SOLVED EXAMPLES

Ex. 1 Construct a 3 × 2 matrix whose elements are given by $a_{ij} = \frac{1}{2}|i - 3j|$.

Sol. In general a 3 × 2 matrix is given by $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$.

$$a_{ij} = \frac{1}{2}|i - 3j|,\ i = 1, 2, 3 \text{ and } j = 1, 2$$

Therefore $a_{11} = \frac{1}{2}|1 - 3 \times 1| = 1$ $\qquad a_{12} = \frac{1}{2}|1 - 3 \times 2| = \frac{5}{2}$

$a_{21} = \frac{1}{2}|2 - 3 \times 1| = \frac{1}{2}$ $\qquad a_{22} = \frac{1}{2}|2 - 3 \times 2| = 2$

$a_{31} = \frac{1}{2}|3 - 3 \times 1| = 0$ $\qquad a_{32} = \frac{1}{2}|3 - 3 \times 2| = \frac{3}{2}$

Hence the required matrix is given hy $A = \begin{bmatrix} 1 & \frac{5}{2} \\ \frac{1}{2} & 2 \\ 0 & \frac{3}{2} \end{bmatrix}$

Ex. 2 If $[1\ \ x\ \ 2]\begin{bmatrix} 2 & 3 & 1 \\ 0 & 4 & 2 \\ 0 & 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ 1 \\ -1 \end{bmatrix} = \mathbf{O}$, then the value of x is :-

Sol. The LHS of the equation

$$= [2 \quad 4x + 9 \quad 2x + 5]\begin{bmatrix} x \\ 1 \\ -1 \end{bmatrix} = [2x + 4x + 9 - 2x - 5] = 4x + 4$$

Thus $4x + 4 = 0 \Rightarrow x = -1$

Ex. 3 Find the value of x, y, z and w which satisfy the matrix equation

$$\begin{bmatrix} x+3 & 2y+x \\ z-1 & 4w-8 \end{bmatrix} = \begin{bmatrix} -x-1 & 0 \\ 3 & 2w \end{bmatrix}$$

Sol. As the given matrices are equal so their corresponding elements are equal.

$x + 3 = -x - 1 \quad \Rightarrow \quad 2x = -4$

$\therefore \quad x = -2$ ......**(i)**

$2y + x = 0 \quad \Rightarrow \quad 2y - 2 = 0$ [from **(i)**]

$\Rightarrow \quad y = 1$ ......**(ii)**

$z - 1 = 3 \quad \Rightarrow \quad z = 4$ ......**(iii)**

$4w - 8 = 2w \quad \Rightarrow \quad 2w = 8$

$\therefore \quad w = 4$ ......**(iv)**

Ex. 4 Prove that if A is non-singular matrix such that A is symmetric then A^{-1} is also symmetric.

Sol. $A^T = A$ [$\because$ A is a symmetric matrix]

$(A^T)^{-1} = A^{-1}$ [since A is non-singular matrix]

$\Rightarrow \quad (A^{-1})^T = A^{-1}$ Hence proved

Ex. 5 If $\begin{bmatrix} x+3 & z+4 & 2y-7 \\ -6 & a-1 & 0 \\ b-3 & -21 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 6 & 3y-2 \\ -6 & -3 & 2c+2 \\ 2b+4 & -21 & 0 \end{bmatrix}$, then find the values of a, b, c, x, y and z.

Sol. As the given matrices are equal, therefore, their corresponding elements must be equal. Comparing the corresponding elements, we get

$x + 3 = 0 \qquad z + 4 = 6 \qquad 2y - 7 = 3y - 2$

$a - 1 = -3 \qquad 0 = 2c + 2 \qquad b - 3 = 2b + 4$

$\Rightarrow \quad a = -2, b = -7, c = -1, x = -3, y = -5, z = 2$

Ex. 6 Let $A = \begin{bmatrix} 1 & -3 & 2 \\ 2 & 1 & -3 \\ 4 & -3 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 & 4 & 1 & 0 \\ 2 & 1 & 1 & 1 \\ 1 & -2 & 1 & 2 \end{bmatrix}$ & $C = \begin{bmatrix} 1 & 1 & -1 & -2 \\ 3 & -2 & -1 & -1 \\ 2 & -5 & -1 & 0 \end{bmatrix}$ be the matrices then, prove that in matrix multiplication cancellation law does not hold.

Sol. We have to show that AB = AC; though B is not equal to C.

We have $AB = \begin{bmatrix} 1 & -3 & 2 \\ 2 & 1 & -3 \\ 4 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & 4 & 1 & 0 \\ 2 & 1 & 1 & 1 \\ 1 & -2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} -3 & -3 & 0 & 1 \\ 1 & 15 & 0 & -5 \\ -3 & 15 & 0 & -5 \end{bmatrix}_{3\times4}$

Now, $AC = \begin{bmatrix} 1 & -3 & 2 \\ 2 & 1 & -3 \\ 4 & -3 & -1 \end{bmatrix}\begin{bmatrix} 2 & 1 & -1 & -2 \\ 3 & -2 & -1 & -1 \\ 2 & -5 & -1 & 0 \end{bmatrix} = \begin{bmatrix} -3 & -3 & 0 & 1 \\ 1 & 15 & 0 & -5 \\ -3 & 15 & 0 & -5 \end{bmatrix}_{3\times4}$

Here, AB = AC though B is not equal to C. Thus cancellation law does not hold in general.

Ex. 7 If $A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $M = AB$, then M^{-1} is equal to-

Sol. $M = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -2 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -2 & 2 \end{bmatrix}$

$|M| = 6$, adj $M = \begin{bmatrix} 2 & -2 \\ 2 & 1 \end{bmatrix}$

$\therefore \quad M^{-1} = \frac{1}{6}\begin{bmatrix} 2 & -2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1/3 & -1/3 \\ 1/3 & 1/6 \end{bmatrix}$

Ex. 8 If A, B are two matrices such that $A + B = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$, $A - B = \begin{bmatrix} 3 & 2 \\ -2 & 0 \end{bmatrix}$ then find AB.

Sol. Given $A + B = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ ....**(i)** & $A - B = \begin{bmatrix} 3 & 2 \\ -2 & 0 \end{bmatrix}$.......**(ii)**

Adding **(i)** & **(ii)**

$$2A = \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$$

Subtracting **(ii)** from **(i)**

$$2B = \begin{bmatrix} -2 & 0 \\ 4 & 4 \end{bmatrix} \Rightarrow B = \begin{bmatrix} -1 & 0 \\ 2 & 2 \end{bmatrix}$$

Now $AB = \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 4 & 4 \end{bmatrix}$

Ex. 9 If A and B are matrices of order $m \times n$ and $n \times m$ respectively, then order of matrix $B^T(A^T)^T$ is -

Sol. Order of B is $n \times m$ so order of B^T will be $m \times n$

Now $(A^T)^T = A$ & its order is $m \times n$. For the multiplication $B^T(A^T)^T$

Number of columns in prefactor $\neq$ Number of rows in post factor.

Hence this multiplication is not defined.

Hence the given matrix A is involutory.

Ex. 10 If $f(x) = x^2 - 3x + 3$ and $A = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}$ be a square matrix then prove that $f(a) = \mathbf{O}$. Hence find A^4.

Sol. $A^2 = A.A = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ -3 & 0 \end{bmatrix}$

Hence $A^2 - 3A + 3I = \begin{bmatrix} 3 & 3 \\ -3 & 0 \end{bmatrix} - 3\begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix} + 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \mathrm{O}$

Ex. 11 If $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, show that $5A^{-1} = A^2 + A - 5I$

Sol. We have the characteristic equation of A.

$|A - xI| = 0$

i.e. $$\begin{vmatrix} 1-x & 2 & 0 \\ 2 & -1-x & 0 \\ 0 & 0 & -1-x \end{vmatrix} = 0$$

i.e. $x^3 + x^2 - 5x - 5 = 0$

Using Cayley – Hamilton theorem

$A^3 + A^2 - 5A - 5I = \mathbf{O} \Rightarrow 5I = A^3 + A^2 - 5A$

Multiplying by A^{-1}, we get $5A^{-1} = A^2 + A - 5I$

Ex. 12 Show that the matrix $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ is idempotent.

Sol. $A^2 = A.A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \times \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

$$= \begin{bmatrix} 2.2+(-2).(-1)+(-4).1 & 2(-2)+(-2).3+(-4).(-2) & 2.(-4)+(-2).4+(-4).(-3) \\ (-1).2+3.(-1)+4.1 & (-1).(-2)+3.3+4.(-2) & (-1).(-4)+3.4+4.(-3) \\ 1.2+(-2).(-1)+(-3).1 & 1.(-2)+(-2).3+(-3).(-2) & 1.(-4)+(-2).4+(-3).(-3) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} = A$$

Hence the matrix A is idempotent.

Ex. 13 Show that the matrix $A = \begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ is involutory.

Sol. $A^2 = A.A = \begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix} \times \begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 25-24+0 & 40-40+0 & 0+0+0 \\ -15+15+0 & -24+25+0 & 0+0+0 \\ -5+6-1 & -8+10-2 & 0+0+1 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Ex. 14 If $A = \begin{bmatrix} 1 & 3 & 5 \\ 3 & 5 & 1 \\ 5 & 1 & 3 \end{bmatrix}$, then adj A is equal to -

Sol. adj. $A = \begin{bmatrix} 14 & -4 & -22 \\ -4 & -22 & 14 \\ -22 & 14 & -4 \end{bmatrix}^T = \begin{bmatrix} 14 & -4 & -22 \\ -4 & -22 & 14 \\ -22 & 14 & -4 \end{bmatrix}$

Ex. 15 $\begin{bmatrix} 1 & -\tan\theta/2 \\ \tan\theta/2 & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta/2 \\ -\tan\theta/2 & 1 \end{bmatrix}^{-1}$ is equal to -

Sol. $\begin{bmatrix} 1 & \tan\theta/2 \\ \tan\theta/2 & 1 \end{bmatrix}^{-1} = \dfrac{1}{\sec^2\theta/2}\begin{bmatrix} 1 & -\tan\theta/2 \\ \tan\theta/2 & 1 \end{bmatrix}$

$\therefore$ Product $= \dfrac{1}{\sec^2\theta/2}\begin{bmatrix} 1 & -\tan\theta/2 \\ \tan\theta/2 & 1 \end{bmatrix}\begin{bmatrix} 1 & -\tan\theta/2 \\ \tan\theta/2 & 1 \end{bmatrix}$

$= \dfrac{1}{\sec^2\theta/2}\begin{bmatrix} 1-\tan^2\theta/2 & -2\tan\theta/2 \\ 2\tan\theta/2 & 1-\tan^2\theta/2 \end{bmatrix}$

$= \begin{bmatrix} \cos^2\theta/2 \;\; \sin^2\theta/2 & -2\sin\theta/2\cos\theta/2 \\ 2\sin\theta/2 \;\; \cos\theta/2 & \cos^2\theta/2-\sin^2\theta/2 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Ex. 16 Solve the system $\begin{matrix} x+y+z=16 \\ x-y+z=2 \\ 2x+y-z=1 \end{matrix}$ using matrix method.

Sol. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ & $B = \begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix}$

Then the system is AX = B.

$|A| = 6$, hence A is non singular,

Cofactor $A = \begin{bmatrix} 0 & 3 & 3 \\ 2 & -3 & 1 \\ 2 & 0 & -2 \end{bmatrix}$

adj $A = \begin{bmatrix} 0 & 2 & 2 \\ 3 & -3 & 0 \\ 3 & 1 & -2 \end{bmatrix}$

$A^{-1} = \dfrac{1}{|A|}\text{adj } A = \dfrac{1}{6}\begin{bmatrix} 0 & 2 & 2 \\ 3 & -3 & 0 \\ 3 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 1/3 & 1/3 \\ 1/2 & -1/2 & 0 \\ 1/2 & 1/6 & -1/3 \end{bmatrix}$

$X = A^{-1}B = \begin{bmatrix} 0 & 1/3 & 1/3 \\ 1/2 & -1/2 & 0 \\ 1/2 & 1/6 & -1/3 \end{bmatrix}\begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix}$ i.e. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$\Rightarrow$ $x = 1, y = 2, z = 3$

Ex. 17 IF $A = \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$, then find the matrix X, such that $2A + 3X = 5B$

Sol. We have $2A + 3X = 5B$.

$\Rightarrow \quad 3X = 5B - 2A$

$\Rightarrow \quad X = \frac{1}{3}(5B - 2A)$

$$\Rightarrow \quad X = \frac{1}{3}\left(5\begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix} - 2\begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}\right) = \frac{1}{3}\left(\begin{bmatrix} 10 & -10 \\ 20 & 10 \\ -25 & 5 \end{bmatrix} + \begin{bmatrix} -16 & 0 \\ -8 & 4 \\ -6 & -12 \end{bmatrix}\right)$$

$$\Rightarrow \quad X = \frac{1}{3}\begin{bmatrix} 10-16 & -10+0 \\ 20-8 & 10+4 \\ -25-6 & 5-12 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} -6 & -10 \\ 12 & 14 \\ -13 & -7 \end{bmatrix} = \begin{bmatrix} -2 & \frac{-10}{3} \\ 4 & \frac{14}{3} \\ \frac{-31}{3} & \frac{-7}{3} \end{bmatrix}$$

Ex. 18 If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$, then show that $A^3 - 23A - 40\,I = O$

Sol. We have $A^2 = A.A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix}$

So $\quad A^3 = AA^2 = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}\begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix} = \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix}$

Now $\quad A^3 - 23A - 40I = \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} - 23\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} - 40\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} + \begin{bmatrix} -23 & -46 & -69 \\ -69 & 46 & -23 \\ -92 & -46 & -23 \end{bmatrix} + \begin{bmatrix} -40 & 0 & 0 \\ 0 & -40 & 0 \\ 0 & 0 & -40 \end{bmatrix}$$

$$= \begin{bmatrix} 63-23-40 & 46-46+0 & 69-69+0 \\ 69-69+0 & -6+46-40 & 23-23+0 \\ 90-92+0 & 46-46+0 & 63-23-40 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$$

Ex. 19 If $A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}$, $B = [1 \ \ 3 \ \ -6]$, verify that $(AB)' = B'A'$.

Sol. We have

$$A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}, B = [1 \ \ 3 \ \ -6]$$

Then $$AB = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix} [1 \ \ 3 \ \ -6] = \begin{bmatrix} -2 & -6 & 12 \\ 4 & 12 & -24 \\ 5 & 15 & -30 \end{bmatrix}$$

Now $$A' = [-2 \ \ 4 \ \ 5], B' = \begin{bmatrix} 1 \\ 3 \\ -6 \end{bmatrix}$$

$$B'A' = \begin{bmatrix} 1 \\ 3 \\ -6 \end{bmatrix} [-2 \ \ 4 \ \ 5] = \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ 12 & -24 & -30 \end{bmatrix} = (AB)'$$

Clearly $(AB)' = B'A'$

Exercise # 1

[Single Correct Choice Type Questions]

1. If number of elements in a matrix is 60 then how many different order of matrix are possible -

(A) 12 **(B)** 6 **(C)** 24 **(D)** none of these

2. If AB = O for the matrices

$$A = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix} \text{ and } B = \begin{bmatrix} \cos^2\varphi & \cos\varphi\sin\varphi \\ \cos\varphi\sin\varphi & \sin^2\varphi \end{bmatrix} \text{ then } \theta - \phi \text{ is}$$

(A) an odd multiple of $\frac{\pi}{2}$ **(B)** an odd multiple of π

(C) an even multiple of $\frac{\pi}{2}$ **(D)** 0

3. If $A_\alpha = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then $A_\alpha A_\beta$ is equal to -

(A) $A_{\alpha+\beta}$ **(B)** $A_{\alpha\beta}$ **(C)** $A_{\alpha-\beta}$ **(D)** none of these

4. A is a (3×3) diagonal matrix having integral entries such that det (A) = 120, number of such matrices is

(A) 360 **(B)** 390 **(C)** 240 **(D)** 270

5. If the product of n matrices $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\ldots\ldots\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ is equal to the matrix $\begin{bmatrix} 1 & 378 \\ 0 & 1 \end{bmatrix}$ then the value of n is equal to -

(A) 26 **(B)** 27 **(C)** 377 **(D)** 378

6. Matrix A has x rows and x + 5 columns. Matrix B has y rows and 11 – y columns. Both AB and BA exist, then -

(A) $x = 3, y = 4$ **(B)** $x = 4, y = 3$ **(C)** $x = 3, y = 8$ **(D)** $x = 8, y = 3$

7. If A = diag (2, –1, 3), B = diag (–1, 3, 2), then A^2B =

(A) diag (5, 4, 11) **(B)** diag (– 4, 3, 18) **(C)** diag (3, 1, 8) **(D)** B

8. If $A - 2B = \begin{bmatrix} 1 & 5 \\ 3 & 7 \end{bmatrix}$ and $2A - 3B = \begin{bmatrix} -2 & 5 \\ 0 & 7 \end{bmatrix}$, then matrix B is equal to -

(A) $\begin{bmatrix} -4 & -5 \\ -6 & -7 \end{bmatrix}$ **(B)** $\begin{bmatrix} 0 & 6 \\ -3 & 7 \end{bmatrix}$ **(C)** $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ **(D)** $\begin{bmatrix} 6 & -1 \\ 0 & 1 \end{bmatrix}$

9. Matrix A is such that $A^2 = 2A - I$, where I is the identity matrix. The for $n \geq 2$, A^n =

(A) $nA - (n-1)I$ **(B)** $nA - I$ **(C)** $2^{n-1}A - (n-1)I$ **(D)** $2^{n-1}A - I$

10. If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $(aI_2 + bA)^2 = A$, then -

(A) $a = b = \sqrt{2}$ **(B)** $a = b = 1/\sqrt{2}$ **(C)** $a = b = \sqrt{3}$ **(D)** $a = b = 1/\sqrt{3}$

11. If A is a skew symmetric matrix such that $A^TA = I$, then A^{4n-1} $(n \in N)$ is equal to -

(A) $-A^T$ **(B)** I **(C)** $-I$ **(D)** A^T

12. Suppose A is a matrix such that $A^2 = A$ and $(I + A)^{10} = I + kA$, then k is

(A) 127 **(B)** 511 **(C)** 1023 **(D)** 1024

13. Which of the following is an orthogonal matrix -

(A) $\begin{bmatrix} 6/7 & 2/7 & -3/7 \\ 2/7 & 3/7 & 6/7 \\ 3/7 & -6/7 & 2/7 \end{bmatrix}$ **(B)** $\begin{bmatrix} 6/7 & 2/7 & 3/7 \\ 2/7 & -3/7 & 6/7 \\ 3/7 & 6/7 & -2/7 \end{bmatrix}$

(C) $\begin{bmatrix} -6/7 & -2/7 & -3/7 \\ 2/7 & 3/7 & 6/7 \\ -3/7 & 6/7 & 2/7 \end{bmatrix}$ **(D)** $\begin{bmatrix} 6/7 & -2/7 & 3/7 \\ 2/7 & 2/7 & -3/7 \\ -6/7 & 2/7 & 3/7 \end{bmatrix}$

14. Given $A = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. If $A - \lambda I$ is a singular matrix then

(A) $\lambda \in \phi$ **(B)** $\lambda^2 - 3\lambda - 4 = 0$ **(C)** $\lambda^2 + 3\lambda + 4 = 0$ **(D)** $\lambda^2 - 3\lambda - 6 = 0$

15. If A is an orthogonal matrix & $|A| = -1$, then A^T is equal to -

(A) $-A$ **(B)** A **(C)** $-(\text{adj } A)$ **(D)** $(\text{adj } A)$

16. If $A = \begin{bmatrix} 1 & -2 \\ 3 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 4 \\ 2 & 3 \end{bmatrix}$ and $ABC = \begin{bmatrix} 4 & 8 \\ 3 & 7 \end{bmatrix}$, then C equals -

(A) $\frac{1}{66}\begin{bmatrix} 72 & -32 \\ 57 & -29 \end{bmatrix}$ **(B)** $\frac{1}{66}\begin{bmatrix} -54 & -110 \\ 3 & 11 \end{bmatrix}$ **(C)** $\frac{1}{66}\begin{bmatrix} -54 & 110 \\ 3 & -11 \end{bmatrix}$ **(D)** $\frac{1}{66}\begin{bmatrix} -72 & 32 \\ -57 & 29 \end{bmatrix}$

17. A is an involutary matrix given by $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$ then the inverse of $\frac{A}{2}$ will be

(A) 2A **(B)** $\frac{A^{-1}}{2}$ **(C)** $\frac{A}{2}$ **(D)** A^2

18. A and B are two given matrices such that the order of A is 3×4, if $A'B$ and BA' are both defined then

(A) order of B' is 3×4 **(B)** order of $B'A$ is 4×4

(C) order of $B'A$ is 3×3 **(D)** $B'A$ is undefined

19. If $A = \begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix}$ and $f(x) = 1 + x + x^2 + ... + x^{16}$, then $f(A) =$

(A) 0 **(B)** $\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}$ **(C)** $\begin{bmatrix} 1 & 5 \\ 0 & 0 \end{bmatrix}$ **(D)** $\begin{bmatrix} 0 & 5 \\ 1 & 1 \end{bmatrix}$

20. If $\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then matrix A equals -

(A) $\begin{bmatrix} 7 & 5 \\ -11 & -8 \end{bmatrix}$ **(B)** $\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ **(C)** $\begin{bmatrix} 7 & 1 \\ 34 & 5 \end{bmatrix}$ **(D)** $\begin{bmatrix} 5 & 3 \\ 13 & 8 \end{bmatrix}$

21. If $F(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $G(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$, then $[F(\alpha)\,G(\beta)]^{-1} =$

(A) $F(\alpha) - G(\beta)$ **(B)** $-F(\alpha) - G(\beta)$ **(C)** $[F(\alpha)]^{-1}\,[G(\beta)]^{-1}$ **(D)** $[G(\beta)]^{-1}\,[F(\alpha)]^{-1}$

22. If $A = \begin{bmatrix} \cos^2\alpha & \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha & \sin^2\alpha \end{bmatrix}$; $B = \begin{bmatrix} \cos^2\beta & \sin\beta\cos\beta \\ \sin\beta\cos\beta & \sin^2\beta \end{bmatrix}$ are such that, AB is a null matrix, then which of the following should necessarily be an odd integral multiple of $\frac{\pi}{2}$.

(A) α **(B)** β **(C)** $\alpha - \beta$ **(D)** $\alpha + \beta$

23. Let $A + 2B = \begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix}$ and $2A - B = \begin{bmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{bmatrix}$

then Tr (A) – Tr (B) has the value equal to

(A) 0 **(B)** 1 **(C)** 2 **(D)** none

24. For a given matrix $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ which of the following statement holds good?

(A) $A = A^{-1}\ \forall\,\theta \in R$ **(B)** A is symmetric, for $\theta = (2n+1)\frac{\pi}{2}$, $n \in I$

(C) A is an orthogonal matrix for $\theta \in R$ **(D)** A is a skew symmetric, for $\theta = n\pi$; $n \in I$

25. Matrix $A = \begin{bmatrix} x & 3 & 2 \\ 1 & y & 4 \\ 2 & 2 & z \end{bmatrix}$, if $xyz = 60$ and $8x + 4y + 3z = 20$, then $A\,(\text{adj}\,A)$ is equal to

(A) $\begin{bmatrix} 64 & 0 & 0 \\ 0 & 64 & 0 \\ 0 & 0 & 64 \end{bmatrix}$ **(B)** $\begin{bmatrix} 88 & 0 & 0 \\ 0 & 88 & 0 \\ 0 & 0 & 88 \end{bmatrix}$ **(C)** $\begin{bmatrix} 68 & 0 & 0 \\ 0 & 68 & 0 \\ 0 & 0 & 68 \end{bmatrix}$ **(D)** $\begin{bmatrix} 34 & 0 & 0 \\ 0 & 34 & 0 \\ 0 & 0 & 34 \end{bmatrix}$

26. $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$ then let us define a function $f(x) = \det.(A^T A^{-1})$ then which of the following can not be the value of $\underbrace{f(f(f(f.........f(x))))}_{\text{n times}}$ is $(n \geq 2)$

(A) $f^n(x)$ **(B)** 1 **(C)** $f^{n-1}(x)$ **(D)** $n f(x)$

27. Consider a matrix $A(\theta) = \begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix}$ then

(A) $A(\theta)$ is symmetric **(B)** $A(\theta)$ is skew symmetric

(C) $A^{-1}(\theta) = A(\pi - \theta)$ **(D)** $A^2(\theta) = A\left(\frac{\pi}{2} - 2\theta\right)$

28. The equation $\begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 4 \\ 3 & 4 & k \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ has a solution for (x, y, z) besides (0, 0, 0). The value of k equals

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3

29. A is a 2 × 2 matrix such that $A\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $A^2\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. The sum of the elements of A, is

(A) –1 **(B)** 0 **(C)** 2 **(D)** 5

30. If A is an idempotent matrix satisfying,

$$(I - 0.4A)^{-1} = I - \alpha A$$

where I is the unit matrix of the same order as that of A then the value of α is equal to

(A) 2/5 **(B)** 2/3 **(C)** – 2/3 **(D)** 1/2

Exercise # 2 Part # I [Multiple Correct Choice Type Questions]

1. If A is a invertible idempotent matrix of order n, then adj A is equal to -

(A) $(\text{adj } A)^2$ **(B)** I **(C)** A^{-1} **(D)** none of these

2. A square matrix can always be expressed as a

(A) sum of a symmetric matrix and skew symmetric matrix of the same order

(B) difference of a symmetric matrix and skew symmetric matrix of the same order

(C) skew symmetric matrix

(D) symmetric matrix

3. Choose the correct answer :

(A) every scalar matrix is an identity matrix.

(B) every identity matrix is a scalar matrix

(C) transpose of transpose of a matrix gives the matrix itself.

(D) for every square matrix A there exists another matrix B such that AB = I = BA.

4. Let A, B, C, D be (not necessarily square) real matrices such that $A^T = BCD$; $B^T = CDA$; $C^T = DAB$ and $D^T = ABC$ for the matrix S = ABCD, then which of the following is/are true

(A) $S^3 = S$ **(B)** $S^2 = S^4$ **(C)** $S = S^2$ **(D)** none of these

5. Let A be an invertible matrix then which of the following is/are true :

(A) $|A^{-1}| = |A|^{-1}$ **(B)** $(A^2)^{-1} = (A^{-1})^2$ **(C)** $(A^T)^{-1} = (A^{-1})^T$ **(D)** none of these

6. Let a_{ij} denote the element of the i^{th} row and j^{th} column in a 3 × 3 matrix and let $a_{ij} = -a_{ji}$ for every i and j then this matrix is an -

(A) orthogonal matrix **(B)** singular matrix

(C) matrix whose principal diagonal elements are all zero **(D)** skew symmetric matrix

7. If A and B are two invertible matrices of the same order, then adj (AB) is equal to -

(A) adj (B) adj (A) **(B)** $|B||A|\, B^{-1} A^{-1}$ **(C)** $|B||A|\, A^{-1} B^{-1}$ **(D)** $|A||B|(AB)^{-1}$

8. If $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, then-

(A) Adj A is zero matrix **(B)** $\text{Adj } A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

(C) $A^{-1} = A$ **(D)** $A^2 = I$

9. If $A = \begin{bmatrix} 1 & 9 & -7 \\ i & \omega^n & 8 \\ 1 & 6 & \omega^{2n} \end{bmatrix}$, where $i = \sqrt{-1}$ and ω is complex cube root of unity, then tr(a) will be-

(A) 1, if $n = 3k, k \in N$ (B) 3, if $n = 3k, k \in N$ (C) 0, if $n \neq 3k, k \in N$ (D) -1, if $n \neq 3k, n \in N$

10. If $A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -1 \end{bmatrix}$, then

(A) $|A| = 2$

(B) A is non-singular

(C) Adj. $A = \begin{bmatrix} 1/2 & -1/2 & 0 \\ 0 & -1 & 1/2 \\ 0 & 0 & -1/2 \end{bmatrix}$

(D) A is skew symmetric matrix

11. Which of the following is true for matrix $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

(A) $A + 4I$ is a symmetric matrix

(B) $A^2 - 4A + 5I_2 = 0$

(C) $A - B$ is a diagonal matrix for any value of α if $B = \begin{bmatrix} \alpha & -1 \\ 2 & 5 \end{bmatrix}$

(D) $A - 4I$ is a skew symmetric matrix

12. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ satisfies the equation $x^2 + k = 0$, then -

(A) $a + d = 0$ (B) $k = -|A|$ (C) $k = a^2 + b^2 + c^2 + d^2$ (D) $k = |A|$

13. Matrix $\begin{bmatrix} a & b & (a\alpha - b) \\ b & c & (b\alpha - c) \\ 2 & 1 & 0 \end{bmatrix}$ is non invertible if -

(A) $\alpha = 1/2$

(B) a, b, c are in A.P.

(C) a, b, c are in G.P.

(D) a, b, c are in H.P.

14. If A and B are 3×3 matrices and $|A| \neq 0$, then which of the following are true?

(A) $|AB| = 0 \Rightarrow |B| = 0$

(B) $|AB| = 0 \Rightarrow B = 0$

(C) $|A^{-1}| = |A|^{-1}$

(D) $|A + A| = 2|A|$

15. If $AB = A$ and $BA = B$, then

(A) $A^2B = A^2$ (B) $B^2A = B^2$ (C) $ABA = A$ (D) $BAB = B$

16. Given the matrices A and B as $A=\begin{bmatrix}1 & -1\\ 4 & -1\end{bmatrix}$ and $B=\begin{bmatrix}1 & -1\\ 2 & -2\end{bmatrix}$.

The two matrices X and Y are such that XA = B and AY = B then which of the following hold(s) true?

(A) $X=\frac{1}{3}\begin{bmatrix}1 & -1\\ -2 & 2\end{bmatrix}$ **(B)** $Y=\frac{1}{3}\begin{bmatrix}3 & 0\\ 4 & 0\end{bmatrix}$ **(C)** det. X = det. Y **(D)** $3(X+Y)=\begin{bmatrix}4 & -1\\ 4 & 2\end{bmatrix}$

17. If A and B are two 3 × 3 matrices such that their product AB is a null matrix then

(A) det. A ≠ 0 ⇒ B must be a null matrix.

(B) det. B ≠ 0 ⇒ A must be a null matrix.

(C) If none of A and B are null matrices then atleast one of the two matrices must be singular.

(D) If neither det. A nor det. B is zero then the given statement is not possible.

18. Let $P=\begin{bmatrix}3 & -5\\ 7 & -12\end{bmatrix}$ and $Q=\begin{bmatrix}12 & -5\\ 7 & -3\end{bmatrix}$ then the matrix $(PQ)^{-1}$ is

(A) nilpotent **(B)** idempotent **(C)** involutory **(D)** symmetric

19. Let $A = a_{ij}$ be a matrix of order 3 where $a_{ij}=\begin{cases}x & \text{if } i=j,\ x\in R\\ 1 & \text{if } |i-j|=1\\ 0 & \text{otherwise}\end{cases}$ then which of the following hold(s) good ?

(A) for x = 2, A is a diagonal matrix.

(B) A is a symmetric matrix

(C) for x = 2, det A has the value equal to 6

(D) Let f (x) = det A, then the function f (x) has both the maxima and minima.

20. If $A=\begin{bmatrix}1 & \tan x\\ -\tan x & 1\end{bmatrix}$ then let us define a function $f(x) = \det(A^TA^{-1})$ then which of the following can be the value of $\underbrace{f(f(f(f\ldots\ldots\ldots f(x))))}_{\text{n times}}$ $(n \geq 2)$

(A) $f^n(x)$ **(B)** 1 **(C)** $f^{n-1}(x)$ **(D)** $nf(x)$

Part # II [Assertion & Reason Type Questions]

These questions contains, Statement I (assertion) and Statement II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.

(C) Statement-I is true, Statement-II is false.

(D) Statement-I is false, Statement-II is true.

1. **Statement - I :** If A is skew symmetric matrix of order 3 then its determinant should be zero

Statement - II : If A is square matrix, then det A = det A' = det(–A')

2. **Statement-I :** If A is a non-singular symmetric matrix, then its inverse is also symmetric.

Statement-II : $(A^{-1})^T = (A^T)^{-1}$, where A is a non-singular symmetric matrix.

3. A and B be 3×3 matrices such that $AB + A + B = 0$

Statement-I : $AB = BA$

Statement-II : $PP^{-1} = I = P^{-1} P$ for every matrix P which is invertible.

4. Let A be any 3×2 matrix.

Statement-I : Inverse of AA^T does not exist.

Statement-II : AA^T is a singular matrix.

5. Let $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & -\cos\theta \end{bmatrix}$

Statement-I: A^{-1} exists for every $\theta \in R$.

Statement-II: A is orthogonal.

6. **Statement - I :** There are only finitely many 2×2 matrices which commute with the matrix $\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$.

Statement - II : If A is non-singular, then it commutes with I, adj A and A^{-1}.

Exercise # 3 Part # I [Matrix Match Type Questions]

Following question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled as A, B, C and D while the statements in Column-II are labelled as p, q, r and s. Any given statement in Column-I can have correct matching with one or more statement(s) in Column-II.

1.

Column-I (Matrix)		Column-II (Type of matrix)	
(A)	$\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$	**(p)**	Idempotent
(B)	$\begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix}$	**(q)**	Involutary
(C)	$\frac{1}{3}\begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ -2 & -2 & -1 \end{bmatrix}$	**(r)**	Nilpotent
(D)	$\begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$	**(s)**	Orthogonal

2.

Column-I		Column-II	
(A)	If A is a square matrix of order 3 and det A = 162 then $\det\left(\frac{A}{3}\right) =$	**(p)**	6
(B)	If A is a matrix such that $A^2 = A$ and $(I + A)^5 = I + \lambda A$ then $\frac{2\lambda+1}{7}$	**(q)**	5
(C)	If $A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$ and $A^2 - xA + yI = 0$ then $y - x =$	**(r)**	0
(D)	If $A = \begin{bmatrix} 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \\ 17 & 18 & 19 & 20 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 & 5 & 7 \\ -3 & -3 & -10 & -10 \\ 5 & 10 & 5 & 0 \\ 7 & 10 & 0 & 7 \end{bmatrix}$ then $(AB)_{23}$	**(s)**	9

Part # II [Comprehension Type Questions]

Comprehension # 1

Consider some special type of matrices.

A square matrix A is said to be an Idempotent Matrix if $A^2 = A$.

A matrix A is said to be a Nilpotent Matrix if $A^k = 0$, for $k \in N$.

A square matrix is said to be an Involutary Matrix, if $A^2 = I$.

Consider the following matrices

$$A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}; \quad B = \begin{bmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{bmatrix}; \quad C = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$$

1. Which one of the following is a Nilpotent Matrix?

(A) A **(B)** B **(C)** C **(D)** AC^2

2. Which one of the following is not an Idempotent Matrix?

(A) A^3C^2 **(B)** A^2C^2 **(C)** BC^2 **(D)** C^2A

3. Which one of the following matrices posses an inverse?

(A) BC^2 **(B)** A^3C^2 **(C)** A^2B **(D)** C^3

Comprehension # 2

If A is a symmetric and B skew symmetric matrix and A + B is non singular and $C = (A+B)^{-1}(A-B)$ then

1. $C^T(A+B)C =$

(A) $A+B$ **(B)** $A-B$ **(C)** A **(D)** B

2. $C^T(A-B)C =$

(A) $A+B$ **(B*)** $A-B$ **(C)** A **(D)** B

3. C^TAC

(A) $A+B$ **(B)** $A-B$ **(C*)** A **(D)** B

Comprehension # 3

Matrix A is called orthogonal matrix if $AA^T = I = A^TA$. Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$ be an orthogonal matrix. Let

$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$. Then $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$ & $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

i.e. $\vec{a}, \vec{b}$ & $\vec{c}$ forms mutually perpendicular triad of unit vectors.

If $abc = p$ and $Q = \begin{bmatrix} a & b & c \\ c & a & b \\ b & c & a \end{bmatrix}$, where Q is an orthogonal matrix. Then.

On the basis of above information, answer the following questions :

1. The values of $a + b + c$ is -

(A) 2 **(B)** p **(C)** 2p **(D)** ± 1

2. The values of $ab + bc + ca$ is -

(A) 0 **(B)** p **(C)** 2p **(D)** 3p

3. The value of $a^3 + b^3 + c^3$ is -

(A) p **(B)** 2p **(C)** 3p **(D)** None of these

4. The equation whose roots are a, b, c is -

(A) $x^3 - 2x^2 + p = 0$ **(B)** $x^3 - px^2 + px + p = 0$

(C) $x^3 - 2x^2 + 2px + p = 0$ **(D)** $x^3 \pm x^2 - p = 0$

Exercise # 4

[Subjective Type Questions]

1. If $A = \begin{bmatrix} 1 & 2 \\ 3 & -4 \\ 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 5 & 6 \\ 7 & -8 & 2 \end{bmatrix}$, will AB be equal to BA. Also find AB & BA.

2. If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ show that $(I + A) = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

3. Let X be the solution set of the equation $A^x = I$, where $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$ and I is the corresponding unit matrix and $x \in N$ then find the minimum value of $\sum(\cos^x\theta + \sin^x\theta), \theta \in R$.

4. If $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$, show that $A^3 = (5A - I)(A - I)$

5. If and B are square matrices of order n, then prove that A and B will commute iff $A - \lambda I$ and $B - \lambda I$ commute for every scalar λ.

6. If $AB = A$ and $BA = B$, then show that $A^2 = A$, $B^2 = B$.

7. Find $\left\{\frac{1}{2}(A - A' + I)\right\}^{-1}$ for $A = \begin{bmatrix} -2 & 3 & 4 \\ 5 & -4 & -3 \\ 7 & 2 & 9 \end{bmatrix}$ using elementary transformation.

8. If $a^2 + b^2 + c^2 = 1$, then prove that $\begin{vmatrix} a^2 + (b^2 + c^2)\cos\phi & ab(1-\cos\phi) & ac(1-\cos\phi) \\ ba(1-\cos\phi) & b^2 + (c^2 + a^2)\cos\phi & bc(1-\cos\phi) \\ ca(1-\cos\phi) & cb(1-\cos\phi) & c^2 + (a^2 + b^2)\cos\phi \end{vmatrix}$ is independent of a, b, c

9. Investigate for what values of λ, μ the simultaneous equations
$x + y + z = 6;\ x + 2y + 3z = 10\ \&\ x + 2y + \lambda z = \mu$ have;

(A) A unique solution **(B)** An infinite number of solutions.

(C) No solution.

10. An amount of Rs 5000 is put into three investments at the rate of interest of 6%, 7%, 8% per annum respectively. The total annual income is Rs 358. If the combined income from the first two investments is Rs 70 more than the income from the third, find the amount of each investment by matrix method.

11. Consider the system of linear equations in x, y, z:

$$(\sin 3\theta)\,x - y + z = 0$$
$$(\cos 2\theta)\,x + 4y + 3z = 0$$
$$2x + 7y + 7z = 0$$

Find the values of θ for which this system has non – trivial solution.

12. If $f(x) = x^2 - 5x + 7$, find f(a) where $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

13. If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, then show that matrix A is a root of polynomial $x^3 - 6x^2 + 7x + 2I = 0$.

14. Given $A = \begin{bmatrix} 2 & 0 & -\alpha \\ 5 & \alpha & 0 \\ 0 & \alpha & 3 \end{bmatrix}$ For what values of α does A^{-1} exists. Find A^{-1} & prove that

$A^{-1} = A^2 - 6A + 11I$, when $\alpha = 1$.

Exercise # 5 Part # I [Previous Year Questions] [AIEEE/JEE-MAIN]

1. If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$ then **[AIEEE 2003]**

(1) $\alpha = a^2 + b^2, \beta = a^2 - b^2$ **(2)** $\alpha = a^2 + b^2, \beta = ab$

(3) $\alpha = a^2 + b^2, \beta = 2ab$ **(4)** $\alpha = 2ab, \beta = a^2 + b^2$

2. If $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$ then- **[AIEEE 2004]**

(1) A^{-1} does not exist **(2)** $A^2 = I$ **(3)** $A = 0$ **(4)** $A = (-1)\,I$

3. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ and $10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$ where $B = A^{-1}$, then α is equal to- **[AIEEE 2004]**

(1) 2 **(2)** –1 **(3)** –2 **(4)** 5

4. If $A^2 - A + I = 0$, then the inverse of A **[AIEEE 2005]**

(1) $I - A$ **(2)** $A - I$ **(3)** A **(4)** $A + I$

5. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which one of the following holds for all $n \geq 1$, (by the principal of mathematical induction) **[AIEEE-2005]**

(1) $A^n = nA - (n-1)I$ **(2)** $A^n = 2^{n-1}A + (n-1)I$

(3) $A^n = nA + (n-1)I$ **(4)** $A^n = 2^{n-1}A - (n-1)I$

6. If A and B are square matrices of size $n \times n$ such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following will be always true- **[AIEEE- 2006]**

(1) $AB = BA$ **(2)** Either of A or B is a zero matrix

(3) Either of A or B is an identity matrix **(4)** $A = B$

7. Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, $a, b \in N$. Then- **[AIEEE- 2006]**

(1) there exist more than one but finite number of B's such that AB = BA

(2) there exist exactly one B such that AB = BA

(3) there exist infinitely many B's such that AB=BA

(4) there cannot exist any B such that AB = BA

8. Let $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$ If $|A^2| = 25$, then $|\alpha|$ equals- **[AIEEE- 2006]**

(1) 5^2 **(2)** 1 **(3)** 1/5 **(4)** 5

9. Let A be a 2×2 matrix with real entries. Let I be the 2×2 identity matrix. Denoted by tr(A), the sum of diagonal entries of A. Assume that $A^2 = I$.

Statement –1: If $A \neq I$ and $A \neq -I$, then det A = –1 **[AIEEE- 2008]**

Statement –2 : If $A \neq I$ and $A \neq -I$, then tr(A) $\neq 0$.

(1) Statement –1 is false, Statement –2 is true.

(2) Statement–1 is true, Statement–2 is true; Statement–2 is a correct explanation for Statement–1.

(3) Statement–1 is true, Statement–2 is true; Statement–2 is not a correct explanation for Statement–1.

(4) Statement–1 is true, Statement–2 is false

10. Let A be a square matrix all of whose entries are integers. Then which one of the following is true ? **[AIEEE- 2008]**

(1) If det A = ± 1, then A^{-1} exists but all its entries are not necessarily integers

(2) If det A $\neq$ ±1, then A^{-1} exists and all its entries are non–integers

(3) If det A = ±1, then A^{-1} exists and all its entries are integers

(4) If det A = ±1, then A^{-1} need not exist

11. Let A be a 2×2 matrix **[AIEEE- 2009]**

Statement–1 : adj (adj A) = A

Statement–2 : |adj A| = |A|

(1) Statement–1 is true, Statement–2 is false.

(2) Statement–1 is false, Statement–2 is true.

(3) Statement–1 is true, Statement–2 is true; Statement–2 is a correct explanation for Statement–1.

(4) Statement–1 is true, Statement–2 is true; Statement–2 is not a correct explanation for statement–1.

12. The number of 3×3 non-singular matrices, with four entries as 1 and all other entries as 0, is :- **[AIEEE-2010]**

(1) Less than 4 **(2)** 5 **(3)** 6 **(4)** At least 7

13. Let A be a 2×2 matrix with non-zero entries and let $A^2 = I$, where I is 2×2 identity matrix. Define Tr(A) = sum of diagonal elements of A and |A| = determinant of matrix A. **[AIEEE-2010]**

Statement–1 : Tr(A) = 0.

Statement–2 : |A| = 1.

(1) Statement–1 is true, Statement–2 is true; Statement–2 is a correct explanation for Statement–1.

(2) Statement–1 is true, Statement–2 is true; Statement–2 is not a correct explanation for statement–1.

(3) Statement–1 is true, Statement–2 is false.

(4) Statement–1 is false, Statement–2 is true.

14. Let A and B be two symmetric matrices of order 3.

Statement-1 : A(BA) and (AB)A are symmetric matrices.

Statement-2 : AB is symmetric matrix if matrix multiplication of A with B is commutative. **[AIEEE-2011]**

(1) Statement-1 is true, Statement-2 is false.

(2) Statement-1 is false, Statement-2 is true

(3) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(4) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

15. **Statement-1 :** Determinant of a skew-symmetric matrix of order 3 is zero.

Statement-2 : For any matrix A, $\det(A^T) = \det(A)$ and $\det(-A) = -\det(A)$.

Where det(B) denotes the determinant of matrix B. Then : **[AIEEE-2011]**

(1) Statement-1 is true and statement-2 is false **(2)** Both statements are true

(3) Both statements are false **(4)** Statement-1 is false and statement-2 is true.

16. If $\omega \neq 1$ is the complex cube root of unity and matrix $H = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$, then H^{70} is equal to: **[AIEEE-2011]**

(1) H **(2)** 0 **(3)** –H **(4)** H^2

17. Let P and Q be 3 × 3 matrices with $P \neq Q$. If $P^3 = Q^3$ and $P^2Q = Q^2P$, then determinant of $(P^2 + Q^2)$ is equal to : **[AIEEE-2012]**

(1) –1 **(2)** –2 **(3)** 1 **(4)** 0

18. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$. If u_1 and u_2 are column matrices such that $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, then $u_1 + u_2$ is equal to : **[AIEEE-2012]**

(1) $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$ **(2)** $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ **(3)** $\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$ **(4)** $\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$

19. If A is an 3 × 3 non - singular matrix such that AA' = A'A and $B = A^{-1}A'$, then BB' equals: **[Main 2014]**

(1) I + B **(2)** I **(3)** B^{-1} **(4)** (B^{-1})

20. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where I is 3 × 3 identify matrix, then the ordered pair (a, b) is equal to **[Main 2015]**

(1) (2, 1) **(2)** (–2, –1) **(3)** (2, – 1) **(4)** (–2, 1)

21. The system of linear equations **[Main 2016]**

$x + \lambda y - z = 0$

$\lambda x - y - z = 0$

$x + y - \lambda z = 0$

has a non-trivial solution for :

(1) exactly one value of λ **(2)** exactly two values of λ

(3) exactly three values of λ **(4)** infinitely many values of λ

22. If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and $A \text{ adj } A = AA^T$, then 5a + b is equal to : **[Main 2016]**

(1) 5 **(2)** 4 **(3)** 13 **(4)** – 1

Part # II [Previous Year Questions][IIT-JEE ADVANCED]

1. If matrix $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$ where a,b,c are real positive numbers, abc = 1 and $A^T A = I$, then find the value of $a^3 + b^3 + c^3$. **[JEE 2003]**

2. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 125$, then α is equal to - **[JEE 2004 (Screening)]**

(A) ±3 **(B)** ±2 **(C)** ±5 **(D)** 0

3. If M is a 3 × 3 matrix, where $M^TM = I$ and det (M) = 1, then prove that det (M–I) = 0. **[JEE 2004 (Mains)]**

4. $A = \begin{bmatrix} a & 1 & 0 \\ 1 & b & d \\ 1 & b & c \end{bmatrix}, B = \begin{bmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{bmatrix}, U = \begin{bmatrix} f \\ g \\ h \end{bmatrix}, V = \begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

If AX = U has infinitely many solutions, then prove that BX = V cannot have a unique solution. If further afd $\neq 0$, then prove that BX = V has no solution **[JEE 2004 (Mains)]**

5. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $A^{-1} = \frac{1}{6}(A^2 + cA + dI)$, then the value of c and d are -

[JEE 2005 (Screening)]

(A) –6, –11 **(B)** 6, 11 **(C)** –6, 11 **(D)** 6, –11

6. If $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$ and $x = P^T Q^{2005} P$, then x is equal to - **[JEE 2005 (Screening)]**

(A) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$ **(B)** $\begin{bmatrix} 4 + 2005\sqrt{3} & 6015 \\ 2005 & 4 - 2005\sqrt{3} \end{bmatrix}$

(C) $\frac{1}{4}\begin{bmatrix} 2 + \sqrt{3} & 1 \\ -1 & 2 - \sqrt{3} \end{bmatrix}$ **(D)** $\frac{1}{4}\begin{bmatrix} 2005 & 2 - \sqrt{3} \\ 2 + \sqrt{3} & 2005 \end{bmatrix}$

Comprehension (3 questions)

7. $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$, if U_1, U_2 and U_3 are columns matrices satisfying. $AU_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $AU_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$, $AU_3 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ and U is 3×3 matrix whose columns are U_1, U_2, U_3 then answer the following questions -

(A) The value of |U| is -

(A) 3 **(B)** –3 **(C)** 3/2 **(D)** 2

(B) The sum of the elements of U^{-1} is -

(A) –1 **(B)** 0 **(C)** 1 **(D)** 3

(C) The value of $\begin{bmatrix} 3 & 2 & 0 \end{bmatrix} U \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$ is -

(A) [5] **(B)** [5/2] **(C)** [4] **(D)** [3/2] **[JEE 2006]**

8. Match the Statement / Expressions in Column I with the Statements / Expressions in Column II and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the ORS.

	Column I		Column II
(A)	The minimum value of $\frac{x^2+2x+4}{x+2}$ is	**(p)**	0
(B)	Let A and B be 3 × 3 matrices of real numbers, where A is symmetric, B is skew-symmetric, and (A+B)(A–B) = (A – B) (A + B). If $(AB)^t = (-1)^k AB$, where $(AB)^t$ is the transpose of the matrix AB, then the possible values of k are	**(q)**	1
(C)	Let $a = \log_3 \log_3 2$. An integer k satisfying $1 < 2^{(-k+3^{-a})} < 2$, must be less than	**(r)**	2
(D)	If $\sin\theta = \cos\phi$, then the possible values of $\frac{1}{\pi}\left(\theta - \phi - \frac{\pi}{2}\right)$ are	**(s)**	3

[JEE 2008]

9. Let A be the set of all 3 × 3 symmetric matrices all of whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0.

(A) The number of matrices in A is -

(A) 12 **(B)** 6 **(C)** 9 **(D)** 3

(B) The number of matrices A in $\mathcal{A}$ for which the system of linear equations $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has a unique solution, is -

(A) less than 4 **(B)** at least 4 but less than 7

(C) at least 7 but less than 10 **(D)** at least 10

(C) The number of matrices A in $\mathcal{A}$ for which the system of linear equations $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is inconsistent, is -

(A) 0 **(B)** more than 2 **(C)** 2 **(D)** 1 **[JEE 2009]**

10. (A) The number of 3×3 matrices A whose entries are either 0 or 1 and for which the system $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has exactly two distinct solutions, is

(A) 0 **(B)** $2^9 - 1$ **(C)** 168 **(D)** 2

(B) Let k be a positive real number and let

$$A = \begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}.$$

If det (adj A) + det(adj B) = 10^6, then [k] is equal to

[**Note :** adj M denotes the adjoint of a square matrix M and [k] denotes the largest integer less than or equal to k].

(C) Let p be an odd prime number and T_p be the following set of 2×2 matrices :

$$T_p = \left\{ A = \begin{bmatrix} a & b \\ c & a \end{bmatrix} : a, b, c \in \{0, 1, 2, \ldots\ldots, p-1\} \right\}$$

(i) The number of A in T_p such that A is either symmetric or skew-symmetric or both, and det(A) divisible by p is

(A) $(p-1)^2$ **(B)** $2(p-1)$ **(C)** $(p-1)^2 + 1$ **(D)** $2p - 1$

(ii) The number of A in T_p such that the trace of A is not divisible by p but det **(A)** is divisible by p is -

[**Note :** The trace of a matrix is the sum of its diagonal entries.]

(A) $(p-1)(p^2-p+1)$ **(B)** $p^3 - (p-1)^2$ **(C)** $(p-1)^2$ **(D)** $(p-1)(p^2-2)$

(iii) The number of A in T_p such that det (A) is not divisible by p is -

(A) $2p^2$ **(B)** $p^3 - 5p$ **(C)** $p^3 - 3p$ **(D)** $p^3 - p^2$ **[JEE 2010]**

11. Let M and N be two 3×3 non-singular skew-symmetric matrices such that MN = NM. If P^T denotes the transpose of P, then $M^2N^2(M^TN)^{-1}(MN^{-1})^T$ is equal to - **[JEE 2011]**

(A) M^2 **(B)** $-N^2$ **(C)** $-M^2$ **(D)** MN

12. Let $\omega \neq 1$ be a cube root of unity and S be the set of all non-singular matrices of the form $\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$, where each of a,b and c is either ω or ω^2. Then the number of distinct matrices in the set S is-

(A) 2 **(B)** 6 **(C)** 4 **(D)** 8 **[JEE 2011]**

13. Let M be 3×3 matrix satisfying

$$M\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix},\ M\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \text{ and } M\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}$$

Then the sum of the diagonal entries of M is **[JEE 2011]**

14. Let $P = [a_{ij}]$ be a 3×3 matrix and let $Q = [b_{ij}]$, where $b_{ij} = 2^{i+j}a_{ij}$ for $1 \leq i, j \leq 3$. If the determinant of P is 2, then the determinant of the matrix Q is - **[JEE 2012]**

(A) 2^{10} **(B)** 2^{11} **(C)** 2^{12} **(D)** 2^{13}

15. If P is a 3×3 matrix such that $P^T = 2P + I$, where P^T is the transpose of P and I is the 3×3 identity matrix, then there exists a column matrix $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ such that **[JEE 2012]**

(A) $PX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ **(B)** $PX = X$ **(C)** $PX = 2X$ **(D)** $PX = -X$

16. If the adjoint of a 3×3 matrix P is $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 2 & 1 & 3 \end{bmatrix}$, then the possible value(s) of the determinant of P is (are) - **[JEE 2012]**

(A) -2 **(B)** -1 **(C)** 1 **(D)** 2

17. Let M be a 2×2 symmetric matrix with integer entries. Then M is invertible if **[JEE Ad. 2014]**

(A) the first column of M is the transpose of the second row of M

(B) the second row of M is the transpose of the first column of M

(C) M is a diagonal matrix with nonzero entries in the main diagonal

(D) the product of entries in the main diagonal of M is not the square of an integer

18. Let X and Y be two arbitrary, 3×3 non-zero, skew-symmetric matrics and z be an arbitrary 3×3, non-zero, symmetric matrix. Then which of the following matrices is (are) skew symmetric ? **[JEE Ad. 2015]**

(A) $Y^3Z^4 - Z^4Y^3$ **(B)** $X^{44} + Y^{44}$

(C) $X^4Z^3 - Z^3X^4$ **(D)** $X^{23} + Y^{23}$

19. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & a \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in R$. Suppose $Q = [q_{ij}]$ is a matric such that $PQ = kI$, where $k \in R$, $k \neq 0$ and I is the identify matrix of order 3. If $q_{23} = -\frac{k}{8}$ and $\det(Q) = \frac{k^2}{2}$, then **[JEE Ad. 2015]**

(A) $a = 0, k = 8$ **(B)** $4a - k + 8 = 0$

(C) $\det(P\,\mathrm{adj}(Q)) = 2^9$ **(D)** $\det(Q\,\mathrm{adj}(P)) = 2^{13}$

20. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix}$ and I be the identity matrix of order 3. If $Q = [q_{ij}]$ is a matrix such that $P^{50} - Q = I$, then $\frac{q_{31} + q_{32}}{q_{21}}$ equal **[JEE Ad. 2016]**

(A) 52 **(B)** 103 **(C)** 201 **(D)** 205

MOCK TEST

SECTION - I : STRAIGHT OBJECTIVE TYPE

1. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ are two matrices such that AB = BA and $c \neq 0$, then value of $\frac{a-d}{3b-c}$ is :

(A) 0 **(B)** 2 **(C)** –2 **(D)** –1

2. If $A = \begin{bmatrix} 3 & 1 & -1 \\ 0 & 1 & 2 \end{bmatrix}$, then AA' is

(A) symmetric matrix
(B) skew - symmetric matrix
(C) orthogonal matrix
(D) none of these

3. Let A and B are two non-singular square matrices, A^T and B^T are the transpose matrices of A and B respectively, then which of the following is correct

(A) $B^T AB$ is symmetric matrix if and only if A is symmetric
(B) $B^T AB$ is symmetric matrix if and only if B is symmetric
(C) $B^T AB$ is skew symmetric matrix for every matrix A
(D) $B^T AB$ is skew symmetric matrix if B is skew symmetric

4. If A and B are two square matrices of order 3×3 which satisfy AB = A and BA = B then $(A + B)^7$ is

(A) $7(A + B)$ **(B)** $7.I_{3 \times 3}$ **(C)** $64(A + B)$ **(D)** $128\, I_{3 \times 3}$

5. If $A^3 = O$, then $I + A + A^2$ equals

(A) $I - A$ **(B)** $(I - A)^{-1}$ **(C)** $(I + A)^{-1}$ **(D)** none of these

6. Let $A = \begin{bmatrix} x+\lambda & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda \end{bmatrix}$, then A^{-1} exists if

(A) $x \neq 0$ **(B)** $\lambda \neq 0$ **(C)** $3x + \lambda \neq 0, \lambda \neq 0$ **(D)** $x \neq 0, \lambda \neq 0$

7. If $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$ then $\lim\limits_{n \to \infty} \frac{1}{n} A^n$ is

(A) $\begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}$ **(B)** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **(C)** $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ **(D)** Does not exist

8. If $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then A =

(A) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ **(B)** $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ **(C)** $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ **(D)** $-\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

9. If A and B are two matrices, then

(A) $AB = BA$ **(B)** $AB = O$

(C) $AB = I$ **(D)** AB cannot necessarily be defined

10. **S_1 :** Square matrix A is non-singular and symmetric then $((A^{-1})^{-1})^{-1}$ is skew symmetric

S_2 : Adjoint of a symmetric matrix is a symmetric matrix

S_3 : Adjoint of a diagonal matrix is diagonal matrix

S_4 : Product of two invertible square matrices of same order is also invertible.

(A) FTFT **(B)** FTTF **(C)** FTTT **(D)** TFFT

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

11. If A is a square matrix, then

(A) AA' is symmetric **(B)** AA' is skew - symmetric

(C) $A'A$ is symmetric **(D)** $A'A$ is skew - symmetric

12. Let $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then

(A) $A^2 - 4A - 5I_3 = 0$ **(B)** $A^{-1} = \frac{1}{5}(A - 4I_3)$ **(C)** A^3 is not invertible **(D)** A^2 is invertible

13. Matrix $\begin{bmatrix} a & b & (a\alpha - b) \\ b & c & (b\alpha - c) \\ 2 & 1 & 0 \end{bmatrix}$ is non invertible if

(A) $\alpha = 1/2$ **(B)** a, b, c are in A.P. **(C)** a, b, c are in G.P. **(D)** a, b, c are in H.P.

14. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then

(A) $\text{adj}(\text{adj}A) = A$ **(B)** $|\text{adj}(\text{adj}A)| = 1$ **(C)** $|\text{adj}A| = 1$ **(D)** None of these

15. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, then

(A) $A^3 = 9A$ **(B)** $A^3 = 27A$ **(C)** $A + A = A^2$ **(D)** A^{-1} does not exist

SECTION - III : ASSERTION AND REASON TYPE

16. **Statement -I** : The inverse of the matrix $A = [a_{ij}]_{n \times n}$ where $a_{ij} = 0$, $i \geq j$ is $B = [a_{ij}^{-1}]_{n \times n}$

Statement -II : The inverse of singular matrix does not exist.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

17. **Statement-I :** If $A = \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$, then adj (adj A) = A

Statement-II : $|\text{adj}(\text{adj } A)| = |A|^{(n-1)^2}$, A be n rowed non singular matrix.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

18. **Statement-I** : If $f_1(x)$, $f_2(x)$ ...., $f_9(x)$ are polynomials whose degree ≥ 1, where

$f_1(\alpha) = f_2(\alpha) = f_2(\alpha)$ .... $= f_9(\alpha) = 0$ and $A(x) = \begin{bmatrix} f_1(x) & f_2(x) & f_3(x) \\ f_4(x) & f_5(x) & f_6(x) \\ f_7(x) & f_8(x) & f_9(x) \end{bmatrix}$ and $\dfrac{A(x)}{x-\alpha}$ is also a matrix of 3×3 whose entries are also polynomials

Statement -II : $x - \alpha$ is a factor of polynomial $f(x)$ if $f(\alpha) = 0$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

19. **Statement-I :** The rank of a unit matrix of order $n \times n$ is n.

Statement-II : The rank of a non singular matrix of order $n \times n$ is not n.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

20. **Statement-I :** If A is a skew symmetric matric of order 3×3, then $\det(A) = 0$ or $|A| = 0$.

Statement-II : If A is square matrix, then $\det(A) = \det(A') = \det(-A')$.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

SECTION - IV : MATRIX - MATCH TYPE

21. **Match the following**

Column - I		**Column - II**	
(A)	A is a real skew symmetric matrix such that $A^2 + I = 0$. Then	**(p)**	$BA - AB$
(B)	A is a matrix such that $A^2 = A$. If $(I + A)^n = I + \lambda A$, then λ equals $(n \in N)$	**(q)**	A is of even order
(C)	If for a matrix A, $A^2 = A$, and $B = I - A$, then $AB + BA + I - (I - A)^2$ equals	**(r)**	A
(D)	A is a matrix with complex entries and A* stands for transpose of complex conjugate of A. If A* = A & B* = B, then (AB – BA)* equals	**(s)**	$2^n - 1$
		(t)	${}^nC_1 + {}^nC_2 .... + {}^nC_n$

22. **Match the following**

Column - I		**Column - II**	
(A)	If A, B and C be 2 × 2 matrices with entires from the set of real numbers. Define * as follows : $A * B = \frac{1}{2}(AB + BA)$, then	**(p)**	$A * B = B * A$
(B)	If A, B and C be 2 × 2 matrices with entires from the set of real numbers. Define * as follows : $A * B = \frac{1}{2}(AB' + A'B)$, then	**(q)**	$A*(B+C)=A*B+A*C$
(C)	If A, B and C be 2 × 2 matrices with entires from the set of real number. Define * as follows : $A * B = \frac{1}{2}(AB - BA)$, then	**(r)**	$A * A = A^2$
		(s)	$A * I = A$
		(t)	$A * I = O$

SECTION - V : COMPREHENSION TYPE

23. **Read the following comprehension carefully and answer the questions.**

Let A and B are two matrices of same order 3 × 3, where $A = \begin{pmatrix} 1 & 3 & \lambda+2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 2 & 4 \\ 3 & 2 & 5 \\ 2 & 1 & 4 \end{pmatrix}$

1. If A is singular matrix, then tr(A + B) is equal to

(A) 6 **(B)** 12 **(C)** 24 **(D)** 17

2. If matrix 2A + 2B is singular, then the value of 2λ is

(A) 11 **(B)** 13 **(C)** 15 **(D)** 17

3. If $\lambda = 3$, then $\frac{1}{7}(tr(AB) + tr(BA))$ is equal to

(A) 34 **(B)** 42 **(C)** 84 **(D)** 63

24. **Read the following comprehension carefully and answer the questions.**

A and B are two matrices of same order 3 × 3, where $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 5 & 6 & 8 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 2 & 5 \\ 2 & 3 & 8 \\ 7 & 2 & 9 \end{pmatrix}$

1. The value of adj(adj A) is equal to

(A) 2A **(B)** 4A **(C)** 8A **(D)** 16A

2. The value of |adj (adj A)| is equal to

(A) 9 **(B)** 16 **(C)** 25 **(D)** 81

3. The value of |adj B| is equal to

(A) 24 **(B)** 24^2 **(C)** 24^3 **(D)** 8^2

25. **Read the following comprehension carefully and answer the questions.**

Let $A = [a_{ij}]_3$ be a square matrix of order 3 whose elements are distinct integers from 1, 2,......9 the matrix is formed so that the sum of numbers in every row, column & diagonal is a multiple of 9.

1. The number of possible combinations of three distinct numbers from 1 to 9 that have a sum of 9 or 18 is

(A) 10 **(B)** 7 **(C)** 8 **(D)** 9

2. The element a_{22} must be a multiple of

(A) 2 **(B)** 3 **(C)** 4 **(D)** 9

3. The maximum value of trace of the matrix A is :

(A) 18 **(B)** 19 **(C)** 12 **(D)** None

SECTION - VI : INTEGER TYPE

26. If $A = \begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$, $abc = 1$, $A'A = I$, then find maximum value of $a^3 + b^3 + c^3$

27. If $A = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$ and $\phi(x) = (1 + x)(1 - x)^{-1}$ and $\phi(A) = -\lambda A$, then find the value of λ.

28. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 1 & 0 \end{bmatrix}$ and B = (adj A) and C = 5A, then find the value of $\dfrac{|\text{adj}B|}{|C|}$.

29. Let $P = \begin{bmatrix} \cos\dfrac{\pi}{9} & \sin\dfrac{\pi}{9} \\ -\sin\dfrac{\pi}{9} & \cos\dfrac{\pi}{9} \end{bmatrix}$ and α, β, γ be non-zero real numbers such that

$\alpha p + \beta p^3 + \gamma I$

is the zero matrix. Then find value of $(\alpha^2 + \beta^2 + \gamma^2)^{(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)}$.

30. Let 'A' is (4×4) matrix such that sum of elements in each row is 1. Find out sum of all the elements in A^{10}.

ANSWER KEY

EXERCISE - 1

1. A 2. A 3. A 4. A 5. B 6. C 7. B 8. A 9. A 10. B 11. D 12. C 13. A
14. B 15. C 16. B 17. A 18. B 19. B 20. A 21. D 22. C 23. C 24. C 25. C 26. D
27. C 28. C 29. D 30. C

EXERCISE - 2 : PART # I

1. ABC 2. AB 3. BC 4. AB 5. ABC 6. BCD 7. ABD 8. BCD 9. BC
10. BC 11. BC 12. AD 13. AC 14. AC 15. ABCD 16. CD 17. ABCD 18. BCD
19. BD 20. ABC

PART - II

1. C 2. A 3. A 4. A 5. A 6. D

EXERCISE - 3 : PART # I

1. $A \to p$ $B \to q$ $C \to s$ $D \to r$ 2. $A \to p$ $B \to s$ $C \to q$ $D \to r$

PART - II

Comprehension # 1 : 1. B 2. C 3. D   **Comprehension # 2 :** 1. A 2. B 3. C
Comprehension # 3 : 1. D 2. A 3. D 4. D

EXERCISE - 5 : PART # I

1. 3 2. 2 3. 4 4. 1 5. 1 6. 1 7. 3 8. 3 9. 4 10. 3 11. 4 12. 4 13. 3
14. 4 15. 1 16. 1 17. 4 18. 1 19. 2 20. 2 21. 3 22. 1

PART - II

1. 4 2. A 5. C 6. A 7. **A.** A **B.** B **C.** A 8. **A** $\to$ R **B** $\to$ q,s **C** $\to$ r,s **D** $\to$ p,r
9. **A.** A **B.** B **C.** B 10. **A.** A **B.** 4 **C.** **(i)** D **(ii)** C **(iii)** D 11. Bonus 12. A 13. 9
14. D 15. D 16. AD 17. CD 18. CD 19. BC 20. B

MOCK TEST

1. D 2. A 3. A 4. C 5. B 6. C 7. A 8. A 9. D
10. C 11. AC 12. ABD 13. AC 14. ABC 15. AD 16. D 17. B 18. A
19. C 20. C 21. **A** $\to$ q **B** $\to$ s,t **C** $\to$ r **D** $\to$ p 22. **A** $\to$ p,q,r,s **B** $\to$ p, q **C** $\to$ q,t
23. 1. C 2. D 3. A 24. 1. A 2. B 3. B 25. 1. A 2. B 3. A
26. 4 27. 1 28. 1 29. 1 30. 4