Elasticity

Deforming Force

A force which produces a change in configuration of the object on applying it, is called a deforming force.

Elasticity

Elasticity is that property of the object by virtue of which it regain its original configuration after the removal of the deforming force.

Elastic Limit

Elastic limit is the upper limit of deforming force upto which, if deforming force is removed, the body regains its original form completely and beyond which if deforming force is increased the body loses its property of elasticity and get permanently deformed.

Perfectly Elastic Bodies

Those bodies which regain its original configuration immediately and completely after the removal of deforming force are called perfectly elastic bodies. *e.g.* quartz, phospher bronze etc.

Perfectly Plastic Bodies

Those bodies which does not regain its original configuration at all on the removal of deforming force are called perfectly plastic bodies. *e.g.* putty, paraffin, wax etc.

Stress

The internal restoring force acting per unit area of a deformed body is called stress.

$$\text{Stress} = \frac{\text{Restoring force}}{\text{Area}}$$

Its SI unit is N/m^2 or pascal and dimensional formula is $[ML^{-1}T^{-2}]$.

Stress is a tensor quantity.

Stress is of three types :

(i) **Normal Stress** If deforming force is applied normally to an object, then the stress is called normal stress.

If there is an increase in length, then stress is called **tensile stress**.

If there is a decrease in length, then stress is called **compression stress**.

(ii) **Volumetric Stress** If deforming force is applied normally on an object all over its surface, that changes its volume, then the stress is called volumetric stress.

(iii) **Tangential Stress** If deforming force is applied tangentially to an object, then the stress is called tangential stress. It changes the shape of the object.

Strain

The fractional change in configuration is called strain.

$$\text{Strain} = \frac{\text{Change in configuration}}{\text{Original configuration}}$$

It has no unit and it is a dimensionless quantity.

According to the change in configuration, the strain is of three types

(i) Longitudinal strain $= \dfrac{\text{Change in length}}{\text{Original length}}$

(ii) Volumetric strain $= \dfrac{\text{Change in volume}}{\text{Original volume}}$

(iii) Shearing strain = Angular displacement of the plane perpendicular to the fixed surface.

Hooke's Law

Within the limit of elasticity, the stress is proportional to the strain.

$$\text{Stress} \propto \text{Strain}$$

or

$$\text{Stress} = E \times \text{Strain}$$

where, E is the **modulus of elasticity** of the material of the body.

Elastic Moduli

The ratio of stress and strain, called modulus of elasticity or elastic moduli.

Types of Modulus of Elasticity

Modulus of elasticity is of three types

1. Young's Modulus of Elasticity

It is defined as the ratio of normal stress to the longitudinal strain within the elastic limit.

$$Y = \frac{\text{Normal stress}}{\text{Longitudinal strain}}$$

$$Y = \frac{F\Delta l}{Al} = \frac{Mg\,\Delta l}{\pi r^2 l}$$

Its SI unit is N/m^2 or pascal and its dimensional formula is [$ML^{-1}T^{-2}$].

Force Constant of Wire

Force required to produce unit elongation in a wire is called force constant of a material of wire. It is denoted by k

$$k = \frac{YA}{l}$$

where, Y = Young's modulus of elasticity

and A = cross-section area of wire.

2. Bulk Modulus of Elasticity

It is defined as the ratio of volumetric stress to the volumetric strain within the elastic limit.

$$K = \frac{\text{Volumetric stress}}{\text{Volumetric strain}}$$

$$K = -\frac{FV}{A\Delta V} = -\frac{\Delta p V}{\Delta V}$$

where, $\Delta p = F / A$ = Change in pressure.

Negative sign implies that when the pressure increases volume decreases and *vice-versa*.

Its SI unit is N/m^2 or pascal and its dimensional formula is [$ML^{-1}T^{-2}$].

Compressibility

Compressibility of a material is the reciprocal of its bulk modulus of elasticity.

$$\text{Compressibility } (C) = \frac{1}{K}$$

Its SI unit is $N^{-1}m^2$ and CGS unit is dyne^{-1} cm^2.

3. **Modulus of Rigidity** (η) (Shear Modulus)

It is defined as the ratio of tangential stress to the shearing strain, within the elastic limit.

$$\eta = \frac{\text{Tangential stress}}{\text{Shearing strain}}$$

$$\eta = \frac{F}{A\,\theta}$$

Its SI unit is N/m^2 or pascal and its dimensional formula is $[ML^{-1}T^{-2}]$.

Factors affecting Elasticity

(i) Modulus of elasticity of materials decreases with the rise in temperature, except for invar.

(ii) By annealing elasticity of material decreases.

(iii) By hammering or rolling elasticity of material increases.

(iv) Addition of impurities affects elastic properties depending on whether impurities are themselves more or less elastic.

Note

- For liquids, modulus of rigidity is zero.
- Young's modulus (Y) and modulus of rigidity (η) are possessed by solid materials only.

Poisson's Ratio

When a deforming force is applied at the free end of a suspended wire of length l and radius R, then its length increases by dl but its radius decreases by dR. Now two types of strains are produced by a single force.

(i) Longitudinal strain $= \Delta l / l$

(ii) Lateral strain $= -\Delta R / R$

$$\therefore \quad \text{Poisson's ratio } (\sigma) = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{-\Delta R / R}{\Delta l / l}$$

The theoretical value of Poisson's ratio lies between -1 and 0.5.

Its practical value lies between 0 and 0.5.

Relation Between Y, K, η and σ

(i) $Y = 3K(1 - 2\sigma)$

(ii) $Y = 2\eta(1 + \sigma)$

(iii) $\sigma = \dfrac{3K - 2\eta}{2\eta + 6K}$

(iv) $\dfrac{9}{Y} = \dfrac{1}{K} + \dfrac{3}{\eta}$ or $Y = \dfrac{9K\eta}{\eta + 3K}$

Important Points

- For the same material, the three coefficients of elasticity γ, η and K have different magnitudes.
- Isothermal elasticity of a gas $E_T = p$ where, $p =$ pressure of the gas.
- Adiabatic elasticity of a gas $E_S = \gamma p$

  where, $\gamma = \frac{C_p}{C_V}$, ratio of specific heats at constant pressure and at constant volume.
- Ratio between isothermal elasticity and adiabatic elasticity $\frac{E_S}{E_T} = \gamma = \frac{C_p}{C_V}$

Stress and Strain Curve

When a wire is stretched by a load as in Fig. (a), it is seen that for small value of load, the extension produced in the wire is proportional to the load as shown in Fig. (b). Hence,

$$\text{Stress} \propto \text{Strain}$$

Beyond the limit of elasticity, the stress and strain are not proportional to each other, on increasing the load further, the wire breaks at point D, known as feature point.

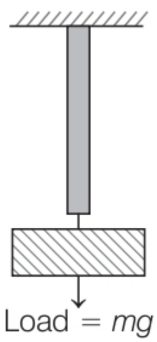
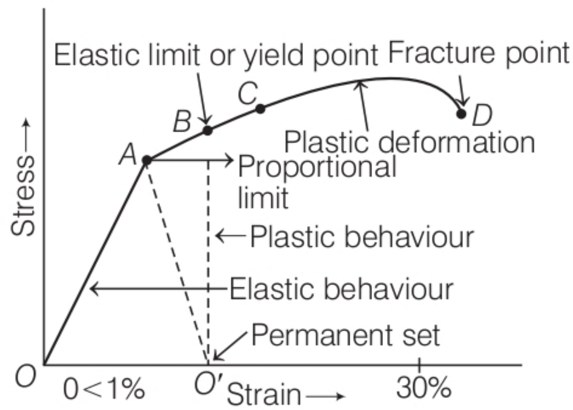


Fig.(a)



Fig(b)

Breaking Stress

The minimum value of stress required to break a wire, is called breaking stress. Breaking stress is fixed for a material but breaking force varies with area of cross-section of the wire.

$$\text{Safety factor} = \frac{\text{Breaking stress}}{\text{Working stress}}$$

Elastic Relaxation Time

The time delay in restoring the original configuration after removal of deforming force is called elastic relaxation time.

For quartz and phospher bronze this time is negligible.

Elastic After Effect

The temporary delay in regaining the original configuration by the elastic body after the removal of deforming force is called elastic after effect.

Elastic Fatigue

The property of an elastic body by virtue of which its behaviour becomes less elastic under the action of repeated alternating deforming force is called elastic fatigue.

Ductile Materials

The materials which show large plastic range beyond elastic limits are called ductile materials. *e.g.* copper, silver, iron, aluminum, etc.

Ductile materials are used for making springs and sheets.

Brittle Materials

The materials which show very small plastic range beyond elastic limits are called brittle materials, *e.g.* glass, cast iron, etc.

Elastomers

The materials for which strain produced is much larger, than the stress applied, within the limit of elasticity are called elastomers. *e.g.* rubber, the elastic tissue of arota, the large vessel carrying blood from heart etc. Elastomers have no plastic range.

Malleability

When a solid is compressed, a stage is reached beyond which it cannot regains its original shape after the deforming force is removed. This quality is called malleability of solid substance.

Elastic hysteresis

As a natural consequence of the elastic after-effect, the strain in the body tends to lag behind the stress applied to the body so that during a rapidly changing stress, the strain is greater for the same value of stress. This lag of strain behind the stress is called elastic hysteresis.

Elastic Potential Energy in a Stretched Wire

The work done in stretching a wire is stored in the form of potential energy of the wire.

Potential energy

$$U = \text{Average force} \times \text{Increase in length} = \frac{1}{2} F\Delta l$$

$$= \frac{1}{2} \text{ Stress} \times \text{Strain} \times \text{Volume of the wire}$$

Elastic potential energy per unit volume

$$U = \frac{1}{2} \times \text{Stress} \times \text{Strain} = \frac{1}{2} (\text{Young's modulus}) \times (\text{Strain})^2$$

Elastic potential energy of a stretched spring $= \frac{1}{2} kx^2$

where, k = Force constant of spring and x = Change in length.

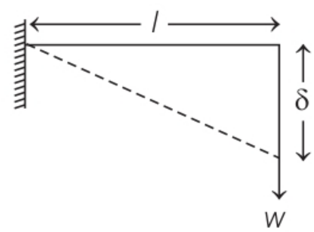
Thermal Stress

When temperature of a rod fixed at its both ends is changed, then the produced stress is called thermal stress.

$$\text{Thermal stress} = \frac{F}{A} = Y\alpha\Delta\theta$$

where, α = Coefficient of linear expansion of the material of the rod.

When temperature of a gas enclosed in a vessel is changed, then the thermal stress produced is equal to change in pressure (Δp) of the gas.



$$\text{Thermal stress} = \Delta p = K\gamma\,\Delta\theta$$

where, K = Bulk modulus of elasticity

and γ = Coefficient of cubical expansion of the gas.

Cantilever

A beam clamped at one end and loaded at free end is called a cantilever. Depression (δ) at the free end of a cantilever is given by

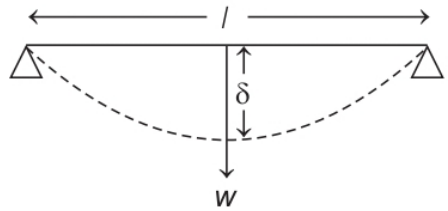
$$\delta = \frac{wl^3}{3YI_G}$$

where, w = Load, l = Length of the cantilever, Y = Young's modulus of elasticity and I_G = Geometrical moment of inertia.

For a beam of rectangular cross-section having breadth b and thicknes d, $I_G = \frac{bd^3}{12}$

For a beam of circular cross-section area having radius r, $I_G = \frac{\pi r^4}{4}$

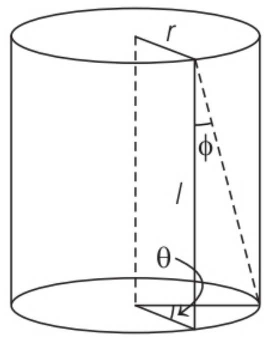
Beam Supported at Two Ends and Loaded at the Middle



Depression at middle, $\delta = \dfrac{wl^3}{48YI_G}$

Torsion of a Cylinder

If the upper end of a cylinder is clamped and a torque is applied at the lower end the cylinder get twisted by angle θ, then



Couple per unit twist, $C = \dfrac{\pi\eta r^4}{2l}$

where, η = Modulus of rigidity of the material of cylinder,

r = Radius of cylinder,

and l = length of cylinder.

Work done in twisting the cylinder through an angle θ

$$W = \frac{1}{2}C\theta^2$$

Relation between angle of twist (θ) and angle of shear (ϕ)

$$r\theta = l\phi$$

or $$\phi = \frac{r}{l}\theta$$