Gravitation

Each object in the universe attracts every other object with a force, which is called the force of **gravitation.**

Gravitation is one of the four classes of interactions found in nature. These are

(i) the gravitational force
(ii) the electromagnetic force
(iii) the strong nuclear force (also called the hadronic force).
(iv) the weak nuclear forces.

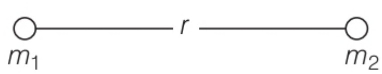
Gravity is the force by which earth attracts the body towards its centre.

Although, of negligible importance in the interactions of elementary particles, gravity is of primary importance in the interactions of large objects. It is gravity that holds the universe together.

Newton's Law of Gravitation

Gravitational force is a attractive force between two masses m_1 and m_2 separated by a distance r.

The gravitational force acting between two point objects is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.



Gravitational force, $F = \dfrac{Gm_1m_2}{r^2}$

where, G is universal gravitational constant.

The value of G is 6.67×10^{-11} N-m^2 kg^{-2} and is same throughout the universe.

The value of G is independent of the nature and size of the bodies as well as the nature of the medium between them.

Dimensional formula of G is $[M^{-1}L^3T^{-2}]$.

Note Newton's law of gravitation holds good for object lying at very large distances and also at very short distances. It fails when the distance between the objects is less than 10^{-9} m *i.e.* of the order of intermolecular distances.

Important Points about Gravitational Force

(i) Gravitational force is a central as well as conservative force.

(ii) It is the weakest force in nature.

(iii) It is 10^{36} times smaller than electrostatic force and 10^{38} times smaller than nuclear force.

(iv) The law of gravitational is applicable for all bodies, irrespective of their size, shape and position.

(v) Gravitational force acting between sun and planet provide it centripetal force for orbital motion.

(vi) Newton's third law of motion holds good for the force of gravitation. It means the gravitational forces between two bodies are action-reaction pairs.

Following three points are important regarding the gravitational force

(i) Unlike the electrostatic force, it is independent of the medium between the particles.

(ii) It is conservative in nature.

(iii) It expresses the force between two point masses (of negligible volume). However, for external points of spherical bodies the whole mass can be assumed to be concentrated at its centre of mass.

Central Forces

Central force is that force which acts along the line joining towards the centres of two interacting bodies. A central force is always directed towards the centre as away from a fixed point.

Acceleration Due to Gravity

The uniform acceleration produced in a freely falling object due to the gravitational pull of the earth is known as **acceleration due to gravity**. It is denoted by g and its SI unit is m/s^2. It is a vector quantity and its direction is towards the centre of the earth.

The value of g is independent of the mass of the object which is falling freely under gravity.

The value of g changes slightly from place to place. The value of g is taken to be 9.8 m/s^2 for all practical purposes. The value of acceleration due to gravity on the moon is about one sixth of that on the earth and on the sun is about 27 times of that on the earth.

Among the planets, the acceleration due to gravity is minimum on the mercury.

Relation between g and G is given by, $g = \frac{GM}{R^2}$

where, M = mass of the earth $= 6.4 \times 10^{24}$ kg

and R = radius of the earth $= 6.38 \times 10^6$ m.

Gravitational mass M_g is defined by Newton's law of gravitation.

$$M_g = \frac{F_g}{g} = \frac{w}{g} = \frac{\text{Weight of body}}{\text{Acceleration due to gravity}}$$

$$\therefore \quad \frac{(M_1)_g}{(M_2)_g} = \frac{F_{g_1} g_2}{F_{g_2} g_1}$$

Inertial mass (= Force/Acceleration) and gravitational mass are equal to each other in magnitude.

Inertial Mass and Gravitational Mass

(a) Inertial mass $= \frac{\text{Force}}{\text{Acceleration}}$

(b) Gravitational mass $= \frac{\text{Weight of body}}{\text{Acceleration due to gravity}}$

(c) They are equal to each other in magnitude.

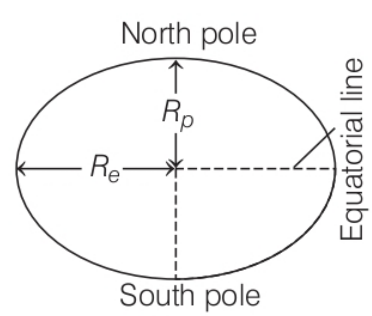
(d) Gravitational mass of a body is affected by the presence of other bodies near it. Inertial mass of a body remains unaffected by the presence of other bodies near it.

Factors Affecting Acceleration Due to Gravity

(i) **Shape of Earth** Acceleration due to gravity $g \propto \frac{1}{R^2}$.

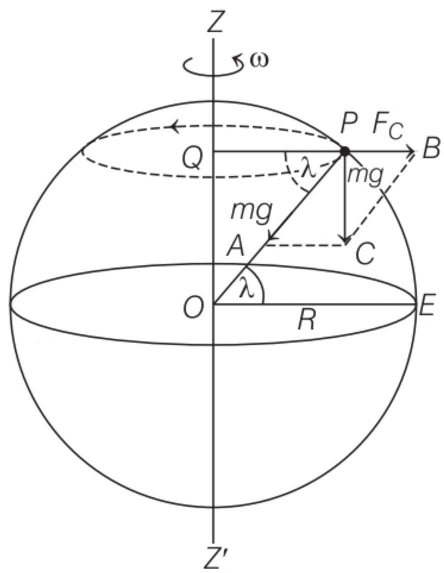
Earth is elliptical in shape. Its diameter at poles is approximately 42 km less than its diameter at equator.

Therefore, g is minimum at equator and maximum at poles.



(ii) **Rotation of Earth about Its Own Axis** If ω is the angular velocity of rotation of earth about its own axis, then acceleration due to gravity at a place having latitude λ is given by

$$g' = g - R\omega^2 \cos^2 \lambda$$



At poles $\lambda = 90°$ and $g' = g$.

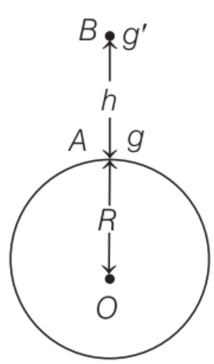
Therefore, there is no effect of rotation of earth about its own axis at poles.

At equator $\lambda = 0°$ and $g' = g - R\omega^2$

The value of g is minimum at equator.

If earth stops its rotation about its own axis, then g will remain unchanged at poles but increases by $R\omega^2$ at equator.

(iii) **Effect of Altitude** The value of g at height h from earth's surface



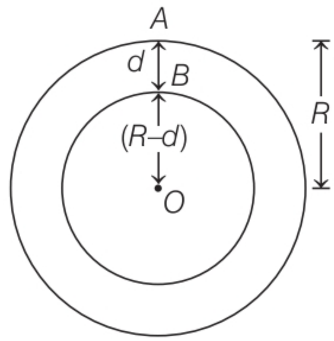
$$g' = \frac{g}{\left(1 + \frac{h}{R}\right)^2}$$

For $h << R$

$$g' = g\left(1 - \frac{2h}{R}\right)$$

Therefore, g decreases with altitude.

(iv) **Effect of Depth** The value of g at depth h from earth's surface



$$g' = g\left(1 - \frac{h}{R}\right)$$

Therefore, g decreases with depth from earth's surface.

The value of g becomes zero at earth's centre.

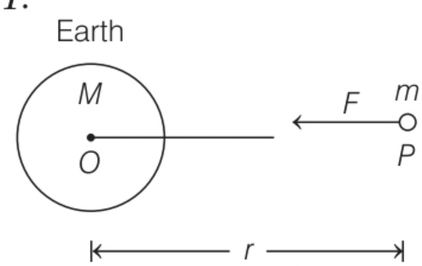
Gravitational Field

The space in the surrounding of any body in which its gravitational pull can be experienced by other bodies is called gravitational field.

Intensity of Gravitational Field

The gravitational force acting per unit mass at any point in gravitational field is called intensity of gravitational field at that point.

It is denoted by E_g or I.



$$E_g \quad \text{or} \quad I = \frac{F}{m}$$

Intensity of gravitational field at a distance r from a body of mass M is given by

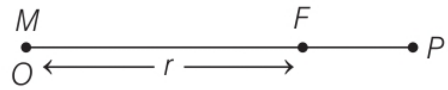
$$E_g \quad \text{or} \quad I = \frac{GM}{r^2}$$

It is a vector quantity and its direction is towards the centre of gravity of the body. Its SI unit is N/m and its dimensional formula is $[LT^{-2}]$.

Gravitational Field Intensity for Different Bodies

1. **Intensity due to a Point Mass**

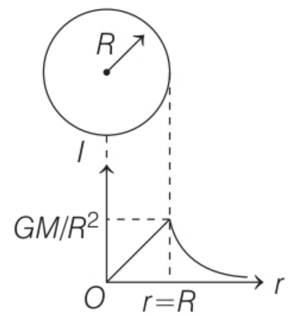
Suppose a point mass M is placed at point O, then gravitational field intensity due to this point mass at point P is given by



$$I = \frac{GM}{r^2}$$

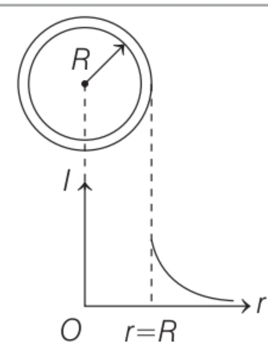
2. **Intensity due to Uniform Solid Sphere**

Outside the surface $r > R$	On the surface $r = R$	Inside the surface $r < R$
$I = \frac{GM}{r^2}$	$I = \frac{GM}{R^2}$	$I = \frac{GMr}{R^3}$



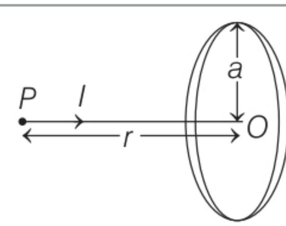
3. **Intensity due to Spherical Shell**

Outside the surface $r > R$	On the surface $r = R$	Inside the surface $r < R$
$I = \frac{GM}{r^2}$	$I = \frac{GM}{R^2}$	$I = 0$



4. **Intensity due to Uniform Circular ring**

At a point on its axis	At the centre of the ring
$I = \frac{GMr}{(a^2 + r^2)^{3/2}}$	$I = 0$



Gravitational Potential

Gravitational potential at any point in gravitational field is equal to the work done per unit mass in bringing a very light body from infinity to that point.

It is denoted by V_g.

Gravitational potential, $V_g = \frac{W}{m} = -\frac{GM}{r}$

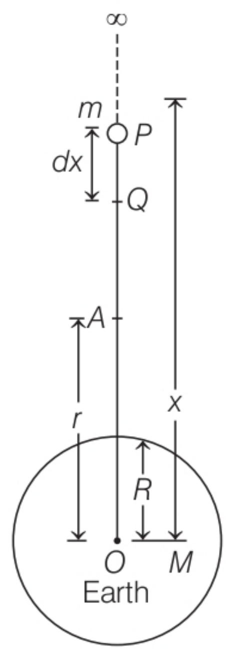
Its SI unit is J/kg and it is a scalar quantity.

Its dimensional formula is $[L^2 T^{-2}]$.

Since, work W is obtained, *i.e.* it is negative, the gravitational potential is always negative.

Gravitational Potential Energy

Gravitational potential energy of any object at any point in gravitational field is equal to the work done in bringing it from infinity to that point. It is denoted by U.



Gravitational potential energy, $U = -\frac{GMm}{r}$

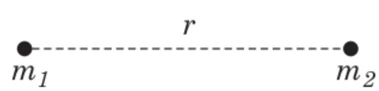
The negative sign shows that the gravitational potential energy decreases with increase in distance.

Gravitational potential energy at height h from surface of earth

$$U_h = -\frac{GMm}{R+h} = \frac{mgR}{1+\frac{h}{R}}$$

Gravitational Potential Energy of a Two Particle System

The gravitational potential energy of two particles of masses m_1 and m_2 separated by a distance r is given by $U = -\frac{Gm_1m_2}{r}$



Gravitational Potential Energy for a System of More than Two Particles

The gravitational potential energy for a system of particles (say m_1, m_2, m_3 and m_4) is given by

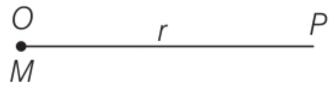
$$U = -G\left[\frac{m_4m_3}{r_{43}} + \frac{m_4m_2}{r_{42}} + \frac{m_4m_1}{r_{41}} + \frac{m_3m_2}{r_{32}} + \frac{m_3m_1}{r_{31}} + \frac{m_2m_1}{r_{21}}\right]$$

Thus, for a n particle system there are $\frac{n(n-1)}{2}$ pairs and the potential energy is calculated for each pair and added to get the total potential energy of the system.

Gravitational Potential for Different Bodies

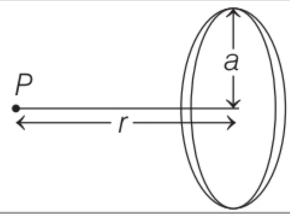
1. Potential due to a Point Mass

Suppose a point mass M is situated at a point O, the gravitational potential due to this mass at point P is given by $V = -\dfrac{GM}{r}$



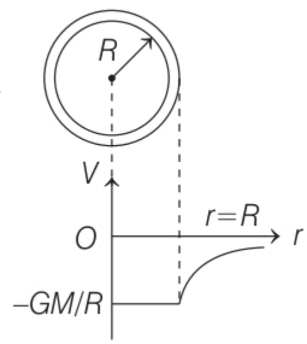
2. Potential due to Uniform Ring

At a point on its axis	At the centre
$V = -\dfrac{GM}{\sqrt{a^2 + r^2}}$	$V = -\dfrac{GM}{a}$



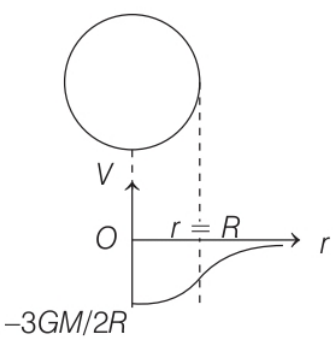
3. Potential due to Spherical Shell

Outside the surface $r > R$	On the surface $r = R$	Inside the surface $r < R$
$V = \dfrac{-GM}{r}$	$V = \dfrac{-GM}{R}$	$V = \dfrac{-GM}{R}$



4. Potential due to Uniform Solid Sphere

Outside the surface $r > R$	On the surface $r = R$	Inside the surface $r < R$
$V = \dfrac{-GM}{r}$	$V_{\text{surface}} = \dfrac{-GM}{R}$	$V = \dfrac{-GM}{2R}\left[3 - \left(\dfrac{r}{R}\right)^2\right]$ At the centre ($r = 0$) $V_{\text{centre}} = \dfrac{-3}{2}\dfrac{GM}{R}$ max $= \dfrac{3}{2}V$ surface



Relation between Gravitational Field and Potential

If change in gravitation potential at a points is dV, gravitational field intensity is E, then during displacement $d\mathbf{r}$ in the field

$$dV = -\mathbf{E} \cdot d\mathbf{r}$$

where, $\mathbf{E} = E_x \hat{\mathbf{i}} + E_y \hat{\mathbf{j}} + E_z \hat{\mathbf{k}}$

$$d\mathbf{r} = dx\,\hat{\mathbf{i}} + dy\,\hat{\mathbf{j}} + dz\,\hat{\mathbf{k}}$$

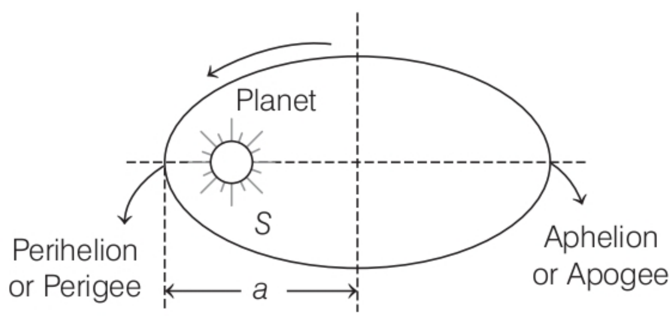
$\therefore$

$$dV = -E_x dx - E_y dy - E_z dz$$

Also we can write $E_x = \dfrac{-\partial V}{\partial x}$, $E_y = \dfrac{-\partial V}{\partial y}$ and $E_z = \dfrac{-\partial V}{\partial z}$

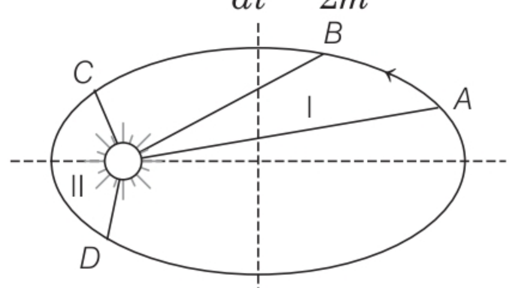
Kepler's Laws of Planetary Motion

(i) **Law of orbit** Every planet revolves around the sun in an elliptical orbit and sun is at its one focus.



(ii) **Law of area** The radius vector drawn from the sun to a planet sweeps out equal areas in equal intervals of time, i.e. the areal velocity of the planet around the sun is constant.

Areal velocity of a planet $\dfrac{d\mathbf{A}}{dt} = \dfrac{\mathbf{L}}{2m} = \text{constant}$



where, L = angular momentum and m = mass of the planet.

(iii) **Law of period** The square of the time period of revolution of a planet around the sun is directly proportional to the cube of semi-major axis of its elliptical orbit.

$$T^2 \propto a^3 \quad \text{or} \quad \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{a_1}{a_2}\right)^3$$

where, a = semi-major axis of the elliptical orbit.

Satellite

A heavenly object which revolves around a planet is called a satellite.

Natural satellites are those heavenly objects which are not man made and revolves around the earth. Artificial satellites are those heavenly objects which are man made and launched for some purposes and revolve around the earth.

Time period of satellite, $T = \dfrac{2\pi r}{\sqrt{\dfrac{GM}{r}}}$,

Here, r = radius of orbital of satellite.

After simplifying, $T = 2\pi\sqrt{\dfrac{r^3}{GM}} = \dfrac{2\pi}{R}\sqrt{\dfrac{(R+h)^3}{g}}$ $\left[\because g = \dfrac{GM}{R^2}\right]$

where, R = radius of earth,

and h = height of satellite above surface of earth.

Near the earth surface, time period of the satellite

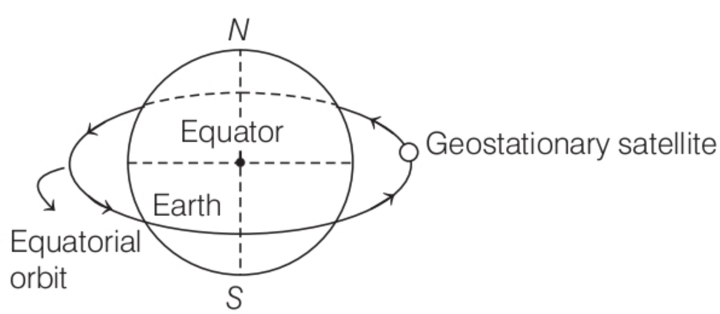
$$T = 2\pi\sqrt{\frac{R^3}{GM}} = \sqrt{\frac{3\pi}{G\rho}}$$

$$T = 2\pi\sqrt{\frac{R}{g}} = 5.08 \times 10^3 \text{ s} = 84 \text{ min} \approx 1.4 \text{ h}$$

where, ρ is the average density of earth.

Artificial satellites are of two types

Geostationary or Parking Satellites A satellite which appears to be at a fixed position at a definite height to an observer on earth is called geostationary or parking satellite.



$$\text{Height, from earth's surface} = 36000 \text{ km}$$
$$\text{Radius of orbit} = 42400 \text{ km}$$
$$\text{Time period} = 24 \text{ h}$$
$$\text{Orbital velocity} = 3.1 \text{ km/s}$$
$$\text{Angular velocity} = \frac{2\pi}{24} = \frac{\pi}{12} \text{ rad/h}$$

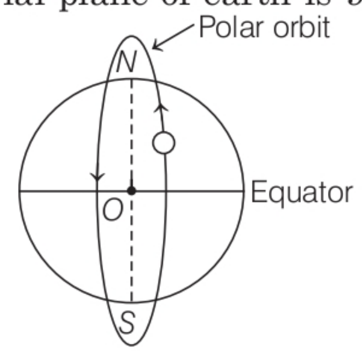
These satellites revolve around the earth in equatorial orbits.

The angular velocity of the satellite is same in magnitude and direction as that of angular velocity of the earth about its own axis.

These satellites are used in communication purpose.

INSAT 2B and INSAT 2C are geostationary satellites of India.

Polar Satellites These are those satellites which revolve in polar orbits around earth. A polar orbit is that orbit whose angle of inclination with equatorial plane of earth is 90°.



$$\text{Height from earth's surface} \approx 880 \text{ km}$$
$$\text{Time period} \approx 84 \text{ min}$$
$$\text{Orbital velocity} = 8 \text{ km/s}$$
$$\text{Angular velocity} = \frac{2\pi}{84} = \frac{\pi}{42} \text{ rad/min}$$

These satellites revolve around the earth in polar orbits.

These satellites are used in forecasting weather, studying the upper region of the atmosphere, in mapping etc.

PSLV series satellites are polar satellites of India.

Orbital Velocity

Orbital velocity of a satellite is the minimum velocity required to put the satellite into a given orbit around earth.

Orbital velocity of a satellite is given by

$$v_o = \sqrt{\frac{GM}{r}} = R\sqrt{\frac{g}{R+h}}$$

where, M = mass of the planet, R = radius of the planet and h = height of the satellite from planet's surface.

If satellite is revolving near the earth's surface, then $r = (R + h) \approx R$.

Now, orbital velocity,

$$v_o = \sqrt{gR} \approx 7.92 \text{ km/h}$$

If v is the speed of a satellite in its orbit and v_o is the required orbital velocity to move in the orbit, then

(i) If $v < v_o$, then satellite will move on a parabolic path and satellite will fall back to earth.

(ii) If $v = v_o$, then satellite will revolve in circular path/orbit around earth.

(iii) If $v_o < v < v_e$, then satellite will revolve around earth in elliptical orbit.

- The orbital velocity of jupiter is less than the orbital velocity of earth.

- For a satellite orbiting near earth's surface

(a) Orbital velocity = 8 km/s

(b) Time period = 84 min approximately

(c) Angular speed, $\omega = \dfrac{2\pi}{84}$ rad/min $= 0.00125$ rad/s

Energy of a Satellite in Orbit

Total energy of a satellite, E = KE + PE

$$= \frac{GMm}{2r} + \left(-\frac{GMm}{r}\right) = -\frac{GMm}{2r}$$

Time Period of Revolution of Satellite

The time taken by a satellite to complete one revolution around the earth, is known as time period of revolution of satellite.

The period of revolution (T) is given by

$$T = \frac{2\pi r}{\sqrt{\dfrac{GM}{r}}} = \frac{2\pi(R + h)}{v_0}$$

Height of Satellite

As it is known that the time period of satellite,

$$T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{(R + h)^3}{gR^2}} \quad \text{...(i)}$$

By squaring on both sides of Eq. (i), we get

$$T^2 = 4\pi^2 \frac{(R+h)^3}{gR^2}$$

$$\Rightarrow \qquad \frac{gR^2T^2}{4\pi^2} = (R+h)^3$$

$$\Rightarrow \qquad h = \left(\frac{T^2gR^2}{4\pi^2}\right)^{1/3} - R$$

By knowing the value of time period, the height of the satellite from the earth surface can be calculated.

Binding Energy

The energy required by a satellite to leave its orbit around the earth (planet) and escape to infinity is called **binding energy** of the satellite. Binding energy of the satellite of mass m is given by

$$\text{BE} = +\frac{GMm}{2r}$$

Escape Velocity

Escape velocity on earth is the minimum velocity with which a body has to be projected vertically upwards from the earth's surface, so that it just crosses the earth's gravitational field and never returns.

Escape velocity of any object (the earth's surface)

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR} = \sqrt{\frac{8\pi\rho\, GR^2}{3}} = R\sqrt{\frac{8}{3}\pi GP}$$

Escape velocity does not depend upon the mass or shape or size of the body as well as the direction of projection of the body. Escape velocity at earth is 11.2 km/s.

Some Important Escape Velocities

Heavenly body	Escape velocity
Moon	2.3 km/s
Mercury	4.28 km/s
Earth	11.2 km/s
Jupiter	60 km/s
Sun	618 km/s
Neutron star	2×10^5 km/s

Relation between escape velocity and orbital velocity of the satellite

$$v_e = \sqrt{2}\, v_o$$

A missile is launched with a velocity less than the escape velocity. The sum of its kinetic energy and potential energy is negative.

Maximum Height Attained by a Particle

When projected vertically upwards from the earth's surface,

$$h = \frac{v^2}{2g - v^2 / R}$$

(i) If velocity of projection v is equal the escape velocity ($v = v_e$), then1$v < v_e$, the body will attain maximum height and then may move around the planet or may fall down back to the planet.

(ii) If velocity of projection v of satellite is greater than the escape velocity ($v > v_e$), then the satellite will escape away following a hyperbolic path.

(iii) If $v < v_e$, the body will attain maximum height and then may move around the planet or may fall down back tothe planet.

Weightlessness

It is a situation in which the effective weight of the body becomes zero. Weightlessness is achieved

(i) during freely falling body under gravity.

(ii) inside a space craft or satellite.

(iii) at the centre of the earth.

(iv) when a body is lying in a freely falling lift.