Magnetic Effect of Current

Oersted's Experiment

HC Oersted by his experiment concluded that a current carrying conductor deflects magnetic compass needle placed near. It means a magnetic field is produced due to current carrying conductor which deflects magnetic compass.

Magnetic Field

The space in the surrounding of a magnet or any current carrying conductor in which its magnetic influence can be experienced is called magnetic field.

SI unit of magnetic field is Wb/m^2 or T (tesla).

The strength of magnetic field is called one tesla, if a charge of one coulomb, when moving with a velocity of 1 ms^{-1} along a direction perpendicular to the direction of the magnetic field experiences a force of one newton.

$$1 \text{ tesla (T)} = 1 \text{ weber metre}^{-2} \text{ (Wbm}^{-2}\text{)}$$

$$= 1 \text{ newton ampere}^{-1} \text{ metre}^{-1} \text{ (NA}^{-1} \text{ m}^{-1}\text{)}$$

CGS units of magnetic field are called gauss or oersted.

$$1 \text{ gauss} = 10^{-4} \text{ tesla.}$$

Rules to Find Direction of Magnetic Field

Following are the few rules that can be used to find out the direction of magnetic field

Maxwell's Cork Screw Rule

If a right handed cork screw is imagined to be rotated in such a direction that tip of the screw points in the direction of the current, then direction of rotation of cork screw gives the direction of magnetic line of force.

The conventional sign for a magnetic field coming out of the plane and normal to it is denoted by u.

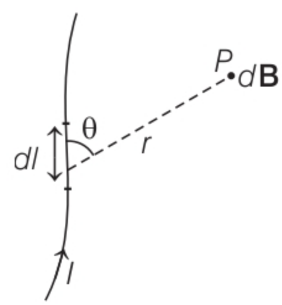
The magnetic field perpendicular to the plane in the downward direction is denoted by q.

Ampere's Swimming Rule

If a man is swimming along the wire in the direction of current with his face turned towards the needle, so that the current enters through his feet, then North pole of the magnetic needle will be deflected towards his left hand.

Biot Savart's Law

The magnetic field produced by a current carrying element of length dl, carrying current I at a point separated by a distance r is given by



$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I(d\mathbf{l}\times\mathbf{r})}{r^3}$$

or

$$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$$

where, θ is the angle between length of the current element and line joining the element to point (**p**) and μ_0 is absolute permeability of the free space.

The direction of magnetic field $d\mathbf{B}$ is that of $I\,d\mathbf{l}\times\mathbf{r}$.

In a medium,

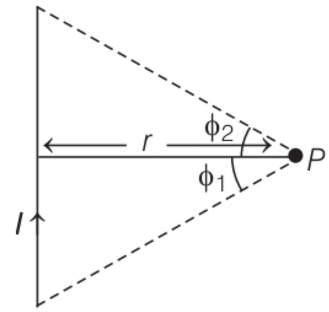
$$\mathrm{d}\mathbf{B} = \frac{\mu}{4\pi}\cdot\frac{I(d\mathbf{l}\times\mathbf{r})}{|\mathbf{r}|^3} = \frac{\mu_0\mu_r}{4\pi}\cdot\frac{I(d\mathbf{l}\times\mathbf{r})}{r^3}$$

Also,

$$\mathrm{d}\mathbf{B} = \frac{\mu_0}{4\pi}\frac{(\mathbf{J}\times\mathbf{r})}{r^3}dv = \frac{\mu_0}{4\pi}\frac{q(\mathbf{v}\times\mathbf{r})}{r^3}$$

Magnetic Field Due to a Straight Current Carrying Conductor

Consider a straight conductor carrying current I in upward direction, then magnetic field at a point P at r distance from it, is given by



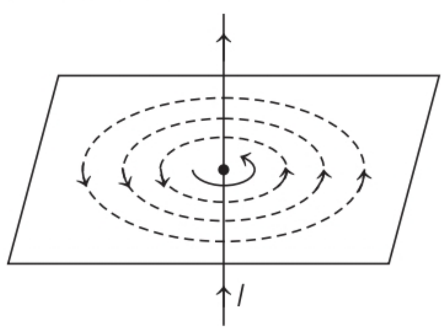
$$B = \frac{\mu_0}{4\pi} \cdot \frac{I}{r} (\sin \phi_1 + \sin \phi_2)$$

where ϕ_1 and ϕ_2 are angles, which the lines joining the two ends of the conductor to the observation point make with the perpendicular from the observation point to the conductor.

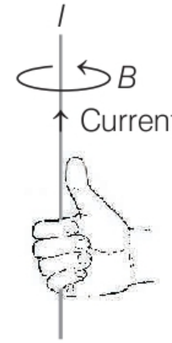
For infinite length conductor and observation point is near the centre of the conductor, $B = \frac{\mu_0}{4\pi} \cdot \frac{2I}{r}$

For infinite length conductor and observation point is near one end of the conductor, $B = \frac{\mu_0}{4\pi} \frac{I}{r}$

The magnetic field lines due to a straight current carrying conductor are concentric circles having centre at conductor and in a plane perpendicular to the conductor.



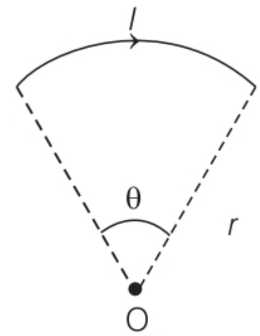
The direction of magnetic field lines can be obtained by Right Hand Thumb Rule.



Right Hand Thumb Rule

If we hold a current carrying conductor in the grip of the right hand in such a way that thumb points in the direction of current, then curling of fingers represents the direction of magnetic field lines.

Magnetic Field at the Centre of a Circular Current Carrying Coil



- The magnetic field at the centre due to the whole circular loop is

$$B = \frac{\mu_0 i}{2\pi a}$$

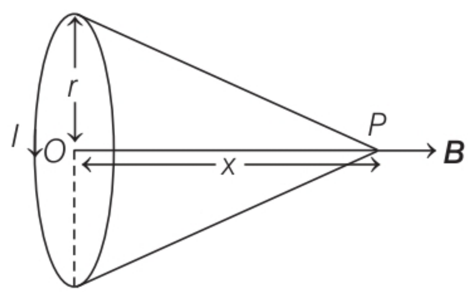
- Magnetic field at the centre of a current carrying coil. Magnetic field due to an arc of circular current carrying coil at the centre is

$$B = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{I}{r}\right)\theta$$

If we look at one face of the coil and the direction of current flowing through the coil is **clockwise**, then that face has South polarity and if direction of current is **anti-clockwise**, then that face has North polarity.

Magnetic Field on the Axis of a Current Carrying Circular Coil

Magnetic field at axis at a distance x from centre O.



$$B = \frac{\mu_0 n I r^2}{2(r^2 + x^2)^{3/2}}$$

where, r = radius of the coil, n = number of turns in the coil and
I = current.

At centre of the coil $(x = 0)$

$$B = \frac{\mu_0 nI}{2r}$$

Ampere's Circuital Law

The line integral of magnetic field induction **B** around any closed path in vacuum is equal to μ_0 times the total current threading the closed path, *i.e.*

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$$

where, **B** is the magnetic field, $d\mathbf{l}$ is small element, μ_0 is the absolute permeability of free space and I is the current.

Ampere's circuital law holds good for a closed path of any size and shape around a current carrying conductor because the relation is independent of distance from conductor.

Magnetic Field Due to a Current Carrying Long Circular Cylindrical Wire

- Outside the cylinder ($r > R$)

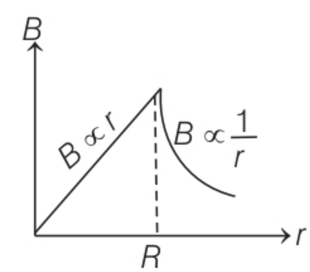
$$B = \frac{\mu_0}{2\pi} \frac{I}{r}$$

- Inside the cylinder when it is made of a thin metal sheet,

$$B = 0$$

- Inside the cylinder when current is uniformly distributed throughout the cross-section of the cylinder ($r < R$)

$$B = \frac{\mu_0 \mu_r}{2\pi} \frac{Ir}{R^2}$$

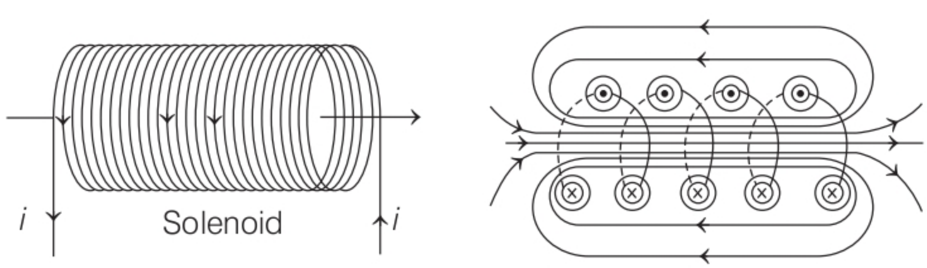


Variation in magnetic field with radius

where, μ_0 and μ_r are permeabilities of free space and material of the cylinder, I is current flowing through the cylinder and r is radius of the cylinder.

Solenoid

A solenoid is a closely wound helix of insulated copper wire.



Magnetic field at a point well inside a long solenoid is given by

$$B = \mu_0 n I$$

where, n = number of turns per unit length

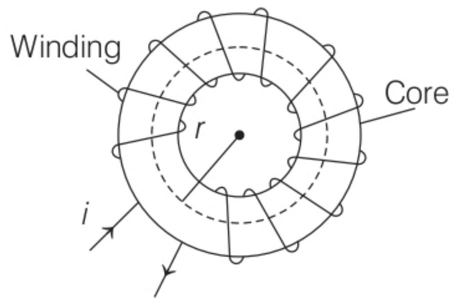
and I = current flowing through the solenoid.

Magnetic field at a point on one end of a long solenoid is given by

$$B = \frac{\mu_0 n I}{2}$$

Toroid

A toroidal solenoid is an anchor ring around which is large number of turns of a copper wire are wrapped.



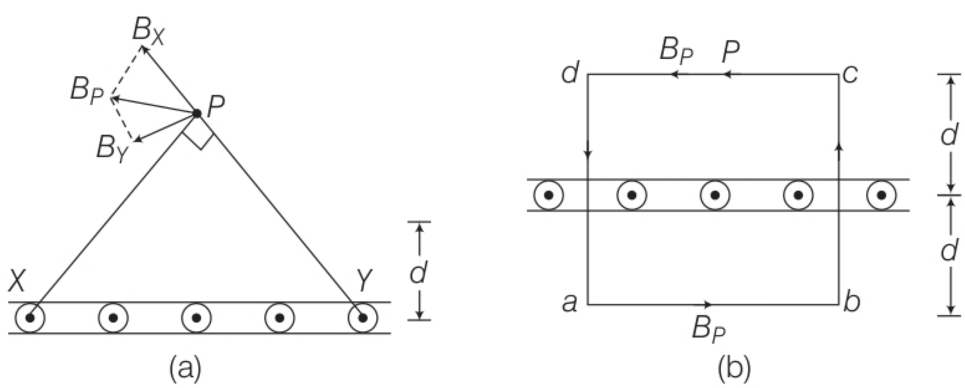
A toroid is an endless solenoid in the form of a ring. Magnetic field inside the turns of toroid is given by $B = \mu_0 n I$

Magnetic field i.nside a toroid is constant and is always tangential to the circular closed path.

Magnetic field at any point inside the empty space surrounded by the toroid and outside the toroid is zero, because net current enclosed by these space is zero.

Magnetic Field due to an Infinitely Large Current Carrying Sheet

Both infinite sheets of current with linear current density J are shown in the figure



Magnetic field due to an infinitely large carrying current sheet is given by

$$B = \frac{\mu_0 J}{2}$$

Force Acting on a Charge Particle Moving in a Uniform Magnetic Field

$$\mathbf{F} = q\,(\mathbf{v} \times \mathbf{B})$$

or $$F = |\mathbf{F}| = Bqv \sin\theta$$

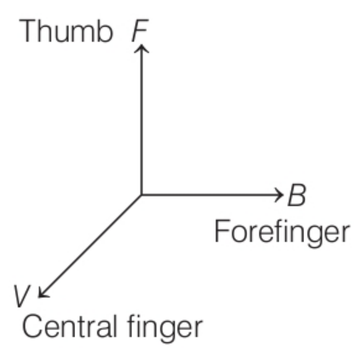
where, B = magnetic field intensity,
q = charge on particle,
v = speed of the particle
and θ = angle between magnetic field and direction of motion.

This force is perpendicular to **B** as well as **v**.

Its direction can be obtained from **Fleming's left hand rule**.

Fleming's Left Hand Rule

If we stretch the thumb, the forefinger and the central finger of left hand in such a way that all three are perpendicular to each other, then if forefinger represents the direction of magnetic field, central finger represents the direction of current flowing through the conductor, then thumb will represent the direction of magnetic force.



Motion of a Charged Particle in a Uniform Magnetic Field

When the charged particle enters parallel or anti-parallel to the magnetic field, then its follows a straight line path.

When charged particle enters normally to the magnetic field it follows a circular path.

The radius of the path, $r = \dfrac{mv}{Bq}$

$$\therefore \quad r \propto mv \text{ and } r \propto \frac{1}{(q/m)}$$

Time period, $T = \dfrac{2\pi m}{Bq}$

When charged particle enters magnetic field at any angle except $0°, 180°$ or $90°$, then it follows helical path.

The radius of the path, $r = \frac{mv \sin \theta}{Bq}$

$$\text{Time period, } T = \frac{2\pi m}{Bq}$$

The distance travelled by the charged particle in one time period due to component of velocity $v \cos \theta$ is called pitch of the path.

$$\text{Pitch } = T \times v \cos \theta$$
$$= \frac{2\pi mv \cos \theta}{Bq}$$

Motion of Charged Particle in Combined Electric and Magnetic Field : Lorentz Force

The total force experienced by a charge moving inside the electric and magnetic fields is called Lorentz force. It is given by

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Case I When **vE** and **B** are all collinear

In this magnetic force on the particle will be zero. So,

$$\mathbf{a} = \frac{q\mathbf{E}}{m}$$

The particle will pass through the field following a straight line path with change in it's speed.

Case II When **v**, **E** and **B** are mutually perpendicular

In this, $\mathbf{F} = \mathbf{F}_e + \mathbf{F}_m = 0$

or $\mathbf{a} = 0$

The particle will pass through the field with same velocity without any deviation in it's path.

Thus, $F_e = F_m$

or $v = \frac{E}{B}$

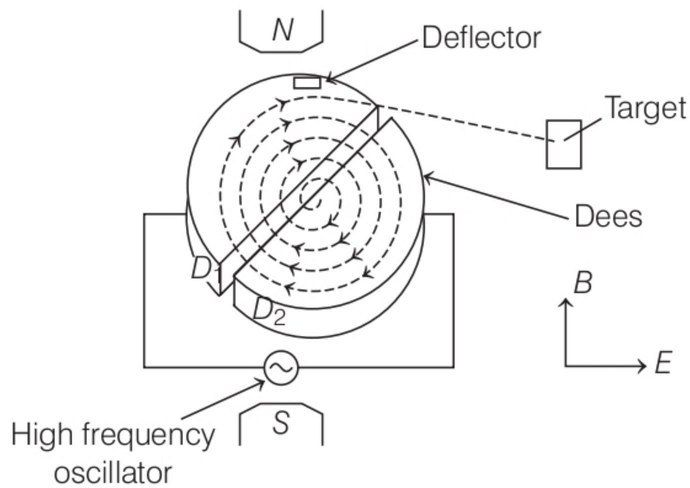
This principle is used in '**velocity selector**' to get a charged beam having a specific velocity.

Cyclotron

Cyclotron is a device used to accelerate positively charged particles such as proton, deuteron etc.

Principle of Cyclotron

A positively charged particle can be accelerated through a moderate electric field by crossing it again and again by use of strong magnetic field.



Radius of circular path, $r = \dfrac{mv}{Bq}$

Cyclotron frequency, $\nu = \dfrac{Bq}{2\pi m}$

where, m and q are mass and charge of the positive ion and B is strength of the magnetic field.

Maximum kinetic energy gained by the particle, $E_{max} = \dfrac{B^2 q^2 r_0^2}{2m}$

where, r_0 = maximum radius of circular path.

When a positive ion is accelerated by the cyclotron, it moves with greater and greater speed. As the speed of ion becomes comparable with that of light, the mass of the ion increases according to the relation.

$$m = \frac{m_0}{\sqrt{1 - \dfrac{v^2}{c^2}}}$$

where, m = mass of the ion, m_0 = maximum mass of the ion,
v = speed of ion and c = speed of light.

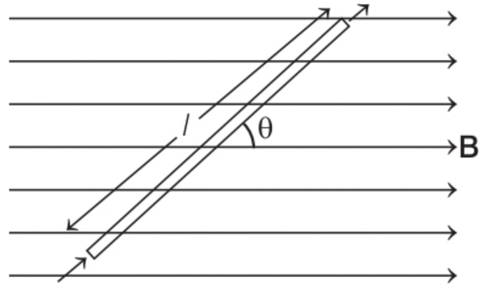
Limitations of the Cyclotron

(i) Cyclotron cannot accelerated uncharged particle like neutron.

(ii) The positively charged particles having large mass, *i.e.* ions cannot move at limitless speed in a cyclotron.

Force on a Current Carrying Conductor in a Magnetic Field

Magnetic force acting on a current carrying conductor in a uniform magnetic field is given by, $\mathbf{F} = I\,(\mathbf{l} \times \mathbf{B})$



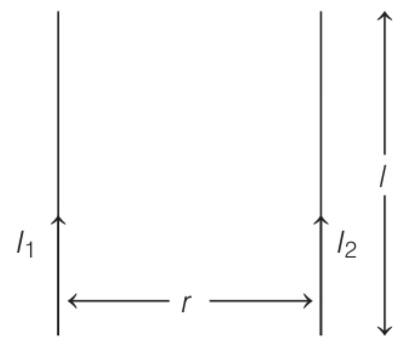
or $$F = BIl \sin\theta$$

Direction of force on a current carrying conductor can be found out by Fleming's left hand rule.

Force between Two Infinitely Long Parallel Current Carrying Conductors

Force between two long parallel current carrying conductors is given by

$$F = \frac{\mu_0}{2\pi} \cdot \frac{I_1 I_2}{r}\, l$$



The force is attractive if current in both conductors is in same direction and repulsive if current in both conductors is in opposite direction.

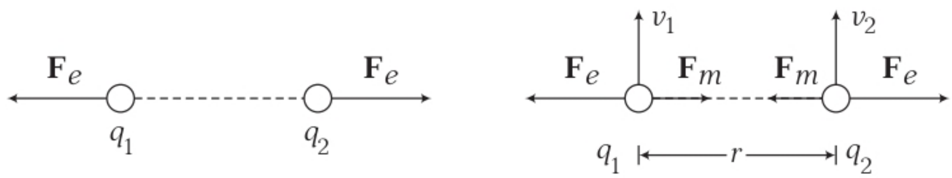
If the currents is both parallel wires are equal and in same direction, then magnetic field at a point exactly half way between the wire is zero.

Definition of Ampere

1 ampere is that steady current which when flowing in each of two infinitely long parallel conductors 1 m apart in vacuum. Produces between them a force of exactly 2×10^{-7} newton per metre of length.

Magnetic Force between two Moving Charges

Consider two charges q_1 and q_2 are moving with velocities v_1 and v_2 respectively and at any instant the distance between them is r.



Two moving charges

A magnetic force F_m will appear between them alongwith the electric force.

i.e.,
$$F_m = \frac{\mu_0}{4\pi} \frac{q_1 q_2 v_1 v_2}{r^2}$$

Magnetic Dipole

Every current carrying loop is a magnetic dipole. It has two poles South (S) and North (N). This is similar to a bar magnet.

Each magnetic dipole has some magnetic moment (M).

The magnitude of M is $|M| = NiA$

where, N = number of turns in the loop,

i = current in the loop

and A = area of cross-section of the loop.

The current carrying loop behaves as a small magnetic dipole placed along the axis one face of the loop behaves as North pole while the other face of loop behaves as South pole.

Torque acting on a Current Carrying Coil Placed Inside a Uniform Magnetic Field

Torque acting on a current carrying coil placed inside a uniform magnetic field is given by

$$\tau = NBIA \sin\theta$$

where, N = number of turns in the coil,

B = magnetic field intensity,

I = current in the coil,

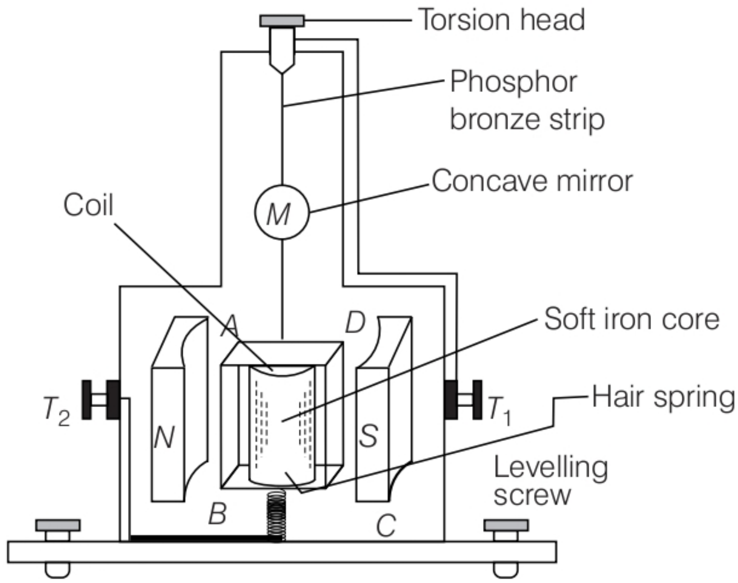
A = area of cross-section of the coil

and θ = angle between magnetic field and normal to the plane of the coil.

Moving Coil Galvanometer

It is a device used for the detection and measurement of the small currents. In equilibrium, deflecting torque = restoring torque

$$NBIA = k\theta \quad \text{or} \quad I = \frac{k}{NBA}\,\theta$$



where, k = restoring torque per unit twist,

N = number of turns in the coil,

B = magnetic field intensity,

A = area of cross-section of the coil and θ = angle of twist.

Current Sensitivity

The deflection produced per unit current in galvanometer is called its current sensitivity.

Current sensitivity, $I_S = \dfrac{\theta}{I} = \dfrac{NBA}{k}$

Voltage Sensitivity

The deflection produced per unit voltage applied across the ends of the galvanometer is called its voltage sensitivity.

Voltage sensitivity, $V_s = \dfrac{\theta}{V} = \dfrac{NBA}{kr}$

where, R is the resistance of the galvanometer.

Therefore for a sensitive galvanometer

(i) N should be large

(ii) B should be large

(iii) A should be large

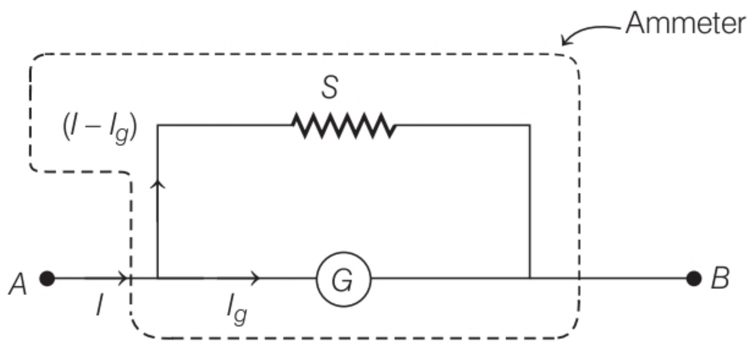
(iv) k should be small.

Ammeter

An ammeter is a low resistance galvanometer used for measuring the current in a circuit. It is always connected in series.

Conversion of a Galvanometer into an Ammeter

A galvanometer can be converted into an ammeter by connecting a low resistance into its parallel. If G is the resistance of a galvanometer and it give full scale deflection for current I_g, then required low resistance S, connected in its parallel for converting it into an ammeter of range I is given by



$$I_g \times G = (I - I_g) \times S \Rightarrow \quad S = \left(\frac{I_g}{I - I_g} \right) G$$

The resistance of an ideal ammeter is zero.

Voltmeter

A voltmeter is a high resistance galvanometer used for measuring the potential difference between two points.

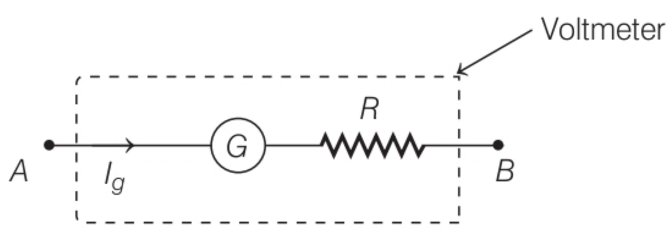
It is always connected in parallel.

The resistance of an ideal voltmeter is infinity.

Conversion of a Galvanometer into a Voltmeter

A galvanometer can be converted into a voltmeter by connecting a high resistance into its series.

If a galvanometer of resistance G shows full scale deflection for current I_g, then required high resistance R, connected in series for converting it into a voltmeter of range V is given by



$$V = I_g (G + R)$$

$$\Rightarrow \quad R = \frac{V}{I_g} - G$$