Hydrostatics

Fluids

Fluids are those substances which can flow when an external force is applied on them.

Liquids and gases are fluids.

The key property of fluids is that they offer very little resistance to shear stress. Hence, fluids do not have finite shap but takes the shape of the containing vessel.

In fluid mechanics, the following properties of fluid would be considered

(i) When the fluid is at rest– **hydrostatics**

(ii) When the fluid is in motion– **hydrodynamics**

Thrust

The total normal force exerted by liquid at rest on a given surface is called **thrust** of liquid.

The SI unit of thrust is newton.

Pressure

Pressure of liquid at a point is $p = \frac{\text{Thrust}}{\text{Area}} = \frac{F}{A}$.

Pressure is a scalar quantity, SI unit is Nm^{-2} and its dimensional formula $[\text{ML}^{-1}\text{T}^{-2}]$.

Pressure Exerted by the Liquid

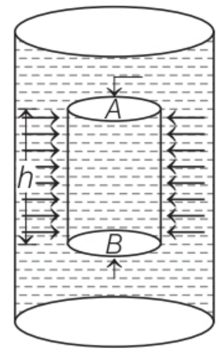
The normal force exerted by a liquid per unit area of the surface in contact is called **pressure of liquid** or **hydrostatic pressure.**

Pressure exerted by a liquid column, $p = h\rho g$

where, h = height of liquid column, ρ = density of liquid
and g = acceleration due to gravity.

Mean pressure on the walls of a vessel containing liquid upto height h is $\left(\frac{h\rho g}{2}\right)$.

Variation of Pressure with Depth



Consider a fluid at rest having density ρ (roh) contained in a cylindrical vessel as shown in figure. Let the two points A and B separated by a vertical distance h.

The pressure p at depth below the surface of a liquid open is given by

Pressure, $$p = p_a + h\rho g$$

where, ρ = density of liquid and g = acceleration due to gravity.

Atmospheric Pressure

The pressure exerted by the atmosphere on earth is called **atmospheric pressure.**

It is equivalent to a weight of 10 tones on 1 m^2.

At sea level, atmospheric pressure is equal to 76 cm of mercury column. Then, atmospheric pressure

$$= hdg = 76 \times 13.6 \times 980 \text{ dyne/cm}^2$$

$$= 0.76 \times 13.6 \times 10^3 \times 9.8 \text{ N/m}^2$$

Thus, 1 atm $= 1.013 \times 10^5$ Nm^{-2} (or Pa)

The atmospheric pressure does not crush our body because the pressure of the blood flowing through our circulatory system is balanced by this pressure.

Atmospheric pressure is also measured in torr and bar.

$$1 \text{ torr} = 1 \text{ mm of mercury column}$$

$$1 \text{ bar} = 10^5 \text{ Pa}$$

Aneroid barometer is used to measure atmospheric pressure.

Pressure measuring devices are open tube manometer, tyre pressure gauge, sphygmomanometer etc.

Gauge Pressure

Gauge pressure at a point in a fluid is the difference of total pressure at that point and atmospheric pressure.

Hydrostatic Paradox

The liquid pressure at a point is independent of the quantity of liquid but depends upon the depth of point below the liquid surface. This is known as hydrostatic paradox.

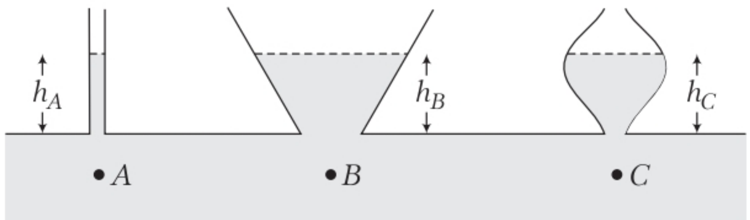
Important Points Related with Fluid Pressure

Important points related with fluid pressure are given below

(i) At a point in the liquid column, the pressure applied on it is same in all directions.

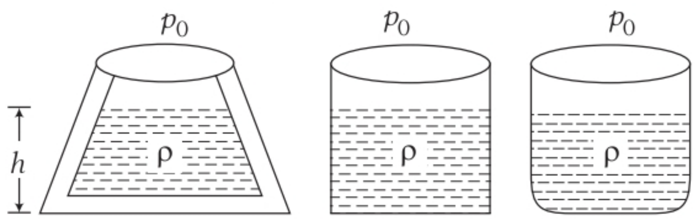
(ii) In a liquid, pressure will be same at all points at the same level.

(iii) The pressure exerted by a liquid depends only on the height of fluid column and is independent of the shape of the containing vessel.



If $h_A = h_B = h_C$, then $p_A = p_B = p_C$

(iv) Consider following shapes of vessels



Pressure at the base of each vessel

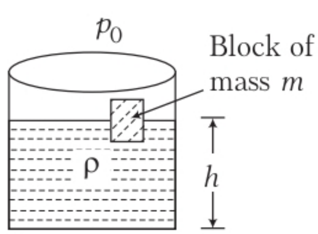
$$p_x = p_y = p_z = p_0 + \rho g h \text{ but } w_x \neq w_y \neq w_z$$

where, ρ = density of liquid in each vessel,

h = height of liquid in each vessel

and p_0 = atmospheric pressure.

(v) In the figure, a block of mass 'm' floats over a fluid surface



If ρ = density of the liquid
A = area of the block

Pressure at the base of the vessel in $p = p_0 + \rho g h + \dfrac{mg}{A}$

Buoyancy

When a body is partially or fully immersed in a fluid, an upward force acts on it, which is called buoyant force, the phenomena is called buoyancy.

The buoyant force acts at the centre of gravity of the liquid displaced by the immersed part of the body and this point is called the centre of buoyancy. The magnitude of buoyant force, $F = v\rho g$.

Pascal's Law

The increase in pressure at a point in the enclosed liquid in equilibrium is transmitted equally in all directions in liquid and to the walls of the container.

The working of hydraulic lift and hydraulic brakes arebased on Pascal's law.

Archimedes' Principle

When a body is partially or fully immersed in a liquid, it loses some of its weight and it is equal to the weight of the liquid displaced by the immersed part of the body. If a is loss of weight of a body in water and b is loss of weight in another liquid, then

$$\frac{a}{b} = \frac{w_{\text{air}} - w_{\text{liquid}}}{w_{\text{air}} - w_{\text{water}}}$$

If T is the observed weight of a body of density σ when it is fully immersed in a liquid of density ρ, then real weight of the body

$$w = \frac{T}{\left(1 - \frac{\rho}{\sigma}\right)}$$

If w_1 = weight of body in air, w_2 = weight of body in liquid,
V_i = immersed of volume of liquid,
ρ_L = density of liquid and g = acceleration due to gravity

$$\Rightarrow \qquad V_i = \frac{w_1 - w_2}{\rho_L g}$$

Laws of Floatation

A body will float in a liquid, if the weight of the body is equal to the weight of the liquid displaced by the immersed part of the body.

If W is the weight of the body and w is the buoyant force, then

(a) If $W > w$, then body will sink to the bottom of the liquid.

(b) If $W < w$, then body will float partially submerged in the liquid.

(c) If $W = w$, then body will float in liquid if its whole volume is just immersed in the liquid.

The floating body will be in stable equilibrium, if meta-centre (centre of buoyancy) lies vertically above the centre of gravity of the body.

The floating body will be in unstable equilibrium, if meta-centre (centre of buoyancy) lies vertically below the centre of gravity of the body. The floating body will be in neutral equilibrium, if meta-centre (centre of buoyancy) coincides with the centre of gravity of the body.

Fraction of volume of a floating body outside the liquid

$$\left(\frac{V_{\text{out}}}{V}\right) = \left[1 - \frac{\rho}{\sigma}\right]$$

where, ρ = density of body and σ = density of liquid

If two different bodies A and B are floating in the same liquid, then

$$\frac{\rho_A}{\rho_B} = \frac{(v_{\text{in}})_A}{(v_{\text{in}})_B}$$

If the same body is made to float in different liquids of densities σ_A and σ_B respectively, then

$$\frac{\sigma_A}{\sigma_B} = \frac{(V_{\text{in}})_B}{(V_{\text{in}})_A}$$

Density and Relative Density

Density of a substance is defined as the ratio of its mass to its volume.

$$\text{Density of a liquid} = \frac{\text{Mass}}{\text{Volume}}$$

$$\text{Density of water} = 1\ \text{g/cm}^3 \text{ or } 10^3\ \text{kg/m}^3$$

In case of homogneous (isotropic) substance it has no directional properties, so it is scalar quantity and its dimensional formula is $[ML^{-3}]$.

Relative density of a substance is defined as the ratio of its density to the density of water at 4°C.

$$\text{Relative density} = \frac{\text{Density of substance}}{\text{Density of water at } 4°\text{C}}$$

$$= \frac{\text{Weight of substance in air}}{\text{Loss of weight in water}}$$

Relative density also known as specific gravity has no unit, no dimensions.

For a solid body, density of body = density of substance.

While for a hollow body, density of body is lesser than that of substance.

When immiscible liquids of different densities are poured in a container, the liquid of highest density will be at the bottom while that of lowest density at the top and interfaces will be plane.

Density of a Mixture of Substances

- When two liquids of masses m_1 and m_2 having densities ρ_1 and ρ_2 are mixed together, then density of mixture is

$$\rho = \frac{m_1 + m_2}{\left(\frac{m_1}{\rho_1}\right) + \left(\frac{m_2}{\rho_2}\right)} = \frac{\rho_1\rho_2 (m_1 + m_2)}{(m_1\rho_2 + m_2\rho_1)}$$

- When two liquids of same mass m but of different densities ρ_1 and ρ_2 are mixed together, then density of mixture is $\rho = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2}$.
- When two liquids of same volume V but of different densities ρ_1 and ρ_2 are mixed together, then density of mixture is $\rho = \frac{\rho_1 + \rho_2}{2}$.

  Density of a liquid varies with pressure, $\rho = \rho_0 \left[1 + \frac{\Delta p}{K}\right]$

  where, ρ_0 = initial density of the liquid, K = bulk modulus of elasticity of the liquid and Δp = change in pressure.
- With rise in temperature (ΔT) due to thermal expansion of a given body, volume will increase while mass will remains constant, so density will decrease $\rho = \frac{\rho_0}{(1 + \gamma \cdot \Delta T)} \simeq \rho_0 (1 - \gamma \cdot \Delta T)$; where γ is volumetric expansion.

Hydrodynamics

Flow of Liquid

(i) **Streamline Flow** The flow of liquid in which each of its particle follows the same path as followed by the preceding particles is called streamline flow.

Two streamlines cannot cross each other and the greater the crowding of streamlines at a place, the greater is the velocity of liquid at that place and *vice-versa*.

(ii) **Laminar Flow** The steady flow of liquid over a horizontal surface in the form of layers of different velocities is called laminar flow.

The laminar flow is generally used synonymously with streamline flow of liquid.

(iii) **Turbulent Flow** The flow of liquid with a velocity greater than its critical velocity is disordered and called turbulent flow.

In case of turbulent flow, maximum part of external energy is spent for producing eddies in the liquid and small part of external energy is available for forward flow.

Reynold's Number

Reynold's number is a pure number. It is equal to the ratio of the inertial force per unit area to the viscous force per unit area for a flowing fluid.

or Reynold number, $K = \dfrac{\text{Inertial force}}{\text{Force of viscosity}} = \dfrac{v_c \rho r}{\eta}$

where, v_c = critical velocity.

For pure water flowing in a cylindrical pipe, K is about 1000.
When $0 < K < 2000$, the flow of liquid is streamlined.

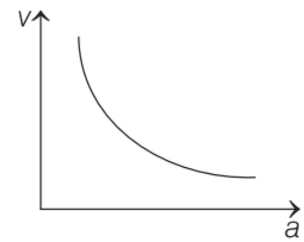
When $2000 < K < 3000$, the flow of liquid is variable between streamlined and turbulent.

When $K > 3000$, the flow of liquid is turbulent.
It has no unit and dimension.

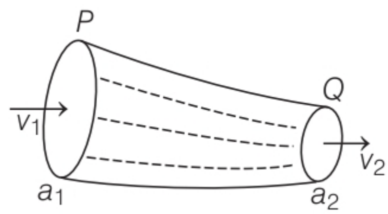
Equation of Continuity

If a liquid is flowing in streamline flow in a pipe of non-uniform cross-sectional area, then rate of flow of liquid across any cross-section remains constant.

i.e. $a_1v_1 = a_2v_2 \Rightarrow av = \text{constant}$ or $a \propto \dfrac{1}{v}$



The velocity of liquid is slower where area of cross-section is larger and faster where area of cross-section is smaller.



The falling stream of water becomes narrower, as the velocity of falling stream of water increases and therefore its area of cross-section decreases. Deep water appears still because it has large cross-sectional area.

Energy of a Liquid

A liquid in motion possess three types of energy

(i) **Pressure Energy**

Pressure energy per unit mass $= \dfrac{p}{\rho}$

where, p = pressure of the liquid
and ρ = density of the liquid.
Pressure energy per unit volume = p

(ii) **Kinetic Energy**

Kinetic energy per unit mass $= \dfrac{1}{2}v^2$

Kinetic energy per unit volume $= \dfrac{1}{2}\rho v^2$

(iii) **Potential Energy**

Potential energy per unit mass $= gh$

Potential energy per unit volume $= \rho gh$

Bernoulli's Theorem

If an ideal liquid is flowing in streamlined flow, then total energy, *i.e.* sum of pressure energy, kinetic energy and potential energy per unit volume of the liquid remains constant at every cross-section of the tube.

Mathematically, $p + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$

It can be expressed as, $\frac{p}{\rho g} + \frac{v^2}{2g} + h = \text{constant}$

where, $\frac{p}{\rho g} =$ pressure head, $\frac{v^2}{2g} =$ velocity head

and $h =$ gravitational head or potential head.

For horizontal flow of liquid, $p + \frac{1}{2}\rho v^2 = \text{constant}$

where, p is called static pressure and $\frac{1}{2}\rho v^2$ is called dynamic pressure.

Therefore in horizontal flow of liquid, if p increases, v decreases and *vice-versa*.

This theorem is applicable to ideal liquid, *i.e.* a liquid which is non-viscous incompressible and irrotational.

Applications of Bernoulli's Theorem

(i) The action of carburetor, paintgun, scent sprayer, atomiser and insect sprayer is based on Bernoulli's theorem.

(ii) The action of Bunsen's burner, gas burner, oil stove and exhaust pump is also based on Bernoulli's theorem.

(iii) Motion of a spinning ball (Magnus effect) is based on Bernoulli's theorem.

(iv) Blowing of roofs by wind storms, attraction between two closely parallel moving boats, fluttering of a flag etc are also based on Bernoulli's theorem.

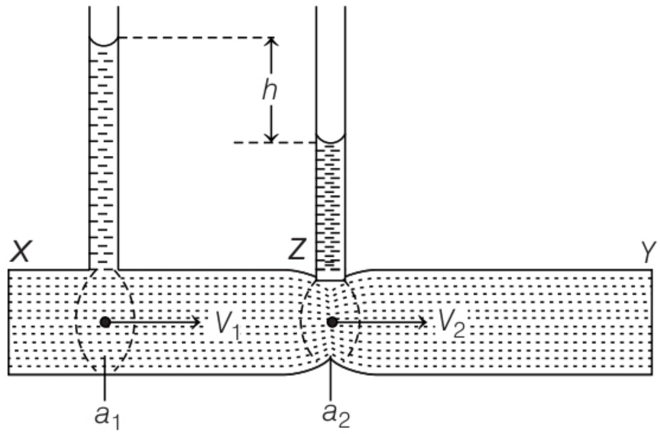
(v) Bernoulli's theorem helps in explaining blood flow in artery.

(vi) Working of an aeroplane is based on Bernoulli's theorem.

Venturimeter

It is a device used for measuring the rate of flow of liquid through pipes. Its working is based on Bernoulli's theorem.

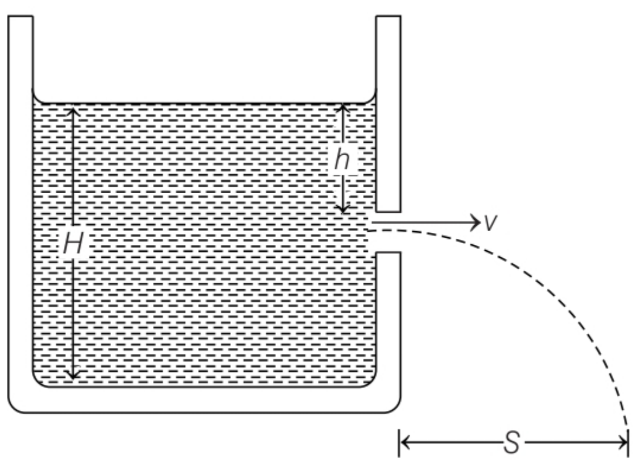
Rate of flow of liquid, $v = a_1 a_2 \sqrt{\dfrac{2gh}{a_1^2 - a_2^2}}$



where, a_1 and a_2 are area of cross-sections of tube at broader and narrower part and h is difference of liquid columns in vertical tubes.

Torricelli's Theorem

Velocity of efflux (the velocity with which the liquid flows out of a orifice or narrow hole) is equal to the velocity acquired by a freely falling body through the same vertical distance equal to the depth of orifice below the free surface of liquid.



Velocity of efflux, $v = \sqrt{2gh}$

where, h = depth of orifice below the free surface of liquid.

Time taken by the liquid to reach the base-level

$$t = \sqrt{\frac{2(H-h)}{g}}$$

$$\text{Horizontal range, } S = \sqrt{4h(H-h)}$$

where, H = height of liquid column.

Horizontal range is maximum, equal to height of the liquid column H, when orifice is at half of the height of liquid column.

If the hole is at the bottom of the tank, then time required to make the tank empty is

$$t = \frac{A}{A_0}\sqrt{\frac{2H}{g}}$$

where, A is area of the container and A_0 is area of orifice.

Volume of liquid coming out from the orifice per second

$$= VA_0 = A_0\sqrt{2gh}$$

Viscosity

The property of a fluid by virtue of which an internal frictional force acts between its different layers which opposes their relative motion is called **viscosity.**

These internal frictional force is called **viscous force.**

Viscous forces are intermolecular forces acting between the molecules of different layers of liquid moving with different velocities.

$$\text{Viscous force } (F) = -\eta A \frac{dv}{dx}$$

or

$$\eta = -\frac{F}{A\left(\frac{dv}{dx}\right)}$$

where, $\frac{dv}{dx}$ = rate of change of velocity with distance called velocity gradient, A = area of cross-section and η = coefficient of viscosity.

SI unit of η is Nsm^{-2} or pascal-second or decapoise. Its dimensional formula is $[ML^{-1}T^{-1}]$.

The knowledge of the coefficient of viscosity of different oils and its variation with temperature helps us to select a suitable lubricant for a given machine.

The cause of viscosity in liquid is due to cohesive force between liquid molecules, while in gases, it is due to diffusion.

Viscosity is due to transport of momentum. The value of viscosity (and compressibility) for ideal liquid is zero.

The viscosity of air and of some liquids is utilised for damping the moving parts of some instruments.

The knowledge of viscosity of some organic liquids is used in determining the molecular weight and shape of large organic moleculars like proteins and cellulose.

In any layer of liquid, the pulling of lower layers backwards while upper layers forward direction is known as laminar flow.

Variation of Viscosity

The viscosity of liquids decreases with increase in temperature

$$\eta_t = \frac{\eta_0}{(1 + \alpha t + \beta t^2)}$$

where, η_0 and η_t are coefficient of viscosities at 0°C and t°C, α and β are constants.

The viscosity of gases increases with increase in temperatures as

$$\eta \propto \sqrt{T}$$

The viscosity of liquids increases with increase in pressure but the viscosity of water decreases with increase in pressure.

The viscosity of gases increases with increase of temperature because when temperature of gas increases, then rate of diffusion increases.

Poiseuille's Formula

The rate of flow (v) of liquid through a horizontal pipe for steady flow is given by

$$v = \frac{\pi}{8} \frac{pr^4}{\eta l}$$

where, p = pressure difference across the two ends of the tube, r = radius of the tube, η = coefficient of viscosity and l = length of the tube.

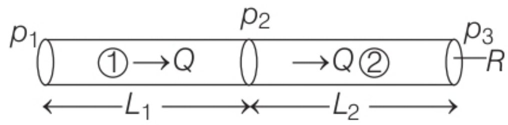
Rate of Flow of Liquid

Rate of flow of liquid through a tube is given by

$$v = \frac{p}{R}$$

where, $R = \frac{8\eta l}{\pi r^4}$ called liquid resistance and p = liquid pressure.

(i) **When two tubes are connected in series**



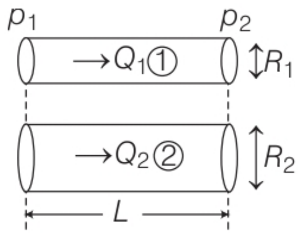
(a) Resultant pressure difference, $p = p_1 + p_2$.

(b) Rate of flow of liquid (v) is same through both tubes.

(c) Equivalent liquid resistance,

$$R = R_1 + R_2.$$

(ii) **When two tubes are connected in parallel**



(a) Pressure difference (p) is same across both tubes.

(b) Rate of flow of liquid $v = v_1 + v_2$.

(c) Equivalent liquid resistance $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$.

Stoke's Law and Terminal Velocity

When a small spherical body falls in a long liquid column, then after sometime it falls with a constant velocity, called **terminal velocity.**

When a small spherical body falls in a liquid column with terminal velocity, then viscous force acting on it is

$$F = 6\pi\eta\, rv$$

where, r = radius of the body, v = terminal velocity and η = coefficient of viscosity.

This is called **Stoke's law.**

Terminal velocity, $v = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$

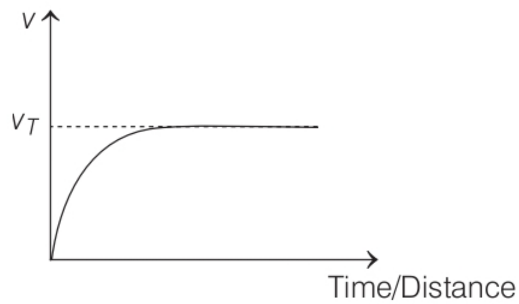
where, ρ = density of body,
σ = density of liquid,
η = coefficient of viscosity of liquid
and g = acceleration due to gravity.

(i) If $\rho > \sigma$, the body falls downwards.

(ii) If $\rho < \sigma$, the body moves upwards with the constant velocity.

(iii) If $\sigma << \rho$, $v = \dfrac{2r^2\rho g}{9\eta}$.

- Terminal velocity depends on the radius of the sphere in such a way that, if radius becomes n times, then terminal velocity will become n^2 times.
- Terminal velocity-Time/distance graph



Importance of Stoke's Law

(i) This law is used in the determination of electronic charge by Millikan in his oil drop experiment.

(ii) This law helps a man coming down with the help of parachute.

(iii) This law accounts for the formation of clouds.

Critical Velocity

The critical velocity is the velocity of liquid flow, below which its flow is streamlined and above which it becomes turbulent.

$$\text{Critical velocity, } v_c = \frac{K\eta}{r\rho}$$

where, K = Reynold's number,

η = coefficient of viscosity of liquid

r = radius of capillary tube

and ρ = density of the liquid.

Surface Tension

Surface tension is the property of any liquid by virtue of which it tries to minimise its free surface area.

Surface tension of a liquid is measured as the force acting per unit length on an imaginary line drawn tangentially on the free surface of the liquid.

Surface tension, $S = \frac{\text{Force}}{\text{Length}} = \frac{F}{l} = \frac{\text{Work done}}{\text{Change in area}}$

Its SI unit is Nm^{-1} or Jm^{-2} and its dimensional formula is $[MT^{-2}]$.

It is a scalar quantity. Surface tension is a molecular phenomenon which is due to cohesive force.

Surface tension of a liquid depends only on the nature of liquid and is independent of the surface area of film or length of the line considered.

Small liquid drops are spherical due to the property of surface tension.

Adhesive Force

The force of attraction acting between the molecules of different substances is called adhesive force, *e.g.* the force of attraction acting between the molecules of paper and ink, water and glass etc.

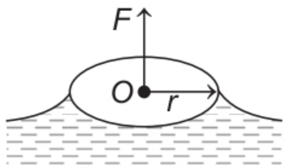
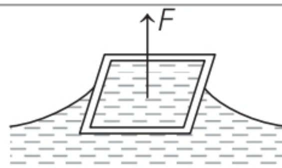
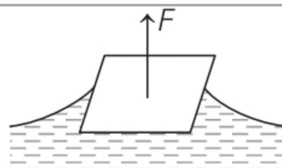
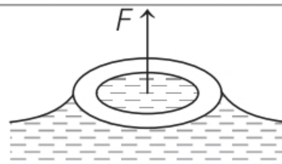
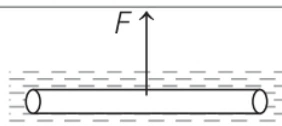
Cohesive Force

The force of attraction acting between the molecules of same substance is called cohesive force, *e.g.* the force of attraction acting between the molecules of water, glass, etc.

Cohesive forces and adhesive forces are van der Waals' forces.

These forces vary inversely as the eighth power of distance between the molecules.

Force of Surface Tension on Different Shape

Shape	Figure	Force of Surface tension
1. Thin ring of radius r		$F = 2\pi(r + r) \cdot S + W$ $= 4\pi\, r \cdot S + W$
2. Circular plate or disc of radius r		$F = 2\pi r \cdot S + W$
3. Square frame of side a		$F = 8a \cdot S + W$
4. Square plate of side a		$F = 4aS + W$
5. Hollow disc of inner radius r_1 and outer radius r_2		$F = 2\pi\,(r_1 + r_2) \cdot S + W$
6. Wire of length l		$F = 2 \cdot l \cdot S + W$

Molecular Range

The maximum distance upto which a molecule can exert a force of attraction on other molecules is called molecular range.

Molecular range is different for different substances.

In solids and liquids, it is of the order of 10^{-9} m.

If the distance between the molecules is greater than 10^{-9} m, the force of attraction between them is negligible.

Factors Affecting Surface Tension

(i) Surface tension of a liquid decreases with increase in temperature and becomes zero at critical temperature.

(ii) At boiling point, surface tension of a liquid becomes zero and becomes maximum at freezing point.

(iii) Surface tension decreases when partially soluble impurities such as soap, detergent, dettol, phenol etc are added in water.

(iv) Surface tension increases when highly soluble impurities such as salt is added in water.