3/2}}+c = \frac{e^x\{1+x^2+x\}}{(1+x^2)^{3/2}}+c$$

Ex. 8 Evaluate $\int \frac{x^4}{(x+2)(x^2+1)}dx$

Sol. $\frac{x^4}{(x+2)(x^2+1)} = (x-2)+\frac{3x^2+4}{(x+2)(x^2+1)}$

Now, $\frac{3x^2+4}{(x+2)(x^2+1)} = \frac{16}{5(x+2)}+\frac{-\frac{1}{5}x+\frac{2}{5}}{x^2+1}$

So, $\frac{x^4}{(x+2)(x^2+1)} = x-2+\frac{16}{5(x+2)}+\frac{-\frac{1}{5}x+\frac{2}{5}}{x^2+1}$

Now, $\int\left((x-2)+\frac{16}{5(x+2)}+\frac{-\frac{1}{5}x+\frac{2}{5}}{x^2+1}\right)dx$

$$= \frac{x^2}{2}-2x+\frac{2}{5}\tan^{-1}x+\frac{16}{5}\ell n|x+2|-\frac{1}{10}\ell n(x^2+1)+c$$

Ex. 9 Evaluate : $\int \frac{1}{x^4+5x^2+1}dx$

Sol. $I = \frac{1}{2}\int\frac{2}{x^4+5x^2+1}dx$

$\Rightarrow I = \frac{1}{2}\int\frac{1+x^2}{x^4+5x^2+1}dx+\frac{1}{2}\int\frac{1-x^2}{x^4+5x^2+1}dx = \frac{1}{2}\int\frac{1+1/x^2}{x^2+5+1/x^2}dx-\frac{1}{2}\int\frac{1-1/x^2}{x^2+5+1/x^2}dx$

{dividing N^r and D^r by x^2}

$$= \frac{1}{2}\int\frac{(1+1/x^2)}{(x-1/x)^2+7}dx-\frac{1}{2}\int\frac{(1-1/x^2)dx}{(x+1/x)^2+3} = \frac{1}{2}\int\frac{dt}{t^2+(\sqrt{7})^2}-\frac{1}{2}\int\frac{du}{u^2+(\sqrt{3})^2}$$

where $t = x-\frac{1}{x}$ and $u = x+\frac{1}{x}$

$I = \frac{1}{2}.\frac{1}{\sqrt{7}}\left(\tan^{-1}\frac{t}{\sqrt{7}}\right)-\frac{1}{2}.\frac{1}{\sqrt{3}}\left(\tan^{-1}\frac{u}{\sqrt{3}}\right)+c \quad\Rightarrow\quad \frac{1}{2}\left[\frac{1}{\sqrt{7}}\tan^{-1}\left(\frac{x-1/x}{\sqrt{7}}\right)-\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x+1/x}{\sqrt{3}}\right)\right]+c$

Ex. 10 Evaluate : $\int\left[\ell n\,(\ell nx)+\frac{1}{(\ell nx)^2}\right]dx$

Sol. Let $I=\int\left[\ell n\,(\ell nx)+\frac{1}{(\ell nx)^2}\right]dx$ $\{$put $x=e^t \Rightarrow dx=e^t\,dt\}$

$\therefore$ $I=\int e^t\left(\ell nt+\frac{1}{t^2}\right)dt=\int e^t\left(\ell nt-\frac{1}{t}+\frac{1}{t}+\frac{1}{t^2}\right)dt$

$$=e^t\left(\ell nt-\frac{1}{t}\right)+C=x\left[\ell n\,(\ell nx)-\frac{1}{\ell nx}\right]+C$$

Ex. 11 Evaluate $\int \sin^2 x\cos^4 x\,dx$

Sol. $\int \sin^2 x\cos^4 x\,dx=\int\left(\frac{1-\cos 2x}{2}\right)\left(\frac{\cos 2x+1}{2}\right)^2dx=\int\frac{1}{8}(1-\cos 2x)\left(\cos^2 2x+2\cos 2x+1\right)dx$

$$=\frac{1}{8}\int\left(\cos^2 2x+2\cos 2x+1-\cos^3 2x-2\cos^2 2x-\cos 2x\right)dx$$

$$=\frac{1}{8}\int\left(-\cos^3 2x-\cos^2 2x+\cos 2x+1\right)dx=-\frac{1}{8}\int\left(\frac{\cos 6x+3\cos 2x}{4}+\frac{1+\cos 4x}{2}-\cos 2x-1\right)dx$$

$$=-\frac{1}{32}\left[\frac{\sin 6x}{6}+\frac{3\sin 2x}{2}\right]-\frac{1}{16}x-\frac{\sin 4x}{64}+\frac{\sin 2x}{16}+\frac{x}{8}+c$$

$$=-\frac{\sin 6x}{192}-\frac{\sin 4x}{64}+\frac{1}{64}\sin 2x+\frac{x}{16}+c$$

Ex. 12 Evaluate : $\int\frac{dx}{2+\sin^2 x}$

Sol. Divide numerator and denominator by $\cos^2 x$

$$I=\int\frac{\sec^2 x\,dx}{2\sec^2 x+\tan^2 x}=\int\frac{\sec^2 x\,dx}{2+3\tan^2 x}$$

Let $\sqrt{3}\tan x=t$ $\therefore$ $\sqrt{3}\sec^2 x\,dx=dt$

So $I=\frac{1}{\sqrt{3}}\int\frac{dt}{2+t^2}=\frac{1}{\sqrt{3}}\cdot\frac{1}{\sqrt{2}}\tan^{-1}\frac{t}{\sqrt{2}}+c=\frac{1}{\sqrt{6}}\tan^{-1}\left(\frac{\sqrt{3}\tan x}{\sqrt{2}}\right)+c$

Ex. 13 Evaluate : $\int\frac{1}{x^2-x+1}dx$

Sol. $\int\frac{1}{x^2-x+1}dx=\int\frac{1}{x^2-x+\frac{1}{4}-\frac{1}{4}+1}dx=\int\frac{1}{(x-1/2)^2+3/4}dx$

$$=\int\frac{1}{(x-1/2)^2+\left(\sqrt{3}/2\right)^2}dx=\frac{1}{\sqrt{3}/2}\tan^{-1}\left(\frac{x-1/2}{\sqrt{3}/2}\right)+C$$

$$=\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+C.$$

Ex. 14 Evaluate $\int \frac{\sqrt{\sin x}}{\cos^{9/2} x} dx$

Sol. Let $I = \int \frac{\sin^{1/2} x}{\cos^{9/2} x} dx = \int \frac{dx}{\sin^{-1/2} x \cos^{9/2} x}$

Here $m + n = \frac{1}{2} - \frac{9}{2} = -4$ (negative even integer).

Divide Numerator & Denominator by $\cos^4 x$.

$$I = \int \sqrt{\tan x} \sec^4 x\, dx = \int \sqrt{\tan x}(1 + \tan^2 x)\sec^2 x dx$$

$$= \int \sqrt{t}(1 + t^2)dt \quad \text{(using } \tan x = t)$$

$$= \frac{2}{3}t^{3/2} + \frac{2}{7}t^{7/2} + c = \frac{2}{3}\tan^{3/2} x + \frac{2}{7}\tan^{7/2} x + c$$

Ex. 15 Evaluate $\int \frac{\cos^4 x dx}{\sin^3 x \{\sin^5 x + \cos^5 x\}^{\frac{3}{5}}}$

Sol. $I = \int \frac{\cos^4 x}{\sin^3 x \{\sin^5 x + \cos^5 x\}^{\frac{3}{5}}} dx = \int \frac{\cos^4 x}{\sin^6 x \{1 + \cot^5 x\}^{\frac{3}{5}}} dx = \int \frac{\cot^4 x \operatorname{cosec}^2 x dx}{(1 + \cot^5 x)^{3/5}}$

Put $1 + \cot^5 x = t$

$5\cot^4 x \operatorname{cosec}^2 x dx = -dt$

$$= -\frac{1}{5}\int \frac{dt}{t^{3/5}} = -\frac{1}{2}t^{2/5} + c = -\frac{1}{2}(1 + \cot^5 x)^{2/5} + c$$

Ex. 16 Evaluate : $\int (\sin x)^{1/3} (\cos x)^{-7/3} dx$

Sol. Here $m + n = \frac{1}{3} - \frac{7}{3} = -2$ (a negative integer)

$\therefore \int (\sin x)^{1/3} (\cos x)^{-7/3} dx = \int (\tan x)^{1/3} \frac{1}{\cos^2 x} dx$ {put $\tan x = t \Rightarrow \sec^2 x\, dx = dt$}

$$= \int t^{1/3} dt = \frac{3}{4} t^{4/3} + C = \frac{3}{4} (\tan x)^{4/3} + C$$

Ex. 17 Evaluate $\int \frac{dx}{(1 + x^2)\sqrt{1 - x^2}}$

Sol. Let, $I = \int \frac{dx}{(1 + x^2)\sqrt{1 - x^2}}$

Put $x = \frac{1}{t}$, So that $dx = \frac{-1}{t^2}dt$

$\therefore I = \int \frac{-1/t^2 dt}{(1 + 1/t^2)\sqrt{1 - 1/t^2}} = -\int \frac{t dt}{(t^2 + 1)\sqrt{t^2 - 1}}$

again let, $t^2 = u$. So that $2t\,dt = du$.

$= \frac{-1}{2}\int \frac{du}{(u+1)\sqrt{u-1}}$ which reduces to the form $\int \frac{dx}{P\sqrt{Q}}$ where both P and Q are linear so that

we put $u - 1 = z^2$ so that $du = 2z\,dz$

$$\therefore \quad I = -\frac{1}{2}\int \frac{2zdz}{(z^2+1+1)\sqrt{z^2}} = -\int \frac{dz}{(z^2+2)}$$

$$I = -\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{z}{\sqrt{2}}\right) + c$$

$$I = -\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{u-1}}{\sqrt{2}}\right) + c = -\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{t^2-1}}{\sqrt{2}}\right) + c = -\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{1-x^2}}{\sqrt{2}x}\right) + c$$

Ex. 18 $\int \frac{dx}{\cos^6 x + \sin^6 x}$ is equal to -

Sol. Let $I = \int \frac{dx}{\cos^6 x + \sin^6 x} = \int \frac{\sec^6 x}{1+\tan^6 x}dx = \int \frac{(1+\tan^2 x)^2 \sec^2 x dx}{1+\tan^6 x}$

If $\tan x = p$, then $\sec^2 x\, dx = dp$

$$\Rightarrow \quad I = \int \frac{(1+p^2)^2 dp}{1+p^6} = \int \frac{(1+p^2)}{p^4-p^2+1}dp = \int \frac{p^2\left(1+\frac{1}{p^2}\right)}{p^2\left(p^2+\frac{1}{p^2}-1\right)}dp$$

$$= \int \frac{dk}{k^2+1} = \tan^{-1}(k) + c \quad \left(\text{where } p - \frac{1}{p} = k, \left(1+\frac{1}{p^2}\right)dp = dk\right)$$

$$= \tan^{-1}\left(p - \frac{1}{p}\right) + c = \tan^{-1}(\tan x - \cot x) + c = \tan^{-1}(-2\cot 2x) + c$$

Ex. 19 Evaluate : $\int \frac{1}{x^4+1}dx$

Sol. We have,

$$I = \int \frac{1}{x^4+1}dx = \int \frac{\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx = \frac{1}{2}\int \frac{\frac{2}{x^2}}{x^2+\frac{1}{x^2}}dx$$

$$= \frac{1}{2}\int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}} - \frac{1-\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx \qquad = \frac{1}{2}\int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx - \frac{1}{2}\int \frac{1-\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx$$

$$= \frac{1}{2}\int \frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+2}\,dx \;-\; \frac{1}{2}\int \frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)^2-2}\,dx$$

Putting $x - \frac{1}{x} = u$ in 1st integral and $x + \frac{1}{x} = v$ in 2^{nd} integral, we get

$$I = \frac{1}{2}\int \frac{du}{u^2+\left(\sqrt{2}\right)^2} - \frac{1}{2}\int \frac{dv}{v^2-\left(\sqrt{2}\right)^2}$$

$$= \frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{u}{\sqrt{2}}\right) - \frac{1}{2}\,\frac{1}{2\sqrt{2}}\,\ell n\left|\frac{v-\sqrt{2}}{v+\sqrt{2}}\right| + C$$

$$= \frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{x-1/x}{\sqrt{2}}\right) - \frac{1}{4\sqrt{2}}\,\ell n\left|\frac{x+1/x-\sqrt{2}}{x+1/x+\sqrt{2}}\right| + C$$

$$= \frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}\,x}\right) - \frac{1}{4\sqrt{2}}\,\ell n\left|\frac{x^2-\sqrt{2}\,x+1}{x^2+x\sqrt{2}+1}\right| + C$$

Ex. 20 Evaluate : $\int \frac{2\sin 2x - \cos x}{6-\cos^2 x - 4\sin x}dx$

Sol. $I = \int \frac{2\sin 2x - \cos x}{6-\cos^2 x - 4\sin x}dx = \int \frac{(4\sin x - 1)\cos x}{6-(1-\sin^2 x)-4\sin x}dx = \int \frac{(4\sin x-1)\cos x}{\sin^2 x - 4\sin x + 5}dx$

Put sinx = t, so that cos x dx = dt.

$\therefore \quad I = \int \frac{(4t-1)dt}{(t^2-4t+5)}$ ...... **(i)**

Now, let $(4t - 1) = \lambda(2t - 4) + \mu$

Comparing coefficients of like powers of t, we get

$2\lambda = 4, -4\lambda + \mu = -1$ ...... **(ii)**

$\lambda = 2, \mu = 7$

$\therefore \quad I = \int \frac{2(2t-4)+7}{t^2-4t+5}dt$ {using **(i)** and **(ii)**}

$$= 2\int \frac{2t-4}{t^2-4t+5}dt + 7\int \frac{dt}{t^2-4t+5} = 2\log\left|t^2-4t+5\right| + 7\int \frac{dt}{t^2-4t+4-4+5}$$

$$= 2\log\left|t^2-4t+5\right| + 7\int \frac{dt}{(t-2)^2+(1)^2} = 2\log|t^2 - 4t + 5| + 7\,.\,\tan^{-1}(t-2) + c$$

$$= 2\log|\sin^2 x - 4\sin x + 5| + 7\tan^{-1}(\sin x - 2) + c.$$

Ex.21 The value of $\int \sqrt{\frac{3-x}{3+x}}.\sin^{-1}\left(\frac{1}{\sqrt{6}}\sqrt{3-x}\right)dx$, is equal to -

Sol. Here, $I = \int \sqrt{\frac{3-x}{3+x}}.\sin^{-1}\left(\frac{1}{\sqrt{6}}\sqrt{3-x}\right)dx$

Put $x = 3\cos 2\theta \Rightarrow dx = -6\sin 2\theta d\theta$

$$= \int \sqrt{\frac{3-3\cos 2\theta}{3+3\cos 2\theta}}.\sin^{-1}\left(\frac{1}{\sqrt{6}}\sqrt{3-3\cos 2\theta}\right)(-6\sin 2\theta)d\theta$$

$$= \int \frac{\sin\theta}{\cos\theta}.\sin^{-1}(\sin\theta).(-6\sin 2\theta)d\theta = -6\int \theta.(2\sin^2\theta)d\theta$$

$$= -6\int \theta(1-\cos 2\theta)d\theta = -6\left\{\frac{\theta^2}{2} - \int \theta\cos 2\theta\, d\theta\right\}$$

$$= -6\left\{\frac{\theta^2}{2} - \left(\theta\frac{\sin 2\theta}{2} - \int 1.\left(\frac{\sin 2\theta}{2}\right)d\theta\right)\right\} = -3\theta^2 + 6\left\{\theta\frac{\sin 2\theta}{2} + \frac{\cos 2\theta}{4}\right\} + c$$

$$= \frac{1}{4}\left\{-3\left(\cos^{-1}\left(\frac{x}{3}\right)\right)^2 + 2\sqrt{9-x^2}.\cos^{-1}\left(\frac{x}{3}\right) + 2x\right\} + c$$

Ex.22 Evaluate : $\int \frac{\tan\left(\frac{\pi}{4}-x\right)}{\cos^2 x\sqrt{\tan^3 x + \tan^2 x + \tan x}}dx$

Sol. $I = \int \frac{\tan\left(\frac{\pi}{4}-x\right)}{\cos^2 x\sqrt{\tan^3 x + \tan^2 x + \tan x}}dx = \int \frac{(1-\tan^2 x)dx}{(1+\tan x)^2\cos^2 x\sqrt{\tan^3 x + \tan^2 x + \tan x}}$

$$I = \int \frac{-\left(1 - \frac{1}{\tan^2 x}\right)\sec^2 x dx}{\left(\tan x + 2 + \frac{1}{\tan x}\right)\sqrt{\tan x + 1 + \frac{1}{\tan x}}}$$

let, $y = \sqrt{\tan x + 1 + \frac{1}{\tan x}} \Rightarrow 2y\, dy = \left(\sec^2 x - \frac{1}{\tan^2 x}.\sec^2 x\right)dx$

$\therefore$ $I = \int \frac{-2ydy}{(y^2+1).y} = -2\int \frac{dy}{1+y^2}$

$$= -2\tan^{-1} y + c = -2\tan^{-1}\left(\sqrt{\tan x + 1 + \frac{1}{\tan x}}\right) + c$$

Exercise # 1

[Single Correct Choice Type Questions]

1. $\int 4\sin x\cos\frac{x}{2}\cos\frac{3x}{2}dx$ is equal to -

(A) $\cos x-\frac{1}{2}\cos 2x+\frac{1}{3}\cos 3x+c$ **(B)** $\cos x-\frac{1}{2}\cos 2x-\frac{1}{3}\cos 3x+c$

(C) $\cos x+\frac{1}{2}\cos 2x+\frac{1}{3}\cos 3x+c$ **(D)** $\cos x+\frac{1}{2}\cos 2x-\frac{1}{3}\cos 3x+c$

2. The value of $\int\frac{dx}{\sin x\,.\sin(x+\alpha)}$ is equal to

(A) $\text{cosec}\,\alpha\;\ell n\left|\frac{\sin x}{\sin(x+\alpha)}\right|+C$ **(B)** $\text{cosec}\,\alpha\;\ell n\left|\frac{\sin(x+\alpha)}{\sin x}\right|+C$

(C) $\text{cosec}\,\alpha\;\ell n\left|\frac{\sec(x+\alpha)}{\sec x}\right|+C$ **(D)** $\text{cosec}\,\alpha\;\ell n\left|\frac{\sec x}{\sec(x+\alpha)}\right|+C$

3. Which one of the following is TRUE.

(A) $x.\int\frac{dx}{x}=x\ln|x|+C$ **(B)** $x.\int\frac{dx}{x}=x\ln|x|+Cx$

(C) $\frac{1}{\cos x}.\int\cos x\;dx=\tan x+C$ **(D)** $\frac{1}{\cos x}.\int\cos x\;dx=x+C$

4. $\int\frac{8x+13}{\sqrt{4x+7}}dx$ is equal to -

(A) $\frac{1}{6}(8x+11)\sqrt{4x+7}+c$ **(B)** $\frac{1}{6}(8x+13)\sqrt{4x+7}+c$

(C) $\frac{1}{6}(8x+9)\sqrt{4x+7}+c$ **(D)** $\frac{1}{6}(8x+15)\sqrt{4x+7}+c$

5. $\int\frac{(2x+1)}{(x^2+4x+1)^{3/2}}dx$

(A) $\frac{x^3}{(x^2+4x+1)^{1/2}}+C$ **(B)** $\frac{x}{(x^2+4x+1)^{1/2}}+C$ **(C)** $\frac{x^2}{(x^2+4x+1)^{1/2}}+C$ **(D)** $\frac{1}{(x^2+4x+1)^{1/2}}+C$

6. If $I=\int\frac{\sin x+\sin^3 x}{\cos 2x}dx=A\cos x+B\,\ell n|f(x)|+C$, then

(A) $A=\frac{1}{4},B=\frac{-1}{\sqrt{2}},f(x)=\frac{\sqrt{2}\cos x-1}{\sqrt{2}\cos x+1}$ **(B)** $A=-\frac{1}{2},B=\frac{-3}{4\sqrt{2}},f(x)=\frac{\sqrt{2}\cos x-1}{\sqrt{2}\cos x+1}$

(C) $A=-\frac{1}{2},B=\frac{3}{\sqrt{2}},f(x)=\frac{\sqrt{2}\cos x+1}{\sqrt{2}\cos x-1}$ **(D)** $A=\frac{1}{2},B=\frac{-3}{4\sqrt{2}},f(x)=\frac{\sqrt{2}\cos x-1}{\sqrt{2}\cos x+1}$

7. $\int \frac{1-x^7}{x(1+x^7)}\,dx$ equals :

(A) $\ln x + \frac{2}{7}\ln(1+x^7) + c$
(B) $\ln x - \frac{2}{7}\ln(1-x^7) + c$
(C) $\ln x - \frac{2}{7}\ln(1+x^7) + c$
(D) $\ln x + \frac{2}{7}\ln(1-x^7) + c$

8. $\int\left(\frac{\cos^8 x - \sin^8 x}{1-2\sin^2 x\cos^2 x}\right)dx$ equals -

(A) $-\frac{\sin 2x}{2} + c$
(B) $\frac{\sin 2x}{2} + c$
(C) $\frac{\cos 2x}{2} + c$
(D) $-\frac{\cos 2x}{2} + c$

9. The value of $\int\left\{\ln(1+\sin x) + x\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right\}dx$ is equal to:

(A) $x\,\ell n\,(1+\sin x) + C$
(B) $\ell n\,(1+\sin x) + C$
(C) $-x\,\ell n\,(1+\sin x) + C$
(D) $\ell n\,(1-\sin x) + C$

10. Suppose $A = \int \frac{dx}{x^2+6x+25}$ and $B = \int \frac{dx}{x^2-6x-27}$.

If $12(A+B) = \lambda\cdot\tan^{-1}\left(\frac{x+3}{4}\right) + \mu\cdot\ln\left|\frac{x-9}{x+3}\right| + C$, then the value of $(\lambda+\mu)$ is

(A) 3
(B) 4
(C) 5
(D) 6

11. The value of $\int\sqrt{\frac{1-\cos x}{\cos\alpha-\cos x}}\,dx$, where $0<\alpha<x<\pi$, is equal to

(A) $2\,\ell n\left(\cos\frac{\alpha}{2}-\cos\frac{x}{2}\right) + C$
(B) $\sqrt{2}\,\ell n\left(\cos\frac{\alpha}{2}-\cos\frac{x}{2}\right) + C$
(C) $2\sqrt{2}\,\ell n\left(\cos\frac{\alpha}{2}-\cos\frac{x}{2}\right) + C$
(D) $-2\sin^{-1}\left(\frac{\cos\frac{x}{2}}{\cos\frac{\alpha}{2}}\right) + C$

12. Primitive of $\sqrt[3]{\frac{x}{(x^4-1)^4}}$ w.r.t. x is -

(A) $\frac{3}{4}\left(1+\frac{1}{x^4-1}\right)^{\frac{1}{3}} + c$
(B) $-\frac{3}{4}\left(1+\frac{1}{x^4-1}\right)^{\frac{1}{3}} + c$
(C) $\frac{4}{3}\left(1+\frac{1}{x^4-1}\right)^{\frac{1}{3}} + c$
(D) $-\frac{4}{3}\left(1+\frac{1}{x^4-1}\right)^{\frac{1}{3}} + c$

13. The value of $\int[1+\tan x\,.\,\tan(x+\alpha)]\,dx$ is equal to

(A) $\cos\alpha\,.\,\ell n\left|\frac{\sin x}{\sin(x+\alpha)}\right| + C$
(B) $\tan\alpha\,.\,\ell n\left|\frac{\sin x}{\sin(x+\alpha)}\right| + C$
(C) $\cot\alpha\,.\,\ell n\left|\frac{\sec(x+\alpha)}{\sec x}\right| + C$
(D) $\cot\alpha\,.\,\ell n\left|\frac{\cos(x+\alpha)}{\cos x}\right| + C$

14. $\int \frac{x\,dx}{\sqrt{1+x^2+\sqrt{(1+x^2)^3}}}$ is equal to :

(A) $\frac{1}{2}\ln\left(1+\sqrt{1+x^2}\right)+c$ **(B)** $2\sqrt{1+\sqrt{1+x^2}}+c$ **(C)** $2\left(1+\sqrt{1+x^2}\right)+c$ **(D)** none of these

15. If $f(x)=\int \frac{2\sin x-\sin 2x}{x^3}dx$, where $x \neq 0$, then $\underset{x\to 0}{\text{Limit}}\ f'(x)$ has the value

(A) 0 **(B)** 1 **(C)** 2 **(D)** not defined

16. If $I_n=\int (\sin x)^n\,dx \quad n\in N$

Then $5I_4-6I_6$ is equal to

(A) $\sin x\cdot(\cos x)^5+C$ **(B)** $\sin 2x\cdot\cos 2x+C$

(C) $\frac{\sin 2x}{8}[\cos^2 2x+1-2\cos 2x]+C$ **(D)** $\frac{\sin 2x}{8}[\cos^2 2x+1+2\cos 2x]+C$

17. $\int \frac{x\quad dx}{\sqrt{1+x^2+\sqrt{\left(1+x^2\right)^3}}}$ is equal to -

(A) $\frac{1}{2}\ln\left(1+\sqrt{1+x^2}\right)+c$ **(B)** $2\sqrt{1+\sqrt{1+x^2}}+c$

(C) $2\left(1+\sqrt{1+x^2}\right)+c$ **(D)** none of these

18. The value of $\int \sqrt{\sec x-1}\ dx$ is equal to

(A) $2\ \ell n\left(\cos\frac{x}{2}+\sqrt{\cos^2\frac{x}{2}-\frac{1}{2}}\right)+C$ **(B)** $\ell n\left(\cos\frac{x}{2}+\sqrt{\cos^2\frac{x}{2}-\frac{1}{2}}\right)+C$

(C) $-2\ \ell n\left(\cos\frac{x}{2}+\sqrt{\cos^2\frac{x}{2}-\frac{1}{2}}\right)+C$ **(D)** none of these

19. If $\int \frac{\cos x-\sin x+1-x}{e^x+\sin x+x}dx=\ln(f(x))+g(x)+C$ where C is the constant of integration and $f(x)$ is positive, then $f(x)+g(x)$ has the value equal to

(A) $e^x+\sin x+2x$ **(B)** $e^x+\sin x$ **(C)** $e^x-\sin x$ **(D)** $e^x+\sin x+x$

20. $\int \frac{\ell n|x|}{x\sqrt{1+\ell n|x|}}dx$ equals -

(A) $\frac{2}{3}\sqrt{1+\ell n|x|}(\ln|x|-2)+c$ **(B)** $\frac{2}{3}\sqrt{1+\ell n|x|}(\ln|x|+2)+c$

(C) $\frac{1}{3}\sqrt{1+\ell n|x|}(\ln|x|-2)+c$ **(D)** $2\sqrt{1+\ell n|x|}(3\ell n|x|-2)+c$

21. The value of $\int \frac{dx}{\cos^3 x\sqrt{\sin 2x}}$ is equal to

(A) $\sqrt{2}\left(\sqrt{\cos x}+\frac{1}{5}\tan^{5/2}x\right)+C$ **(B)** $\sqrt{2}\left(\sqrt{\tan x}+\frac{1}{5}\tan^{5/2}x\right)+C$

(C) $\sqrt{2}\left(\sqrt{\tan x}-\frac{1}{5}\tan^{5/2}x\right)+C$ **(D)** none of these

22. $\int x.\frac{\ln\left(x+\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}dx$ equals :

(A) $\sqrt{1+x^2}\ \ln\left(x+\sqrt{1+x^2}\right)-x+c$ **(B)** $\frac{x}{2}.\ln^2\left(x+\sqrt{1+x^2}\right)-\frac{x}{\sqrt{1+x^2}}+c$

(C) $\frac{x}{2}.\ln^2\left(x+\sqrt{1+x^2}\right)+\frac{x}{\sqrt{1+x^2}}+c$ **(D)** $\sqrt{1+x^2}\ \ln\left(x+\sqrt{1+x^2}\right)+x+c$

23. If $\int \frac{x^4+1}{x\left(x^2+1\right)^2}dx = A\ \ell n|x|+\frac{B}{1+x^2}$ + c, where c is the constant of integration then :

(A) A = 1; B = –1 **(B)** A = –1; B = 1 **(C)** A = 1; B = 1 **(D)** A = –1 ; B = –1

24. $\int\left(\frac{x}{1+x^5}\right)^{3/2}dx$ equals -

(A) $\frac{2}{5}\sqrt{\frac{x^5}{1+x^5}}+c$ **(B)** $\frac{2}{5}\sqrt{\frac{x}{1+x^5}}+c$ **(C)** $\frac{2}{5}\frac{1}{\sqrt{1+x^5}}+c$ **(D)** none of these

25. $\int\frac{(x^2-1)\,dx}{(x^4+3x^2+1)\ \tan^{-1}\left(\frac{x^2+1}{x}\right)} = \ln|f(x)|+C$ then f(x) is

(A) $\ln\left(x+\frac{1}{x}\right)$ **(B)** $\tan^{-1}\left(x+\frac{1}{x}\right)$ **(C)** $\cot^{-1}\left(x+\frac{1}{x}\right)$ **(D)** $\ln\left(\tan^{-1}\left(x+\frac{1}{x}\right)\right)$

26. The value of $\int\sqrt{\frac{e^x-1}{e^x+1}}dx$ is equal to

(A) $\ell n\left(e^x+\sqrt{e^{2x}-1}\right)-\sec^{-1}(e^x)+C$ **(B)** $\ell n\left(e^x+\sqrt{e^{2x}-1}\right)+\sec^{-1}(e^x)+C$

(C) $\ell n\left(e^x-\sqrt{e^{2x}-1}\right)-\sec^{-1}(e^x)+C$ **(D)** none of these

27. If $I_n = \int \cot^n x \, dx$, then $I_0 + I_1 + 2(I_2 + I_3 + ... + I_8) + I_9 + I_{10}$ equals to :

(where $u = \cot x$)

(A) $u + \frac{u^2}{2} + ... + \frac{u^9}{9}$

(B) $-\left(u + \frac{u^2}{2} + ... + \frac{u^9}{9}\right)$

(C) $-\left(u + \frac{u^2}{2!} + ... + \frac{u^9}{9!}\right)$

(D) $\frac{u}{2} + \frac{2u^2}{3} + ... + \frac{9u^9}{10}$

28. The value of $\int \frac{1}{\cos^6 x + \sin^6 x} dx$ is equal to

(A) $\tan^{-1}(\tan x + \cot x) + C$

(B) $-\tan^{-1}(\tan x + \cot x) + C$

(C) $\tan^{-1}(\tan x - \cot x) + C$

(D) $-\tan^{-1}(\tan x - \cot x) + C$

29. Let $\int \frac{dx}{x^{2008} + x} = \frac{1}{p} \ln\left(\frac{x^q}{1 + x^r}\right) + C$

where p, q, $r \in N$ and need not be distinct, then the value of (p + q + r) equals

(A) 6024 **(B)** 6022 **(C)** 6021 **(D)** 6020

30. If $\int \sqrt{\frac{\cos^3 x}{\sin^{11} x}} dx = -2\left(A \tan^{\frac{-9}{2}} x + B \tan^{\frac{-5}{2}} x\right) + C$, then

(A) $A = \frac{1}{9}, B = \frac{-1}{5}$ **(B)** $A = \frac{1}{9}, B = \frac{1}{5}$ **(C)** $A = -\frac{1}{9}, B = \frac{1}{5}$ **(D)** none of these

31. The integral $\int \sqrt{\cot x}\, e^{\sqrt{\sin x}} \sqrt{\cos x}\, dx$ equals

(A) $\frac{\sqrt{\tan x}\, e^{\sqrt{\sin x}}}{\sqrt{\cos x}} + C$ **(B)** $2e^{\sqrt{\sin x}} + C$ **(C)** $-\frac{1}{2} e^{\sqrt{\sin x}} + C$ **(D)** $\frac{\sqrt{\cot x}\, e^{\sqrt{\sin x}}}{2\sqrt{\cos x}} + C$

32. $\int \frac{x^2 - 4}{x^4 + 24x^2 + 16} dx$ equals -

(A) $\frac{1}{4} \tan^{-1}\left(\frac{(x^2 + 4)}{4x}\right) + c$

(B) $-\frac{1}{4} \cot^{-1}\left(\frac{(x^2 + 4)}{x}\right) + c$

(C) $-\frac{1}{4} \cot^{-1}\left(\frac{4(x^2 + 4)}{x}\right) + c$

(D) $\frac{1}{4} \cot^{-1}\left(\frac{(x^2 + 4)}{x}\right) + c$

33. $\int \frac{x^4 - 4}{x^2 \sqrt{4 + x^2 + x^4}} dx$ equals-

(A) $\frac{\sqrt{4 + x^2 + x^4}}{x} + c$ **(B)** $\sqrt{4 + x^2 + x^4} + c$ **(C)** $\frac{\sqrt{4 + x^2 + x^4}}{2} + c$ **(D)** $\frac{\sqrt{4 + x^2 + x^4}}{2\ x} + c$

34. $\int \frac{x^2(1-\ln x)}{\ln^4 x - x^4} dx$ equals

(A) $\frac{1}{2}\ln\left(\frac{x}{\ln x}\right)-\frac{1}{4}\ln\left(\ln^2 x - x^2\right)+C$

(B) $\frac{1}{4}\ln\left(\frac{\ln x - x}{\ln x + x}\right)-\frac{1}{2}\tan^{-1}\left(\frac{\ln x}{x}\right)+C$

(C) $\frac{1}{4}\ln\left(\frac{\ln x + x}{\ln x - x}\right)+\frac{1}{2}\tan^{-1}\left(\frac{\ln x}{x}\right)+C$

(D) $\frac{1}{4}\left(\ln\left(\frac{\ln x - x}{\ln x + x}\right)+\tan^{-1}\left(\frac{\ln x}{x}\right)\right)+C$

35. If $\int \frac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}} dx = Ax + B\,\ell n\,|9e^{2x} - 4| + C$, then

(A) $A = -\frac{3}{2}, B = \frac{35}{36}, C = 0$

(B) $A = \frac{35}{36}, B = -\frac{3}{2}, C \in R$

(C) $A = -\frac{3}{2}, B = \frac{35}{36}, C \in R$

(D) $A = \frac{3}{2}, B = \frac{35}{36}, C \in R$

36. If $\int \frac{(2x+3)dx}{x(x+1)(x+2)(x+3)+1} = C - \frac{1}{f(x)}$ where $f(x)$ is of the form of $ax^2 + bx + c$ then $(a + b + c)$ equals

(A) 4 **(B)** 5 **(C)** 6 **(D)** none

37. $\int \frac{x^9}{(x^2+4)^6} dx$ is equal to -

(A) $\frac{1}{5x}\left(4+\frac{1}{x^2}\right)^{-5} + c$

(B) $\frac{1}{5}\left(4+\frac{1}{x^2}\right)^{-5} + c$

(C) $\frac{1}{10x}(1 + 4x^2)^{-5} + c$

(D) $\frac{1}{40}(1 + 4x^{-2})^{-5} + c$

38. $\int \left(\sin(101x)\cdot\sin^{99} x\right)dx$ equals

(A) $\frac{\sin(100x)(\sin x)^{100}}{100} + C$

(B) $\frac{\cos(100x)(\sin x)^{100}}{100} + C$

(C) $\frac{\cos(100x)(\cos x)^{100}}{100} + C$

(D) $\frac{\sin(100x)(\sin x)^{101}}{101} + C$

39. If $\int \frac{dx}{\sqrt{\sin^3 x \cos^5 x}} = a\sqrt{\cot x} + b\sqrt{\tan^3 x} + C$, where C is an arbitrary constant of integration, then the values of 'a' and 'b' are respectively:

(A) -2 & $\frac{2}{3}$ **(B)** 2 & $-\frac{2}{3}$ **(C)** 2 & $\frac{2}{3}$ **(D)** none

Exercise # 2 Part # I [Multiple Correct Choice Type Questions]

1. Primitive of $\sqrt{1+2\tan x\left(\sec x+\tan x\right)}$ w.r.t.x is -

(A) $\ell n\left|\sec x\right|-\ell n\left|\sec x-\tan x\right|+c$ **(B)** $\ell n\left|\sec x+\tan x\right|+\ell n\left|\sec x\right|+c$

(C) $2\ell n\left|\sec\frac{x}{2}+\tan\frac{x}{2}\right|+c$ **(D)** $\ell n\left|1+\tan x(\sec x+\tan x)\right|+c$

2. If $\int e^{3x}\cos 4x\,dx=e^{3x}(A\sin 4x+B\cos 4x)+C$ then:

(A) $4A=3B$ **(B)** $2A=3B$ **(C)** $3A=4B$ **(D)** $4A+3B=1$

3. Let $f'(x)=3x^2.\sin\frac{1}{x}-x\cos\frac{1}{x}$, $x\neq 0$, $f(0)=0$, $f\left(\frac{1}{\pi}\right)=0$, then which of the following is/are not correct.

(A) f(x) is continuous at x = 0 **(B)** f(x) is non-differentiable at x = 0
(C) f'(x) is discontinuous at x = 0 **(D)** f'(x) is differentiable at x = 0

4. $\int\frac{\sin 2x}{\sin^4 x+\cos^4 x}dx$ is equal to -

(A) $\cot^{-1}\left(\cot^2 x\right)+c$ **(B)** $-\cot^{-1}\left(\tan^2 x\right)+c$ **(C)** $\tan^{-1}\left(\tan^2 x\right)+c$ **(D)** $-\tan^{-1}\left(\cos 2x\right)+c$

5. $\int\frac{\ell n(\tan x)}{\sin x\ \cos x}dx$ equal:

(A) $\frac{1}{2}ln^2(\cot x)+c$ **(B)** $\frac{1}{2}ln^2(\sec x)+c$

(C) $\frac{1}{2}ln^2(\sin x\ \sec x)+c$ **(D)** $\frac{1}{2}ln^2(\cos x\ \mathrm{cosec}\,x)+c$

6. The value of $\int 2^{mx}.\,3^{nx}\,dx$ (when m, n ∈ N) is equal to :

(A) $\frac{2^{mx}+3^{nx}}{m\,\ell n2+n\,\ell n3}+C$ **(B)** $\frac{e^{(m\,\ell n2+n\,\ell n3)\,x}}{m\,\ell n2+n\,\ell n3}+C$

(C) $\frac{2^{mx}.\,3^{nx}}{\ell n\left(2^m.3^n\right)}+C$ **(D)** $\frac{(mn).\,2^x.\,3^x}{m\,\ell n2+n\,\ell n3}+C$

7. $\int\frac{dx}{x^3\left(1-\frac{1}{2x^2}\right)}$ equals-

(A) $\ell n\left|2x^2-1\right|+2\ell n\,|x|+c$ **(B)** $\ell n\left|2x^2-1\right|-2\ell n\,|x|+c$

(C) $\ell n\left|2x^2-1\right|-\ell n(x^2)-\ell n2+c$ **(D)** $\ell n\left|1-\frac{1}{2x^2}\right|+c$

8. $\int \frac{1}{x^2-1}\ \ell n\ \frac{x-1}{x+1}\ dx$ equals -

(A) $\frac{1}{2}\ell n^2\left|\frac{x-1}{x+1}\right|+c$ **(B)** $\frac{1}{4}\ell n^2\left|\frac{x-1}{x+1}\right|+c$ **(C)** $\frac{1}{2}\ell n^2\left|\frac{x+1}{x-1}\right|+c$ **(D)** $\frac{1}{4}\ell n^2\left|\frac{x+1}{x-1}\right|+c$

9. If $\int e^u . \sin 2x\ dx$ can be found in terms of known functions of x then u can be:

(A) x **(B)** sin x **(C)** cos x **(D)** cos 2x

10. If $\int \frac{dx}{5+4\cos x} = I\tan^{-1}\left(m\tan\frac{x}{2}\right)+C$ then :

(A) I = 2/3 **(B)** m = 1/3 **(C)** I = 1/3 **(D)** m = 2/3

11. $\int \sin 2x\, dx$ equals -

(A) $-\frac{\cos 2x}{2}+c$ **(B)** $\frac{\sin^2 x}{2}+c$ **(C)** $-\frac{\cos^2 x}{2}+c$ **(D)** $\frac{\cos 2x}{2}+c$

12. The value of $\int \frac{x^2+\cos^2 x}{1+x^2}\ \mathrm{cosec}^2 x\ dx$ is equal to:

(A) $\cot x - \cot^{-1}x + C$ **(B)** $C - \cot x + \cot^{-1}x$

(C) $-\tan^{-1}x - \frac{\mathrm{cosec}\, x}{\sec x} + C$ **(D)** $-e^{\ell n\ \tan^{-1}x} - \cot x + C$

13. If $I_n = \int \cot^n x\ dx$ and $I_0 + I_1 + 2(I_2 + ... + I_8) + I_9 + I_{10} = A\left(u + \frac{u^2}{2} + ... + \frac{u^9}{9}\right) + C$, where u = cotx and C is an arbitrary constant, then

(A) A is constant **(B)** A = – 1 **(C)** A = 1 **(D)** A is dependent on x

14. Suppose $J = \int \frac{\sin^2 x + \sin x}{1+\sin x+\cos x}dx$ and $K = \int \frac{\cos^2 x+\cos x}{1+\sin x+\cos x}dx$. If C is an arbitrary constant of integration then which of the following is/are correct?

(A) $J = \frac{1}{2}(x - \sin x + \cos x) + C$ **(B)** $J = K - (\sin x + \cos x) + C$

(C) $J = x - K + C$ **(D)** $K = \frac{1}{2}(x - \sin x + \cos x) + C$

15. $\int \frac{dx}{\sqrt{x-x^2}}$ equals, where $x \in \left(\frac{1}{2},1\right)$ -

(A) $2\sin^{-1}\sqrt{x}+c$ **(B)** $\sin^{-1}(2x-1)+c$ **(C)** $c-\cos^{-1}(2x-1)$ **(D)** $\cos^{-1}2\sqrt{x-x^2}+c$

Part # II [Assertion & Reason Type Questions]

In each of the following questions, a statement of Assertion (A) is given followed by a corresponding statement of Reason (R) just below it . Of the statements mark the correct answer as

(A) Statement-I is True, Statement-II is True ; Statement-II is a correct explantion for Statement-I

(B) Statement-I is True, Statement-II is True ; Statement-II is NOT a correct explantion for Statement-I

(C) Statement-I is True, Statement-II is False.

(D) Statement-I is False, Statement-II is True.

1. If $D(x) = \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, where f_1, f_2, f_3 are differentiable function and a_2, b_2, c_2, a_3, b_3, c_3 are constants.

Statement - I $\int D(x)\,dx = \begin{vmatrix} \int f_1(x)dx & \int f_2(x)dx & \int f_3(x)dx \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + c$

Statement - II Integration of sum of several function is equal to sum of integration of individual functions.

2. **Statement - I** $\int \frac{\ell n(e^x+1)}{e^x}\,dx = x - \left(\frac{1+e^x}{e^x}\right)\ell n(e^x+1) + C$

Statement - II $\int \frac{f'(x)}{f(x)}\,dx = \ell n|f(x)| + C$

3. **Statement - I** If a > 0 and $b^2 - 4ac < 0$, then the values of integral $\int \frac{dx}{ax^2+bx+c}$ will be of the type $\mu \tan^{-1}\frac{x+A}{B}$ + c. where A, B, C, μ are constants.

Statement - II If a > 0, $b^2 - 4ac < 0$, then $ax^2 + bx + c$ can be written as sum of two squares.

4. **Statement - I** The function $F(x) = \int \frac{x}{(x-1)(x^2+1)}dx$ is discontinuous at x = 1

Statement - II If $F(x) = \int f(x)\,dx$ and $f(x)$ is discontinuous at x = a then F (x) is also discontinuous at x = a.

5. **Statement - I** If $x > 0, x \neq 1$ then $\int (\log_x e - (\log_x e)^2)\,dx = x\log_x e + C$

Statement - II $\int e^x(f(x) + f'(x))\,dx = e^x f(x) + C$ and $e^t = x$ iff $t = \ell nx$

6. **Statement - I** $\int (\sin x)^5 \cos x\,dx = \frac{\sin^6 x}{6} + C$

Statement - II $\int (f(x))^n f'(x)\,dx = \frac{(f(x))^{n+1}}{n+1} + C,\ n \in I$

7. If y is a function of x such that $y(x - y)^2 = x$.

Statement - I : $\int \frac{dx}{x-3y} = \frac{1}{2}\log[(x-y)^2 - 1]$

Statement - II : $\int \frac{dx}{x-3y} = \log(x - 3y) + c$.

Exercise # 3 Part # I [Matrix Match Type Questions]

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with one or more statement(s) in **Column-II**.

1. The antiderivative of

Column-I	Column-II
(A) $f(x) = \dfrac{1}{(a^2+b^2)-(a^2-b^2)\cos x}$ is	**(p)** $\dfrac{1}{ab}\tan^{-1}\left(\dfrac{a}{b}\tan\dfrac{x}{2}\right) + c$
(B) $f(x) = \dfrac{1}{a^2\sin^2 x + b^2\cos^2 x}$ is	**(q)** $\dfrac{1}{a^2\sin\alpha}\tan^{-1}\left(\dfrac{\tan x}{\sin\alpha}\right) + c,\ \alpha = \cos^{-1}\dfrac{b}{a}$
(C) $f(x) = \dfrac{1}{a\cos x + b\sin x}$ is	**(r)** $\dfrac{1}{ab}\tan^{-1}\left(\dfrac{a}{b}\tan x\right) + c$
(D) $f(x) = \dfrac{1}{a^2 - b^2\cos^2 x}$ is ; $(a^2 > b^2)$	**(s)** $\dfrac{1}{\sqrt{a^2+b^2}}\log\left|\tan\dfrac{1}{2}\left(x+\tan^{-1}\dfrac{a}{b}\right)\right| + c$

2. If $I = \displaystyle\int \frac{dx}{a + b\cos x}$, where a, b > 0 and a + b = u, a – b = v, then match the following column

Column – I	Column – II
(A) $v = 0$	**(p)** $I = \dfrac{1}{\sqrt{uv}}\ \ell n\left|\dfrac{\sqrt{u}+\sqrt{v}\ \tan\dfrac{x}{2}}{\sqrt{u}-\sqrt{v}\ \tan\dfrac{x}{2}}\right| + C$
(B) $v > 0$	**(q)** $I = \dfrac{2}{\sqrt{uv}}\tan^{-1}\left(\sqrt{\dfrac{v}{u}}\ \tan\dfrac{x}{2}\right) + C$
(C) $v < 0$	**(r)** $I = \dfrac{1}{\sqrt{-uv}}\ \ell n\left|\dfrac{\sqrt{u}+\sqrt{-v}\ \tan\dfrac{x}{2}}{\sqrt{u}-\sqrt{-v}\ \tan\dfrac{x}{2}}\right| + C$
	(s) $\dfrac{2}{u}\tan\dfrac{x}{2} + C$

3.

Column-I	Column-II
(A) Let $f(x) = \int x^{\sin x}(1 + x\cos x\cdot\ln x + \sin x)dx$ and $f\left(\dfrac{\pi}{2}\right) = \dfrac{\pi^2}{4}$ then the value of $f(\pi)$ is	**(p)** rational
(B) Let $g(x) = \displaystyle\int \frac{1+2\cos x}{(\cos x + 2)^2}dx$ and $g(0) = 0$	**(q)** irrational
then the value of $g\left(\dfrac{\pi}{2}\right)$ is	**(r)** integral

(C) If real numbers x and y satisfy $(x + 5)^2 + (y - 12)^2 = (14)^2$ then the minimum value of $\sqrt{(x^2 + y^2)}$ is **(s)** prime

(D) Let $k(x) = \int \frac{(x^2+1)dx}{\sqrt[3]{x^3+3x+6}}$ and $k(-1) = \frac{1}{\sqrt[3]{2}}$ then the value of $k(-2)$ is

4. **Column – I** **Column – II**

(A) If $F(x) = \int \frac{x + \sin x}{1 + \cos x}\,dx$ and $F(0) = 0$, then the value of $F(\pi/2)$ is **(p)** $\frac{\pi}{2}$

(B) Let $F(x) = \int e^{\sin^{-1}x}\left(1 - \frac{x}{\sqrt{1-x^2}}\right)dx$ and $F(0) = 1$, **(q)** $\frac{\pi}{3}$

If $F(1/2) = \frac{k\sqrt{3}\,e^{\pi/6}}{\pi}$, then the value of k is

(C) Let $F(x) = \int \frac{dx}{(x^2+1)(x^2+9)}$ and $F(0) = 0$, **(r)** $\frac{\pi}{4}$

if $F(\sqrt{3}) = \frac{5}{36}k$, then the value of k is

(D) Let $F(x) = \int \frac{\sqrt{\tan x}}{\sin x \cos x}\,dx$ and $F(0) = 0$ **(s)** π

if $F(\pi/4) = \frac{2k}{\pi}$, then the value of k is

5. $\int f(x)\,dx$ when

Column-I	**Column-II**
(A) $f(x) = \frac{1}{(a^2+x^2)^{3/2}}$	**(p)** $c - \frac{1}{a}\sin^{-1}\frac{a}{\lvert x \rvert}$
(B) $f(x) = \frac{x^2}{\sqrt{a^2 - x^2}}$	**(q)** $\frac{a^2}{2}\sin^{-1}\frac{x}{a} - \frac{x}{2}\sqrt{a^2 - x^2} + c$
(C) $f(x) = \frac{1}{(x^2 - a^2)^{3/2}}$	**(r)** $c - \frac{x}{a^2\sqrt{x^2 - a^2}}$
(D) $f(x) = \frac{1}{x\sqrt{x^2 - a^2}}$	**(s)** $\frac{x}{a^2\sqrt{x^2 + a^2}} + c$

Part # II [Comprehension Type Questions]

Comprehension # 1

In calculating a number of integrals we had to use the method of integration by parts several times in succession. The result could be obtained more rapidly and in a more concise form by using the so-called generalized formula for integration by parts

$$\int u(x)v(x)dx = u(x)v_1(x) - u'(x)v_2(x) + u''(x)v_3(x) - ..... + (-1)^{n-1}u^{n-1}(x)\,v_n(x) - (-1)^{n-1}\int u^n(x)\,v_n(x)\,dx$$

where $v_1(x) = \int v(x)dx,\ v_2(x) = \int v_1(x)\,dx\ ....,\ v_n(x) = \int v_{n-1}(x)\,dx$

Of course, we assume that all derivatives and integrals appearing in this formula exist. The use of the generalized formula for integration by parts is especially useful when calculating $\int P_n(x)\,Q(x)\,dx$, where $P_n(x)$, is polynomial of degree n and the factor Q(x) is such that it can be integrated successively n + 1 times.

1. If $\int (x^3 - 2x^2 + 3x - 1)\cos 2x\,dx = \frac{\sin 2x}{4}u(x) + \frac{\cos 2x}{8}v(x) + c$, then -

(A) $u(x) = x^3 - 4x^2 + 3x$
(B) $u(x) = 2x^3 - 4x^2 + 3x$
(C) $v(x) = 3x^2 - 4x + 3$
(D) $v(x) = 6x^2 - 8x$

2. If $\int e^{2x}.x^4\,dx = \frac{e^{2x}}{2}f(x) + C$ then f(x) is equal to -

(A) $\left(x^4 - 2x^3 + 3x^2 - 3x + \frac{3}{2}\right)\frac{1}{2}$
(B) $x^4 - x^3 + 2x^2 - 3x + 2$
(C) $x^4 - 2x^3 + 3x^2 - 3x + \frac{3}{2}$
(D) $x^4 - 2x^3 + 2x^2 - 3x + \frac{3}{2}$

Comprehension # 2

It is known that

$$\sqrt{\tan x} + \sqrt{\cot x} = \begin{cases} \frac{\sqrt{\sin x}}{\sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x}} & \text{if } 0 < x < \frac{\pi}{2} \\ \frac{\sqrt{-\sin x}}{\sqrt{-\cos x}} + \frac{\sqrt{-\cos x}}{\sqrt{-\sin x}} & \text{if } \pi < x < \frac{3\pi}{2} \end{cases},$$

$$\frac{d}{dx}\left(\sqrt{\tan x} - \sqrt{\cot x}\right) = \frac{1}{2}\left(\sqrt{\tan x} + \sqrt{\cot x}\right)(\tan x + \cot x),\ \forall\ x \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$$

and $\frac{d}{dx}\left(\sqrt{\tan x} + \sqrt{\cot x}\right) = \frac{1}{2}\left(\sqrt{\tan x} - \sqrt{\cot x}\right)(\tan x + \cot x),\ \forall\ x \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$.

1. Value of integral $I = \int (\sqrt{\tan x} + \sqrt{\cot x})\, dx$, where $x \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$ is

(A) $\sqrt{2}\, \tan^{-1}\left(\frac{\sqrt{\tan x} - \sqrt{\cot x}}{\sqrt{2}}\right) + C$

(B) $\sqrt{2}\, \tan^{-1}\left(\frac{\sqrt{\tan x} + \sqrt{\cot x}}{\sqrt{2}}\right) + C$

(C) $-\sqrt{2}\, \tan^{-1}\left(\frac{\sqrt{\tan x} - \sqrt{\cot x}}{\sqrt{2}}\right) + C$

(D) $-\sqrt{2}\, \tan^{-1}\left(\frac{\sqrt{\tan x} + \sqrt{\cot x}}{\sqrt{2}}\right) + C$

2. Value of the integral $I = \int (\sqrt{\tan x} + \sqrt{\cot x})\, dx$, where $x \in \left(0, \frac{\pi}{2}\right)$, is

(A) $\sqrt{2}\, \sin^{-1}(\cos x - \sin x) + C$

(B) $\sqrt{2}\, \sin^{-1}(\sin x - \cos x) + C$

(C) $\sqrt{2}\, \sin^{-1}(\sin x + \cos x) + C$

(D) $-\sqrt{2}\, \sin^{-1}(\sin x + \cos x) + C$

3. Value of the integral $I = \int (\sqrt{\tan x} + \sqrt{\cot x})\, dx$, where $x \in \left(\pi, \frac{3\pi}{2}\right)$, is

(A) $\sqrt{2}\, \sin^{-1}(\cos x - \sin x) + C$

(B) $\sqrt{2}\, \sin^{-1}(\sin x - \cos x) + C$

(C) $\sqrt{2}\, \sin^{-1}(\sin x + \cos x) + C$

(D) $-\sqrt{2}\, \sin^{-1}(\sin x + \cos x) + C$

Comprehension # 3

Let $I_{n, m} = \int \sin^n x \cos^m x . dx$. Then we can relate $I_{n, m}$ with each of the following

(i) $I_{n-2, m}$ **(ii)** $I_{n+2, m}$ **(iii)** $I_{n, m-2}$

(iv) $I_{n, m+2}$ **(v)** $I_{n-2, m+2}$ **(vi)** $I_{n+2, m-2}$

Suppose we want to establish a relation between $I_{n, m}$ and $I_{n, m-2}$, then we set

$P(x) = \sin^{n+1}x \cos^{m-1}x$ ....**(1)**

In $I_{n,m}$ and $I_{n, m-2}$ the exponent of cosx is m and m – 2 respectively, the minimum of the two is m – 2, adding 1 to the minimum we get m – 2 + 1 = m – 1. Now choose the exponent m – 1 of cosx in P(x). Similarly choose the exponent of sin x for P (x)

Now differentiating both sides of (1), we get

$P'(x) = (n + 1) \sin^n x \cos^m x - (m - 1) \sin^{n+2} x \cos^{m-2} x$

$= (n + 1) \sin^n x \cos^m x - (m - 1) \sin^n x (1 - \cos^2 x) \cos^{m-2} x$

$= (n + 1) \sin^n x \cos^m x - (m - 1) \sin^n x \cos^{m-2} x + (m - 1) \sin^n x \cos^m x$

$= (n + m) \sin^n x \cos^m x - (m - 1) \sin^n x \cos^{m-2} x$

Now integrating both sides, we get

$\sin^{n+1}x \cos^{m-1}x = (n + m)\, I_{n, m} - (m - 1)\, I_{n, m-2}$.

Similarly we can establish the other relations.

1. The relation between $I_{4,2}$ and $I_{2,2}$ is

(A) $I_{4,2} = \frac{1}{6}(-\sin^3x\cos^3x + 3I_{2,2})$

(B) $I_{4,2} = \frac{1}{6}(\sin^3x\cos^3x + 3I_{2,2})$

(C) $I_{4,2} = \frac{1}{6}(\sin^3x\cos^3x - 3I_{2,2})$

(D) $I_{4,2} = \frac{1}{4}(-\sin^3x\cos^3x + 2I_{2,2})$

2. The relation between $I_{4,2}$ and $I_{6,2}$ is

(A) $I_{4,2} = \frac{1}{5}(\sin^5x\cos^3x + 8I_{6,2})$

(B) $I_{4,2} = \frac{1}{5}(-\sin^5x\cos^3x + 8I_{6,2})$

(C) $I_{4,2} = \frac{1}{5}(\sin^5x\cos^3x - 8I_{6,2})$

(D) $I_{4,2} = \frac{1}{6}(\sin^5x\cos^3x + 8I_{6,2})$

3. The relation between $I_{4,2}$ and $I_{4,4}$ is

(A) $I_{4,2} = \frac{1}{3}(\sin^5x\cos^3x + 8\,I_{4,4})$

(B) $I_{4,2} = \frac{1}{3}(-\sin^5x\cos^3x + 8\,I_{4,4})$

(C) $I_{4,2} = \frac{1}{3}(\sin^5x\cos^3x - 8\,I_{4,4})$

(D) $I_{4,2} = \frac{1}{3}(\sin^5x\cos^3x + 6\,I_{4,4})$

Comprehension # 4

Integrals of class of functions following a definite pattern can be found by the method of reduction and recursion. Reduction formulas make it possible to reduce an integral dependent on the index n > 0, called the order of the integral, to an integral of the same type with a smaller index. Integration by parts helps us to derive reduction formulas. (Add a constant in each question)

1. If $I_n = \int \frac{dx}{(x^2+a^2)^n}$ then $I_{n+1} + \frac{1-2n}{2n}\cdot\frac{1}{a^2}\,I_n$ is equal to -

(A) $\frac{x}{(x^2+a^2)^n}$

(B) $\frac{1}{2na^2}\,\frac{1}{(x^2+a^2)^{n-1}}$

(C) $\frac{1}{2na^2}\cdot\frac{x}{(x^2+a^2)^n}$

(D) $\frac{1}{2na^2}\cdot\frac{1}{(x^2+a^2)}$

2. If $I_{n,-m} = \int \frac{\sin^n x}{\cos^m x}dx$ then $I_{n,-m} + \frac{n-1}{m-1}I_{n-2,\,2-m}$ is equal to-

(A) $\frac{\sin^{n-1}x}{\cos^{m-1}x}$

(B) $\frac{1}{(m-1)}\,\frac{\sin^{n-1}x}{\cos^{m-1}x}$

(C) $\frac{1}{(n-1)}\,\frac{\sin^{n-1}x}{\cos^{m-1}x}$

(D) $\frac{n-1}{m-1}\,\frac{\sin^{n-1}x}{\cos^{m-1}x}$

3. If $u_n = \int \frac{x^n}{\sqrt{ax^2+2bx+c}}dx$, then $(n+1)au_{n+1} + (2n+1)bu_n + ncu_{n-1}$ is equal to -

(A) $x^{n-1}\sqrt{ax^2+2bx+c}$

(B) $\frac{x^{n-2}}{\sqrt{ax^2+2bx+c}}$

(C) $\frac{x^n}{\sqrt{ax^2+2bx+c}}$

(D) $x^n\sqrt{ax^2+2bx+c}$

Exercise # 4

[Subjective Type Questions]

1. $\int \frac{dx}{\sin(x-a)\sin(x-b)}$

2. $\int \sqrt{x+\sqrt{x^2+2}}\,dx$

3. $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\,dx$

4. $\int \frac{x\ell n x}{(x^2-1)^{3/2}}\,dx$

5. $\int \frac{5x^4+4x^5}{(x^5+x+1)^2}\,dx$

6. $\int \sqrt{\frac{\sin(x-a)}{\sin(x+a)}}\,dx$

7. Find the value of $\int \frac{d(x^2+1)}{\sqrt{(x^2+2)}}$.

8. $\int \frac{\cot x\,dx}{(1-\sin x)(\sec x+1)}$

9. $\int \frac{x^3+3x+2}{(x^2+1)^2(x+1)}\,dx$

10. $\int \frac{\sqrt{\cot x}-\sqrt{\tan x}}{1+3\sin 2x}\,dx$

11. $\int \left[\frac{\sqrt{x^2+1}\,\left[\ell n(x^2+1)-2\ \ell n x\right]}{x^4}\right]dx$

12. $\int \frac{dx}{\sin x\sqrt{\sin(2x+\alpha)}}$

13. $\int \frac{x^2}{(x\sin x+\cos x)^2}\,dx$

14. $\int \frac{e^x\left(2-x^2\right)}{(1-x)\sqrt{1-x^2}}\,dx$

15. $\int x\sqrt{\frac{a^2-x^2}{a^2+x^2}}\,dx$

16. Integrate $\frac{1}{2}f'(x)$ w.r.t. x^4, where $f(x) = \tan^{-1}x+\ell n\sqrt{1+x}-\ell n\sqrt{1-x}$

17. $\int \frac{\ell n\left(\cos x+\sqrt{\cos 2x}\right)}{\sin^2 x}\,dx$

18. $\int \frac{dx}{x^2(x^4+1)^{3/4}}$

19. $\int \frac{(\cos 2x)^{1/2}}{\sin x}\,dx$

20. The antiderivative of $f(x) = \ln(\ln x) + (\ln x)^{-2}$ whose graph passes through (e, e) is $x\ln(\ln x) - x(\ln x)^{-1} + 2$.

21. $\int \sqrt{\frac{\operatorname{cosec}x-\cot x}{\operatorname{cosec}x+\cot x}}\cdot\frac{\sec x}{\sqrt{1+2\sec x}}\,dx$

22. Let $\begin{bmatrix}1&0&0\\6&2&0\\5&4&3\end{bmatrix}\begin{bmatrix}x\\x^2\\1\end{bmatrix}=\begin{bmatrix}x\\2\alpha x+\beta x^2\\5x+\gamma x^2+3\end{bmatrix}$, $\forall\ x \in R$ and $f(x)$ is a differentiable function satisfying,

$f(xy) = f(x) + x^2(y^2-1) + x(y-1)$; $\forall\ x, y \in R$ and $f(1) = 3$. Evaluate $\int \frac{\alpha x^2+\beta x+\gamma}{f(x)}\,dx$

23. $\int \frac{dx}{\sin^2 x + \sin 2x}$

24. $\int \frac{(ax^2 - b)\ dx}{x\sqrt{c^2x^2 - (ax^2 + b)^2}}$

25. $\int (\sin x)^{-11/3} (\cos x)^{-1/3} dx$

26. $\int \sin^{-1} \sqrt{\frac{x}{a+x}}\ dx$

27. $\int \frac{2 \sin 2\phi - \cos \phi}{6 - \cos^2 \phi - 4 \sin \phi}\ d\phi$

28. $\int \frac{1}{\sqrt[3]{x} + \sqrt[4]{x}} dx$

29. Let f(x) is a quadratic function such that f(0) = 1 and $\int \frac{f(x)dx}{x^2(x+1)^3}$ is a rational function, find the value of f'(0)

30. $\int \cos 2\theta . \ell n \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} d\theta$

31. $\int \left[\left(\frac{x}{e}\right)^x + \left(\frac{e}{x}\right)^x\right] \ell n\, x\, dx$

32. $\int \frac{x}{(7x - 10 - x^2)^{3/2}}\ dx$

33. $\int \frac{1 + x \cos x}{x\left(1 - x^2 e^{2\sin x}\right)}\ dx$

34. $\int \frac{e^{\cos x}(x \sin^3 x + \cos x)}{\sin^2 x}\ dx$

35. $\int \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx$

Exercise # 5 Part # I [Previous Year Questions] [AIEEE/JEE-MAIN]

1. $\int \frac{\cos 2x - 1}{\cos 2x + 1} dx =$ **[AIEEE-2002]**

(1) $\tan x - x + C$ (2) $x + \tan x + C$ (3) $x - \tan x + C$ (4) $-x - \cot x + C$

2. $\int \frac{(\log x)}{x^2} dx$ **[AIEEE-2002]**

(1) $\frac{1}{2}(\log x + 1) + C$ (2) $-\frac{1}{x}(\log x + 1) + C$ (3) $\frac{1}{x}(\log x - 1) + C$ (4) $\log(x + 1) + C$

3. If $\int \frac{\sin x}{\sin(x-\alpha)} dx = Ax + B \log\sin(x - \alpha) + C$ then values of (A, B) is - **[AIEEE-2004]**

(1) $(\sin\alpha, \cos\alpha)$ (2) $(\cos\alpha, \sin\alpha)$ (3) $(-\sin\alpha, \cos\alpha)$ (4) $(-\cos\alpha, \sin\alpha)$

4. $\int \frac{dx}{\cos x - \sin x}$ is equal to- **[AIEEE-2004]**

(1) $\frac{1}{\sqrt{2}} \log\left|\tan\left(\frac{x}{2} - \frac{\pi}{8}\right)\right| + C$ (2) $\frac{1}{\sqrt{2}} \log\left|\cot\left(\frac{x}{2}\right)\right| + C$

(3) $\frac{1}{\sqrt{2}} \log\left|\tan\left(\frac{x}{2} - \frac{3\pi}{8}\right)\right| + C$ (4) $\frac{1}{\sqrt{2}} \log\left|\tan\left(\frac{x}{2} + \frac{3\pi}{8}\right)\right| + C$

5. $\int \left\{\frac{(\log x - 1)}{1 + (\log x)^2}\right\}^2 dx$ is equals to - **[AIEEE-2005]**

(1) $\frac{\log x}{(\log x)^2 + 1} + C$ (2) $\frac{x}{x^2 + 1} + C$ (3) $\frac{xe^x}{1 + x^2} + C$ (4) $\frac{x}{(\log x)^2 + 1} + C$

6. $\int \frac{dx}{\cos x + \sqrt{3}\sin x}$ equals- **[AIEEE-2007]**

(1) $\frac{1}{2}\log\tan\left(\frac{x}{2} + \frac{\pi}{12}\right) + C$ (2) $\frac{1}{2}\log\tan\left(\frac{x}{2} - \frac{\pi}{12}\right) + C$

(3) $\log\tan\left(\frac{x}{2} + \frac{\pi}{12}\right) + C$ (4) $\log\tan\left(\frac{x}{2} - \frac{\pi}{12}\right) + C$

7. The value of $\sqrt{2}\int \frac{\sin x\, dx}{\sin\left(x - \frac{\pi}{4}\right)}$ is - **[AIEEE-2008]**

(1) $x + \log\left|\cos\left(x - \frac{\pi}{4}\right)\right| + c$ (2) $x - \log\left|\sin\left(x - \frac{\pi}{4}\right)\right| + c$ (3) $x + \log\left|\sin\left(x - \frac{\pi}{4}\right)\right| + c$ (4) $x - \log\left|\cos\left(x - \frac{\pi}{4}\right)\right| + c$

8. If the integral $\int \frac{5\tan x}{\tan x - 2}dx = x + a\ln|\sin x - 2\cos x| + k$ then a is equal to : **[AIEEE-2012]**

(1) 2 **(2)** –1 **(3)** –2 **(4)** 1

9. If $\int f(x)dx = \Psi(x)$, then $\int x^5 f(x^3)dx$ is equal to : **[JEE Main 2013]**

(1) $\frac{1}{3}\left[x^3\Psi(x^3) - \int x^2\Psi(x^3)dx\right] + C$ **(2)** $\frac{1}{3}x^3\Psi(x^3) - 3\int x^3\Psi(x^3)dx + C$

(3) $\frac{1}{3}x^3\Psi(x^3) - \int x^2\Psi(x^3)dx + C$ **(4)** $\frac{1}{3}\left[x^3\Psi(x^3) - \int x^3\Psi(x^3)dx\right] + C$

10 . The integral $\int\left(1 + x - \frac{1}{x}\right)e^{x+\frac{1}{x}}dx$ is equal to : **[JEE Main 2014]**

(1) $(x-1)e^{x+\frac{1}{x}} + c$ **(2)** $x\,e^{x+\frac{1}{x}} + C$ **(3)** $(x+1)e^{x+\frac{1}{x}} + c$ **(4)** $-x\,e^{x+\frac{1}{x}} + c$

11. The integral $\int \frac{dx}{x^2\left(x^4+1\right)^{3/4}}$ equals : **[JEE Main 2015]**

(1) $-\left(x^4+1\right)^{1/4} + c$ **(2)** $-\left(\frac{x^4+1}{x^4}\right)^{1/4} + c$ **(3)** $\left(\frac{x^4+1}{x^4}\right)^{1/4} + c$ **(4)** $\left(x^4+1\right)^{1/4} + c$

12. The integral $\int \frac{2x^{12}+5x^9}{\left(x^5+x^3+1\right)^3}dx$ is equal to : **[JEE Main 2016]**

(1) $\frac{x^{10}}{2\left(x^5+x^3+1\right)^2} + C$ **(2)** $\frac{x^5}{2\left(x^5+x^3+1\right)^2} + C$ **(3)** $\frac{-x^{10}}{2\left(x^5+x^3+1\right)^2} + C$ **(4)** $\frac{-x^5}{\left(x^5+x^3+1\right)^2} + C$

Part # II [Previous Year Questions][IIT-JEE ADVANCED]

1. Evaluate : $\int \sin^{-1}\left(\frac{2x+2}{\sqrt{4x^2+8x+13}}\right)dx$. **[JEE 2001]**

2. For any natural number m, evaluate $\int (x^{3m} + x^{2m} + x^m)(2x^{2m} + 3x^m + 6)^{\frac{1}{m}}\,dx$ where $x > 0$

[JEE 2002]

3. $\int \frac{x^2-1}{x^3\sqrt{2x^4-2x^2+1}}dx$ is equal to - **[JEE 2006]**

(A) $\frac{\sqrt{2x^4-2x^2+1}}{x^2} + c$ **(B)** $\frac{\sqrt{2x^4-2x^2+1}}{x^3} + c$

(C) $\frac{\sqrt{2x^4-2x^2+1}}{x} + c$ **(D)** $\frac{\sqrt{2x^4-2x^2+1}}{2x^2} + c$

4. Let $f(x)=\dfrac{x}{(1+x^n)^{1/n}}$ for $n\ge 2$ and $g(x)=\underbrace{(f\circ f\circ\ldots\circ f)}_{f \text{ occurs } n \text{ times}}(x)$. Then $\int x^{n-2}g(x)dx$ equals. **[JEE 2007]**

(A) $\dfrac{1}{n(n-1)}(1+nx^n)^{1-\frac{1}{n}}+K$ **(B)** $\dfrac{1}{n-1}(1+nx^n)^{1-\frac{1}{n}}+K$

(C) $\dfrac{1}{n(n+1)}(1+nx^n)^{1+\frac{1}{n}}+K$ **(D)** $\dfrac{1}{n+1}(1+nx^n)^{1+\frac{1}{n}}+K$

5. Let F(x) be an indefinite integral of $\sin^2x$. **[JEE 2007]**

Statement-1 : The function F(x) satisfies $F(x+\pi)=F(x)$ for all real x.

Statement-2 : $\sin^2(x+\pi)=\sin^2x$ for all real x.

(A) Statement-1 is True, Statement-2 is True ; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True ; Statement-2 is NOT a correct explanation for Statement-1.

(C) Statement-1 is True, Statement-2 is False.

(D) Statement-1 is False, Statement-2 is True.

6. Let $I=\int\dfrac{e^x}{e^{4x}+e^{2x}+1}dx$, $J=\int\dfrac{e^{-x}}{e^{-4x}+e^{-2x}+1}dx$. **[JEE 2008]**

Then, for an arbitrary constant c, the value of J – I equals

(A) $\dfrac{1}{2}\log\left(\dfrac{e^{4x}-e^{2x}+1}{e^{4x}+e^{2x}+1}\right)+c$ **(B)** $\dfrac{1}{2}\log\left(\dfrac{e^{4x}+e^{2x}+1}{e^{2x}-e^{2x}+1}\right)+c$

(C) $\dfrac{1}{2}\log\left(\dfrac{e^{2x}-e^{x}+1}{e^{2x}+e^{x}+1}\right)+c$ **(D)** $\dfrac{1}{2}\log\left(\dfrac{e^{4x}+e^{2x}+1}{e^{4x}-e^{2x}+1}\right)+c$

7. The integral $\int\dfrac{\sec^2x}{(\sec x+\tan x)^{9/2}}dx$ equals (for some arbitrary constant K) **[JEE 2012]**

(A) $-\dfrac{1}{(\sec x+\tan x)^{11/2}}\left\{\dfrac{1}{11}-\dfrac{1}{7}(\sec x+\tan x)^2\right\}+K$

(B) $\dfrac{1}{(\sec x+\tan x)^{11/2}}\left\{\dfrac{1}{11}-\dfrac{1}{7}(\sec x+\tan x)^2\right\}+K$

(C) $-\dfrac{1}{(\sec x+\tan x)^{11/2}}\left\{\dfrac{1}{11}+\dfrac{1}{7}(\sec x+\tan x)^2\right\}+K$

(D) $\dfrac{1}{(\sec x+\tan x)^{11/2}}\left\{\dfrac{1}{11}+\dfrac{1}{7}(\sec x+\tan x)^2\right\}+K$

MOCK TEST

SECTION - I : STRAIGHT OBJECTIVE TYPE

1. If $\int \frac{2\cos x - \sin x + \lambda}{\cos x + \sin x - 2} dx = A\,\ell n\,|\cos x + \sin x - 2| + Bx + C.$

Then the ordered triplet A, B, λ is

(A) $\left(\frac{1}{2}, \frac{3}{2}, -1\right)$ **(B)** $\left(\frac{3}{2}, \frac{1}{2}, -1\right)$ **(C)** $\left(\frac{1}{2}, -1, -\frac{3}{2}\right)$ **(D)** $\left(\frac{3}{2}, -1, \frac{1}{2}\right)$

2. The value of $\int x.\frac{\ell n\left(x + \sqrt{1+x^2}\right)}{\sqrt{1+x^2}} dx$ equals:

(A) $\sqrt{1+x^2}\;\ell n\left(x + \sqrt{1+x^2}\right) - x + C$

(B) $\frac{x}{2}.\ell n^2\left(x + \sqrt{1+x^2}\right) - \frac{x}{\sqrt{1+x^2}} + C$

(C) $\frac{x}{2}.\ell n^2\left(x + \sqrt{1+x^2}\right) + \frac{x}{\sqrt{1+x^2}} + C$

(D) $\sqrt{1+x^2}\;\ell n\left(x + \sqrt{1+x^2}\right) + x + C$

3. If $\int \frac{e^{x-1}}{(x^2 - 5x + 4)} 2x\,dx = A\,F(x-1) + B\,F(x-4) + C$ and $F(x) = \int \frac{e^x}{x} dx$, then A & B ordered set is

(A) $\left(-\frac{2}{3}, \frac{8}{3}\right)$ **(B)** $\left(-\frac{2}{3}, \frac{8e^3}{3}\right)$ **(C)** $\left(\frac{8}{3}, \frac{2}{3}\right)$ **(D)** $\left(-\frac{2}{3}, -\frac{8e^3}{3}\right)$

4. $I_n = \int (\log x)^n dx$, then $I_n + nI_{n-1} =$

(A) $n(\log x)^n$ **(B)** $(n\log x)^{n-1}$ **(C)** $(\log x)^{n-1}$ **(B)** $(n\log x)^n$

5. The value of $\int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x} dx$ is :

(A) $\frac{1}{2}\sin 2x + C$ **(B)** $-\frac{1}{2}\sin 2x + C$ **(C)** $-\frac{1}{2}\sin x + C$ **(D)** $-\sin^2 x + C$

6. If A is square matrix and e^A is defined as $e^A = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} \;..... = \frac{1}{2}\begin{bmatrix} f(x) & g(x) \\ g(x) & f(x) \end{bmatrix}$, where

$A = \begin{bmatrix} x & x \\ x & x \end{bmatrix}$ and $0 < x < 1$, I is an identity matrix , then $\int \frac{g(x)}{f(x)}.dx$ is equal to

(A) $\log(e^x + e^{-x}) + C$ **(B)** $\log(e^x - e^{-x}) + C$ **(C)** $\log(e^{2x} - 1) + C$ **(D)** None of these

7. The value of $\int \{1+2\tan x(\tan x+\sec x)\}^{1/2}\, dx$ is equal to

(A) $\ell n\,|\sec x\,(\sec x - \tan x)| + C$
(B) $\ell n\,|\text{cosec}\, x\,(\sec x + \tan x)| + C$
(C) $\ell n\,|\sec x\,(\sec x + \tan x)| + C$
(D) $\ell n\,|(\sec x + \tan x)| + C$

8. The value of $\int e^{(x\sin x+\cos x)}\left(\dfrac{x^4\cos^3 x - x\sin x + \cos x}{x^2\cos^2 x}\right).dx$, is equal to

(A) $e^{x\sin x+\cos x}.\left(x+\dfrac{1}{x\cos x}\right)+C$
(B) $e^{x\sin x+\cos x}.\left(x\cos x+\dfrac{1}{x}\right)+C$
(C) $e^{x\sin x+\cos x}.\left(x-\dfrac{1}{x\cos x}\right)+C$
(D) none of these

9. The value of $2\int \sin x\,.\,\text{cosec}4x\, dx$ is equal to

(A) $\dfrac{1}{2\sqrt{2}}\,\ell n\left|\dfrac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x}\right| - \dfrac{1}{4}\,\ell n\left|\dfrac{1+\sin x}{1-\sin x}\right| + C$
(B) $\dfrac{1}{2\sqrt{2}}\,\ell n\left|\dfrac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x}\right| + \dfrac{1}{4}\,\ell n\left|\dfrac{1+\sin x}{1-\sin x}\right| + C$
(C) $\dfrac{1}{2\sqrt{2}}\,\ell n\left|\dfrac{1-\sqrt{2}\sin x}{1+\sqrt{2}\sin x}\right| - \dfrac{1}{4}\,\ell n\left|\dfrac{1+\sin x}{1-\sin x}\right| + C$
(D) none of these

10. If $\int \dfrac{dx}{a+\cos x} = f\left(\tan\dfrac{x}{2}\right)+C$, then –

S_1 : f is a log function for a = 0
S_2 : f is a inverse trigonometric function for |a| < 1
S_3 : f is a polynomial function for a = 1
S_4 : f is rational function but not polynomial for a = 1

(A) T T T F **(B)** T F T F **(C)** F T T F **(D)** T F F T

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

11. If $\int \dfrac{3\cot 3x - \cot x}{\tan x - 3\tan 3x}\, dx = p\, f(x) + q\, g(x) + C$, where 'C' is a constant of integration, then

(A) $p = 1;\ q = \dfrac{1}{\sqrt{3}};\ f(x) = x;\ g(x) = \ell n\left|\dfrac{\sqrt{3}-\tan x}{\sqrt{3}+\tan x}\right|$
(B) $p = 1;\ q = -\dfrac{1}{\sqrt{3}};\ f(x) = x;\ g(x) = \ell n\left|\dfrac{\sqrt{3}-\tan x}{\sqrt{3}+\tan x}\right|$
(C) $p = 1;\ q = -\dfrac{2}{\sqrt{3}};\ f(x) = x;\ g(x) = \ell n\left|\dfrac{\sqrt{3}+\tan x}{\sqrt{3}-\tan x}\right|$
(D) $p = 1;\ q = -\dfrac{1}{\sqrt{3}};\ f(x) = x;\ g(x) = \ell n\left|\dfrac{\sqrt{3}+\tan x}{\sqrt{3}-\tan x}\right|$

12. If $\int \dfrac{(x-1)\, dx}{x^2\sqrt{2x^2-2x+1}} = \dfrac{\sqrt{f(x)}}{g(x)} + C$, then

(A) $f(x) = 2x^2 - 2x + 1$ **(B)** $g(x) = x + 1$ **(C)** $g(x) = x$ **(D)** $f(x) = \sqrt{2x^2-2x}$

13. If $I_n = \int \cot^n x \, dx$ and $I_0 + I_1 + 2(I_2 + ... + I_8) + I_9 + I_{10} = A\left(u + \frac{u^2}{2} + ... + \frac{u^9}{9}\right) + C$, where $u = \cot x$ and C is an arbitrary constant, then

(A) A is constant (B) $A = -1$ (C) $A = 1$ (D) A is dependent on x

14. If $\int e^{3x} \cos 4x \, dx = e^{3x}(A \sin 4x + B \cos 4x) + C$ then:

(A) $4A = 3B$ (B) $2A = 3B$ (C) $3A = 4B$ (D) $4A + 3B = 1$

15. The value of $\int 2^{mx} . 3^{nx} \, dx$ (when $m, n \in N$) is equal to :

(A) $\frac{2^{mx} + 3^{nx}}{m \ell n 2 + n \ell n 3} + C$ (B) $\frac{e^{(m \ell n 2 + n \ell n 3) x}}{m \ell n 2 + n \ell n 3} + C$ (C) $\frac{2^{mx} . 3^{nx}}{\ell n (2^m . 3^n)} + C$ (D) $\frac{(mn) . 2^x . 3^x}{m \ell n 2 + n \ell n 3} + C$

SECTION - III : ASSERTION AND REASON TYPE

16. **Statement-I :** If $x > 0, x \neq 1$ then $\int (\log_x e - (\log_x e)^2) dx = x \log_x e + C$

Statement-II : $\int e^x (f(x) + f'(x)) dx = e^x f(x) + C$ and $e^t = x$ iff $t = \ell nx$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

17. **Statement-I :** If $y = \sin^{-1}x$, then $\int \sin^{-1} x \, dx = \int y \cos y \, dy + c$

Statement-II : If $y = f^{-1}(x)$, then $\int f^{-1}(x) \, dx = \int y f'(y) \, dy + c$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

18. **Statement-I :** $\int \frac{\ell n (e^x + 1)}{e^x} dx = x - \left(\frac{1 + e^x}{e^x}\right) \ell n (e^x + 1) + C$

Statement-II : $\int \frac{f'(x)}{f(x)} dx = \ell n |f(x)| + C$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

19. **Statement-I :** $\int 2^{\tan^{-1}x}\, d(\cot^{-1}x) = \dfrac{2^{\tan^{-1}x}}{\ell n\, 2} + c$

Statement-II : $\dfrac{d}{dx}(a^x + c) = a^x\, \ell n\, a$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

20. **Statement-I :** $\int e^{\sin^{-1}x}\left(1 - \dfrac{x}{\sqrt{1-x^2}}\right)dx = e^{\sin^{-1}x}.\sqrt{1-x^2} + c$

Statement-II : $\int e^{g(x)}\left(g'(x)f(x) + f'(x)\right)dx = e^{g(x)}f(x) + c$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

SECTION - IV : MATRIX - MATCH TYPE

21.

Column – I	Column – II
(A) If $I = \int \dfrac{\sin x - \cos x}{\lvert \sin x - \cos x \rvert}dx$, where $\dfrac{\pi}{4} < x < \dfrac{3\pi}{8}$, then I equal to	**(p)** $\sin x$
(B) If $\int \dfrac{x^2}{(x^3+1)(x^3+2)}dx = \dfrac{1}{3} f\left(\dfrac{x^3+1}{x^3+2}\right) + C$, then f(x) is equal to	**(q)** $x + c$
(C) If $\int \sin^{-1}x . \cos^{-1}x\ dx = f^{-1}(x)\left[\dfrac{\pi}{2}x - x\, f^{-1}(x) - 2\sqrt{1-x^2}\right] + \dfrac{\pi}{2}\sqrt{1-x^2} + 2x + C$, then f(x) is equal to	**(r)** $\ell n\lvert x\rvert$
(D) If $\int \dfrac{dx}{xf(x)} = f(f(x)) + C$, then f(x) is equal to	**(s)** $\sin^{-1}x$
	(t) $-x + c$

22.

Column – I	Column – II
(A) If $F(x) = \int \dfrac{x + \sin x}{1 + \cos x}dx$ and $F(0) = 0$, then $F(\pi/2) =$	**(p)** $-\dfrac{\pi}{2}$
(B) Let $F(x) = \int e^{\sin^{-1}x}\left(1 - \dfrac{x}{\sqrt{1-x^2}}\right)dx$ and $F(0) = 1$, If $F(1/2) = \dfrac{k\sqrt{3}\,e^{\pi/6}}{\pi}$, then $k =$	**(q)** $\dfrac{\pi}{3}$

(C) Let $F(x)=\int \frac{dx}{(x^2+1)(x^2+9)}$ and $F(0)=0$, **(r)** $\frac{\pi}{4}$

if $F(\sqrt{3})=\frac{5}{36}k$, then k =

(D) Let $F(x)=\int \frac{\sqrt{\tan x}}{\sin x \cos x}dx$ and $F(0)=0$ **(s)** π

if $F(\pi/4)=\frac{2k}{\pi}$, then k = **(t)** $\frac{\pi}{2}$

SECTION - V : COMPREHENSION TYPE

23. **Read the following comprehension carefully and answer the questions.**

Integrals of the form $\int \frac{p_m(x)}{\sqrt{ax^2+bx+c}}dx$, where $p_m(x)$ is a polynomial of degree m, are calculated by the reduction formula.

$$\int \frac{p_m(x)}{\sqrt{ax^2+bx+c}}dx = p_{m-1}(x)\sqrt{ax^2+bx+c}+\lambda\int\frac{dx}{\sqrt{ax^2+bx+c}}$$

where $p_{m-1}(x)$ is a polynomial of degree (m – 1) and λ is some constant number.

e.g. $I=\int \frac{x^3-x-1}{\sqrt{x^2+2x+2}}dx$ then applying the above formula, we can write

$$\int \frac{x^3-x-1}{\sqrt{x^2+2x+2}}dx=(Ax^2+Bx+C)\sqrt{x^2+2x+2}+\lambda\int\frac{dx}{\sqrt{x^2+2x+2}}$$

differentiate both sides, we get

$$\frac{x^3-x-1}{\sqrt{x^2+2x+2}}=(Ax^2+Bx+C).\frac{2(x+1)}{2\sqrt{x^2+2x+2}}+(2Ax+B)\sqrt{x^2+2x+2}+\frac{\lambda}{\sqrt{x^2+2x+2}}$$

$x^3-x-1=(Ax^2+Bx+C)(x+1)+(2Ax+B)(x^2+2x+2)+\lambda$

On comparing coefficeints of like powers of x we obtain the values of A, B, C and λ.

1. If $\int \frac{x^3-6x^2+11x-6}{\sqrt{x^2+4x+3}}dx=(Ax^2+Bx+C)\sqrt{x^2+4x+3}+\lambda\int\frac{dx}{\sqrt{x^2+4x+3}}$, then value of 'A' is

(A) $\frac{1}{3}$ **(B)** 1 **(C)** 3 **(D)** – 1/3

2. In Q.No. 1 value of 'C' is

(A) – 37 **(B)** $-\frac{14}{3}$ **(C)** $\frac{14}{3}$ **(D)** 37

3. In Q.No. 1 value of 'λ', is

(A) 66 **(B)** – 66 **(C)** $\frac{37}{3}$ **(D)** $-\frac{37}{3}$

24. Read the following comprehension carefully and answer the questions.

It is known that

$$\sqrt{\tan x}+\sqrt{\cot x} = \begin{cases} \dfrac{\sqrt{\sin x}}{\sqrt{\cos x}}+\dfrac{\sqrt{\cos x}}{\sqrt{\sin x}} & \text{if } \quad 0<x<\dfrac{\pi}{2} \\ \dfrac{\sqrt{-\sin x}}{\sqrt{-\cos x}}+\dfrac{\sqrt{-\cos x}}{\sqrt{-\sin x}} & \text{if } \quad \pi<x<\dfrac{3\pi}{2} \end{cases},$$

$$\frac{d}{dx}\left(\sqrt{\tan x}-\sqrt{\cot x}\right)=\frac{1}{2}\left(\sqrt{\tan x}+\sqrt{\cot x}\right)(\tan x+\cot x),\ \forall\ x\in\left(0,\frac{\pi}{2}\right)\cup\left(\pi,\frac{3\pi}{2}\right)$$

and $$\frac{d}{dx}\left(\sqrt{\tan x}+\sqrt{\cot x}\right)=\frac{1}{2}\left(\sqrt{\tan x}-\sqrt{\cot x}\right)(\tan x+\cot x),\ \forall\ x\in\left(0,\frac{\pi}{2}\right)\cup\left(\pi,\frac{3\pi}{2}\right).$$

1. Value of integral $I=\int(\sqrt{\tan x}+\sqrt{\cot x})\,dx$, where $x\in\left(0,\frac{\pi}{2}\right)\cup\left(\pi,\frac{3\pi}{2}\right)$ is

(A) $\sqrt{2}\tan^{-1}\left(\dfrac{\sqrt{\tan x}-\sqrt{\cot x}}{\sqrt{2}}\right)+C$
(B) $\sqrt{2}\tan^{-1}\left(\dfrac{\sqrt{\tan x}+\sqrt{\cot x}}{\sqrt{2}}\right)+C$
(C) $-\sqrt{2}\tan^{-1}\left(\dfrac{\sqrt{\tan x}-\sqrt{\cot x}}{\sqrt{2}}\right)+C$
(D) $-\sqrt{2}\tan^{-1}\left(\dfrac{\sqrt{\tan x}+\sqrt{\cot x}}{\sqrt{2}}\right)+C$

2. Value of the integral $I=\int(\sqrt{\tan x}+\sqrt{\cot x})\,dx$, where $x\in\left(0,\frac{\pi}{2}\right)$, is

(A) $\sqrt{2}\sin^{-1}(\cos x-\sin x)+C$
(B) $\sqrt{2}\sin^{-1}(\sin x-\cos x)+C$
(C) $\sqrt{2}\sin^{-1}(\sin x+\cos x)+C$
(D) $-\sqrt{2}\sin^{-1}(\sin x+\cos x)+C$

3. Value of the integral $I=\int(\sqrt{\tan x}+\sqrt{\cot x})\,dx$, where $x\in\left(\pi,\frac{3\pi}{2}\right)$, is

(A) $\sqrt{2}\sin^{-1}(\cos x-\sin x)+C$
(B) $\sqrt{2}\sin^{-1}(\sin x-\cos x)+C$
(C) $\sqrt{2}\sin^{-1}(\sin x+\cos x)+C$
(D) $-\sqrt{2}\sin^{-1}(\sin x+\cos x)+C$

25. Read the following comprehension carefully and answer the questions.

Let $I_{n,m}=\int\sin^n x\cos^m x.dx$. Then we can relate $I_{n,m}$ with each of the following

(i) $I_{n-2,m}$ **(ii)** $I_{n+2,m}$ **(iii)** $I_{n,m-2}$
(iv) $I_{n,m+2}$ **(v)** $I_{n-2,m+2}$ **(vi)** $I_{n+2,m-2}$

Suppose we want to establish a relation between $I_{n,m}$ and $I_{n,m-2}$, then we set

$P(x)=\sin^{n+1}x\cos^{m-1}x$ ....**(i)**

In $I_{n,m}$ and $I_{n,m-2}$ the exponent of cosx is m and $m-2$ respectively, the minimum of the two is $m-2$, adding 1 to the minimum we get $m-2+1=m-1$. Now choose the exponent $m-1$ of cosx in P(x). Similarly choose the exponent of sin x for P(x)

Now differentiating both sides of (1), we get

$P'(x) = (n+1)\sin^n x \cos^m x - (m-1)\sin^{n+2}x \cos^{m-2}x$

$= (n+1)\sin^n x \cos^m x - (m-1)\sin^n x\,(1-\cos^2 x)\cos^{m-2}x$

$= (n+1)\sin^n x \cos^m x - (m-1)\sin^n x \cos^{m-2} x + (m-1)\sin^n x \cos^m x$

$= (n+m)\sin^n x \cos^m x - (m-1)\sin^n x \cos^{m-2}x$

Now integrating both sides, we get

$\sin^{n+1}x \cos^{m-1}x = (n+m)\, I_{n,m} - (m-1)\, I_{n,m-2}$.

Similarly we can establish the other relations.

1. The relation between $I_{4,2}$ and $I_{2,2}$ is

(A) $I_{4,2} = \frac{1}{6}(-\sin^3 x \cos^3 x + 3I_{2,2})$ **(B)** $I_{4,2} = \frac{1}{6}(\sin^3 x \cos^3 x + 3I_{2,2})$

(C) $I_{4,2} = \frac{1}{6}(\sin^3 x \cos^3 x - 3I_{2,2})$ **(D)** $I_{4,2} = \frac{1}{4}(-\sin^3 x \cos^3 x + 2I_{2,2})$

2. The relation between $I_{4,2}$ and $I_{6,2}$ is

(A) $I_{4,2} = \frac{1}{5}(\sin^5 x \cos^3 x + 8I_{6,2})$ **(B)** $I_{4,2} = \frac{1}{5}(-\sin^5 x \cos^3 x + 8I_{6,2})$

(C) $I_{4,2} = \frac{1}{5}(\sin^5 x \cos^3 x - 8I_{6,2})$ **(D)** $I_{4,2} = \frac{1}{6}(\sin^5 x \cos^3 x + 8I_{6,2})$

3. The relation between $I_{4,2}$ and $I_{4,4}$ is

(A) $I_{4,2} = \frac{1}{3}(\sin^5 x \cos^3 x + 8\, I_{4,4})$ **(B)** $I_{4,2} = \frac{1}{3}(-\sin^5 x \cos^3 x + 8\, I_{4,4})$

(C) $I_{4,2} = \frac{1}{3}(\sin^5 x \cos^3 x - 8\, I_{4,4})$ **(D)** $I_{4,2} = \frac{1}{3}(\sin^5 x \cos^3 x + 6\, I_{4,4})$

SECTION - VI : INTEGER TYPE

26. Evaluate : $\int \cos 2x \,\ell n\,(1+\tan x)\, dx$

27. $\int \frac{x\cos\alpha + 1}{\left(x^2 + 2x\cos\alpha + 1\right)^{3/2}}\, dx = \frac{\lambda x}{\sqrt{x^2 + 2x\cos\alpha + 1}} + c$, then find λ

28. Evaluate : $\int \sqrt{\frac{\operatorname{cosec} x - \cot x}{\operatorname{cosec} x + \cot x}} \cdot \frac{\sec x}{\sqrt{1 + 2\sec x}}\, dx$

29. $\int \left(\frac{x-1}{x+1}\right)\frac{dx}{\sqrt{x^3 + x^2 + x}} = \lambda \tan^{-1}\sqrt{\left(x + \frac{1}{x} + 1\right)} + c$, then find λ

30. Evaluate : $\int e^x \frac{1 + n x^{n-1} - x^{2n}}{\left(1 - x^n\right)\sqrt{1 - x^{2n}}}\, dx$

ANSWER KEY

EXERCISE - 1

1. B **2.** A **3.** B **4.** A **5.** B **6.** D **7.** C **8.** B **9.** A **10.** B **11.** D **12.** B **13.** C
14. B **15.** B **16.** C **17.** B **18.** C **19.** B **20.** A **21.** B **22.** A **23.** C **24.** A **25.** B **26.** A
27. B **28.** C **29.** C **30.** B **31.** B **32.** A **33.** A **34.** B **35.** C **36.** B **37.** D **38.** A **39.** A

EXERCISE - 2 : PART # I

1. ABD **2.** CD **3.** BCD **4.** ABCD **5.** ACD **6.** BC **7.** BCD **8.** BD **9.** ABCD
10. AB **11.** ABC **12.** BCD **13.** AB **14.** BC **15.** ABCD

PART - II

1. A **2.** A **3.** A **4.** C **5.** A **6.** C **7.** C

EXERCISE - 3 : PART # I

1. $A \to p$ $B \to r$ $C \to s$ $D \to q$ **2.** $A \to s$ $B \to q$ $C \to r$ **3.** $A \to q$ $B \to p$ $C \to p,r$ $D \to p,r,s$
4. $A \to p$ $B \to p$ $C \to r$ $D \to s$ **5.** $A \to s$ $B \to q$ $C \to r$ $D \to p$

PART - II

Comprehension # 1 : **1.** B **2.** C **Comprehension # 2 :** **1.** A **2.** B **3.** A
Comprehension # 3 : **1.** A **2.** A **3.** B **Comprehension # 4 :** **1.** C **2.** B **3.** D

EXERCISE - 5 : PART # I

1. 3 **2.** 2 **3.** 2 **4.** 4 **5.** 4 **6.** 1 **7.** 3 **8.** 1 **9.** 3 **10.** 2 **11.** 2 **12.** 1

PART - II

1. $(x+1)\tan^{-1}\frac{2(x+1)}{3} - \frac{3}{4}\ell n(4x^2+8x+13)+c$ **2.** $\frac{\left(2x^{3m}+3x^{2m}+6x^m\right)^{\frac{m+1}{m}}}{6\ (m+1)}+C$

3. D **4.** A **5.** D **6.** C **7.** C

MOCK TEST

1. B	**2.** A	**3.** B	**4.** A	**5.** B	**6.** A	**7.** C	**8.** C	**9.** A
10. B	**11.** AD	**12.** AC	**13.** AB	**14.** CD	**15.** BC	**16.** A	**17.** A	**18.** A
19. D	**20.** A							

21. $A \to q \; B \to r \; C \to p \; D \to r$ **22.** $A \to t \; B \to t \; C \to r \; D \to s$

23. **1.** A **2.** D **3.** B **24.** **1.** A **2.** B **3.** A **25.** **1.** A **2.** A **3.** B

26. $\frac{1}{2}[\sin 2x . \ell n(1+\tan x) - x + \ell n|\sin x + \cos x|] + C$ **27.** 1 **28.** $\sin^{-1}\left(\frac{1}{2}\sec^2\frac{x}{2}\right) + C$ **29.** 2

30. $e^x\sqrt{\frac{1+x^n}{1-x^n}} + C$