SOLVED EXAMPLES

Ex. 1 Prove that $\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} = \begin{vmatrix} y & b & q \\ x & a & p \\ z & c & r \end{vmatrix}$

Sol. $D = \begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} = \begin{vmatrix} a & x & p \\ b & y & q \\ c & z & r \end{vmatrix}$ (By interchanging rows & columns)

$$= -\begin{vmatrix} x & a & p \\ y & b & q \\ z & c & r \end{vmatrix} \qquad (C_1 \leftrightarrow C_2)$$

$$= \begin{vmatrix} y & b & q \\ x & a & p \\ z & c & r \end{vmatrix} \qquad (R_1 \leftrightarrow R_2)$$

Ex. 2 If $D_r = \begin{vmatrix} r & r^3 & 2 \\ n & n^3 & 2n \\ \frac{n(n+1)}{2} & \left(\frac{n(n+1)}{2}\right)^2 & 2(n+1) \end{vmatrix}$, find $\sum_{r=0}^{n} D_r$.

Sol. $$\sum_{r=0}^{n} D_r = \begin{vmatrix} \sum_{r=0}^{n} r & \sum_{r=0}^{n} r^3 & \sum_{r=0}^{n} 2 \\ n & n^3 & 2n \\ \frac{n(n+1)}{2} & \left(\frac{n(n+1)}{2}\right)^2 & 2(n+1) \end{vmatrix} = \begin{vmatrix} \frac{n(n+1)}{2} & \left(\frac{n(n+1)}{2}\right)^2 & 2(n+1) \\ n & n^3 & 2n \\ \frac{n(n+1)}{2} & \left(\frac{n(n+1)}{2}\right)^2 & 2(n+1) \end{vmatrix} = 0$$

Ex. 3 Simplify $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

Sol. Let $R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ b & c & a \\ c & a & b \end{vmatrix} = (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ b & c & a \\ c & a & b \end{vmatrix}$$

Apply $C_1 \to C_1 - C_2,\ C_2 \to C_2 - C_3$

$$= (a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ b-c & c-a & a \\ c-a & a-b & b \end{vmatrix}$$

$$= (a+b+c)\,((b-c)(a-b)-(c-a)^2)$$

$$= (a+b+c)\,(ab+bc-ca-b^2-c^2+2ca-a^2)$$

$$= (a+b+c)\,(ab+bc+ca-a^2-b^2-c^2) \equiv 3abc-a^3-b^3-c^3$$

Ex. 4 Determinant $\begin{vmatrix} a+b+nc & (n-1)a & (n-1)b \\ (n-1)c & b+c+na & (n-1)b \\ (n-1)c & (n-1)a & c+a+nb \end{vmatrix}$ is equal to -

Sol. Applying $C_1 \to C_1 + (C_2 + C_3)$

$$D = n(a+b+c)\begin{vmatrix} 1 & (n-1)a & (n-1)b \\ 1 & b+c+na & (n-1)b \\ 1 & (n-1)a & c+a+nb \end{vmatrix}$$

$$D = n(a+b+c)\begin{vmatrix} 1 & (n-1)a & (n-1)b \\ 0 & a+b+c & 0 \\ 0 & 0 & a+b+c \end{vmatrix} \quad \begin{Bmatrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{Bmatrix}$$

$$= n(a+b+c)^3$$

Ex. 5 Prove that $\begin{vmatrix} (x-a)^2 & (x-b)^2 & (x-c)^2 \\ (y-a)^2 & (y-b)^2 & (y-c)^2 \\ (z-a)^2 & (z-b)^2 & (z-c)^2 \end{vmatrix} = 2(x-y)(y-z)(z-x)(a-b)(b-c)(c-a)$

Sol. $D = \begin{vmatrix} (x-a)^2 & (x-b)^2 & (x-c)^2 \\ (y-a)^2 & (y-b)^2 & (y-c)^2 \\ (z-a)^2 & (z-b)^2 & (z-c)^2 \end{vmatrix}$

Using factor theorem, Put $x = y$

$$D = \begin{vmatrix} (y-a)^2 & (y-b)^2 & (y-c)^2 \\ (y-a)^2 & (y-b)^2 & (y-c)^2 \\ (z-a)^2 & (z-b)^2 & (z-c)^2 \end{vmatrix}$$

R_1 and R_2 are identical which makes $D = 0$. Therefore, $(x-y)$ is a factor of D.

Similarly $(y-z)$ & $(z-x)$ are factors of D

Now put $a = b$

$$D = \begin{vmatrix} (x-b)^2 & (x-b)^2 & (x-c)^2 \\ (y-b)^2 & (y-b)^2 & (y-c)^2 \\ (z-b)^2 & (z-b)^2 & (z-c)^2 \end{vmatrix}$$

C_1 and C_2 become identical which makes D = 0. Therefore, (a–b) is a factor of D.
Similarly (b–c) and (c–a) are factors of D.
Therefore, $D = \lambda(x-y)(y-z)(z-x)(a-b)(b-c)(c-a)$
To get the value of λ put $x = -1 = a$, $y = 0 = b$ and $z = 1 = c$

$$D = \begin{vmatrix} 0 & 1 & 4 \\ 1 & 0 & 1 \\ 4 & 1 & 0 \end{vmatrix} = \lambda(-1)(-1)(2)(-1)(-1)(2)$$

$\Rightarrow \quad 4\lambda = 8 \quad \Rightarrow \quad \lambda = 2$

$\therefore \quad D = 2(x-y)(y-z)(z-x)(a-b)(b-c)(c-a)$

Ex. 6 Prove that $\begin{vmatrix} 1 & \alpha & \alpha^2 \\ \alpha & \alpha^2 & 1 \\ \alpha^2 & 1 & \alpha \end{vmatrix} = -(1-\alpha^3)^2.$

Sol. This is a cyclic determinant.

$$\Rightarrow \quad \begin{vmatrix} 1 & \alpha & \alpha^2 \\ \alpha & \alpha^2 & 1 \\ \alpha^2 & 1 & \alpha \end{vmatrix} = -(1+\alpha+\alpha^2)(1+\alpha^2+\alpha^4-\alpha-\alpha^2-\alpha^3)$$

$$= -(1+\alpha+\alpha^2)(-\alpha+1-\alpha^3+\alpha^4) = -(1+\alpha+\alpha^2)(1-\alpha)^2(1+\alpha+\alpha^2)$$

$$= -(1-\alpha)^2(1+\alpha+\alpha^2)^2 = -(1-\alpha^3)^2$$

Ex. 7 If the system of equations $x + \lambda y + 1 = 0$, $\lambda x + y + 1 = 0$ & $x + y + \lambda = 0$. is consistent then find the value of λ.

Sol. For consistency of the given system of equations

$$D = \begin{vmatrix} 1 & \lambda & 1 \\ \lambda & 1 & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$

$\Rightarrow \quad 3\lambda = 1 + 1 + \lambda^3$ or $\lambda^3 - 3\lambda + 2 = 0 \quad \Rightarrow \quad (\lambda-1)^2(\lambda+2) = 0$

$\Rightarrow \quad \lambda = 1$ or $\lambda = -2$

Ex. 8 For a 3×3 skew-symmetric matrix A, show that adj A is a symmetric matrix.

Sol. $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} \qquad \text{cof } A = \begin{bmatrix} c^2 & -bc & ca \\ -bc & b^2 & -ab \\ ca & -ab & a^2 \end{bmatrix}$

$$\text{adj } A = (\text{cof } A)' = \begin{bmatrix} c^2 & -bc & ca \\ -bc & b^2 & -ab \\ ca & -ab & a^2 \end{bmatrix} \text{ which is symmetric.}$$

Ex. 9 If a, b, c x, y, z ∈ R, then prove that $\begin{vmatrix} (a-x)^2 & (b-x)^2 & (c-x)^2 \\ (a-y)^2 & (b-y)^2 & (c-y)^2 \\ (a-z)^2 & (b-z)^2 & (c-z)^2 \end{vmatrix} = \begin{vmatrix} (1+ax)^2 & (1+bx)^2 & (1+cx)^2 \\ (1+ay)^2 & (1+by)^2 & (1+cy)^2 \\ (1+az)^2 & (1+bz)^2 & (1+cz)^2 \end{vmatrix}$

Sol. L.H.S. $= \begin{vmatrix} (a-x)^2 & (b-x)^2 & (c-x)^2 \\ (a-y)^2 & (b-y)^2 & (c-y)^2 \\ (a-z)^2 & (b-z)^2 & (c-z)^2 \end{vmatrix} = \begin{vmatrix} a^2-2ax+x^2 & b^2-2bx+x^2 & c^2-2cx+x^2 \\ a^2-2ay+y^2 & b^2-2by+y^2 & c^2-2cy+y^2 \\ a^2-2az+z^2 & b^2-2bz+z^2 & c^2-2az+z^2 \end{vmatrix}$

$$= \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \times \begin{vmatrix} a^2 & -2a & 1 \\ b^2 & -2b & 1 \\ c^2 & -2c & 1 \end{vmatrix} \text{ (Row by Row)}$$

$$= \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \times (-1) \begin{vmatrix} a^2 & 2a & 1 \\ b^2 & 2b & 1 \\ c^2 & 2c & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \times (-1)(-1) \begin{vmatrix} 1 & 2a & a^2 \\ 1 & 2b & b^2 \\ 1 & 2c & c^2 \end{vmatrix} \; (C_1 \leftrightarrow C_3)$$

$$= \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \times \begin{vmatrix} 1 & 2a & a^2 \\ 1 & 2b & b^2 \\ 1 & 2c & c^2 \end{vmatrix}$$

Multiplying row by row

$$= \begin{vmatrix} 1+2ax+a^2x^2 & 1+2bx+b^2x^2 & 1+2cx+c^2x^2 \\ 1+2ay+a^2y^2 & 1+2by+b^2y^2 & 1+2cy+c^2y^2 \\ 1+2az+a^2z^2 & 1+2bz+b^2z^2 & 1+2cz+c^2z^2 \end{vmatrix}$$

$$= \begin{vmatrix} (1+ax)^2 & (1+bx)^2 & (1+cx)^2 \\ (1+ay)^2 & (1+by)^2 & (1+cy)^2 \\ (1+az)^2 & (1+bz)^2 & (1+cz)^2 \end{vmatrix} = \text{R.H.S.}$$

Ex. 10 For two non-singular matrices A & B, show that adj (AB) = (adj B) (adj A)

Sol. We have $(AB)\,(\text{adj}\,(AB)) = |AB|\, I_n$

$= |A|\,|B|\, I_n$

$A^{-1}\,(AB)(\text{adj}\,(AB)) = |A|\,|B|\,A^{-1}$

$\Rightarrow$ $B\,\text{adj}\,(AB) = |B|\,\text{adj}\,A$ $\quad\left(\because\ A^{-1} = \dfrac{1}{|A|}\,\text{adj}\,A\right)$

$\Rightarrow$ $B^{-1}B\,\text{adj}\,(AB) = |B|\,B^{-1}\,\text{adj}\,A$

$\Rightarrow$ $\text{adj}\,(AB) = (\text{adj}B)\,(\text{adj}\,A)$

Ex. 11 If $D_1 = \begin{vmatrix} y^5z^6(z^3-y^3) & x^4z^6(x^3-z^3) & x^4y^5(y^3-x^3) \\ y^2z^3(y^6-z^6) & xz^3(z^6-x^6) & xy^2(x^6-y^6) \\ y^2z^3(z^3-y^3) & xz^3(x^3-z^3) & xy^2(y^3-x^3) \end{vmatrix}$ and $D_2 = \begin{vmatrix} x & y^2 & z^3 \\ x^4 & y^5 & z^6 \\ x^7 & y^8 & z^9 \end{vmatrix}$. Then D_1D_2 is equal to -

Sol. The given determinant D_1 is obtained by corresponding cofactors of determinant D_2.

Hence $D_1 = D_2^2$

$\Rightarrow$ $D_1D_2 = D_2^2D_2 = D_2^3$

Ex. 12 Obtain the inverse of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ using elementary operations.

Sol. Write A = IA, i.e., $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

or $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$ (applying $R_1 \leftrightarrow R_2$)

or $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} A$ (applying $R_3 \rightarrow R_3 - 3R_1$)

or $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} A$ (applying $R_1 \rightarrow R_1 - 2R_2$)

or $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix} A$ (applying $R_3 \rightarrow R_3 + 5R_2$)

or $$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix} A \quad \left(\text{applying } R_3 \to \frac{1}{2} R_3\right)$$

or $$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix} A \quad (\text{Applying } R_1 \to R_1 + R_3)$$

or $$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix} A \quad (\text{Applying } R_2 \to R_2 - 2R_3)$$

Hence $$A^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$$

Exercise # 1

[Single Correct Choice Type Questions]

1. If A and B are square matrices of order 3 such that |A| = – 1, |B| = 3, then |3AB| is equal to

(A) – 9 (B) – 81 (C) – 27 (D) 81

2. If $\begin{vmatrix} \sin 2x & \cos^2 x & \cos 4x \\ \cos^2 x & \cos 2x & \sin^2 x \\ \cos^4 x & \sin^2 x & \sin 2x \end{vmatrix} = a_0 + a_1 (\sin x) + a_2 (\sin^2 x) + ... + a_n (\sin^n x)$ then the value of a_0 is -

(A) –1 (B) 1 (C) 0 (D) 2

3. Let $A = \begin{bmatrix} \cos^{-1} x & \cos^{-1} y & \cos^{-1} z \\ \cos^{-1} y & \cos^{-1} z & \cos^{-1} x \\ \cos^{-1} z & \cos^{-1} x & \cos^{-1} y \end{bmatrix}$ such that |A| = 0, then maximum value of x + y + z is

(A) 3 (B) 0 (C) 1 (D) 2

4. If $\begin{vmatrix} x^2 - 2x + 3 & 7x + 2 & x + 4 \\ 2x + 7 & x^2 - x + 2 & 3x \\ 3 & 2x - 1 & x^2 - 4x + 7 \end{vmatrix} = ax^6 + bx^5 + cx^4 + dx^3 + ex^2 + fx + g$ the value of g is

(A) 2 (B) 1 (C) – 2 (D) none of these

5. If $D_p = \begin{vmatrix} p & 15 & 8 \\ p^2 & 35 & 9 \\ p^3 & 25 & 10 \end{vmatrix}$, then $D_1 + D_2 + D_3 + D_4 + D_5$ is equal to -

(A) 0 (B) 25 (C) 625 (D) none of these

6. For any ΔABC, the value of determinant $\begin{vmatrix} \sin^2 A & \cot A & 1 \\ \sin^2 B & \cot B & 1 \\ \sin^2 C & \cot C & 1 \end{vmatrix}$ is equal to -

(A) 0 (B) 1 (C) sin A sin B sin C (D) sin A + sin B + sin C

7. If a_1, a_2, a_3, 5, 4, a_6, a_7, a_8, a_9 are in H.P. and $D = \begin{vmatrix} a_1 & a_2 & a_3 \\ 5 & 4 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$, then the value of [D] is

(Where [.] represents, the greatest integer fn.)

(A) 0 (B) 1 (C) 2 (D) 3

8. Let $\Delta = \begin{vmatrix} \sin\theta\cos\varphi & \sin\theta\sin\varphi & \cos\theta \\ \cos\theta\cos\varphi & \cos\theta\sin\varphi & -\sin\theta \\ -\sin\theta\sin\varphi & \sin\theta\cos\varphi & 0 \end{vmatrix}$, then

(A) Δ is independent of θ (B) Δ is indepedent of ϕ

(C) Δ is a constant (D) none of these

9. If A is a square matrix of order 3 and A′ denotes transpose of matrix A, A′ A = I and det A = 1, then det (A – I) must be equal to

(A) 0 **(B)** – 1 **(C)** 1 **(D)** none of these

10. The number of real values of x satisfying $\begin{vmatrix} x & 3x+2 & 2x-1 \\ 2x-1 & 4x & 3x+1 \\ 7x-2 & 17x+6 & 12x-1 \end{vmatrix} = 0$ is -

(A) 3 **(B)** 0 **(C)** 1 **(D)** infinite

11. The value of the determinant $\begin{vmatrix} a^2 & a & 1 \\ \cos(nx) & \cos(n+1)x & \cos(n+2)x \\ \sin(nx) & \sin(n+1)x & \sin(n+2)x \end{vmatrix}$ is independent of :

(A) n **(B)** a **(C)** x **(D)** a , n and x

12. Three distinct points $P(3u^2, 2u^3)$; $Q(3v^2, 2v^3)$ and $R(3w^2, 2w^3)$ are collinear then

(A) $uv + vw + wu = 0$ **(B)** $uv + vw + wu = 3$

(C) $uv + vw + wu = 2$ **(D)** $uv + ww + wu = 1$

13. The determinant $\Delta = \begin{vmatrix} a^2(1+x) & ab & ac \\ ab & b^2(1+x) & bc \\ ac & bc & c^2(1+x) \end{vmatrix}$ is divisible by

(A) $1 + x$ **(B)** $(1 + x)^2$ **(C)** x^2 **(D)** $x^2 + 1$

14. $\Delta = \begin{vmatrix} 1+a^2+a^4 & 1+ab+a^2b^2 & 1+ac+a^2c^2 \\ 1+ab+a^2b^2 & 1+b^2+b^4 & 1+bc+b^2c^2 \\ 1+ac+a^2c^2 & 1+bc+b^2c^2 & 1+c^2+c^4 \end{vmatrix}$ is equal to

(A) $(a – b)^2 (b – c)^2 (c – a)^2$ **(B)** $2(a – b) (b – c) (c – a)$

(C) $4(a – b) (b – c) (c – a)$ **(D)** $(a + b + c)^3$

15. If $D = \begin{vmatrix} a^2+1 & ab & ac \\ ba & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$ then D =

(A) $1 + a^2 + b^2 + c^2$ **(B)** $a^2 + b^2 + c^2$ **(C)** $(a + b + c)^2$ **(D)** none

16. Let $f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 1+\cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 1+4\sin 2x \end{vmatrix}$ then the maximum value of f(x) is

(A) 4 **(B)** 6 **(C)** 8 **(D)** 12

17. The determinant $\begin{vmatrix} b_1+c_1 & c_1+a_1 & a_1+b_1 \\ b_2+c_2 & c_2+a_2 & a_2+b_2 \\ b_3+c_3 & c_3+a_3 & a_3+b_3 \end{vmatrix} =$

(A) $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ **(B)** $2\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ **(C)** $3\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ **(D)** none of these

18. If a, b, c are pth, qth and rth terms of a GP, then $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix}$ is equal to -

(A) 0 **(B)** 1 **(C)** log abc **(D)** pqr

19. If $a_1, a_2, \ldots\ldots a_n, a_{n+1}, \ldots\ldots$ are in GP and $a_i > 0\ \forall i$, then $\begin{vmatrix} \log a_n & \log a_{n+2} & \log a_{n+4} \\ \log a_{n+6} & \log a_{n+8} & \log a_{n+10} \\ \log a_{n+12} & \log a_{n+14} & \log a_{n+16} \end{vmatrix}$ is equal to -

(A) 0 **(B)** $n \log a_n$ **(C)** $n(n+1) \log a_n$ **(D)** none of these

20. The determinant $\begin{vmatrix} \cos(\theta+\varphi) & -\sin(\theta+\varphi) & \cos 2\varphi \\ \sin\theta & \cos\theta & \sin\varphi \\ -\cos\theta & \sin\theta & \cos\varphi \end{vmatrix}$ is:

(A) 0 **(B)** independent of θ **(C)** independent of ϕ **(D)** independent of θ & ϕ both

21. If A is a non singular maxrix satisfying AB – BA = A, then which one of the following holds true

(A) det. B = 0 **(B)** B = 0 **(C)** det. A = 1 **(D)** det. (B + I) = det. (B – I)

22. Let a determinant is given by $A = \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$ and suppose det. A = 6. If $B = \begin{vmatrix} p+x & q+y & r+z \\ a+x & b+y & c+z \\ a+p & b+q & c+r \end{vmatrix}$ then

(A) det. B = 6 **(B)** det. B = – 6 **(C)** det. B = 12 **(D)** det. B = – 12

23. Let $D_1 = \begin{vmatrix} a & b & a+b \\ c & d & c+d \\ a & b & a-b \end{vmatrix}$ and $D_2 = \begin{vmatrix} a & c & a+c \\ b & d & b+d \\ a & c & a+b+c \end{vmatrix}$ then the value of $\frac{D_1}{D_2}$ where $b \neq 0$ and $ad \neq bc$, is

(A) – 2 **(B)** 0 **(C)** – 2b **(D)** 2b

24. If $\Delta_1 = \begin{vmatrix} 2a & b & e \\ 2d & e & f \\ 4x & 2y & 2z \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} f & 2d & e \\ 2z & 4x & 2y \\ e & 2a & b \end{vmatrix}$, then the value of $\Delta_1 - \Delta_2$ is

(A) $x + \frac{y}{2} + z$ **(B)** 2 **(C)** 0 **(D)** 3

25. If a, b, c are non zeros, then the system of equations

$$(\alpha + a)x + \alpha y + \alpha z = 0$$
$$\alpha x + (\alpha + b)y + \alpha z = 0$$
$$\alpha x + \alpha y + (\alpha + c)z = 0$$

has a non-trivial solution if

(A) $\alpha^{-1} = -(a^{-1} + b^{-1} + c^{-1})$ **(B)** $\alpha^{-1} = a + b + c$
(C) $\alpha + a + b + c = 1$ **(D)** none of these

26. If A, B are two $n \times n$ non-singular matrices, then

(A) AB is non-singular **(B)** AB is singular
(C) $(AB)^{-1} = A^{-1} B^{-1}$ **(D)** $(AB)^{-1}$ does not exist

27. The equation $\begin{vmatrix} (1+x)^2 & (1-x)^2 & -(2+x^2) \\ 2x+1 & 3x & 1-5x \\ x+1 & 2x & 2-3x \end{vmatrix} + \begin{vmatrix} (1+x)^2 & 2x+1 & x+1 \\ (1-x)^2 & 3x & 2x \\ 1-2x & 3x-2 & 2x-3 \end{vmatrix} = 0$

(A) has no real solution **(B)** has 4 real solutions
(C) has two real and two non-real solutions **(D)** has infinite number of solutions, real or non-real

28. The system of linear equations $x + y - z = 6$, $x + 2y - 3z = 14$ and $2x + 5y - \lambda z = 9$ $(\lambda \in R)$ has a unique solution if

(A) $\lambda = 8$ **(B)** $\lambda \neq 8$ **(C)** $\lambda = 7$ **(D)** $\lambda \neq 7$

29. The value of ' k ' for which the set of equations $3x + ky - 2z = 0$, $x + ky + 3z = 0$, $2x + 3y - 4z = 0$ has a non – trivial solution over the set of rational is:

(A) 33/2 **(B)** 31/2 **(C)** 16 **(D)** 15

30. If $U_n = \begin{vmatrix} n & 1 & 5 \\ n^2 & 2N+1 & 2N+1 \\ n^3 & 3N^2 & 3N+1 \end{vmatrix}$, then $\sum_{n=1}^{N} U_n$ is equal to

(A) $2\sum_{n=1}^{N} n$ **(B)** $2\sum_{n=1}^{N} n^2$ **(C)** $\frac{1}{2}\sum_{n=1}^{N} n^2$ **(D)** 0

31. If the system of equations $x + 2y + 3z = 4$, $x + py + 2z = 3$, $x + 4y + \mu z = 3$ has an infinite number of solutions, then:

(A) $p = 2, \mu = 3$ **(B)** $p = 2, \mu = 4$ **(C)** $3p = 2\mu$ **(D)** $p = 4, \mu = 2$

32. If $a \neq b \neq c$ and $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix} = 0$ then -

(A) $a + b + c = 0$ **(B)** $ab + bc + ca = 0$

(C) $a^2 + b^2 + c^2 = ab + bc + ca$ **(D)** $abc = 0$

33. If a, b, & c are nonzero real numbers, then $\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix}$ is equal to -

(A) $a^2b^2c^2(a + b + c)$ **(B)** $abc(a + b + c)^2$ **(C)** zero **(D)** none of these

34. An equilateral triangle has each of its sides of length 6 cm. If (x_1, y_1); (x_2, y_2) & (x_3, y_3) are its vertices then the value of the determinant, $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2$ is equal to -

(A) 192 **(B)** 243 **(C)** 486 **(D)** 972

35. If the system of equations $x + 2y + 3z = 4$, $x + py + 2z = 3$, $\mu x + 4y + z = 3$ has an infinite number of solutions, then -

(A) $p = 2, \mu = 3$ **(B)** $p = 2, \mu = 4$ **(C)** $3p = 2\mu$ **(D)** none of these

36. The values of θ, λ for which the following equations

$\sin\theta x - \cos\theta y + (\lambda+1)z = 0$; $\cos\theta x + \sin\theta y - \lambda z = 0$; $\lambda x + (\lambda + 1)y + \cos\theta\, z = 0$ have non trivial solution, is

(A) $\theta = n\pi, \lambda \in R - \{0\}$ **(B)** $\theta = 2n\pi$, λ is any rational number

(C) $\theta = (2n + 1)\pi, \lambda \in R^+, n \in I$ **(D)** $\theta = (2n + 1)\frac{\pi}{2}, \lambda \in R, n \in I$

37. The set of equations

$$\lambda x - y + (\cos\theta)\, z = 0$$
$$3x + y + 2z = 0$$
$$(\cos\theta)x + y + 2z = 0$$

$0 \leq \theta < 2\pi$, has non- trivial solution(s)

(A) for no value of λ and θ **(B)** for all values of λ and θ

(C) for all values of λ and only two values of θ **(D)** for only one value of λ and all values of θ

38. If A, B and C are $n \times n$ matrices and det(A) = 2, det(B) = 3 and det(C) = 5, then the value of the $\det(A^2BC^{-1})$ is equal to

(A) $\frac{6}{5}$ **(B)** $\frac{12}{5}$ **(C)** $\frac{18}{5}$ **(D)** $\frac{24}{5}$

39. If a, b, c are real then the value of determinant $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1$ if

(A) $a + b + c = 0$ **(B)** $a + b + c = 1$ **(C)** $a + b + c = -1$ **(D)** $a = b = c = 0$

40. The system of equations $(\sin\theta)x + 2z = 0$, $(\cos\theta)x + (\sin\theta)y = 0$, $(\cos\theta)y + 2z = a$ has

(A) no unique solution

(B) a unique solution which is a function of a and θ

(C) a unique solution which is independent of a and θ

(D) a unique solution which is independent of θ only

41. If the system of equations

$x - 2y + z = a$

$2x + y - 2z = b$

and $x + 3y - 3z = c$

have atleast one solution, then the relationship between a, b and c is

(A) $a + b + c = 0$ **(B)** $a - b + c = 0$ **(C)** $-a + b + c = 0$ **(D)** $a + b - c = 0$

42. The equation $\begin{vmatrix} (1+x)^2 & (1-x)^2 & -(2+x^2) \\ 2x+1 & 3x & 1-5x \\ x+1 & 2x & 2-3x \end{vmatrix} + \begin{vmatrix} (1+x)^2 & 2x+1 & x+1 \\ (1-x)^2 & 3x & 2x \\ 1-2x & 3x-2 & 2x-3 \end{vmatrix} = 0$

(A) has no real solution

(B) has 4 real solutions

(C) has two real and two non-real solutions

(D) has infinite number of solutions , real or non-real

43. Three digit numbers x17, 3y6 and 12z where x, y, z are integers from 0 to 9, are divisible by a fixed constant k. Then

the determinant $\begin{vmatrix} x & 3 & 1 \\ 7 & 6 & z \\ 1 & y & 2 \end{vmatrix}$ must be divisible by

(A) k **(B)** k^2 **(C)** k^3 **(D)** None

Exercise # 2 Part # I [Multiple Correct Choice Type Questions]

1. Which of the following determinant(s) vanish(es) ?

(A) $\begin{vmatrix} 1 & bc & bc(b+c) \\ 1 & ca & ca(c+a) \\ 1 & ab & ab(a+b) \end{vmatrix}$ **(B)** $\begin{vmatrix} 1 & ab & \frac{1}{a}+\frac{1}{b} \\ 1 & bc & \frac{1}{b}+\frac{1}{c} \\ 1 & ca & \frac{1}{c}+\frac{1}{a} \end{vmatrix}$

(C) $\begin{vmatrix} 0 & a-b & a-c \\ b-a & 0 & b-c \\ c-a & c-b & 0 \end{vmatrix}$ **(D)** $\begin{vmatrix} \log_x xyz & \log_x y & \log_x z \\ \log_y xyz & 1 & \log_y z \\ \log_z xyz & \log_z y & 1 \end{vmatrix}$

2. If $D(x) = \begin{vmatrix} \sin 2x & e^x \sin x + x\cos x & \sin x + x^2 \cos x \\ \cos x + \sin x & e^x + x & 1 + x^2 \\ e^x \cos x & e^{2x} & e^x \end{vmatrix}$, then the value of $|\ell n \cos (Dx)|$ will be -

(A) independent of x **(B)** dependent on x

(C) 0 **(D)** non-existent

3. The determinant $\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix}$ is equal to zero, if -

(A) a, b, c are in AP **(B)** a, b, c are in GP

(C) α is a root of the equation $ax^2 + bx + c = 0$ **(D)** $(x-\alpha)$ is a factor of $ax^2 + 2bx + c$

4. If the system of linear equations x + ay + az = 0, x + by + bz = 0, x + cy + cz = 0 has a non-zero solution then

(A) System has always non-trivial solutions.

(B) System is consistent only when a = b = c

(C) If $a \neq b \neq c$ then $x = 0, y = t, z = -t \ \forall\ t \in R$

(D) If a = b = c then $y = t_1, z = t_2, x = -a(t_1 + t_2) \ \forall\ t_1, t_2 \in R$

5. The value of θ lying between $\theta = 0$ & $\theta = \pi/2$ & satisfying the equation :

$$\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0 \text{ are :}$$

(A) $\frac{7\pi}{24}$ **(B)** $\frac{5\pi}{24}$ **(C)** $\frac{11\pi}{24}$ **(D)** $\frac{\pi}{24}$

6. Let a, b > 0 and $\Delta = \begin{vmatrix} -x & a & b \\ b & -x & a \\ a & b & -x \end{vmatrix}$, then

(A) $a + b - x$ is a factor of Δ
(B) $x^2 + (a + b)x + a^2 + b^2 - ab$ is a factor of Δ
(C) $\Delta = 0$ has three real roots if a = b
(D) none of these

7. The determinant $\Delta = \begin{vmatrix} b & c & b\alpha + c \\ c & d & c\alpha + d \\ b\alpha + c & c\alpha + d & a\alpha^3 - c\alpha \end{vmatrix}$ is equal to zero if

(A) b, c, d are in A.P.
(B) b, c, d are in G.P.
(C) b, c, d are in H.P.
(D) α is a root of $ax^3 - bx^2 - 3cx - d = 0$

8. Let $\Delta = \begin{vmatrix} a & a^2 & 0 \\ 1 & 2a + b & (a + b)^2 \\ 0 & 1 & 2a + 3b \end{vmatrix}$, then

(A) a + b is a factor of Δ
(B) a + 2b is a factor of Δ
(C) 2a + 3b is a factor of Δ
(D) a^2 is a factor of Δ

9. The value of θ lying between $-\frac{\pi}{4}$ & $\frac{\pi}{2}$ and $0 \le A \le \frac{\pi}{2}$ and satisfying the equation

$$\begin{vmatrix} 1 + \sin^2 A & \cos^2 A & 2\sin 4\theta \\ \sin^2 A & 1 + \cos^2 A & 2\sin 4\theta \\ \sin^2 A & \cos^2 A & 1 + 2\sin 4\theta \end{vmatrix} = 0 \text{ are -}$$

(A) $A = \frac{\pi}{4}, \quad \theta = -\frac{\pi}{8}$
(B) $A = \frac{3\pi}{8} = \theta$
(C) $A = \frac{\pi}{5}, \quad \theta = -\frac{\pi}{8}$
(D) $A = \frac{\pi}{6}, \quad \theta = \frac{3\pi}{8}$

10. If $\begin{vmatrix} a & a & x \\ m & m & m \\ b & x & b \end{vmatrix} = 0$, then x may be equal to -

(A) a
(B) b
(C) a + b
(D) m

11. The value of the determinant $\begin{vmatrix} \alpha & \beta & \ell \\ \alpha & x & n \\ \alpha & \beta & x \end{vmatrix}$ is

(A) independent of ℓ
(B) independent of n
(C) $\alpha(x - \ell)(x - \beta)$
(D) $\alpha\beta(x - \ell)(x - n)$

12. The determinant $\begin{vmatrix} a^2 & a^2 - (b - c)^2 & bc \\ b^2 & b^2 - (c - a)^2 & ca \\ c^2 & c^2 - (a - b)^2 & ab \end{vmatrix}$ is divisible by -

(A) a + b + c
(B) (a + b) (b + c) (c + a)
(C) $a^2 + b^2 + c^2$
(D) (a – b)(b – c) (c – a)

13. Let A and B be two 2 × 2 matrix with real entries. If AB = O and tr(A) = tr(B) = 0 then

(A) A and B are comutative w.r.t. operation of multiplication.

(B) A and B are not commutative w.r.t. operation of multiplication.

(C) A and B are both null matrices.

(D) BA = 0

14. If $A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -1 \end{bmatrix}$, then

(A) $|A| = 2$ **(B)** A is non-singular

(C) $\text{Adj. } A = \begin{bmatrix} 1/2 & -1/2 & 0 \\ 0 & -1 & 1/2 \\ 0 & 0 & -1/2 \end{bmatrix}$ **(D)** A is skew symmetric matrix

15. If the system of equations $x + y - 3 = 0, (1 + K)x + (2 + K)y - 8 = 0$ & $x - (1 + K)y + (2 + K) = 0$ is consistent then the value of K may be -

(A) 1 **(B)** $\frac{3}{5}$ **(C)** $-\frac{5}{3}$ **(D)** 2

16. The set of equations $x - y + 3z = 2, 2x - y + z = 4, x - 2y + \alpha z = 3$ has -

(A) unique solution only for $\alpha = 0$

(B) unique solution for $\alpha \neq 8$

(C) infinite number of solutions of $\alpha = 8$

(D) no solution for $\alpha = 8$

17. The determinant

$$\Delta = \begin{vmatrix} a^2 + x^2 & ab & ac \\ ab & b^2 + x^2 & bc \\ ac & bc & c^2 + x^2 \end{vmatrix} \text{ is divisible by}$$

(A) x **(B)** x^2 **(C)** x^3 **(D)** x^4

18. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ (where $bc \neq 0$) satisfies the equations $x^2 + k = 0$, then

(A) $a + d = 0$ **(B)** $k = -|A|$ **(C)** $k = |A|$ **(D)** none of these

19. If the system of equations, $a^2x - by = a^2 - b$ & $bx - b^2y = 2 + 4b$ possess an infinite number of solutions then the possible values of 'a' and 'b' are

(A) $a = 1, b = -1$ **(B)** $a = 1, b = -2$

(C) $a = -1, b = -1$ **(D)** $a = -1, b = -2$

20. If p, q, r, s are in A.P. and $f(x) = \begin{vmatrix} p+\sin x & q+\sin x & p-r+\sin x \\ q+\sin x & r+\sin x & -1+\sin x \\ r+\sin x & s+\sin x & s-q+\sin x \end{vmatrix}$

such that $\int_0^2 f(x)dx = -4$ then the common difference of the A.P. can be :

(A) -1 **(B)** $\frac{1}{2}$ **(C)** 1 **(D)** 2

Part # II [Assertion & Reason Type Questions]

These questions contains, Statement I (assertion) and Statement II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.
(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.
(C) Statement-I is true, Statement-II is false.
(D) Statement-I is false, Statement-II is true.

1. **Statement - I :** Consider the system of equations,

$2x + 3y + 4z = 5$ $x + y + z = 1$ $x + 2y + 3z = 4$

This system of equations has infinite solutions.

Statement - II : If the system of equations is

$e_1 : a_1x + b_1y + c_1z - d_1 = 0$

$e_2 : a_2x + b_2y + c_2z - d_2 = 0$

$e_3 : e_1 + \lambda e_2 = 0$, where $\lambda \in R$ & $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Then such system of equations has infinite solutions.

2. **Statement - I :** Consider $D = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

Let B_1, B_2, B_3 be the co-factors of b_1, b_2, and b_3 respectively then $a_1B_1 + a_2B_2 + a_3B_3 = 0$

Statement - II : If any two rows (or columns) in a determinant are identical then value of determinant is zero.

3. **Statement - I :** If a, b, c $\in$ R and $a \neq b \neq c$ and x,y,z are non zero. Then the system of equations

$ax + by + cz = 0$

$bx + cy + az = 0$

$cx + ay + bz = 0$ has infinite solutions.

Statement - II : If the homogeneous system of equations has non trivial solution, then it has infinitely many solutions.

4. **Statement-I :** If $A = \begin{bmatrix} 2 & 1+2i \\ 1-2i & 7 \end{bmatrix}$ then det(A) is real.

Statement-II : If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, a_{ij} being complex numbers then det(A) is always real.

Exercise # 3 Part # I [Matrix Match Type Questions]

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with one or more statement(s) in **Column-II**.

1. **Column – I** **Column – II**

Column – I		Column – II	
(A)	If A is a skew–symmetric matrix of odd order then det(A) is	**(p)**	7
(B)	If A is a square matrix such that $A^2 = A$ and $(I + A)^3 = I + kA$, then k is equal to	**(q)**	8
(C)	If A and B are two invertible matrices such that $AB = C$ and $\|A\| = 2$, $\|C\| = -2$,then det(B) is	**(r)**	0
(D)	If $A = [a_{ij}]_{3\times3}$ is a scalar matrix with $a_{11} = a_{22} = a_{33} = 2$ and $A(\text{adj}A) = kI_3$ then k is	**(s)**	– 1

2. **Column-I** **Column-II**

(A) If the determinant $\begin{vmatrix} a+p & \ell+x & u+f \\ b+q & m+y & v+g \\ c+r & n+z & w+h \end{vmatrix}$ splits into exactly K determinants of order 3, each element of which contains only one term, then the values of K is — **(p)** 3

(B) The values of λ for which the system of equations

$x + y + z = 6,$

$x + 2y + 3z = 10$

& $x + 2y + \lambda z = 12$

is inconsistent — **(q)** 8

(C) If x, y, z are in A.P. then the value of the determinant

$\begin{vmatrix} a+2 & a+3 & a+2x \\ a+3 & a+4 & a+2y \\ a+4 & a+5 & a+2z \end{vmatrix}$ is — **(r)** 5

(D) Let p be the sum of all possible determinants of order 2 having 0, 1, 2 & 3 as their four elements (without repeatition of digits). The value of 'p' is — **(s)** 0

Part # II [Comprehension Type Questions]

Comprehension # 1

Consider the system of linear equations

$\alpha x + y + z = m$

$x + \alpha y + z = n$

and $x + y + \alpha z = p$

On the basis of above information, answer the following questions :

1. If $\alpha \neq 1, -2$ then the system has -

(A) no solution **(B)** infinte solutions

(C) unique solution **(D)** trivial solution if $m \neq n \neq p$

2. If $\alpha = -2$ & $m + n + p \neq 0$ then system of linear equations has -

(A) no solution **(B)** infinite solutions **(C)** unique solution **(D)** finitely many solution

3. If $\alpha = 1$ & $m \neq p$ then the system of linear equations has -

(A) no solution **(B)** infinite solutions **(C)** unique solution **(D)** unique solution if $p = n$

Comprehension # 2

Let $x, y, z \in R^{+}$ & $D = \begin{vmatrix} x & x^3 & x^4-1 \\ y & y^3 & y^4-1 \\ z & z^3 & z^4-1 \end{vmatrix}$

On the basis of above information, answer the following questions

1. If $x \neq y \neq z$ & x, y, z are in GP and D = 0, then y is equal to -

(A) 1 **(B)** 2 **(C)** 4 **(D)** none of these

2. If x, y, z are the roots of $t^3 - 21t^2 + bt - 343 = 0$, $b \in R$, then D is equal to-

(A) 1 **(B)** 0 **(C)** dependent on x, y, z **(D)** data inadequate

3. If $x \neq y \neq z$ & x, y, z are in A.P. and D = 0, then $2xy^2z + x^2z^2$ is equal to-

(A) 1 **(B)** 2 **(C)** 3 **(D)** none of these

Exercise # 4

[Subjective Type Questions]

1. Without expanding the determinant prove that :

(A) $\begin{vmatrix} 0 & b & -c \\ -b & 0 & a \\ c & -a & 0 \end{vmatrix} = 0$ **(B)** $\begin{vmatrix} 0 & p-q & p-r \\ q-p & 0 & q-r \\ r-p & r-q & 0 \end{vmatrix} = 0$

2. Prove that :

(A) $\begin{vmatrix} ax & by & cz \\ x^2 & y^2 & z^2 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} a & b & c \\ x & y & z \\ yz & zx & xy \end{vmatrix}$ **(B)** $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix} = 0$

3. If $a^2 + b^2 + c^2 = 1$, then prove that $\begin{vmatrix} a^2+(b^2+c^2)\cos\phi & ab(1-\cos\phi) & ac(1-\cos\phi) \\ ba(1-\cos\phi) & b^2+(c^2+a^2)\cos\phi & bc(1-\cos\phi) \\ ca(1-\cos\phi) & cb(1-\cos\phi) & c^2+(a^2+b^2)\cos\phi \end{vmatrix}$

is independent of a, b, c

4. Prove that $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$.

5. If a, b, c, x, y, z $\in$ R, then prove that,

$$\begin{vmatrix} (a-x)^2 & (b-x)^2 & (c-x)^2 \\ (a-y)^2 & (b-y)^2 & (c-y)^2 \\ (a-z)^2 & (b-z)^2 & (c-z)^2 \end{vmatrix} = \begin{vmatrix} (1+ax)^2 & (1+bx)^2 & (1+cx)^2 \\ (1+ay)^2 & (1+by)^2 & (1+cy)^2 \\ (1+az)^2 & (1+bz)^2 & (1+cz)^2 \end{vmatrix}.$$

6. Using properties of determinants, prove that $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 2(a+b+c)(ab+bc+ca-a^2-b^2-c^2)$.

7. Prove that : $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$

8. Prove that $\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2)$

9. For a fixed positive integer n, if $D = \begin{vmatrix} n! & (n+1)! & (n+2)! \\ (n+1)! & (n+2)! & (n+3)! \\ (n+2)! & (n+3)! & (n+4)! \end{vmatrix}$ then show that $\left[\dfrac{D}{(n!)^3} - 4\right]$ is divisible by n.

10. If $D_r = \begin{vmatrix} 2^{r-1} & 2(3^{r-1}) & 4(5^{r-1}) \\ x & y & z \\ 2^n - 1 & 3^n - 1 & 5^n - 1 \end{vmatrix}$ then prove that $\sum_{r=1}^{n} D_r = 0$.

11. Using the properties of determinants, evaluate the following : $\begin{vmatrix} 0 & ab^2 & ac^2 \\ a^2b & 0 & bc^2 \\ a^2c & cb^2 & 0 \end{vmatrix}$.

12. Using properties of determinants prove the following : $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3$

13. Show that $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$, where a, b, c are in A.P.

14. Solve the following sets of equations using Cramer's rule and remark about their consistency.

(A) $\begin{matrix} x + y + z - 6 = 0 \\ 2x + y - z - 1 = 0 \\ x + y - 2z + 3 = 0 \end{matrix}$

(B) $\begin{matrix} x + 2y + z = 1 \\ 3x + y + z = 6 \\ x + 2y = 0 \end{matrix}$

(C) $\begin{matrix} x - 3y + z = 2 \\ 3x + y + z = 6 \\ 5x + y + 3z = 3 \end{matrix}$

(D) $\begin{matrix} 7x - 7y + 5z = 3 \\ 3x + y + 5z = 7 \\ 2x + 3y + 5z = 5 \end{matrix}$

15. Let α_1, α_2 and β_1, β_2 be the roots of $ax^2 + bx + c = 0$ and $px^2 + qx + r = 0$ respectively. If the system of equations $\alpha_1 y + \alpha_2 z = 0$ and $\beta_1 y + \beta_2 z = 0$ has a non-trivial solution, then prove that $\frac{b^2}{q^2} = \frac{ac}{pr}$.

Exercise # 5

Part # I [Previous Year Questions] [AIEEE/JEE-MAIN]

1. If a, b, c are pth, qth and rth terms of a GP, and all are positive then $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix}$ is equal to- **[AIEEE-2002]**

(1) 0 **(2)** 1 **(3)** log abc **(4)** pqr

2. If 1, ω, ω^2 are cube roots of unity and $n \neq 3p$, $p \in Z$, then $\begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^{2n} & 1 & \omega^n \\ \omega^n & \omega^{2n} & 1 \end{vmatrix}$ is equal to- **[AIEEE-2003]**

(1) 0 **(2)** ω **(3)** ω^2 **(4)** 1

3. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and vectors $(1, a, a^2)$, $(1, b, b^2)$ and $(1, c, c^2)$ are non-coplanar, then the product abc equals-

(1) 1 **(2)** 0 **(3)** 2 **(4)** –1 **[AIEEE-2003]**

4. If $a_1, a_2, \ldots\ldots a_n, a_{n+1}, \ldots\ldots$ are in GP and $a_i > 0\ \forall i$, then $\begin{vmatrix} \log a_n & \log\ a_{n+2} & \log\ a_{n+4} \\ \log\ a_{n+6} & \log\ a_{n+8} & \log\ a_{n+10} \\ \log\ a_{n+12} & \log\ a_{n+14} & \log\ a_{n+16} \end{vmatrix}$ is equal to-

[AIEEE-2004,05]

(1) 0 **(2)** n log a_n **(3)** n(n + 1) log a_n **(4)** none of these

5. If $a^2 + b^2 + c^2 = -2$ and $f(x) = \begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$, then f(x) is a polynomial of degree- **[AIEEE 2005]**

(1) 2 **(2)** 3 **(3)** 0 **(4)** 1

6. The system of equations $\alpha x + y + z = \alpha - 1;\ \ x + \alpha y + z = \alpha - 1;\ \ x + y + \alpha z = \alpha - 1$ has no solution, If α is
[AIEEE 2005]

(1) 1 **(2)** not –2 **(3)** either -2 or 1 **(4)** –2

7. If $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ for $x \neq 0$, $y \neq 0$ then D is- **[AIEEE - 2007]**

(1) Divisible by both x and y **(2)** Divisible by x but not y
(3) Divisible by y but not x **(4)** Divsible by neither x nor y

8. Let $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$, if $|A^2| = 25$ then $|\alpha|$ equals- **[AIEEE - 2007]**

(1) 5 **(2)** 5^2 **(3)** 1 **(4)** 1/5

9. Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that

$x = cy + bz$, $y = az + cx$ and $z = bx + ay$, then $a^2 + b^2 + c^2 + 2abc$ is equal to **[AIEEE - 2008]**

(1) 2 **(2)** –1 **(3)** 0 **(4)** 1

10. Let a, b, c be such that $b(a + c) \neq 0$. If $\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix} = 0,$

then the value of n is :- **[AIEEE - 2009]**

(1) Any odd integer **(2)** Any integer **(3)** Zero **(4)** Any even integer

11. Consider the system of linear equations :

$x_1 + 2x_2 + x_3 = 3$

$2x_1 + 3x_2 + x_3 = 3$

$3x_1 + 5x_2 + 2x_3 = 1$

The system has **[AIEEE - 2010]**

(1) Infinite number of solutions **(2)** Exactly 3 solutions

(3) A unique solution **(4)** No solution

12. The number of values of k for which the linear equations

$4x + ky + 2z = 0$

$kx + 4y + z = 0$

$2x + 2y + z = 0$

possess a non-zero solution is :- **[AIEEE - 2011]**

(1) 1 **(2)** zero **(3)** 3 **(4)** 2

13. If the trivial solution is the only solution of the system of equations

$x - ky + z = 0$

$kx + 3y - kz = 0$

$3x + y - z = 0$

Then the set of all values of k is: **[AIEEE - 2011]**

(1) $\{2, -3\}$ **(2)** $R - \{2, -3\}$ **(3)** $R - \{2\}$ **(4)** $R - \{-3\}$

14. If $\alpha, \beta \neq 0$, and $f(n) = \alpha^n + \beta^n$ and $\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix} = K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$, then K is equal to :

(1) $\alpha\beta$ **(2)** $\dfrac{1}{\alpha\beta}$ **(3)** 1 **(4)** -1 **[Main 2014]**

15. The set of all values of λ for which the system of linear equations : **[Main 2015]**

$2x_1 - 2x_2 + x_3 = \lambda x_1$

$2x_1 - 3x_2 + 2x_3 = \lambda x_2$

$-x_1 + 2x_2 = \lambda x_3$

has a non-trivial solution,

(1) contains two elements **(2)** contains more than two elements..

(3) is an empty set. **(4)** is a singleton

Part # II [Previous Year Questions][IIT-JEE ADVANCED]

1. Solve for x the equation $\begin{vmatrix} a^2 & a & 1 \\ \sin(n+1)x & \sin nx & \sin(n-1)x \\ \cos(n+1)x & \cos nx & \cos(n-1)x \end{vmatrix} = 0$ **[REE 2001, (Mains)]**

2. Test the consistency and solve them when consistent, the following system of equations for all values of λ :

$x + y + z = 1$

$x + 3y - 2z = \lambda$

$3x + (\lambda + 2)y - 3z = 2\lambda + 1$ **[JEE 2001, (Mains)]**

3. Let a, b, c, be real numbers with $a^2 + b^2 + c^2 = 1$, Show that the equation **[JEE 2001, (Mains)]**

$\begin{vmatrix} ax-by-c & bx+ay & cx+a \\ bx+ay & -ax+by-c & cy+b \\ cx+a & cy+b & -ax-by+c \end{vmatrix} = 0$ represents a straight line.

4. The number of values of k for which the system of equations

$(k+1)x + 8y = 4k$

$kx + (k+3)y = 3k - 1$

has infinitely many solutions is **[JEE 2002, (Screening)]**

(A) 0 **(B)** 1 **(C)** 2 **(D)** infinite

5. The value of λ for which the system of equations $2x - y - z = 12$, $x - 2y + z = -4$, $x + y + \lambda z = 4$ has no solution is **[JEE 2004 (Screening)]**

(A) 3 **(B)** –3 **(C)** 2 **(D)** –2

6. (a) Consider three point $P = (-\sin(\beta - \alpha), -\cos\beta)$, $Q = (\cos(\beta - \alpha), \sin\beta)$ and

$R = (\cos(\beta - \alpha + \theta), \sin(\beta - \theta))$, where $0 < \alpha, \beta, \theta < \pi/4$

(A) P lies on the line segment RQ **(B)** Q lies on the line segment PR

(C) R lies on the line segment QP **(D)** P, Q, R are non collinear

(b) Consider the system of equations $x - 2y + 3z = -1$; $-x + y - 2z = k$; $x - 3y + 4z = 1$.

Statement-I : The system of equations has no solution for $k \neq 3$.

Statement-II : The determinant $\begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} \neq 0$, for $k \neq 3$. **[JEE 2008]**

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.

(C) Statement-I is true, Statement-II is false.

(D) Statement-I is false, Statement-II is true.

7. The number of all possible values of θ, where $0 < \theta < \pi$, for which the system of equations

$$(y+z)\cos 3\theta = (xyz)\sin 3\theta$$

$$x \sin 3\theta = \frac{2\cos 3\theta}{y} + \frac{2\sin 3\theta}{z}$$

$$(xyz)\sin 3\theta = (y+2z)\cos 3\theta + y\sin 3\theta$$

have a solution (x_0, y_0, z_0) with $y_0 z_0 \neq 0$, is **[JEE 2010]**

8. Let M and N be two 3×3 matrices such that MN = NM. Further, if $M \neq N^2$ and $M^2 = N^4$, then **[JEE Ad. 2014]**

(A) determinant of $(M^2 + MN^2)$ is 0

(B) there is a 3×3 non-zero matrix U such that $(M^2 + MN^2)U$ is the zero matrix

(C) determinant of $(M^2 + MN^2) \geq 1$

(D) for a 3×3 matrix U, if $(M^2 + MN^2)U$ equal the zero matrix then U is the zero matrix

9. Which of the following values of α satisfy the equation **[JEE Ad. 2015]**

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2 \\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2 \end{vmatrix} = 648\alpha$$

(A) – 4 **(B)** 9 **(C)** – 9 **(D)** 4

10. The total number of distinct $x \in R$ for which $\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10$ is **[JEE Ad. 2016]**

11. Let $a, \lambda, \mu, \in R$. Consider the system of linear equations

$$ax + 2y = \lambda$$

$$3x - 2y = \mu$$

Which of the following statements (s) is (are) correct ? **[JEE Ad. 2016]**

(A) If $a = -3$, then the system has infinitely many solutions for all values of λ and μ

(B) If $a \neq -3$, then the system has a unique solution for all values of λ and μ

(C) If $\lambda + \mu, = 0$, then the system has infinitely many solutions for $a = -3$

(D) If $\lambda + \mu, \neq 0$, then the system has no solution for $a = -3$

MOCK TEST

SECTION - I : STRAIGHT OBJECTIVE TYPE

1. If A is a square matrix of order 3 such that |A| = 2 then $|(\text{adj } A^{-1})^{-1}|$ is

(A) 1 **(B)** 2 **(C)** 4 **(D)** 8

2. If $\begin{vmatrix} x^k & x^{k+2} & x^{k+3} \\ y^k & y^{k+2} & y^{k+3} \\ z^k & z^{k+2} & z^{k+3} \end{vmatrix} = (x-y)(y-z)(z-x)\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$, then:

(A) $k=-3$ **(B)** $k=-1$ **(C)** $k=1$ **(D)** $k=3$

3. If $p+q+r=0=a+b+c$, then the value of the determinant $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix}$ is

(A) 0 **(B)** $pa+qb+rc$ **(C)** 1 **(D)** none of these

4. If the determinant $\begin{vmatrix} a+p & \ell+x & u+f \\ b+q & m+y & v+g \\ c+r & n+z & w+h \end{vmatrix}$ splits into exactly K determinants of order 3, each element of which contains only one term, then the value of K =

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9

5. $|A_{3\times3}| = 3$, $|B_{3\times3}| = -1$ and $|C_{2\times2}| = +2$ then $|2ABC| =$

(A) $2^3(6)$ **(B)** $2^3(-6)$ **(C)** $2(-6)$ **(D)** none of these

6. If a, b, c are all different and $\begin{vmatrix} a & a^3 & a^4-1 \\ b & b^3 & b^4-1 \\ c & c^3 & c^4-1 \end{vmatrix} = 0$, then:

(A) $abc(ab+bc+ca) = a+b+c$ **(B)** $(a+b+c)(ab+bc+ca) = abc$

(C) $abc(a+b+c) = ab+bc+ca$ **(D)** none of these

7. If A is a non-diagonal involutory matrix, then

(A) $A-I=O$ **(B)** $A+I=O$

(C) $A-I$ is non zero singular **(D)** none of these

8. If $f(x) = \log_{10}x$ and $g(x) = e^{i\pi x}$ and $h(x) = \begin{vmatrix} f(x)g(x) & [f(x)]^{g(x)} & 1 \\ f(x^2)g(x^2) & [f(x^2)]^{g(x^2)} & 0 \\ f(x^3)g(x^3) & [f(x^3)]^{g(x^3)} & 1 \end{vmatrix}$, then the value of h(10) is

(A) 0 **(B)** 2 **(C)** 1 **(D)** 4

9. If a, b, c, are real numbers, and $D = \begin{vmatrix} a & 1+2i & 3-5i \\ 1-2i & b & -7-3i \\ 3+5i & -7+3i & c \end{vmatrix}$ then D is

(A) purely real **(B)** purely imaginary **(C)** non real **(D)** integer

10. **S_1 :** The value of the determinant $D = \begin{vmatrix} \ell nx & \ell ny & \ell nz \\ \ell n2x & \ell n2y & \ell n2z \\ \ell n3x & \ell n3y & \ell n3z \end{vmatrix}$ is $\ell n\,216\,xyz$.

S_2 : The roots of $\begin{vmatrix} x & a & b & 1 \\ \lambda & x & b & 1 \\ \lambda & \mu & x & 1 \\ \lambda & \mu & \nu & 1 \end{vmatrix} = 0$ are independent of λ, μ, ν, a, b

S_3 : If a, b, c, are sides of a scalene triangle, then value of $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is negative

S_4 : Let $f(x) = \begin{vmatrix} \frac{1}{x} & \ell n\,x & x^n \\ 1 & -\frac{1}{n} & (-1)^n \\ 1 & a & a^2 \end{vmatrix}$, if $f^n(x)$ is the n^{th} derivative of f(x) then $f^n(1)$ is independent of a.

(A) FFTT **(B)** FTTT **(C)** FFFT **(D)** TTTT

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

11. If A and B are invertible square matrices of the same order, then which of the following is correct ?

(A) adj(AB) = (adjB) (adjA)

(B) (adjA)′ = (adjA′)

(C) $|adjA| = |A|^{n-1}$, where n is the order of matrix A

(D) $adj(adjB) = |B|^{n-2} B$, where n is the order of matrix B

12. Let $f(x) = \begin{vmatrix} 2\sin x & \sin^2 x & 0 \\ 1 & 2\sin x & \sin^2 x \\ 0 & 1 & 2\sin x \end{vmatrix}$, then

(A) f(x) is independent of x **(B)** $f'(\pi/2) = 0$

(C) $\int_{-\pi/2}^{\pi/2} f(x)dx = 0$ **(D)** tangent to the curve y = f(x) at x = 0 is y = 0

13. System of equation $x + 3y + 2z = 6$ $x + \lambda y + 2z = 7$
$x + 3y + 2z = \mu$ has

(A) unique solution if $\lambda = 2, \mu \neq 6$ **(B)** infinitely many solution if $\lambda = 4, \mu = 6$

(C) no solution if $\lambda = 5, \mu = 7$ **(D)** no solution if $\lambda = 3, \mu = 5$

14. Let $f(x) = \begin{vmatrix} 1/x & \log x & x^n \\ 1 & -1/n & (-1)^n \\ 1 & a & a^2 \end{vmatrix}$, then

(A) $f^n(1)$ is indepedent of a

(B) $f^n(1)$ is indepedent of n

(C) $f^n(1)$ depends on a and n

(D) $y = a(x - f^n(1))$ represents a straight line through the origin

15. Which of the following statement is always true

(A) Adjoint of a symmetric matrix is a symmetric matrix

(B) Adjoint of a unit matrix is unit matrix

(C) A (adj A) = (adj A) A

(D) Adjoint of a diagonal matrix is diagonal matrix

SECTION - III : ASSERTION AND REASON TYPE

16. **Statement-I :** The system of equations possess a non trivial solution over the set of rationals $x + ky + 3z = 0$, $3x + ky - 2z = 0$, $2x + 3y - 4z = 0$, then the value of k is 31/2.

Statement-II : For non trivial solution $\Delta = 0$.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

17. **Statement-I :** The determinants of a matrix $A = [a_{ij}]_{5 \times 5}$ where $a_{ij} + a_{ji} = 0$ for i and j is zero

Statement-II : The determinant of a skew symmetric matrix of odd order is zero.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

18. **Statement-I :** For a singular square matrix A, if AB = AC $\Rightarrow$ B = C

Statement-II : If $|A| = 0$ then A^{-1} does not exist

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

19. **Statement-I :** $(a_{11}, a_{22}, \ldots\ldots\ldots, a_{nn})$ is a diagonal matrix then $A^{-1} = \text{dia}(a_{11}^{-1}, a_{22}^{-1}, \ldots\ldots, a_{nn}^{-1})$

Statement-II : If A = dia (2, 1, –3) and B = dia (1, 1, 2) then $\det(AB^{-1}) = 3$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

20. **Statement-I :** If a, b, c are distinct and x, y, z are not all zero and $ax + by + cz = 0$, $bx + cy + az = 0$, $cx + ay + bz = 0$, then $a + b + c = 0$.

Statement-II : $a^2 + b^2 + c^2 > ab + bc + ca$, if a, b, c are distinct.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

SECTION - IV : MATRIX - MATCH TYPE

21. **Match the following**

Column - I		Column - II	
(A)	If $\Delta(x) = \begin{vmatrix} x^2-5x+3 & 2x-5 & 3 \\ 3x^2+x+4 & 6x+1 & 9 \\ 7x^2+6x+9 & 14x-6 & 21 \end{vmatrix} = ax^3 + bx^2 + cx + d$, then	**(p)**	$3a + 4b + 5c + d = 141$
(B)	If $\Delta(x) = \begin{vmatrix} x-1 & 5x & 7 \\ x^2-1 & x-1 & 8 \\ 2x & 3x & 0 \end{vmatrix} = ax^3 + bx^2 + cx + d$, then	**(q)**	$a + 2b + 3c + 5d = 156$
(C)	If $\Delta(x) = \begin{vmatrix} 2x^3-3x^2 & 5x+7 & 2 \\ 4x^3-7x & 3x+2 & 1 \\ 7x^3-8x^2 & x-1 & 3 \end{vmatrix}$ $a + bx + cx^2 + dx^3 + ex^4$, then	**(r)**	$c - d = 19$
		(s)	$b - c = 25$
		(t)	$3a + 2b + 5c + 5d = 187$

22. **Match the following**

Column - I		Column - II	
(A)	Let $\lvert A\rvert = \lvert a_{ij}\rvert_{3\times 3} \neq 0$. Each element a_{ij} is multiplied by k^{i-j}. Let $\lvert B\rvert$ the resulting determinant, where $k_1\lvert A\rvert + k_2\lvert B\rvert = 0$. Then $k_1 + k_2 =$	**(p)**	0
(B)	The maximum value of a third order determinant each of its entries are ± 1 equals	**(q)**	4
(C)	$\begin{vmatrix} 1 & \cos\alpha & \cos\beta \\ \cos\alpha & 1 & \cos\gamma \\ \cos\beta & \cos\gamma & 1 \end{vmatrix} = \begin{vmatrix} 0 & \cos\alpha & \cos\beta \\ \cos\alpha & 0 & \cos\gamma \\ \cos\beta & \cos\gamma & 0 \end{vmatrix}$ if $\cos^2\alpha + \cos^2\beta + \cos^2\gamma =$	**(r)**	1
(D)	$\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = Ax + B$ where A and B are determinants of order 3. Then $A + 2B =$	**(s)**	2
		(t)	$\begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix}$

SECTION - V : COMPREHENSION TYPE

23. Read the following comprehension carefully and answer the questions.

Consider the determinant $\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ d_1 & d_2 & d_3 \end{vmatrix}$

M_{ij} = Minor of the element of i^{th} row and j^{th} column

C_{ij} = Cofactor of the element of i^{th} row and j^{th} column

1. Value of $b_1 . C_{31} + b_2 . C_{32} + b_3 . C_{33}$ is

(A) 0 **(B)** Δ **(C)** 2Δ **(D)** Δ^2

2. If all the elements of the determinant are multiplied by 2, then the value of new determinant is

(A) 0 **(B)** 8Δ **(C)** 2Δ **(D)** $2^9 . \Delta$

3. $a_3 M_{13} - b_3 . M_{23} + d_3 . M_{33}$ is equal to

(A) 0 **(B)** 4Δ **(C)** 2Δ **(D)** Δ

24. Read the following comprehension carefully and answer the questions.

Let $\Delta \neq 0$ and Δ^c denotes the determinant of cofactors, then $\Delta = \Delta^{n-1}$, where n(> 0) is the order of Δ.

1. If a, b, c are the roots of the equation $x^3 - px^2 + r = 0$, then the value of $\begin{vmatrix} bc-a^2 & ca-b^2 & c^2 \\ ca-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ca-b^2 \end{vmatrix}$ is

(A) p^2 **(B)** p^4 **(C)** p^6 **(D)** p^9

2. If $l_1, m_1, n_1 ; l_2, m_2, n_2 ; l_3, m_3, n_3$ are real quantities satisfying the six relations :

$l_1^2 + m_1^2 + n_1^2 = l_2^2 + m_2^2 + n_2^2 = l_3^2 + m_3^2 + n_3^2 = 1$

$l_2 l_3 + m_2 m_3 + n_2 n_3 = l_3 l_1 + m_3 m_1 + n_3 n_1 = l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$, then the value of $\begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix}$ is

(A) 0 **(B)** ±1 **(C)** ±2 **(D)** ±3

3. If a, b, c are the roots of the equation $x^3 - 3x^2 + 3x + 7 = 0$, then the value of $\begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ac-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix}$ is

(A) 9 **(B)** 27 **(C)** 81 **(D)** 0

25. Read the following comprehension carefully and answer the questions.

Let A be a m × n matrix. If there exists a matrix L of type n × m such that $LA = I_n$, then L is called left inverse of A. Similarly, if there exists a matrix R of type n × m such that $AR = I_m$, then R is called right inverse of A. For example to find right inverse of matrix

$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 2 & 3 \end{bmatrix}$ we take $R = \begin{bmatrix} x & y & z \\ u & v & w \end{bmatrix}$ and solve $AR = I_3$ i.e.

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x & y & z \\ u & v & w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow$ $x - u = 1$ $\quad$ $y - v = 0$ $\quad$ $z - w = 0$

$x + u = 0$ $\quad$ $y + v = 1$ $\quad$ $z + w = 0$

$2x + 3u = 0$ $\quad$ $2y + 3v = 0$ $\quad$ $2z + 3w = 1$

As this system of equations is inconsistent, we say there is no right inverse for matrix A.

1. Which of the following matrices is NOT left inverse of matrix $\begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 2 & 3 \end{bmatrix}$

(A) $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$ **(B)** $\begin{bmatrix} 2 & -7 & 3 \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$ **(C)** $\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$ **(D)** $\begin{bmatrix} 0 & 3 & -1 \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$

2. The number of right inverses for the matrix $\begin{bmatrix} 1 & -1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$

(A) 0 **(B)** 1 **(C)** 2 **(D)** infinite

3. For which of the following matrices number of left inverses is greater than the number of right inverses

(A) $\begin{bmatrix} 1 & 2 & 4 \\ -3 & 2 & 1 \end{bmatrix}$ **(B)** $\begin{bmatrix} 3 & 2 & 1 \\ 3 & 2 & 1 \end{bmatrix}$ **(C)** $\begin{bmatrix} 1 & 4 \\ 2 & -3 \\ 5 & 4 \end{bmatrix}$ **(D)** $\begin{bmatrix} 3 & 3 \\ 1 & 1 \\ 4 & 4 \end{bmatrix}$

SECTION - VI : INTEGER TYPE

26. If f(x) satisfies the equation $\begin{vmatrix} f(x+1) & f(x+8) & f(x+1) \\ 1 & 2 & -5 \\ 2 & 3 & \lambda \end{vmatrix} = 0$ for all real x. If f is periodic with period 7, then find the value of $|\lambda|$.

27. Let $f(x) = \begin{vmatrix} x & 1 & 1 \\ \sin 2\pi x & 2x^2 & 1 \\ x^3 & 3x^4 & 1 \end{vmatrix}$. If f(x) be an odd function and its odd values is equal g(x), then find the value of λ.

If $f(1)\,g(1) = -4\lambda$

28. If $f(x) = \begin{vmatrix} \cos(x+\alpha) & \cos(x+\beta) & \cos(x+\gamma) \\ \sin(x+\alpha) & \sin(x+\beta) & \sin(x+\gamma) \\ \sin(\beta-\gamma) & \sin(\gamma-\alpha) & \sin(\alpha-\beta) \end{vmatrix}$ and $f(2) = 6$, then find $\frac{1}{5}\sum_{r=1}^{25} f(r)$,

29. Find the coefficient of x in the determinant $\begin{vmatrix} (1+x)^{a_1b_1} & (1+x)^{a_1b_2} & (1+x)^{a_1b_3} \\ (1+x)^{a_2b_1} & (1+x)^{a_2b_2} & (1+x)^{a_2b_3} \\ (1+x)^{a_3b_1} & (1+x)^{a_3b_2} & (1+x)^{a_3b_3} \end{vmatrix}$, where $a_i, b_j \in N$

30. If $p, q \in R$ and $p^2 + q^2 - pq - p - q + 1 \le 0$, $\alpha + \beta + \gamma = 0$, then $\begin{vmatrix} 1 & \cos\gamma & \cos\beta \\ \cos\gamma & p & \cos\alpha \\ \cos\beta & \cos\alpha & q \end{vmatrix} = 0$.

ANSWER KEY

EXERCISE - 1

1. B **2.** A **3.** A **4.** D **5.** D **6.** A **7.** C **8.** B **9.** A **10.** D **11.** A **12.** A **13.** C
14. A **15.** A **16.** B **17.** B **18.** A **19.** A **20.** B **21.** D **22.** C **23.** A **24.** C **25.** A **26.** A
27. D **28.** B **29.** A **30.** B **31.** D **32.** A **33.** C **34.** D **35.** D **36.** D **37.** A **38.** B **39.** D
40. B **41.** B **42.** D **43.** A

EXERCISE - 2 : PART # I

1. ABCD **2.** AC **3.** BD **4.** ACD **5.** AC **6.** ABC **7.** BD **8.** AB **9.** ABCD
10. AB **11.** BC **12.** ACD **13.** AD **14.** BC **15.** AC **16.** BD **17.** ABCD **18.** AC
19. ABCD **20.** AC

PART - II

1. A **2.** A **3.** A **4.** C

EXERCISE - 3 : PART # I

1. $A \to r$ $B \to s$ $C \to p$ $D \to q$

2. $A \to q$ $B \to p$ $C \to s$ $D \to s$

PART - II

Comprehension # 1 : **1.** C **2.** A **3.** A

Comprehension # 2 : **1.** A **2.** D **3.** C

Comprehension # 3 : **1.** A **2.** B **3.** C

EXERCISE - 5 : PART # I

1. 1 **2.** 1 **3.** 4 **4.** 1 **5.** 1 **6.** 4 **7.** 1 **8.** 4 **9.** 4 **10.** 1 **11.** 4 **12.** 4 **13.** 2
14. 3 **15.** 2

PART - II

1. $x = n\pi, n \in I$

2. If $\lambda = 5$, system is consistent with infinite solution given by $z = K$,

$y = \frac{1}{2}(3K + 4)$ and $x = -\frac{1}{2}(5K + 2)$ where $K \in R$

If $\lambda \neq 5$, system is consistent with unique solution given by $z = \frac{1}{3}(1 - \lambda)$; $x = \frac{1}{3}(\lambda + 2)$ and $y = 0$.

4. B **5.** D **6.** **A**. D **B**. A **7.** 3 **8.** AB **9.** BC **10.** 2 **11.** BCD

MOCK TEST

1. C	**2.** B	**3.** A	**4.** C	**5.** D	**6.** A	**7.** C	**8.** A	**9.** A
10. A	**11.** ABCD	**12.** BCD	**13.** BCD	**14.** ABD	**15.** ABCD	**16.** D	**17.** A	**18.** D
19. C	**20.** A							

21. $A \rightarrow p$ $B \rightarrow q,s$ $C \rightarrow r,t$ **22.** $A \rightarrow p,t$ $B \rightarrow q$ $C \rightarrow r$ $D \rightarrow p,t$

23. **1.** A **2.** B **3.** D **24.** **1.** C **2.** B **3.** D **25.** **1.** C **2.** D **3.** C

26. 4 **27.** 1 **28.** 3 **29.** 0 **30.** 0