# HINTS & SOLUTIONS

## EXERCISE - 1

### Single Choice

**7.** $D=\begin{vmatrix} a_1 & a_2 & a_3 \\ 5 & 4 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$

Since $a_n=\frac{20}{n}, d=\frac{1}{20}$

Hence, $D=\begin{vmatrix} 20 & \frac{20}{2} & \frac{20}{3} \\ \frac{20}{4} & \frac{20}{5} & \frac{20}{6} \\ \frac{20}{7} & \frac{20}{8} & \frac{20}{9} \end{vmatrix}=\frac{(20)^3}{4\times 7}\begin{vmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ 1 & \frac{4}{5} & \frac{2}{3} \\ 1 & \frac{7}{8} & \frac{7}{9} \end{vmatrix}$

$R_1\to R_1-R_2$ and $R_2\to R_2-R_3$

$=\frac{(20)^3}{4\times 7}\begin{vmatrix} 0 & \frac{-3}{10} & \frac{-1}{3} \\ 1 & \frac{-3}{40} & \frac{-1}{9} \\ 1 & \frac{7}{8} & \frac{7}{9} \end{vmatrix}=\frac{50}{21}$ $\Rightarrow$ $[D]=2$ (Sol)

**8.** $D=\begin{vmatrix} \sin\theta\cos\varphi & \sin\theta\sin\varphi & \cos\theta \\ \cos\theta\cos\varphi & \cos\theta\sin\varphi & -\sin\theta \\ -\sin\theta\sin\varphi & \sin\theta\cos\varphi & 0 \end{vmatrix}$

$\Delta=\sin^2\theta\cos\theta\begin{vmatrix} \cos\varphi & \sin\varphi & \cos\theta \\ \cos\varphi & \sin\varphi & -\tan\theta \\ -\sin\varphi & \cos\varphi & 0 \end{vmatrix}$

Applying $R_1\to R_1-R_2$

$\Delta=\sin^2\theta\cos\theta\begin{vmatrix} 0 & 0 & \cot\theta+\tan\theta \\ \cos\varphi & \sin\varphi & -\tan\theta \\ -\sin\varphi & \cos\varphi & 0 \end{vmatrix}$

$\Delta=\sin\theta$

**10.** Applying $R_3\to R_3-3R_1-2R_2$ we get $\Delta=0$

$\Rightarrow$ infinite solution.

**11.** Directly open by $R_1$ to get a form of $\sin(A-B)$ etc

**12.** $\begin{vmatrix} 3u^2 & 2u^3 & 1 \\ 3v^2 & 2v^3 & 1 \\ 3w^2 & 2w^3 & 1 \end{vmatrix}=0$

$R_1\to R_1-R_2$ and $R_2\to R_2-R_3$

$\Rightarrow\begin{vmatrix} u^2-v^2 & u^3-v^3 & 0 \\ v^2-w^2 & v^3-w^3 & 0 \\ w^2 & w^3 & 1 \end{vmatrix}=0$

$\Rightarrow\begin{vmatrix} u+v & u^2+v^2+vu & 0 \\ v+w & v^2+w^2+vw & 0 \\ w^2 & w^3 & 1 \end{vmatrix}=0$

$R_1\to R_1-R_2$

$\Rightarrow\begin{vmatrix} u-w & (u^2-w^2)+v(u-w) & 0 \\ v+w & v^2+w^2+vw & 0 \\ w^2 & w^3 & 1 \end{vmatrix}=0$

$\Rightarrow\begin{vmatrix} 1 & u+w+v & 0 \\ v+w & v^2+w^2+vw & 0 \\ w^2 & w^3 & 1 \end{vmatrix}=0$

$\Rightarrow$ $(v^2+w^2+vw)-(v+w)[(v+w)+u]=0$

$\Rightarrow$ $v^2+w^2+vw=(v+w)^2+u(v+w)$

$\Rightarrow$ $uv+vw+wu=0$

**13.** $\Delta=\begin{vmatrix} a^2(1+x) & ab & ac \\ ab & b^2(1+x) & bc \\ ac & bc & c^2(1+x) \end{vmatrix}$

$=a^2b^2c^2\begin{vmatrix} (1+x) & 1 & 1 \\ 1 & (1+x) & 1 \\ 1 & 1 & (1+x) \end{vmatrix}$

Applying $C_1\to C_2+C_3$

$a^2b^2c^2(3+x)\begin{vmatrix} 1 & 1 & 1 \\ 1 & (1+x) & 1 \\ 1 & 1 & (1+x) \end{vmatrix}$

Applying $R_1\to R_1-R_2$, $R_2\to R_2-R_3$

$a^2b^2c^2(3+x)\,x^2$

Which is divisible by $x^2$

**16.** $f(x)=\begin{vmatrix}1+\sin^2 x & \cos^2 x & 4\sin 2x\\ \sin^2 x & 1+\cos^2 x & 4\sin 2x\\ \sin^2 x & \cos^2 x & 1+4\sin 2x\end{vmatrix}$

Applying $C_1\to C_1+C_2+C_3$

$$=(2+4\sin 2x)\begin{vmatrix}1 & \cos^2 x & 4\sin 2x\\ 1 & 1+\cos^2 x & 4\sin 2x\\ 1 & \cos^2 x & 1+4\sin 2x\end{vmatrix}=(2+4\sin 2x)$$

$$\begin{vmatrix}1 & \cos^2 x & 4\sin 2x\\ 0 & 1 & 0\\ 0 & 0 & 1\end{vmatrix}$$

$f(x)=(2+4\sin 2x)$

$f(x)_{max}=6$

**18.** $a=a_0.r_1^{\,p-1}\quad\Rightarrow\ \log a=(p-1)\log r_1+\log a_0$

$b=a_0.r_1^{\,q-1}\quad\Rightarrow\ \log b=(q-1)\log r_1+\log a_0$

$c=a_0r_1^{\,r-1}\quad\Rightarrow\ \log c=(r-1)\log r_1+\log a_0$

$$\begin{vmatrix}\log a_0+(p-1)\log r_1 & p & 1\\ \log a_0+(q-1)\log r_1 & q & 1\\ \log a_0+(r-1)\log r_1 & r & 1\end{vmatrix}$$

$$=\begin{vmatrix}\log a_0 & p & 1\\ \log a_0 & q & 1\\ \log a_0 & r & 1\end{vmatrix}+\log r_1\begin{vmatrix}p-1 & p & 1\\ q-1 & q & 1\\ r-1 & r & 1\end{vmatrix}=0$$

**21.** A is non singular $\det A\neq 0$

Given $AB-BA=A$ hence $AB=A+BA=A(I+B)$

$\det. A\cdot\det. B=\det. A\cdot\det.(I+B)$

$\det. B=\det.(I+B)$ ....(1)

(as A is non singular)

again $AB-A=BA$

$A(B-I)=BA$

$(\det. A)\cdot\det.(B-I)=\det. B\cdot\det. A$

$\Rightarrow\ \det.(B-I)=\det.(B)$ ....(2)

from (1) and (2)

$\det.(B-I)=\det.(B+I)$

**22.** Consider the det. B, using $R_1\to R_1+R_2+R_3$

$$B=2\begin{vmatrix}a+p+x & b+q+y & c+r+z\\ a+x & b+y & c+z\\ a+p & b+q & c+r\end{vmatrix}$$

using $R_2\to R_2-R_1$ and $R_3\to R_3-R_1$

$$=2\begin{vmatrix}a+p+x & b+q+y & c+r+z\\ -p & -q & -r\\ -x & -y & -z\end{vmatrix}$$

using $R_1\to R_1+R_2+R_3$

$B=2\det. A=2\cdot 6=12$

**23.** Using: $C_3\to C_3-(C_1+C_2)$, $D_1=\begin{vmatrix}a & b & a+b\\ c & d & c+d\\ a & b & a-b\end{vmatrix}$

and $D_2=\begin{vmatrix}a & c & a+c\\ b & d & b+d\\ a & c & a+b+c\end{vmatrix}$

$$\therefore\ \frac{D_1}{D_2}=\frac{-2b(ad-bc)}{b(ad-bc)}=-2$$

**27.** $\begin{vmatrix}(1+x)^2 & (1-x)^2 & -(2+x^2)\\ 2x+1 & 3x & 1-5x\\ x+1 & 2x & 2-3x\end{vmatrix}+\begin{vmatrix}(1+x)^2 & (1-x)^2 & 1-2x\\ 2x+1 & 3x & 3x-2\\ x+1 & 2x & 2x-3\end{vmatrix}$

Since two columns are same in above determinants therefore we can add them along $C_3$.

$$\Rightarrow\ \begin{vmatrix}(1+x)^2 & (1-x)^2 & -(x+1)^2\\ 2x+1 & 3x & -(1+2x)\\ x+1 & 2x & -(1+x)\end{vmatrix}=0$$

$\Rightarrow\ 0=0\quad\Rightarrow$ infinite solution

**30.** $U_n=\begin{vmatrix}n & 1 & 5\\ n^2 & 2N+1 & 2N+1\\ n^3 & 3N^2 & 3N+1\end{vmatrix}$

$$\Rightarrow\ \sum_{n=1}^{N}U_n=\begin{vmatrix}\frac{N(N+1)}{2} & 1 & 5\\ \frac{N(N+1)(2N+1)}{6} & (2N+1) & (2N+1)\\ \left[\frac{N(N+1)}{2}\right]^2 & 3N^2 & (3N+1)\end{vmatrix}$$

$$=\frac{N(N+1)(2N+1)}{2}\begin{vmatrix}1 & 1 & 5\\ \frac{1}{3} & 1 & 1\\ \frac{N(N+1)}{2} & 3N^2 & 3N+1\end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$

$$\frac{N(N+1)(2N+1)}{2}\begin{vmatrix} \frac{2}{3} & 0 & 4 \\ \frac{1}{3} & 1 & 1 \\ \frac{N(N+1)}{2} & 3N^2 & 3N+1 \end{vmatrix} = 2\sum_{n=1}^{N} n^2$$

**31.** $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 1 & p & 2 \\ 1 & 4 & \mu \end{vmatrix}$ $\quad \Delta_x = \begin{vmatrix} 4 & 2 & 3 \\ 3 & p & 2 \\ 3 & 4 & \mu \end{vmatrix}$

$\Delta_y = \begin{vmatrix} 1 & 4 & 3 \\ 1 & 3 & 2 \\ 1 & 3 & \mu \end{vmatrix}$ $\quad \Delta_z = \begin{vmatrix} 1 & 2 & 4 \\ 1 & p & 3 \\ 1 & 4 & 3 \end{vmatrix}$

For infinite no. of solution $\Delta = \Delta_x = \Delta_y = \Delta_z = 0$

$\Rightarrow \quad \mu = 2, p = 4$

**32.** $R_3 \to R_1 + R_3$

**33.** $D = \frac{1}{abc}\begin{vmatrix} ab^2c^2 & abc & a(b+c) \\ bc^2a^2 & bca & b(c+a) \\ ca^2b^2 & cab & c(a+b) \end{vmatrix}$

$(R_1 \to aR_1, R_2 \to bR_2, R_3 \to cR_3)$

$= abc\begin{vmatrix} bc & 1 & ab+ac \\ ca & 1 & bc+ab \\ ab & 1 & ca+cb \end{vmatrix}$

$= abc(ab+bc+ca)\begin{vmatrix} bc & 1 & 1 \\ ca & 1 & 1 \\ ab & 1 & 1 \end{vmatrix}$ $(C_3 \to C_3 + C_1)$

$= 0$

**35.** $\Delta = 0$

**36.** for non trivial solution $\begin{vmatrix} \sin\theta & -\cos\theta & \lambda+1 \\ \cos\theta & \sin\theta & -\lambda \\ \lambda & \lambda+1 & \cos\theta \end{vmatrix} = 0$ ; this

gives $2\cos\theta(\lambda^2 + \lambda + 1) = 0$

**37.** $D = \cos\theta - \cos^2\theta + 6 \neq 0$ since $D \neq 0$

$\Rightarrow$ only trivial solution is possible $\Rightarrow$ (A)

**38.** $|A| = 2$ ; $|B| = 3$ ; $|C| = 5$

$\det(A^2BC^{-1}) = |A^2BC^{-1}| = \frac{|A|^2|B|}{|C|} = \frac{4 \cdot 3}{5} = \frac{12}{5}$

**39.** Multiply $R_1$ by a, $R_2$ by b & $R_3$ by c & divide the determinant by abc. Now take a, b & c common from $c_1$, $c_2$ & $c_3$. Now use $C_1 \to C_1 + C_2 + C_3$ to get

$(a^2+b^2+c^2+1)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix} = 1.$

Now use $c_1 \to c_1 - c_2$ & $c_2 \to c_2 - c_3$

we get $1 + a^2 + b^2 + c^2 = 1$

$\Rightarrow \quad a = b = c = 0 \quad \Rightarrow$ **(D)**

**40.** $D = \begin{vmatrix} \sin\theta & 0 & 2 \\ \cos\theta & \sin\theta & 0 \\ 0 & \cos\theta & 2 \end{vmatrix}$

$\sin\theta(2\sin\theta) + 2\cos^2\theta = 2$

**41.** $D = \begin{vmatrix} 1 & -2 & 1 \\ 2 & 1 & -2 \\ 1 & 3 & -3 \end{vmatrix}$ which vanishes ;

hencefor atleast one solution $D_1 = D_2 = D_3 = 0$

$\therefore \quad D_1 = \begin{vmatrix} a & -2 & 1 \\ b & 1 & -2 \\ c & 3 & -3 \end{vmatrix} = 0 \quad \Rightarrow \quad a - b + c = 0$

**Note :** Same condition is obtained by putting

$D_2 = 0$ or $D_3 = 0$

**42.** $1^{st}$ two columns of $1^{st}$ determinant are same as $1^{st}$ two rows of $2^{nd}$. Hence transpose the $2^{nd}$. Add the two determinants and use $C_1 \to C_1 + C_3 \quad \Rightarrow \quad D = 0$

**43.** $x17 = kl = 100x + 10 + 7$

$3y6 = km = 300 + 10y + 6$

$12z = kn = 100 + 20 + z$

use $R_2 \to R_2 + 100R_1 + 10R_3$ to get the result

## EXERCISE - 2

### Part # I : Multiple Choice

**5.** Get Result $R_1 \to 2R_1 - R_2$ and $R_2 \to R_2 - R_3$

**6.** $\Delta = \begin{vmatrix} -x & a & b \\ b & -x & a \\ a & b & -x \end{vmatrix} = (a+b-x)\begin{vmatrix} 1 & a & b \\ 1 & -x & a \\ 1 & b & -x \end{vmatrix}$

Applying $R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1$

$= (a+b-x)\begin{vmatrix} 1 & a & b \\ 0 & -(x+a) & a-b \\ 0 & b-a & -(x+b) \end{vmatrix} = (a+b-x)\,\{(x+a)$

$(x+b) + (a-b)^2\}$

If $a = b$ then $x = a, -b, (a+b)$

**8.** $\Delta = \begin{vmatrix} a & a^2 & 0 \\ 1 & 2a+b & (a+b)^2 \\ 0 & 1 & 2a+2b \end{vmatrix} = a\begin{vmatrix} 1 & a & 0 \\ 1 & (2a+b) & (a+b)^2 \\ 0 & 1 & 2a+3b \end{vmatrix}$

$R_2 \to R_2 - R_1$

$= a(a+b)\begin{vmatrix} 1 & a & 0 \\ 0 & ab & (a+b)^2 \\ 0 & 1 & 2a+3b \end{vmatrix}$

$= a(a+b)\begin{vmatrix} 1 & a & 0 \\ 0 & 1 & (a+b) \\ 0 & 1 & 2a+3b \end{vmatrix}$

$= a(a+b)\begin{vmatrix} 1 & a & 0 \\ 0 & 1 & (a+b) \\ 0 & 1 & 2a+3b \end{vmatrix} = a\,(a+b)\,(2a+3b-a-b)$

$= a(a+b)\,(a+2b)$

**9.** $\begin{vmatrix} 1+\sin^2 A & \cos^2 A & 2\sin 4\theta \\ \sin^2 A & 1+\cos^2 A & 2\sin 4\theta \\ \sin^2 A & \cos^2 A & 1+2\sin 4\theta \end{vmatrix} = 0$

$C_1 \to C_1 + C_2$

$\Rightarrow \begin{vmatrix} 2 & \cos^2 A & 2\sin 4\theta \\ 2 & 1+\cos^2 A & 2\sin 4\theta \\ 1 & \cos^2 A & 1+2\sin 4\theta \end{vmatrix} = 0$

$R_1 \to R_1 - R_2$

$\Rightarrow \begin{vmatrix} 0 & -1 & 0 \\ 2 & 1+\cos^2 A & 2\sin 4\theta \\ 1 & \cos^2 A & 1+2\sin 4\theta \end{vmatrix} = 0$

$\Rightarrow 2(1+2\sin 4\theta) - 2\sin 4\theta = 0$

$\Rightarrow 1 + \sin 4\theta = 0$

$\sin 4\theta = -1$

$4\theta = -\dfrac{\pi}{2}$ or $4\theta = \dfrac{3\pi}{2}$

$\theta = -\dfrac{\pi}{8}$ or $\theta = \dfrac{3\pi}{8}$ & $A \in R$

**13.** Let $A = \begin{bmatrix} a_1 & b_1 \\ c_1 & -a_1 \end{bmatrix}$ and $B = \begin{bmatrix} a_2 & b_2 \\ c_2 & -a_2 \end{bmatrix}$

$\Rightarrow AB = \begin{bmatrix} a_1a_2 + b_2c_2 & a_1b_2 + b_1a_2 \\ c_1a_2 - a_1c_2 & c_1b_2 + a_1a_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow a_1a_2 + b_1c_2 = a_1b_2 - b_1a_2 = c_1a_2 - a_1c_2 = c_1b_2 + a_1a_2 = 0$

$BA = \begin{bmatrix} a_2a_1 + b_2c_1 & a_2b_1 - b_2a_1 \\ c_2a_1 - a_2c_1 & c_2b_1 + a_2a_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

**15.** $x + y = 3$ .......(i)

$(1+K)x + (K+2)y = 8$ .......(ii)

$x - (1+K)y = -K - 2$ .......(iii)

If system is consistent then $\Delta = 0$

on solving we get

$K = 1, \dfrac{-5}{3}$

**19.** $\dfrac{a^2}{b} = \dfrac{b}{b^2} = \dfrac{a^2 - b}{2 + 4b}$

$\Rightarrow a^2 b^2 = b^2 \quad \Rightarrow b^2(a^2 - 1) = 0$

$\Rightarrow a = 1$ or $-1$ & $b = 0,$

if $a = 1$ then $2 + 4b = b - b^2 \Rightarrow b^2 + 3b + 2 = 0$

$\Rightarrow b = -2$ or $-1$ ;

if $a = -1$ then $b = -2$ or $-1$

**20.** Start : $p = a$ ; $q = a + d$ ; $r = a + 2d$ ; $s = a + 3d$

$\Rightarrow f(x) = -2d^2$

Also use $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

## Part # II : Assertion & Reason

**2.** Statement-I : $B_1 = c_2a_3 - a_2c_3$

$B_2 = a_1c_3 - c_1a_3$, $B_3 = a_2c_1 - a_1c_2$

$a_1B_1 = a_1a_3c_2 - a_1a_2c_3$

$a_2B_2 = a_1a_2c_3 - a_2a_3c_1$

$a_3B_3 = a_1a_2c_3 - a_1a_3c_2$

Statement-II is obviously true.

## EXERCISE - 3

## Part # II : Comprehension

### Comprehension # 1

$$\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = (\alpha - 1)^2 (\alpha + 2)$$

**1.** $\Delta \neq 0 \Rightarrow$ unique solution

**2.** $\alpha = -2 \Rightarrow \Delta = 0,\ \Delta_1 = \begin{vmatrix} m & -2 & 1 \\ n & 1 & -2 \\ p & 1 & 1 \end{vmatrix}$

$\Delta_1 = 3(m + n + p) \neq 0$

$\Rightarrow \Delta_1 \neq 0$

Hence no solution

**3.** $x + y + z = m$

$x + y + z = p$

$\therefore\ m \neq p \Rightarrow$ no solution

### Comprehension # 3

**Hint :**

$\Delta = (x-y)(y-z)(z-x)[xyz(xy+yz+zx)-(x+y+z)]$

## EXERCISE - 4

### Subjective Type

**3.** $a^2 + b^2 + c^2 = 1$

$$\Delta = \begin{vmatrix} a^2+(b^2+c^2)\cos\phi & ab(1-\cos\phi) & ac(1-\cos\phi) \\ ba(1-\cos\phi) & b^2+(c^2+a^2)\cos\phi & bc(1-\cos\phi) \\ ca(1-\cos\phi) & cb(1-\cos\phi) & c^2+(a^2+b^2)\cos\phi \end{vmatrix}$$

$$\Delta = \frac{1}{abc}$$

$$\begin{vmatrix} a^3+a(b^2+c^2)\cos\varphi & b^2a(1-\cos\varphi) & ac^2(1-\cos\varphi) \\ ba^2(1-\cos\varphi) & b^3+b(c^2+a^2)\cos\varphi & bc^2(1-\cos\varphi) \\ ca^2(1-\cos\varphi) & cb(1-\cos\varphi) & c^3+c(a^2+b^2)\cos\varphi \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$

$$\frac{abc}{abc}\begin{vmatrix} a^2+(b^2+c^2)\cos\varphi & b^2(1-\cos\varphi) & c^2(1-\cos\varphi) \\ a^2(1-\cos\varphi) & b^2+(c^2+a^2)\cos\varphi & c^2(1-\cos\varphi) \\ a^2(1-\cos\varphi) & b^2(1-\cos\varphi) & c^2+(a^2+b^2)\cos\varphi \end{vmatrix}$$

$= (a^2 + b^2 + c^2)$

$$\begin{vmatrix} 1 & b^2(1-\cos\varphi) & c^2(1-\cos\varphi) \\ 1 & b^2+(c^2+a^2)\cos\varphi & c^2(1-\cos\varphi) \\ 1 & b^2(1-\cos\varphi) & c^2+(a^2+b^2)\cos\varphi \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$(a^2 + b^2 + c^2)$

$$\begin{vmatrix} 1 & b^2(1-\cos\varphi) & c^2(1-\cos\varphi) \\ 0 & (a^2+b^2+c^2)\cos\varphi & 0 \\ 0 & 0 & (a^2+b^2+c^2)\cos\varphi \end{vmatrix}$$

$= (a^2 + b^2 + c^2)\cos^2\phi = \cos^2\phi$

This is independent from a, b, c.

**6.** Let $\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$

Operate : $C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow \Delta = \begin{vmatrix} 2(a+b+c) & c+a & a+b \\ 2(a+b+a) & a+b & b+c \\ 2(a+b+c) & b+c & c+a \end{vmatrix} = 2(a + b + c)$$

$$\begin{vmatrix} 1 & c+a & a+b \\ 1 & a+b & b+c \\ 1 & b+c & c+a \end{vmatrix}$$

Operate : $R_2 \to R_2 - R_1$ ; $R_3 \to R_3 - R_1$

$$\Rightarrow D = 2(a+b+c)\begin{vmatrix} 1 & c+a & a+b \\ 0 & b-c & c-a \\ 0 & b-a & c-b \end{vmatrix}$$

$= 2(a+b+c)\,1.\begin{vmatrix} b-c & c-a \\ b-a & c-b \end{vmatrix}$ open w.r.t. $R_1$

$= 2(a + b + c) [(b - c) (c - b) - (c - a) (b - a)]$

$= 2(a + b + c) [bc - b^2 - c^2 + cb - (cb - ac - ab + a^2)]$

$= 2(a + b + c) (ab + bc + ca - a^2 - b^2 - c^2)$

**10.** $$\sum_{r=1}^{n} D_r = \begin{vmatrix} \sum_{r=1}^{n} 2^{r-1} & \sum_{r=1}^{n} 2(3^{r-1}) & \sum_{r=1}^{n} 4(5^{r-1}) \\ x & y & z \\ 2^n-1 & 3^n-1 & 5^n-1 \end{vmatrix}$$

$$= \begin{vmatrix} 1+2+\ldots 2^{n-1} & 2\{1+3\ldots 3^{n-1}\} & 4(1+5+\ldots 5^{n-1}) \\ x & y & z \\ 2^n-1 & 3^n-1 & 5^n-1 \end{vmatrix}$$

$$= \begin{vmatrix} \frac{2^n-1}{2-1} & \frac{2(3^n-1)}{3-1} & \frac{4(5^n-1)}{5-1} \\ x & y & z \\ 2^n-1 & 3^n-1 & 5^n-1 \end{vmatrix} = 0$$

**11.** Let $\Delta = \begin{vmatrix} 0 & ab^2 & ac^2 \\ a^2b & 0 & bc^2 \\ a^2c & cb^2 & 0 \end{vmatrix}$

Take : $a^2$, $b^2$ and $c^2$ respectively common from $C_1$, $C_2$ and $C_3$.

$$\Delta = a^2 b^2 c^2 \begin{vmatrix} 0 & a & a \\ b & 0 & b \\ c & c & 0 \end{vmatrix}$$

operate : $C_2 \to C_2 - C_3$

$$\Delta = a^2 b^2 c^2 \begin{vmatrix} 0 & 0 & a \\ b & -b & b \\ c & c & 0 \end{vmatrix}$$

Expand by $R_1$

$$\Delta = a^2 b^2 c^2.a = \begin{vmatrix} b & -b \\ c & c \end{vmatrix} = 2a^3 b^3 c^3.$$

**12.** Let $\Delta = \begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix}$

Operate : $R_2 \to R_2 - 2R_1$; $R_3 \to R_3 - 3R_1$

$$\Delta = \begin{vmatrix} x+y & x & x \\ 3x+2y & 2x & 0 \\ 7x+5y & 5x & 0 \end{vmatrix}$$

Expand by $C_3$

$$\Delta = x\begin{vmatrix} 3x+2y & 2x \\ 7x+5y & 5x \end{vmatrix}$$

$$= x[5x(3x+2y) - 2x(7x+5y)]$$
$$= x[15x^2 + 10xy - (14x^2 + 10xy)] = x^3$$

**13.** $\because$ a, b, c are in A.P.

$\therefore$ $b - a = c - b$ ...(i)

Now L.H.S. $= \begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$

Operate : $R_1 \to R_2 - R_1$ ; $R_3 \to R_3 - R_2$

$$= \begin{vmatrix} x+1 & x+2 & x+a \\ 1 & 1 & b-a \\ 1 & 1 & c-b \end{vmatrix}$$

$$= \begin{vmatrix} x+1 & x+2 & x+a \\ 1 & 1 & b-a \\ 1 & 1 & b-a \end{vmatrix} \quad \text{[Using (1) ]}$$

$= 0$ [$\because$ $R_2$ and $R_3$ are identical]

= R.H.S.

**14. (a)** $D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 1 & 1 & -2 \end{vmatrix} = 3$

$$\Rightarrow D_1 = \begin{vmatrix} 6 & 1 & 1 \\ 1 & 1 & -1 \\ -3 & 1 & -2 \end{vmatrix} = 3$$

$$D_2 = \begin{vmatrix} 1 & 6 & 1 \\ 2 & 1 & -1 \\ 1 & -3 & -2 \end{vmatrix} = 6$$

Similarly $D_3 = 9$

so consistent having x = 1, y = 2, z = 3

**(d)** $D = \begin{vmatrix} 7 & -7 & 5 \\ 3 & 1 & 5 \\ 2 & 3 & 5 \end{vmatrix} = 0$

$$D_1 = \begin{vmatrix} 3 & -7 & 1 \\ 7 & 1 & 1 \\ 5 & 3 & 1 \end{vmatrix} = 24$$

Inconsistent system

**15.** $\frac{\alpha_1}{\alpha_2} = \frac{\beta_1}{\beta_2} \Rightarrow \frac{(\alpha_1+\alpha_2)^2}{(\alpha_1+\alpha_2)^2 - 4\alpha_1\alpha_2} = \frac{(\beta_1+\beta_2)^2}{(\beta_1+\beta_2)^2 - 4\beta_1\beta_2}$

$$\Rightarrow \frac{b^2/a^2}{b^2/a^2 - 4c/a} = \frac{q^2/p^2}{q^2/p^2 - 4r/p}$$

$$\Rightarrow \frac{b^2}{q^2} = \frac{b^2 - 4ac}{q^2 - 4rp} \Rightarrow \frac{b^2}{q^2} = \frac{4ac}{4rp}$$

## EXERCISE - 5

### Part # I : AIEEE/JEE-MAIN

**3.** $\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = 0$

$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + abc \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$

$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} (1 + abc) = 0$

$(a - b)(b - c)(c - a)(1 + abc) = 0$

but $a \neq b \neq c$ so $abc = -1$

**4.** If $a_1, a_2, \ldots\ldots a_n, \ldots\ldots$ are in G.P.

then $\log a_1, \log a_2, \ldots\ldots \log a_n, \ldots\ldots$ are in A.P.

A, A + D, ................

Let common difference of A.P. is D

so $\begin{vmatrix} \log a_n & \log a_{n+2} & \log a_{n+4} \\ \log a_{n+6} & \log a_{n+8} & \log a_{n+10} \\ \log a_{n+12} & \log a_{n+14} & \log a_{n+16} \end{vmatrix}$

$\begin{vmatrix} A & A+2D & A+4D \\ A+6D & A+8D & A+10D \\ A+12D & A+14D & A+16D \end{vmatrix}$

$C_2 \to 2C_2 - (C_1 + C_3)$

$\begin{vmatrix} A & 0 & A+4D \\ A+6D & 0 & A+10D \\ A+12D & 0 & A+16D \end{vmatrix} = 0$

**5.** $a^2 + b^2 + c^2 = -2$

applying $C_1 \to C_1 + C_2 + C_3$

$= \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 1 & 1+b^2x & (1+c^2)x \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix}$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$\Rightarrow \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 0 & 1-x & 0 \\ 0 & 0 & 1-x \end{vmatrix}$

$= (1 - x)^2$ so degree is $\to 2$

**6.** For no solution

$\Delta - 0$ and $\Delta_x$ or $\Delta_y$ or $\Delta_z$ at least one is not zero.

$\Delta = \begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0$

at $\alpha = 1$ their are 3 row are identical so factor of determinant $(\alpha - 1)^2$

and other factor will be find out by $R_1 \to R_1 + R_3 + R_3$

$\lambda(\alpha + 2)(\alpha - 1)^2 = 0$

$\alpha = -2, 1$

but at $\alpha = 1$

all equation are same so at $\alpha = 1$ system of equation infinite solution and

at $\alpha = -2$

$\Delta x = \begin{vmatrix} -3 & 1 & 1 \\ -3 & -2 & 1 \\ -3 & 1 & -2 \end{vmatrix}$

$-3(4 - 1) - 1(6 + 3) + 1(-3 + 6)$

$-9 - 9 + 3 = -15 \neq 0$

so at $\alpha = -2$ system have no solution.

**9.** $x - cy - bz = 0$

$cx - y + az = 0$ $\quad x \neq 0 ; y \neq 0, z \neq 0$

$bx + ay - z = 0$

these system is homogeneous

so $\Delta_x = \Delta_y = \Delta_z = 0$

and at $\Delta = 0 \to$ system have non zero solution.

$\Delta = \begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$

$1 - a^2 + c(-c - ab) - b(ac + b) = 0$

$1 - a^2 - b^2 - c^2 - abc - abc = 0$

$a^2 + b^2 + c^2 + 2abc = 1$

**12.** $\Delta = 0$ (For Non zero solution)

$$\begin{vmatrix} 4 & K & 2 \\ K & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0$$

$8 - K(K-2) + 2(2K-8) = 0$

$8 - K^2 + 2K + 4K - 16 = 0$

$-K^2 + 6K - 8 = 0$

$K^2 - 6K + 8 = 0$

$(K-4)(K-2) = 0$

$K = 2, 4$ Two solution

**13.** For Trivial sol$^n$ $\Delta \neq 0$

$$\begin{vmatrix} 1 & -K & 1 \\ K & 3 & -K \\ 3 & 1 & -1 \end{vmatrix} \neq 0$$

$(-3 + K) + K(-K + 3K) + (K - 9) \neq 0$

$2K^2 + 2K - 12 \neq 0$

$K^2 + K - 6 \neq 0$

$(K+3)(K-2) \neq 0$

$K \neq -3$

$K \neq 2$

So Ans. $R - \{2, -3\}$

## Part # II : IIT-JEE ADVANCED

**6. (a) Method : 1**

$P \equiv (-\sin(\beta - \alpha), -\cos\beta) \equiv (x_1, y_1)$,

$Q \equiv (\cos(\beta - \alpha), \sin\beta) \equiv (x_2, y_2)$

and $R \equiv (x_2\cos\theta + x_1\sin\theta, y_2\cos\theta + y_1\sin\theta)$

We see that

$$T \equiv \left(\frac{x_2\cos\theta + x_1\sin\theta}{\cos\theta + \sin\theta}, \frac{y_2\cos\theta + y_1\sin\theta}{\cos\theta + \sin\theta}\right) \text{ and}$$

P, Q, T are collinear $\Rightarrow$ P, Q, R are non–collinear

**Method : 2**

$$\begin{vmatrix} -\sin(\beta-\alpha) & -\cos\beta & 1 \\ \cos(\beta-\alpha) & \sin\beta & 1 \\ \cos(\beta-\alpha+\theta) & \sin(\beta-\theta) & 1 \end{vmatrix}$$

Applying $R_3 \to R_3 + R_1\sin\theta - R_2\cos\theta$

$$\begin{vmatrix} -\sin(\beta-\alpha) & -\cos\beta & 1 \\ \cos(\beta-\alpha) & \sin\beta & 1 \\ 0 & 0 & 1+\sin\theta-\cos\theta \end{vmatrix}$$

$= (1 + \sin\theta - \cos\theta)\,[-\sin\beta\,\sin(\beta - \alpha) + \cos\beta\,\cos(\beta - \alpha)]$

$= (1 + \sin\theta - \cos\theta)\cos(2\beta - \alpha) \neq 0$

Hence P, Q, R are non collinear.

**(b)** $x - 2y + 3z = -1, -x + y - 2z = k$

& $x - 3y + 4z = 1$

$$\Delta = \begin{vmatrix} 1 & -2 & 3 \\ -1 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix} = 0 \;\&\; \begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} = 3 - k$$

Hence if k = 3 then system will have infinite solutions and $k \neq 3$ then system will have no solution. so S(I) & S(II) both are true & (II) is correct explaination for (I).

**7.** $(y + z)\cos3\theta = (xyz)\sin3\theta$ ...(i)

$$x\sin3\theta = \frac{2\cos3\theta}{y} + \frac{2\sin3\theta}{z} \quad \text{...(ii)}$$

$(xyz)\sin3\theta = (y + 2z)\cos3\theta + y\sin3\theta$ ...(iii)

where $yz \neq 0$ and $0 < \theta < \pi$

from (i) & (iii)

$(y + z)\cos3\theta = (y + 2z)\cos3\theta + y\sin3\theta$

$\Rightarrow z\cos3\theta + y\sin3\theta = 0$ ...(iv)

from eq$^n$ (ii)

$2z\cos3\theta + 2y\sin3\theta = xyz\sin3\theta$ ...(v)

from equation (iv) & (v)

$\Rightarrow xyz\sin3\theta = 0$

$\Rightarrow x\sin3\theta = 0$ as $yz \neq 0$

Possible cases are either $x = 0$ or $\sin 3\theta = 0$

Case (1) : if $x = 0$

$\Rightarrow y + z = 0 \Rightarrow y = -z$

from eq$^n$ (iv) $\cos 3\theta = \sin 3\theta$

$$\Rightarrow 3\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4} \Rightarrow \theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{9\pi}{12}$$

Case (2) : if $\sin 3\theta = 0$

$$\Rightarrow \theta = \frac{\pi}{3}, \frac{2\pi}{3}$$

But these values does not satisfy given equations. Hence, total number of possible values of $\theta$ are 3.

**10.** $x^3\begin{vmatrix}1 & 1 & 1+x^3\\ 2 & 4 & 1+8x^3\\ 3 & 9 & 1+27x^3\end{vmatrix}=10$

$$x^3\begin{vmatrix}1 & 1 & 1\\ 2 & 4 & 1\\ 3 & 9 & 1\end{vmatrix}+x^6\begin{vmatrix}1 & 1 & 1\\ 2 & 4 & 8\\ 3 & 9 & 27\end{vmatrix}=10$$

$$x^3\begin{vmatrix}1 & 0 & 0\\ 2 & 2 & -1\\ 3 & 6 & -2\end{vmatrix}+x^6\begin{vmatrix}1 & 0 & 0\\ 2 & 2 & 6\\ 3 & 6 & 24\end{vmatrix}=10$$

$6x^3+x^3-5=0$

$\Rightarrow \quad 6x^6+6x^3-5x^3-5=0$

$(6x^3-5)(x^3+1)=0$

$x^3=\dfrac{5}{6}$ or $x^3=-1$. Two real distinct values of x.

**11.** $ax+2y=\lambda$

$3x-2y=\mu$

$\Delta=\begin{vmatrix}\alpha & 2\\ 3 & -2\end{vmatrix}=-2\alpha-6$

$\Delta=0 \quad\Rightarrow\quad \propto=-3$

$\Delta_1=\begin{vmatrix}\lambda & 2\\ \mu & -2\end{vmatrix}=-2\lambda-2\mu=-(\lambda+\mu)$

$\Delta_2=\begin{vmatrix}-3 & \lambda\\ 3 & \mu\end{vmatrix}=-3\mu-3\lambda=-(\lambda+\mu)$

## MOCK TEST

**1. (C)**

$|\text{adj } A^{-1}|=|A^{-1}|^2=\dfrac{1}{|A|^2}$

$|(\text{adj } A^{-1})^{-1}|=\dfrac{1}{|\text{adj } A^{-1}|}=|A|^2=2^2=4$

**2.** $\begin{vmatrix}x^k & x^{k+2} & x^{k+3}\\ y^k & y^{k+2} & y^{k+3}\\ z^k & z^{k+2} & z^{k+3}\end{vmatrix}=x^ky^kz^k\begin{vmatrix}1 & x^2 & x^3\\ 1 & y^2 & y^3\\ 1 & z^2 & z^3\end{vmatrix}$

$=(x-y)(y-z)(z-x)(xy+zy+zx)\,x^ky^kz^k$ at $k=-1$

$=(x-y)(y-z)(z-x)\left(\dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}\right)$

**3. (A)**

$\begin{vmatrix}pa & qb & rc\\ qc & ra & pb\\ rb & pc & qa\end{vmatrix}=pqr(a^3+b^3+c^3)-abc(p^3+q^3+r^3)=pqr$

$(3\ abc)-abc\ (3\ pqr)=0$

**4.** $\begin{vmatrix}a+p & \ell+x & u+f\\ b+q & m+y & v+g\\ c+r & n+z & w+h\end{vmatrix}=\begin{vmatrix}a+p & \ell+x & u\\ b+q & m+y & v\\ c+r & n+z & w\end{vmatrix}$

$+\begin{vmatrix}a+p & \ell+x & f\\ b+q & m+y & g\\ c+r & n+z & h\end{vmatrix}$

$=\begin{vmatrix}a+p & \ell & u\\ b+q & m & v\\ c+r & n & w\end{vmatrix}+\begin{vmatrix}a+p & x & u\\ b+q & y & v\\ c+r & z & w\end{vmatrix}$

$+\begin{vmatrix}a+p & \ell & f\\ b+q & m & g\\ c+r & n & h\end{vmatrix}+\begin{vmatrix}a+p & x & f\\ b+q & y & g\\ c+r & z & h\end{vmatrix}$

Now each det. may splits in 2 det.

So, total determinants are 8.

**5. (D)**

2ABC is not defined

$\therefore$ there is no solution

**6.** $\begin{vmatrix} a & a^3 & a^4-1 \\ b & b^3 & b^4-1 \\ c & c^3 & c^4-1 \end{vmatrix} = 0$

$$\begin{vmatrix} a & a^3 & a^4 \\ b & b^3 & b^4 \\ c & c^3 & c^4 \end{vmatrix} = \begin{vmatrix} a & a^3 & 1 \\ b & b^3 & 1 \\ c & c^3 & 1 \end{vmatrix}$$

abc (a – b) (b – c) (c – a) (ab + bc + ca)

= (a – b) (b – c) (c – a) (a + b + c)

$\therefore$ abc (ab + bc + ca) = (a + b + c)

**7. (C)**

$A^2 = I \Rightarrow A^2 - I = O$

$\Rightarrow (A + I)(A - I) = O$

$\therefore$ either $|A + I| = 0$ or $|A - I| = 0$

If $|A - I| \neq 0$, then $(A + I)(A - I) = O$

$\Rightarrow$ A + I = O which is not so

$\therefore$ $|A - I| = 0$ and $A - I \neq O$.

**8.** $f(x) = \log_{10}x$, $g(x) = e^{i\pi x} = \cos\pi x + i\sin\pi x$

$f(10) = 1$ $g(10) = 1$

$$h(10) = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 0 \\ 3 & 3 & 1 \end{vmatrix} = 0$$

**9.** $D = \begin{vmatrix} a & 1+2i & 3-5i \\ 1-2i & b & -7-3i \\ 3+5i & -7+3i & c \end{vmatrix}$

$= abc - a(58) - (1+2i)\{c(1-2i) - (-21-35i-9i+15)\} + (3-5i)\{(-7+14i+3i+6) - 6(3+5i)\}$

$= abc - 58a - (1+2i)(c - 2ic + 6 + 44i) - (3-5i)(36 + 56i) + (3-5i)(-1+17i)$

$= abc - 58a - c - 2ic + 2ic - 6 - 44i - 4c - 12i + 88 - 3 - 5i - 51i + 85 - 96 - 25b + 15ib - 15ib$

$= abc - 58a - 34b - c + 85$ Purely real.

Aliter : observe that $D = D^T$

and $\bar{D} = D^T = D$ i.e. $\bar{D} = D$

i.e. D is real

$\bar{D}$ is a conjugate of D

**10. (A)**

**$S_1$ :** $D = \begin{vmatrix} \ell n\, x & \ell n\, y & \ell n\, z \\ \ell n\, 2x & \ell n\, 2y & \ell n\, 2y \\ \ell n\, 3x & \ell n\, 3y & \ell n\, 3z \end{vmatrix}$

$$= \begin{vmatrix} \ell n\, x & \ell n\, y & \ell n\, z \\ \ell n\, 2 + \ell n\, x & \ell n 2 + \ell n y & \ell n 2 + \ell n z \\ \ell n\, 3 + \ell n\, x & \ell n\, 3 + \ell n\, y & \ell n\, 3 + \ell n\, z \end{vmatrix}$$

$$= \begin{vmatrix} \ell n\, x & \ell n\, y & \ell n\, z \\ \ell n\, 2 & \ell n\, 2 & \ell n\, 2 \\ \ell n\, 3 & \ell n\, 3 & \ell n\, 3 \end{vmatrix} = 0$$

**$S_2$ :** $0 = \begin{vmatrix} x & a & b & 1 \\ \lambda & x & b & 1 \\ \lambda & \mu & x & 1 \\ \lambda & \mu & \nu & 1 \end{vmatrix} = \begin{vmatrix} x & a & b & 1 \\ \lambda - x & x - a & 0 & 0 \\ \lambda - x & \mu - a & x - b & 0 \\ \lambda - x & \mu - a & \nu - b & 0 \end{vmatrix}$

$$\begin{cases} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \\ R_4 \to R_4 - R_1 \end{cases}$$

$$= (\lambda - x)\begin{vmatrix} 1 & x-a & 0 \\ 1 & \mu - a & x - b \\ 1 & \mu - a & \nu - b \end{vmatrix} \quad \begin{cases} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{cases}$$

$$= (\lambda - x)\begin{vmatrix} 1 & x-a & 0 \\ 0 & \mu - x & x - b \\ 0 & \mu - x & \nu - b \end{vmatrix} = (\lambda - x)(\mu - x)(\nu - x)$$

**$S_3$ :** $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

$$= -\frac{a+b+c}{2}\left((a-b)^2 + (b-c)^2 + (c-a)^2\right) < 0$$

$S_4$ : $f(x) = \begin{vmatrix} \frac{1}{x} & \ell nx & x^n \\ 1 & -\frac{1}{n} & (-1)^n \\ 1 & a & a^2 \end{vmatrix} = f'(x)$

$= \begin{vmatrix} -\frac{1}{x^2} & \frac{1}{x} & nx^{n-1} \\ 1 & -\frac{1}{n} & (-1)^n \\ 1 & a & a^2 \end{vmatrix} f^{(n)}(x)$

$= \begin{vmatrix} (-1)^n \frac{n!}{x^{n+1}} & (-1)^{n-1}\frac{(n-1)!}{x^n} & n! \\ 1 & -\frac{1}{n} & (-1)^n \\ 1 & a & a^2 \end{vmatrix}$

$f^{(n)}(1) = \begin{vmatrix} (-1)^n n! & (-1)^{n-1}(n-1)! & n! \\ 1 & -\frac{1}{n} & (-1)^n \\ 1 & a & a^2 \end{vmatrix}$

$= (-1)^n n! \begin{vmatrix} 1 & -\frac{1}{n} & (-1)^n \\ 1 & -\frac{1}{n} & (-1)^n \\ 1 & a & a^2 \end{vmatrix} = 0$

**11. (A, B, C, D)**

Here, (a), (b), (c), (d) are the properties of adjoint.

**12.** $f(x) = \begin{vmatrix} 2\sin x & \sin^2 x & 0 \\ 1 & 2\sin x & \sin^2 x \\ 0 & 1 & 2\sin x \end{vmatrix} = 2\sin x(4\sin^2 x - \sin^2 x)$

$- \sin^2 x(2\sin x) = 6\sin^3 x - 2\sin^3 x$

$f(x) = 4\sin^3 x$

$f'(\pi) = 12\sin^2 x \cos x$

**(B)** $f'(\pi/2) = 12\sin^2(\pi/2)\cos(\pi/2) = 0$

**(C)** $f(-x) = -f(x)$ odd function

$\therefore \int_{-\pi/2}^{\pi/2} f(x)dx = 0$

**(D)** at $x = 0, y = 0$

$\left(\frac{dy}{dx}\right)_{(0,0)} = 0$ tangent at (0, 0)

$y - 0 = \left(\frac{dy}{dx}\right)_{(0,0)}(x - 0)$

$y = 0$

**13. (B, C, D)**

$x + 3y + 2z = 6$ .......**(i)**

$x + \lambda y + 2z = 7$ .......**(ii)**

$x + 3y + 2z = \mu$ .......**(iii)**

**(A)** If $\lambda = 2$, then $D = 0$, therefore unique solution is not possible

**(B)** If $\lambda = 4, \mu = 6$

$x + 3y = 6 - 2z$

$x + 4y = 7 - 2z$

$\therefore$ $y = 1$ and $x = 3 - 2z$

substituting in equation **(iii)**

$3 - 2z + 3 + 2z = 6$ is satisfied

$\therefore$ infinite solutions

**(C)** $\lambda = 5, \mu = 7$

consider equation **(ii)** and **(iii)**

$x + 5y = 7 - 2z$

$x + 3y = 7 - 2z$

$\therefore$ $y = 0$ $x = 7 - 2z$ are solution

sub. in **(i)**

$7 - 2z + 2z = 6$ does not satisfy

$\therefore$ no solution

**(D)** if $\lambda = 3$, $\mu = 5$

then equation **(i)** and **(ii)** have no solution

$\therefore$ no solution

**14.** $f(x) = \begin{vmatrix} 1/x & \log x & x^n \\ 1 & -1/n & (-1)^n \\ 1 & a & a^2 \end{vmatrix}$

$f'(x) = \begin{vmatrix} -1/x^2 & 1/x & nx^{n-1} \\ 1 & -1/n & (-1)^n \\ 1 & a & a^2 \end{vmatrix}$

$f''(x) = \begin{vmatrix} 2/x^3 & -1/x^2 & n(n-1)x^{n-2} \\ 1 & -1/n & (-1)^n \\ 1 & a & a^2 \end{vmatrix}$

$f^n(x) = \begin{vmatrix} (-1)^n \dfrac{n!}{x^{n+1}} & \dfrac{(-1)^{n-1}(n-1)!}{x^n} & n! \\ 1 & -1/n & (-1)^n \\ 1 & a & a^2 \end{vmatrix}$

$f^n(1) = \begin{vmatrix} (-1)^n n! & (-1)^{n-1}(n-1)! & n! \\ 1 & -1/n & (-1)^n \\ 1 & a & a^2 \end{vmatrix}$

$= (-1)^n n! \begin{vmatrix} 1 & -1/n & (-1)^n \\ 1 & -1/n & (-1)^n \\ 1 & a & a^2 \end{vmatrix} = 0$

and $y = a(x - f^n(1))$

$y = ax$

**15. (A, B, C, D)**

Obvious ( using properties)

**16.** $\Delta = \begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -4 \end{vmatrix}$

Applying $R_2 \to R_2 - 3R_1$

and $R_3 \to R_3 - 2R_1$ then,

$\Delta = \begin{vmatrix} 1 & \ldots\ldots & k & \ldots\ldots & 3 \\ \vdots & & & & \\ 0 & & -2k & & -11 \\ \vdots & & & & \\ 0 & & 3-2k & & -10 \end{vmatrix}$

$= 20k + 33 - 22k = 0 \quad \therefore \quad k = 33/2$

**17. (A)**

$A = -A^T \Rightarrow |A| = -|A^T| = -|A|$

$\Rightarrow \quad 2|A| = 0$

$\Rightarrow \quad |A| = 0$

**18. (D)**

$A^{-1}$ exists only for non-singular matrix

$AB = AC$

$\Rightarrow \quad B = C$ if $A^{-1}$ exists

If $A^{-1}$ exists

**19. (C)**

$\det(AB^{-1}) = \det A \,.\, \det B^{-1} = \dfrac{\det A}{\det B} = \dfrac{-6}{2} = -3$

**20.** $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a^3 + b^3 + c^3 - 3abc)$

$= -(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = 0$

$(\because a + b + c = 0)$ (non trivial)

and $\quad a^2 + b^2 + c^2 - ab - bc - ca$

$= \dfrac{1}{2}\{(a-b)^2 + (b-c)^2 + (c-a)^2\} > 0$

**22. (A) → (p, t), (B) → (q), (C) → (r), (D) → (p, t)**

**(A)** $|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$|B| = \begin{vmatrix} a_{11} & k^{-1}a_{12} & k^{-2}a_{13} \\ k\,a_{21} & a_{22} & k^{-1}a_{23} \\ k^2a_{31} & k\,a_{32} & a_{33} \end{vmatrix} = \dfrac{1}{k^3}\begin{vmatrix} k^2a_{11} & k\,a_{12} & a_{13} \\ k^2a_{21} & ka_{22} & a_{23} \\ k^2a_{31} & ka_{32} & a_{33} \end{vmatrix}$

$= |A|$

$k_1|A| + k_2|B| = 0$

$k_1 + k_2 = 0$

**(B)** $\begin{vmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix} = 4$

**(C)** $\begin{vmatrix} 1 & \cos\alpha & \cos\beta \\ \cos\alpha & 1 & \cos\gamma \\ \cos\beta & \cos\gamma & 1 \end{vmatrix} = \begin{vmatrix} 0 & \cos\alpha & \cos\beta \\ \cos\alpha & 0 & \cos\gamma \\ \cos\beta & \cos\gamma & 0 \end{vmatrix}$

$\Rightarrow \sin^2\gamma - \cos\alpha(\cos\alpha - \cos\beta\cos\gamma) + \cos\beta(\cos\alpha\cos\gamma - \cos\beta)$

$= -\cos\alpha(-\cos\beta\cos\gamma) + \cos\beta(\cos\alpha\cos\gamma)$

$\Rightarrow \sin^2\gamma - \cos^2\alpha + 2\cos\alpha\cos\beta\cos\gamma - \cos^2\beta = 2\cos\alpha\cos\beta\cos\gamma$

$\Rightarrow \sin^2\gamma = \cos^2\alpha + \cos^2\beta$

$\Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

**(D)** $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix}$

$R_2 \to R_2 - (R_1 + R_3)$

$= \begin{vmatrix} x^2+x & x+1 & x-2 \\ -4 & 0 & 0 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = 4\begin{vmatrix} x+1 & x-2 \\ 2x-1 & 2x-1 \end{vmatrix}$

$= 4\begin{vmatrix} x+1 & -3 \\ 2x-1 & 0 \end{vmatrix} = (24x - 12)$

$\therefore$ A = 24, B = –12

$\therefore$ A + 2B = 0

**23.**

**1. (A)**

$b_1 . C_{31} + b_2 . C_{32} + b_3 . C_{33} =$

$b_1\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - b_2\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + b_3\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = 0$

**2. (B)**

Value of new determinant $= 2^3\Delta = 8\Delta$

**3. (D)**

$a_3 M_{13} - b_3 . M_{23} + d_3 . M_{33} = a_3 C_{13} + b_3 . C_{23} + d_3 . C_{33} = \Delta$ by definition

**24.**

**1.** a + b + c = p, ab + bc + ca = 0

$\therefore a^2 + b^2 + c^2 = (a + b + c)^2 - 2(ab + bc + ca)$

$= p^2 - 0$

$= p^2$

If $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

$\therefore \Delta^c = \begin{vmatrix} bc-a^2 & ca-b^2 & ab-c^2 \\ ca-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ca-b^2 \end{vmatrix} = \Delta^{3-1}$

$= \Delta^2 = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \times \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

$= \begin{vmatrix} a^2+b^2+c^2 & ab+bc+ca & ab+bc+ca \\ ab+bc+ca & a^2+b^2+c^2 & ab+bc+ca \\ ab+bc+ca & ab+bc+ca & a^2+b^2+c^2 \end{vmatrix}$

$= \begin{vmatrix} p^2 & 0 & 0 \\ 0 & p^2 & 0 \\ 0 & 0 & p^2 \end{vmatrix} = p^6$

**2.** $\begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix} \times \begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix}$

$= \begin{vmatrix} l_1^2+m_1^2+n_1^2 & l_1l_2+m_1m_3+n_1n_2 & l_1l_3+m_1m_3+n_1n_3 \\ l_1l_2+m_1m_2+n_1n_2 & l_2^2+m_2^2+n_2^2 & l_2l_3+m_2m_3+n_2n_3 \\ l_3l_1+m_3m_1+n_3n_1 & l_2l_3+m_2m_3+n_2n_3 & l_3^2+m_3^2+n_3^2 \end{vmatrix}$

$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$

$\therefore \begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix}^2 = 1$

$$\therefore \begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix} = \pm 1$$

**3.** $x^3 - 3x^2 + 3x + 7 = 0$

$\Rightarrow (x-1)^3 + 8 = 0$

$\Rightarrow (x-1)^3 = (-2)^3$

$$\Rightarrow \left(\frac{x-1}{-2}\right)^3 = 1$$

$$\Rightarrow \frac{x-1}{-2} = (1)^{1/3} = 1, \omega, \omega^2$$

$\Rightarrow x - 1 = -2, -2\omega, -2\omega^2$

or $x = -1, 1 - 2\omega, 1 - 2\omega^2$

$\therefore a = -1, b = 1 - 2\omega, c = 1 - 2\omega^2$

$$\because \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2$$

$$= \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \times \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$= \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \times \begin{vmatrix} -a & c & b \\ -b & a & c \\ -c & b & a \end{vmatrix} \quad \text{(row by row)}$$

$$= \begin{vmatrix} 2bc - a^2 & c^2 & b^2 \\ c^2 & 2ac - b^2 & a^2 \\ b^2 & a^2 & c^2 \end{vmatrix}$$

$$= \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2$$

$= (a^3 + b^3 + c^3 - 3abc)^2$

$= \{(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)\}^2$

$$= \frac{1}{4}(a+b+c)^2\{(a-b)^2 + (b-c)^2 + (c-a)^2\}^2$$

$$= \frac{9}{4}\{-12(1 + \omega + \omega^2)\} = 0$$

**25.**

**1. (C)**

As $2^{nd}$ row of all the options is same, we are to look at the elements of the first row.

Let left inverse be $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$, then

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\therefore \quad a + b + 2c = 1$

$$-a + b + 3c = 0 \quad \text{i.e. } b = \frac{1-5c}{2}, \; a = \frac{1+c}{2}$$

Thus matrices in the options A, B and D are the inverses and matrix in option C is not the left inverse.

**2. (D)**

Let right inverse is

$$\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$$

Now $\quad a - c + 2e = 1$

$b - d + 2f = 0$

$2a - c + e = 0$

$2b - d + f = 1$

infinite solution

so answer is (D)

**3. (C)**

By observation there can't be any left inverse for (B) & (D) so we will check for (A) & (C) only.

For (A) let left inverse be $\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$, then

$$\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 \\ -3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now $\quad a - 3b = 1, 2a + 2b = 0$

and $\quad 4a + b = 0$ which is not possible.

For (C) $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 2 & -3 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow$ $a + 2b + 5c = 1, 4a - 3b + 4c = 0,$

$d + 2e + 5f = 0, 4d - 3e + 4f = 1$

$\therefore$ there are infinite number of left inverses.

$$\begin{bmatrix} 1 & 4 \\ 2 & -3 \\ 5 & 4 \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow$ $a + 4d = 1, 2a - 3d = 0$ and $5a + 4d = 0$

which is not possible

$\therefore$ There is no right inverse.

**26. (4)**

On solving we get

$(2\lambda + 15) f(x+1) - (\lambda + 10) f(x+8) - f(x+1) = 0$

$(2\lambda + 14) f(x+1) = (\lambda + 10) f(x+8)$

Since f is periodic with period 7

$\therefore$ $f(x+1) = f(x+8)$

$\Rightarrow$ $2\lambda + 14 = \lambda + 10$

$\Rightarrow$ $|\lambda| = 4$

**27. (1)** $f(-x) = -f(x) = g(x)$

$\therefore$ $f(x).g(x) = -(f(x))^2$

or $f(1)\,g(1) = -(f(1))^2 = -\begin{vmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 1 & 3 & 1 \end{vmatrix}^2 = -4$

$\Rightarrow$ $\lambda f(1)\,g(1) = 4 \Rightarrow \lambda(-4) = 4 \Rightarrow \lambda = -1$

**28.** Clearly $f'(x) = 0$

$\therefore$ $f(x) = c = 6$

$\therefore$ $\sum_{r=1}^{25} f(r) = \sum_{r=1}^{25} 6 = 150$

**29. (0)** Let

$$\begin{vmatrix} (1+x)^{a_1b_1} & (1+x)^{a_1b_2} & (1+x)^{a_1b_3} \\ (1+x)^{a_2b_1} & (1+x)^{a_2b_2} & (1+x)^{a_2b_3} \\ (1+x)^{a_3b_1} & (1+x)^{a_3b_2} & (1+x)^{a_3b_3} \end{vmatrix}$$

$= \lambda_0 + \lambda_1 x + \lambda_2 x^2 + \lambda_3 x^3 + .....$

for $\lambda_1$ differentiate w.r.t. x and put $x = 0$

so $\lambda_1 = 0$

**30.** $p^2 + q^2 - pq - p - q + 1 \le 0$

or $2p^2 + 2q^2 - 2pq - 2p - 2q + 2 \le 0$

or $(p^2 + q^2 - 2pq) + (p^2 - 2p + 1) + (q^2 - 2q + 1) \le 0$

$\Rightarrow$ $(p - q)^2 + (p - 1)^2 + (q - 1)^2 \le 0$

which is possible

$(p - q)^2 + (p - 1)^2 + (q - 1)^2 = 0$

or $p - q = 0, p - 1 = 0, q - 1 = 0$

$\therefore$ $p = 1, q = 1$

Then, $\begin{vmatrix} 1 & \cos\gamma & \cos\beta \\ \cos\gamma & p & \cos\alpha \\ \cos\beta & \cos\alpha & q \end{vmatrix} = \begin{vmatrix} 1 & \cos\gamma & \cos\beta \\ \cos\gamma & 1 & \cos\alpha \\ \cos\beta & \cos\alpha & 1 \end{vmatrix}$

$= 1 + 2\cos\alpha\cos\beta\cos\gamma - \cos^2\alpha - \cos^2\beta - \cos^2\gamma$

$= 2\cos\alpha\cos\beta\cos\gamma - (\cos^2\alpha - \sin^2\beta) - \cos^2\gamma$

$= 2\cos\alpha\cos\beta\cos\gamma - \cos(\alpha + \beta)\cos(\alpha - \beta) - \cos^2\gamma$

$= 2\cos\alpha\cos\beta\cos\gamma - \cos(-\gamma)\cos(\alpha - \beta) - \cos^2\gamma$

$(\because \alpha + \beta + \gamma = 0)$

$= \cos\gamma\{2\cos\alpha\cos\beta - \cos(\alpha - \beta) - \cos\gamma\}$

$= \cos\gamma\{2\cos\alpha\cos\beta - \cos(\alpha - \beta) - \cos(\alpha + \beta)\}$

$= \cos\gamma(2\cos\alpha\cos\beta - 2\cos\alpha\cos\beta)$

$= 0$