Current Electricity

Electric Current (I)

The rate of flow of charge through any cross-section of a wire w.r.t. time is called electric current flowing through it.

Electric current $(I) = \frac{q}{t}$. Its SI unit is ampere (A).

The conventional direction of electric current is the direction of motion of positive charge.

The current is the same for all cross-sections of a conductor of non-uniform cross-section. Similar to the water flow, charge flows faster where the conductor is smaller in cross-section and slower where the conductor is larger in cross-section, so that charge rate remains unchanged.

If a charge q revolves in a circle with frequency f, the equivalent current,

$$i = qf$$

In a metallic conductor current flows due to motion of free electrons while in electrolytes and ionised gases current flows due to electrons and positive ions.

According to its magnitude and direction electric current is of two types

(i) **Direct Current (DC)** Its magnitude and direction do not change with time. A cell, battery or DC dynamo are the sources of direct current.

(ii) **Alternating Current (AC)** An electric current whose magnitude changes continuously and changes its direction periodically is called alternating current. AC dynamo is the source of alternating current.

Thermal Velocity of Free Electrons

Free electrons in a metal move randomly with a very high speed of the order of 10^5 ms^{-1}. This speed is called thermal velocity of free electrons. Average thermal velocity of free electrons in any direction remains zero.

Relaxation Time (τ)

The time interval between two successive collisions of electrons with the positive ions in the metallic lattice is defined as relaxation time.

$$\tau = \frac{\text{mean free path}}{\text{rms velocity of electrons}} = \frac{\lambda}{v_{\text{rms}}}$$

Drift Velocity of Free Electrons

When a potential difference is applied across the ends of a conductor, the free electrons in it move with an average velocity opposite to the direction of electric field, which is called drift velocity of free electrons.

Drift velocity $$v_d = \frac{eE\tau}{m} = \frac{eV\tau}{ml}$$

where, τ = relaxation time,
e = charge on electron,
E = electric field intensity,
l = length of the conductor,
V = potential difference across the ends of the conductor
and m = mass of the electron.

Relation between electric current and drift velocity is given by

$$v_d = \frac{I}{An\,e}$$

Current Density

The electric current flowing per unit area of cross-section of a conductor is called current density.

Current density $(J) = \dfrac{I}{A} = nev_d$

Its SI unit is ampere metre^{-2} (Am^{-2}) and dimensional formula is [AT^{-2}]. It is a vector quantity and its direction is in the direction of motion of positive charge or in the direction of flow of current.

Mobility

The drift velocity of electron per unit electric field applied is called mobility of electron.

Mobility of electron $(\mu) = \frac{v_d}{E} = \frac{e\tau}{m}$

Its SI unit is $m^2 s^{-1} V^{-1}$ and its dimensional formula is $[M^{-1}T^2A]$.

Ohm's Law

If physical conditions of a conductor such as temperature remains unchanged, then the electric current (I) flowing through the conductor is directly proportional to the potential difference (V) applied across its ends.

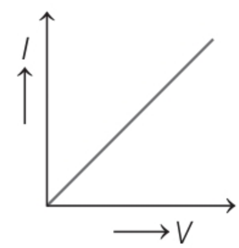
$$I \propto V \quad \text{or} \quad V = IR$$

where, R is the electrical resistance of the conductor and $R = \frac{ml}{Ane^2\tau}$.

Ohmic Conductors

Those conductors which obey Ohm's law, are called ohmic conductors, *e.g.* all metallic conductors are ohmic conductor.

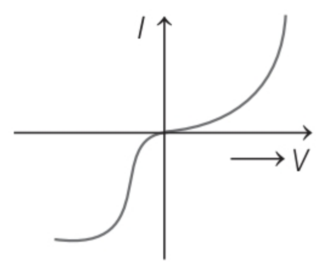
For ohmic conductors V-I graph is a straight line.



Non-ohmic Conductors

Those conductors which do not obey Ohm's law, are called non-ohmic conductors, *e.g.* diode valve, triode valve, transistor , vacuum tubes etc.

For non-ohmic conductors V-I graph is not a straight line.



Electrical Resistance

The obstruction offered by any conductor in the path of flow of current is called its electrical resistance.

Electrical resistance, $R = \frac{V}{I}$

Its SI unit is ohm (Ω) and its dimensional formula is $[ML^2T^{-3}A^{-2}]$.

Electrical resistance of a conductor, $R = \frac{\rho l}{A}$

where, l = length of the conductor, A = cross-section area

and ρ = resistivity of the material of the conductor.

If a resistance wire is stretched to a greater length, keeping volume constant, then

$$R \propto l^2 \Rightarrow \frac{R_1}{R_2} = \left(\frac{l_1}{l_2}\right)^2$$

and
$$R \propto \frac{1}{r^4} \Rightarrow \frac{R_1}{R_2} = \left(\frac{r_2}{r_1}\right)^4$$

where, l is the length of wire and r is the radius of cross-section area of wire.

Resistivity

Resistivity of a material of a conductor is given by

$$\rho = \frac{m}{ne^2\tau}$$

where, n = number of free electrons per unit volume.

Resistivity is low for metals, more for semiconductors and very high for alloys like nichrome, constantan etc.

Resistivity of a material depend on temperature and nature of the material. It is independent of dimensions of the conductor, *i.e.* length, area of cross-section etc.

Temperature Dependence of Resistivity

Resistivity of metals increases with increase in temperature as

$$\rho_t = \rho_0 (1 + \alpha t)$$

where, ρ_0 and ρ_t are resistivity of metals at 0°C and t°C and α = temperature coefficient of resistivity of the material.

For metals α is positive, for some alloys like nichrome, manganin and constantan, α is positive but very low.

For semiconductors and insulators, α is negative.

Therefore, resistivity of metal increases with increase in temperature. However, for semiconductors, it decreases with increase in temperature. But in the case of alloy, dependence on temperature is weak.

In magnetic field, the resistivity of metals increases. But resistivity of ferromagnetic materials such as iron, nickel, cobalt etc decreases in magnetic field.

Electrical Conductivity

The reciprocal of resistivity is called electrical conductivity.

Electrical conductivity $(\sigma) = \frac{1}{\rho} = \frac{l}{RA} = \frac{ne^2\tau}{m}$

Its SI units is $\text{ohm}^{-1}\text{m}^{-1}$ or mho m^{-1} or siemen m^{-1}.

Relation between current density (J) and electrical conductivity **(σ)** is given by $J = \sigma E$

where, E = electric field intensity.

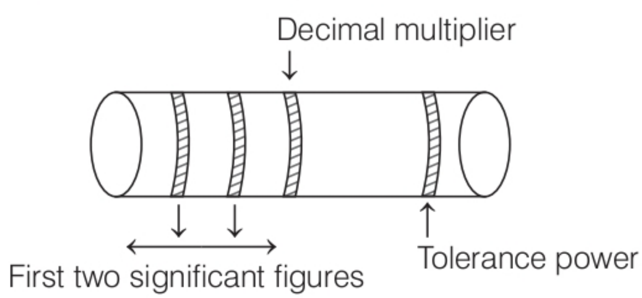
Superconductors

When few metals are cooled, then below a certain critical temperature their electrical resistance suddenly becomes zero. In this state, these substances are called **superconductors** and this phenomena is called **superconductivity.**

Mercury become superconductor at 4.2 K, lead at 7.25 K and niobium at 9.2 K.

Colour Coding of Carbon Resistors

The resistance of a carbon resistor can be calculated by the code given on it in the form of coloured strips.



Colour coding

Colour	Figure	Multiplier
Black	0	1
Brown	1	10^1
Red	2	10^2
Orange	3	10^3
Yellow	4	10^4
Green	5	10^5
Blue	6	10^6
Violet	7	10^7
Grey	8	10^8
White	9	10^9

Tolerance power

Colour	Tolerance
Gold	5%
Silver	10%
No colour	20%

This colour coding can be easily learned in the sequence "B B ROY Great Britain Very Good Wife".

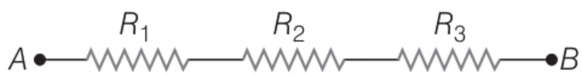
Combination of Resistors

1. In Series

(i) Equivalent resistance, $R = R_1 + R_2 + R_3$

(ii) Current through each resistor is same.

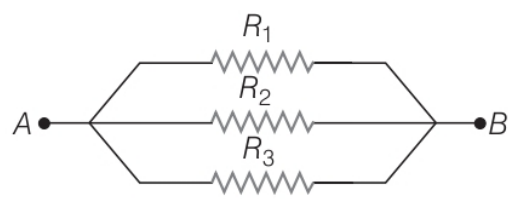
(iii) Sum of potential differences across individual resistors is equal to the potential difference applied by the source.



2. In Parallel

(i) Equivalent resistance,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$



(ii) Potential difference across each resistor is same.

(iii) Sum of electric currents flowing through individual resistors is equal to the electric current drawn from the source.

If n identical resistances are first connected in series and then in parallel, the ratio of the equivalent resistance

$$\frac{R_s}{R_p} = \frac{n^2}{1}$$

If a skeleton cube is made with 12 equal resistors, each having a resistance R, then the net resistance across

(a) the diagonal of cube $= \frac{5}{6}R$

(b) the diagonal of a face $= \frac{3}{4}R$

(c) along a side $= \frac{7}{12}R$

Electric Cell

An electric cell is a device which converts chemical energy into electrical energy.

Electric cells are of two types

(i) **Primary Cells** Primary cells cannot be charged again. Voltaic, Daniel and Leclanche cells are primary cells.

(ii) **Secondary Cells** Secondary cells can be charged again and again. Acid and alkali accumulators are secondary cells.

Electromotive Force (emf) of a Cell

The energy given by a cell in flowing unit positive charge throughout the circuit completely one time is equal to the emf of a cell.

Emf of a cell $(e) = \frac{W}{q}$.

Its SI unit is volt.

Terminal Potential Difference of a Cell

The energy given by a cell in flowing unit positive charge through the outer circuit one time from one terminal of the cell to the other terminal of the cell.

Its SI unit is also volt. It is always less than the emf of a cell.

Internal Resistance of a Cell

The obstruction offered by the electrolyte of a cell in the path of electric current is called internal resistance (r) of the cell. Internal resistance of a cell

(i) increases with increase in concentration of the electrolyte.

(ii) increases with increase in distance between the electrodes.

(iii) decreases with increase in area of electrodes dipped in electrolyte.

Relation between e, V and r

$$e = V + Ir$$

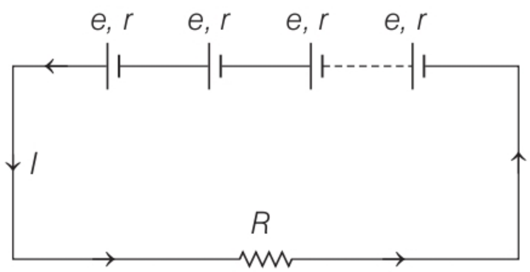
$$r = \left(\frac{e}{V} - 1\right) R$$

If cell is in charging state, then

$$e = V - Ir$$

Grouping of Cells

(i) **In Series** If n cells, each of emf e and internal resistance r are connected in series to a resistance R, then equivalent emf



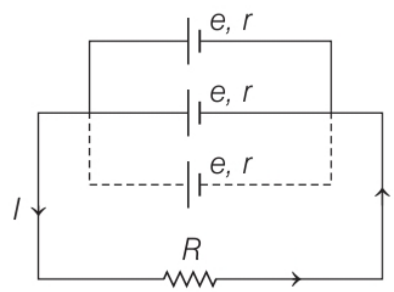
$$e_{eq} = e_1 + e_2 + \ldots + e_n = ne$$

Equivalent internal resistance,

$$r_{eq} = r_1 + r_2 + \ldots + r_n = nr$$

Current in the circuit $I = \dfrac{e_{eq}}{(R + r_{eq})} = \dfrac{ne}{(R + nr)}$

(ii) **In Parallel** If n cells, each of emf e and internal resistance r are connected to in parallel, then equivalent emf, $e_{eq} = e$

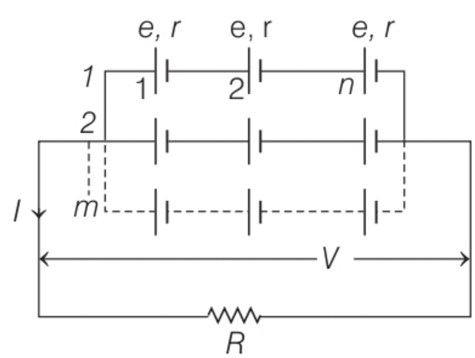


Equivalent internal resistance,

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_1} + \ldots + \frac{1}{r_n} = \frac{n}{r} \quad \text{or} \quad r_{eq} = \frac{r}{n}$$

Current in the circuit, $I = \dfrac{e}{(R + r/n)}$

(iii) **Mixed Grouping of Cells** If n cells, each of emf e and internal resistance r are connected in series and such m rows are connected in parallel, then



Equivalent emf, $e_{\text{eq}} = ne$

Equivalent internal resistance,

$$r_{\text{eq}} = \frac{nr}{m}$$

Current in the circuit, $I = \dfrac{ne}{\left(R + \dfrac{nr}{m}\right)}$ or $I = \dfrac{mne}{mR + nr}$

Note Current in this circuit will be maximum when external resistance is equal to the equivalent internal resistance, i.e.,

$$R = \frac{nr}{m} \Rightarrow mR = nr$$

Kirchhoff's Laws

There are two Kirchhoff's laws for solving complicated electrical circuits

(i) **Junction Rule** The algebraic sum of all currents meeting at a junction in a closed circuit is zero, *i.e.* $\Sigma I = 0$.

This law follows law of conservation of charge.

(ii) **Loop Rule** The algebraic sum of all the potential differences in any closed circuit is zero, *i.e.*

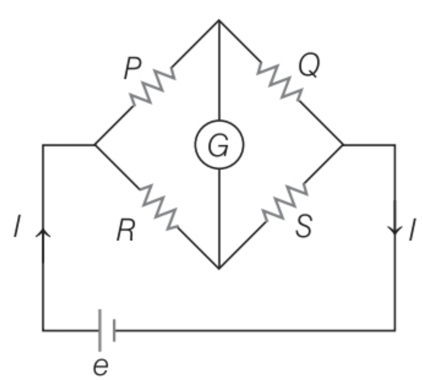
$$\Sigma \Delta V = 0$$

This law follows law of conservation of energy.

Wheatstone Bridge

Wheatstone bridge is also known as a **meter bridge** or **slide wire bridge.**

This is an arrangement of four resistances in which one resistance is unknown but rest are known. The Wheatstone bridge is as shown in figure below.



Principle of Wheatstone Bridge

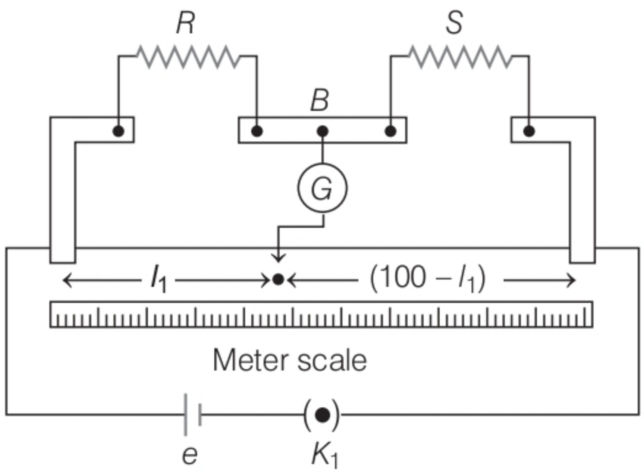
The bridge is said to be balanced when deflection in galvanometer is zero, *i.e.* $I_G = 0$.

Thus, we have the balance condition as $\frac{P}{Q} = \frac{R}{S}$

The value of unknown resistance S can found, as we know the value of P, Q and R. It may be remembered that the bridge is most sensitive, when all the four resistances are of the same order.

Meter Bridge

This is the simplest form of Wheatstone bridge. It is specially useful for comparing resistances more accurately and for measuring an unknown resistance.



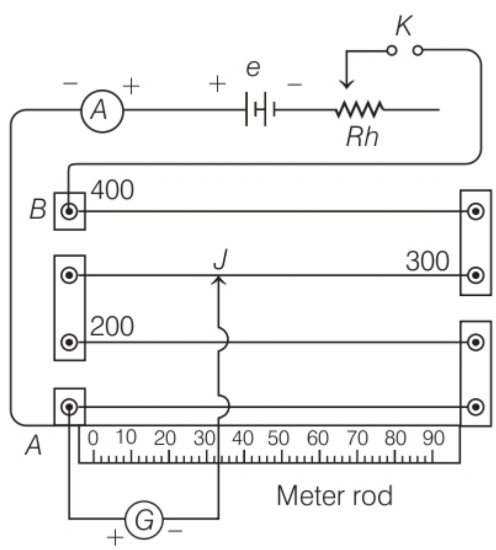
At balancing situation of bridge, $\frac{R}{S} = \frac{l_1}{(100 - l_1)}$

where, l_1 is the length of wire from one end where null point is obtained.

Potentiometer

Potentiometer is an ideal device voltmeter to measure the potential difference between two points or the internal resistance of an unknown source.

It consists of a long resistance wire AB of uniform cross-section in which a steady direct current is set up by means of a battery.



The principle of potentiometer states that, when a constant amount of current flows through a wire of uniform cross-section, then the potential drop across the wire is directly proportional to its length, *i.e.*

$$V \propto l$$

$$\Rightarrow \qquad V = kl$$

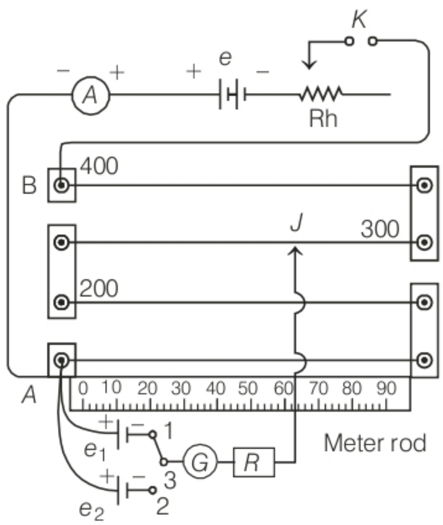
where, k is known as potential gradient.

SI unit of k is Vm^{-1}.

Sensitivity of potentiometer is increased by increasing length of potentiometer wire.

To Compare the emf's of two Cells using Potentiometer

The arrangement of two cells of emfs e_1 and e_2 which are to be compared is shown in the figure below.



If the plug is put in the gap between 1 and 3, we get

$$e_1 = (x\, l_1)\, I \qquad \text{...(i)}$$

where, x = resistance per unit length

Similarly, when the plug is put in the gap between 2 and 3, we get

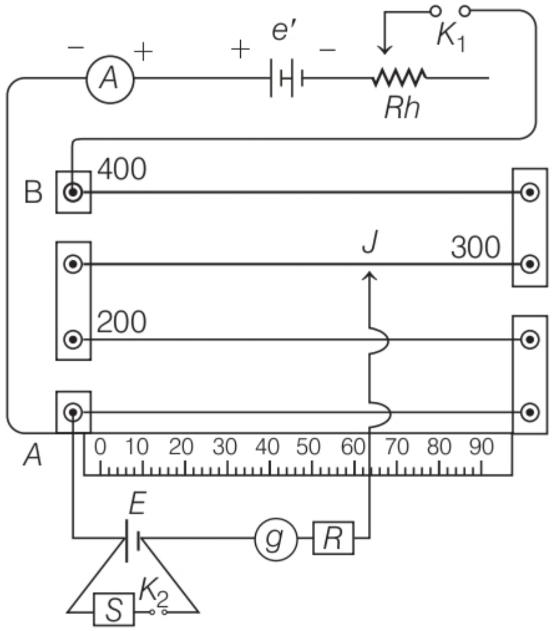
$$e_2 = (x\, l_2)\, I \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

or $$\frac{e_1}{e_2} = \frac{l_1}{l_2}$$

Determination of Internal Resistance of a Cell using Potentiometer

The arrangement is shown in figure.



When K_2 is kept out, $e = xl_1 I$

But if by inserting key K_2 and introducing some resistance S (say), then potential difference V is balanced by a length l_2, where

$$V = kl_2$$

Internal resistance of cell,

$$r = \frac{e - V}{V} R = \frac{l_1 - l_2}{l_2} R$$