Thermometry and Calorimetry

The branch dealing with measurement of temperature is called **thermometry** and the devices used to measure temperature are called **thermometers**.

Heat

Heat is a form of energy called thermal energy which flows from a (hotter) higher temperature body to a lower temperature body (colder) when they are placed in contact.

Heat or thermal energy of a body is the sum of kinetic energies of all its constituent particles, on account of translational, vibrational and rotational motion.

The SI unit of heat energy is joule (J).

The practical unit of heat energy is calorie.

$$1 \text{ cal} = 4.18 \text{ J}$$

1 calorie is the quantity of heat required to raise the temperature of 1 g of water by 1°C (from 14.5°C to 15.5°C).

Mechanical energy or work (W) can be converted into heat (Q) by

$$W = JQ$$

where, $J =$ Joule's mechanical equivalent of heat.

J is a conversion factor (not a physical quantity) and its value is 4.186 J/cal.

Temperature and its Measurement

Temperature of a body is the degree of hotness or coldness of the body.

Highest possible temperature achieved in laboratory is about 10^8 K, while lowest possible temperature attained is 10^{-8} K.

Branch of Physics dealing with production and measurement of temperature close to 0 K is known as **cryagenics**, while that dealing with the measurement of very high temperature is called **pyrometry**.

Temperature of the core of the sun is 10^7 K while that of its surface is 6000 K.

NTP or STP implies 273.15 K (0°C = 32°F).

Different Scale of Temperature

(i) **Celsius Scale** In this scale of temperature, the melting point of ice is taken as 0°C and the boiling point of water as 100°C and the space between these two points is divided into 100 equal parts.

(ii) **Fahrenheit Scale** In this scale of temperature, the melting point of ice is taken as 32°F and the boiling point of water as 212°F and the space between these two points is divided into 180 equal parts.

(iii) **Kelvin Scale** In this scale of temperature, the melting point of ice is taken as 273 K and the boiling point of water as 373 K and the space between these two points is divided into 100 equal parts.

(iv) **Reaumer Scale** In this scale of temperature, the melting point of ice is taken as 0° R and the boiling point of water as 80° R and the space between these two points is divided into 80 equal parts.

Relation between Different Scales of Temperatures

$$\frac{C}{100} = \frac{F - 32}{180} = \frac{K - 273}{100} = \frac{R}{80}$$

Absolute Temperature

There is no limit for maximum temperature but there is a sharp point for minimum temperature that nobody can have the temperature lower than this minimum value of temperature, which is known as absolute temperature.

Thermometric Property

The property of an object which changes with temperature is called a thermometric property. Different thermometric properties and thermometers have been given below

(i) **Pressure of a Gas at Constant Volume**

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \text{ and } p_t = p_0 \left(1 + \frac{t}{273}\right)$$

$$t = \left(\frac{p_t - p_0}{p_{100} - p_0} \times 100\right) °\text{C}$$

where, p_0, p_{100} and p_t are pressure of a gas at constant volume at 0°C, 100°C and t°C.

A constant volume gas thermometer can measure temperature from – 200°C to 500°C.

(ii) **Electrical Resistance of Metals**

$$R_t = R_0 (1 + \alpha t + \beta t^2)$$

where, α and β are constants for a metal.

As β is too small therefore, we can take

$$R_t = R_0 (1 + \alpha t)$$

where, α = temperature coefficient of resistance and

R_0 and R_t are electrical resistances at 0°C and t°C.

$$\alpha = \frac{R_2 - R_1}{R_1 t_2 - R_2 t_1}$$

where, R_1 and R_2 are electrical resistances at temperatures t_1 and t_2.

or $$t = \frac{R_t - R_0}{R_{100} - R_0} \times 100°\text{C}$$

where, R_{100} is the resistance at 100°C.

Platinum resistance thermometer can measure temperature from –200°C to 1200°C.

(iii) **Length of Mercury Column in a Capillary Tube**

$$l_t = l_0 (1 + \alpha t)$$

where α = coefficient of linear expansion and l_0, l_t are lengths of mercury column at 0°C and t°C.

Thermometers

The thermometers work on the thermometric property, *i.e.,* the property which changes with temperature like any physical quantity such as length, volume, pressure and resistance etc., which varies linearly with a certain range of temperature.

Different Types of Thermometers

Some different types of thermometers are given below

(i) **Mercury thermometer** In this thermometer, the length of a mercury column from some fixed point is taken as thermometric property.

If length of mercury column at 0° and 100° are l_0 and l_{100} respectively and at $t°$ the length of mercury is l_t. Then

$$t = \left(\frac{l_t - l_0}{l_{100} - l_0}\right) \times 100°\text{C}$$

(ii) **Constant volume gas thermometer** It works on the principle of change in pressure with temperature when the volume is kept constant. If p_0, p_{100} and p_t are the pressures of gas at temperatures 0° C, 100°C, and unknown temperature (t°C) respectively keeping the volume constant, then

$$t = \left(\frac{p_t - p_0}{p_{100} - p_0} \times 100\right)°\text{C}$$

Note For constant pressure gas thermometers, $t = \dfrac{V_t - V_0}{V_{100} - V_0} \times 100°\text{C}$

(iii) **Platinum resistance thermometer** It works on the principle of variation of resistance of metals with temperature.

If R_0, R_{100} and R_t are the resistances of a platinum wire at temperature 0°C, 100°C and unknown temperature (t°C) respectively.

Then,

$$t = \left(\frac{R_t - R_0}{R_{100} - R_0} \times 100\right)°\text{C} = \left(\frac{R_t}{R_{tr}} \times 273.16\right)\text{K}$$

Here, temperature coefficient of resistance (α) is given by

$$\alpha = \frac{R_{100} - R_0}{R_0 \times 100}$$

(iv) **Pyrometers** Pyrometers are the devices used to measure the temperature by measuring the intensity of radiations received from the body.

Thermal Expansion

When matter is heated without any change in its state, it usually expand. This phenomena of expansion of matter on heating, is called thermal expansion. There are three types of thermal expansion

1. Expansion of Solids

Three types of expansion takes place in solid

(i) **Linear Expansion** Expansion in length on heating is called linear expansion.

Increase in length, $l_2 = l_1 (1 + \alpha\, \Delta t)$

where, l_1 and l_2 are initial and final lengths, Δt = change in temperature and α = coefficient of linear expansion.

Coefficient of linear expansion, $\alpha = \dfrac{\Delta l}{l \times \Delta t}$

where, l = real length and Δl = change in length

and Δt = change in temperature.

(ii) **Superficial Expansion** Expansion in area on heating is called superficial expansion.

Increase in area, $A_2 = A_1 (1 + \beta \Delta t)$

where, A_1 and A_2 are initial and final areas and β is a coefficient of superficial expansion.

Coefficient of superficial expansion, $\beta = \dfrac{\Delta A}{A \times \Delta t}$

where, A = area, ΔA = change in area and Δt = change in temperature.

(iii) **Cubical Expansion** Expansion in volume on heating is called cubical expansion.

Increase in volume, $0V_2 = V_1 (1 + \gamma \Delta t)$

where, V_1 and V_2 are initial and final volumes and γ is a coefficient of cubical expansion.

Coefficient of cubical expansion, $\gamma = \dfrac{\Delta V}{V \times \Delta t}$

where V = real volume, ΔV = change in volume and Δt = change in temperature.

Dimension of α, β and γ are same $[\theta^{-1}]$ and units are K^{-1} or $(°C)^{-1}$

Relation between coefficients of linear, superficial and cubical expansions $\beta = 2\alpha$ and $\gamma = 3\alpha$

or $\alpha : \beta : \gamma = 1 : 2 : 3$

2. Expansion of Liquids

In liquids only expansion in volume takes place on heating.

(i) **Apparent Expansion of Liquids** When expansion of the container containing liquid, on heating is not taken into account, then observed expansion is called apparent expansion of liquids.

Coefficient of apparent expansion of a liquid

$$(\gamma_a) = \frac{\text{apparent (or observed) increase in volume}}{\text{original volume } \times \text{ change in temperature}}$$

(ii) **Real Expansion of Liquids** When expansion of the container, containing liquid, on heating is also taken into account, then observed expansion is called real expansion of liquids.

Coefficient of real expansion of a liquid

$$(\gamma_r) = \frac{\text{real increase in volume}}{\text{original volume } \times \text{ change in temperature}}$$

Both γ_r and γ_a are measured in $^\circ C^{-1}$.

We can show that

$$\boxed{\gamma_r = \gamma_a + \gamma_g}$$

where, γ_r and γ_a are coefficient of real and apparent expansion of liquids and γ_g is coefficient of cubical expansion of the container (vessel).

Note Some substances contract with rising temperature because transverse vibration of atoms of substance dominate on the longitudinal vibration which is responsible for contraction.

Anamalous Expansion of Water

When temperature of water is increased from 0°C, then its volume decreases upto 4°C, becomes minimum at 4°C and then increases. This behaviour of water around 4°C is called anamalous expansion of water.

3. Thermal Expansion of Gases

There are two types of coefficient of expansion in gases

(i) **Volume Coefficient (γ_V)** At constant pressure, the change in volume per unit volume per degree celsius is called volume coefficient.

$$\gamma_V = \frac{V_2 - V_1}{V_0 (t_2 - t_1)}$$

where V_0, V_1 and V_2 are volumes of the gas at 0°C, t_1°C and t_2°C.

(ii) **Pressure Coefficient (γ_p)** At constant volume, the change in pressure per unit pressure per degree celsius is called pressure coefficient.

$$\gamma_p = \frac{p_2 - p_1}{p_0 (t_2 - t_1)}$$

where p_0, p_1 and p_2 are pressure of the gas at 0°C, t_1 °C and t_2 °C.

Variation of Density with Temperature

Most substances expand when they are heated *i.e.* volume of a given mass of a substance increases on heating, so density decreases. Hence $\rho \propto \frac{1}{V}$,

$$\rho' = \rho(1 + \gamma \Delta T)^{-1}, \text{ as } \gamma \text{ is small } (1 + \gamma \Delta T)^{-1} \approx 1 - \gamma \Delta T$$

$$\rho' \approx \rho(1 - \gamma \Delta T)$$

Practical Applications of Thermal Expansion

(i) When rails are laid down on the ground, space is left between the end of two rails.

(ii) The transmission cables are not tightly fixed to the poles.

(iii) The iron rim to be put on a cart wheel is always of slightly smaller diameter than that of wheel.

(iv) A glass stopper jammed in the neck of a glass bottle can be taken out by heating the neck of the bottle.

Thermal Equilibrium

When there is no transfer of heat between two bodies in contact, then the bodies are called in thermal equilibrium.

Triple Point of Thermal Water

The values of pressure and temperature at which water coexists in equilibrium in all three states of matter, *i.e.* ice, water and vapour is called triple point of water.

Triple point of water is 273 K temperature and 0.46 cm of mercury pressure.

Specific Heat

The amount of heat required to raise the temperature of unit mass of the substance through 1°C is called its specific heat.

It is denoted by c or s.

Its SI unit is 'joule/kilogram-°C' (J/kg-°C) or $\text{Jkg}^{-1}\text{K}^{-1}$ and its dimensional formula is $[\text{L}^2\text{T}^{-2}\theta^{-1}]$.

The specific heat of water is 4200 J $kg^{-1}°C^{-1}$ or 1 cal g^{-1} C^{-1} which is high as compared with most other substances.

Gases have two types of specific heat

(i) The specific heat capacity at constant volume (C_V).

(ii) The specific heat capacity at constant pressure (C_p).

Specific heat at constant pressure (C_p) is greater than specific heat at constant volume (C_V), *i.e.* $C_p > C_V$.

For molar specific heats, $C_p - C_V = R$

where, $R =$ gas constant and this relation is called **Mayer's formula**.

The ratio of two principal sepecific heats of a gas is represented by γ, *i.e.*

$$\gamma = \frac{C_p}{C_V}$$

The value of γ depends on atomicity of the gas.

Amount of heat energy required to change the temperature of any substance is given by

$$Q = mc\Delta t$$

where, $m =$ mass of the substance,

$c =$ specific heat of the substance

and $\Delta t =$ change in temperature.

Thermal (Heat) Capacity

Heat capacity of any body is equal to the amount of heat energy required to increase its temperature through 1°C.

Heat capacity $= mc$

where, $c =$ specific heat of the substance of the body and $m =$ mass of the body.

Its SI unit is joule/kelvin (J/K) and dimensional formula $[ML^2T^{-2}K^{-1}]$.

Molar specific heat capacity, $c = \frac{s}{\mu} = \frac{\Delta Q}{\mu \Delta T}$

where, $\mu =$ number of moles of substances (gas).

Relation between c and M are

$$c = MS$$

where, $M =$ molecular mass of substance

and $S =$ specific heat capacity.

Water Equivalent

It is the quantity of water whose thermal capacity is same as the heat capacity of the body. It is denoted by W.

$$W = ms = \text{heat capacity of the body.}$$

Its expressed in the unit gram.

Latent Heat (Change of State)

The heat energy absorbed or released at constant temperature per unit mass for change of state is called latent heat.

Heat energy absorbed or released during change of state is given by

$$Q = mL$$

where, m = mass of the substance and L = latent heat.

Its unit is cal/g or J/kg and its dimensional formula is $[L^2 T^{-2}]$.

For water at its normal boiling point or condensation temperature (100°C), the latent heat of vaporisation is

$$L = 540 \text{ cal/g} = 40.8 \text{ kJ/mol}$$
$$= 2260 \text{ kJ/kg}$$

For water at its normal freezing temperature or melting point (0°C), the latent heat of fusion is

$$L = 80 \text{ cal/g} = 60 \text{ kJ/mol}$$
$$= 336 \text{ kJ/kg}$$

It is more painful to get burnt by steam rather than by boiling water at 100°C. Steam converted to water at 100°C, then it gives out 536 cal of heat, so, it is clear that steam at 100°C has more heat than water at 100°C (*i.e.* boiling of water).

After snow falls, the temperature of the atmosphere becomes very low. This is because the snow absorbs the heat from the atmosphere to melt down. So, in the mountains, when snow falls, one does not feel too cold but when ice melts, he feels too cold.

There is more shivering effect of icecream on teeth as compared to that of water (obtained from ice). This is because when icecream melts down, it absorbs large amount of heat from teeth.

Joule's Law

According to Joule, whenever heat is converted into work or work is converted into heat, then the ratio of between work and heat in constant.

$\frac{W}{Q} = J$, where J is mechanical equivalent of heat.

When water falls from height h, then increase in temperature dT at the bottom is

$$dT = \left(\frac{gh}{J \cdot C}\right)^\circ C$$

- When m kg of ice-block falls from height h and its some part m' is melt down, then

$$h = \frac{m'}{m}\left(\frac{JL}{g}\right) \text{ meter}$$

If ice-block melts completely, then $m = m'$ and hence $h = \frac{JL}{g}$.

Melting

Conversion of solid into liquid state at constant temperature is called melting.

Fusion and Freezing Point

The process of change of state from liquid to solid is called fusion. The temperature at which liquid starts to freeze is known as the freezing point of the liquid.

Evaporation

Conversion of liquid into vapour at all temperatures (even below its boiling point) is called evaporation.

Boiling

When a liquid is heated gradually, at a particular temperature the saturated vapour pressure of the liquid becomes equal to the atmospheric pressure, now bubbles of vapour rise to the surface of the liquid. This process is called boiling of the liquid.

The temperature at which liquid boils is called **boiling point**.

The boiling point of water increases with increase in pressure and decreases with decrease in pressure.

Sublimation

The conversion of a solid into vapour state is called sublimation.

Hoar Frost

The conversion of vapours into solid state is called hoar frost.

Calorimetry

This is the branch of heat transfer that deals with the measurement of heat. The heat is usually measured in calories or kilo calories.

Principle of Calorimetry

When a hot body is mixed with a cold body, then heat lost by hot body is equal to the heat gained by cold body.

$$\text{Heat lost} = \text{Heat gain}$$

i.e. principle of calorimetry follows the law of conservation of heat energy.

If two substances having masses m_1 and m_2, specific heats c_1 and c_2 kept at temperatures T_1 and T_2 ($T_1 > T_2$) are mixed, such that temperature of mixture at equilibrium is T_{mix}.

Then, $$m_1 . c_1 (T_1 - T_{mix}) = m_2 c_2 (T_{mix} - T_2)$$

or $$T_{mix} = \frac{m_1 c_1 T_1 + m_2 c_2 T_2}{m_1 c_1 + m_2 c_2}$$

Temperature of Mixture in Different Cases

Condition	Temperture of mixture
If bodies are of same material, *i.e.* $c_1 = c_2$	$T_{mix} = \frac{m_1 T_1 + m_2 T_2}{m_1 + m_2}$
If bodies are of same mass, *i.e.* $m_1 = m_2$	$T_{mix} = \frac{c_1 T_1 + c_2 T_2}{c_1 + c_2}$
If bodies are of same mass and same material, *i.e.* $m_1 = m_2$ and $c_1 = c_2$	$T_{mix} = \frac{T_1 + T_2}{2}$
If water at T_w°C is mixed with ice at 0°C. First, ice will melt and then its temperature rises to attain thermal equilibrium (say final temperature is T_{mix})	$T_{mix} = \frac{m_w T_w - \frac{m_i L_i}{C_W}}{m_w + m_i}$
If $m_w = m_i$	$T_{mix} = \frac{T_w - \frac{L_i}{c_w}}{2}$
If $T_{mix} < T_i$	$T_{mix} = 0°C$