DCAM classes
Dynamic Classes for Academic Mastery

Wave Optics

Wave optics describes the connection between waves and rays of light. According to wave theory of light, the light is a form of energy which travels through a medium in the form of transverse wave motion. The speed of light in a medium depends upon the nature of medium.

Newtons' Corpuscular Theory of Light

Light consists of very small invisible elastic particles called corpuscles, which travel in vacuum with a speed of 3×10^8 m/s.

The size of corpuscular of different colours of light are different.

The theory could explain reflection and refraction.

But it could not explain interference, diffraction, polarisation, photoelectric effect and Compton effect.

The theory failed as it could not explain why light travels faster in a rarer medium than in a denser medium.

Huygens' Wave Theory of Light

Light travel in the form of waves. These waves travel in all direction with the velocity of light.

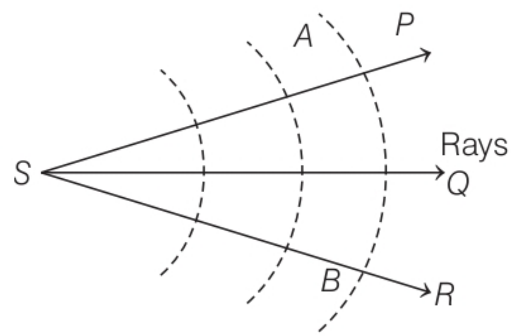
The waves of light of different colours have different wavelengths.

Huygens' theory could explain reflection, refraction interference, diffraction, polarisation but could not explain photoelectric effect and compton's effect.

Wave theory introduced the concept of wavefront.

Wavefront

A wavefront is defined as the continuous locus of all the particles of a medium, which are vibrating in the same phase.



S = source of light, AB = wavefront and SP, SQ and SR are rays of light.

These are three types

(i) Spherical Wavefront

When source of light is a point source, the wavefront is spherical. Amplitude (A) is inversely proportional to distance (x) i.e. $A \propto \frac{1}{x}$.

$$\therefore \qquad \text{Intensity } (I) \propto (\text{Amplitude})^2$$

(ii) Cylindrical wavefront

When source of light is linear, the wavefront is cylindrical.

$$\text{Amplitude (A)} \propto \frac{1}{\sqrt{x}}$$

$$\therefore \qquad \text{Intensity} \propto (\text{Amplitude})^2 \propto \frac{1}{x}$$

(iii) Plane wavefront

When the source of light is very far off, the wavefront is plane.

$$\text{Amplitude } (A) \propto x^0$$

$$\text{Intensity } (I) \propto x^0$$

Huygens' Principle

(i) Every point on given wavefront (called primary wavefront) acts as a fresh source of new disturbance called secondary wavelets.

(ii) The secondary wavelets travels in all the directions with the speed of light in the medium.

(iii) A surface touching these secondary wavelets tangentially in the forward direction at any instant gives the new (secondary) wave front of that instant.

Superposition of Waves

When two similar waves propagate in a medium simultaneously, then at any point the resultant displacement is equal to the vector sum of displacement produced by individual waves.

$$\mathbf{y} = \mathbf{y}_1 + \mathbf{y}_2$$

Interference of Light

When two light waves of similar frequency having a zero or constant phase difference propagate in a medium simultaneously in the same direction, then due to their superposition maximum intensity is obtained at few points and minimum intensity at other few points. This phenomenon of redistribution of energy due to superposition of waves is called interference of light waves.

The interference taking place at points of maximum intensity is called **constructive interference.**

The interference taking place at points of minimum intensity is called **destructive interference.**

Conditions for Constructive and Destructive Interference

For Constructive Interference

Phase difference, $\phi = 2n\pi$

Path difference, $\Delta x = n\lambda$ where, $n = 0, 1, 2, 3, \ldots$

For Destructive Interference

Phase difference, $\phi = (2n - 1)\pi$

Path difference, $\Delta x = \dfrac{(2n - 1)\lambda}{2}$ where, $n = 1, 2, 3, \ldots$

If two waves of exactly same frequency and of amplitude a and b interfere, then **amplitude of resultant wave** is given by

$$R = \sqrt{a^2 + b^2 + 2ab\cos\phi}$$

where, ϕ is the phase difference between two waves.

$$R_{\text{max}} = (a + b)$$

$$R_{\text{min}} = (a - b)$$

Intensity of wave,

$$\therefore \quad I = a^2 + b^2 + 2ab\cos\phi$$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

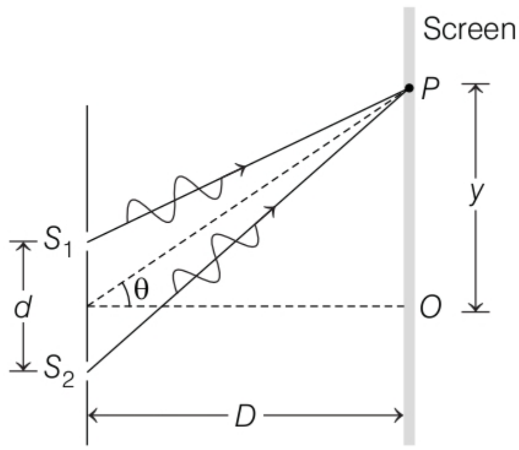
where, I_1 and I_2 are intensities of two waves.

Coherent Sources of Light

The sources of light emitting light of same wavelength, same frequency having a zero or constant phase difference are called coherent sources of light.

Young's Double Slit Experiment (YDSE)

The arrangement of YDSE to produce interference is shown below



Position of bright fringe, $y_{\text{bright}} = \dfrac{n\lambda}{d} D$

Position of dark fringe, $y_{\text{dark}} = \dfrac{(2n-1)\lambda D}{2d}$

Fringe Width

The distance between the centres of two consecutive bright or dark fringes is called the fringe width, $\beta = \dfrac{\lambda D}{d}$

The angular fringe width is given by $\theta = \dfrac{\lambda}{d} = \dfrac{\beta}{d}$.

where, λ is the wavelength of light d is the distance between two coherent sources.

Important Points Related with Fringe Width

As we know that fringe width (β) is the distance between two successive maxima or minima. It is given by

$$\beta = \frac{\lambda D}{d} \text{ or } \beta \propto \lambda$$

Two conclusions can be drawn from this relation

(i) If YDSE apparatus is immersed in a liquid of refractive index μ, then wavelength of light and hence fringe width decreases μ times.

(ii) If white light is used in place of a monochromatic light, then coloured fringes are obtained on the screen with red fringes of larger size than that of violet because $\lambda_{\text{red}} > \lambda_{\text{violet}}$.

But note that centre is still white because path difference there is zero for all colours. Hence, all the wavelengths interfere constructively. At other places light will interfere destructively for those wavelengths for whom path difference is $\lambda/2, 3\lambda/2, \ldots$, etc, and they will interfere constructively for the wavelengths for whom path difference is $\lambda, 2\lambda, \ldots$, etc.

Note Shape of fringes on the screen is hyperbolic. But, if the screen is placed at very large distance from slits, then the hyperbola nearly looks straight line in shape.

Angular Width of Fringes

Let angular position of nth bright fringe is θ_n and because of its small value $\tan\theta_n \approx \theta_n$

$$\therefore \qquad \theta_n = \frac{Y_n}{D} = \frac{nD\lambda/d}{D} = \frac{n\lambda}{d}$$

Similarly, if angular position of $(n+1)$th bright fringe is θ_{n+1}, then

$$\theta_{n+1} = \frac{Y_{n+1}}{D} = \frac{(n+1)D\lambda/d}{D} = \frac{(n+1)\lambda}{d}$$

$\therefore$ Angular width of a fringe,

$$\theta = \theta_{n+1} - \theta_n = \frac{(n+1)\lambda}{d} - \frac{n\lambda}{d} = \frac{\lambda}{d}$$

Also,
$$\beta = \frac{D}{d}\lambda$$

$$\therefore \qquad \theta = \frac{\lambda}{d} = \frac{\beta}{D}$$

It is independent of n, *i.e.* angular width of all fringes are same.

Intensity of Fringes

Intensity of bright frine $= 4I$ [as $I_1 = I_2 = I$ is YDSE]

Intensity of dark fringe $= 0$

Also,
$$\frac{I_1}{I_2} = \frac{a^2}{b^2} = \frac{W_1}{W_2}$$

where, W_1 and W_2 are width of slits, which emanates light of intensity I_1 and I_2.

Maximum Intensity

From above expression, we can see that intensity is maximum at points, where

$$\cos\frac{\phi}{2} = \pm 1 \text{ or } \frac{\phi}{2} = n\pi,\ n = 0, \pm 1, \pm 2, \ldots$$

or $$\phi = 2n\pi \text{ or } \frac{2\pi}{\lambda}\Delta x = 2n\pi$$

or $\Delta x = n\lambda$, we know that this path difference is for maxima.
Thus, intensity of bright points are maximum and given by

$$I_{max} = 4I_0$$

Therefore, we can write $I = I_{max}\cos^2\frac{\phi}{2}$

Minimum Intensity

Minimum intensity on the screen is found at points, where

$$\cos\frac{\phi}{2} = 0 \text{ or } \frac{\phi}{2} = \left(n - \frac{1}{2}\right)\pi$$

(where, $n = \pm 1, \pm 2, \pm 3, \ldots$)

or $$\phi = (2n-1)\pi$$

or $$\frac{2\pi}{\lambda}\Delta x = (2n-1)\pi$$

$\Rightarrow$ $$\Delta x = (2n-1)\frac{\lambda}{2}$$

We know that this path difference corresponds to minima. Thus, intensity of minima are minimum and given by

$$I_{min} = 0$$

Note If both the slits are of equal width, $I_1 \approx I_2 = I_0$ and in that cases,

$$I_{max} = 4I_0 \text{ and } I_{min} = 0$$

If the slits are of unequal width, then $I_1 \neq I_2$ $I_{min} \neq 0$

Insertion of Transparent Slab in YDSE

When a transparent slab (sheet) of refractive index μ and of thickness t is introduced in one of the path of interfering waves, then fringe pattern shifts in that direction by a distance Y

$$Y = \frac{D}{d}(\mu - 1)t = \frac{\beta}{\lambda}(\mu - 1)t$$

where, β = fringe width.

Interference in Thin Flim

Interference effects are commonly observed in thin films, such as thin layers of coil on water or the thin surface of a soap bubble.

The varied colours observed when white light is incident on such films as the result of the interference of waves reflected from the two surfaces of the film.

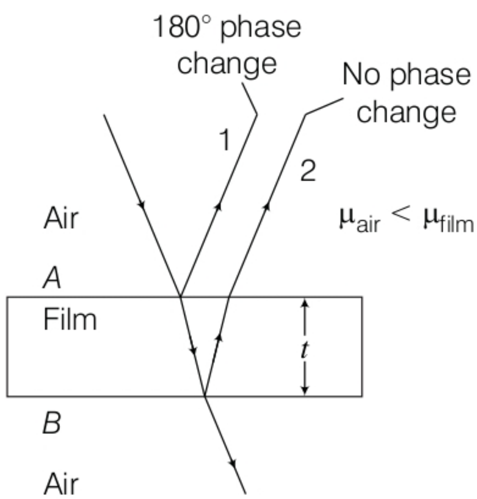
Let us assume that the light rays travelling in air are nearly normal to the surfaces of the film. Then

(i) The wavelength of light in a medium whose refractive index is μ is $\lambda_\mu = \dfrac{\lambda}{\mu}$

(ii) If wave is reflected from a denser medium, then it undergoes a phase change 180°.

The path difference betwen the two rays 1 and 2 is $2t$.

Hence, condition of constructive interference will be



$$2t = (2n - 1)\frac{\lambda_\mu}{2}$$

where, $n = 1, 2, 3, \ldots$

or
$$2\mu t = \left(n - \frac{1}{2}\right)\lambda \quad \text{as} \quad \lambda_\mu = \frac{\lambda}{\mu}$$

Similarly, condition of destructive interference will be

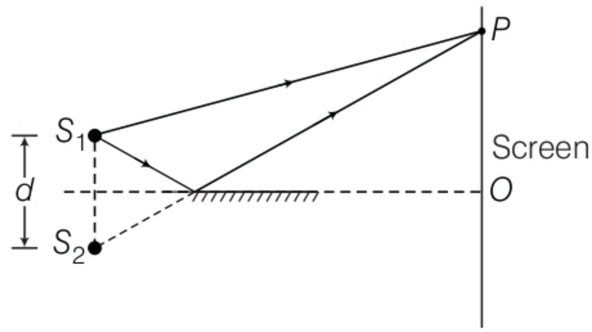
$$2\mu t = n\lambda, \qquad \text{where } n = 0, 1, 2, \ldots$$

- A soap bubble or oil film on water appears coloured in white light due to interference of light reflected from upper and lower surfaces of soap bubble or oil film.
- In interference fringe pattern, all bright and dark fringes are of same width.

Fresnel's Biprism

It is a combination of two prisms of very small refracting angles placed base to base. It is used to obtain two coherent sources from a single light source.

Llyod's Mirror



The shape of interference fringes obtained from Llyod's mirror are usually hyperbolic.

When screen is held at 90° to the line joining foci of the hyperbola, the fringes are circular.

When distance of screen (D) is very large compare to the distance between the slits (d), the fringes are straight.

Diffraction

The bending of light waves around the corners of an obstacle or aperture of the size of the wavelength is called diffraction of light. The phenomenon of diffraction is divided mainly in the following two classes

(a) Fresnel class (b) Fraunhofer class

S.No.	Fresnel Class	Fraunhofer Class
1.	The source is at a finite distance.	The source is at infinite distance.
2.	No opticals are required.	Opticals in the form of collimating lens and focusing lens are required.
3.	Fringes are not sharp and well defined.	Fringes are sharp and well defined.

Fraunhoffer Diffraction at a Single Slit

For Secondary Minima

(a) Path difference $= n\lambda$

(b) Linear distance $= \dfrac{nD\lambda}{a} = \dfrac{nf\lambda}{a}$

where, λ = wavelength of light, a = width of single slit, D = distance of screen from the slit and f = focal length of convex lens.

(c) Angular spread $= \dfrac{n\lambda}{a}$ where, $n = 1, 2, 3, \ldots$

For Secondary Maxima

(a) Path difference $= \dfrac{(2n+1)\lambda}{2}$

(b) Linear distance $= \dfrac{(2n+1)\,D\lambda}{2a} = \dfrac{(2n+1)\,f\lambda}{2a}$

(c) Angular spread $= \dfrac{(2n+1)\,\lambda}{2a}$

For Central Maxima

Linear width of central maximum $\dfrac{2D\lambda}{a} = \dfrac{2f\lambda}{a}$

Angular width of central maximum, $2\theta = \dfrac{2\lambda}{a}$

Fresnel's Distance

It is given as, $Z_F = \dfrac{a^2}{\lambda}$, where, is a is the size of slit or hole.

Image formation can be explained by ray optics for distance less than Z_F.

Diffraction Grating

It consists of large number of equally spaced parallel slits. If light is incident normally on a transmission grating, then the direction of principal maxima is given by $d \sin\theta = n\lambda$

Here, d is the distance between two consecutive slits and is called grating element. $n = 1, 2, 3, \ldots$ is the order of principal maxima.

Note Diffraction grating is based on combined phenomena of interference and diffraction. It is the results of interference in diffracted waves.

Important Points

- In diffraction fringe pattern central bright fringe is brightest and widest and remaining secondary maximas are of gradually decreasing intensities.
- The difference between interference and diffraction is that the interference is the superposition between the wavelets coming from two coherent sources while the diffraction is the superposition between the wavelets coming from the single wavefront.

Polarisation

The phenomenon of restricting of electric vectors of light into a single direction is called **polarisation.** Ordinary light has electric vectors in all possible directions in a plane perpendicular to the direction of propagation of light.

When ordinary light is passed through a tourmaline, calcite or quartz crystal the transmitted light will have electric vectors in a particular direction parallel to the axis of crystal. This light is then known as plane polarised light.

A plane containing the vibrations of polarised light is called **plane of vibration**. A plane perpendicular to the plane of vibration is called **plane of polarisation**. Polarisation can take place only in transverse waves.

Nicol Prism

A nicol prism is an optical device which is used for producing plane polarised light and analysing light the same.

The nicol prism consists of two calcite crystal cut at 68° with its principal axis joined by a glue called Canada balsam.

Law of Malus

When a beam of completely plane polarised light is incident on an analyser, the intensity of transmitted light from analyser is directly proportional to the square of the cosine of the angle between plane of transmission of analyser and polariser, i.e. $I \propto \cos^2 \theta$

When ordinary light is incident on a polariser the intensity of transmitted light is half of the intensity of incident light.

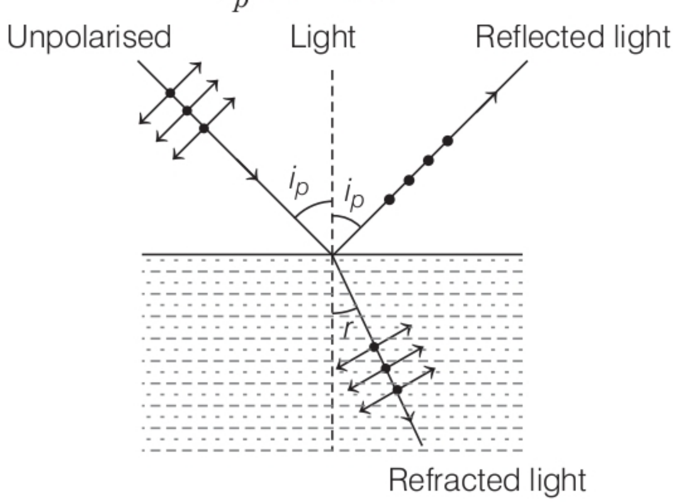
When a polariser and a analyser are perpendicular to each other, then intensity of transmitted light from analyser becomes 0.

Brewster's Law

When unpolarised light is incident at an angle of polarisation (i_p) on the interface separating air from a medium of refractive index μ, then reflected light becomes fully polarised, provided

$$\mu = \tan i_p$$

Here, $$i_p + r = 90^\circ$$



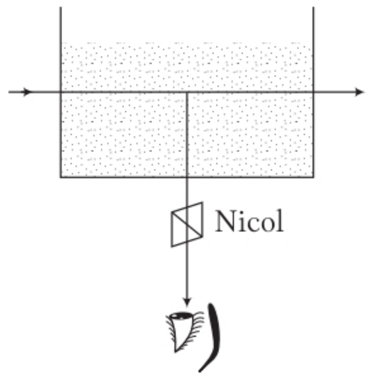
Double Refraction

When unpolarised light is incident on a calcite or quartz crystal it splits up into two refracted rays, one of which follows laws of refraction, called **ordinary ray** (O-ray) and other do not follow laws of refraction, called **extraordinary ray** (E-ray). This phenomenon is called double refraction.

Polarisation of Light by Scattering

The scattering of light can be demonstrated by a simple experiment. A few drops of dilute sulphuric acid is added to a dilute solution of hypo (sodium thiosulphate) prepared in a glass tank.

A precipitate of fine sulphur particles is formed, the particles grow in size with time. Now, a beam of light from a bright source is sent through the tank. The light scattered by the sulphur particles in a direction at right angles to the incident light appears bluish shown in figure below.



When the scattered bluish light is seen through an analyser (for example, a rotating Nicol) a variation in intensity with minimum intensity zero is found. This shows that the light scattered in a direction perpendicular to the incident light is plane polarised.

Dichroism

Few double refracting crystals have a property of absorbing one of the two refracted rays and allowing the other to emerge out. This property of crystal is called dichroism.

Polaroid

It is a polarising film mounted between two glass plates, which is used to produce polarised light.

Uses of Polaroid

(i) Polaroids are used in sun glasses. They protect the eyes from glare.

(ii) The pictures taken by a stercoscopic camera. When seen with the help of polaroid spectacles, it helps in creating three dimensional effect.

(iii) The wind shield of an automobile is made of polaroid. Such a wind shield protects the eyes of the driver of the automobile from the **dazzling light** of the approaching vehicles.

Doppler's Effect in Light

The phenomenon of apparent change in frequency (or wavelength) of the light due to relative motion between the source of light and the observer is called **Doppler's Effect in Light**.

Doppler's Shift

Apparent wavelength > actual wavelength

So spectrum of the radiation from the source of light shifts towards the red end of spectrum. This is called **red shift**.

However, when waves are received from a source moving towards the observer, there is an apparent decrease in wavelength, this is reffered as **blue shift**

$$\text{Doppler's shift, } \Delta\lambda = \lambda \cdot \frac{v}{c}$$

where, v = speed of source w.r.t. stationary observer

and c = speed of light.