Electrostatics

Charge

Charge is that property of an object by virtue of which it apply electrostatic force of interaction on other objects.

Charges are of two types

(i) Positive charge

(ii) Negative charge

Like charges repel and unlike charges attract each other.

Conductors

Conductors are those substances which can be used to carry or conduct electric charge from one point to other, e.g. silver, copper, aluminium etc.

Insulators

Insulators are those substances which cannot conduct electric charge, e.g. glass, rubber, plastic etc.

Charging by Induction

The process of charging a neutral body by bringing a charged body nearby it without making contact between the two bodies is known as charging by induction.

Basic Properties of Electric Charges

Additivity of Charge

If a system consists of n charges q_1, q_2, q_3 .... q_n, then the total charge of the system will be $q_1 + q_2 + q_3 + q_4 + \ldots\ldots + q_n$.

Quantisation of Charge

Charge on any object can be an integer multiple of a smallest charge (e).

$$Q = \pm ne$$

where, $n = 1, 2, 3, \dots$ and $e = 1.6 \times 10^{-19}$ C.

The protons and neutrons are combination of other entities called quarks, which have charges $\frac{1}{3}$e. However, isolated quarks have not been observed, so, quantum of charge is still e.

Conservation of Charge

Charge can neither be created nor be destroyed, but can be transferred from one object to another object.

Coulomb's Law of Electrostatics

Coulomb's law is a quantitative statement about the force between two point charges. It states that "the force of interaction between any two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them".

Suppose two point charges q_1 and q_2 are separated in vacuum by a distance r, then force between two charges is given by

$$F_e = \frac{K|q_1 q_2|}{r^2}$$

The constant K is usually put as $K = \frac{1}{4\pi\varepsilon_0}$, where ε_0 is called the permittivity of free space and has the value $\varepsilon_0 = 8.854 \times 10^{-12} \text{C}^2/\text{N-m}^2$.

For all practical purposes we will take $\frac{1}{4\pi\varepsilon_0} \simeq 9 \times 10^9 \text{N-m}^2/\text{C}^2$.

If there is another medium between the point charges except air or vacuum, then ε_0 is replaced by $\varepsilon_0 K$ or $\varepsilon_0 \varepsilon_r$ or ε.

Here K or ε_r is called dielectric constant or relative permittivity of the medium.

$$K = \varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

where, ε = permittivity of the medium.

For air or vacuum, $K = 1$

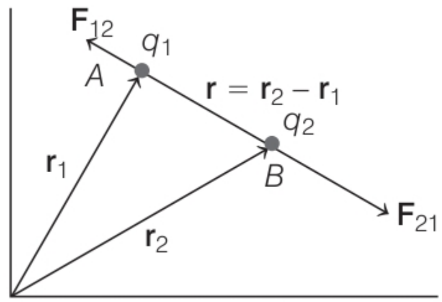
For water, $K = 81$

For metals, $K = \infty$

Coulomb's Law in Vector Form

Let q_1 and q_2 both are positive.

Force on q_2 due to q_1,



$$\mathbf{F}_{21} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}\,\hat{\mathbf{r}}_{21}$$

$$\mathbf{F}_{21} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^3}\,\mathbf{r}_{21}$$

$$\mathbf{F}_{21} = \frac{q_1 q_2}{4\pi\varepsilon_0} \cdot \frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|^3}$$

The above equations give the Coulomb's law in vector form.

Also, force on q_1 due to q_2 is

$$\mathbf{F}_{12} = \frac{q_1 q_2}{4\pi\varepsilon_0} \cdot \frac{\mathbf{r}_1 - \mathbf{r}_2}{|\mathbf{r}_1 - \mathbf{r}_2|^3}$$

$\therefore$ Force on q_1 due to q_2 = – Force on q_2 due to q_1

or $\mathbf{F}_{12} = -\mathbf{F}_{21}$

The forces due to two point charges are parallel to the line joining the point charges; such forces are called **central forces** and so electrostatic forces are **conservative forces**.

Forces between Multiple Charges : Superposition Principle

According to the principle of superposition, "total force on a given charge due to number of charges is the vector sum of individual forces acting on that charge due to the presence of other charges".

Consider a system of n point charges q_1, q_2, q_3 .....q_n distributed in space. Let the charges be q_2, q_3 ...q_n, exert forces $\mathbf{F}_{12}, \mathbf{F}_{13}, \ldots\ldots \mathbf{F}_{1n}$ on charge q_1. The total force on charge q_1 is given by

$$\mathbf{F}_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}^2}\,\hat{\mathbf{r}}_{12} + \frac{q_1 q_2}{r_{13}^2}\,\hat{\mathbf{r}}_{13} + \ldots\ldots + \frac{q_1 q_2}{r_{1n}^2}\,\hat{\mathbf{r}}_{1n}\right)$$

Contiuous Charge Distribution

The region in which charges are closely spaced is said to have continuous distribution of charge. It is of three types; linear charge distribution, surface charge distribution and volume charge distribution.

Linear Charge Density (λ) It is defined as the charge per unit length of linear charge distribution. Its unit is coulomb/metre.

$$\lambda = \frac{dq}{dL}$$

Surface Charge Density (σ) It is defined as the charge per unit surface area of surface charge distribution. Its unit is coulomb/metre^2.

$$\sigma = \frac{dq}{dS}$$

Volume Charge Density (ρ) It is defined as the charge per unit volume of volume charge distribution. Its unit is coulomb/metre^3.

$$\rho = \frac{dq}{dV}$$

Electric Field

The space in the surrounding of any charge in which its influence can be experienced by other charges is called electric field.

Electric Field Intensity (E)

The electrostatic force acting per unit positive charge at a point in electric field is called electric field intensity at that point.

Electric field intensity $\mathbf{E} = \lim_{q_0 \to 0} \frac{\mathbf{F}}{q_0}$

where $\mathbf{F}$ = force experienced by the test charge q_0.

Its SI unit is NC^{-1} or V/m and its dimensions are $[MLT^{-3}A^{-1}]$.

It is a vector quantity and its direction is in the direction of electrostatic force acting on positive charge.

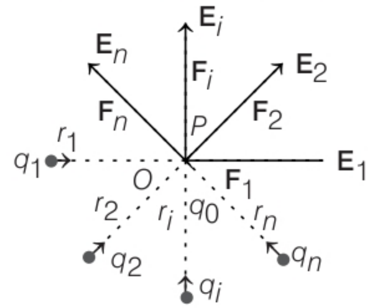
Electric Field due to a Point Charge

Electric field intensity due to a point charge q at a distance r is given by

$$E = \frac{1}{4\pi \varepsilon_0} \frac{q}{r^2}$$

Electric Field due to System of Charges

Electric field **E** at point P due to the systems of charges is given by

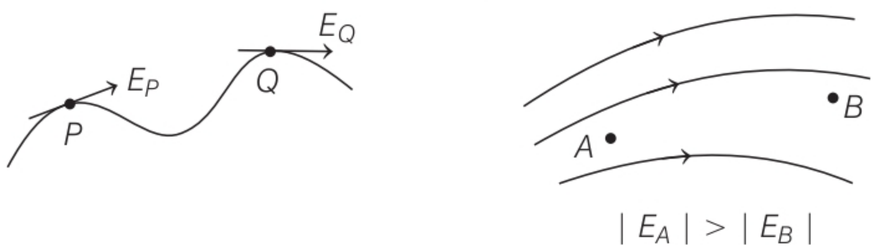


$$\mathbf{E}(r) = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i$$

Electric Field Lines

"An electric field line is an imaginary line or curve drawn through a region of space so that its tangent at any point is in the direction of the electric field vector at that point.

The relative closeness of the lines at some place give an idea about the intensity of electric field at that point."



Properties of Electric Field Lines

(i) Electric field lines (lines of force) start from positive charge and terminate on negative charge.

(ii) Two electric lines of force never intersect each other.

(iii) Electric field lines do not form any closed loops.

(iv) In a charge-free region, electric field lines can be taken to be continuous curves without breaks.

Electric Field Due to Continuous Charge Distribution

(i) Electric field due to the line charge distribution at the location of charge q_0 is

$$\mathbf{E}_L = \frac{1}{4\pi\varepsilon_0} \int_L \frac{\lambda}{r^2}\, dL\, \hat{\mathbf{r}}$$

(ii) Electric field due to the surface charge distribution at the location of charge q_0 is

$$\mathbf{E}_S = \frac{1}{4\pi\varepsilon_0}\int_S \frac{\sigma}{r^2}\, dS\,\hat{\mathbf{r}}$$

(iii) Electric field due to the volume charge distribution at the location of charge q_0 is

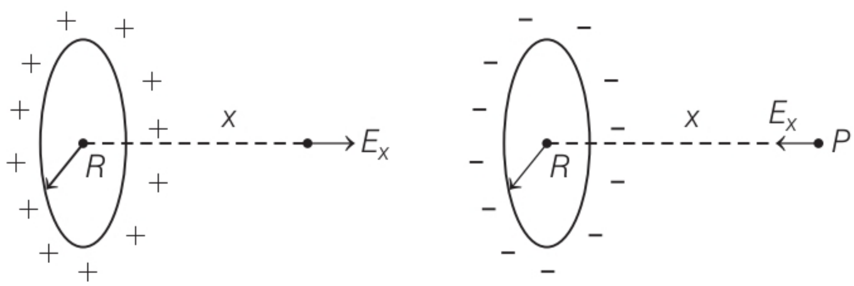
$$\mathbf{E}_V = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho}{r^2}\, dV\hat{\mathbf{r}}$$

Electric Field of a Ring of Charge

Electric field at distance x from the centre of uniformly charged ring of total charge q on its axis is given by

$$E_x = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{qx}{(x^2 + R^2)^{3/2}}$$

Direction of this electric field is along the axis and away from the ring in case of positively charged ring and towards the ring in case of negatively charged ring.



Special cases

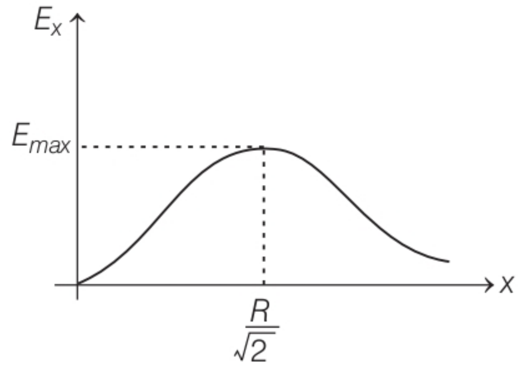
From the above expression, we can see that

(i) $E_x = 0$ at $x = 0$ *i.e.*, **field is zero at the centre of the ring.** This would occur because charges on opposite sides of the ring would push in opposite directions on a test charge at the centre, and the forces would add to zero.

(ii) $E_x = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{x^2}$ for $x \gg R$, *i.e.*, when the point P is much farther from the ring, its field is the same as that of a point charge.

To an observer far from the ring, the ring would appear like a point, and the electric field reflects this.

(iii) E_x will be maximum where $\frac{dE_x}{dx} = 0$. Differentiating E_x w.r.t. x and putting it equal to zero, we get $x = \frac{R}{\sqrt{2}}$ and E_{max} comes out to be, $\frac{2}{\sqrt[3]{3}}\left(\frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^2}\right)$.



Electric Potential (V)

Electric potential at any point is equal to the work done per unit positive charge in carrying it from infinity to that point in electric field.

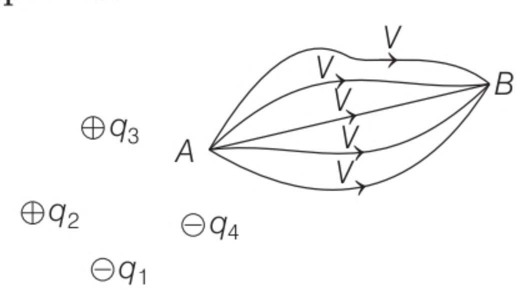
Electric potential, $V = \frac{W}{q}$

Its SI unit is J/C or volt and its dimensions are $[ML^2T^{-3}A^{-1}]$.

It is a scalar quantity.

Electric Potential Difference

The electric potential difference between two points A and B is equal to the work done by the external force in moving a unit positive charge against the electrostatic force from point B to A along any path between these two points.



If V_A and V_B be the electric potential at point A and B respectively, then $\Delta V = V_A - V_B$

or $$\Delta V = \frac{W_{AB}}{q}$$

The SI unit of potential difference is volt (V).

The dimensional formula for electric potential difference is given by $[ML^2T^{-3}A^{-1}]$.

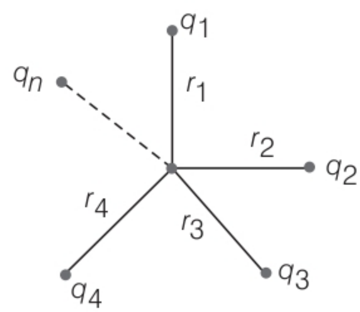
Electric Potential due to a Point Charge

Electric potential due to a point charge at a distance r is given by

$$V = \frac{1}{4\pi \varepsilon_0} \frac{q}{r}$$

Potential due to System of Charges

Let there be n number of point charges $q_1, q_2, q_3, \ldots, q_n$ at distances $r_1, r_2, r_3, \ldots, r_n$ respectively from the point P, where electric potential is given by



$$V = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i}$$

Potential Gradient

The rate of change of potential with distance in electric field is called potential gradient.

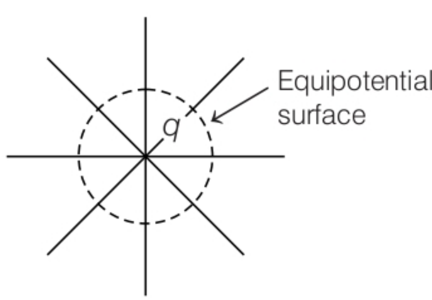
Potential gradient $= \frac{dV}{dr}$. Its unit is V/m.

Relation between potential gradient and electric field intensity is given by

$$E = -\left(\frac{dV}{dr}\right)$$

Equipotential Surface

Equipotential surface is an imaginary surface joining the points of same potential in an electric field. So, we can say that the potential difference between any two points on an equipotential surface is zero. The electric lines of force at each point of an equipotential surface are normal to the surface.



Properties of equipotential surfaces are as follows

(i) Equipotential surface may be planer, solid etc. But equipotential surface can never be point size.

(ii) Electric field is always perpendicular to equipotential surface.

(iii) Equipotential surface due to an isolated point charge is spherical.

(iv) Equipotential surface are planer in an uniform electric field.

(v) Equipotential suface due to a line charge is cylindrical.

Motion of Charged Particle in Electric Field

Consider a charged particle having charge q and mass m is initially at rest in an electric field of strength E. The particle will experience an electric force which causes its motion.

The force experienced by the charged particle is F, where

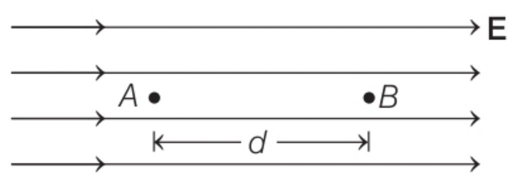
$$F = qE$$

$\therefore$ Acceleration produced by this force is

$$a = \frac{F}{m} = \frac{qE}{m} \quad \text{...(i)}$$

Suppose at point A particle is at rest and after some time, t it reaches the point B and attains velocity v.

$$\therefore \quad v = at$$



If potential difference between A and B be ΔV and the distance between them is d, then

$$v = \frac{qEt}{m} = \sqrt{\frac{2q\Delta V}{m}} \quad \text{... (ii)}$$

As momentum, $p = mv$

$$\therefore \quad p = m\left(\frac{qEt}{m}\right) \quad \text{[from Eq. (ii)]}$$

$$p = qEt$$

$$= m \times \sqrt{\frac{2q\Delta V}{m}} = \sqrt{2mq\Delta V}$$

Kinetic Energy of a Charged Particle

Kinetic energy gained by the particle in time t is

$$K = \frac{1}{2} mv^2 = \frac{1}{2} m \left(\frac{qEt}{m} \right)^2 \quad \text{[from Eq. (ii)]}$$

$$= \frac{q^2 E^2 t^2}{2m} \text{ or } K = \frac{1}{2} m \times \frac{2q\Delta V}{m} = q\Delta V$$

Electric Flux (ϕ_E)

Electric flux over an area is equal to the total number of electric field lines crossing this area.

Electric flux through a small area element dS is given by $\phi_E = \mathbf{E} \cdot d\mathbf{S}$

where, $\mathbf{E}$ = electric field intensity and $d\mathbf{S}$ = area vector.

Its SI unit is N-m^{2}C^{-1}.

Gauss' Theorem

The electric flux over any closed surface is $\frac{1}{\varepsilon_0}$ times the total charge enclosed by that surface, i.e. $\phi_E = \oint_s \mathbf{E} \cdot d\mathbf{S} = \frac{1}{\varepsilon_0} \Sigma q$

If a charge q is placed at the centre of a cube, then

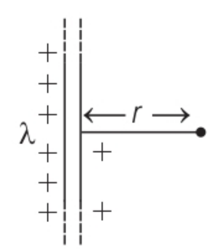
total electric flux linked with the whole cube $= \frac{q}{\varepsilon_0}$

and electric flux linked with one face of the cube $= \frac{q}{6\,\varepsilon_0}$

Applications of Gauss' Theorem

(i) **Electric Field Intensity due to an Infinite Line Charge**

$$E = \frac{1}{2\pi\,\varepsilon_0} \frac{\lambda}{r}$$



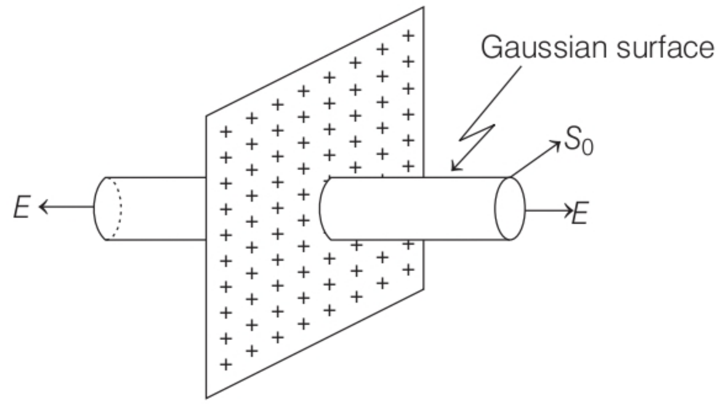
where, λ is linear charge density and r is distance from the line charge.

(ii) **Electric Field Near an Infinite Plane Sheet of Charge**

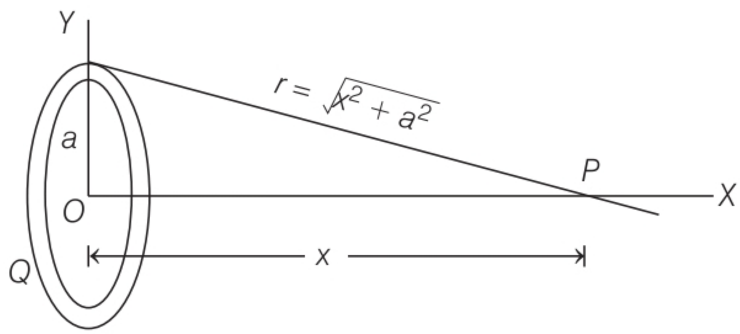
$$E = \frac{\sigma}{2\,\varepsilon_0}$$

where, σ = surface charge density.

If there are two uniformly charged parallel sheets with surface charge densities σ and $-\sigma$, then $E = \frac{\sigma}{\varepsilon_0}$



(iii) **Electric Field and potential at Any Point on the Axis of a Uniformly Charged Ring** A ring-shaped conductor with radius a carries a total charge Q uniformly distributed around it. Let us calculate the electric field at a point P that lies on the axis of the ring at a distance x from its centre.



(a) $E_x = \frac{1}{4\pi\varepsilon_0} \cdot \frac{xQ}{(x^2 + a^2)^{3/2}}$

The maximum value of electric field, $E = \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{3\sqrt{3}a^2}\right)$

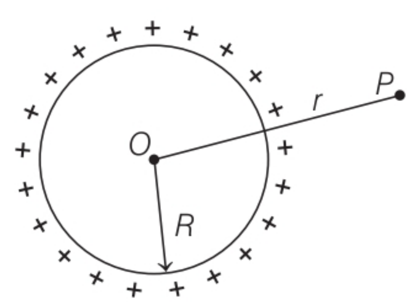
(b) $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{(x^2 + a^2)^{1/2}}$

(iv) **Electric Field and Potential due to a Charged Spherical Shell**

At an extreme point ($r > R$)

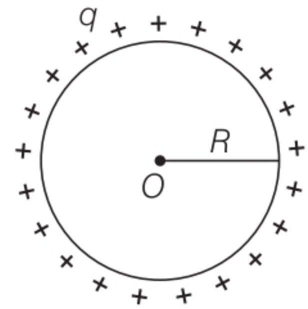
(a) Electric field intensity,

$$E = \frac{1}{4\pi\,\varepsilon_0}\frac{q}{r^2}$$



(b) Electric potential, $V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$

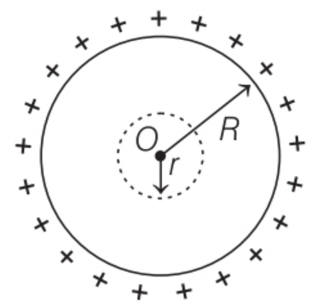
At the surface of a shell $(r = R)$



(a) Electric field intensity, $E = \frac{1}{4\pi\varepsilon_0} \frac{q}{R^2}$

(b) Electric potential, $V = \frac{1}{4\pi\varepsilon_0} \frac{q}{R}$

At an internal point $(r < R)$

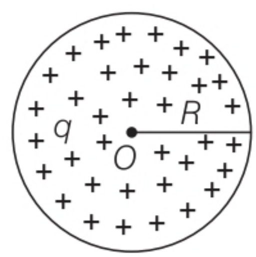


(a) Electric field intensity, $E = 0$

(b) Electric potential, $V = \frac{1}{4\pi\varepsilon_0} \frac{q}{R}$

Therefore, potential inside a charged conducting spherical shell is equal to the potential at its surface.

(v) Electric Field and Potential due to a Charged Non-Conducting Sphere



At an extreme point, $(r > R)$

(a) Electric field intensity $E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$

(b) Electric potential $V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$

On the surface, $(r = R)$

(a) Electric field intensity $E = \frac{1}{4\pi\varepsilon_0} \frac{q}{R^2}$

(b) Electric potential $V = \frac{1}{4\pi\varepsilon_0} \frac{q}{R}$

Inside the sphere, $(r < R)$

(a) Electric field intensity $E = \frac{1}{4\pi\varepsilon_0} \frac{qr}{R^3}$

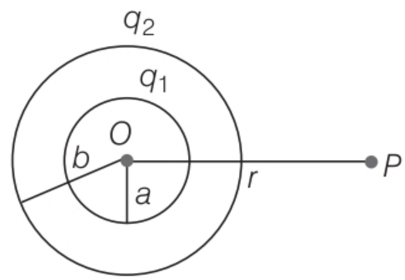
(b) Electric potential $V = \frac{1}{4\pi\varepsilon_0} \frac{q\,(3R^2 - r^2)}{2R^3}$

At the centre of the sphere, $(r = 0)$

(a) Electric field intensity $E = 0$

(b) Electric potential $V = \frac{3}{2} \times \frac{1}{4\pi\varepsilon_0} \frac{q}{R}$

(vi) **Potential due to a Spherical Shell**



(a) At point P, where $OP = r$

$$V = \frac{q_1 + q_2}{4\pi\varepsilon_0 r}$$

(b) At a point, where $a < r < b$

$$V = \frac{q_1}{4\pi\varepsilon_0 r} + \frac{q_2}{4\pi\varepsilon_0 b}$$

(c) At a point, where $r < a$

$$V = \frac{q_1}{4\pi\varepsilon_0 a} + \frac{q_2}{4\pi\varepsilon_0 b}$$

Electric Potential Energy

Electric potential energy of a system of point charges is defined as the total amount of work done in bringing the different charges to their respective positions from infinitely large mutual separations.

It is represented by U. Thus, electric potential can also be written as potential energy per unit charge, *i.e.*,

$$V = \frac{W}{q} = \frac{U}{q}$$

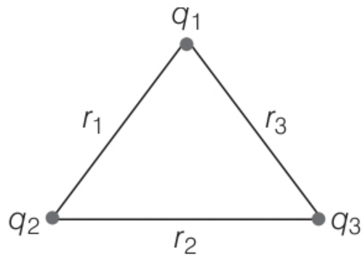
Electric potential energy is defined only in a conservative field.

Potential Energy of Charge System

Potential energy of two point charges system, that contains charges q_1 and q_2 separated by a distance r is given by

$$U = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$$

Three point charges system



$$U = \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{q_1 q_2}{r_1} + \frac{q_2 q_3}{r_2} + \frac{q_3 q_1}{r_3} \right]$$

Potential Energy in an External Field

Potential energy of a single charge q at a point with position vector $\mathbf{r}$ in an external field $= q \cdot V(\mathbf{r})$, where $V(\mathbf{r})$ is the potential at the point due to external electric field $\mathbf{E}$.

For a system of two charges q_1 and q_2, the potential energy is given as

$$U = q_1 \cdot V(\mathbf{r}_1) + q_2 \cdot V(\mathbf{r}_2) + \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$$

where, $q_1, q_2 =$ two point charges at position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$, respectively

$V(\mathbf{r}_1) =$ potential at $\mathbf{r}_1$ due to the external field

and $V(\mathbf{r}_2) =$ potential at $\mathbf{r}_2$ due to the external field.

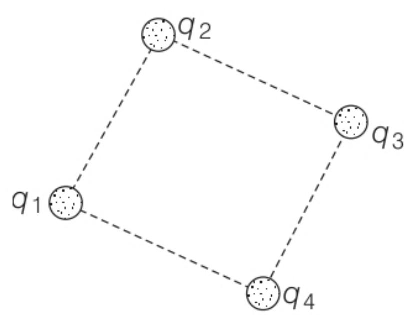
Potential Energy for a Collection of More than Two Charges

The potential energy of a system of n charges is given by

$$U = \frac{K}{2} \sum_{\substack{i,j \\ i \neq j}}^{n} \frac{q_i q_j}{r_{ij}} \qquad \left(\text{here, } K = \frac{1}{4\pi\varepsilon_0}\right)$$

The factor of 1/2 is applied only with the summation sign because on expanding the summation, each pair is counted twice. It is reqresented by U.

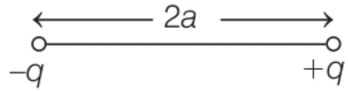
For example electric potential energy of four point charges q_1, q_2, q_3 and q_4 would be given by



$$U = \frac{1}{4\pi\varepsilon_0} \left[\frac{q_4 q_3}{r_{43}} + \frac{q_4 q_2}{r_{42}} + \frac{q_4 q_1}{r_{41}} + \frac{q_3 q_2}{r_{32}} + \frac{q_3 q_1}{r_{31}} + \frac{q_2 q_1}{r_{21}} \right]$$

Electric Dipole

An electric dipole consists of two equal and opposite point charges separated by a very small distance. e.g. a molecule of HCl, a molecule of water etc.



Electric Dipole Moment, $p = q \times 2a$

Its SI unit is 'coulomb-metre' and its dimensions are [LTA].

It is a vector quantity and its direction is from negative charge towards positive charge.

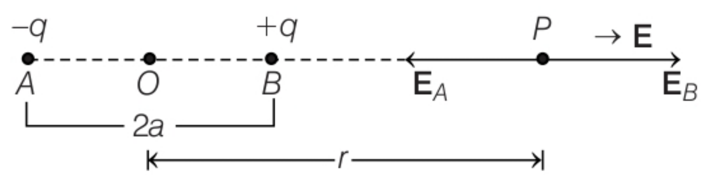
Electric Field Intensity and Potential due to an Electric Dipole

(i) **On Axial Line**

Electric field intensity, $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2pr}{(r^2 - a^2)^2}$

If $r >> 2a$, then $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2p}{r^3}$

Electric potential, $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{p}{(r^2 - a^2)}$



If $r >> 2a$, then $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{p}{r^2}$

(ii) **On Equatorial Line**

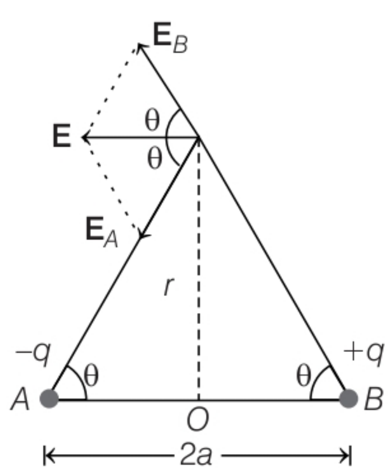
Electric field intensity

$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{(r^2 + a^2)^{3/2}}$$

If $r >> 2a$, then

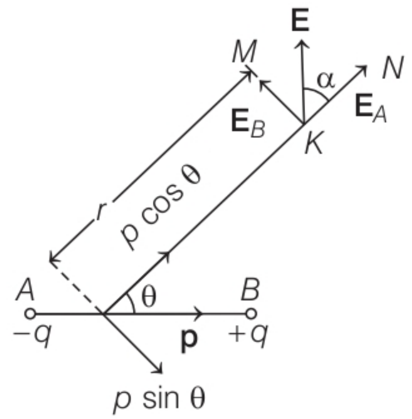
$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$$

Electric potential, $V = 0$



(iii) At any Point along a Line Making θ Angle with Dipole Axis

Electric field intensity $E = \frac{1}{4\pi\varepsilon_0} \frac{p\sqrt{(1+3\cos^2\theta)}}{r^3}$



Electric potential $V = \frac{1}{4\pi\varepsilon_0} \frac{p\cos\theta}{(r^2 - a^2\cos^2\theta)}$

If $r >> 2a$, then $V = \frac{1}{4\pi\varepsilon_0} \frac{p\cos\theta}{r^2}$

Potential Energy of a Dipole in a Uniform Electric Field

The work done in rotating the dipole through small angle $d\theta$, then $dW = \tau d\theta$.

$$dW = -pE\sin\theta d\theta$$

Suppose initially dipole is kept in a uniform electric field at angle θ_1. Now, to turn it through an angle θ_2 (with the field). Then, work done

$$\int_{U_1}^{U_2} dW = \int_{\theta_1}^{\theta_2} pE\sin\theta d\theta$$

$$W = -pE\,[\cos\theta_2 - \cos\theta_1]$$

If $\theta_1 = 0°$ and $\theta_2 = \theta$, *i.e.*, Initially dipole is kept along the field then it turns through θ so work done,

$$W = pE(1 - \cos\theta)$$

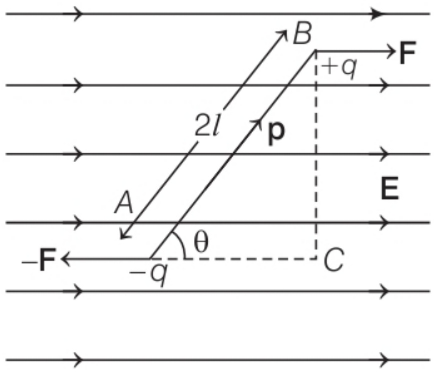
Potential energy of dipole is defined as work done in rotating a dipole from a direction perpendicular to the field to the given direction.

If the dipole is rotated by an angle $\theta_1 = 90°$ to $\theta_2 = \theta$, then energy is given by

$$W = U = pE(\cos 90° - \cos\theta)$$
$$= -pE\cos\theta$$
$$= -\mathbf{p}\cdot\mathbf{E}$$

Torque on Dipole in a Uniform External Field

Torque acting on an electric dipole placed in uniform electric field is given by



$$\tau = (q \times 2l)E \sin\theta = Ep \sin\theta$$

or $$\tau = \mathbf{p} \times \mathbf{E}$$

When $\theta = 90°$, then $\tau_{max} = Ep$

When electric dipole is parallel to electric field, it is in stable equilibrium and when it is anti-parallel to electric field, it is in unstable equilibrium.

Work Done

Work done is rotating an electric dipole in a uniform electric field from angle θ_1 to θ_2 is given by

$$W = Ep(\cos\theta_1 - \cos\theta_2)$$

If initially it is in the direction of electric field, then work done in rotating through an angle θ

$$W = Ep(1 - \cos\theta)$$

Potential Energy

Potential energy of an electric dipole in a uniform electric field in rotating from an angle of θ_1 to θ_2 is given by $U = -pE(\cos\theta_1 - \cos\theta_2)$.

Dipole in Non-uniform Electric Field

When an electric dipole is placed in a non-uniform electric field, then a resultant force as well as a torque act on it.

Net force on electric dipole $= (qE_1 - qE_2)$, along the direction of greater electric field intensity.

Therefore, electric dipole undergoes rotational as well as linear motion.

Behaviour of a Conductor in an Electrostatic Field

(i) Electric field at any point inside the conductor is zero.

(ii) Electric field at any point on the surface of charged conductor is directly proportional to the surface density of charge at that point, but electric potential does not depend upon the surface density of charge.

(iii) Electric potential at any point inside the conductor is constant and equal to potential on its surface.

Electrostatic Shielding

The process of protecting certain region from external electric field is called electrostatic shielding.

Electrostatic shielding is achieved by enclosing that region in a closed hollow metallic chamber.

Lightning Conductor

When a charged cloud passes by a tall building, the charge on the cloud passes to the earth through the building. This causes huge damage to the building. Thus to protect the tall building from lightning, the lightning conductors, (which are pointed metal rods) are installed at the top of these buildings. They help in passing over the charge on the clouds to earth, thus protecting the building.

Dielectric

Dielectrics are insulating (non-conducting) materials that can produce electric effect without conduction.

Dielectrics are of two types

Non-polar Dielectric

The non-polar dielectrics (like N_2, O_2, benzene, methane etc.) are made up of non-polar atoms/molecules, in which the centre of positive charge coincides with the centre of negative charge of the atom/molecule.

Polar Dielectric

The polar dielectric (like H_2O, CO_2, NH_3 etc) are made up of polar atoms/molecules, in which the centre of positive charge does not coincide with the centre of negative charge of the atom.

Dielectric Constant (K)

The ratio of the strength of the applied electric field to the strength of the reduced value of electric field on placing the dielectric between the plates of a capacitor is the dielectric constant. It is denoted by K (or ε_r). $K = \frac{E_0}{E}$

Polarisation (P) and Electric Susceptibility (χ_e)

The induced dipole moment developed per unit volume in a dielectric slab on placing it in an electric field is called polarisation. It is denoted by P.

$$P = \chi_e E$$

where, χ_e is known as electric susceptibility of the dielectric medium.

It is a dimensionless constant.It describes the electrical behaviour of a dielectric. It has different values for different dielectrics.

For vacuum, $\chi = 0$. Relation between dielectric constant and electric susceptibility can be given as

$$\boxed{K = 1 + \chi}$$

Capacitor

A capacitor is a device which is used to store huge charge over it, without changing its dimensions.

It is a pair of two conductors of any shape, close to each other and have equal and opposite charges.

Capacitance of a conductor, $C = \frac{q}{V}$

Its SI unit is coulomb/volt or farad.

Its other units are $1\,\mu\text{F} = 10^{-6}\,\text{F}$

$$1\,\mu\mu\text{F} = 1\,\text{pF} = 10^{-12}\,\text{F}$$

Its dimensional formula is $[\text{M}^{-1}\text{L}^{-2}\text{T}^{4}\text{A}^{2}]$.

When an earthed conductor is placed near a charged conductor, then it decreases its potential and therefore more charge can be stored over it.

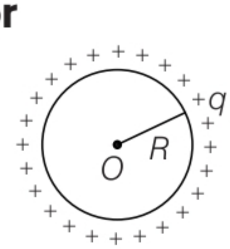
Capacitance of an Isolated Spherical Conductor

$$C = 4\pi\varepsilon_0 K R$$

For air $K = 1$

$\therefore$

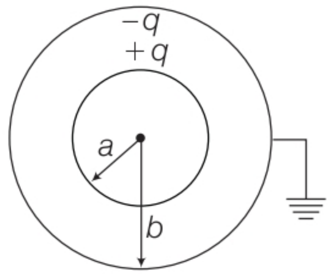
$$C = 4\pi\varepsilon_0 R = \frac{R}{9 \times 10^9}$$



Capacity of a Spherical Conductor Enclosed by an Earthed Concentric Spherical Shell

It consists of two concentric conducting spheres of radii a and b ($a < b$). Inner sphere is given charge q while outer sphere is earthed. Potential difference between the spheres is given by

$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) \quad \text{...(i)}$$



Hence, the capacitance of this system will be

$$C = \frac{q}{V}$$

or $$C = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right) \quad \text{[From Eq. (i)]}$$

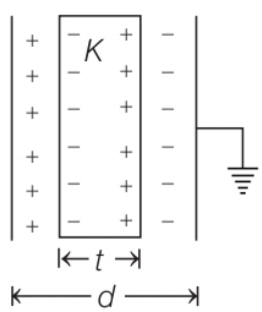
Parallel Plate Capacitor

The parallel plate capacitor consists of two metal plates parallel to each other and separated by a distance d.

Capacitance $$C = \frac{KA\varepsilon_0}{d}$$

For air $$C_0 = \frac{A\varepsilon_0}{d}$$

When a dielectric slab is inserted between the plates partially, then its capacitance



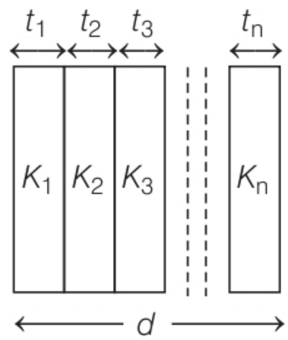
$$C = \frac{A\varepsilon_0}{\left(d - t + \frac{t}{K}\right)}$$

If a conducting (metal) slab is inserted between the plates, then

$$C = \frac{A\varepsilon_0}{(d-t)}$$

When more than one dielectric slabs are placed fully between the plates, then

$$C = \frac{A\varepsilon_0}{\left(\frac{t_1}{K_1} + \frac{t_2}{K_2} + \frac{t_3}{K_3} + \ldots + \frac{t_n}{K_n}\right)}$$



The plates of a parallel plate capacitor attract each other with a force,

$$F = \frac{Q^2}{2A\varepsilon_0}$$

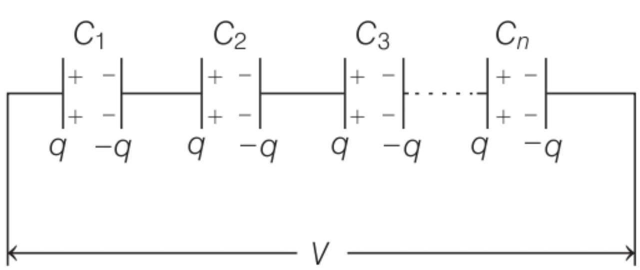
When a dielectric slab is placed between the plates of a capacitor than charge induced on its side due to polarization of dielectric is

$$q' = q\left(1 - \frac{1}{K}\right)$$

Capacitors Combinations

(i) **In Series**

Resultant capacitance, $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$



In series, charge is same on each capacitor, which is equal to the charge supplied by the source.

If $V_1, V_2, V_3, \ldots$ are potential differences across the plates of the capacitors then total voltage applied by the source

$$V = V_1 + V_2 + V_3 + \ldots$$

(ii) **In Parallel**

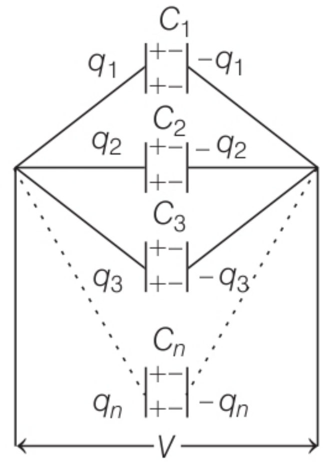
Resultant capacitance

$$C = C_1 + C_2 + C_3 + \dots$$

In parallel, potential differences across the plates of each capacitor is same.

If $q_1, q_2, q_3, \dots$ are charges on the plate of capacitors connected in parallel, then total charge given by the source

$$q = q_1 + q_2 + q_3 + \dots$$



Potential Energy Stored in a Capacitor

Electric potential energy of a charged conductor or a capacitor is given by

$$U = \frac{1}{2} Vq = \frac{1}{2} CV^2 = \frac{1}{2} \frac{q^2}{C}$$

Energy density between the plates

The energy stored per unit volume of space in a capacitor is called energy density. Charge on either plate of capacitor is

$$Q = \sigma A = \varepsilon_0 EA$$

Energy stored in the capaitor is

$$U = \frac{Q}{2C} = \frac{(\varepsilon_0 EA)^2}{2 \cdot \varepsilon_0 A/d} = \frac{1}{2} \varepsilon_0 E^2 \cdot Ad$$

Energy density, $$u = \frac{\text{Energy stored}}{\text{Volume of capacitor}} = \frac{U}{Ad}$$

$\therefore$ $$u = \frac{1}{2} \varepsilon_0 E^2$$

Redistribution of Charge

When two isolated charged conductors are connected to each other then charge is redistributed in the ratio of their capacitances.

Common potential $$V = \frac{q_1 + q_2}{C_1 + C_2} = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

Energy loss $$= \frac{1}{2} \frac{C_1 C_2 (V_1 - V_2)^2}{(C_1 + C_2)}$$

This energy is lost in the form of heat in connecting wires.

When n small drops, each of capacitance C, charged to potential V with charge q, surface charge density σ and potential energy U coalesce to form a single drop.

Then for new drop,

Total charge $= nq$

Total capacitance $= n^{1/3}C$

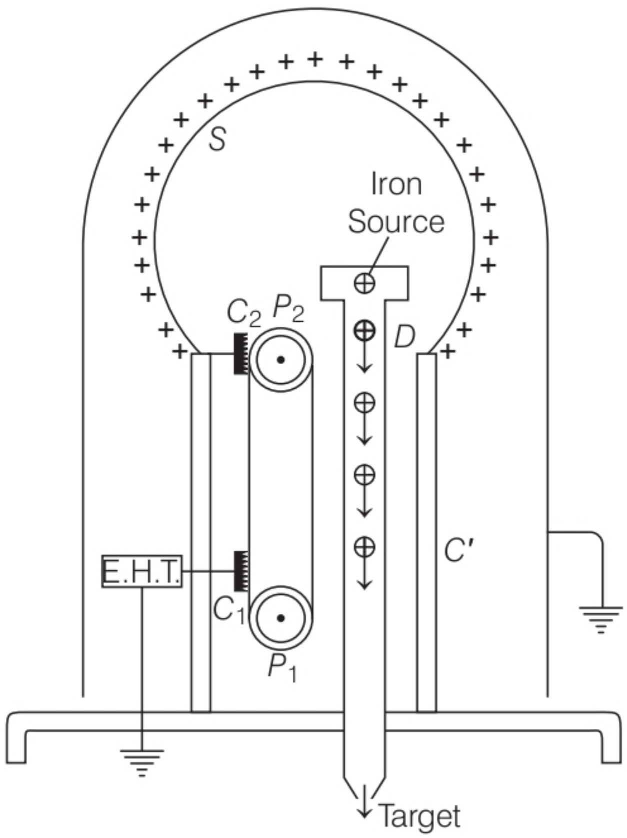
Total potential $= n^{2/3}\ V$

Surface charge density $= n^{1/3}\ \sigma$

Total potential energy $= n^{2/3}\ U$

Van-de-Graff Generator

It is a device used to build up very high potential difference of the order of few million volt.



Its working is based on two points

(i) The action of sharp points (corona discharge)

(ii) Total charge given to a spherical shell resides on its outer surface.